AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 3 Mechanics

Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification.
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

For Examiner's Use	
Question	Mark
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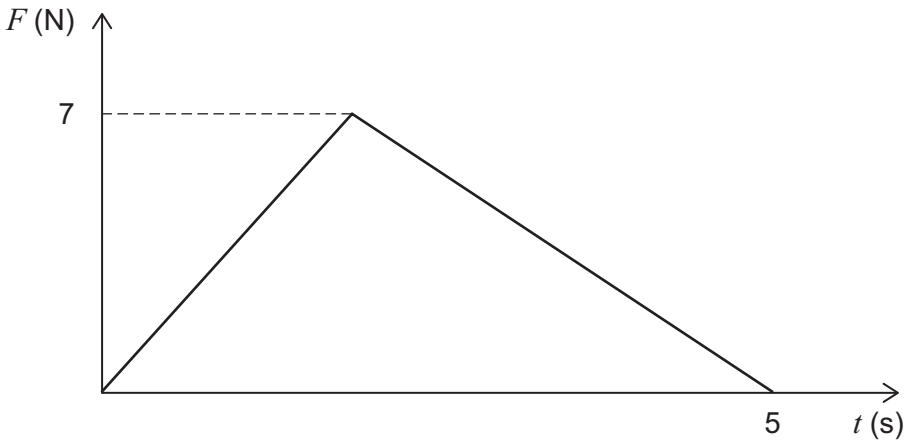
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The graph shows how a force, F newtons, varies during a 5 second period of time.



Calculate the magnitude of the impulse of the force.

Circle your answer.

[1 mark]

(17.5 N s) 25 N s 35 N s 70 N s

Impulse = area under graph = 1/2 × 5 × 7 = 17.5 N s

2 A car of mass 1200 kg is travelling at a constant speed of 18 m s^{-1} on a straight horizontal road.

The car experiences a total resistive force of 240 newtons.

Calculate the power of the car's engine.

Circle your answer.

[1 mark]

900 W (4320 W) 16 000 W 21 600 W

Driving force = resistance = 240 N, P = Fv = 240 × 18 = 4320 W

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3 Three particles are attached to a light rod, AB, of length 40 cm

The particles are attached at A, B and the midpoint of the rod.

The particle at A has mass 5 kg

The particle at B has mass 1 kg

The particle at the midpoint has mass 4 kg

Find the distance of the centre of mass of this system from the midpoint of the rod.

Circle your answer.

[1 mark]

4 cm | (8 cm) | 12 cm | 28 cm

Moments about midpoint (A side +): $(5 \times 20 - 1 \times 20) \div 10 = 8$ cm

Turn over for the next question

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4 (a) State the dimensions of force.

[1 mark]

$[\text{Force}] = MLT^{-2}$

4 (b) The velocity of an object in a circular orbit can be calculated using the formula

$$v = G^a m^b r^c$$

where:

$G =$ Universal constant of gravitation in $\text{N m}^2\,\text{kg}^{-2}$

$m =$ Mass of the Earth in kg

$r =$ Radius of the orbit in metres

Use dimensional analysis to find the values of a, b and c

[4 marks]

$[G] = [F][r]^2 \div [m]^2 = MLT^{-2} \times L^2 \times M^{-2} = M^{-1}L^3T^{-2}$

$[v] = LT^{-1}, \quad [m] = M, \quad [r] = L$

$LT^{-1} = (M^{-1}L^3T^{-2})^a \times M^b \times L^c = M^{-a+b}\, L^{3a+c}\, T^{-2a}$

$T: \quad -2a = -1 \quad \Rightarrow \quad a = 1/2$

$M: \quad -a + b = 0 \quad \Rightarrow \quad b = a = 1/2$

$L: \quad 3a + c = 1 \quad \Rightarrow \quad c = 1 - 3/2 = -1/2$

$\therefore \; a = 1/2, \quad b = 1/2, \quad c = -1/2$

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5 A train of mass 10 000 kg is travelling at $0.3\,\text{m}\,\text{s}^{-1}$ when it collides with a buffer. The buffer brings the train to rest.

As the buffer brings the train to rest it compresses by 0.2 metres.

When the buffer is compressed by a distance of x metres it exerts a force of magnitude F newtons, where

$$F = Ax + 9000x^2$$

where A is a constant.

5 (a) Find, in terms of A, the work done in compressing the buffer by 0.2 metres.

