AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Discrete

Wednesday 14 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
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TOTAL	

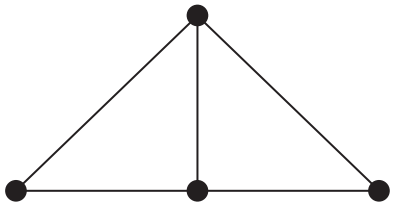
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The simple-connected graph G is shown below.



The graph G has n faces.

State the value of n

Circle your answer.

[1 mark]

2 (3) 4 5

2 Jonathan and Hoshi play a zero-sum game.

The game is represented by the following pay-off matrix for Jonathan.

		Hoshi		
	Strategy	**H_1**	**H_2**	**H_3**
Jonathan	**J_1**	-2	3	2
	J_2	3	2	0
	J_3	4	-1	3
	J_4	3	1	0

The game does not have a stable solution.

Which strategy should Jonathan **never** play?

Circle your answer.

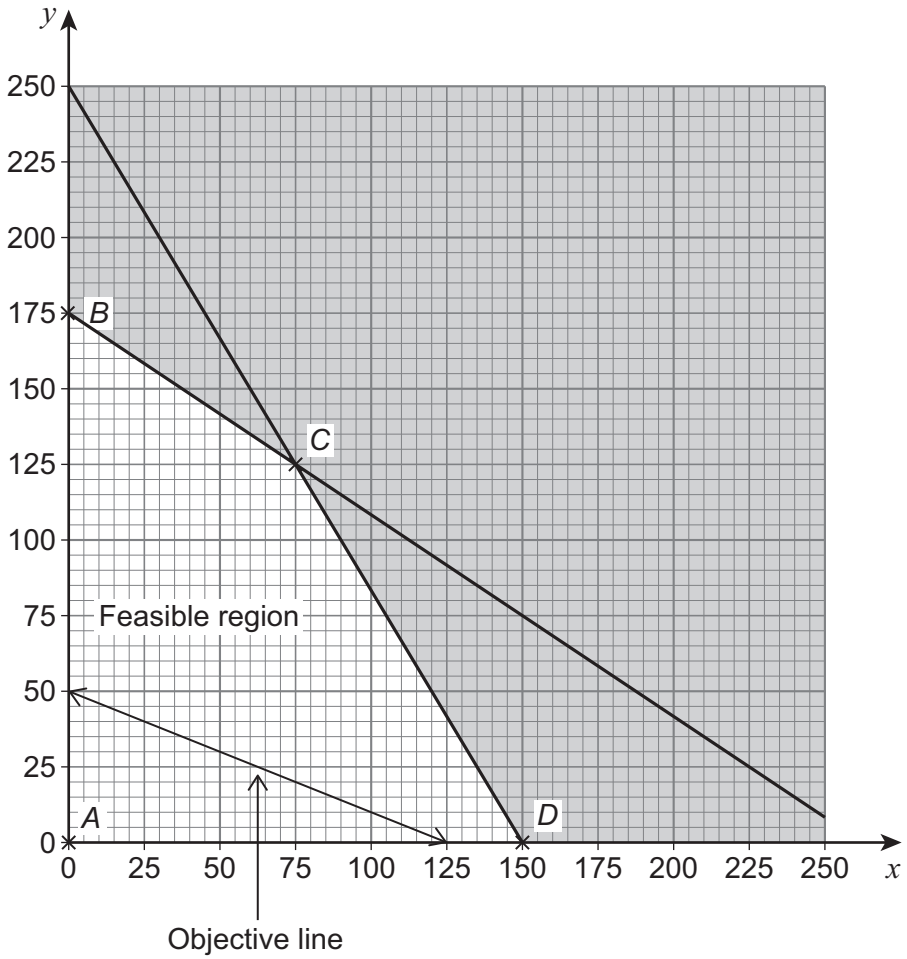
[1 mark]

J_1 **J_2** **J_3** (**J_4**)

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3 A student is solving a maximising linear programming problem.

The graph below shows the constraints, feasible region and objective line for the student's linear programming problem.



Which vertex is the optimal vertex?

Circle your answer.

