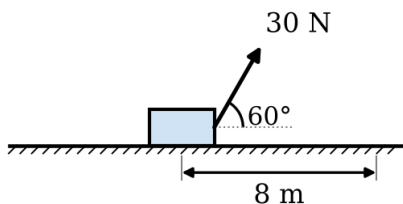


Work Done

- 1 Find the work done by a force of 40 N that moves its point of application a distance of 12 m in the direction of the force. [2]

- 2 A block of mass 5 kg is lifted at constant speed through a vertical height of 3 m. Take $g = 9.8 \text{ m s}^{-2}$.
Find the work done by the lifting force. [2]

- 3 A box is pulled 8 m along horizontal ground by a rope. The tension in the rope is 30 N and the rope makes an angle of 60° with the horizontal.
Find the work done by the tension.

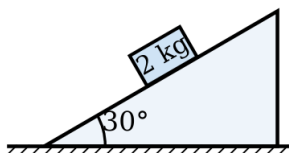


[2]

- 4 A crate slides 5 m across a rough horizontal floor. The friction force on the crate is 12 N. Find the work done against friction. [2]

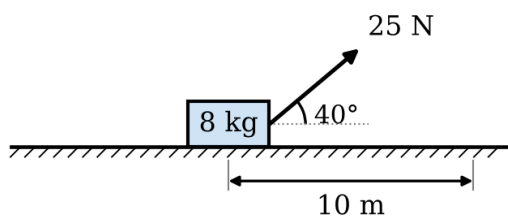
- 5 A shopper carries a bag of mass 3 kg at constant speed along a horizontal path for 20 m.
Explain why the work done by the upward force of her hand on the bag is zero. [2]

- 6 A block of mass 2 kg slides 5 m down a smooth plane inclined at 30° to the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
Find the work done by the weight of the block.



[3]

- 7 A box of mass 8 kg is pulled 10 m along rough horizontal ground by a rope inclined at 40° above the horizontal. The tension in the rope is 25 N and the friction force is 12 N.

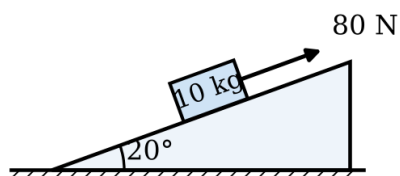


- (a) Find the work done by the tension. [2]
(b) Find the work done against friction. [1]
(c) Find the net work done on the box by these two forces. [1]

- 8** A crane lifts a load of mass 250 kg at constant speed through a vertical height of 12 m. Take $g = 9.8 \text{ m s}^{-2}$.
Find the work done by the crane on the load, in kJ. [3]

- 9** A car of mass 1200 kg accelerates uniformly from rest at 1.5 m s^{-2} along a straight horizontal road for 8 s. The resistance to motion is a constant 300 N.
Find the work done by the engine during the 8 s. [5]

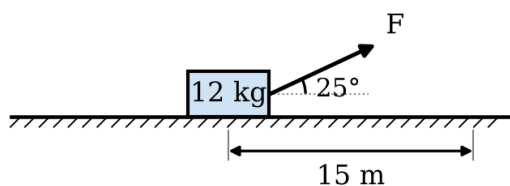
- 10** A crate of mass 10 kg is pushed 6 m up a rough plane inclined at 20° to the horizontal by a force of 80 N acting parallel to the plane. The friction force is 15 N. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the work done by the 80 N force. [1]
(b) Find the work done against gravity. [2]
(c) Find the work done against friction. [1]
- 11** A skier of mass 70 kg descends a mountain slope and ends 150 m vertically below her starting point. Take $g = 9.8 \text{ m s}^{-2}$.
(a) Find the work done by the skier's weight. [2]
(b) Explain why the answer to part (a) does not depend on the length or shape of the path taken. [1]

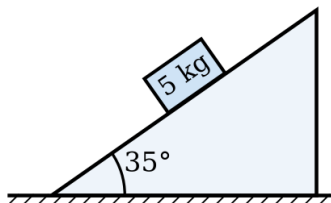
- 12** A woman of mass 60 kg climbs three flights of stairs, each flight rising 3.2 m. Take $g = 9.8 \text{ m s}^{-2}$.
(a) Find the total height climbed. [1]
(b) Find the work done against gravity. [2]
(c) She repeats the climb carrying a 5 kg bag. Find the total work done against gravity. [2]

- 13** A box of mass 12 kg is pulled along rough horizontal ground at constant speed through a distance of 15 m by a rope inclined at 25° above the horizontal. The tension in the rope is F newtons and the coefficient of friction is 0.3. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the value of F . [5]
(b) Find the work done by the tension. [2]

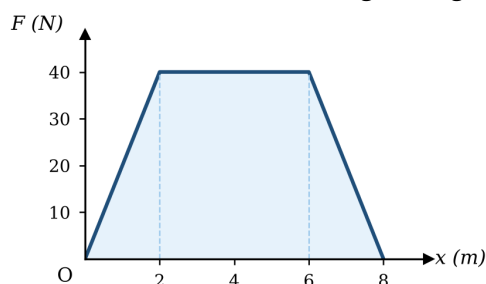
- 14** A block of mass 5 kg is released from rest on a rough plane inclined at 35° to the horizontal and slides 8 m down the line of greatest slope. The coefficient of friction is 0.2. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the work done by the weight. [2]
- (b) Find the work done against friction. [3]
- (c) Hence find the speed of the block after sliding 8 m. [2]

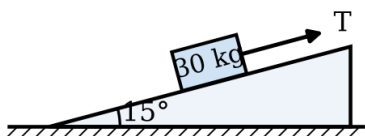
- 15** A car of mass 900 kg travels at constant speed up a straight road that rises 1 m for every 20 m travelled along the road. The resistance to motion is 400 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done by the engine in travelling 2 km along the road. [4]

- 16** The graph shows the force F newtons acting on a particle in the direction of its motion, as the particle moves a distance x metres along a straight line.



- (a) Find the work done by the force while the particle moves from $x = 0$ to $x = 8$. [3]
- (b) Find the constant force that would do the same work over the same 8 m. [1]

- 17** A trolley of mass 30 kg is pulled at constant speed up a ramp of length 5 m inclined at 15° to the horizontal, by a rope parallel to the ramp. The friction force on the trolley is 20 N. Take $g = 9.8 \text{ m s}^{-2}$.

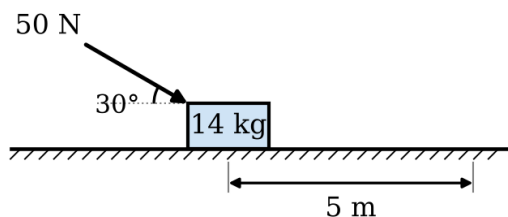


- (a) Find the work done against gravity. [2]
- (b) Find the work done by the tension in the rope. [2]

- 18** A load of mass 25 kg is raised 6 m from rest by a rope that exerts a constant force of 270 N. Take $g = 9.8 \text{ m s}^{-2}$.

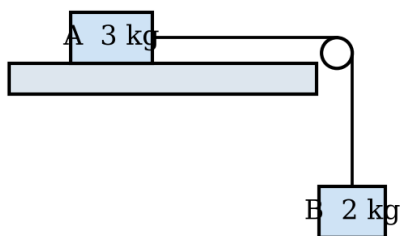
- (a) Find the work done by the tension in the rope. [1]
- (b) Find the work done against gravity. [1]
- (c) Use the work-energy principle to find the speed of the load after it has been raised 6 m. [3]

- 19** A man pushes a trolley of mass 14 kg along horizontal ground through a distance of 5 m by exerting a force of 50 N directed 30° below the horizontal. The coefficient of friction between the trolley and the ground is 0.2. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the work done by the man. [2]
 (b) Find the work done against friction. [3]

- 20** Particle A of mass 3 kg lies on a rough horizontal table and is connected by a light inextensible string over a smooth pulley at the table edge to particle B of mass 2 kg, which hangs freely. The coefficient of friction between A and the table is 0.25. The system is released from rest and B descends 1.5 m. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the acceleration of the system and the tension in the string. [4]
 (b) Find the work done by the tension on A and on B during the 1.5 m. [2]
 (c) Verify that the net work done on A equals its gain in kinetic energy. [3]

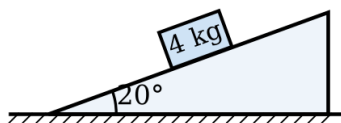
- 21** A bucket of mass 5 kg is raised at constant speed from the bottom of a well 20 m deep by a rope of mass 0.4 kg per metre. When the bucket is x metres below the top of the well, x metres of rope are hanging. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Show that the force needed at the top of the rope when the bucket is x metres below the top is $(5 + 0.4x)g$ newtons. [2]
 (b) Hence use integration to find the work done in raising the bucket to the top of the well. [4]
 (c) Check your answer by considering the centre of mass of the rope. [2]

- 22** A particle of mass 2 kg moves along the x -axis. When the particle is at the point with coordinate x metres it is acted on by a force of $(3x^2 + 2x)$ newtons in the positive x -direction. It passes $x = 1$ with speed 2 m s^{-1} .

