

Vector Notation and Operations

1 (2 marks)

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \text{B1}$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \quad \text{B1}$$

$$\text{Answer: } \mathbf{a} + \mathbf{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \mathbf{a} - \mathbf{b} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \quad \text{2 marks}$$

2 (3 marks)

$$5\mathbf{i} - 2\mathbf{j} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \text{B1}$$

$$3(5\mathbf{i} - 2\mathbf{j}) - (\mathbf{i} + 4\mathbf{j}) = 15\mathbf{i} - 6\mathbf{j} - \mathbf{i} - 4\mathbf{j} \quad \text{M1}$$

$$= 14\mathbf{i} - 10\mathbf{j} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} 5 \\ -2 \end{pmatrix}; 14\mathbf{i} - 10\mathbf{j} \quad \text{3 marks}$$

3 (4 marks)

$$\mathbf{BC} = \mathbf{AD} = \mathbf{d} \quad \text{B1}$$

$$\mathbf{DC} = \mathbf{AB} = \mathbf{a} \quad \text{B1}$$

$$\mathbf{BD} = \mathbf{BA} + \mathbf{AD} = -\mathbf{a} + \mathbf{d} \quad \text{B1}$$

$$\mathbf{CA} = -\mathbf{AC} = -\mathbf{a} - \mathbf{d} \quad \text{B1}$$

$$\text{Answer: } \mathbf{BC} = \mathbf{d}, \mathbf{DC} = \mathbf{a}, \mathbf{BD} = \mathbf{d} - \mathbf{a}, \mathbf{CA} = -\mathbf{a} - \mathbf{d} \quad \text{4 marks}$$

4 (2 marks)

$$\text{Equate the } \mathbf{i} \text{ components: } x + 2 = 5, \text{ so } x = 3 \quad \text{B1}$$

$$\text{Equate the } \mathbf{j} \text{ components: } 3 - y = -2, \text{ so } y = 5 \quad \text{B1}$$

$$\text{Answer: } x = 3, y = 5 \quad \text{2 marks}$$

5 (2 marks)

$$\text{Required vector} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \text{2 marks}$$

6 (3 marks)

$$\mathbf{q} = 3\mathbf{p} \quad \text{B1}$$

$$\mathbf{r} = -2\mathbf{p} \quad \text{B1}$$

$$s \text{ is not a multiple of } \mathbf{p} \quad (3 \div 2 \neq 2 \div (-3)) \quad \text{B1}$$

$$\text{Answer: } \mathbf{q} \text{ (3p) and } \mathbf{r} \text{ (-2p) are parallel to } \mathbf{p} \quad \text{3 marks}$$

7 (2 marks)

$$\mathbf{q} = \lambda\mathbf{p}: \text{comparing } \mathbf{j} \text{ components, } 2\lambda = -6 \text{ so } \lambda = -3 \quad \text{M1}$$

$$\text{Comparing } \mathbf{i} \text{ components, } k = 3\lambda = -9 \quad \text{A1}$$

$$\text{Answer: } k = -9 \quad \text{2 marks}$$

8 (4 marks)

$$\text{(a) } \mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \mathbf{a} + \mathbf{c} \text{ (since } \mathbf{AB} = \mathbf{OC}) \quad \text{B1}$$

$$\text{Answer: } \mathbf{a} + \mathbf{c} \quad \text{1 mark}$$

$$\text{(b) } \mathbf{AC} = \mathbf{AO} + \mathbf{OC} = -\mathbf{a} + \mathbf{c} \quad \text{B1}$$

$$\text{Answer: } \mathbf{c} - \mathbf{a} \quad \text{1 mark}$$

$$\text{(c) } \mathbf{AB} = \mathbf{OC}, \text{ so } \mathbf{AM} = \frac{1}{2}\mathbf{c} \quad \text{M1}$$

$$\mathbf{OM} = \mathbf{OA} + \mathbf{AM} = \mathbf{a} + \frac{1}{2}\mathbf{c}$$

A1

$$\text{Answer: } \mathbf{a} + \frac{1}{2}\mathbf{c}$$

2 marks

9 (3 marks)

$$2\mathbf{x} - \mathbf{a} = 3\mathbf{x} + 3\mathbf{b}$$

M1

$$\mathbf{x} = -\mathbf{a} - 3\mathbf{b}$$

M1

$$= \begin{pmatrix} -1 \\ -2 \end{pmatrix} - \begin{pmatrix} -9 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 8 \\ -5 \end{pmatrix}$$

3 marks

10 (4 marks)

$$2m + n = 8 \text{ and } m + 3n = 9$$

M1

$$\text{From the first, } n = 8 - 2m; \text{ substitute: } m + 3(8 - 2m) = 9$$

M1

$$24 - 5m = 9, \text{ so } m = 3$$

A1

$$n = 8 - 6 = 2$$

A1

$$\text{Answer: } m = 3, n = 2$$

4 marks

11 (4 marks)

$$\mathbf{AC} = \mathbf{AB} + \mathbf{BC} = \mathbf{p} + \mathbf{q}$$

B1

$$\mathbf{CA} = -\mathbf{AC} = -\mathbf{p} - \mathbf{q}$$

B1

$$\mathbf{CB} = -\mathbf{q}$$

B1

$$\mathbf{AB} - \mathbf{CB} = \mathbf{p} + \mathbf{q}$$

B1

$$\text{Answer: (a) } \mathbf{p} + \mathbf{q} \text{ (b) } -\mathbf{p} - \mathbf{q} \text{ (c) } \mathbf{p} + \mathbf{q}$$