[1 mark]

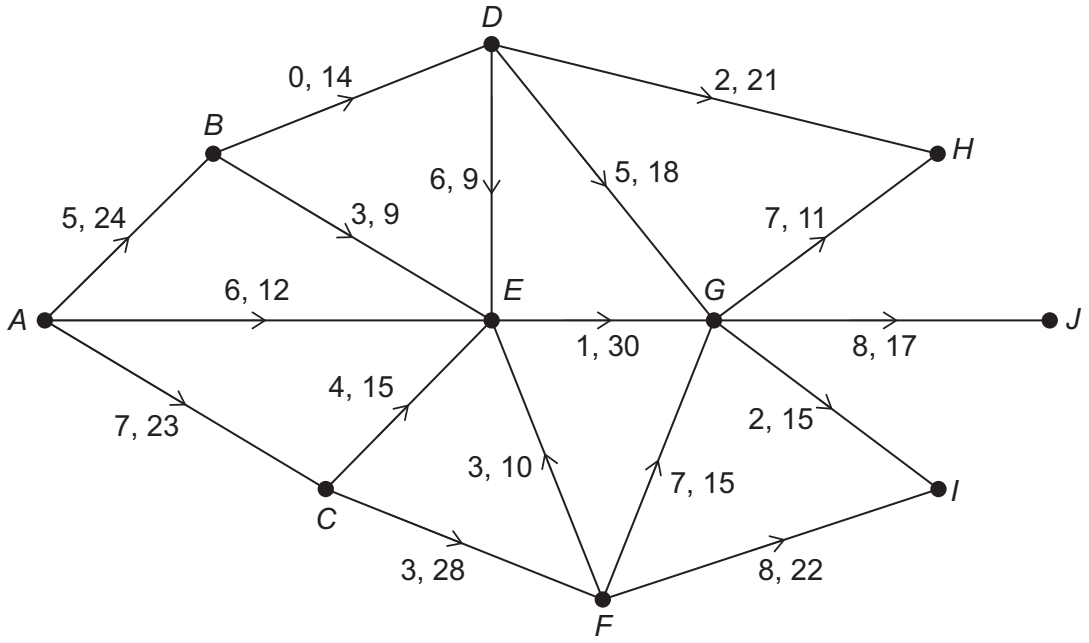
A (*B*) *C* *D*

Turn over for the next question

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4 The network below represents a system of water pipes in a geothermal power station.

The numbers on each arc represent the lower and upper capacity for each pipe in gallons per second.



The water is taken from a nearby river at node *A*
The water is then pumped through the system of pipes and passes through one of three treatment facilities at nodes *H*, *I* and *J* before returning to the river.

4 (a) The senior management at the power station want all of the water to undergo a final quality control check at a new facility before it returns to the river.

Using the language of networks, explain how the network above could be modified to include the new facility.

[2 marks]

Add a supersink (the new facility) and join H, I and J to it by arcs directed from H, I and J into the supersink, so all water leaving the sinks H, I and J passes through it (arcs of large enough capacity).

4 (b) Find the value of the cut {*A*, *B*, *C*, *D*, *E*} {*F*, *G*, *H*, *I*, *J*}

[1 mark]

Arcs crossing: DH + DG + EG + CF − FE(lower capacity)

= 21 + 18 + 30 + 28 − 3

= 94 gallons per second

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4 (c) Tim, a trainee engineer at the power station, correctly calculates the value of the cut $\{A, B, C, D, E, F\}$ $\{G, H, I, J\}$ to be 106 gallons per second.

Tim then claims that the maximum flow through the network of pipes is 106 gallons per second.

Comment on the validity of Tim's claim.

[2 marks]

From (b), the cut {A, B, C, D, E} {F, G, H, I, J} has value 94 and 94 < 106, so Tim's cut is not the minimum cut. By the maximum flow minimum cut theorem, the maximum flow equals the value of the minimum cut, so max flow ≤ 94 gallons/s. Hence Tim's claim is not valid: the maximum flow cannot be 106 gallons per second (it is at most 94 gallons per second).

