

Work Done

1 (2 marks)

Work done = force $\times$ distance

M1

$$= 40 \times 12 = 480 \text{ J}$$

A1

Answer: 480 J

2 marks

2 (2 marks)

At constant speed the lifting force equals the weight: $5 \times 9.8 = 49 \text{ N}$

M1

$$\text{Work done} = 49 \times 3 = 147 \text{ J}$$

A1

Answer: 147 J

2 marks

3 (2 marks)

Work done = $F d \cos \theta$

M1

$$= 30 \times 8 \times \cos 60^\circ = 120 \text{ J}$$

A1

Answer: 120 J

2 marks

4 (2 marks)

Work done against friction = friction $\times$ distance

M1

$$= 12 \times 5 = 60 \text{ J}$$

A1

Answer: 60 J

2 marks

5 (2 marks)

The force of the hand is vertical (equal to the weight) and the displacement is horizontal

E1

Work = $F d \cos 90^\circ = 0$, because the force has no component in the direction of motion

E1

Answer: Force perpendicular to motion, so work = 0

2 marks

6 (3 marks)

Component of weight down the slope: $2 \times 9.8 \times \sin 30^\circ = 9.8 \text{ N}$

M1 A1

$$\text{Work done} = 9.8 \times 5 = 49 \text{ J}$$

A1

Answer: 49 J

3 marks

7 (4 marks)

(a) $W = T d \cos 40^\circ = 25 \times 10 \times \cos 40^\circ$

M1

$$= 192 \text{ J}$$

A1

Answer: 192 J

2 marks

(b) $12 \times 10 = 120 \text{ J}$

B1

Answer: 120 J

1 mark

(c) $192 - 120 = 71.5 \text{ J}$

A1

Answer: 71.5 J

1 mark

8 (3 marks)

Tension = weight = $250 \times 9.8 = 2450 \text{ N}$

M1 A1

Work done = $2450 \times 12 = 29400 \text{ J} = 29.4 \text{ kJ}$

A1

Answer: 29.4 kJ

3 marks

9 (5 marks)

Newton's second law: $F - 300 = 1200 \times 1.5$, so $F = 2100 \text{ N}$

M1 A1

Distance: $s = \frac{1}{2} \times 1.5 \times 8^2 = 48 \text{ m}$

M1 A1

Work done = $2100 \times 48 = 100800 \text{ J}$

A1

Answer: 100800 J

5 marks

10 (4 marks)

(a) $80 \times 6 = 480 \text{ J}$

B1

Answer: 480 J

1 mark

(b) Component of weight down the slope = $10 \times 9.8 \times \sin 20^\circ$

M1

Work = $10 \times 9.8 \times \sin 20^\circ \times 6 = 201 \text{ J}$

A1

Answer: 201 J	2 marks
(c) $15 \times 6 = 90 \text{ J}$	B1
Answer: 90 J	1 mark
11 (3 marks)	
(a) Work = $mg \times \text{vertical drop}$	M1
$= 70 \times 9.8 \times 150 = 102900 \text{ J}$	A1
Answer: 102900 J	2 marks
(b) Only the vertical displacement matters, since gravity acts vertically and the work done is mg multiplied by the vertical drop	E1
Answer: Depends on vertical drop only	1 mark
12 (5 marks)	
(a) $3 \times 3.2 = 9.6 \text{ m}$	B1
Answer: 9.6 m	1 mark
(b) Work = $mgh = 60 \times 9.8 \times 9.6$	M1
$= 5640 \text{ J}$	A1
Answer: 5640 J	2 marks
(c) Total mass 65 kg: $65 \times 9.8 \times 9.6$	M1
$= 6120 \text{ J}$	A1
Answer: 6120 J	2 marks
13 (7 marks)	
(a) Vertical: $R + F \sin 25^\circ = 12 \times 9.8$, so $R = 117.6 - F \sin 25^\circ$	M1 A1
Horizontal (constant speed): $F \cos 25^\circ = 0.3R$	M1
$F \cos 25^\circ = 0.3(117.6 - F \sin 25^\circ)$	A1
$F = 34.1$	A1
Answer: 34.1 N	5 marks
(b) $W = F d \cos 25^\circ = 34.1 \times 15 \times \cos 25^\circ$	M1
$= 464 \text{ J}$	A1
Answer: 464 J	2 marks
14 (7 marks)	
(a) $W = 5 \times 9.8 \times \sin 35^\circ \times 8$	M1
$= 225 \text{ J}$	A1
Answer: 225 J	2 marks
(b) $R = 5 \times 9.8 \times \cos 35^\circ$, so $F = 0.2R = 8.03 \text{ N}$	M1 A1
Work = $8.03 \times 8 = 64.2 \text{ J}$	A1
Answer: 64.2 J	3 marks
(c) Net work = $225 - 64.2 = 161 \text{ J} = \frac{1}{2} \times 5 \times v^2$	M1
$v = 8.02 \text{ m s}^{-1}$	A1
Answer: 8.02 m s⁻¹	2 marks
15 (4 marks)	
Component of weight along the road: $900 \times 9.8 \times \frac{1}{20} = 441 \text{ N}$	M1 A1
Driving force at constant speed: $441 + 400 = 841 \text{ N}$	M1
Work = $841 \times 2000 = 1.68 \times 10^6 \text{ J}$	A1
Answer: $1.68 \times 10^6 \text{ J}$	4 marks
16 (4 marks)	
(a) Work done = area under the graph	M1
Triangles: $\frac{1}{2} \times 2 \times 40 = 40 \text{ J}$ each, rectangle: $4 \times 40 = 160 \text{ J}$	M1
Total = $40 + 160 + 40 = 240 \text{ J}$	A1
Answer: 240 J	3 marks

(b) $\frac{240}{8} = 30 \text{ N}$

A1

Answer: 30 N

1 mark

17 (4 marks)

(a) Height gained = $5 \sin 15^\circ = 1.29 \text{ m}$

M1

Work = $30 \times 9.8 \times 1.29 = 380 \text{ J}$

A1

Answer: 380 J

2 marks

(b) Work against friction = $20 \times 5 = 100 \text{ J}$

B1

Work by tension = $380 + 100 = 480 \text{ J}$

A1

Answer: 480 J

2 marks

18 (5 marks)

(a) $270 \times 6 = 1620 \text{ J}$

B1

Answer: 1620 J

1 mark

(b) $25 \times 9.8 \times 6 = 1470 \text{ J}$

B1

Answer: 1470 J

1 mark

(c) Net work done on the load = $1620 - 1470 = 150 \text{ J}$

M1

$150 = \frac{1}{2} \times 25 \times v^2$

M1

$v = 3.46 \text{ m s}^{-1}$

A1

Answer: 3.46 m s⁻¹

3 marks

19 (5 marks)

(a) $W = F d \cos 30^\circ = 50 \times 5 \times \cos 30^\circ$

M1

$= 217 \text{ J}$

A1

Answer: 217 J

2 marks

(b) $R = 14 \times 9.8 + 50 \sin 30^\circ = 162 \text{ N}$

M1 A1

Friction = $0.2R = 32.4 \text{ N}$, so work = $32.4 \times 5 = 162 \text{ J}$

A1

Answer: 162 J

3 marks

20 (9 marks)

(a) B: $2g - T = 2a$; A: $T - 0.25 \times 3g = 3a$

M1 A1

Add: $2g - 0.75g = 5a$, so $a = 2.45 \text{ m s}^{-2}$

M1

$T = 2(9.8 - 2.45) = 14.7 \text{ N}$

A1

Answer: $a = 2.45 \text{ m s}^{-2}$, $T = 14.7 \text{ N}$

4 marks

(b) On A: $14.7 \times 1.5 = 22.05 \text{ J}$

B1

On B: -22.05 J (the tension opposes the motion of B)

B1

Answer: +22.05 J on A, -22.05 J on B

2 marks

(c) Work against friction = $0.25 \times 3 \times 9.8 \times 1.5 = 11.025 \text{ J}$

M1

Net work on A = $22.05 - 11.025 = 11.025 \text{ J}$

A1

$v^2 = 2 \times 2.45 \times 1.5 = 7.35$, so $\text{KE} = \frac{1}{2} \times 3 \times 7.35 = 11.025 \text{ J}$, equal

B1

Answer: Both equal 11.025 J

3 marks

21 (8 marks)

(a) Tension at the top supports the bucket and the hanging rope

M1

$T = 5g + 0.4x \times g = (5 + 0.4x)g$

A1

Answer: shown

2 marks

(b) Work = $\int_0^{20} (5 + 0.4x) \times 9.8 \, dx$

M1 A1

$= 9.8[5x + 0.2x^2]_0^{20}$

M1

$= 9.8 \times (100 + 80) = 1764 \text{ J}$

A1

Answer: 1764 J

4 marks

(c) Bucket: $5 \times 9.8 \times 20 = 980 \text{ J}$

M1

Rope of mass 8 kg has its centre of mass raised 10 m: $8 \times 9.8 \times 10 = 784 \text{ J}$, total 1764 J

A1

Answer: 1764 J

2 marks

22 (6 marks)

(a) Work = $\int_1^4 (3x^2 + 2x) \, dx$

M1

$$= [x^3 + x^2]_1^4$$

A1

$$= (64 + 16) - (1 + 1) = 78 \text{ J}$$

A1

Answer: 78 J

3 marks

(b) Gain in KE = 78 J

M1

$$\frac{1}{2} \times 2 \times v^2 - \frac{1}{2} \times 2 \times 2^2 = 78$$

M1

$$v^2 = 82, \text{ so } v = 9.06 \text{ m s}^{-1}$$

A1

Answer: 9.06 m s⁻¹

3 marks

23 (8 marks)

(a) Perpendicular to the plane: $R = 4g \cos 20^\circ + 50 \sin 20^\circ$

M1 A1

$$= 53.9 \text{ N}$$

A1

Answer: 53.9 N

3 marks

(b) Component of the force along the plane: $50 \cos 20^\circ$, so $W = 50 \cos 20^\circ \times 5 = 235 \text{ J}$

M1 A1

Against gravity: $4 \times 9.8 \times \sin 20^\circ \times 5 = 67 \text{ J}$

A1

Answer: 235 J and 67 J

3 marks

(c) Friction = $0.3R = 16.2 \text{ N}$, work against friction = 80.9 J

M1

$$\text{Net} = 235 - 67 - 80.9 = 87 \text{ J}$$

A1

Answer: 87 J

2 marks

24 (7 marks)

(a) Driving force = $1100 \times 9.8 \times \frac{1}{25} + 250 = 681.2 \text{ N}$

M1 A1

$$\text{Work} = 681.2 \times 3000 = 2\,043\,600 \text{ J}$$

A1

Answer: 2 043 600 J

3 marks

(b) Gravity does $-1100 \times 9.8 \times 120$ on the uphill and $+1100 \times 9.8 \times 100$ on the downhill

M1

$$\text{Net} = 1100 \times 9.8 \times (-20) = -215\,600 \text{ J}$$

A1

Answer: -215 600 J

2 marks

(c) Braking force = $1100 \times 9.8 \times \frac{1}{20} - 250 = 289 \text{ N}$

M1

$$\text{Work} = 289 \times 2000 = 578\,000 \text{ J}$$

A1

Answer: 578 000 J

2 marks

25 (9 marks)

(a) Work = area under the graph

M1

$$\frac{1}{2} \times 3 \times 24 + 4 \times 24 + \frac{1}{2} \times 3 \times 24$$