4 marks

12 (4 marks)

$$\mathbf{AB} = \mathbf{AO} + \mathbf{OB} = \mathbf{b} - \mathbf{a}$$

B1

$$\mathbf{AM} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

M1

$$\mathbf{OM} = \mathbf{OA} + \mathbf{AM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

M1

$$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

A1

$$\text{Answer: } \mathbf{AB} = \mathbf{b} - \mathbf{a}, \mathbf{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

4 marks

13 (3 marks)

$$\text{Resultant} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

M1

$$= \begin{pmatrix} -2 \\ 6 \end{pmatrix} \text{ km}$$

A1

$$\text{Return displacement} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \text{ km}$$

B1

$$\text{Answer: } \begin{pmatrix} -2 \\ 6 \end{pmatrix} \text{ km; } \begin{pmatrix} 2 \\ -6 \end{pmatrix} \text{ km}$$

3 marks

14 (4 marks)

$$\mathbf{p} = \lambda\mathbf{q}: \text{comparing the } \mathbf{j} \text{ components, } 6 = 3\lambda, \text{ so } \lambda = 2$$

M1

$$\text{Comparing the } \mathbf{i} \text{ components: } x^2 - 1 = 2(x + 1)$$

M1

$$x^2 - 2x - 3 = 0, \text{ so } (x - 3)(x + 1) = 0$$

A1

$$x = 3 \text{ or } x = -1$$

A1

$$\text{Answer: } x = 3 \text{ or } x = -1$$

4 marks

15 (4 marks)

$$(a) \text{ Position after leg 2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

M1

$$d = \begin{pmatrix} 8 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

2 marks

$$(b) \text{ Fourth leg} = \mathbf{O} - \begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

M1

$$= \begin{pmatrix} -8 \\ -5 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} -8 \\ -5 \end{pmatrix}$$

2 marks

16 (4 marks)

Equate coefficients (a and b are not parallel): $p + q = 5$ and $2p - q = 1$

M1 A1

Adding: $3p = 6$, so $p = 2$

M1

$q = 5 - 2 = 3$

A1

Answer: $p = 2$, $q = 3$

4 marks

17 (5 marks)

$$AC = AD + DC = d + \frac{3}{2}a$$

M1 A1

$$BD = BA + AD = -a + d$$

B1

$$BC = BA + AC = -a + d + \frac{3}{2}a$$

M1

$$= \frac{1}{2}a + d$$

A1

$$\text{Answer: (a) } d + \frac{3}{2}a \text{ (b) } d - a \text{ (c) } \frac{1}{2}a + d$$

5 marks

18 (5 marks)

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

M1

$$\mathbf{F}_3 = -\begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} N$$

A1

$$\mathbf{F}_1 + t\mathbf{F}_2 = \begin{pmatrix} 3 - 5t \\ -2 + 4t \end{pmatrix}$$

M1

Parallel to $(1, 0)$ means the second component is 0: $-2 + 4t = 0$

M1

$$t = \frac{1}{2}$$

A1

$$\text{Answer: } \mathbf{F}_3 = \begin{pmatrix} 2 \\ -2 \end{pmatrix} N; t = \frac{1}{2}$$

5 marks

19 (7 marks)

Let O be the centre. OABC is a rhombus (all four sides equal), so $\mathbf{OC} = \mathbf{AB} = \mathbf{a}$

B1

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \mathbf{a} - \mathbf{b}$, and $\mathbf{OA} = \mathbf{OB} + \mathbf{BA} = -\mathbf{b}$

M1 A1

O is the centre, so $\mathbf{OD} = -\mathbf{OA} = \mathbf{b}$, and $\mathbf{CD} = \mathbf{OD} - \mathbf{OC} = \mathbf{b} - \mathbf{a}$

A1

$$\mathbf{AD} = 2\mathbf{OD} = 2\mathbf{b}$$

B1

$\mathbf{OE} = -\mathbf{OB} = \mathbf{b} - \mathbf{a}$, so $\mathbf{AE} = \mathbf{OE} - \mathbf{OA}$

M1

$$= \mathbf{b} - \mathbf{a} + \mathbf{b} = 2\mathbf{b} - \mathbf{a}$$

A1

Answer: (a) $\mathbf{b} - \mathbf{a}$ (b) $2\mathbf{b}$ (c) $2\mathbf{b} - \mathbf{a}$

7 marks

20 (4 marks)

$$\mathbf{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{BC} = \begin{pmatrix} 9 \\ 18 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{AB} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \mathbf{BC} = 5\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \text{ so AB and BC are parallel with a common point B, hence collinear} \quad \text{A1}$$

$$\mathbf{AB} : \mathbf{BC} = 3 : 5 \quad \text{A1}$$

Answer: Collinear, AB : BC = 3 : 5 4 marks

21 (9 marks)

(a) If either bracket were non-zero, then **a** would be a multiple of **b**, contradicting that they are not parallel; so both brackets are zero. E1