Turn over for the next question

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5 A student is solving the following linear programming problem.

$$\begin{aligned} &\text{Minimise} && Q = -4x - 3y \\ &\text{subject to} && x + y \leq 520 \\ & && 2x - 3y \leq 570 \\ &\text{and} && x \geq 0,\ y \geq 0 \end{aligned}$$

5 (a) The student wants to use the simplex algorithm to solve the linear programming problem.

They modify the linear programming problem by introducing the objective function

$$P = 4x + 3y$$

and the slack variables r and s

State **one** further modification that must be made to the linear programming problem so that it can be solved using the simplex algorithm.

[1 mark]

The objective function P = 4x + 3y must be maximised

(minimising Q = −4x − 3y is the same as maximising P = −Q).

5 (b) (i) Complete the initial simplex tableau for the modified linear programming problem.

[2 marks]

P	x	y	r	s	**value**
1	−4	−3	0	0	0
0	1	1	1	0	520
0	2	−3	0	1	570

Pivot column x (most negative value −4 in the objective row)

θ ratios: 520 ÷ 1 = 520, 570 ÷ 2 = 285 (least) ⇒ pivot = 2

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5 (b) (ii) Hence, perform **one** iteration of the simplex algorithm.

[2 marks]

P	x	y	r	s	**value**
1	0	−9	0	2	1140
0	0	$\frac{5}{2}$	1	$-\frac{1}{2}$	235
0	1	$-\frac{3}{2}$	0	$\frac{1}{2}$	285

R3 ÷ 2 (new pivot row), R1 + 4 × new R3, R2 − new R3

5 (c) The student performs one further iteration of the simplex algorithm, which results in the following correct simplex tableau.

P	x	y	r	s	**value**
1	0	0	$\frac{18}{5}$	$\frac{1}{5}$	1986
0	0	1	$\frac{2}{5}$	$-\frac{1}{5}$	94
0	1	0	$\frac{3}{5}$	$\frac{1}{5}$	426

5 (c) (i) Explain how the student can tell that the optimal solution to the modified linear programming problem can be determined from the above simplex tableau.

[1 mark]

There are no negative values in the objective (P) row, so the tableau is optimal.

5 (c) (ii) Find the optimal solution of the **original** linear programming problem.

[2 marks]