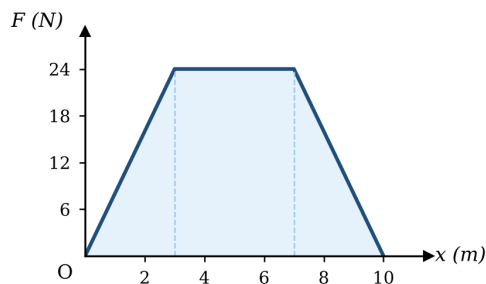
- (a) Find the work done by the force as the particle moves from $x = 1$ to $x = 4$. [3]
 (b) Find the speed of the particle at $x = 4$. [3]

- 23** A block of mass 4 kg is pushed 5 m up a rough plane inclined at 20° to the horizontal by a horizontal force of 50 N, the force acting in the vertical plane containing a line of greatest slope. The coefficient of friction is 0.3. Take $g = 9.8 \text{ m s}^{-2}$.

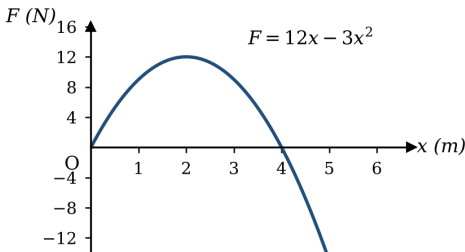


- Show that the normal reaction is 53.9 N, correct to 3 significant figures. [3]
 - Find the work done by the 50 N force and the work done against gravity. [3]
 - Find the net work done on the block. [2]
- 24** A car of mass 1100 kg travels at constant speed up a straight road of length 3 km inclined at an angle with $\sin \alpha = \frac{1}{25}$, and then at constant speed down a straight road of length 2 km inclined with $\sin \beta = \frac{1}{20}$. The resistance to motion is 250 N throughout. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the work done by the engine on the uphill section. [3]
 - Find the net work done by gravity over the whole journey. [2]
 - On the downhill section the engine is switched off and the driver brakes to maintain constant speed. Find the work done by the brakes. [2]

- 25** A particle of mass 5 kg is at rest at $x = 0$ on a rough horizontal surface. A horizontal force F newtons acts along the x -axis; its magnitude varies with x as shown in the graph. The friction force is a constant 6 N.



- Find the work done by F as the particle moves from $x = 0$ to $x = 10$ m. [3]
 - Find the speed of the particle at $x = 10$ m. [3]
 - The force F is zero for $x > 10$. Find how much further the particle travels before coming to rest. [3]
- 26** A particle moves in a horizontal plane from $A(2, 1)$ to $B(8, 5)$, where the units are metres. It is acted on by two constant forces $F_1 = (4\mathbf{i} + 3\mathbf{j}) \text{ N}$ and $F_2 = (-\mathbf{i} + 5\mathbf{j}) \text{ N}$.
- Find the displacement AB and the work done by each force. [4]
 - Find the resultant force, and verify that the work done by the resultant is the sum of the work done by the two forces. [3]
 - Find the angle between F_2 and AB , to the nearest 0.1° . [3]
- 27** A uniform chain of length 3 m and mass 6 kg lies on a smooth horizontal table with 1 m of its length hanging over the edge, vertically. Take $g = 9.8 \text{ m s}^{-2}$.
- When a length x metres hangs over the edge, find the weight of the hanging part. [2]
 - Use integration to find the work done in slowly pulling the chain completely on to the table, starting from the position where 1 m hangs. [3]
 - A length h of a chain of mass λ per unit length hangs over the edge. Show that the work done in pulling it on to the table is $\frac{1}{2}\lambda g h^2$, and hence find the work done if 2 m of the chain originally hung over the edge. [3]

- 28** A particle moves from A(1, 2) to B(7, 4), where the units of distance are metres, under the action of a constant force $F = (3i + 4j)$ N.
- Find the displacement vector AB. [1]
 - Find the work done by the force, using the scalar product. [2]
 - Find the angle between the force and the displacement. [3]
 - The particle instead travels from A to C(7, 2) and then to B. Show that the total work done is the same, and explain why. [2]
-
- 29** A car of mass 1000 kg tows a trailer of mass 400 kg up a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$. The tow bar is light and parallel to the road. The resistances to motion are constant at 300 N on the car and 150 N on the trailer. Take $g = 9.8 \text{ m s}^{-2}$.
- At constant speed the car travels 2 km up the road. Find the driving force of the car's engine and the work it does. [3]
 - Find the tension in the tow bar and the work done by the tow bar on the trailer over the 2 km. [3]
 - The car now accelerates uniformly at 0.2 m s^{-2} up the same road. Find the new driving force and the work done by the tow bar on the trailer over the next 500 m. [4]
-
- 30** A particle of mass 4 kg is at rest at the origin of an x-axis. When it is at position x metres it is acted on by a force $F = (12x - 3x^2)$ newtons in the direction of the positive x-axis.
- 
- Find the values of x for which $F = 0$, and the greatest value of F for $0 \leq x \leq 4$. [3]
 - Find the work done by the force from $x = 0$ to $x = 4$. [3]
 - Find the speed of the particle at $x = 4$. [2]
 - Show that the total work done from $x = 0$ to $x = 6$ is zero, and describe the motion of the particle at $x = 6$. [2]

Kinetic and Potential Energy

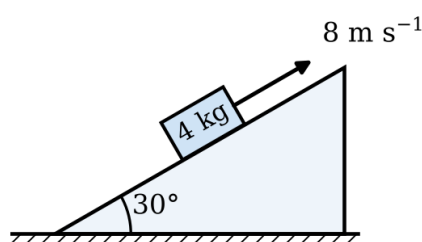
- Find the kinetic energy of a car of mass 1500 kg travelling at 20 m s^{-1} . [2]
- Find the gravitational potential energy gained by a 4 kg mass raised through 12 m. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- A ball of mass 0.2 kg has kinetic energy 40 J. Find its speed. [2]
- A 6 kg mass is raised from a height of 2 m to a height of 9 m above the ground. Take $g = 9.8 \text{ m s}^{-2}$. Find the increase in its gravitational potential energy. [2]
- A particle of mass 2 kg speeds up from 3 m s^{-1} to 7 m s^{-1} . Find the gain in its kinetic energy. [2]
- A book of mass 0.5 kg gains 14.7 J of potential energy when it is lifted. Take $g = 9.8 \text{ m s}^{-2}$. Find the height through which it was lifted. [2]

- 7 A car of mass 900 kg slows from 25 m s^{-1} to 10 m s^{-1} over a distance of 60 m on a horizontal road, under a constant resisting force. Find the magnitude of the resisting force. [4]
- 8 A ball of mass 0.4 kg is thrown vertically upwards with speed 15 m s^{-1} . Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the initial kinetic energy of the ball. [1]
 (b) Use energy considerations to find the greatest height reached. [3]
- 9 A stone of mass 3 kg is dropped from rest from a point 20 m above the ground. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
 Find the kinetic energy and the speed of the stone just before it hits the ground. [4]
- 10 A bullet of mass 0.02 kg travelling at 400 m s^{-1} embeds itself in a wooden block and comes to rest after penetrating 0.25 m. Find the average resistance of the wood. [4]
- 11 A cyclist and her bicycle have a combined mass of 80 kg. She ascends a hill, gaining 6 m in height, while her speed falls from 12 m s^{-1} to 8 m s^{-1} . Ignore resistances. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the change in kinetic energy. [2]
 (b) Find the gain in potential energy. [1]
 (c) Find the work done by the cyclist. [2]
- 12 A box of mass 5 kg, initially at rest on a rough horizontal floor, is pushed 6 m by a horizontal force of 30 N. The friction force is 10 N. Find the speed of the box after 6 m. [4]
- 13 A particle of mass 0.5 kg is released from rest at a height of 10 m. It falls freely past the points A, B and C, at heights 7 m, 4 m and 0 m above the ground. Take $g = 9.8 \text{ m s}^{-2}$. Complete the table of potential and kinetic energies, taking the ground as the zero of potential energy, and the starting KE as zero.