Answer: Both coefficients must be 0. 1 mark

(b) $\lambda + 2\mu = 3$ (1) $2\lambda - \mu = -4$ (2)

(1) $\times 2 -$ (2): $5\mu = 10$, so $\mu = 2$ M1 A1

Then $\lambda = 3 - 4 = -1$ M1 A1

Answer: $\lambda = -1$, $\mu = 2$ 4 marks

(c) $(k+1)\mathbf{a} + 6\mathbf{b} = t(2\mathbf{a} + k\mathbf{b})$ for some scalar t M1

$k+1 = 2t$ and $6 = tk$ M1

Eliminating t : $k+1 = 12/k$, so $k^2 + k - 12 = 0$ M1

$(k+4)(k-3) = 0$, so $k = 3$ or $k = -4$ A1

Answer: $k = 3$ or $k = -4$ 4 marks

22 (9 marks)

(a) $\mathbf{AB} = \mathbf{AO} + \mathbf{OB} = \mathbf{b} - \mathbf{a}$ M1

OABC is a rhombus ($OA = AB = BC = CO$), so $\mathbf{OC} = \mathbf{AB} = \mathbf{b} - \mathbf{a}$ E1

Answer: $\mathbf{OC} = \mathbf{b} - \mathbf{a}$ 2 marks

(b) $\mathbf{AC} = \mathbf{OC} - \mathbf{OA}$ M1

$= (\mathbf{b} - \mathbf{a}) - \mathbf{a} = \mathbf{b} - 2\mathbf{a}$ A1

Answer: $\mathbf{b} - 2\mathbf{a}$ 2 marks

(c) F is opposite C, so $\mathbf{OF} = -\mathbf{OC} = \mathbf{a} - \mathbf{b}$; D is opposite A, so $\mathbf{OD} = -\mathbf{a}$ B1

$\mathbf{FD} = \mathbf{OD} - \mathbf{OF} = -\mathbf{a} - (\mathbf{a} - \mathbf{b}) = \mathbf{b} - 2\mathbf{a}$ M1

Equal to AC, so AC and FD are parallel and equal in length. A1

Answer: $\mathbf{FD} = \mathbf{b} - 2\mathbf{a}$; AC and FD are equal and parallel 3 marks

(d) $\mathbf{FC} = \mathbf{OC} - \mathbf{OF} = (\mathbf{b} - \mathbf{a}) - (\mathbf{a} - \mathbf{b}) = 2(\mathbf{b} - \mathbf{a})$ M1

$= 2\mathbf{AB}$ A1

Answer: Shown 2 marks

23 (3 marks)

$\mathbf{CA} = -\mathbf{AC}$, not $\mathbf{AC}$: reversing the direction changes the sign. E1

$\mathbf{AB} + \mathbf{BC} + \mathbf{CA} = \mathbf{AC} - \mathbf{AC}$ M1

$= \mathbf{0}$: the vectors form a closed triangle. A1

Answer: Correct sum is the zero vector 3 marks

24 (6 marks)

$$k\mathbf{a} + \mathbf{b} = \begin{pmatrix} k+3 \\ 2k-1 \end{pmatrix}, \mathbf{a} + k\mathbf{b} = \begin{pmatrix} 1+3k \\ 2-k \end{pmatrix} \quad \text{B1 B1}$$

Parallel: $(k+3)(2-k) = (2k-1)(1+3k)$ M1

LHS $= -k^2 - k + 6$, RHS $= 6k^2 - k - 1$ M1

$7k^2 = 7$, so $k^2 = 1$ M1

$k = 1$ or $k = -1$ A1

Answer: $k = 1$ or $k = -1$ 6 marks

25 (6 marks)

$$\mathbf{OP} = \frac{1}{3}\mathbf{a}, \mathbf{OQ} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) \quad \text{B1 B1}$$

$$\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP} = \frac{1}{6}\mathbf{a} + \frac{1}{2}\mathbf{b} \quad \text{M1}$$

$$\mathbf{OR} = \mathbf{OP} + \mu\mathbf{PQ} = \left(\frac{1}{3} + \frac{\mu}{6}\right)\mathbf{a} + \frac{\mu}{2}\mathbf{b} \quad \text{M1}$$

$$\text{R is on line OB, so there is no a component: } \frac{1}{3} + \frac{\mu}{6} = 0, \text{ giving } \mu = -2 \quad \text{M1}$$

$$\mathbf{OR} = -\mathbf{b}; \text{ R is the reflection of B in O.} \quad \text{A1}$$

$$\text{Answer: } \mathbf{OR} = -\mathbf{b} \quad \text{6 marks}$$

26 (5 marks)

$$\text{Let OABC be a parallelogram with } \mathbf{OA} = \mathbf{a} \text{ and } \mathbf{OC} = \mathbf{c}, \text{ so } \mathbf{OB} = \mathbf{a} + \mathbf{c} \quad \text{B1}$$

$$\text{Midpoint of OB: } \frac{1}{2}(\mathbf{a} + \mathbf{c}) \quad \text{M1}$$

$$\text{Midpoint of AC: } \frac{1}{2}(\mathbf{a} + \mathbf{c}) \quad \text{M1 A1}$$

$$\text{Both diagonals have the same midpoint, so each bisects the other.} \quad \text{E1}$$

$$\text{Answer: Proved} \quad \text{5 marks}$$

27 (9 marks)