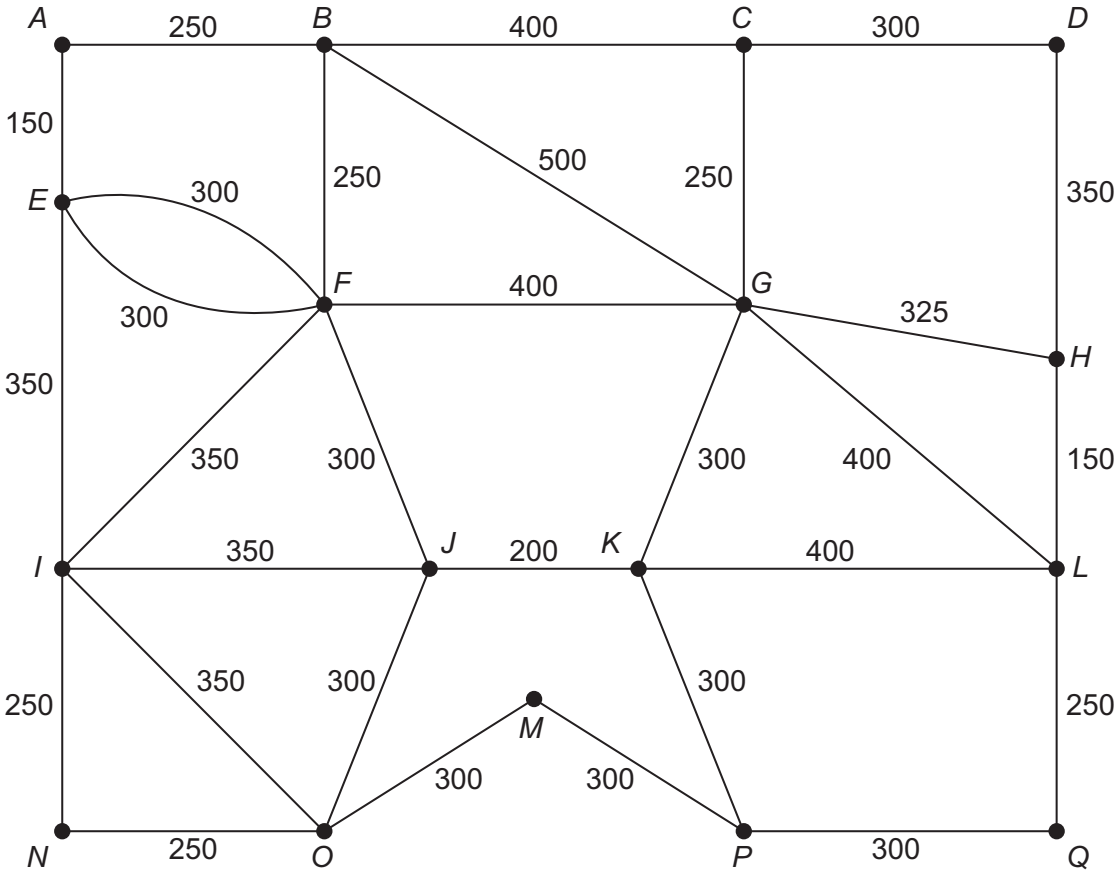
Max P = 1986 when x = 426, y = 94

Q = −P ⇒ minimum Q = −1986 when x = 426, y = 94

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6 A council wants to grit all of the roads on a housing estate.

The network shows the roads on a housing estate. Each node represents a junction between two or more roads and the weight of each arc represents the length, in metres, of the road.



The total length of all of the roads on the housing estate is 9175 metres.

In order to grit all of the roads, the council requires a gritter truck to travel along each road at least once. The gritter truck starts and finishes at the same junction.

6 (a) The gritter truck starts gritting the roads at 7:00 pm and moves with an average speed of 5 metres per second during its journey.

Find the earliest time for the gritter truck to have gritted each road at least once and arrived back at the junction it started from, giving your answer to the nearest minute.

Fully justify your answer.

[6 marks]

Route inspection problem: odd degree vertices are C, H, I, P

Shortest distances:

C–H: CGH = 250 + 325 = 575 I–P: IJKP = 350 + 200 + 300 = 850

C–I: CGFI = 250 + 400 + 350 = 1000 H–P: HLQP = 150 + 250 + 300 = 700

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C–P: CGKP = 250 + 300 + 300 = 850 H–I: HGFI = 325 + 400 + 350 = 1075

Pairings:

(C–H) + (I–P) = 575 + 850 = 1425 ← minimum

(C–I) + (H–P) = 1000 + 700 = 1700

(C–P) + (H–I) = 850 + 1075 = 1925

Repeat CGH and IJKP.

Minimum distance = 9175 + 1425 = 10 600 m

Time = 10 600 ÷ 5 = 2120 seconds

2120 ÷ 60 = 35.33... minutes ≈ 35 minutes

7:00 pm + 35 minutes

Earliest time = 7:35 pm (to the nearest minute)

6 (b) Explain how a refinement to the council's requirement, that the gritter truck must start and finish at the same junction, could reduce the time taken to grit all of the roads at least once.

[2 marks]

If the truck did not have to finish where it started, it could start at one odd vertex and finish at another (semi-Eulerian route), e.g. start at I and finish at P, so only the shortest path C–H (575 m) is repeated.