	Start	A	B	C
Height above ground (m)	10	7	4	0
Potential energy (J)				
Kinetic energy (J)				

[4]

- 14 A box of mass 4 kg is projected up a rough plane inclined at 30° to the horizontal with initial speed 8 m s^{-1} . The coefficient of friction between the box and the plane is 0.2. Take $g = 9.8 \text{ m s}^{-2}$.
 Use the work-energy principle to find how far up the plane the box travels before coming to rest.



[6]

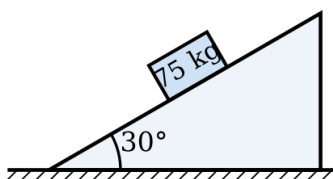
- 15** A car of mass 1200 kg is braked to rest by a constant braking force of 4800 N on a horizontal road.
- (a) Find the stopping distance from a speed of 15 m s^{-1} . [3]
- (b) Show that the stopping distance from 30 m s^{-1} is four times the stopping distance from 15 m s^{-1} , and explain why this happens. [3]

- 16** A lorry of mass 8000 kg travels 200 m down a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{40}$. Its speed increases from 6 m s^{-1} to 12 m s^{-1} . The resistance to motion is a constant 500 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the driving force of the engine, assumed constant. [6]

- 17** A stone of mass 0.3 kg enters a pond with speed 5 m s^{-1} . After sinking a vertical distance of 2 m its speed is 3 m s^{-1} . Find the average resistance of the water to the stone's motion. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 18** A hammer of mass 50 kg falls from rest through 1.8 m and strikes a nail, driving it 0.02 m into a block of wood before the hammer comes to rest. Assume that the resistance of the wood is constant. Take $g = 9.8 \text{ m s}^{-2}$. Find the resistance of the wood to the nail, to 3 significant figures. [5]

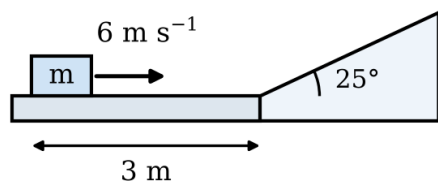
- 19** A skier of mass 75 kg starts from rest and slides 80 m down a straight slope inclined at 30° to the horizontal. The coefficient of friction between the skis and the snow is 0.1. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the skier at the bottom of the slope.



[6]

- 20** A ball of mass 0.5 kg is dropped from rest from a height of 2 m above a hard floor and rebounds to a height of 1.2 m. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the speed of the ball just before it hits the floor, and just after it leaves the floor. [3]
- (b) Find the kinetic energy lost in the impact. [4]

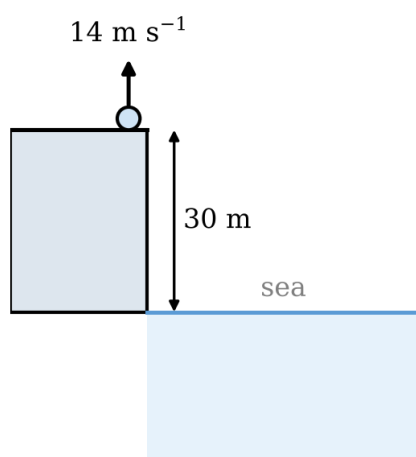
- 21** A block slides in a straight line across a rough horizontal table, with a coefficient of friction of 0.3, and then up a smooth ramp inclined at 25° to the horizontal. The block has speed 6 m s^{-1} at the start of the 3 m rough section. Assume the speed is unchanged where the table meets the ramp. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the speed of the block when it reaches the ramp. [3]
- (b) Find the distance the block travels up the ramp before it comes to rest. [4]

- 22** A stone is thrown from a point 15 m above horizontal ground with speed 12 m s^{-1} . Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
- Show that the speed with which the stone hits the ground is the same whatever the direction of projection, and find this speed. [4]
 - Explain why the time of flight does depend on the direction of projection. [1]
 - The stone is thrown horizontally. Find the vertical component of its velocity when it hits the ground. [3]
-
- 23** A ball of mass 0.2 kg is projected vertically upwards with speed 20 m s^{-1} . Throughout the motion the air resistance is a constant 0.5 N. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the greatest height reached by the ball. [4]
 - Find the speed of the ball when it returns to its starting point. [4]
-
- 24** A van of mass 2000 kg descends a straight road, inclined at an angle α to the horizontal with $\sin \alpha = \frac{1}{25}$, travelling 400 m while its speed increases from 10 m s^{-1} to 20 m s^{-1} . The driving force of the engine is a constant 600 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the constant resistance to motion. [6]
-
- 25** A particle is projected with speed 7 m s^{-1} up a line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction is 0.25. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the distance the particle travels up the plane before coming instantaneously to rest. [4]
 - Show that the particle slides back down the plane, and find its speed when it returns to its starting point. [4]
-
- 26** A particle of mass m falls from rest through a height h and then penetrates a distance d into sand, coming to rest. The resistance of the sand is a constant R .
- Show that $R = \frac{mg(h+d)}{d}$. [3]
 - Find R when $m = 1.5 \text{ kg}$, $h = 4 \text{ m}$ and $d = 0.2 \text{ m}$. Take $g = 9.8 \text{ m s}^{-2}$. [2]
 - Write down, with a reason, how R changes if the particle penetrates less far, with h unchanged. [1]
-
- 27** A car of mass M travelling at speed v on a horizontal road is brought to rest by a constant braking force F .
- Show that the stopping distance is $s = \frac{Mv^2}{2F}$. [2]
 - Write down the factor by which s changes if (i) v is doubled, (ii) M is doubled, with F unchanged. [2]
 - The car is now travelling down a hill of slope α ($\sin \alpha = \frac{1}{10}$). Show that the stopping distance from speed v is $s = \frac{Mv^2}{2(F - Mg \sin \alpha)}$, and find s when $M = 1200 \text{ kg}$, $v = 20 \text{ m s}^{-1}$ and $F = 5000 \text{ N}$. [5]

- 28** A ball of mass 0.5 kg is thrown vertically upwards with speed 14 m s^{-1} from the top of a cliff 30 m above the sea. The ball later falls into the sea, where it sinks vertically 4 m before coming to rest. Ignore air resistance, and model the resistance of the water as a constant force. Take $g = 9.8 \text{ m s}^{-2}$.

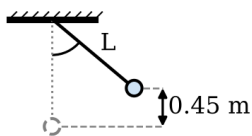


- (a) Show that the speed of the ball as it enters the water is 28 m s^{-1} . [3]
 (b) Find the resistance of the water. [4]
 (c) The ball is instead thrown vertically downwards with speed 14 m s^{-1} . Explain, without further calculation, why the answer to part (b) is unchanged. [2]
-
- 29** A rubber ball is dropped from rest at a height of 2.5 m above a hard floor. Each time it hits the floor it retains 64% of its kinetic energy (the rest being lost as heat and sound). Ignore air resistance.
- (a) Show that the ball rises to a height of 1.6 m after the first bounce. [2]
 (b) Write down the height reached after the n th bounce. [1]
 (c) Find the total distance travelled by the ball from release until it comes to rest. [5]
 (d) State one reason why this model is unrealistic. [1]
-
- 30** A particle of mass m is released from rest on a rough plane inclined at an angle α to the horizontal, with coefficient of friction μ , and slides a distance L down a line of greatest slope.
- (a) Show that the speed v of the particle after sliding the distance L satisfies $v^2 = 2gL(\sin \alpha - \mu \cos \alpha)$, stating the condition on μ for the particle to move. [4]
 (b) Find v when $L = 50 \text{ m}$, $\alpha = 30^\circ$ and $\mu = 0.3$. Take $g = 9.8 \text{ m s}^{-2}$. [2]
 (c) Find the least value of μ for which the speed after 50 m on the 30° slope is at most 10 m s^{-1} . [3]

Conservation of Energy

- 1** A ball is dropped from rest from a point 5 m above the ground. Ignore air resistance and use conservation of energy to find its speed on reaching the ground. Take $g = 9.8 \text{ m s}^{-2}$. [3]
-
- 2** A block slides from rest down a smooth slope. It descends a vertical height of 3 m . Find its speed at the bottom of the slope. Take $g = 9.8 \text{ m s}^{-2}$. [3]
-
- 3** A stone is thrown vertically upwards with speed 14 m s^{-1} . Use energy to find the greatest height it reaches above the point of projection. Take $g = 9.8 \text{ m s}^{-2}$. [2]

- 4 A pendulum bob is released from rest at a point 0.45 m above its lowest position, as shown. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the bob as it passes through its lowest position.