$$\text{(a) } \mathbf{OP} = \frac{1}{2}(\mathbf{a} + \mathbf{b}), \mathbf{OQ} = \frac{1}{2}(\mathbf{b} + \mathbf{c}), \mathbf{OR} = \frac{1}{2}(\mathbf{c} + \mathbf{d}), \mathbf{OS} = \frac{1}{2}(\mathbf{d} + \mathbf{a}) \quad \text{B1}$$

$$\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP} = \frac{1}{2}(\mathbf{c} - \mathbf{a}) \quad \text{M1 A1}$$

$$\mathbf{SR} = \mathbf{OR} - \mathbf{OS} = \frac{1}{2}(\mathbf{c} - \mathbf{a}) \quad \text{M1 A1}$$

$$\text{So PQ and SR are equal in length and parallel, hence PQRS is a parallelogram.} \quad \text{E1}$$

$$\text{Answer: Proved} \quad \text{6 marks}$$

$$\text{(b) } \mathbf{QR} = \mathbf{OR} - \mathbf{OQ} = \frac{1}{2}(\mathbf{d} - \mathbf{b}) = \frac{1}{2}\mathbf{BD} \quad \text{M1}$$

$$|\mathbf{PQ}| = \frac{1}{2}|\mathbf{AC}| \text{ and } |\mathbf{QR}| = \frac{1}{2}|\mathbf{BD}| \quad \text{A1}$$

$$\text{AC = BD, so } |\mathbf{PQ}| = |\mathbf{QR}|; \text{ a parallelogram with two equal adjacent sides is a rhombus.} \quad \text{E1}$$

$$\text{Answer: Proved} \quad \text{3 marks}$$

28 (8 marks)

$$\text{(a) Parallel means the vector is a scalar multiple of } \mathbf{i} + 2\mathbf{j}, \text{ not equal to it.} \quad \text{B1}$$

$$\text{The student should have written } (2 + \lambda)\mathbf{i} + (1 - 3\lambda)\mathbf{j} = \mu(\mathbf{i} + 2\mathbf{j}). \quad \text{E1}$$

$$\text{Answer: Parallel means a multiple, not equal.} \quad \text{2 marks}$$

$$\text{(b) } (2 + \lambda)\mathbf{i} + (1 - 3\lambda)\mathbf{j} = \mu(\mathbf{i} + 2\mathbf{j}) \quad \text{M1}$$

$$2 + \lambda = \mu \text{ and } 1 - 3\lambda = 2\mu \quad \text{M1}$$

$$1 - 3\lambda = 2(2 + \lambda), \text{ so } 1 - 3\lambda = 4 + 2\lambda \quad \text{A1}$$

$$5\lambda = -3, \text{ so } \lambda = -\frac{3}{5} \quad \text{A1}$$

$$\text{Answer: } \lambda = -\frac{3}{5} \quad \text{4 marks}$$

$$\text{(c) } \mathbf{a} + \lambda\mathbf{b} = \left(2 - \frac{3}{5}\right)\mathbf{i} + \left(1 + \frac{9}{5}\right)\mathbf{j} \quad \text{M1}$$

$$= \frac{7}{5}\mathbf{i} + \frac{14}{5}\mathbf{j} = \frac{7}{5}(\mathbf{i} + 2\mathbf{j}) \quad \text{A1}$$

$$\text{Answer: } \frac{7}{5}(\mathbf{i} + 2\mathbf{j}) \quad \text{2 marks}$$

29 (8 marks)

$$(a) \mathbf{BC} = \mathbf{BA} + \mathbf{AD} + \mathbf{DC} = (\lambda - 1)\mathbf{a} + \mathbf{d}$$

M1 A1

$$\mathbf{AN} = \mathbf{AB} + \frac{1}{2}\mathbf{BC} = \mathbf{a} + \frac{(\lambda - 1)}{2}\mathbf{a} + \frac{1}{2}\mathbf{d}$$

M1

$$\mathbf{AM} = \frac{1}{2}\mathbf{d}$$

B1

$$\mathbf{MN} = \mathbf{AN} - \mathbf{AM} = \frac{(\lambda + 1)}{2}\mathbf{a}$$

A1

$$\text{Answer: } \mathbf{MN} = \frac{(\lambda + 1)}{2}\mathbf{a}$$

5 marks

(b) $\mathbf{MN}$ is a scalar multiple of $\mathbf{a}$, so it is parallel to $\mathbf{AB}$ and $\mathbf{DC}$.