[2 marks]

$WD = \int_0^{0.2} (Ax + 9000x^2)\,dx$

$= [\,Ax^2/2 + 3000x^3\,]_0^{0.2}$

$= A(0.2)^2/2 + 3000(0.2)^3$

$= 0.02A + 24$

$= A/50 + 24 \quad J$

5 (b) Find the value of A

[2 marks]

Work done by buffer = KE lost by train

$A/50 + 24 = 1/2 \times 10000 \times 0.3^2$

$A/50 + 24 = 450$

$A/50 = 426$

$A = 21300$

Turn over for the next question

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6 A particle, of mass 5 kg, moves on a circular path so that at time t seconds it has position vector **r** metres, where

$$\mathbf{r} = (2\sin 3t)\mathbf{i} + (2\cos 3t)\mathbf{j}$$

6 (a) Prove that the velocity of the particle is perpendicular to its position vector.

[4 marks]

$v = dr/dt = (6\cos 3t)i - (6\sin 3t)j$

$v \cdot r = (6\cos 3t)(2\sin 3t) + (-6\sin 3t)(2\cos 3t)$

$= 12\sin 3t \cos 3t - 12\sin 3t \cos 3t$

$= 0$

The scalar product of v and r is zero (and neither vector is zero, since $|r| = 2$ and $|v| = 6$), so the velocity is perpendicular to the position vector.

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6 (b) Prove that the magnitude of the resultant force on the particle is constant.

[3 marks]

$a = dv/dt = (-18\sin 3t)i - (18\cos 3t)j$

$F = ma = 5a = (-90\sin 3t)i - (90\cos 3t)j$

$|F| = \sqrt{((-90\sin 3t)^2 + (-90\cos 3t)^2)}$

$= \sqrt{(90^2(\sin^2 3t + \cos^2 3t))}$

$= \sqrt{(90^2 \times 1)} = 90$ N

90 N does not depend on t, so the magnitude of the resultant force is constant.

Turn over for the next question

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7 Two snooker balls, one white and one red, have equal mass.

The balls are on a horizontal table *ABCD*

The white ball is struck so that it moves at a speed of $2\,m\,s^{-1}$ parallel to *AB*

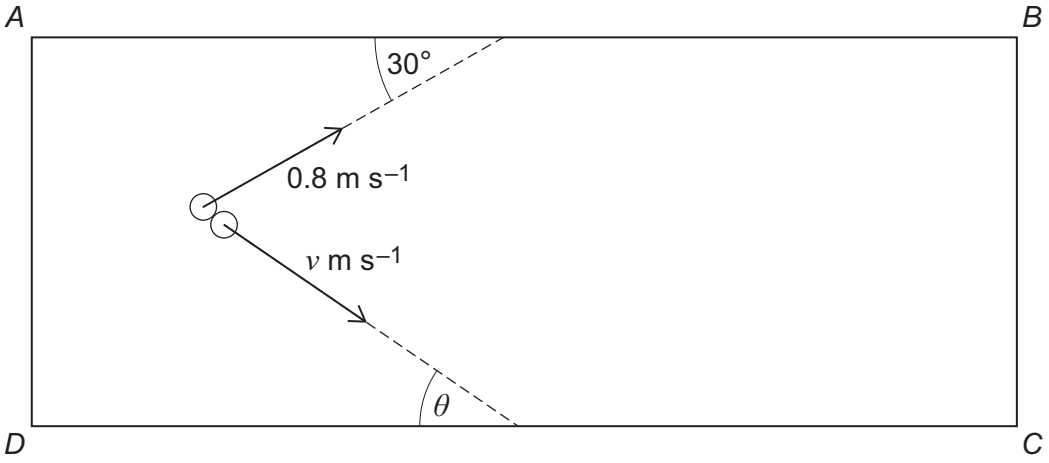
The white ball hits a stationary red ball.

After the collision, the white ball moves at a speed of $0.8\,m\,s^{-1}$ and at an angle of 30° to *AB*

After the collision, the red ball moves at a speed $v\,m\,s^{-1}$ and at an angle θ to *CD*

When the collision takes place, the white ball is the same distance from *AB* as the distance the red ball is from *CD*

The diagram below shows the table and the velocities of the balls after the collision.



Not to scale

After the collision, the white ball hits *AB* and the red ball hits *CD*

Model the balls as particles that do not experience any air resistance.

7 (a) Explain why the two balls hit the sides of the table at the same time.

[2 marks]

Momentum is conserved perpendicular to AB. Before the collision it is zero in this direction, so afterwards the components of momentum of the two balls perpendicular to AB and CD are equal in size. The masses are equal, so these velocity components are equal (and stay constant, no resistance). Each ball is the same distance from its side, so both reach the sides at the same time.

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7 (b) Show that $\theta = 17.0°$ correct to one decimal place.