A1

$$= 168 \text{ J}$$

A1

Answer: 168 J

3 marks

(b) Work against friction = $6 \times 10 = 60 \text{ J}$

B1

$$\text{Net work} = 108 \text{ J} = \frac{1}{2} \times 5 \times v^2$$

M1

$$v = 6.57 \text{ m s}^{-1}$$

A1

Answer: 6.57 m s⁻¹

3 marks

(c) KE at $x = 10$ is 108 J

B1

Work against friction over a further distance s : $6s = 108$

M1

$$s = 18 \text{ m}$$

A1

Answer: 18 m

3 marks

26 (10 marks)

(a) $\mathbf{AB} = (6\mathbf{i} + 4\mathbf{j}) \text{ m}$

B1

$$W_1 = \mathbf{F}_1 \cdot \mathbf{AB} = 4 \times 6 + 3 \times 4 = 36 \text{ J}$$

M1 A1

$$W_2 = \mathbf{F}_2 \cdot \mathbf{AB} = -1 \times 6 + 5 \times 4 = 14 \text{ J}$$

A1

Answer: $\mathbf{AB} = (6\mathbf{i} + 4\mathbf{j}) \text{ m}$, $W_1 = 36 \text{ J}$, $W_2 = 14 \text{ J}$

4 marks

$$(b) F = F_1 + F_2 = (3\mathbf{i} + 8\mathbf{j}) \text{ N}$$

B1

$$F \cdot AB = 3 \times 6 + 8 \times 4 = 50 \text{ J}$$

M1

$$= 36 + 14, \text{ as required}$$

A1

$$\text{Answer: } 50 \text{ J} = 36 \text{ J} + 14 \text{ J}$$

3 marks

$$(c) \cos \theta = \frac{F_2 \cdot AB}{|F_2||AB|} = \frac{14}{\sqrt{26} \times \sqrt{52}}$$

M1 A1

$$\theta = 67.6^\circ$$

A1

$$\text{Answer: } 67.6^\circ$$

3 marks

27 (8 marks)

$$(a) \text{ Mass per unit length} = \frac{6}{3} = 2 \text{ kg m}^{-1}$$

M1

$$\text{Weight} = 2 \times 9.8 \times x = 19.6x \text{ N}$$

A1

$$\text{Answer: } 19.6x \text{ N}$$

2 marks

$$(b) \text{ The pulling force when } x \text{ metres hangs equals } 19.6x$$

M1

$$\text{Work} = \int_0^1 19.6x \, dx = [9.8x^2]_0^1$$

M1

$$= 9.8 \text{ J}$$

A1

$$\text{Answer: } 9.8 \text{ J}$$

3 marks

$$(c) \text{ Work} = \int_0^h \lambda g x \, dx$$

M1

$$= \frac{1}{2} \lambda g h^2$$

A1

$$h = 2: \frac{1}{2} \times 2 \times 9.8 \times 4 = 39.2 \text{ J (four times as much)}$$

A1

$$\text{Answer: } 39.2 \text{ J}$$

3 marks

28 (8 marks)

$$(a) AB = (7 - 1)\mathbf{i} + (4 - 2)\mathbf{j} = (6\mathbf{i} + 2\mathbf{j}) \text{ m}$$

B1

$$\text{Answer: } 6\mathbf{i} + 2\mathbf{j}$$

1 mark

$$(b) W = F \cdot AB = 3 \times 6 + 4 \times 2$$

M1

$$= 26 \text{ J}$$

A1

$$\text{Answer: } 26 \text{ J}$$

2 marks

$$(c) |F| = 5 \text{ and } |AB| = \sqrt{40}$$

B1

$$\cos \theta = \frac{26}{5\sqrt{40}}$$

M1

$$\theta = 34.7^\circ$$

A1

$$\text{Answer: } 34.7^\circ$$

3 marks

$$(d) AC = 6\mathbf{i}; W = 3 \times 6 = 18 \text{ J}; CB = 2\mathbf{j}; W = 4 \times 2 = 8 \text{ J}; \text{ total } 26 \text{ J}$$

M1

Work by a constant force depends only on the displacement vector AB, not the path

E1

$$\text{Answer: } 26 \text{ J}$$

2 marks

29 (10 marks)

$$(a) \text{ Treat car and trailer together: } D = 1400 \times 9.8 \times \frac{1}{20} + 450$$

M1

$$D = 686 + 450 = 1136 \text{ N}$$

A1

$$\text{Work} = 1136 \times 2000 = 2.27 \times 10^6 \text{ J}$$

A1

$$\text{Answer: } 1136 \text{ N, } 2.27 \times 10^6 \text{ J}$$

3 marks

$$(b) \text{ Trailer: } T = 400 \times 9.8 \times \frac{1}{20} + 150$$

M1 A1

$$T = 346 \text{ N, work} = 346 \times 2000 = 6.92 \times 10^5 \text{ J}$$

A1

$$\text{Answer: } 346 \text{ N, } 6.92 \times 10^5 \text{ J}$$

3 marks

$$(c) \text{ Whole system: } D - 450 - 686 = 1400 \times 0.2, \text{ so } D = 1416 \text{ N}$$

M1 A1

$$\text{Trailer: } T - 196 - 150 = 400 \times 0.2, \text{ so } T = 426 \text{ N}$$

M1

$$\text{Work} = 426 \times 500 = 2.13 \times 10^5 \text{ J}$$

A1

$$\text{Answer: } D = 1416 \text{ N, work} = 2.13 \times 10^5 \text{ J}$$

4 marks

30 (10 marks)

(a) $12x - 3x^2 = 3x(4 - x) = 0$, so $x = 0$ or $x = 4$ M1 A1

Maximum at $x = 2$ by symmetry (or $dF/dx = 12 - 6x = 0$): $F = 24 - 12 = 12$ N A1

Answer: $x = 0, 4$; greatest $F = 12$ N 3 marks

(b) Work = $\int_0^4 (12x - 3x^2) dx$ M1

$= [6x^2 - x^3]_0^4$ A1

$= 96 - 64 = 32$ J A1

Answer: 32 J 3 marks

(c) $\frac{1}{2} \times 4 \times v^2 = 32$ M1

$v = 4$ m s⁻¹ A1

Answer: 4 m s⁻¹ 2 marks

(d) Work = $[6x^2 - x^3]_0^6 = 216 - 216 = 0$ M1

Net work is zero so its kinetic energy is zero: the particle is instantaneously at rest at $x = 6$ E1

Answer: Work = 0; at rest 2 marks

Kinetic and Potential Energy

1 (2 marks)

KE = $\frac{1}{2}mv^2$ M1

$= \frac{1}{2} \times 1500 \times 20^2 = 300000$ J (300 kJ) A1

Answer: 300 kJ 2 marks

2 (2 marks)

PE = mgh M1

$= 4 \times 9.8 \times 12 = 470.4$ J A1

Answer: 470.4 J 2 marks

3 (2 marks)

$\frac{1}{2} \times 0.2 \times v^2 = 40$ M1

$v^2 = 400$, so $v = 20$ m s⁻¹ A1

Answer: 20 m s⁻¹ 2 marks

4 (2 marks)

Increase = $mg \times (\text{change in height})$ M1

$= 6 \times 9.8 \times 7 = 411.6$ J A1

Answer: 411.6 J 2 marks

5 (2 marks)

Gain = $\frac{1}{2} \times 2 \times (7^2 - 3^2)$ M1

$= 40$ J A1

Answer: 40 J 2 marks

6 (2 marks)

$0.5 \times 9.8 \times h = 14.7$ M1

$h = 3$ m A1

Answer: 3 m 2 marks

7 (4 marks)

Loss of KE = $\frac{1}{2} \times 900 \times (25^2 - 10^2)$ M1
 = 236250 J A1
 Work done by resistance = $F \times 60$ = loss of KE M1
 $F = 3937.5$ N A1
Answer: 3937.5 N 4 marks

8 (4 marks)

(a) $\frac{1}{2} \times 0.4 \times 15^2 = 45$ J B1
Answer: 45 J 1 mark
 (b) At the highest point KE = 0 and all the KE has become PE M1
 $0.4 \times 9.8 \times h = 45$ M1
 $h = 11.5$ m A1
Answer: 11.5 m 3 marks

9 (4 marks)

PE lost = $3 \times 9.8 \times 20 = 588$ J M1
 KE gained = 588 J A1
 $\frac{1}{2} \times 3 \times v^2 = 588$ M1
 $v = 19.8$ m s⁻¹ A1
Answer: 588 J, 19.8 m s⁻¹ 4 marks

10 (4 marks)

Initial KE = $\frac{1}{2} \times 0.02 \times 400^2 = 1600$ J M1 A1
 Work done by resistance: $R \times 0.25 = 1600$ M1
 $R = 6400$ N A1
Answer: 6400 N 4 marks

11 (5 marks)

(a) $\frac{1}{2} \times 80 \times (8^2 - 12^2)$ M1
 = -3200 J A1
Answer: -3200 J 2 marks
 (b) $80 \times 9.8 \times 6 = 4704$ J B1
Answer: 4704 J 1 mark
 (c) Work done by cyclist = change in KE + change in PE M1
 = -3200 + 4704 = 1504 J A1
Answer: 1504 J 2 marks

12 (4 marks)

Net force = $30 - 10 = 20$ N M1
 Net work = $20 \times 6 = 120$ J A1
 $\frac{1}{2} \times 5 \times v^2 = 120$ M1
 $v = 6.93$ m s⁻¹ A1
Answer: 6.93 m s⁻¹ 4 marks

13 (4 marks)

Start: PE = $0.5 \times 9.8 \times 10 = 49$ J, KE = 0 B1
 A: PE = $0.5 \times 9.8 \times 7 = 34.3$ J B1
 B: PE = 19.6 J and C: PE = 0 B1
 KE = $49 -$ PE at each point: 14.7 J, 29.4 J, 49 J (total energy stays 49 J) B1
Answer: See table 4 marks

	Start	A	B	C
Height above ground (m)	10	7	4	0
Potential energy (J)	49	34.3	19.6	0
Kinetic energy (J)	0	14.7	29.4	49

14 (6 marks)

$$\text{Initial KE} = \frac{1}{2} \times 4 \times 8^2 = 128 \text{ J}$$

B1

$$R = 4 \times 9.8 \times \cos 30^\circ = 33.9 \text{ N, friction } F = 0.2R = 6.79 \text{ N}$$

M1 A1

$$\text{Resistive force along the plane} = 4 \times 9.8 \times \sin 30^\circ + F = 26.4 \text{ N}$$

M1

$$\text{Work done against it over } d \text{ metres equals initial KE: } 26.4 d = 128$$

M1

$$d = 4.85 \text{ m}$$

A1

Answer: 4.85 m

6 marks

15 (6 marks)

$$\text{(a) Work done by brakes} = \text{initial KE: } 4800 \text{ J} = \frac{1}{2} \times 1200 \times 15^2$$

M1 A1

$$s = 28.125 \text{ m}$$

A1

Answer: 28.125 m

3 marks

$$\text{(b) } s = \frac{mv^2}{2F} = \frac{1200 \times 900}{9600} = 112.5 \text{ m}$$

M1 A1

KE is proportional to v^2 , so doubling the speed quadruples the KE that must be removed, and the braking force does work at the same rate per metre

E1

Answer: Four times

3 marks

16 (6 marks)

$$\text{Gain in KE} = \frac{1}{2} \times 8000 \times (12^2 - 6^2) = 432000 \text{ J}$$

M1 A1

$$\text{Work by gravity} = 8000 \times 9.8 \times 200 \times \frac{1}{40} = 392000 \text{ J}$$