E1

$$|\mathbf{MN}| = \frac{1}{2}(1 + \lambda)|\mathbf{a}|$$

M1

$$|\mathbf{AB}| = |\mathbf{a}| \text{ and } |\mathbf{DC}| = \lambda|\mathbf{a}|, \text{ so } |\mathbf{MN}| = \frac{1}{2}(|\mathbf{AB}| + |\mathbf{DC}|).$$

A1

Answer: Proved

3 marks

30 (9 marks)

$$(a) \mathbf{PA} = -\mathbf{AP}, \mathbf{PB} = \mathbf{AB} - \mathbf{AP} \text{ and } \mathbf{PC} = \mathbf{AC} - \mathbf{AP}$$

B1

$$2(-\mathbf{AP}) + (\mathbf{AB} - \mathbf{AP}) + 3(\mathbf{AC} - \mathbf{AP}) = \mathbf{0}$$

M1

$$\mathbf{AB} + 3\mathbf{AC} = 6\mathbf{AP}$$

A1

$$\mathbf{AP} = \frac{1}{6}\mathbf{AB} + \frac{1}{2}\mathbf{AC}$$

A1

$$\text{Answer: } \mathbf{AP} = \frac{1}{6}\mathbf{AB} + \frac{1}{2}\mathbf{AC}$$

4 marks

$$(b) \mathbf{BD} = \frac{3}{4}\mathbf{BC} = \frac{3}{4}(\mathbf{AC} - \mathbf{AB})$$

M1

$$\mathbf{AD} = \mathbf{AB} + \mathbf{BD} = \frac{1}{4}\mathbf{AB} + \frac{3}{4}\mathbf{AC}$$

A1

$$\mathbf{AP} = \frac{1}{6}\mathbf{AB} + \frac{1}{2}\mathbf{AC} = \frac{2}{3}\left(\frac{1}{4}\mathbf{AB} + \frac{3}{4}\mathbf{AC}\right) = \frac{2}{3}\mathbf{AD}$$

M1

$\mathbf{AP}$ is parallel to $\mathbf{AD}$ and they share the point $\mathbf{A}$, so $\mathbf{A}$, $\mathbf{P}$ and $\mathbf{D}$ are collinear

E1

$$\mathbf{AP} = \frac{2}{3}\mathbf{AD}, \text{ so } \mathbf{PD} = \frac{1}{3}\mathbf{AD} \text{ and } \mathbf{AP} : \mathbf{PD} = 2 : 1$$

A1

Answer: Collinear; $\mathbf{AP} : \mathbf{PD} = 2 : 1$

5 marks

Magnitude and Direction**1 (2 marks)**

$$\left| \begin{pmatrix} 6 \\ -8 \end{pmatrix} \right| = \sqrt{6^2 + (-8)^2}$$

M1

$$= \sqrt{100} = 10$$

A1

Answer: 10

2 marks

2 (2 marks)

$$\text{Magnitude} = \sqrt{8^2 + 15^2} = 17$$

M1

$$\text{Unit vector} = \frac{1}{17} \begin{pmatrix} 8 \\ -15 \end{pmatrix} = \begin{pmatrix} \frac{8}{17} \\ -\frac{15}{17} \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} \frac{8}{17} \\ -\frac{15}{17} \end{pmatrix}$$

2 marks

3 (3 marks)

The forces are $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ N and $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$ N

B1

Resultant = $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ N

M1

Magnitude = $\sqrt{6^2 + 8^2} = 10$ N

A1

Answer: $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$; 10 N

3 marks

4 (4 marks)

$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

B1

$|\mathbf{a} - \mathbf{b}| = \sqrt{25 + 144} = 13$

B1

$|\mathbf{a}| = \sqrt{49 + 1} = 5\sqrt{2}$

B1

$|\mathbf{b}| = \sqrt{4 + 121} = 5\sqrt{5}$, so $|\mathbf{a}| + |\mathbf{b}| = 5\sqrt{2} + 5\sqrt{5}$

B1

Answer: (a) 13 (b) $5\sqrt{2} + 5\sqrt{5}$

4 marks

5 (2 marks)

$\mathbf{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

M1

$AB = \sqrt{36 + 64} = 10$

A1

Answer: 10

2 marks

6 (2 marks)

Speed = $\sqrt{(-5)^2 + 12^2}$

M1

= 13 m s^{-1}

A1

Answer: 13 m s^{-1}

2 marks

7 (3 marks)

Acute angle with the x-axis: $\tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

M1

$\mathbf{v}$ points into the second quadrant, so the angle is $180^\circ - 53.1^\circ$

M1

= 126.9°

A1

Answer: 126.9°

3 marks

8 (3 marks)

$9 + k^2 = 49$

M1

$k^2 = 40$

A1

$k = \pm\sqrt{40} = \pm 2\sqrt{10}$

A1

Answer: $k = \pm 2\sqrt{10}$

3 marks

9 (3 marks)

$|(3, 4)| = 5$

B1

Unit vector = $\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

M1

Vector = $20 \times \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$

A1

Answer: $\begin{pmatrix} 12 \\ 16 \end{pmatrix}$

3 marks

10 (3 marks)

$\tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$

M1 A1

Both components are positive, so $\theta = 60^\circ$

A1

Answer: 60°

3 marks

11 (3 marks)

$$x \text{ component} = 8 \cos 30^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

M1 A1

$$y \text{ component} = 8 \sin 30^\circ = 4$$

A1

$$\text{Answer: } \begin{pmatrix} 4\sqrt{3} \\ 4 \end{pmatrix}$$

3 marks

12 (3 marks)

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 7 \\ -5 \end{pmatrix}$$

M1

$$= \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

A1

$$OB = \sqrt{3^2 + (-4)^2} = 5$$

A1

$$\text{Answer: } OB = 5$$

3 marks

13 (4 marks)

$$PQ = \sqrt{9 + 16} = 5 \text{ and } PR = \sqrt{16 + 9} = 5, \text{ so } PQ = PR \text{ (isosceles)}$$

