AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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Candidate signature _____

A-level
FURTHER MATHEMATICS

Paper 3 Statistics

Thursday 13 June 2019 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions in the spaces provided.

1 The discrete random variable X has $\text{Var}(X) = 5$

Find $\text{Var}(4X - 3)$

Circle your answer.

[1 mark]

17 20 77 (80)

$\text{Var}(4X - 3) = 4^2\,\text{Var}(X) = 16 \times 5 = 80$

2 Amy takes a sample of size 50 from a normal distribution with mean μ and variance 16

She conducts a hypothesis test with hypotheses:

$H_0 : \mu = 52$

$H_1 : \mu > 52$

She rejects the null hypothesis if her sample has a mean greater than 53

The actual population mean is 53.5

Find the probability that Amy makes a Type II error.

Circle your answer.

[1 mark]

0.4% 3.9% (18.9%) 45.0%

$\bar{x} \sim N(53.5, 16/50)$ when $\mu = 53.5$

$P(\text{Type II}) = P(\bar{x} \leq 53) = P(Z \leq (53 - 53.5)/\sqrt{0.32})$

$= P(Z \leq -0.884) = 0.1884 = 18.9\%$

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3 Alan's journey time to work can be modelled by a normal distribution with standard deviation 6 minutes.

Alan measures the journey time to work for a random sample of 5 journeys. The mean of the 5 journey times is 36 minutes.

3 (a) Construct a 95% confidence interval for Alan's mean journey time to work, giving your values to one decimal place.

[2 marks]

$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}, \quad z = 1.96$

$= 36 \pm 1.96 \times \frac{6}{\sqrt{5}}$

$= 36 \pm 5.259...$

95% confidence interval = (30.7, 41.3)

3 (b) Alan claims that his mean journey time to work is 30 minutes.

State, with a reason, whether or not the confidence interval found in part **(a)** supports Alan's claim.

[1 mark]

No, it does not support the claim, because 30 minutes lies outside the confidence interval (30.7, 41.3).

3 (c) Suppose that the standard deviation is not known but a sample standard deviation is found from Alan's sample and calculated to be 6

Explain how the working in part **(a)** would change.

[1 mark]

A t distribution would be used instead of the normal distribution: 1.96 is replaced by the t_4 value 2.776, with $s = 6$ used for σ.

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4 A random variable X has a rectangular distribution.

The mean of X is 3 and the variance of X is 3

4 (a) Determine the probability density function of X.

Fully justify your answer.

[5 marks]

$X \sim R[a, b]$ with $a < b$

$E(X) = \frac{a+b}{2} = 3 \quad \Rightarrow \quad a + b = 6$

$Var(X) = \frac{(b-a)^2}{12} = 3$

Sub $a = 6 - b$: $\frac{(b-6+b)^2}{12} = 3$

$(2b - 6)^2 = 36$

$2b - 6 = 6$ (taking the positive root since $b > a$)

$b = 6$ and $a = 6 - 6 = 0$

Check: $E(X) = \frac{0+6}{2} = 3$ ✓ $\quad Var(X) = \frac{6^2}{12} = 3$ ✓

Height of pdf $= \frac{1}{b-a} = \frac{1}{6}$

$$f(x) = \begin{cases} \frac{1}{6} & 0 \leqslant x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

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4 (b) A 6 metre clothes line is connected between the point P on one building and the point Q on a second building.

Roy is concerned the clothes line may break. He uses the random variable X to model the distance in metres from P where the clothes line breaks.