M1

$$\text{Work against resistance} = 500 \times 200 = 100\,000 \text{ J}$$

B1

$$432000 = 200F + 392000 - 100\,000, \text{ so } 200F = 140000$$

M1

$$F = 700 \text{ N}$$

A1

Answer: 700 N

6 marks

17 (5 marks)

$$\text{Change in KE} = \frac{1}{2} \times 0.3 \times (3^2 - 5^2) = -2.4 \text{ J}$$

M1 A1

$$\text{Work by weight} = 0.3 \times 9.8 \times 2 = 5.88 \text{ J}$$

B1

$$5.88 - 2R = -2.4$$

M1

$$R = 4.14 \text{ N}$$

A1

Answer: 4.14 N

5 marks

18 (5 marks)

$$\text{Total distance fallen by the hammer} = 1.8 + 0.02 = 1.82 \text{ m}$$

B1

$$\text{Total PE lost} = 50 \times 9.8 \times 1.82 = 891.8 \text{ J}$$

M1

$$\text{KE is zero at the start and the end, so this equals work against resistance: } R \times 0.02$$

M1 A1

$$R = 44600 \text{ N}$$

A1

Answer: 44600 N

5 marks

19 (6 marks)

$$R = 75 \times 9.8 \times \cos 30^\circ, \text{ so friction} = 0.1R = 63.7 \text{ N}$$

M1 A1

$$\text{Work by weight component} = 75 \times 9.8 \times \sin 30^\circ \times 80 = 29400 \text{ J}$$

M1

$$\text{Work against friction} = 63.7 \times 80 = 5090 \text{ J}$$

M1

$$\text{Net work} = \frac{1}{2} \times 75 \times v^2, \text{ so } v^2 = 648$$

M1

$$v = 25.5 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 25.5 \text{ m s}^{-1}$$

6 marks

20 (6 marks)

$$\text{(a) Just before: } \frac{1}{2}mv^2 = mg \times 2, \text{ so } v = \sqrt{2 \times 9.8 \times 2} = 6.26 \text{ m s}^{-1}$$

M1 A1

$$\text{Just after: } v = \sqrt{2 \times 9.8 \times 1.2} = 4.85 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 6.26 \text{ and } 4.85 \text{ m s}^{-1}$$

3 marks

$$\text{(b) KE before} = 0.5 \times 9.8 \times 2 = 9.8 \text{ J}$$

M1

$$\text{KE after} = 0.5 \times 9.8 \times 1.2 = 5.88 \text{ J}$$

A1

$$\text{Energy lost} = 9.8 - 5.88 = 3.92 \text{ J}$$

A1

$$\text{Answer: } 3.92 \text{ J}$$

3 marks

21 (7 marks)

$$\text{(a) Per unit mass, work against friction} = 0.3 \times 9.8 \times 3 = 8.82 \text{ J kg}^{-1}$$

M1

$$\frac{1}{2}v^2 = \frac{1}{2} \times 36 - 8.82$$

M1

$$v = 4.28 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 4.28 \text{ m s}^{-1}$$

3 marks

$$\text{(b) Smooth ramp: KE converts to PE: } \frac{1}{2}v^2 = gh$$

M1

$$h = \frac{18.4}{2 \times 9.8} = 0.937 \text{ m}$$

A1

$$\text{Distance along the ramp: } d \sin 25^\circ = h$$

M1

$$d = 2.22 \text{ m}$$

A1

$$\text{Answer: } 2.22 \text{ m}$$

4 marks

22 (8 marks)

$$\text{(a) Energy: } \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgH$$

M1

$$v^2 = u^2 + 2gH \text{ depends only on } u \text{ and } H, \text{ not on the direction of the velocity (energy is a scalar)}$$

E1

$$v^2 = 144 + 2 \times 9.8 \times 15 = 438$$

M1

$$v = 20.9 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 20.9 \text{ m s}^{-1}$$

4 marks

$$\text{(b) Time depends on the vertical component of the initial velocity, which changes with direction}$$

E1

$$\text{Answer: Vertical component changes}$$

1 mark

$$\text{(c) Horizontal component remains } 12 \text{ m s}^{-1}$$

B1

$$v_y^2 = 438 - 144$$

M1

$$v_y = 17.1 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 17.1 \text{ m s}^{-1}$$

3 marks

23 (8 marks)

$$\text{(a) Initial KE} = \frac{1}{2} \times 0.2 \times 20^2 = 40 \text{ J}$$

B1

$$\text{Resistive force while rising} = \text{weight} + \text{air resistance} = 1.96 + 0.5 = 2.46 \text{ N}$$

M1

$$40 = 2.46h$$

M1

$$h = 16.3 \text{ m}$$

A1

$$\text{Answer: } 16.3 \text{ m}$$

4 marks

$$\text{(b) While falling, net downward force} = 1.96 - 0.5 = 1.46 \text{ N}$$

M1 A1

$$\frac{1}{2} \times 0.2 \times v^2 = 1.46 \times 16.3$$

M1

$$v = 15.4 \text{ m s}^{-1}$$

A1

Answer: 15.4 m s⁻¹

4 marks

24 (6 marks)

$$\text{Gain in KE} = \frac{1}{2} \times 2000 \times (20^2 - 10^2) = 300\,000 \text{ J}$$

M1 A1

$$\text{Work by gravity} = 2000 \times 9.8 \times 400 \times \frac{1}{25} = 313600 \text{ J}$$

M1

$$\text{Work by engine} = 600 \times 400 = 240\,000 \text{ J}$$

B1

$$300\,000 = 313600 + 240\,000 - 400R$$

M1

$$R = 634 \text{ N}$$

A1

Answer: 634 N

6 marks

25 (8 marks)

$$\text{(a) } \sin \alpha = \frac{3}{5}, \cos \alpha = \frac{4}{5}; \text{ resistive force per unit mass} = g(\sin \alpha + 0.25 \cos \alpha) = 7.84$$

M1 A1

$$\frac{1}{2} \times 7^2 = 7.84 d$$

M1

$$d = 3.125 \text{ m}$$

A1

Answer: 3.125 m

4 marks

(b) Weight component down the plane per unit mass $0.6g = 5.88$ exceeds the maximum friction

E1

 $0.25 \times 0.8g = 1.96$, so it slides back

$$\text{Net force per unit mass down} = 5.88 - 1.96 = 3.92$$

M1

$$\frac{1}{2}v^2 = 3.92 \times 3.125$$

M1

$$v = 4.95 \text{ m s}^{-1}$$

A1

Answer: 4.95 m s⁻¹

4 marks

26 (6 marks)**(a)** Total fall = $h + d$, so PE lost = $mg(h + d)$

M1

KE is zero at the start and at the end, so work against resistance $Rd = mg(h + d)$

M1

$$R = \frac{mg(h + d)}{d}$$

A1

Answer: shown

3 marks

$$\text{(b) } R = \frac{1.5 \times 9.8 \times 4.2}{0.2}$$

M1

$$= 308.7 \text{ N}$$

A1

Answer: 308.7 N

2 marks

(c) R increases, since the same energy must be removed over a shorter distance

E1

Answer: Increases

1 mark

27 (9 marks)

$$\text{(a) Work done by brakes} = \text{loss of KE: } Fs = \frac{1}{2}Mv^2$$

M1

$$s = \frac{Mv^2}{2F}$$

A1

Answer: shown

2 marks

(b) (i) $s \propto v^2$, so s is multiplied by 4

B1

(ii) $s \propto M$, so s is multiplied by 2

B1

Answer: ×4 and ×2

2 marks

(c) Gravity does positive work $Mg \sin \alpha \times s$ while braking does $-Fs$

M1

$$0 - \frac{1}{2}Mv^2 = Mg \sin \alpha s - Fs$$

M1

$$s = \frac{Mv^2}{2(F - Mg \sin \alpha)}$$

A1

$$F - Mg \sin \alpha = 5000 - 1200 \times 9.8 \times 0.1 = 3824$$

M1

$$s = \frac{1200 \times 400}{2 \times 3824} = 62.8 \text{ m}$$

Answer: 62.8 m

A1

5 marks

28 (9 marks)

(a) Energy from the start to the sea: $\frac{1}{2}v^2 = \frac{1}{2} \times 14^2 + 9.8 \times 30$

M1 A1

$$v^2 = 196 + 588 = 784, \text{ so } v = 28 \text{ m s}^{-1}$$

A1

Answer: shown

3 marks

(b) KE on entry = $\frac{1}{2} \times 0.5 \times 784 = 196 \text{ J}$

B1

Weight does $0.5 \times 9.8 \times 4 = 19.6 \text{ J}$ of work while sinking

M1

$$196 + 19.6 = 4R$$

M1

$$R = 53.9 \text{ N}$$

A1

Answer: 53.9 N

4 marks

(c) Energy conservation gives the same entry speed 28 m s^{-1} since $v^2 = u^2 + 2gH$ does not depend on direction

E1

The motion in the water starts from the same state, so R is the same

E1

Answer: Same entry speed

2 marks

29 (9 marks)

(a) KE just after impact = $0.64 \times \text{KE just before} = 0.64mg \times 2.5$

M1

This becomes PE: $mgh = 0.64mg \times 2.5$, so $h = 1.6 \text{ m}$

A1

Answer: shown

2 marks

(b) Each bounce multiplies the height by 0.64: $h_n = 2.5 \times 0.64^n$

B1

Answer: 2.5×0.64^n

1 mark

(c) Down 2.5 m, then up and down for each bounce

B1

$$\text{Total} = 2.5 + 2(2.5 \times 0.64 + 2.5 \times 0.64^2 + \dots)$$

M1

Geometric series with $a = 1.6$ and $r = 0.64$: $\text{sum} = \frac{1.6}{1 - 0.64} = \frac{1.6}{0.36}$

M1 A1

$$\text{Total} = 2.5 + 2 \times 4.444 = 11.39 \text{ m}$$

A1

Answer: 11.39 m

5 marks

(d) A real ball stops bouncing after a finite time and air resistance is not zero (or the energy fraction is not constant)

E1

Answer: e.g. air resistance

1 mark

30 (9 marks)

(a) $R = mg \cos \alpha$, so friction = $\mu mg \cos \alpha$

B1

Work-energy: $\frac{1}{2}mv^2 = mgL \sin \alpha - \mu mg \cos \alpha L$

M1 A1

Hence $v^2 = 2gL(\sin \alpha - \mu \cos \alpha)$, which requires $\sin \alpha > \mu \cos \alpha$, i.e. $\mu < \tan \alpha$

E1

Answer: shown

4 marks

(b) $v^2 = 2 \times 9.8 \times 50 \times (\sin 30^\circ - 0.3 \cos 30^\circ)$

M1

$$v = 15.3 \text{ m s}^{-1}$$

A1

Answer: 15.3 m s^{-1}

2 marks

(c) $100 \geq 2 \times 9.8 \times 50 \times (0.5 - \mu \cos 30^\circ)$

M1

$$0.5 - 0.866\mu \leq 0.10204, \text{ so } \mu \geq \frac{(0.5 - 0.10204)}{\cos 30^\circ}$$

M1

$$\mu \geq 0.46$$

A1

Answer: $\mu \geq 0.46$

3 marks

Conservation of Energy

1 (3 marks)

$$\text{KE gained} = \text{PE lost: } \frac{1}{2}mv^2 = mg \times 5$$

M1

$$v^2 = 2 \times 9.8 \times 5 = 98$$

A1

$$v = 9.9 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 9.9 \text{ m s}^{-1}$$

3 marks

2 (3 marks)

$$\text{Smooth slope, so mechanical energy is conserved: } \frac{1}{2}mv^2 = mg \times 3$$

M1

$$v^2 = 58.8$$

A1

$$v = 7.67 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 7.67 \text{ m s}^{-1}$$