B1

$$\mathbf{QR} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}, \text{ so } QR = \sqrt{49 + 1} = 5\sqrt{2}$$

M1 A1

$$PQ^2 + PR^2 = 25 + 25 = 50 = QR^2, \text{ so angle } QPR = 90^\circ \text{ (Pythagoras)}$$

E1

$$\text{Answer: } QR = 5\sqrt{2}$$

4 marks

14 (5 marks)

$$\text{(a) Speed} = \sqrt{(-6)^2 + 8^2}$$

M1

$$= 10 \text{ km h}^{-1}$$

A1

$$\text{Answer: } 10 \text{ km h}^{-1}$$

2 marks

$$\text{(b) Angle west of north} = \tan^{-1}\left(\frac{6}{8}\right) = 36.9^\circ$$

M1

$$\text{The ship heads north-west, so bearing} = 360^\circ - \text{angle}$$

M1

$$= 323.1^\circ$$

A1

$$\text{Answer: } 323.1^\circ$$

3 marks

15 (7 marks)

$$\text{(a) East component} = \text{distance} \times \sin(\text{bearing}), \text{ north component} = \text{distance} \times \cos(\text{bearing})$$

M1

$$\mathbf{SP} = \begin{pmatrix} 7.61 \\ 2.47 \end{pmatrix}$$

A1

$$\mathbf{PQ} = \begin{pmatrix} -1.71 \\ -4.70 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{SP} = \begin{pmatrix} 7.61 \\ 2.47 \end{pmatrix}, \mathbf{PQ} = \begin{pmatrix} -1.71 \\ -4.70 \end{pmatrix}$$

3 marks

$$\text{(b) } \mathbf{SQ} = \mathbf{SP} + \mathbf{PQ} = \begin{pmatrix} 5.90 \\ -2.23 \end{pmatrix}$$

M1

$$SQ = \sqrt{5.90^2 + (-2.23)^2} = 6.30 \text{ km}$$

M1 A1

$$Q \text{ is east and south of S. Bearing} = 90^\circ + \tan^{-1}\left(\frac{2.23}{5.90}\right) = 110.7^\circ$$

A1

$$\text{Answer: } SQ = 6.30 \text{ km, bearing } 110.7^\circ$$

4 marks

16 (5 marks)

Take the 7 N force along the x-axis: components (7, 0) and $(5 \cos 60^\circ, 5 \sin 60^\circ) = (2.5, 4.330)$

M1 A1

$$\text{Resultant} = \begin{pmatrix} 9.5 \\ 4.330 \end{pmatrix} \text{ N}$$

A1

$$\text{Magnitude} = \sqrt{9.5^2 + 4.330^2} = 10.4 \text{ N}$$

M1

$$\text{Angle} = \tan^{-1}\left(\frac{4.330}{9.5}\right) = 24.5^\circ$$

A1

Answer: 10.4 N at 24.5° to the 7 N force

5 marks

17 (5 marks)

$$\mathbf{OA} \cdot \mathbf{OB} = 3 \times 5 \times \cos 60^\circ = \frac{15}{2}$$

B1

$$|\mathbf{OP}|^2 = 4|\mathbf{OA}|^2 - 4\mathbf{OA} \cdot \mathbf{OB} + |\mathbf{OB}|^2 = 36 - 30 + 25 = 31$$

M1

$$|\mathbf{OP}| = \sqrt{31}$$

A1

$$\mathbf{OP} \cdot \mathbf{OA} = 2|\mathbf{OA}|^2 - \mathbf{OA} \cdot \mathbf{OB} = 18 - \frac{15}{2} = \frac{21}{2}$$

M1

$$\cos \varphi = \frac{7}{2\sqrt{31}} = 0.6286, \text{ so } \varphi = 51.1^\circ$$

A1

Answer: $|\mathbf{OP}| = \sqrt{31}$, angle 51.1°

5 marks

18 (5 marks)

$$\mathbf{BA} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \text{ and } \mathbf{BC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

B1

$$\mathbf{BA} \cdot \mathbf{BC} = 12 - 4 = 8$$

M1

$$|\mathbf{BA}| = \sqrt{17}, |\mathbf{BC}| = 5, \text{ so } \cos B = \frac{8}{5\sqrt{17}} = 0.3881$$

M1

$$\text{Angle ABC} = 67.2^\circ$$

A1

$$\text{Area} = \frac{1}{2} \times \sqrt{17} \times 5 \times \sin 67.2^\circ = 9.5$$

A1

Answer: Angle ABC = 67.2°, area 9.5

5 marks

19 (4 marks)

Let a perpendicular vector be $\begin{pmatrix} x \\ y \end{pmatrix}$: gradient of $(-8, 6)$ is $-\frac{3}{4}$, so the perpendicular direction has

M1

gradient $\frac{4}{3}$

Perpendicular vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, with magnitude 5

A1

Unit vectors: $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

A1

and $\begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$

A1

Answer: $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$ and $\begin{pmatrix} -\frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$

4 marks

20 (5 marks)

$$|\mathbf{a}| = 5 \text{ and } |\mathbf{b}| = 13$$

B1

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} -2 \\ 16 \end{pmatrix}, \text{ so } |\mathbf{a} - \mathbf{b}|^2 = 4 + 256 = 260$$

