A-level
FURTHER MATHEMATICS
7367/3S

Paper 3 Statistics

Mark scheme

June 2019

Version: 1.0 Final

196A7367/3S/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

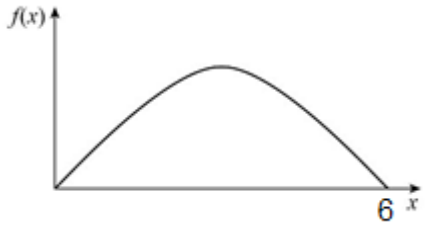
AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	80
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	AO1.1b	B1	18.9%
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Uses formula for confidence interval with AWRT 1.96 or 1.64 or 1.645 (PI)	AO1.1a	M1	$\bar{x} \pm z\sqrt{\frac{\sigma^2}{n}}$ $= 36 \pm 1.96\frac{6}{\sqrt{5}}$
	Obtains correct confidence interval AWRT (30.7, 41.3) Condone poor notation	AO1.1b	A1	= (30.7, 41.3)
3(b)	Infers no as 30 minutes is outside the confidence interval. Follow through their confidence interval	AO2.2b	E1F	The confidence interval does not support Alan's claim as 30 minutes is outside the confidence interval.
3(c)	Explains that their *z* value would be replaced by a *t* value or gives correct formula Condone use a t-test rather than a z-test Ignore any values given following a correct statement Implied by AWRT 1.96 replaced by AWRT 2.78	AO3.5c	E1	A t distribution will be used instead of a normal distribution
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Forms one correct equation using E(X) = 3 by using the formulae for a rectangular distribution or by first principles	AO3.1a	M1	$\frac{1}{2}(a+b)=3$ $\frac{1}{12}(b-a)^2=3$
	Forms one correct equation using Var(X) = 3 by using the formulae for a rectangular distribution or by first principles	AO1.1a	M1	$\frac{1}{12}(b-6+b)^2=3$ $b = 6$ $a = 0$
	Forms a correct equation just in terms of a or b (PI) $\frac{1}{12}(b-6+b)^2=3$ (OE) or $\frac{1}{12}(6-a-a)^2=3$ (OE)	AO1.1b	A1	$f(x)=\begin{cases}\frac{1}{6} & 0\le x\le 6\\ 0 & \text{otherwise}\end{cases}$
	Find correct values of a and b	AO1.1b	A1	
	Fully defined and correct probability density function Follow through their a and b	AO3.2a	A1F	
4(b)(i)	Makes valid criticism of model by giving a reason why the clothes line is more or less likely to snap at particular points Accept references to clothes on the line or tension in the line	AO3.5b	E1	The clothes line is more likely to snap in the middle.
4(b)(ii)	Sketch a symmetrical pdf with maximums consistent with their criticism in (b)(i) Their 6 or 6 must be seen Do not award for pdfs with non-zero values at x > their 6 or 6 Allow a non-symmetrical pdf with clear maximums if their criticism in (b)(i) relates to the placement of clothes on the clothes line	AO3.3	B1F	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Forms an integral with correct upper limit using P(X ≤ 200) and set equal to 0.5 (may be implied by later work) Condone missing dx	AO1.1a	M1	$\int_1^{200} \frac{k}{x} dx = 0.5$ $\left[k \ln x\right]_1^{200} = 0.5$ k ln 200 = 0.5 k = $\frac{0.5}{\ln 200} = \frac{1}{2\ln 200}$
	Integrates correctly Condone missing limits	AO1.1b	M1	
	Shows substitution of limits and solves to show that k = $\frac{1}{2\ln 200}$ Mark awarded if they have a completely correct solution, which is clear, easy to follow, uses correct notation and contains no slips	AO2.1	R1	
5(b)	States the correct integral to find P(X < 2000) (PI) Condone missing dx	AO1.1a	M1	$\frac{1}{2\ln 200} \int_1^{2000} \frac{1}{x} dx$ = 0.717
	Finds correct probability (AWRT 0.717)	AO1.1b	A1	
5(c)	Explains that the model is restricted to values from 1 to 2000 Implied by set a = 2000	AO3.5c	E1	Set a = 2000 and change the value of k so that P(X < 2000) = 1
	Either suggests model with pdf unchanged for (1,2000) and has probability 1 – their (b) for 2000 or Suggests same pdf with a different or increased value of k or a different pdf so that P(X < 2000) = 1 Implied by k = 1/ln 2000 or AWRT 0.132	AO3.5c	E1	
	Total		**7**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="9">6(a)</td><td>States both hypotheses using correct language
Variables must be included in at least the null hypothesis</td><td>AO2.5</td><td>B1</td><td rowspan="9">H_0: There is no association between driving test result and driving test centre
H_1: There is an association between driving test result and driving test centre

<table><tr><td>Observed</td><td>P</td><td>F</td><td>Total</td></tr><tr><td>A</td><td>60</td><td>42</td><td>102</td></tr><tr><td>B</td><td>80</td><td>30</td><td>110</td></tr><tr><td>Total</td><td>140</td><td>72</td><td>212</td></tr></table>
<table><tr><td>Expected</td><td>P</td><td>F</td></tr><tr><td>A</td><td>67.4</td><td>34.6</td></tr><tr><td>B</td><td>72.6</td><td>37.4</td></tr></table>
ts = $\frac{(\lvert 60-67.4\rvert-0.5)^2}{67.4}+\frac{(\lvert 42-34.6\rvert-0.5)^2}{34.6}+\frac{(\lvert 80-72.6\rvert-0.5)^2}{72.6}+\frac{(\lvert 30-37.4\rvert-0.5)^2}{37.4}$