[2]

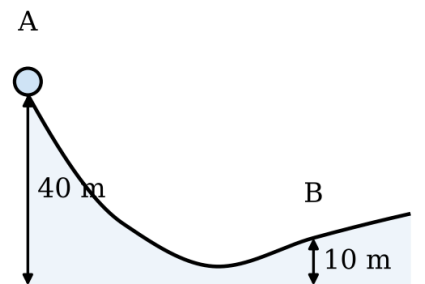
- 5 Find the minimum speed with which a stone must be thrown vertically upwards from ground level in order to just reach a height of 20 m. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.

[2]

- 6 A ball of mass 0.5 kg is thrown vertically upwards from the ground with speed 8 m s^{-1} . Find its speed when it is 2 m above the ground, ignoring air resistance. Take $g = 9.8 \text{ m s}^{-2}$.

[3]

- 7 A roller-coaster car of mass 400 kg passes the top A of a hill, 40 m above the ground, with speed 3 m s^{-1} . It then runs down and up to a point B on another hill, 10 m above the ground. Ignore resistances. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the car at B.

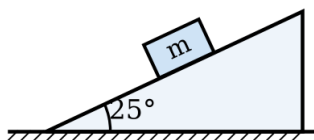


[4]

- 8 A stone is thrown from ground level with speed 20 m s^{-1} at 30° above the horizontal. Ignore air resistance. Use energy to find its speed when it is at a height of 8 m. Take $g = 9.8 \text{ m s}^{-2}$.

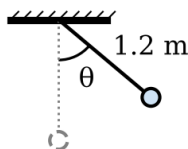
[3]

- 9 A particle is released from rest at the top of a smooth plane inclined at 25° to the horizontal and slides 6 m down the line of greatest slope. Find its speed at the bottom. Take $g = 9.8 \text{ m s}^{-2}$.



[4]

- 10** A pendulum has a bob of mass 0.3 kg on a light string of length 1.2 m. The bob is released from rest with the string taut. Find the speed of the bob at its lowest point when it is released from



- (a) the horizontal position (string horizontal), [2]
 (b) the position where the string makes an angle of 60° with the downward vertical. [3]

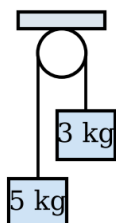
- 11** A block of mass 2 kg slides from rest down a rough slope of length 6 m, descending a vertical height of 3 m, and reaches the bottom with speed 5 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the energy lost to friction. [3]
 (b) Find the average friction force. [2]

- 12** A particle of mass 2 kg is projected vertically upwards from the ground with speed 12 m s^{-1} . Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.

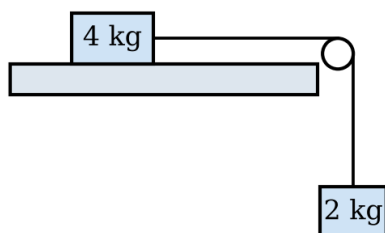
Find the height at which the kinetic energy of the particle equals its potential energy (taking the ground as the level of zero potential energy). [4]

- 13** Two particles of mass 5 kg and 3 kg are connected by a light inextensible string passing over a smooth fixed pulley. The system is released from rest with the strings vertical and taut, and the heavier particle descends 2 m. Use energy to find the speed of the particles at this instant. Take $g = 9.8 \text{ m s}^{-2}$.



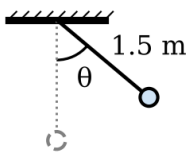
[5]

- 14** Particle A of mass 4 kg lies on a smooth horizontal table and is connected by a light inextensible string, passing over a smooth pulley at the edge of the table, to particle B of mass 2 kg, which hangs freely. The system is released from rest. Find the speed of the particles when B has descended 1.5 m. Take $g = 9.8 \text{ m s}^{-2}$.



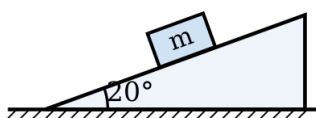
[5]

- 15** A pendulum bob hangs from a light string of length 1.5 m. When the bob is released from rest with the string taut at an angle θ to the downward vertical, its speed at the lowest point is 3 m s^{-1} . Find θ to the nearest 0.1° . Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 16** A block is released from rest on a rough plane inclined at 20° to the horizontal. The coefficient of friction between the block and the plane is 0.15. Use an energy method to find the speed of the block after it has slid 5 m down the plane. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 17** A ball is thrown from a point 10 m above horizontal ground with speed 15 m s^{-1} at 40° above the horizontal. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the speed with which the ball hits the ground. [2]
- (b) Find the speed of the ball at its highest point. [2]
- (c) Find the greatest height of the ball above the ground. [3]

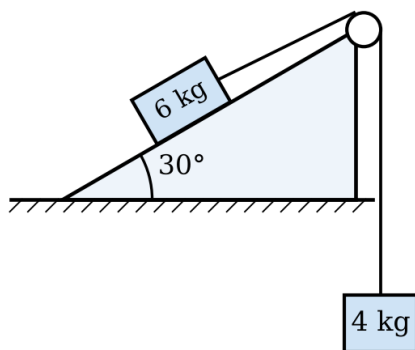
- 18** A cyclist and bicycle, of total mass 85 kg, freewheel (no pedalling) from rest down a hill, losing 25 m in height over a road distance of 400 m, and reach a speed of 15 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the energy lost to resistive forces. [3]
- (b) Find the average resistive force. [2]

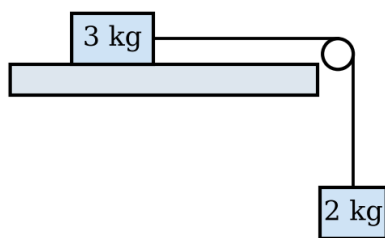
- 19** A lift of mass 600 kg is moving downwards at 3 m s^{-1} when it is brought to rest in a distance of 2 m by a constant tension T newtons in the cable. Take $g = 9.8 \text{ m s}^{-2}$. Use the work-energy principle to find T .

[5]

- 20** Particle A of mass 6 kg lies on a plane inclined at 30° to the horizontal and is connected by a light inextensible string over a smooth pulley at the top of the plane to particle B of mass 4 kg, which hangs freely. Take $g = 9.8 \text{ m s}^{-2}$.

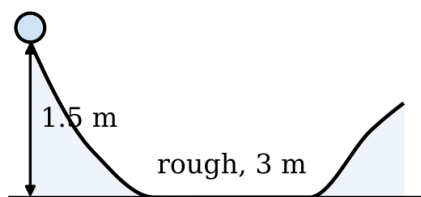


- The plane is smooth. The system is released from rest and B descends 1.2 m. Find the speed of the particles at this instant. [4]
 - The plane is now rough with coefficient of friction 0.2. Show that the system does not move when released from rest. [3]
 - Find the smallest coefficient of friction that prevents motion. [2]
-
- 21** A stone is projected from level ground with speed u at an angle θ above the horizontal. Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
- Use conservation of energy to show that the greatest height reached is $H = \frac{u^2 \sin^2 \theta}{2g}$. [3]
 - Find H when $u = 25 \text{ m s}^{-1}$ and $\theta = 40^\circ$. [2]
 - Find the speed of the stone when it is 6 m above the ground, for the values in part (b). [2]
-
- 22** Particle A of mass 3 kg lies on a rough horizontal table, with coefficient of friction 0.4, and is connected by a light inextensible string over a smooth pulley at the edge of the table to particle B of mass 2 kg. B is held 1.5 m above the floor and the system is released from rest. B hits the floor and does not rebound. A does not reach the pulley. Take $g = 9.8 \text{ m s}^{-2}$.