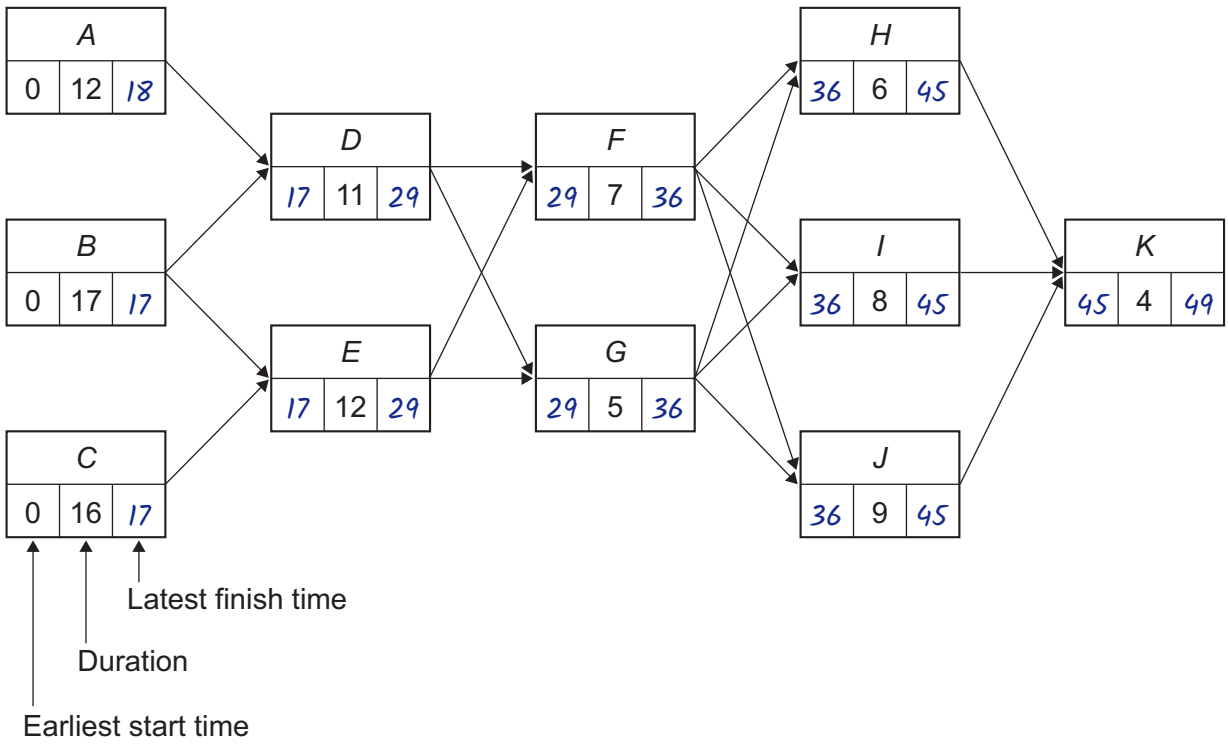
Distance = 9175 + 575 = 9750 m < 10 600 m, so the truck travels less distance and takes less time to grit all of the roads.

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7 Nova Merit Construction are planning a building project.

The planning involves producing an activity network for the project, which is shown in **Figure 1** below. The duration of each activity is given in weeks.

Figure 1



7 (a) (i) Find the earliest start time and the latest finish time for each activity and write these values on the activity network in **Figure 1**

[2 marks]

7 (a) (ii) Write down the critical path.

[1 mark]