3 marks

3 (2 marks)

$$\frac{1}{2}mu^2 = mgh$$

M1

$$h = \frac{14^2}{2 \times 9.8} = 10 \text{ m}$$

A1

$$\text{Answer: } 10 \text{ m}$$

2 marks

4 (2 marks)

$$\frac{1}{2}v^2 = g \times 0.45$$

M1

$$v = 2.97 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 2.97 \text{ m s}^{-1}$$

2 marks

5 (2 marks)

$$\frac{1}{2}mu^2 = mg \times 20$$

M1

$$u = 19.8 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 19.8 \text{ m s}^{-1}$$

2 marks

6 (3 marks)

$$\text{Energy: } \frac{1}{2}v^2 + g \times 2 = \frac{1}{2} \times 8^2$$

M1

$$v^2 = 64 - 39.2 = 24.8$$

A1

$$v = 4.98 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 4.98 \text{ m s}^{-1}$$

3 marks

7 (4 marks)

$$\text{Energy conserved: } \frac{1}{2}v^2 + g \times 10 = \frac{1}{2} \times 3^2 + g \times 40$$

M1 A1

$$v^2 = 9 + 2 \times 9.8 \times 30 = 597$$

M1

$$v = 24.4 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 24.4 \text{ m s}^{-1}$$

4 marks

8 (3 marks)

$$\frac{1}{2}v^2 + g \times 8 = \frac{1}{2} \times 20^2$$

M1 A1

$$v = \sqrt{400 - 156.8} = 15.6 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 15.6 \text{ m s}^{-1}$$

3 marks

9 (4 marks)

$$\text{Vertical drop} = 6 \sin 25^\circ = 2.54 \text{ m}$$

M1 A1

$$\frac{1}{2}v^2 = g \times 6 \sin 25^\circ$$

M1

$$v = 7.05 \text{ m s}^{-1}$$

A1

Answer: 7.05 m s⁻¹

4 marks

10 (5 marks)

$$\text{(a) Drop} = 1.2 \text{ m: } \frac{1}{2}v^2 = g \times 1.2$$

M1

$$v = 4.85 \text{ m s}^{-1}$$

A1

Answer: 4.85 m s⁻¹

2 marks

$$\text{(b) Drop} = 1.2 - 1.2 \cos 60^\circ = 0.6 \text{ m}$$

M1 A1

$$v = \sqrt{2 \times 9.8 \times 0.6} = 3.43 \text{ m s}^{-1}$$

A1

Answer: 3.43 m s⁻¹

3 marks

11 (5 marks)

$$\text{(a) PE lost} = 2 \times 9.8 \times 3 = 58.8 \text{ J}$$

B1

$$\text{KE gained} = \frac{1}{2} \times 2 \times 5^2 = 25 \text{ J}$$

B1

$$\text{Energy lost} = 58.8 - 25 = 33.8 \text{ J}$$

A1

Answer: 33.8 J

3 marks

$$\text{(b) Work done against friction} = F \times 6 = 33.8$$

M1

$$F = 5.63 \text{ N}$$

A1

Answer: 5.63 N

2 marks

12 (4 marks)

$$\text{Total mechanical energy} = \frac{1}{2} \times 2 \times 12^2 = 144 \text{ J}$$

M1 A1

$$\text{When KE} = \text{PE, each equals } 72 \text{ J}$$

M1

$$2 \times 9.8 \times h = 72, \text{ so } h = 3.67 \text{ m}$$

A1

Answer: 3.67 m

4 marks

13 (5 marks)

$$\text{Loss of PE of the 5 kg mass} = 5 \times 9.8 \times 2 = 98 \text{ J}$$

M1

$$\text{Gain in PE of the 3 kg mass} = 3 \times 9.8 \times 2 = 58.8 \text{ J}$$

M1

$$\text{Net PE lost} = 39.2 \text{ J} = \text{KE gained} = \frac{1}{2}(5 + 3)v^2$$

M1 A1

$$v = 3.13 \text{ m s}^{-1}$$

A1

Answer: 3.13 m s⁻¹

5 marks

14 (5 marks)

$$\text{Only B changes its PE: loss} = 2 \times 9.8 \times 1.5 = 29.4 \text{ J}$$

M1 A1

$$\text{Both particles gain KE: } \frac{1}{2}(4 + 2)v^2$$

M1

$$3v^2 = 29.4$$

A1

$$v = 3.13 \text{ m s}^{-1}$$

A1

Answer: 3.13 m s⁻¹

5 marks

15 (5 marks)

$$\text{Energy: } \frac{1}{2} \times 3^2 = g h, \text{ so } h = 0.4592 \text{ m}$$

M1 A1

$$h = 1.5 - 1.5 \cos \theta$$

M1

$$\cos \theta = 1 - \frac{0.4592}{1.5} = 0.6939$$

A1

$$\theta = 46.1^\circ$$

A1

Answer: 46.1°

5 marks

16 (5 marks)

$$R = mg \cos 20^\circ, \text{ friction} = 0.15mg \cos 20^\circ$$

B1

$$\text{Work by weight} = mg \times 5 \sin 20^\circ; \text{ work against friction} = 0.15mg \cos 20^\circ \times 5$$

M1

$$\frac{1}{2}mv^2 = 5mg(\sin 20^\circ - 0.15 \cos 20^\circ)$$

M1 A1

$$v = 4.44 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 4.44 \text{ m s}^{-1}$$

5 marks

17 (7 marks)

$$(a) \frac{1}{2}v^2 = \frac{1}{2} \times 15^2 + 9.8 \times 10$$

M1

$$v = 20.5 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 20.5 \text{ m s}^{-1}$$

2 marks

$$(b) \text{ The horizontal component } 15 \cos 40^\circ \text{ is constant and the vertical component is zero at the top}$$

M1

$$\text{Speed} = 11.5 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 11.5 \text{ m s}^{-1}$$

2 marks

$$(c) \text{ Energy: } \frac{1}{2}(15 \sin 40^\circ)^2 = g h \text{ gives } h \text{ above the start}$$

M1

$$h = \frac{(15 \sin 40^\circ)^2}{2 \times 9.8} = 4.74 \text{ m}$$

A1

$$\text{Height above ground} = 14.7 \text{ m}$$

A1

$$\text{Answer: } 14.7 \text{ m}$$

3 marks

18 (5 marks)

$$(a) \text{ PE lost} = 85 \times 9.8 \times 25 = 20825 \text{ J}$$

M1

$$\text{KE gained} = \frac{1}{2} \times 85 \times 15^2 = 9562.5 \text{ J}$$

M1

$$\text{Energy lost} = 11262.5 \text{ J}$$

A1

$$\text{Answer: } 11262.5 \text{ J}$$

3 marks

$$(b) \text{ Work against resistance} = R \times 400$$

M1

$$R = \frac{11262.5}{400} = 28.2 \text{ N}$$

A1

$$\text{Answer: } 28.2 \text{ N}$$

2 marks

19 (5 marks)

$$\text{Loss of KE} = \frac{1}{2} \times 600 \times 3^2 = 2700 \text{ J}$$

M1 A1

$$\text{Loss of PE} = 600 \times 9.8 \times 2 = 11\,760 \text{ J}$$

M1

$$\text{Work done by tension (against motion)} = 2700 + 11\,760 = 2T$$

M1

$$T = 7230 \text{ N}$$

A1

$$\text{Answer: } 7230 \text{ N}$$

5 marks

20 (9 marks)

$$(a) \text{ PE lost by B} = 4 \times 9.8 \times 1.2 = 47.04 \text{ J}; \text{ PE gained by A} = 6 \times 9.8 \times 1.2 \sin 30^\circ = 35.28 \text{ J}$$

M1 A1

$$\text{Net} = 11.76 \text{ J} = \frac{1}{2}(6 + 4)v^2$$

M1

$$v = 1.53 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 1.53 \text{ m s}^{-1}$$

4 marks

$$(b) \text{ Driving force} = 4g - 6g \sin 30^\circ = 39.2 - 29.4 = 9.8 \text{ N}$$

M1 A1

$$\text{Maximum friction} = 0.2 \times 6g \cos 30^\circ = 10.2 \text{ N, which exceeds } 9.8 \text{ N, so there is no motion}$$

E1

$$\text{Answer: No motion}$$

3 marks

$$(c) \mu \times 6 \times 9.8 \cos 30^\circ \geq 9.8$$

M1

$$\mu \geq 0.192$$

A1

$$\text{Answer: } \mu \geq 0.192$$

2 marks

21 (7 marks)**(a)** At the highest point the speed is the horizontal component $u \cos \theta$

B1

$$\frac{1}{2}m(u \cos \theta)^2 + mgH = \frac{1}{2}mu^2$$

M1

$$gH = \frac{1}{2}u^2(1 - \cos^2\theta) = \frac{1}{2}u^2 \sin^2\theta, \text{ so } H = \frac{u^2 \sin^2\theta}{2g}$$

A1

Answer: shown

3 marks

$$\text{(b)} H = \frac{625 \times \sin^2 40^\circ}{2 \times 9.8}$$

M1

$$= 13.2 \text{ m}$$

A1

Answer: 13.2 m

2 marks

$$\text{(c)} \frac{1}{2}v^2 + g \times 6 = \frac{1}{2} \times 625$$

M1

$$v = 22.5 \text{ m s}^{-1}$$

A1

Answer: 22.5 m s⁻¹

2 marks

22 (8 marks)**(a)** Work by weight of B = $2 \times 9.8 \times 1.5 = 29.4 \text{ J}$

M1

Work against friction = $0.4 \times 3 \times 9.8 \times 1.5 = 17.64 \text{ J}$

M1 A1

$$29.4 - 17.64 = \frac{1}{2} \times 5 \times v^2$$

M1

$$v^2 = 4.704, \text{ so } v = 2.17 \text{ m s}^{-1}$$

A1

Answer: 2.17 m s⁻¹

5 marks

(b) After B lands the string is slack; friction decelerates A at $0.4g = 3.92 \text{ m s}^{-2}$

M1

$$0 = v^2 - 2 \times 3.92 \times s$$

M1

$$s = 0.6 \text{ m}$$

A1

Answer: 0.6 m

3 marks

23 (7 marks)**(a)** Energy: PE lost = work against friction

M1

$$mg \times 1.5 = 0.3mg \times d$$

M1

$$d = 5 \text{ m}$$

A1

Answer: 5 m

3 marks

(b) Work against friction over 3 m = $0.3 \times 2 \times 9.8 \times 3 = 17.64 \text{ J}$

M1 A1

Energy remaining as PE: $2 \times 9.8 \times 1.5 - 17.64 = 11.76 \text{ J}$

M1

$$2 \times 9.8 \times h = 11.76, \text{ so } h = 0.6 \text{ m}$$

A1

Answer: 0.6 m

4 marks

24 (7 marks)

$$\text{(a)} \text{ Energy: } \frac{1}{2}u^2 = \frac{1}{2} \times 14^2 + 9.8 \times 8$$

M1 A1

$$u^2 = 352.8, u = 18.8 \text{ m s}^{-1}$$

A1

Answer: 18.8 m s⁻¹

3 marks

(b) At the highest point the speed is $u \cos \theta = 10$

M1

$$\theta = 57.8^\circ$$

A1

Answer: 57.8°

2 marks

$$\text{(c)} \frac{1}{2} \times 10^2 + gH = \frac{1}{2}u^2$$

M1

$$H = 12.9 \text{ m}$$

A1

Answer: 12.9 m

2 marks

25 (7 marks)**(a)** PE lost = $50 \times 9.8 \times 3 = 1470 \text{ J}$

B1

$$\text{KE gained} = \frac{1}{2} \times 50 \times 6^2 = 900 \text{ J}$$

M1

Energy lost = 570 J

A1

Answer: 570 J

3 marks

(b) Work against resistance = $F \times 5 = 570$

M1

$F = 114 \text{ N}$

A1

Answer: 114 N

2 marks

(c) $\frac{1}{2}mv^2 = \mu mg \times 9$, so $18 = 88.2\mu$

M1

$\mu = 0.204$

A1

Answer: 0.204

2 marks

26 (10 marks)