- Find the speed of the particles at the instant B hits the floor. [5]
- Find the further distance travelled by A before it comes to rest. [3]

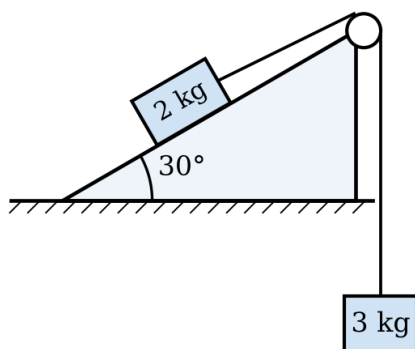
- 23** A block of mass 2 kg is released from rest at a height of 1.5 m on a smooth curved track, which leads to a horizontal section. The coefficient of friction on the horizontal section is 0.3. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) The horizontal rough section is very long. Find the distance the block slides along it before stopping. [3]
- (b) In fact the rough section is only 3 m long and is followed by a smooth ramp. Find the greatest height above the horizontal section reached by the block on the ramp. [4]
-
- 24** A stone is thrown from ground level at an angle θ above the horizontal with speed u . When the stone is 8 m above the ground its speed is 14 m s^{-1} , and at its highest point its speed is 10 m s^{-1} . Ignore air resistance. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find u . [3]
- (b) Find θ . [2]
- (c) Find the greatest height reached. [2]

- 25** A child of mass 50 kg slides from rest down a playground slide of length 5 m whose top is 3 m above its lower end, and leaves the bottom of the slide horizontally with speed 6 m s^{-1} . She then slides along level ground and comes to rest after 9 m. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the energy lost as she travels down the slide. [3]
- (b) Find the average resistance on the slide. [2]
- (c) Find the coefficient of friction between the child and the level ground. [2]

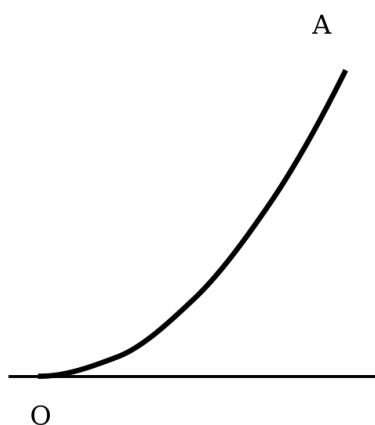
- 26** Particle A of mass 2 kg lies on a smooth plane inclined at 30° to the horizontal. It is connected by a light inextensible string over a smooth pulley at the top of the plane to particle B of mass 3 kg, which hangs 1 m above a horizontal floor. The system is released from rest. B hits the floor and does not rebound, and A does not reach the pulley. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that A has speed 2.8 m s^{-1} when B reaches the floor. [4]
- (b) Find the further distance A travels up the plane before it first comes to rest. [3]
- (c) Find the total time from release until A first comes to rest. [3]

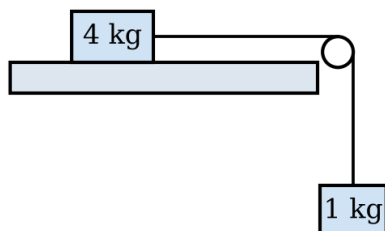
- 27** A pole-vaulter of mass 70 kg runs up to the vaulting box at a speed of 9 m s^{-1} and uses the pole to raise his centre of mass. Model the vaulter as a particle. Take $g = 9.8 \text{ m s}^{-2}$.
- Find his kinetic energy at take-off. [1]
 - If all of this energy were converted into gravitational potential energy, find the height through which his centre of mass would be raised. [2]
 - In a more realistic model, 20% of the kinetic energy is lost, and he is still moving horizontally at 1.2 m s^{-1} at the highest point. Find the height through which his centre of mass is raised. [4]
 - State two assumptions made in the first model. [2]

- 28** A smooth wire is bent into the shape of the curve $y = \frac{x^2}{4}$ for $0 \leq x \leq 4$, where x and y are measured in metres, with the y -axis vertical. A small bead of mass m is released from rest at the point A(4, 4) and slides down the wire. Take $g = 9.8 \text{ m s}^{-2}$.



- Show that the speed of the bead when it is at the point with coordinate x is $v = \sqrt{\frac{g}{2}(16 - x^2)}$. [4]
- Find the speed of the bead when $x = 2$ and when it reaches the origin. [3]
- Taking the x -axis as the level of zero potential energy, find the coordinates of the point on the wire where the kinetic energy equals the potential energy. [3]

- 29** Particle A of mass 4 kg lies on a rough horizontal table and is connected by a light inextensible string over a smooth pulley at the edge of the table to particle B of mass 1 kg. B is held 0.8 m above the floor and the system is released from rest. B hits the floor without rebounding, and A continues to slide a further 0.6 m before coming to rest (without reaching the pulley). Let μ be the coefficient of friction between A and the table. Take $g = 9.8 \text{ m s}^{-2}$.

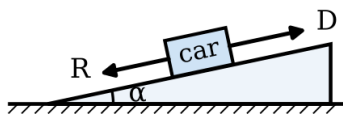


- Show that when B reaches the floor the speed v of the particles satisfies $2.5v^2 = 7.84 - 31.36\mu$. [3]
 - By considering the motion of A after B lands, find a second expression for v^2 in terms of μ . [2]
 - Hence find μ , to 3 significant figures. [3]
 - Find the tension in the string while B is falling. [3]
- 30** A smooth bead B of mass 0.2 kg is threaded on a smooth vertical wire. A light inextensible string is attached to the bead, passes over a smooth peg P which is at a horizontal distance 0.8 m from the wire, and carries a particle C of mass 0.5 kg hanging freely at its other end. The bead is released from rest at the point of the wire level with P. Take $g = 9.8 \text{ m s}^{-2}$.
- When the bead has descended 0.6 m, show that the length of string between the bead and P is 1 m, and that the speed of C is 0.6 times the speed of B. [3]
 - Find the speed of the bead at this instant. [4]
 - Show that the bead first comes to rest after descending a distance $\frac{16}{21} \text{ m}$. [5]

Power

- A car's engine produces a driving force of 800 N while the car travels at a constant speed of 15 m s^{-1} . Find the power of the engine. [2]
- An engine does 360 kJ of work in 20 s. Find its average power in kW. [2]
- A car has a maximum speed on a level road of 20 m s^{-1} when its engine works at 40 kW. Find the resistance to motion, assumed constant. [3]
- A motor raises a load of mass 80 kg at a constant speed of 0.5 m s^{-1} . Find the power of the motor. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- A car of mass 1200 kg is travelling along a straight horizontal road with its engine working at 30 kW. The resistance to motion is 800 N. Find the acceleration of the car when its speed is 10 m s^{-1} . [4]

- 6** A car of mass 900 kg climbs a straight hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{15}$, at a constant speed of 18 m s^{-1} . The resistance to motion is 500 N. Take $g = 9.8 \text{ m s}^{-2}$.
Find the power developed by the engine.



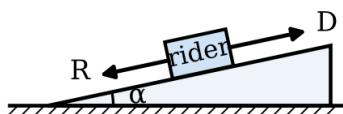
[4]

- 7** A boat's engine works at 36 kW. The resistance to the boat's motion is $40v$ newtons when its speed is $v \text{ m s}^{-1}$. Find the maximum speed of the boat. [4]
- 8** A lorry of mass 6000 kg moves on a straight horizontal road with its engine working at a constant 60 kW. The resistance to motion is a constant 1500 N.
(a) Find the maximum speed of the lorry. [2]
(b) Find the acceleration when the lorry is travelling at 20 m s^{-1} . [3]

- 9** A pump raises 600 kg of water per minute through a vertical height of 12 m and delivers it with a speed of 4 m s^{-1} . Find the power required to do this, ignoring all other losses. Take $g = 9.8 \text{ m s}^{-2}$. [4]

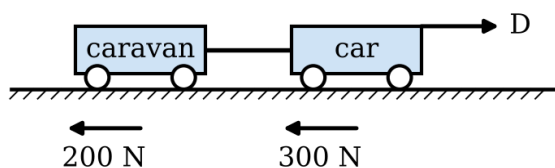
- 10** A hoist has an efficiency of 80%. It raises a load of mass 150 kg at a constant speed of 0.6 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
Find the power that must be supplied to the hoist. [4]

- 11** A cyclist and bicycle have a combined mass of 70 kg. The cyclist rides up a road inclined at an angle α to the horizontal, where $\sin \alpha = 0.04$, working at a constant rate of 250 W against a resistance of 15 N. Take $g = 9.8 \text{ m s}^{-2}$.
Find the steady speed at which the cyclist climbs the hill.



