AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
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8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The random variable T follows a discrete uniform distribution and can take values

1, 2, 3, …, 16

Find the variance of T

Circle your answer.

[1 mark]

1.25 | 18.75 | (21.25) | 21.33

2 The random variable X has probability density function

$$f(x) = \begin{cases} 1 & 0 < x \leq \frac{1}{2} \\ \frac{3}{8}x^{-2} & \frac{1}{2} < x \leq \frac{3}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X < 1)$

Circle your answer.

[1 mark]

$\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | ($\frac{7}{8}$)

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3 The random variable X has an exponential distribution with probability density function $\mathrm{f}(x) = \lambda \mathrm{e}^{-\lambda x}$ where $x \geq 0$

3 (a) Show that the cumulative distribution function, for $x \geq 0$, is given by $\mathrm{F}(x) = 1 - \mathrm{e}^{-\lambda x}$

[3 marks]

$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt$

$= [\, -e^{-\lambda t} \,]_0^x$

$= (-e^{-\lambda x}) - (-e^0)$

$= -e^{-\lambda x} + 1$

$\therefore\ F(x) = 1 - e^{-\lambda x}$ for $x \geq 0$, as required

3 (b) Given that $\lambda = 2$, find $\mathrm{P}(X > 1)$, giving your answer to three decimal places.

[2 marks]

$P(X > 1) = 1 - F(1)$

$= 1 - (1 - e^{-2\times 1}) = e^{-2}$

$= 0.135335\ldots$

$= 0.135$ (3 d.p.)

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4 Daisies and dandelions are the only flowers growing in a field.

The number of daisies per square metre in the field has a mean of 16

The number of dandelions per square metre in the field has a mean of 10

The number of daisies per square metre and the number of dandelions per square metre are independent.

4 (a) Using a Poisson model, find the probability that a randomly selected square metre from the field has a total of at least 30 flowers, giving your answer to three decimal places.

[3 marks]

Let T = total number of flowers per square metre

$T \sim Po(16 + 10)$ i.e. $T \sim Po(26)$

$P(T \geqslant 30) = 1 - P(T \leqslant 29)$

$= 1 - 0.75926...$

$= 0.24073...$ $= 0.241$ (3 d.p.)

4 (b) A survey of the entire field is taken.

The standard deviation of the total number of flowers per square metre is 10

State, with a reason, whether the model used in part **(a)** is valid.

[2 marks]

Poisson model: variance = mean = 26 (s.d. = $\sqrt{26} \approx 5.10$)

Survey: s.d. = 10, so variance = $10^2 = 100$

The variance 100 is not approximately equal to the mean 26

(it is much larger), so the Poisson model is not valid.

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5 The mass, X, in grams of a particular type of apple is modelled using a normal distribution.

A random sample of 12 apples is collected and the summarised results are

$$\sum x = 1038 \quad \text{and} \quad \sum x^2 = 90\,100$$

5 (a) A 99% confidence interval for the population mean of the masses of the apples is constructed using the random sample.

Show that the confidence interval is (81.7, 91.3) with values correct to three significant figures.

[4 marks]

$\bar{x} = \frac{1038}{12} = 86.5$

$s^2 = \frac{1}{11}\left(90100 - \frac{1038^2}{12}\right)$

$= \frac{313}{11} = 28.4545...$ $\quad (s = 5.3342...)$

$n - 1 = 11$ degrees of freedom, 99%: $t_{11} = 3.106$

CI: $\bar{x} \pm t \times \sqrt{(s^2 \div n)}$

$= 86.5 \pm 3.106 \times \sqrt{(28.4545... \div 12)}$

$= 86.5 \pm 3.106 \times 1.53987...$

$= 86.5 \pm 4.7825...$

Lower = 81.717... $\quad$ Upper = 91.282...

∴ 99% confidence interval is (81.7, 91.3) to 3 s.f.

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5 (b) Padraig claims that the population mean mass of the apples is 85 grams.

He carries out a hypothesis test at the 1% level of significance using the random sample of 12 apples.

The hypotheses are

H_0: $\mu = 85$

H_1: $\mu \neq 85$

State, with a reason, whether the null hypothesis is accepted or rejected.

[1 mark]

85 lies within the 99% confidence interval (81.7, 91.3),

so H_0 is accepted (not rejected) at the 1% level.