(a) PE lost by B = $3 \times 9.8 \times 1 = 29.4 \text{ J}$, PE gained by A = $2 \times 9.8 \times 1 \times \sin 30^\circ = 9.8 \text{ J}$

M1 A1

$$19.6 = \frac{1}{2} \times 5 \times v^2$$

M1

$$v^2 = 7.84, v = 2.8 \text{ m s}^{-1}$$

A1

Answer: shown

4 marks

(b) String slack: A decelerates at $g \sin 30^\circ = 4.9 \text{ m s}^{-2}$

M1

$$0 = 2.8^2 - 2 \times 4.9 \times s$$

M1

$$s = 0.8 \text{ m}$$

A1

Answer: 0.8 m

3 marks

(c) First stage: $a = \frac{3g - 2g \sin 30^\circ}{5} = 3.92$, so $t_1 = \frac{2.8}{3.92} = 0.714 \text{ s}$

M1 A1

Second stage: $t_2 = \frac{2.8}{4.9} = 0.571 \text{ s}$, total = 1.29 s

A1

Answer: 1.29 s

3 marks

27 (9 marks)

(a) $\frac{1}{2} \times 70 \times 9^2 = 2835 \text{ J}$

B1

Answer: 2835 J

1 mark

(b) $70 \times 9.8 \times h = 2835$

M1

$$h = 4.13 \text{ m}$$

A1

Answer: 4.13 m

2 marks

(c) Energy available = $0.8 \times 2835 = 2268 \text{ J}$

M1 A1

$$\text{Energy at the top: } mgh + \frac{1}{2} \times 70 \times 1.2^2 = 2268$$

M1

$$h = 3.23 \text{ m}$$

A1

Answer: 3.23 m

4 marks

(d) No energy lost, for example to the flexing of the pole or to air resistance

E1

All the kinetic energy (including the horizontal component) can be converted to potential energy, and the vaulter is a particle

E1

Answer: No losses; all KE converted

2 marks

28 (10 marks)

(a) Height above the x-axis is $y = \frac{x^2}{4}$, so the drop from A is $4 - \frac{x^2}{4}$

M1

$$\frac{1}{2}v^2 = g(4 - \frac{x^2}{4})$$

M1 A1

$$v^2 = g(8 - \frac{x^2}{2}) = \frac{g}{2}(16 - x^2)$$

A1

Answer: shown

4 marks

(b) $x = 2: v^2 = \frac{9.8}{2}(12) = 58.8$, $v = 7.67 \text{ m s}^{-1}$

M1 A1

$$x = 0: v^2 = \frac{9.8}{2} \times 16 = 78.4, v = 8.85 \text{ m s}^{-1}$$

A1

Answer: 7.67 and 8.85 m s⁻¹

3 marks

(c) Total energy = $4mg$

B1

$$KE = PE = 2mg, \text{ so } y = 2$$

M1

$$\frac{x^2}{4} = 2, x = \sqrt{8} = 2.83, \text{ so the point is } (2.83, 2)$$

A1

Answer: (2.83, 2)**3 marks****29 (11 marks)****(a)** Work by weight of B = $1 \times 9.8 \times 0.8 = 7.84 \text{ J}$ **B1**Work against friction = $\mu \times 4 \times 9.8 \times 0.8 = 31.36\mu$ **M1**

$$\frac{1}{2}(4 + 1)v^2 = 7.84 - 31.36\mu$$

A1**Answer: shown****3 marks****(b)** A decelerates at μg : $0 = v^2 - 2\mu g \times 0.6$ **M1**

$$v^2 = 11.76\mu$$

A1**Answer: $v^2 = 11.76\mu$** **2 marks****(c)** $2.5 \times 11.76\mu = 7.84 - 31.36\mu$ **M1**

$$60.76\mu = 7.84$$

A1

$$\mu = 0.129$$

A1**Answer: 0.129****3 marks****(d)** System: $a = \frac{1 \times 9.8 - \mu \times 4 \times 9.8}{5} = 0.948 \text{ m s}^{-2}$ **M1 A1**B: $1 \times 9.8 - T = 1 \times a$, so $T = 8.85 \text{ N}$ **A1****Answer: 8.85 N****3 marks****30 (12 marks)****(a)** Length of string BP = $\sqrt{0.6^2 + 0.8^2} = 1 \text{ m}$ **B1**C has risen $1 - 0.8 = 0.2 \text{ m}$ **B1**The component of the velocity of B along the string is $v \cos \phi = 0.6v$, which is the speed of C**B1****Answer: shown****3 marks****(b)** Loss of PE of B = $0.2 \times 9.8 \times 0.6$; gain of PE of C = $0.5 \times 9.8 \times 0.2$ **M1**

$$\text{Energy: } 0.2g \times 0.6 - 0.5g \times 0.2 = \frac{1}{2} \times 0.2v^2 + \frac{1}{2} \times 0.5 \times (0.6v)^2$$

M1 A1

$$0.196 = 0.19v^2 \text{ so } v = 1.02 \text{ m s}^{-1}$$

A1**Answer: 1.02 m s^{-1}** **4 marks****(c)** Let the bead descend $d \text{ m}$: the string BP has length $\sqrt{d^2 + 0.64}$, so C rises $\sqrt{d^2 + 0.64} - 0.8$ **B1**At rest KE = 0, so $0.2g d = 0.5g(\sqrt{d^2 + 0.64} - 0.8)$ **M1**

$$\sqrt{d^2 + 0.64} = 0.4d + 0.8$$

A1

$$d^2 + 0.64 = 0.16d^2 + 0.64d + 0.64, \text{ so } 0.84d^2 = 0.64d$$

M1

$$d = \frac{0.64}{0.84} = \frac{16}{21} \text{ m (d = 0 is the starting position)}$$

A1**Answer: $\frac{16}{21} \text{ m}$** **5 marks****Power****1 (2 marks)**

$$P = Fv$$

M1

$$= 800 \times 15 = 12\,000 \text{ W} = 12 \text{ kW}$$

A1**Answer: 12 kW****2 marks****2 (2 marks)**

$$\text{Power} = \frac{\text{work done}}{\text{time}}$$

M1

$$= \frac{360}{20} = 18 \text{ kW}$$

A1**Answer: 18 kW****2 marks**

3 (3 marks)

At maximum speed the acceleration is zero, so driving force = resistance

M1

$$F = \frac{P}{v} = \frac{40\,000}{20}$$

M1

$$R = 2000 \text{ N}$$

A1

Answer: 2000 N

3 marks

4 (3 marks)At constant speed, force = weight = $80 \times 9.8 = 784 \text{ N}$

M1 A1

$$P = 784 \times 0.5 = 392 \text{ W}$$

A1

Answer: 392 W

3 marks

5 (4 marks)

$$\text{Driving force } F = \frac{30\,000}{10} = 3000 \text{ N}$$

M1 A1

$$\text{N2L: } 3000 - 800 = 1200a$$

M1

$$a = 1.83 \text{ m s}^{-2}$$

A1

Answer: 1.83 m s⁻²

4 marks

6 (4 marks)

$$\text{Constant speed: } F = 500 + 900 \times 9.8 \times \frac{1}{15}$$

M1 A1

$$F = 500 + 588 = 1088 \text{ N}$$

A1

$$P = 1088 \times 18 = 19584 \text{ W}$$

A1

Answer: 19584 W (19.6 kW)

4 marks

7 (4 marks)At maximum speed, driving force = resistance: $\frac{P}{v} = 40v$

M1 A1

$$P = 40v^2, \text{ so } v^2 = \frac{36\,000}{40} = 900$$

M1

$$v = 30 \text{ m s}^{-1}$$

A1

Answer: 30 m s⁻¹

4 marks

8 (5 marks)(a) At maximum speed $F = R = 1500 \text{ N}$

M1

$$v = \frac{60\,000}{1500} = 40 \text{ m s}^{-1}$$

A1

Answer: 40 m s⁻¹

2 marks

$$\text{(b) } F = \frac{60\,000}{20} = 3000 \text{ N}$$

B1

$$3000 - 1500 = 6000a$$

M1

$$a = 0.25 \text{ m s}^{-2}$$

A1

Answer: 0.25 m s⁻²

3 marks

9 (4 marks)

Per second the pump raises 10 kg of water

B1

$$\text{PE gained per second} = 10 \times 9.8 \times 12 = 1176 \text{ J}$$

M1

$$\text{KE gained per second} = \frac{1}{2} \times 10 \times 4^2 = 80 \text{ J}$$

M1

$$\text{Power} = 1176 + 80 = 1256 \text{ W}$$

A1

Answer: 1256 W

4 marks

10 (4 marks)

$$\text{Useful (output) power} = 150 \times 9.8 \times 0.6 = 882 \text{ W}$$

M1 A1

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}, \text{ so input} = \frac{882}{0.8}$$

M1

$$= 1102.5 \text{ W}$$

A1

Answer: 1102.5 W

4 marks

11 (4 marks)Driving force at constant speed: $F = 70 \times 9.8 \times 0.04 + 15$

M1

$$F = 42.44 \text{ N}$$

A1

$$v = \frac{P}{F} = \frac{250}{42.44}$$

M1

$$= 5.89 \text{ m s}^{-1}$$

A1

Answer: 5.89 m s⁻¹

4 marks

12 (7 marks)

(a) Driving force = $\frac{36\,000}{12} = 3000 \text{ N}$

B1

Whole system: $3000 - 500 = 1600a$

M1 A1

$$a = 1.5625 \text{ m s}^{-2}$$

A1

Answer: 1.5625 m s⁻²

4 marks

(b) Caravan: $T - 200 = 600a$

M1

$$T = 200 + 600 \times 1.5625$$

A1

$$= 1137.5 \text{ N}$$

A1

Answer: 1137.5 N

3 marks

13 (9 marks)

(a) Level: $P = 30R$

B1

Hill: $P = 20(R + 1000 \times 9.8 \times \frac{1}{20}) = 20(R + 490)$

M1 A1

$$30R = 20R + 9800, \text{ so } R = 980 \text{ N}$$

A1

$$P = 30 \times 980 = 29400 \text{ W}$$

A1

Answer: R = 980 N, P = 29400 W

5 marks

(b) Driving force = $\frac{29\,400}{20} = 1470 \text{ N}$

B1

Down the hill: $1470 + 490 - 980 = 1000a$

M1 A1

$$a = 0.98 \text{ m s}^{-2}$$

A1

Answer: 0.98 m s⁻²

4 marks

14 (7 marks)

(a) (i) $F = 4500 \text{ N}$, so $4500 - 1000 = 1500a$, $a = 2.333 \text{ m s}^{-2}$

M1 A1

(ii) $F = 2250 \text{ N}$, so $2250 - 1000 = 1500a$, $a = 0.833 \text{ m s}^{-2}$

M1 A1

Answer: 2.333 and 0.833 m s⁻²

4 marks

(b) $a = 0$: $F = R = 1000 \text{ N}$

M1

$$v = \frac{45\,000}{1000} = 45 \text{ m s}^{-1}$$

A1

Answer: 45 m s⁻¹

2 marks

(c) Driving force = $\frac{P}{v}$ decreases as v increases, so the net force decreases