[4]

- 12** A car of mass 1000 kg tows a caravan of mass 600 kg along a straight horizontal road. The resistances to motion are constant at 300 N for the car and 200 N for the caravan. The tow bar is light and horizontal. When the car's engine works at 36 kW and the speed is 12 m s^{-1} , find



- (a) the acceleration of the car, [4]
(b) the tension in the tow bar. [3]

- 13** A car of mass 1000 kg has a maximum speed of 30 m s^{-1} on a level road and a maximum speed of 20 m s^{-1} up a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$. The engine works at the same constant rate in both cases, and the resistance to motion is the same constant R in both cases. Take $g = 9.8 \text{ m s}^{-2}$.

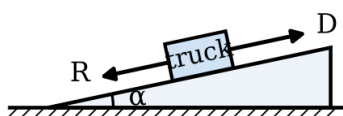
- (a) Find R and the power of the engine. [5]
 (b) The car travels down the same hill with the engine working at the same rate. Find its acceleration when its speed is 20 m s^{-1} . [4]

- 14** A car of mass 1500 kg moves on a straight horizontal road with its engine working at a constant 45 kW against a constant resistance of 1000 N .

- (a) Find the acceleration when the speed is (i) 10 m s^{-1} , (ii) 20 m s^{-1} . [4]
 (b) Find the maximum speed of the car. [2]
 (c) Explain why the acceleration decreases as the speed increases. [1]

- 15** A lorry of mass 5000 kg travels along a straight horizontal road with its engine working at a constant 40 kW . The resistance to motion is a constant 2000 N . The lorry's speed increases from 5 m s^{-1} to 15 m s^{-1} while it travels 200 m . Use the work-energy principle to find the time taken. [5]

- 16** A truck of mass 4000 kg descends a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$, at a constant speed of 20 m s^{-1} with the engine switched off. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that the resistance to motion is 1960 N . [2]
 (b) Find the power of the engine when the truck climbs the same road at a constant speed of 20 m s^{-1} , with the same resistance. [3]
 (c) Find the steady speed up the road when the engine works at 60 kW . [2]

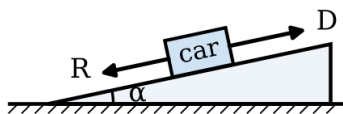
- 17** A crane raises a load of mass 500 kg from rest with a constant acceleration of 0.4 m s^{-2} for 5 s . Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the tension in the cable and the speed of the load after 5 s . [3]
 (b) Find the power being developed by the crane at the end of the 5 s . [1]
 (c) Find the average power during the 5 s . [3]

- 18** Water leaves a hose nozzle of cross-sectional area $5 \times 10^{-4} \text{ m}^2$ with speed 8 m s^{-1} . The density of water is 1000 kg m^{-3} . Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Show that the mass of water leaving the nozzle per second is 4 kg . [2]
 (b) Find the rate at which the kinetic energy of the water is increased. [2]
 (c) The nozzle is on a pump that also raises the water 5 m . Find the total power the pump supplies to the water. [2]

- 19** A car of mass 1500 kg has an engine that works at a constant 50 kW. The resistance to motion at speed v m s⁻¹ is kv newtons, where k is a constant. The maximum speed of the car on a horizontal road is 40 m s⁻¹. Take $g = 9.8$ m s⁻².

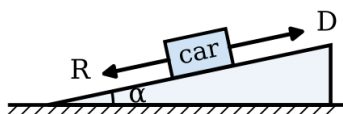


- (a) Show that $k = 31.25$. [2]
 (b) Find the acceleration of the car on the horizontal road when its speed is 20 m s⁻¹. [3]
 (c) The car climbs a hill inclined at an angle α with $\sin \alpha = \frac{1}{20}$. Show that the maximum speed up the hill satisfies $31.25v^2 + 735v - 50\,000 = 0$, and find its value. [4]

- 20** A locomotive of mass 40 000 kg pulls three trucks, each of mass 10 000 kg, up a straight track inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{80}$, at a constant speed of 12 m s⁻¹. The resistance to motion is 2000 N on the locomotive and 500 N on each truck. The couplings are light and parallel to the track. Take $g = 9.8$ m s⁻².
- (a) Find the tension in the coupling between the locomotive and the first truck. [3]
 (b) Find the tension in the coupling between the first and second trucks. [2]
 (c) Find the power developed by the locomotive. [4]

- 21** A lift of total mass 800 kg is raised by a motor working at a constant 12 kW. The resistance to the motion of the lift is a constant 400 N. Take $g = 9.8$ m s⁻².
- (a) Find the speed of the lift when its acceleration is 0.5 m s⁻². [4]
 (b) Find the maximum speed of the lift. [3]

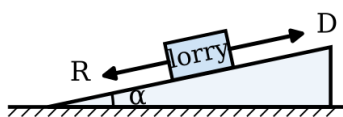
- 22** A car of mass 1200 kg moves up a straight hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$, at a constant speed of 10 m s⁻¹, with the engine working at 14 kW. The resistance to motion is a constant R newtons. Take $g = 9.8$ m s⁻².



- (a) Find R . [3]
 (b) Find the power the engine must develop for the car to travel down the same hill at a constant speed of 10 m s⁻¹. [3]
 (c) Find the maximum speed of the car on a level road when the engine works at 14 kW. [2]
- 23** A lorry of mass 8000 kg works at a constant 60 kW going up a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{40}$, against a constant resistance of 1500 N. Its speed increases from 8 m s⁻¹ to 12 m s⁻¹ while it travels 250 m along the road. Take $g = 9.8$ m s⁻². Find the time taken. [6]

- 24** The resistance to the motion of a motor boat is proportional to the square of its speed. When the boat travels at a constant speed of 20 m s^{-1} the engine works at 8 kW .
- Show that the power required for a constant speed v is proportional to v^3 . [2]
 - Find the power required for a constant speed of 30 m s^{-1} . [2]
 - Find the maximum speed of the boat if the engine's greatest power is 40 kW . [2]
-
- 25** A cyclist and bicycle of total mass 80 kg freewheel down a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{25}$, at a constant speed of 7 m s^{-1} . The resistance to motion is kv^2 newtons when the speed is $v \text{ m s}^{-1}$, where k is a constant. Take $g = 9.8 \text{ m s}^{-2}$.
- Show that $k = 0.64$. [3]
 - Find the power the cyclist must develop to climb the same road at a constant speed of 7 m s^{-1} . [3]
 - The cyclist's power output is limited to 500 W . Show that the greatest steady speed v up the road satisfies $0.64v^3 + 31.36v - 500 = 0$ and find its value. [4]
-
- 26** A man of mass 75 kg jogs up a straight road inclined at 5° to the horizontal at a constant speed of 3 m s^{-1} . He experiences a constant wind assistance of 20 N in the direction of motion and a constant resistance of 30 N opposing the motion. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the power the man develops. [4]
 - The man works at a constant 300 W on the same road. Find his steady speed up the road. [3]
-
- 27** A car of mass 1000 kg travels along a straight horizontal road with its engine working at a constant 20 kW against a constant resistance of 800 N . Let $v \text{ m s}^{-1}$ be its speed at time t seconds.
- Show that $1000v \frac{dv}{dt} = 20\,000 - 800v$. [2]
 - Show that the time taken to accelerate from 5 m s^{-1} to 15 m s^{-1} is given by
$$t = 1000 \int_5^{15} \frac{v}{20\,000 - 800v} dv.$$
 [2]
 - Hence find the time, giving your answer to 3 significant figures. [6]
-
- 28** A car of mass 1500 kg has an engine whose driving force cannot exceed 4000 N (because of limited grip) and whose power cannot exceed 60 kW . The resistance to motion is a constant 1000 N . The car accelerates from rest along a straight horizontal road, using the greatest possible driving force at each stage.
- Show that the driving force is a constant 4000 N until the speed reaches 15 m s^{-1} , and find the time taken to reach this speed. [5]
 - Find the maximum speed of the car. [2]
 - After reaching 15 m s^{-1} , the car travels at constant power 60 kW and its speed increases to 30 m s^{-1} over a distance of 300 m . Find the time taken for this stage. [5]
-
- 29** A sprinter of mass 80 kg accelerates from rest along a straight track, and her acceleration at time t seconds is $(6 - t) \text{ m s}^{-2}$ for $0 \leq t \leq 6$. Model the net force on her as m times her acceleration, and define the power as that force multiplied by her speed.
- Show that her speed at time t is $v = 6t - \frac{1}{2}t^2$. [3]
 - Find the power at $t = 3$. [3]
 - Show that the power is a maximum when $t^2 - 12t + 24 = 0$, and find this maximum power to 3 significant figures. [5]

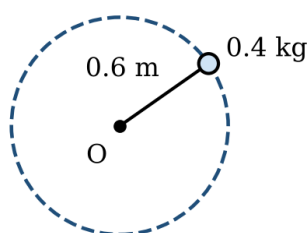
- 30 A lorry of mass 2000 kg has an engine working at a constant rate P . The resistance to motion at speed $v \text{ m s}^{-1}$ is kv newtons. The lorry moves at a constant speed of 10 m s^{-1} up a hill inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$, and at a constant speed of 20 m s^{-1} down the same hill. Take $g = 9.8 \text{ m s}^{-2}$.