5 (c) Interpret, in context, the conclusion to the hypothesis test in part **(b)**.

[1 mark]

There is insufficient evidence to suggest that the population

mean mass of the apples is different from 85 grams.

Turn over for the next question

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6 The discrete random variable X has probability distribution function

$$\mathrm{P}(X = x) = \begin{cases} a & x = 0 \\ b & x = 1 \\ c & x = 2 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

The mean of X is 1.2 and the variance of X is 0.56

6 (a) Deduce the values of a, b and c

[6 marks]

$\Sigma P(X = x) = 1$: $a + b + c = 1$ (1)

$E(X) = 0a + 1b + 2c = 1.2 \Rightarrow b + 2c = 1.2$ (2)

$E(X^2) = 0^2a + 1^2b + 2^2c = b + 4c$

$Var(X) = E(X^2) - [E(X)]^2$

$b + 4c - 1.2^2 = 0.56$

$b + 4c = 0.56 + 1.44 = 2$ (3)

(3) − (2): $2c = 0.8 \Rightarrow c = 0.4$

(2): $b = 1.2 - 2(0.4) = 0.4$

(1): $a = 1 - 0.4 - 0.4 = 0.2$

$\therefore a = 0.2, \quad b = 0.4, \quad c = 0.4$

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6 (b) The continuous random variable Y is independent of X and has variance 15

Find $\text{Var}(X - 2Y - 11)$

[2 marks]

$\text{Var}(X - 2Y - 11) = \text{Var}(X) + (-2)^2 \text{Var}(Y)$ (independent)

$= 0.56 + 4 \times 15$

$= 60.56$

Turn over for the next question

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7 A scientist is investigating the air quality in two countries, *A* and *B*

Measurements of the air quality are taken at sites throughout both countries and assigned a rank.

The air quality is given Rank 1 if the level of air pollution is lower than a specified threshold.

The air quality is given Rank 2 if the level of air pollution is higher than the specified threshold.

A random sample of 500 measurements is collected.

The results are summarised in the following table.

		Air Quality		
		Rank 1	Rank 2	Total
Country	*A*	87	101	188
	B	167	145	312
	Total	254	246	500

The scientist claims that there is an association between country and air quality.

7 (a) Test the scientist's claim, using the 10% level of significance.

[8 marks]

H_0: there is no association between country and air quality

H_1: there is an association between country and air quality

Expected frequencies E = (row total × column total) ÷ 500

A: Rank 1 $\frac{188 \times 254}{500} = 95.504$ Rank 2 92.496

B: Rank 1 158.496 Rank 2 153.504

2 × 2 table, $\nu = 1$, so use Yates' correction:

$$\chi^2 = \Sigma \frac{(|O - E| - 0.5)^2}{E}$$

$$= \frac{(|87 - 95.504| - 0.5)^2}{95.504} + \frac{(|101 - 92.496| - 0.5)^2}{92.496}$$

$$+ \frac{(|167 - 158.496| - 0.5)^2}{158.496} + \frac{(|145 - 153.504| - 0.5)^2}{153.504}$$

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$= 0.6708 + 0.6926 + 0.4042 + 0.4173$

$= 2.1849... \approx 2.185$

Critical value χ^2_1 (10%) = 2.706

2.185 < 2.706, so the test statistic is not in the critical region

$\therefore$ Accept H_0

There is insufficient evidence at the 10% level to support the claim that there is an association between country and air quality.

7 (b) For the context of the test carried out in part **(a)**, state the meaning of a Type I error.

[1 mark]

Concluding that there is an association between country and air quality when in fact there is no association.

Turn over for the next question

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8 The continuous random variable X has cumulative distribution function $\mathrm{F}(x)$ where

$$\mathrm{F}(x) = \begin{cases} 0 & x = 0 \\ \mathrm{e}^{kx} - 1 & 0 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

8 (a) Show that $k = \frac{1}{5}\ln 2$

[2 marks]

$F(5) = 1 \Rightarrow e^{5k} - 1 = 1$

$e^{5k} = 2$

$5k = \ln 2$

$k = \frac{1}{5}\ln 2$ as required

8 (b) Show that the median of X is $a\frac{\ln b}{\ln 2} - c$, where a, b and c are integers to be found.