E1

Answer: F = P/v decreases

1 mark

15 (5 marks)

Gain in KE = $\frac{1}{2} \times 5000 \times (15^2 - 5^2) = 500000 \text{ J}$

M1 A1

Work done against resistance = $2000 \times 200 = 400\,000 \text{ J}$

B1

Work done by engine = $40\,000t = 500000 + 400\,000$

M1

$$t = 22.5 \text{ s}$$

A1

Answer: 22.5 s

5 marks

16 (7 marks)

(a) Constant speed: resistance = component of weight down the slope

M1

$$R = 4000 \times 9.8 \times \frac{1}{20} = 1960 \text{ N}$$

A1

Answer: shown

2 marks

(b) Driving force = $1960 + 1960 = 3920 \text{ N}$

M1 A1

$P = 3920 \times 20 = 78400 \text{ W}$

A1

Answer: 78400 W

3 marks

(c) $v = \frac{60\,000}{3920}$

M1

$= 15.3 \text{ m s}^{-1}$

A1

Answer: 15.3 m s⁻¹

2 marks

17 (7 marks)

(a) $T - 500 \times 9.8 = 500 \times 0.4$, so $T = 5100 \text{ N}$

M1 A1

$v = 0 + 0.4 \times 5 = 2 \text{ m s}^{-1}$

B1

Answer: T = 5100 N, v = 2 m s⁻¹

3 marks

(b) $P = Tv = 5100 \times 2 = 10200 \text{ W}$

A1

Answer: 10200 W

1 mark

(c) Distance = $\frac{1}{2} \times 0.4 \times 5^2 = 5 \text{ m}$

B1

Work done = $5100 \times 5 = 25500 \text{ J}$

M1

Average power = $\frac{25500}{5} = 5100 \text{ W}$

A1

Answer: 5100 W

3 marks

18 (6 marks)

(a) Volume per second = $5 \times 10^{-4} \times 8 = 4 \times 10^{-3} \text{ m}^3$

M1

Mass per second = $1000 \times 4 \times 10^{-3} = 4 \text{ kg}$

A1

Answer: shown

2 marks

(b) $\frac{1}{2} \times 4 \times 8^2$

M1

$= 128 \text{ W}$

A1

Answer: 128 W

2 marks

(c) Raising: $4 \times 9.8 \times 5 = 196 \text{ W}$

M1

Total = $128 + 196 = 324 \text{ W}$

A1

Answer: 324 W

2 marks

19 (9 marks)

(a) At maximum speed: $\frac{P}{v} = kv$, so $50\,000 = k \times 40^2$

M1

$k = 31.25$

A1

Answer: shown

2 marks

(b) $F = \frac{50\,000}{20} = 2500 \text{ N}$, $R = 31.25 \times 20 = 625 \text{ N}$

M1 A1

$a = \frac{2500 - 625}{1500} = 1.25 \text{ m s}^{-2}$

A1

Answer: 1.25 m s⁻²

3 marks

(c) Steady speed: $\frac{P}{v} = 31.25v + 1500 \times 9.8 \times \frac{1}{20} = 31.25v + 735$

M1 A1

Multiply by v: $31.25v^2 + 735v - 50\,000 = 0$

A1

$v = \frac{-735 + \sqrt{735^2 + 4 \times 31.25 \times 50\,000}}{62.5} = 29.9 \text{ m s}^{-1}$

A1

Answer: 29.9 m s⁻¹

4 marks

20 (9 marks)

(a) Force needed to pull the three trucks at constant speed: $3 \times (10\,000 \times 9.8 \times \frac{1}{80} + 500)$

M1 A1

$= 3 \times 1725 = 5175 \text{ N}$

A1

Answer: 5175 N

3 marks

(b) Two trucks behind: 2×1725

M1

$= 3450 \text{ N}$

A1

Answer: 3450 N

2 marks

(c) Locomotive: $D = T_1 + 2000 + 40\,000 \times 9.8 \times \frac{1}{80}$

M1 A1

$$= 5175 + 2000 + 4900 = 12075 \text{ N}$$

A1

$$P = 12075 \times 12 = 144900 \text{ W (144.9 kW)}$$

A1

Answer: 144900 W

4 marks

21 (7 marks)

(a) Tension T: $T - 800 \times 9.8 - 400 = 800 \times 0.5$

M1 A1

$$T = 8640 \text{ N}$$

A1

$$v = \frac{12\,000}{8640} = 1.39 \text{ m s}^{-1}$$

A1

Answer: 1.39 m s⁻¹

4 marks

(b) At maximum speed $a = 0$: $T = 7840 + 400 = 8240 \text{ N}$

M1 A1

$$v = \frac{12\,000}{8240} = 1.46 \text{ m s}^{-1}$$

A1

Answer: 1.46 m s⁻¹

3 marks

22 (8 marks)

(a) Driving force = $\frac{14\,000}{10} = 1400 \text{ N}$

B1

$$1400 = R + 1200 \times 9.8 \times \frac{1}{20} = R + 588$$

M1

$$R = 812 \text{ N}$$

A1

Answer: 812 N

3 marks

(b) Down the hill: $F + 588 = R$, so $F = 812 - 588 = 224 \text{ N}$

M1 A1

$$P = 224 \times 10 = 2240 \text{ W}$$

A1

Answer: 2240 W

3 marks

(c) $F = R = 812 \text{ N}$

M1

$$v = \frac{14\,000}{812} = 17.2 \text{ m s}^{-1}$$

A1

Answer: 17.2 m s⁻¹

2 marks

23 (6 marks)

$$\text{Gain in KE} = \frac{1}{2} \times 8000 \times (12^2 - 8^2) = 320000 \text{ J}$$

M1 A1

$$\text{Gain in PE} = 8000 \times 9.8 \times 250 \times \frac{1}{40} = 490000 \text{ J}$$

M1 A1

$$\text{Work against resistance} = 1500 \times 250 = 375000 \text{ J}$$

B1

$$60\,000t = 320000 + 490000 + 375000, \text{ so } t = 19.75 \text{ s}$$

A1

Answer: 19.75 s

6 marks

24 (6 marks)

(a) Resistance $R = kv^2$

B1

$$\text{At constant speed } P = Rv = kv^3$$

A1

Answer: shown

2 marks

(b) $P \propto v^3$: $P = 8 \times \left(\frac{30}{20}\right)^3$

M1

$$= 27 \text{ kW}$$

A1

Answer: 27 kW

2 marks

(c) $\frac{v^3}{20^3} = \frac{40}{8} = 5$

M1

$$v = 20 \times 5^{1/3} = 34.2 \text{ m s}^{-1}$$

A1

Answer: 34.2 m s⁻¹

2 marks

25 (10 marks)**(a)** No pedalling and constant speed, so resistance = component of weight down the slope

M1

$$80 \times 9.8 \times \frac{1}{25} = 31.36 = k \times 7^2$$

M1

$$k = 0.64$$

A1

Answer: $k = 0.64$

3 marks

(b) Driving force = resistance + weight component = $31.36 + 31.36 = 62.72$ N

M1 A1

$$\text{Power} = 62.72 \times 7 = 439 \text{ W}$$

A1

Answer: 439 W

3 marks

(c) $500 = Fv$ where $F = 0.64v^2 + 31.36$

M1 A1

$$0.64v^3 + 31.36v - 500 = 0$$

A1

Solving numerically, $v = 7.46 \text{ m s}^{-1}$

A1

Answer: 7.46 m s^{-1}

4 marks

26 (7 marks)**(a)** Driving force $F + 20 = 75 \times 9.8 \times \sin 5^\circ + 30$

M1 A1

$$F = 64.06 + 10 = 74.06 \text{ N}$$

A1

$$\text{Power} = 74.06 \times 3 = 222 \text{ W}$$

A1

Answer: 222 W

4 marks

(b) $F = \frac{300}{v}$ and $F + 20 = 64.06 + 30$

M1

$$\frac{300}{v} = 74.06$$

A1

$$v = 4.05 \text{ m s}^{-1}$$

A1

Answer: 4.05 m s^{-1}

3 marks

27 (10 marks)**(a)** Driving force = $\frac{P}{v}$; N2L: $\frac{20\,000}{v} - 800 = 1000 \frac{dv}{dt}$

M1

Multiply by v

A1

Answer: shown

2 marks

(b) Separate the variables: $dv = \frac{1000v}{20\,000 - 800v} dt$

M1

Integrate between $v = 5$ and $v = 15$

A1

Answer: shown

2 marks

(c) Write $\frac{v}{20\,000 - 800v} = -\frac{1}{800} + \frac{\frac{25}{800}}{25 - v}$

M1 A1

$$\text{Integrate: } -\frac{v}{800} - \frac{25}{800} \ln(25 - v) \text{ [for } v < 25]$$

M1 A1

$$t = 1000 \left[-\frac{v}{800} - \frac{25}{800} \ln(25 - v) \right] \text{ from } v = 5 \text{ to } v = 15$$

M1

$$t = 1000 \left(-0.0125 + 0.03125 \ln \frac{20}{10} \right) = 1000(-0.0125 + 0.03125 \ln 2) = 9.16 \text{ s}$$

A1

Answer: 9.16 s

6 marks

28 (12 marks)**(a)** Power = Fv reaches 60 kW when $v = \frac{60\,000}{4000} = 15 \text{ m s}^{-1}$

M1 A1

$$a = \frac{4000 - 1000}{1500} = 2 \text{ m s}^{-2}$$

M1 A1

$$t = \frac{15}{2} = 7.5 \text{ s}$$

A1

Answer: 7.5 s

5 marks

(b) At the maximum speed $F = R = 1000$ N, with $P = 60$ kW

M1

$$v = \frac{60\,000}{1000} = 60 \text{ m s}^{-1}$$

A1

Answer: 60 m s^{-1}

2 marks

$$(c) \text{ Gain in KE} = \frac{1}{2} \times 1500 \times (30^2 - 15^2) = 506250 \text{ J}$$

M1 A1

$$\text{Work against resistance} = 1000 \times 300 = 300\,000 \text{ J}$$

B1

$$60\,000t = 506250 + 300\,000$$

M1

$$t = 13.44 \text{ s}$$

A1

Answer: 13.44 s

5 marks

29 (11 marks)

$$(a) v = \int_0^t (6 - s) \, ds$$

M1

$$= [6s - \frac{1}{2}s^2]_0^t$$

A1

$$= 6t - \frac{1}{2}t^2$$

A1

Answer: shown

3 marks

$$(b) \text{ At } t = 3: a = 3 \text{ and } v = 18 - 4.5 = 13.5$$

M1 A1

$$P = 80 \times 3 \times 13.5 = 3240 \text{ W}$$

A1

Answer: 3240 W

3 marks

$$(c) P = 80(6 - t)(6t - \frac{1}{2}t^2) = 80(36t - 9t^2 + \frac{1}{2}t^3)$$

M1 A1

$$\frac{dP}{dt} = 80(36 - 18t + \frac{3}{2}t^2) = 0, \text{ so } t^2 - 12t + 24 = 0$$

M1

$$t = 6 - \sqrt{12} = 2.536 \text{ (the root in } [0, 6])$$

A1

$$P_{\max} = 3330 \text{ W}$$

A1

Answer: 3330 W

5 marks

30 (10 marks)

$$(a) \text{ Up: } \frac{P}{10} = 10k + 2000 \times 9.8 \times \frac{1}{20} = 10k + 980$$

M1 A1

$$\text{Down: } \frac{P}{20} + 980 = 20k$$

M1 A1

$$\text{Eliminate } P: 10(10k + 980) = 20(20k - 980), \text{ so } 300k = 29\,400, k = 98; P = 19\,600 \text{ W}$$