M1 A1

$$\cos \theta = \frac{5^2 + 13^2 - 260}{2 \times 5 \times 13} = \frac{-66}{130}$$

M1

$$\theta = 120.5^\circ$$

A1

Answer: 120.5°

5 marks

21 (6 marks)

$$\mathbf{a} + \lambda \mathbf{b} = \begin{pmatrix} 3 + \lambda \\ 1 + 2\lambda \end{pmatrix}$$

M1

$$|\mathbf{a} + \lambda \mathbf{b}|^2 = (3 + \lambda)^2 + (1 + 2\lambda)^2 = 9 + 6\lambda + \lambda^2 + 1 + 4\lambda + 4\lambda^2 = 5\lambda^2 + 10\lambda + 10 \text{ (shown)}$$

M1

A1

$$\text{Complete the square: } 5(\lambda + 1)^2 + 5$$

M1

$$\text{Least when } \lambda = -1$$

A1

$$\text{Least value of } |\mathbf{a} + \lambda \mathbf{b}| = \sqrt{5}$$

A1

Answer: $\lambda = -1$, least value $\sqrt{5}$

6 marks

22 (7 marks)

$$(a) k^2 + (k + 1)^2 = 25$$

M1

$$2k^2 + 2k - 24 = 0, \text{ so } k^2 + k - 12 = 0$$

M1

$$(k + 4)(k - 3) = 0$$

A1

$$k = 3 \text{ gives } \begin{pmatrix} 3 \\ 4 \end{pmatrix}; k = -4 \text{ gives } \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$

A1

Answer: $k = 3 \left(\begin{pmatrix} 3 \\ 4 \end{pmatrix} \right)$ or $k = -4 \left(\begin{pmatrix} -4 \\ -3 \end{pmatrix} \right)$

4 marks

$$(b) \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ makes } \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ \text{ with the positive x-axis}$$

M1

$$\begin{pmatrix} -4 \\ -3 \end{pmatrix} \text{ is in the third quadrant, at } 180^\circ + \tan^{-1}\left(\frac{3}{4}\right) = 216.9^\circ$$

M1

$$\text{Angle between} = 216.9^\circ - 53.1^\circ = 163.7^\circ$$

A1

Answer: 163.7°

3 marks

23 (10 marks)

$$(a) \text{ Velocity} = \begin{pmatrix} 0 \\ 1.5 \end{pmatrix} + \begin{pmatrix} 0.8 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 1.5 \end{pmatrix} \text{ m s}^{-1}$$

M1 A1

$$\text{Speed} = \sqrt{0.8^2 + 1.5^2} = 1.70 \text{ m s}^{-1}$$

A1

$$\text{Bearing} = \tan^{-1}\left(\frac{0.8}{1.5}\right) = 28.1^\circ$$

A1

Answer: $\begin{pmatrix} 0.8 \\ 1.5 \end{pmatrix}$; 1.70 m s⁻¹; bearing 28.1°

4 marks

$$(b) \text{ Time} = 60 \div 1.5 = 40 \text{ s}$$

M1 A1

$$\text{Drift} = 0.8 \times 40 = 32.0 \text{ m}$$

A1

Answer: 32.0 m

3 marks

(c) Let her heading make angle ϕ west of north. East component of her velocity must cancel the current: $1.5 \sin \phi = 0.8$

M1

$$\sin \phi = \frac{0.8}{1.5}, \phi = 32.2^\circ$$

A1

$$\text{Bearing} = 360^\circ - 32.2^\circ = 327.8^\circ$$

A1

Answer: 327.8°

3 marks

24 (8 marks)

(a) At $t = 0$, $r = (1, 2)$, so the distance is $\sqrt{5}$ m

B1

$$\text{Velocity} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

M1

$$\text{Speed} = 5 \text{ m s}^{-1}$$

A1

Answer: $\sqrt{5}$ m; 5 m s^{-1}

3 marks

(b) $|r|^2 = (1 + 3t)^2 + (2 - 4t)^2 = 1 + 6t + 9t^2 + 4 - 16t + 16t^2$

M1

$$= 25t^2 - 10t + 5$$

A1

$$= 25\left(t - \frac{1}{5}\right)^2 + 4$$

M1

$$\text{Least when } t = \frac{1}{5} \text{ s}$$

A1

$$\text{Least distance} = \sqrt{4} = 2 \text{ m}$$

A1

Answer: 2 m at $t = 0.2$ s

5 marks

25 (7 marks)

(a) The bearings differ by 90° , so angle $APB = 90^\circ$

B1

After 2 hours $PA = 24$ km and $PB = 18$ km

B1

$$AB = \sqrt{24^2 + 18^2} = 30 \text{ km}$$

A1

Answer: 30 km

3 marks

(b) $\mathbf{PA} = \begin{pmatrix} 20.78 \\ 12.00 \end{pmatrix}$, $\mathbf{PB} = \begin{pmatrix} 9.000 \\ -15.59 \end{pmatrix}$

M1

$$\mathbf{AB} = \begin{pmatrix} -11.78 \\ -27.59 \end{pmatrix}$$

A1

$$\text{Acute angle west of south} = \tan^{-1}\left(\frac{11.78}{27.59}\right) = 23.1^\circ$$