[3 marks]

Median m satisfies $F(m) = 0.5$

$e^{km} - 1 = 0.5 \Rightarrow e^{km} = 1.5$

$km = \ln 1.5 = \ln 3 - \ln 2$

$m = \frac{\ln 3 - \ln 2}{k} = \frac{5(\ln 3 - \ln 2)}{\ln 2}$ (using $k = \frac{1}{5}\ln 2$)

$m = 5\frac{\ln 3}{\ln 2} - 5$

$\therefore\ a = 5,\quad b = 3,\quad c = 5$

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8 (c) Show that the mean of X is $p - \frac{q}{\ln 2}$, where p and q are integers to be found.

[6 marks]

$f(x) = F'(x) = k\,e^{kx}$ for $0 \leq x \leq 5$, where $k = \frac{1}{5}\ln 2$

$E(X) = \int_0^5 x\,f(x)\,dx = \int_0^5 kx\,e^{kx}\,dx$

By parts: $u = x$, $dv/dx = k\,e^{kx} \Rightarrow du/dx = 1$, $v = e^{kx}$

$E(X) = [\,x\,e^{kx}\,]_0^5 - \int_0^5 e^{kx}\,dx$

$= [\,x\,e^{kx}\,]_0^5 - [\,\frac{1}{k}\,e^{kx}\,]_0^5$

$= (5e^{5k} - 0) - \frac{1}{k}(e^{5k} - e^0)$

Using $e^{5k} = 2$ from part (a):

$= 5 \times 2 - \frac{1}{k}(2 - 1)$

$= 10 - \frac{1}{k} = 10 - \frac{5}{\ln 2}$

$\therefore\ p = 10,\quad q = 5$

Turn over ▶

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9 Lianne models the maximum time in hours that a rechargeable battery can be used, before needing to be recharged, with a rectangular distribution with values between 8 and 12

9 (a) The probability that the maximum time the battery can be used before needing to be recharged is more than 10.5 hours is equal to p

Lianne will only buy the battery if p is more than 0.4

Determine whether Lianne will buy the battery.

[2 marks]

$X \sim R[8, 12]$, so $f(x) = \frac{1}{4}$ for $8 \leq x \leq 12$

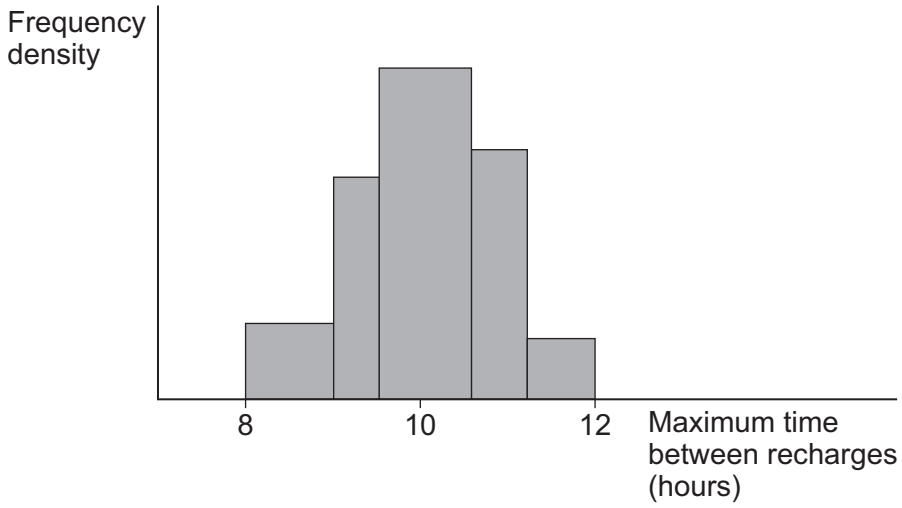
$p = P(X > 10.5) = (12 - 10.5) \times \frac{1}{4} = 0.375$

$0.375 < 0.4$, so p is not more than 0.4

$\therefore$ Lianne will not buy the battery.

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9 (b) A histogram is plotted for 100 recharges showing the maximum time the battery can be used before needing to be recharged.



Explain why the model used in part **(a)** may not be valid and suggest the name of a different distribution that could be used to model the maximum time between recharges.

[2 marks]

The frequency density on the histogram is not approximately level between 8 and 12 hours: it peaks around 10 hours and is roughly symmetrical, so a rectangular distribution is not a good model. A normal distribution could be used instead.

END OF QUESTIONS

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