A1

Answer: k = 98, P = 19 600 W

5 marks

$$(b) P = kv^2: 98v^2 = 19\,600$$

M1

$$v = \sqrt{200} = 14.1 \text{ m s}^{-1}$$

A1

Answer: 14.1 m s⁻¹

2 marks

$$(c) \text{ Steady speed: } kv = 2000 \times 9.8 \times \frac{1}{20} = 980$$

M1

$$v = \frac{980}{98} = 10 \text{ m s}^{-1}$$

A1

Answer: 10 m s⁻¹

2 marks

$$(d) \text{ Dividing by 98: } v^2 + 10v - 200 = (v - 10)(v + 20) = 0, \text{ so } v = 10$$

E1

Answer: Consistent

1 mark

Uniform Circular Motion**1 (3 marks)**

$$\omega = \frac{2\pi \times 5}{20} = \frac{\pi}{2} = 1.571 \text{ rad s}^{-1}$$

M1 A1

$$v = r\omega = 0.8 \times 1.571 = 1.26 \text{ m s}^{-1}$$

A1

Answer: $\omega = 1.571 \text{ rad s}^{-1}$, $v = 1.26 \text{ m s}^{-1}$

3 marks

2 (2 marks)

$$a = \frac{v^2}{r}$$

M1

$$= \frac{64}{2} = 32 \text{ m s}^{-2}, \text{ directed towards the centre}$$

A1

Answer: 32 m s⁻² towards the centre

2 marks

3 (2 marks)

$$T = \frac{2\pi}{\omega}$$

M1

$$= \frac{2\pi}{3} = 2.09 \text{ s}$$

A1

Answer: 2.09 s

2 marks

4 (2 marks)

$$(a) a = \frac{v^2}{r} = \frac{100}{50} = 2 \text{ m s}^{-2}$$

B1

Answer: 2 m s⁻²

1 mark

$$(b) F = ma = 1200 \times 2 = 2400 \text{ N}$$

B1

Answer: 2400 N

1 mark

5 (3 marks)

The tension provides the centripetal force

E1

$$T = mr\omega^2 = 0.4 \times 0.6 \times 5^2$$

M1

$$= 6 \text{ N}$$

A1

Answer: 6 N

3 marks

6 (4 marks)Friction provides the centripetal force; at the limit $F = \mu R = \mu mg$

M1

$$\mu mg = mr\omega^2, \text{ so } \omega^2 = \frac{\mu g}{r} = \frac{0.4 \times 9.8}{0.12}$$

M1

$$\omega = 5.72 \text{ rad s}^{-1}$$

A1

$$= 5.72 \times \frac{60}{2\pi} = 54.6 \text{ rev min}^{-1}$$

A1

Answer: 5.72 rad s⁻¹ = 54.6 rev min⁻¹

4 marks

7 (4 marks)Friction provides the centripetal force: $F = \frac{mv^2}{r}$

M1

$$R = mg, F_{\max} = \mu mg$$

B1

$$\mu mg = \frac{mv^2}{60}, \text{ so } v^2 = 0.5 \times 9.8 \times 60 = 294$$

M1

$$v = 17.1 \text{ m s}^{-1}$$

A1

Answer: 17.1 m s⁻¹

4 marks

8 (4 marks)Resolve vertically: $R \cos \theta = mg$; horizontally: $R \sin \theta = \frac{mv^2}{r}$

M1 A1

$$\text{Divide: } \tan \theta = \frac{v^2}{rg} = \frac{64}{20 \times 9.8}$$

M1

$$\theta = 18.1^\circ$$

A1

Answer: 18.1°

4 marks

9 (6 marks)

$$(a) \text{ Vertically: } T \cos 30^\circ = mg$$

M1

$$T = \frac{0.4 \times 9.8}{\cos 30^\circ} = 4.53 \text{ N}$$

A1

Answer: 4.53 N

2 marks

$$(b) \text{ Radius } r = 1.2 \sin 30^\circ = 0.6 \text{ m}$$

B1

$$\text{Horizontally: } T \sin 30^\circ = m r \omega^2, \text{ so } \omega^2 = \frac{g \tan 30^\circ}{r} = 9.43$$

M1 A1

$$\omega = 3.07 \text{ rad s}^{-1}, \text{ period} = \frac{2\pi}{\omega} = 2.05 \text{ s}$$

A1

$$\text{Answer: } \omega = 3.07 \text{ rad s}^{-1}, T = 2.05 \text{ s}$$

4 marks

10 (5 marks)

The only forces are the weight and the normal reaction R

B1

$$\text{Vertically: } R \cos 12^\circ = mg$$

M1

$$\text{Horizontally (towards the centre): } R \sin 12^\circ = \frac{mv^2}{80}$$

M1

$$\tan 12^\circ = \frac{v^2}{80g}$$

A1

$$v = 12.9 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 12.9 \text{ m s}^{-1}$$

5 marks

11 (6 marks)

(a) Limiting friction μmg provides the centripetal force: $\mu mg = m r \omega^2$

M1

$$\omega^2 = \frac{0.6 \times 9.8}{0.5} = 11.76$$

A1

$$\omega = 3.43 \text{ rad s}^{-1}$$

A1

$$\text{Answer: } 3.43 \text{ rad s}^{-1}$$

3 marks

(b) Centripetal force needed = $m r \omega^2 = 0.3 \times 0.5 \times 16 = 2.4 \text{ N}$

M1

Friction (limiting) = $0.6 \times 0.3 \times 9.8 = 1.764 \text{ N}$ acts towards the centre

B1

$$T + 1.764 = 2.4, \text{ so } T = 0.636 \text{ N}$$

A1

$$\text{Answer: } 0.636 \text{ N}$$

3 marks

12 (5 marks)

$$\text{(a) Towards the centre (downwards): } mg - R = \frac{mv^2}{r}$$

M1 A1

$$R = 9800 - \frac{1000 \times 100}{40} = 7300 \text{ N}$$

A1

$$\text{Answer: } 7300 \text{ N}$$

3 marks

$$\text{(b) } R = 0: mg = \frac{mv^2}{r}, \text{ so } v^2 = gr = 392$$

M1

$$v = 19.8 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 19.8 \text{ m s}^{-1}$$

2 marks

13 (6 marks)

$$\text{(a) } r = 3 + 4 \sin 30^\circ = 5 \text{ m}$$

B1

$$\text{Answer: shown}$$

1 mark

$$\text{(b) } T \cos 30^\circ = 80 \times 9.8$$

M1

$$T = 905 \text{ N}$$

A1

$$\text{Answer: } 905 \text{ N}$$

2 marks

$$\text{(c) } T \sin 30^\circ = m r \omega^2, \text{ so } mg \tan 30^\circ = m \times 5 \times \omega^2$$

M1 A1

$$\omega = 1.06 \text{ rad s}^{-1}$$

A1

$$\text{Answer: } 1.06 \text{ rad s}^{-1}$$

3 marks

14 (5 marks)

$$\text{(a) Towards the centre: } T + mg = \frac{mv^2}{r}$$

M1 A1

$$T = \frac{0.2 \times 16}{0.9} - 1.96 = 1.6 \text{ N}$$

A1

$$\text{Answer: } 1.6 \text{ N}$$

3 marks

$$\text{(b) } T = 0: mg = \frac{mv^2}{r}$$

M1

$$v = \sqrt{gr} = 2.97 \text{ m s}^{-1}$$

A1

Answer: 2.97 m s^{-1} **2 marks****15 (5 marks)****(a)** M is in equilibrium: $T = 2 \times 9.8 = 19.6 \text{ N}$ **B1****Answer: 19.6 N** **1 mark****(b)** $T = \frac{mv^2}{r}$, so $19.6 = \frac{0.5v^2}{0.4}$ **M1** $v = 3.96 \text{ m s}^{-1}$ **A1****Answer: 3.96 m s^{-1}** **2 marks****(c)** Period = $\frac{2\pi r}{v} = \frac{2\pi \times 0.4}{3.96}$ **M1** $= 0.635 \text{ s}$ **A1****Answer: 0.635 s** **2 marks****16 (7 marks)****(a)** Radius = $\sqrt{0.5^2 - 0.3^2} = 0.4 \text{ m}$ **B1**The reaction R acts along the radius towards O, at angle α to the vertical with $\cos \alpha = \frac{0.3}{0.5} = 0.6$ **B1**Vertical: $R \cos \alpha = mg$ so $R = \frac{0.05 \times 9.8}{0.6}$ **M1** $R = 0.817 \text{ N}$ **A1****Answer: $r = 0.4 \text{ m}$, $R = 0.817 \text{ N}$** **4 marks****(b)** Horizontal: $R \sin \alpha = \frac{mv^2}{r}$, with $\sin \alpha = 0.8$ **M1** $0.8167 \times 0.8 = \frac{0.05v^2}{0.4}$ **A1** $v = 2.29 \text{ m s}^{-1}$ **A1****Answer: 2.29 m s^{-1}** **3 marks****17 (9 marks)****(a)** At the greatest speed friction acts down the slope with $F = 0.2R$ **B1**Vertically: $R \cos 15^\circ - 0.2R \sin 15^\circ = mg$ **M1 A1**Horizontally: $R \sin 15^\circ + 0.2R \cos 15^\circ = \frac{mv^2}{50}$ **M1**Dividing: $v^2 = 50 \times 9.8 \times \frac{\tan 15^\circ + 0.2}{1 - 0.2 \tan 15^\circ}$, so $v = 15.6 \text{ m s}^{-1}$ **A1****Answer: 15.6 m s^{-1}** **5 marks****(b)** At the least speed friction acts up the slope: $R \cos 15^\circ + 0.2R \sin 15^\circ = mg$ **M1 A1** $R \sin 15^\circ - 0.2R \cos 15^\circ = \frac{mv^2}{50}$ **M1** $v^2 = 50 \times 9.8 \times \frac{\tan 15^\circ - 0.2}{1 + 0.2 \tan 15^\circ}$, $v = 5.62 \text{ m s}^{-1}$ **A1****Answer: 5.62 m s^{-1}** **4 marks****18 (10 marks)****(a)** P is 0.8 m vertically from each of A and B (by symmetry), with horizontal distance 0.6 m**M1** $\cos = \frac{0.8}{1} = 0.8$ **A1****Answer: shown****2 marks****(b)** Vertically: $0.8T_A - 0.8T_B = 0.5 \times 9.8$, so $T_A - T_B = 6.125$ **M1 A1**Horizontally: $0.6(T_A + T_B) = 0.5 \times 0.6 \times 36$, so $T_A + T_B = 18$ **M1 A1** $T_A = 12.062 \text{ N}$, $T_B = 5.938 \text{ N}$ **A1****Answer: $T_A = 12.062 \text{ N}$, $T_B = 5.938 \text{ N}$** **5 marks****(c)** $T_B \geq 0$: $T_A + T_B = m\omega^2 \geq T_A - T_B = 6.125$ **M1 A1** $\omega^2 \geq \frac{6.125}{0.5} = 12.25$, so $\omega \geq 3.5 \text{ rad s}^{-1}$ **A1****Answer: $\omega \geq 3.5 \text{ rad s}^{-1}$** **3 marks**

19 (10 marks)

$$(a) T - mg = \frac{mv^2}{r} \quad \text{M1}$$

$$T = 0.4 \times 9.8 + \frac{0.4 \times 100}{1.5} = 30.6 \text{ N} \quad \text{A1}$$