[4 marks]

Conservation of momentum (equal masses m cancel):

Parallel to AB: $2m = 0.8m\cos30° + mv\cos\theta$

Perpendicular to AB: $0 = 0.8m\sin30° - mv\sin\theta$

$v\cos\theta = 2 - 0.8\cos30° = 2 - 2\sqrt{3}/5 = 1.30717...$

$v\sin\theta = 0.8\sin30° = 0.4$

$\tan\theta = 0.4 \div 1.30717... = 0.30600...$

$\theta = \tan^{-1}(0.30600...) = 17.0142...°$

$\theta = 17.0°$ (1 d.p.)

7 (c) Find v

[2 marks]

$v = 0.8\sin30° \div \sin17.0142...° = 0.4 \div 0.292609...$

$v = 1.36701...$

$v = 1.37\ m\ s^{-1}$

7 (d) Determine which ball travels the greater distance after the collision and before hitting the side of the table.

Fully justify your answer.

[2 marks]

Both balls take the same time t to reach the sides (from part (a)).

Speeds are constant, so distance = speed × time:

red: 1.37t, white: 0.8t

1.37 > 0.8, so the red ball travels the greater distance.

7 (e) State **one** possible refinement to the model that you have used.

[1 mark]

Include the rotational motion (spin) of the balls in the model.

Turn over ▶

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8 **In this question use g as 9.8 m s^{-2}**

A rope is used to pull a crate, of mass 60 kg, along a rough horizontal surface.

The coefficient of friction between the crate and the surface is 0.4

The crate is at rest when the rope starts to pull on it.

The tension in the rope is 240 N and the rope makes an angle of 30° to the horizontal.

When the crate has moved 5 metres, the rope becomes detached from the crate.

8 (a) Use an energy method to find the maximum speed of the crate.

[4 marks]

Resolving vertically: $R + 240\sin30° = 60 \times 9.8$

$R = 588 - 120 = 468$ N

Friction $F = \mu R = 0.4 \times 468 = 187.2$ N

The crate speeds up while the rope pulls, then only friction acts and it slows down, so the maximum speed is when the rope detaches after 5 m.

Work done by tension − work done against friction = gain in KE

$240\cos30° \times 5 - 187.2 \times 5 = 1/2 \times 60 \times v^2$

$600\sqrt{3} - 936 = 30v^2$

$1039.230... - 936 = 30v^2$

$103.230... = 30v^2$

$v^2 = 3.44101...$

$v = 1.85499...$

Maximum speed = 1.85 m s^{-1}

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8 (b) Use an energy method to find the total distance travelled by the crate.

[2 marks]

After the rope detaches: $R = 60 \times 9.8 = 588$ N

Friction $= 0.4 \times 588 = 235.2$ N

KE lost = work done against friction

$103.230... = 235.2 \times d$

$d = 0.43890...$ m

Total distance $= 5 + 0.43890... = 5.43890...$

Total distance = 5.4 m (2 s.f.)

8 (c) A student claims that in reality the crate is unlikely to travel more than 5.3 metres in total. Comment on the validity of this claim.

[2 marks]

The model ignores air resistance. In reality air resistance would also act on the crate and slow it down, so it would travel less than the 5.4 m predicted. Hence the student's claim that it is unlikely to travel more than 5.3 m is reasonable (valid).

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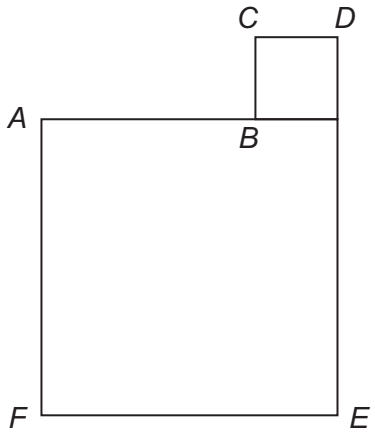
9 Two blocks have square cross sections.

One block has mass 9 kg and its cross section has sides of length 20 cm

The other block has mass 1 kg and its cross section has sides of length 4 cm

The blocks are fixed together to form the composite body shown in **Figure 1**.

Figure 1



9 (a) Find the distance of the centre of mass of the composite body from *AF*

[2 marks]

Distances from AF: large block 10 cm, small block 16 + 2 = 18 cm

$(9 + 1)d = 9 \times 10 + 1 \times 18$

$d = 108 \div 10$

$d = 10.8$ cm

Question 9 continues on the next page

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9 (b) A uniform rod has mass 12 kg and length 1 metre.

One end of the rod rests against a smooth vertical wall.

The other end of the rod rests on the composite body at point B

The composite body is on a horizontal surface.