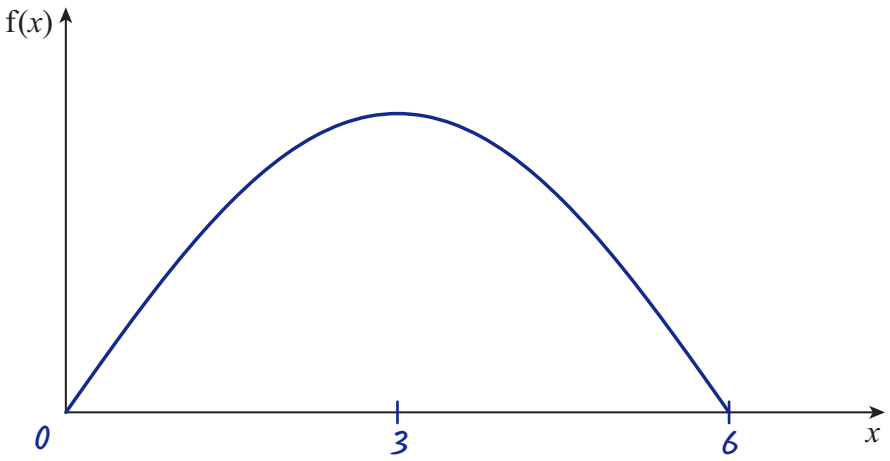
4 (b) (i) State a criticism of Roy's model.

[1 mark]

The model assumes the line is equally likely to break at any point, but it is more likely to snap in the middle, where the tension/sag is greatest (and clothes hang), than near the ends P and Q.

4 (b) (ii) On the axes below, sketch the probability density function for an alternative model for the clothes line.

[1 mark]



Turn over for the next question

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5 An insurance company models the claims it pays out in pounds (£) with a random variable X which has probability density function

$$f(x) = \begin{cases} \frac{k}{x} & 1 < x < a \\ 0 & \text{otherwise} \end{cases}$$

5 (a) The median claim is £200

Show that $k = \frac{1}{2\ln 200}$

[3 marks]

Median = 200 $\Rightarrow$ P(X < 200) = 0.5

$\int_1^{200} \frac{k}{x}\,dx = 0.5$

$[k \ln x]_1^{200} = 0.5$

$k \ln 200 - k \ln 1 = 0.5$

$k \ln 200 = 0.5$ $(\ln 1 = 0)$

$k = \frac{0.5}{\ln 200} = \frac{1}{2 \ln 200}$ as required

5 (b) Find $P(X < 2000)$, giving your answer to three significant figures.

[2 marks]

$P(X < 2000) = \frac{1}{2 \ln 200} \int_1^{2000} \frac{1}{x}\,dx$

$= \frac{1}{2 \ln 200} [\ln x]_1^{2000} = \frac{\ln 2000}{2 \ln 200}$

$= 0.71729...$

$= 0.717$ (3 s.f.)

(valid as $k \ln a = 1$ gives $a = 200^2 = 40\,000 > 2000$)

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5 (c) The insurance company finds that the maximum possible claim is £2000 and they decide to refine their probability density function.

Suggest how this could be done.

[2 marks]

Restrict the model to $1 < x < 2000$ (set $a = 2000$) and change the value of k so that $P(X < 2000) = 1$:

$$\int_1^{2000} \frac{k}{x}\,dx = 1 \quad \Rightarrow \quad k \ln 2000 = 1 \quad \Rightarrow \quad k = \frac{1}{\ln 2000} = 0.132$$

so $f(x) = \frac{1}{x \ln 2000}$ for $1 < x < 2000$, 0 otherwise.

Turn over for the next question

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6 During August, 102 candidates took their driving test at centre *A* and 60 passed.

During the same month, 110 candidates took their driving test at centre *B* and 80 passed.

6 (a) Test whether the driving test result is independent of the driving test centre using the 5% level of significance.

[8 marks]

H_0: there is no association between driving test result and driving test centre (they are independent)

H_1: there is an association between test result and test centre

Observed	Pass	Fail	Total
A	60	42	102
B	80	30	110
Total	140	72	212

E = row total × column total ÷ 212, e.g. $102 \times 140 \div 212$

Expected	Pass	Fail
A	67.36	34.64
B	72.64	37.36

All $E \geq 5$; 2 × 2 table so $\nu = 1$ and use Yates' correction

$\chi^2 = \Sigma \frac{(|O - E| - 0.5)^2}{E}$ $|O - E| = 7.358$ in every cell

$= \frac{6.858^2}{67.36} + \frac{6.858^2}{34.64} + \frac{6.858^2}{72.64} + \frac{6.858^2}{37.36}$

$= 0.698 + 1.358 + 0.648 + 1.259$

$= 3.9628... = 3.96$

Critical value χ^2_1 (5%) = 3.841

$3.96 > 3.841$ so reject H_0

There is sufficient evidence at the 5% level to suggest that the driving test result and the driving test centre are not independent.