Answer: 30.6 N 2 marks

$$(b) \text{ Energy: } \frac{1}{2}v^2 = \frac{1}{2} \times 100 - g \times 3 \quad \text{M1 A1}$$

$$v^2 = 100 - 58.8 = 41.2 \quad \text{A1}$$

$$T + mg = \frac{mv^2}{r}, \text{ so } T = \frac{0.4 \times 41.2}{1.5} - 3.92 \quad \text{M1}$$

$$T = 7.07 \text{ N} \quad \text{A1}$$

Answer: $v = 6.42 \text{ m s}^{-1}$, $T = 7.07 \text{ N}$ 5 marks

$$(c) \text{ At the top } T \geq 0: v^2 \geq gr = 14.7 \quad \text{M1}$$

$$\text{Energy: } u^2 = v^2 + 4gr \geq 5gr \quad \text{M1}$$

$$u \geq 8.57 \text{ m s}^{-1} \quad \text{A1}$$

Answer: $u \geq 8.57 \text{ m s}^{-1}$ 3 marks

20 (8 marks)

$$(a) \text{ At the top, } R \geq 0: mg \leq \frac{mv^2}{r} \quad \text{M1}$$

$$v \geq \sqrt{gr} = \sqrt{78.4} = 8.85 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 8.85 m s^{-1} 2 marks

$$(b) \text{ Speed at top: } v^2 = gr \quad \text{B1}$$

$$\text{Energy: } mgh = mg(2r) + \frac{1}{2}m(gr) \quad \text{M1 A1}$$

$$h = 2.5r = 20 \text{ m} \quad \text{A1}$$

Answer: 20 m 4 marks

$$(c) R + mg = \frac{mv^2}{r}, \text{ so } R = \frac{400 \times 144}{8} - 3920 \quad \text{M1}$$

$$R = 3280 \text{ N, directed downwards} \quad \text{A1}$$

Answer: 3280 N 2 marks

21 (8 marks)

$$(a) \text{ Gravity provides the centripetal force: } \frac{GMm}{r^2} = \frac{mv^2}{r} \quad \text{M1}$$

$$v^2 = \frac{GM}{r} \quad \text{A1}$$

Answer: shown 2 marks

$$(b) r = 6.371 \times 10^6 + 2.0 \times 10^6 = 8.371 \times 10^6 \text{ m} \quad \text{B1}$$

$$v = \sqrt{3.99 \times 10^{14} / 8.371 \times 10^6} \quad \text{M1}$$

$$= 6900 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 6900 m s^{-1} 3 marks

$$(c) T = \frac{2\pi r}{v} = \frac{2\pi \times 8.371 \times 10^6}{6900} \quad \text{M1 A1}$$

$$= 7620 \text{ s} = 2.12 \text{ hours} \quad \text{A1}$$

Answer: 7620 s = 2.12 hours 3 marks

22 (8 marks)

$$(a) \text{ The normal reaction of the wall provides the centripetal force} \quad \text{E1}$$

$$\text{Friction acting upwards along the wall balances the weight} \quad \text{E1}$$

Answer: R centripetal, $F = mg$ 2 marks

$$(b) \text{ Vertical: } F = mg; \text{ limiting } F = \mu R, \text{ so } \mu R \geq mg \quad \text{M1}$$

$$\text{Horizontal: } R = mr\omega^2, \text{ so } \mu r\omega^2 \geq g \quad \text{M1}$$

$$\omega \geq \sqrt{\frac{9.8}{0.4 \times 2.5}} = 3.13 \text{ rad s}^{-1}$$

A1

$$= 29.9 \text{ rev min}^{-1}$$

A1

Answer: $3.13 \text{ rad s}^{-1} = 29.9 \text{ rev min}^{-1}$

4 marks

(c) $R = mr\omega^2 = 70 \times 2.5 \times 16 = 2800 \text{ N}$

M1

$F = mg = 686 \text{ N}$ (less than $\mu R = 1120 \text{ N}$, so no slipping)

A1

Answer: $R = 2800 \text{ N}$, $F = 686 \text{ N}$

2 marks

23 (5 marks)

Section BA: the only mass beyond it is $2m$ at A

M1

$$T_{BA} = 2m \times 1.2\omega^2 = 2.4m\omega^2$$

A1

Section OB: $T_{OB} - T_{BA} = m \times 0.4\omega^2$

M1

$$T_{OB} = 2.4m\omega^2 + 0.4m\omega^2 = 2.8m\omega^2$$

A1

$$T_{OB} : T_{BA} = 2.8 : 2.4 = 7 : 6$$

A1

Answer: $7 : 6$

5 marks

24 (7 marks)

(a) The surface makes 30° with the vertical, so the normal reaction R makes 30° with the horizontal

B1

Vertical: $R \sin 30^\circ = mg$

M1

$$R = 2 \times 0.4 \times 9.8 = 7.84 \text{ N}$$

A1

Answer: shown

3 marks

(b) Horizontal: $R \cos 30^\circ = \frac{mv^2}{r}$

M1

$$7.84 \times \cos 30^\circ = \frac{0.4v^2}{0.5}, \text{ so } v^2 = 8.487$$

A1

$$v = 2.91 \text{ m s}^{-1}$$

A1

$$T = \frac{2\pi \times 0.5}{v} = 1.08 \text{ s}$$

A1

Answer: $v = 2.91 \text{ m s}^{-1}$, $T = 1.08 \text{ s}$

4 marks

25 (5 marks)

Angular speed $\omega = \frac{2\pi}{86164} \text{ rad s}^{-1}$

B1

$$\frac{GMm}{r^2} = mr\omega^2, \text{ so } r^3 = \frac{GM}{\omega^2}$$

M1

$$r^3 = \frac{3.986 \times 10^{14} \times 86164^2}{4\pi^2}$$

M1

$$r = 4.22\text{e}+07 \text{ m}$$

A1

$$\text{Height} = r - 6.371 \times 10^6 = 3.58\text{e}+07 \text{ m}$$

A1

Answer: $r = 4.22\text{e}+07 \text{ m}$, height = $3.58\text{e}+07 \text{ m}$

5 marks

26 (10 marks)

(a) P has fallen a vertical distance $a - a \cos \theta$

B1

Energy: $\frac{1}{2}mv^2 = mga(1 - \cos \theta)$

M1

$$v^2 = 2ga(1 - \cos \theta)$$

A1

Answer: shown

3 marks

(b) Towards the centre: $mg \cos \theta - R = \frac{mv^2}{a}$

M1 A1

$$R = mg \cos \theta - 2mg(1 - \cos \theta) = mg(3 \cos \theta - 2)$$

M1

P leaves the surface when $R = 0$, so $\cos \theta = \frac{2}{3}$

A1

Answer: shown

4 marks

(c) $v^2 = 2 \times 9.8 \times 1.5 \times \frac{1}{3} = 9.8$

M1

$$v = 3.13 \text{ m s}^{-1} \quad \text{A1}$$

$$\text{Height} = a \cos \theta = 1.5 \times \frac{2}{3} = 1 \text{ m} \quad \text{A1}$$

Answer: $v = 3.13 \text{ m s}^{-1}$, height = 1 m 3 marks

27 (12 marks)

(a) Energy: $v^2 = u^2 - 2gL(1 - \cos \theta)$ M1 A1

Towards O: $T - mg \cos \theta = \frac{mv^2}{L}$ M1 A1

$$T = \frac{mu^2}{L} - 2mg(1 - \cos \theta) + mg \cos \theta = \frac{mu^2}{L} + mg(3 \cos \theta - 2) \quad \text{A1}$$

Answer: shown 5 marks

(b) $T = 3mg + mg(3 \cos \theta - 2) = mg(1 + 3 \cos \theta)$ M1 A1

$T = 0$ gives $\cos \theta = -\frac{1}{3}$, $\theta = 109^\circ$ A1

Angle above the horizontal = $\theta - 90^\circ = 19.5^\circ$ A1

Answer: $\theta = 109^\circ$, 19.5° above horizontal 4 marks

(c) $u = \sqrt{3 \times 9.8 \times 0.8} = 4.85 \text{ m s}^{-1}$ M1 A1

$T = 0.3 \times 9.8 \times (1 + 3 \cos 60^\circ) = 7.35 \text{ N}$ A1

Answer: $u = 4.85 \text{ m s}^{-1}$, $T = 7.35 \text{ N}$ 3 marks

28 (10 marks)

(a) Friction $F = \mu R$ acts down the slope at the limit B1

Vertical: $R \cos \theta - \mu R \sin \theta = mg$ M1 A1

Horizontal: $R \sin \theta + \mu R \cos \theta = \frac{mv^2}{r}$ M1

Dividing and then dividing top and bottom by $\cos \theta$ gives the result A1

Answer: shown 5 marks

(b) $v_{\max}^2 = 120 \times 9.8 \times \frac{\tan 8^\circ + 0.35}{1 - 0.35 \tan 8^\circ}$ M1 A1

$v_{\max} = 24.6 \text{ m s}^{-1}$ A1

No friction: $v_0^2 = rg \tan \theta$, $v_0 = 12.9 \text{ m s}^{-1}$, much smaller than $v_{\max}$ A1

Answer: $v_{\max} = 24.6 \text{ m s}^{-1}$, $v_0 = 12.9 \text{ m s}^{-1}$ 4 marks

(c) The denominator becomes zero or negative: friction can then hold the car at any speed, so there is no upper limit E1

Answer: No limit on speed 1 mark

29 (12 marks)

(a) Horizontally: $T \sin \theta = m(L \sin \theta)\omega^2$, so $T = mL\omega^2$ M1 A1

Vertically: $T \cos \theta = mg$ M1

$$\cos \theta = \frac{mg}{T} = \frac{g}{L\omega^2} \quad \text{A1}$$

Answer: shown 4 marks

(b) $\cos \theta < 1$ requires $\frac{g}{L\omega^2} < 1$ M1

$$\omega > \sqrt{\frac{g}{L}} \quad \text{A1}$$

Answer: $\omega > \sqrt{g/L}$ 2 marks

(c) $T = 0.25 \times 0.8 \times 36 = 7.2 \text{ N}$ A1

$$\cos \theta = \frac{9.8}{0.8 \times 36} = 0.34 \quad \text{M1}$$

$\theta = 70.1^\circ$ A1

Answer: $T = 7.2 \text{ N}$, $\theta = 70.1^\circ$ 3 marks

(d) Period = $\frac{2\pi}{\omega}$, with $\omega^2 = \frac{g}{L \cos \theta}$ M1

$$\text{Period} = 2\pi \sqrt{\frac{L \cos \theta}{g}} \quad \text{A1}$$

As $\theta \rightarrow 0$, $\cos \theta \rightarrow 1$: period $\rightarrow 2\pi \sqrt{\frac{L}{g}} = 1.8 \text{ s}$

A1

Answer: 1.8 s

3 marks

30 (10 marks)

(a) Least speed at the top, greatest at the bottom (height difference $2a$)

B1

Energy: $\frac{1}{2}m(3u)^2 = \frac{1}{2}mu^2 + mg(2a)$

M1 A1

$4u^2 = 2ga$ so $u^2 = \frac{ga}{2}$

A1

Answer: shown

4 marks

(b) Towards O (downwards): $mg - S = \frac{mu^2}{a}$, where S is the upward thrust

M1

$S = mg - \frac{mg}{2} = \frac{mg}{2}$

A1

A thrust (the rod pushes P upwards) of $\frac{1}{2}mg$

A1

Answer: thrust of $\frac{1}{2}mg$

3 marks

(c) At the bottom $v^2 = 9u^2 = \frac{9ga}{2}$

B1

$T - mg = \frac{mv^2}{a} = \frac{9mg}{2}$

M1

$T = \frac{11mg}{2}$

A1

Answer: $\frac{11mg}{2}$

3 marks