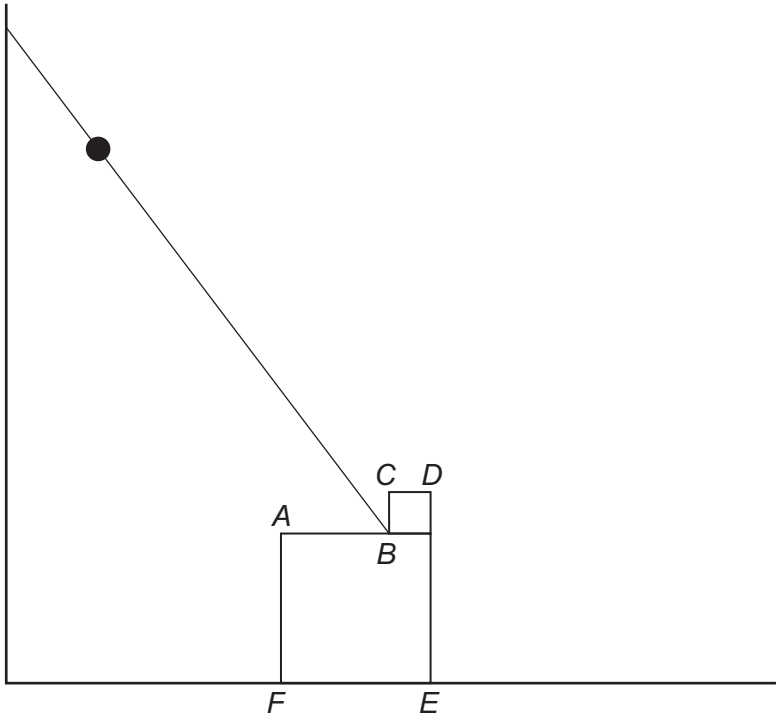
The coefficient of friction between the composite body and the horizontal surface is 0.3

The angle between the rod and AB is 60°

A particle of mass m kg is fixed to the rod at a distance of 75 cm from B

The rod, particle and composite body are shown in **Figure 2**.

Figure 2



9 (b) (i) Write down the magnitude of the vertical reaction force acting on the rod at B in terms of m and g

[1 mark]

Resolving vertically for the rod and particle (smooth wall):

$R = (12 + m)g$

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9 (b) (ii) Show that the magnitude of the horizontal reaction force acting on the rod at B is

$$\frac{g(6+0.75m)}{\sqrt{3}}$$

[3 marks]

The wall is smooth, so its reaction P on the rod is horizontal. Resolving horizontally: S = P

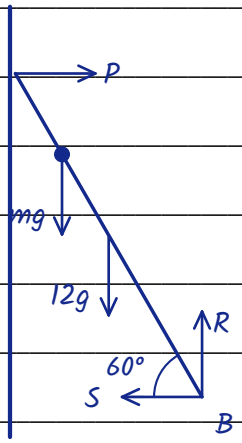
Moments about B (rod 1 m long, at 60° to the horizontal):

P × 1sin60° = 12g × 0.5cos60° + mg × 0.75cos60°

P sin60° = g(6 + 0.75m)cos60°

P = g(6 + 0.75m) ÷ tan60°

S = P = g(6 + 0.75m)/√3 as required



Question 9 continues on the next page

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9 (b) (iii) Find the maximum value of m for which the composite body does not slide or topple.

Fully justify your answer.

[6 marks]

Forces on the body from the rod at B (20 cm high, 4 cm from E):

$(12 + m)g$ down and $g(6 + 0.75m)/\sqrt{3}$ horizontally away from the wall.

Centre of mass of body is $20 - 10.8 = 9.2$ cm from E (horizontally).

Toppling: on the point of toppling about E, normal reaction acts at E.

No toppling: $20 \times g(6 + 0.75m)/\sqrt{3} \leq 10g \times 9.2 + (12 + m)g \times 4$

$20(6 + 0.75m) \leq \sqrt{3}(92 + 48 + 4m)$

$120 + 15m \leq 140\sqrt{3} + 4\sqrt{3}m$

$m(15 - 4\sqrt{3}) \leq 140\sqrt{3} - 120$

$m \leq (140\sqrt{3} - 120) \div (15 - 4\sqrt{3}) = 15.1747...$

Sliding: normal reaction from ground $= 10g + (12 + m)g = (22 + m)g$

No sliding: $g(6 + 0.75m)/\sqrt{3} \leq 0.3(22 + m)g$

$6 + 0.75m \leq 0.3\sqrt{3}(22 + m) = 6.6\sqrt{3} + 0.3\sqrt{3}m$

$m(0.75 - 0.3\sqrt{3}) \leq 6.6\sqrt{3} - 6$

$m \leq (6.6\sqrt{3} - 6) \div (0.75 - 0.3\sqrt{3}) = 23.5759...$

$15.17... < 23.58...$, so as m increases the body topples before it slides. Both conditions hold only for $m \leq 15.17...$

Maximum value of $m = 15.2$

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