B E F J K

Forward pass: K ES = 36 + 9 = 45, finish = 45 + 4 = 49 weeks

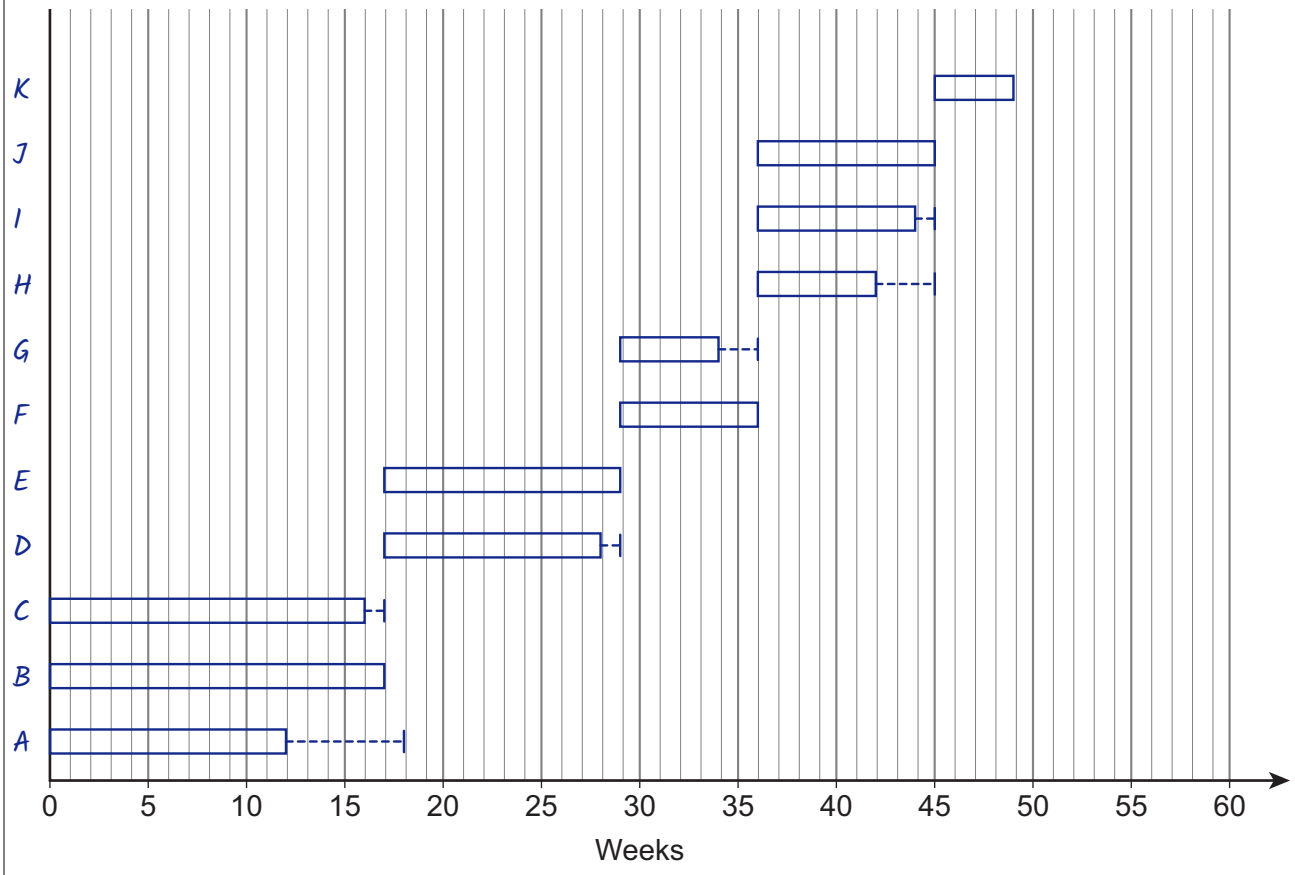
Backward pass: D LS = 29 − 11 = 18 ⇒ LFT of A = 18

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7 (b) On **Figure 2** below, draw a cascade diagram (Gantt chart) for the planned building project, assuming that each activity starts as early as possible.

[3 marks]

Figure 2



7 (c) During further planning of the building project, Nova Merit Construction find that activity *F* is not necessary and they remove it from the project.

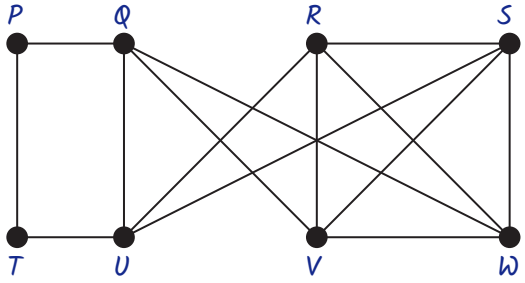
Explain the effect removing activity *F* has on the minimum completion time of the project.

[2 marks]

Removing F makes G critical. G has duration 5, which is 2 weeks less than F (7), so H, I and J can now start at 29 + 5 = 34 weeks. K then starts at 34 + 9 = 43, finishing at 43 + 4 = 47 weeks. So the minimum completion time reduces by 2 weeks, from 49 weeks to 47 weeks.

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8 The graph G is shown below.



8 (a) (i) State, with a reason, whether or not G is simple.

[2 marks]

G has no loops and no multiple edges (each pair of vertices is joined by at most one edge), therefore G is simple.

8 (a) (ii) A student states that G is Eulerian.

Explain why the student is correct.

[2 marks]

G is connected, and every vertex of G has even degree (degrees 2, 2, 4, 4, 4, 4, 4, 4).