- Find k and P . [5]
- Find the maximum speed of the lorry on a level road. [2]
- Find the speed at which the lorry descends the hill with the engine switched off. [2]
- Check that your value of P is consistent with the maximum speed up the hill being 10 m s^{-1} , by showing that $v = 10$ is the positive root of $98v^2 + 980v - 19\,600 = 0$. [1]

Uniform Circular Motion

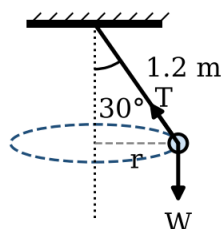
- A particle moves in a horizontal circle of radius 0.8 m , completing 5 revolutions in 20 s . Find its angular speed and its speed. [3]
- A particle moves at a constant speed of 8 m s^{-1} in a circle of radius 2 m . Find its acceleration. [2]
- A particle moves in a circle with a constant angular speed of 3 rad s^{-1} . Find the time taken for one complete revolution. [2]
- A car of mass 1200 kg travels at a constant speed of 10 m s^{-1} round a circular bend of radius 50 m .
 - Find the acceleration of the car. [1]
 - Find the horizontal resultant force on the car. [1]
- A particle of mass 0.4 kg moves in a horizontal circle of radius 0.6 m on a smooth horizontal table. It is attached to a fixed point O on the table by a light inextensible string, and moves with constant angular speed 5 rad s^{-1} . Find the tension in the string.



- Find the tension in the string. [3]
- A coin lies on a rough horizontal turntable at a distance of 0.12 m from the axis of rotation. The coefficient of friction between the coin and the turntable is 0.4 . The turntable rotates with gradually increasing angular speed. Take $g = 9.8 \text{ m s}^{-2}$. Find the greatest angular speed at which the coin stays at rest relative to the turntable, in rad s^{-1} and in revolutions per minute. [4]
- A car travels round a flat circular bend of radius 60 m . The coefficient of friction between the tyres and the road is 0.5 . Take $g = 9.8 \text{ m s}^{-2}$. Find the greatest speed at which the car can take the bend without skidding. [4]

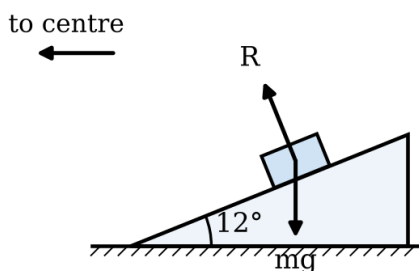
- 8** A cyclist rides round a circular bend of radius 20 m at a constant speed of 8 m s^{-1} . To balance, the cyclist and bicycle lean at an angle θ to the vertical, the resultant contact force from the ground acting along the line of the bicycle. Take $g = 9.8 \text{ m s}^{-2}$. Find θ . [4]

- 9** A particle of mass 0.4 kg is attached to one end of a light inextensible string of length 1.2 m. The other end is fixed, and the particle moves in a horizontal circle with the string making a constant angle of 30° with the vertical (a conical pendulum). Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the tension in the string. [2]
 (b) Find the angular speed and the period of the motion. [4]

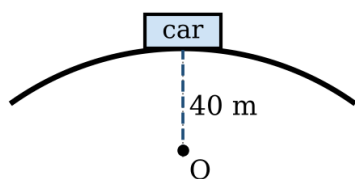
- 10** A car travels at a constant speed round a circular bend of radius 80 m on a road banked at 12° to the horizontal. At the design speed the car has no tendency to slip sideways, so no friction is needed. Take $g = 9.8 \text{ m s}^{-2}$. Find the design speed.



[5]

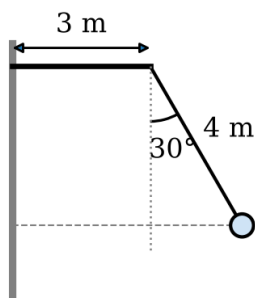
- 11** A particle of mass 0.3 kg lies on a rough horizontal disc, 0.5 m from its centre, where the coefficient of friction is 0.6. The disc rotates about its vertical axis. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the greatest angular speed for which the particle remains at rest relative to the disc. [3]
 (b) The particle is now joined to the centre of the disc by a light string, which is taut. The disc rotates at 4 rad s^{-1} and the particle is on the point of slipping outwards. Find the tension in the string. [3]

- 12** A car of mass 1000 kg passes over the top of a humpback bridge, the top of the bridge being an arc of a vertical circle of radius 40 m. Take $g = 9.8 \text{ m s}^{-2}$.



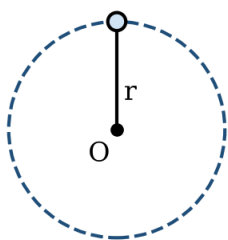
- (a) Find the normal reaction of the road on the car when its speed at the top is 10 m s^{-1} . [3]
 (b) Find the speed at the top at which the car just loses contact with the road. [2]

- 13** In a fairground ride, a seat of mass 80 kg (including its rider) hangs from a rotating horizontal arm of radius 3 m by a light chain of length 4 m. When the ride rotates at constant angular speed ω the chain makes a constant angle of 30° with the vertical. Treat the seat and rider as a particle. Take $g = 9.8 \text{ m s}^{-2}$.



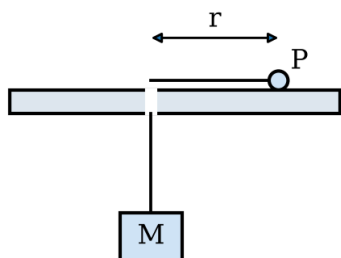
- (a) Show that the particle moves in a circle of radius 5 m. [1]
 (b) Find the tension in the chain. [2]
 (c) Find ω . [3]

- 14** A particle of mass 0.2 kg is attached to one end of a light inextensible string of length 0.9 m and moves in a vertical circle. The other end of the string is fixed at O. At the highest point of the circle the speed of the particle is 4 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the tension in the string at the highest point. [1]
 (b) Find the least speed at the highest point if the string is to remain taut. [2]

- 15** A particle P of mass 0.5 kg moves in a horizontal circle of radius 0.4 m on a smooth table. It is attached to a light inextensible string which passes through a small smooth hole in the table and carries a particle M of mass 2 kg hanging in equilibrium below the table. Take $g = 9.8 \text{ m s}^{-2}$.

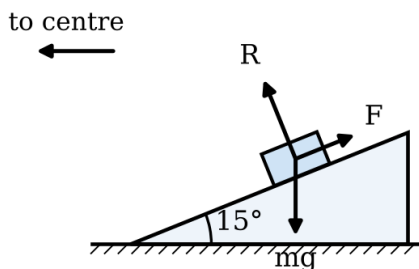


- (a) Find the tension in the string. [1]
 (b) Find the speed of P. [2]
 (c) Find the time taken for P to complete one revolution. [2]

- 16** A smooth hollow hemispherical bowl of radius 0.5 m is fixed with its rim horizontal and uppermost. A marble of mass 0.05 kg moves with constant speed in a horizontal circle on the inside surface of the bowl, in a plane 0.3 m below the centre O of the bowl. Take $g = 9.8 \text{ m s}^{-2}$.