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6 (b) Rebecca claims that if the result of the test in part **(a)** is to reject the null hypothesis then it is easier to pass a driving test at centre *B* than centre *A*.

State, with a reason, whether or not you agree with Rebecca's claim.

[1 mark]

No. Rejecting H_0 only shows an association between result and centre, not that the test is easier at B. The higher pass rate at B may have another cause, e.g. better prepared candidates or more lessons taken.

Turn over for the next question

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7 A shopkeeper sells chocolate bars which are described by the manufacturer as having an average mass of 45 grams.

The shopkeeper claims that the mass of the chocolate bars, X grams, is getting smaller on average.

A random sample of 6 chocolate bars is taken and their masses in grams are measured. The results are

$$\sum x = 246 \qquad \text{and} \qquad \sum x^2 = 10\,198$$

Investigate the shopkeeper's claim using the 5% level of significance.

State any assumptions that you make.

[9 marks]

H_0: $\mu = 45$ H_1: $\mu < 45$ (μ = population mean mass, grams)

Assume the masses X can be modelled by a normal distribution.

$\bar{x} = \frac{246}{6} = 41$

$s^2 = \frac{1}{6-1}\left(10198 - \frac{246^2}{6}\right) = \frac{112}{5} = 22.4$

$s = 4.733$ (4 s.f.)

$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{41 - 45}{\sqrt{(22.4 \div 6)}} = \frac{-4}{1.9322...}$

$= -2.070... = -2.07$

Under H_0, $t \sim t_5$ ($\nu = 6 - 1 = 5$)

5% one-tailed critical value $= -2.015$

$-2.07 < -2.015$, so the test statistic lies in the critical region

Reject H_0

There is sufficient evidence at the 5% level to support the shopkeeper's claim that the mean mass of the chocolate bars is getting smaller (less than 45 grams).

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Turn over for the next question

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8 The number of telephone calls received by an office can be modelled by a Poisson distribution with mean 3 calls per 10 minutes.

8 (a) Find the probability that:

8 (a) (i) the office receives exactly 2 calls in 10 minutes;

[1 mark]

$X \sim Po(3)$: $P(X = 2) = \frac{e^{-3} \times 3^2}{2!} = 0.22404... = 0.224$

8 (a) (ii) the office receives more than 30 calls in an hour.

[3 marks]

Number of calls in an hour: $Y \sim Po(3 \times 6) = Po(18)$

$P(Y > 30) = P(Y \geq 31) = 1 - P(Y \leq 30)$

$= 1 - 0.99667$

$= 0.003330... = 0.00333$ (3 s.f.)

8 (b) The office manager splits an hour into 6 periods of 10 minutes and records the number of telephone calls received in each of the 10 minute periods.

Find the probability that the office receives exactly 2 calls in a 10 minute period exactly twice within an hour.

[3 marks]

C = number of 10 minute periods with exactly 2 calls

$C \sim B(6, 0.224)$

$P(C = 2) = {}^6C_2 \times 0.224^2 \times (1 - 0.224)^4$

$= 15 \times 0.050176 \times 0.36262...$

$= 0.2729... = 0.273$ (3 s.f.)

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8 (c) The office has just received a call.

8 (c) (i) Find the probability that the next call is received more than 10 minutes later.

[3 marks]

Time T between calls, in units of 10 minutes: $T \sim Exp(3)$

$P(T > 1) = e^{-3 \times 1}$

$= e^{-3} = 0.049787...$

$= 0.0498$ (3 s.f.)

(Equivalently P(no calls in 10 minutes) = $P(X = 0) = e^{-3}$)

8 (c) (ii) Mahah arrives at the office 5 minutes after the last call was received.

State the probability that the next call received by the office is received more than 10 minutes later.

Explain your answer.

[2 marks]

Probability = 0.0498

The exponential distribution is memoryless: the time already waited (5 minutes) does not affect the probability that the next call takes more than 10 more minutes, so it is the same as in (c)(i).

END OF QUESTIONS

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