A connected graph with all vertices of even degree is Eulerian, so G satisfies both conditions and the student is correct.

8 (b) The graph H has 8 vertices with degrees 2, 2, 4, 4, 4, 4, 4 and 4

Comment on whether H is isomorphic to G

[2 marks]

G also has 8 vertices with degrees 2, 2, 4, 4, 4, 4, 4 and 4, so G and H have the same degree sequence (and both have 14 edges).

So H may be isomorphic to G, but having the same degrees does not guarantee it, so we cannot be sure that H is isomorphic to G without more information.

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8 (c) The formula $v - e + f = 2$, where

$v =$ number of vertices

$e =$ number of edges

$f =$ number of faces

can be used with graphs which satisfy certain conditions.

Prove that *G* does not satisfy the conditions for the above formula to apply.

[3 marks]

The formula v − e + f = 2 can only be used with connected planar graphs.

Labelling the vertices of G as on the diagram:

Q is joined to U, V and W

R is joined to U, V and W

S is joined to U, V and W

So G contains a subgraph isomorphic to the complete bipartite graph

$K_{3,3}$ with vertex sets {Q, R, S} and {U, V, W}.

By Kuratowski's theorem, a graph containing a subgraph isomorphic to

$K_{3,3}$ is non-planar, so G is non-planar.

Hence G does not satisfy the conditions for v − e + f = 2 to apply.

Turn over for the next question

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9 The group $(C, +_4)$ contains the elements 0, 1, 2 and 3

9 (a) (i) Show that C is a cyclic group.

[2 marks]

C is cyclic if it has a generator. Repeated addition of 1 modulo 4:

$1 = 1$

$1 +_4 1 = 2$

$1 +_4 1 +_4 1 = 3$

$1 +_4 1 +_4 1 +_4 1 = 0$

So 1 generates every element of C, hence C is a cyclic group.

9 (a) (ii) State the group of symmetries of a regular polygon that is isomorphic to C

[1 mark]

The rotations of a square (rotational symmetry group of a square)

9 (b) The group $(V, \otimes)$ contains the elements (1, 1), (1, −1), (−1, 1) and (−1, −1)

The binary operation $\otimes$ between elements of V is defined by

$$(a, b) \otimes (c, d) = (a \times c, b \times d)$$

9 (b) (i) Find the element in V that is the inverse of (−1, 1)

Fully justify your answer.

[2 marks]

Identity of V is (1, 1). Let the inverse of (−1, 1) be (a, b):

$(-1, 1) \otimes (a, b) = (-a, b) = (1, 1)$

$\Rightarrow \quad a = -1 \quad \text{and} \quad b = 1$

Check: $(-1, 1) \otimes (-1, 1) = ((-1)(-1), 1 \times 1) = (1, 1)$
$\therefore$ the inverse of (−1, 1) is (−1, 1) (it is self-inverse)

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9 (b) (ii) Determine, with a reason, whether or not $C \cong V$

[2 marks]

In V, (a, b) ⊗ (a, b) = (a^2, b^2) = (1, 1) for every element, so all elements of V are self-inverse (order 1 or 2). So no element of V has order 4: V has no generator and is not cyclic. C is cyclic (1 has order 4). A cyclic group of order 4 can only be isomorphic to another cyclic group, so C is not isomorphic to V.

9 (c) The group *G* has order 16

Rachel claims that as 1, 2, 4, 8 and 16 are the only factors of 16 then, by Lagrange's theorem, the group *G* will have exactly 5 distinct subgroups, including the trivial subgroup and *G* itself.

Comment on the validity of Rachel's claim.

[2 marks]

Lagrange's theorem only says the order of any subgroup of G must divide 16. It does not say there is exactly one subgroup of each order. G could have more than one subgroup of order 2 (or of order 4 or 8), e.g. if G has several self-inverse elements, each one generates a different subgroup of order 2. For example, if G is cyclic of order 16 there are exactly 5 subgroups, but if every non-identity element of G is self-inverse then G has 15 subgroups of order 2 alone. So Rachel's reasoning is not valid: there is not enough information about G to tell whether or not it has exactly 5 subgroups.

END OF QUESTIONS

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