= 4.0

χ^2 cv for 1 df = 3.84
4.0 > 3.84

Reject H_0

Some evidence to suggest/support that driving test result and driving test centre are not independent</td></tr>
<tr><td>Constructs correct contingency table with frequencies shown (PI)
Condone one slip</td><td>AO3.3</td><td>M1</td></tr>
<tr><td>Finds expected frequencies (PI)
AWRT 3 significant figures</td><td>AO3.4</td><td>M1</td></tr>
<tr><td>Calculates χ^2-test statistic with their observed and expected frequencies
Condone no Yates correction used
Condone missing modulus sign
Accept use of their $\frac{212(\lvert 60\times 30-42\times 80\rvert-0.5\times 212)^2}{140\times 72\times 102\times 110}$</td><td>AO1.1a</td><td>M1</td></tr>
<tr><td>Calculates χ^2-test statistic correctly
AWRT 4.0</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Evaluates χ^2-test statistic by comparing their ts with the cv (AWRT 3.84)
(or p = AWFW 0.045 to 0.047 with 0.05)</td><td>AO3.5a</td><td>R1</td></tr>
<tr><td>Infers H_0 rejected
Follow through their test statistic</td><td>AO2.2b</td><td>E1F</td></tr>
<tr><td>Concludes in context, based on their hypotheses
(not definite)
Should be consistent with decision to accept or reject H_0 if stated or ts (or p value) and cv (or 0.05) if not</td><td>AO3.2a</td><td>E1F</td></tr>
<tr><td>6(b)</td><td>States that they do not agree with Rebecca's claim that it is easier at centre B giving valid reason or example of alternative possible cause</td><td>AO3.5b</td><td>E1</td><td>No, may be another underlying cause for the higher pass rate at B</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
7	States both hypotheses using correct language	AO2.5	B1	H_0: $\mu = 45$ H_1: $\mu < 45$
	States required assumption to use a *t*-test	AO2.4	E1	Assume that *X* can be modelled by a normal distribution
	Calculates sample mean	AO1.1b	B1	$\bar{x} = \frac{246}{6} = 41$
	Calculates sample variance or sample standard deviation (AWRT 4.73)	AO1.1b	B1	$s^2 = \frac{1}{6-1}\left(10198 - \frac{246^2}{6}\right) = 22.4$
	Calculates the *t*-test statistic with their sample mean and variance or confidence interval (with their t value) (PI) Condone absolute value	AO1.1a	M1	$\frac{41-45}{\sqrt{\frac{22.4}{6}}} = -2.07$ t_5 at 95% = 2.015 –2.07 < –2.015 Reject H_0
	Calculates *t*-test statistic or confidence interval correctly or finds *p* (AWRT *p* = 0.0466) Condone absolute value	AO1.1b	A1	Some evidence to suggest/support that the shopkeeper is correct that the mass of chocolate bars is getting lighter
	Evaluates *t* model by comparing their negative test statistic and correct critical value (AWRT -2.02) or by comparing *p* value with 0.05 or by comparing sample mean with confidence interval calculated using correct t value (AWRT 2.02) Condone comparing AWRT 2.02 with their positive test statistic	AO3.5a	R1	
	Infers H_0 rejected, follow through their comparison with consistent signs	AO2.2b	E1F	
	Concludes in context based on their hypotheses (not definite) Should be consistent with decision to accept or reject the H_0 if stated or ts (or *p* value) and cv (or 0.05) if not	AO3.2a	E1F	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)(i)	Obtains correct probability AWRT 0.224	AO1.1b	B1	P(X = 2) = 0.224
8(a)(ii)	Selects Poisson model with $\lambda = 3 \times 6 = 18$ (PI) Implied by 0.0059 for P(Y ≥ 30)	AO3.3	B1	Y ~ Po(18) P(Y > 30) = P(Y ≥ 31) = 0.0033
	Identifies correct probability P(Y ≥ 31) or 1 – P(Y ≤ 30)	AO1.1a	M1	
	Obtains correct probability AWRT 0.0033	AO1.1b	A1	
8(b)	Selects binomial model with n = 6 and p their probability from **8(a)(i)** (PI)	AO3.3	B1	C ~ B(6, 0.224) P(C = 2) $= \binom{6}{2} 0.224^2 (1-0.224)^4$ = 0.273
	Calculates $0.224^2(1-0.224)^4$ with their 0.224	AO1.1a	M1	
	Obtains correct probability AWRT 0.273	AO1.1b	A1	
8(c)(i)	Selects exponential or Poisson model with λ = 3 calls per 10 minutes or equivalent (PI)	AO3.3	B1	T ~ Exp(3) P(T > 1) = $e^{-3 \times 1}$ = 0.0498
	Calculates correct probability, consistent with their exponential or Poisson model Condone confusion between mean and parameter of exponential	AO1.1a	M1	
	Obtains correct probability AWRT 0.0498	AO1.1b	A1	
8(c)(ii)	States correct probability Follow through their answer to part **8(c)(i)**	AO1.2	B1F	0.0498 Due to the memoryless property of the exponential distribution probability is unaffected by the current time since the last call
	Deduces that it is the same because of the memoryless property of the exponential distribution	AO2.2a	E1	
	Total		**12**	