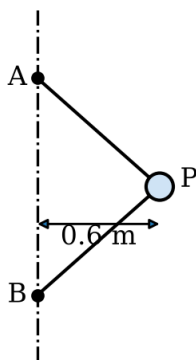
- (a) Show that the radius of the circle is 0.4 m, and find the normal reaction between the marble and the bowl. [4]
 (b) Find the speed of the marble. [3]

- 17** A road is banked at 15° to the horizontal at a circular bend of radius 50 m. The coefficient of friction between the tyres of a car and the road is 0.2. Take $g = 9.8 \text{ m s}^{-2}$.



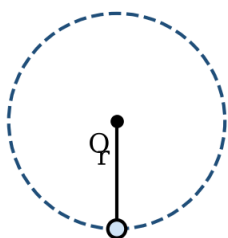
- (a) Show that the greatest speed at which the car can take the bend without slipping up the slope satisfies $v^2 = \frac{rg(\tan 15^\circ + 0.2)}{1 - 0.2 \tan 15^\circ}$, and find this speed. [5]
- (b) Find the least speed at which the car can take the bend without slipping down the slope. [4]

- 18** A particle P of mass 0.5 kg is attached by two light inextensible strings, each of length 1 m, to points A and B on a vertical axis, with A 1.6 m above B. P moves in a horizontal circle of radius 0.6 m about the axis with constant angular speed ω , with both strings taut. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that each string makes an angle with the vertical whose cosine is 0.8. [2]
- (b) Given that $\omega = 6 \text{ rad s}^{-1}$, find the tensions T_A and T_B . [5]
- (c) Find the least value of ω for which both strings remain taut. [3]

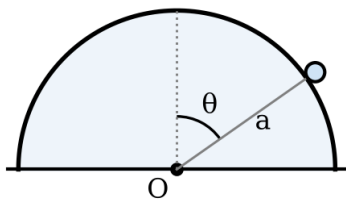
- 19** A particle of mass 0.4 kg is attached to one end of a light inextensible string of length 1.5 m, the other end being fixed at O. The particle is projected horizontally from its lowest point with speed 10 m s^{-1} and moves in a vertical circle. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the tension in the string at the lowest point. [2]
- (b) Find the speed of the particle at the highest point, and the tension there. [5]
- (c) Find the least speed of projection at the lowest point for the particle to complete the circle. [3]

- 20** A roller-coaster car of mass 400 kg runs along a smooth track and round a vertical circular loop of radius 8 m. Model the car as a particle. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the least speed of the car at the top of the loop if it is to stay in contact with the track. [2]
 - The car starts from rest at a point on the smooth track at height h above the bottom of the loop. Find the least value of h for the car to complete the loop. [4]
 - At the top of the loop the car has speed 12 m s^{-1} . Find the force exerted by the track on the car. [2]
-
- 21** A satellite of mass m moves in a circular orbit of radius r about the Earth under the gravitational force $\frac{GMm}{r^2}$, where $GM = 3.99 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$.
- Show that the speed of the satellite is $v = \sqrt{\frac{GM}{r}}$. [2]
 - The satellite orbits $2.0 \times 10^6 \text{ m}$ above the Earth's surface. The radius of the Earth is $6.371 \times 10^6 \text{ m}$. Find the speed of the satellite, to 3 significant figures. [3]
 - Find the period of the orbit in seconds and in hours. [3]
-
- 22** In a fairground rotor ride, riders stand with their backs against the inside wall of a vertical cylinder of radius 2.5 m. The cylinder rotates about its vertical axis and the floor is then lowered, so riders are held up by friction. The coefficient of friction between rider and wall is 0.4. Model a rider of mass 70 kg as a particle. Take $g = 9.8 \text{ m s}^{-2}$.
- Explain what provides the centripetal force and what balances the rider's weight. [2]
 - Find the least angular speed needed so that the rider does not slip, in rad s^{-1} and revolutions per minute. [4]
 - The cylinder rotates at 4 rad s^{-1} . Find the normal reaction on the rider and the friction force. [2]
-
- 23** A light rod OA of length 1.2 m rotates in a horizontal plane about the fixed end O with constant angular speed ω . Particles of mass m and $2m$ are attached to the rod at B and A respectively, where $OB = 0.4 \text{ m}$ and $OA = 1.2 \text{ m}$. Find the ratio of the tension in OB to the tension in BA. [5]
-
- 24** A smooth hollow cone has its axis vertical and its vertex downwards, and its semi-vertical angle is 30° . A particle of mass 0.4 kg moves with constant speed in a horizontal circle of radius 0.5 m on the inside surface of the cone. Take $g = 9.8 \text{ m s}^{-2}$.
- Show that the normal reaction between the particle and the cone is 7.84 N. [3]
 - Find the speed of the particle and the time taken for one complete circle. [4]
-
- 25** A geostationary satellite orbits the Earth in a circle in the equatorial plane, and stays above the same point of the Earth's surface. The Earth turns through one revolution in 86 164 s. The gravitational force on the satellite is $\frac{GMm}{r^2}$, where $GM = 3.986 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ and the radius of the Earth is $6.371 \times 10^6 \text{ m}$. Find the radius of the orbit and the height of the satellite above the Earth's surface, each to 3 significant figures. [5]

- 26** A particle P is placed at the highest point of a smooth fixed hemisphere of radius a and is given a very small horizontal displacement, so that it slides down the surface starting effectively from rest. Let θ be the angle between OP and the upward vertical, where O is the centre of the hemisphere. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that while P remains on the surface, $v^2 = 2ga(1 - \cos \theta)$. [3]
- (b) Show that P leaves the surface when $\cos \theta = \frac{2}{3}$. [4]
- (c) For $a = 1.5 \text{ m}$, find the speed of P and its height above the base plane of the hemisphere at the instant it leaves the surface. [3]
-
- 27** A particle of mass m is attached to one end of a light inextensible string of length L . The other end is fixed at O. The particle hangs at rest vertically below O and is given a horizontal speed u , so that it moves in a vertical circle. When the string makes an angle θ with the downward vertical (and remains taut) the tension is T . Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that $T = \frac{mu^2}{L} + mg(3 \cos \theta - 2)$. [5]
- (b) Given that $u^2 = 3gL$, show that the string goes slack when $\cos \theta = -\frac{1}{3}$, and find the angle of the string above the horizontal at that point. [4]
- (c) For $L = 0.8 \text{ m}$ and $m = 0.3 \text{ kg}$ with $u^2 = 3gL$, find u , and the tension when $\theta = 60^\circ$. [3]
-
- 28** A car travels at constant speed v round a circular bend of radius r on a road banked at an angle θ to the horizontal. The coefficient of friction between the tyres and the road is μ , where $\mu \tan \theta < 1$.
- (a) Show that the greatest speed at which the car can take the bend without slipping outwards is given by $v^2 = rg \frac{\tan \theta + \mu}{1 - \mu \tan \theta}$. [5]
- (b) Find this greatest speed for $r = 120 \text{ m}$, $\theta = 8^\circ$ and $\mu = 0.35$, and compare it with the speed v_0 for which no friction is needed. [4]
- (c) Explain what happens if $\mu \tan \theta \geq 1$. [1]
-
- 29** A conical pendulum consists of a particle of mass m on a light inextensible string of length L , moving in a horizontal circle with constant angular speed ω , with the string making an angle θ with the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the tension is $T = mL\omega^2$, and that $\cos \theta = \frac{g}{L\omega^2}$. [4]
- (b) Deduce the least angular speed for which this motion is possible. [2]
- (c) For $L = 0.8 \text{ m}$, $m = 0.25 \text{ kg}$ and $\omega = 6 \text{ rad s}^{-1}$, find T and θ . [3]
- (d) Show that the period is $2\pi \sqrt{\frac{L \cos \theta}{g}}$ and find its limiting value as θ tends to 0, evaluating it for $L = 0.8 \text{ m}$. [3]

- 30** A particle P of mass m is attached to one end of a light rigid rod of length a , whose other end is freely hinged at a fixed point O. P describes complete vertical circles about O, and its greatest speed is three times its least speed.
Let u be the least speed.
- (a) Show that $u^2 = \frac{ga}{2}$. [4]
- (b) Find the force exerted by the rod on P when P is at the highest point, stating whether it is a tension or a thrust. [3]
- (c) Find the tension in the rod when P is at the lowest point. [3]
-