M1

$$\text{Bearing} = 180^\circ + 23.1^\circ = 203.1^\circ$$

A1

Answer: 203.1°

4 marks

26 (6 marks)

$\tan \theta = -5/12$ in the second quadrant gives $\cos \theta = -12/13$ and $\sin \theta = 5/13$

M1

$$\mathbf{F} = 26 \begin{pmatrix} -\frac{12}{13} \\ \frac{5}{13} \end{pmatrix} = \begin{pmatrix} -24 \\ 10 \end{pmatrix} \text{ N}$$

A1

$$\mathbf{F} + \mathbf{G} = \begin{pmatrix} -24 \\ 10 \end{pmatrix} + \begin{pmatrix} 34 \\ -20 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$$

M1 A1

$$\text{Magnitude} = \sqrt{100 + 100} = 10\sqrt{2} \text{ N}$$

A1

$$\text{Angle below the x-axis} = \tan^{-1}(1) = 45^\circ$$

A1

Answer: $\mathbf{F} = \begin{pmatrix} -24 \\ 10 \end{pmatrix} \text{ N}$; resultant $10\sqrt{2} \text{ N}$ at 45° below the positive x-axis

6 marks

27 (11 marks)

(a) Velocity through the air = $\begin{pmatrix} 200 \sin \theta \\ 200 \cos \theta \end{pmatrix}$

Ground velocity = $\begin{pmatrix} 200 \sin \theta - 30 \\ 200 \cos \theta + 40 \end{pmatrix}$; due north means the $\mathbf{i}$ component is 0

M1

$$\sin \theta = \frac{30}{200} = 0.15$$

M1

$$\theta = 8.6^\circ \text{ (bearing } 008.6^\circ)$$

A1

Answer: $\theta = 8.6^\circ$

3 marks

(b) $\mathbf{j}$ component = $200 \cos \theta + 40 = 200 \times 0.9887 + 40$

M1

$$= 238 \text{ km h}^{-1}$$

A1

Answer: 238 km h^{-1}

2 marks

(c) Time = $600 \div 237.7 = 2.524$ hours

M1

$$= 2 \text{ h } 31 \text{ min}$$

Answer: 2 h 31 min

A1
2 marks

$$\text{(d) Ground velocity} = \begin{pmatrix} 0 \\ 200 \end{pmatrix} + \begin{pmatrix} -30 \\ 40 \end{pmatrix} = \begin{pmatrix} -30 \\ 240 \end{pmatrix}$$

M1

$$\text{Speed} = \sqrt{30^2 + 240^2} = 241.9 \text{ km h}^{-1}$$

A1

$$\text{Angle west of north} = \tan^{-1}\left(\frac{30}{240}\right) = 7.1^\circ$$

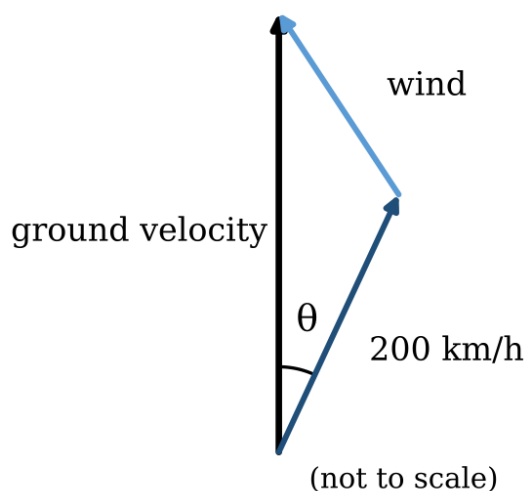
M1

$$\text{Bearing} = 360^\circ - 7.1^\circ = 352.9^\circ$$

A1

Answer: 241.9 km h⁻¹ on a bearing of 352.9°

4 marks



28 (10 marks)

$$\text{(a) } OA^2 = 2^2 + 1^2 = 5, \text{ so } OB^2 = AB^2 = 5$$

B1

$$x^2 + y^2 = 5 \quad (1) \quad (x - 2)^2 + (y - 1)^2 = 5 \quad (2)$$

M1

$$(2) - (1): -4x - 2y + 5 = 0, \text{ so } y = \frac{(5 - 4x)}{2}$$

M1

$$\text{Substitute in (1): } x^2 + \frac{(5 - 4x)^2}{4} = 5, \text{ so } 20x^2 - 40x + 5 = 0, \text{ i.e. } 4x^2 - 8x + 1 = 0$$

M1 A1

$$x = \frac{8 \pm \sqrt{48}}{8} = 1 \pm \frac{\sqrt{3}}{2}$$

M1

$$y = \frac{(5 - 4x)}{2} = \frac{1}{2} \mp \sqrt{3}$$

A1

$$B = \begin{pmatrix} 1 - \frac{\sqrt{3}}{2} \\ \frac{1}{2} + \sqrt{3} \end{pmatrix} \text{ or } \begin{pmatrix} 1 + \frac{\sqrt{3}}{2} \\ \frac{1}{2} - \sqrt{3} \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 1 - \frac{\sqrt{3}}{2} \\ \frac{1}{2} + \sqrt{3} \end{pmatrix} \text{ or } \begin{pmatrix} 1 + \frac{\sqrt{3}}{2} \\ \frac{1}{2} - \sqrt{3} \end{pmatrix}$$

8 marks

$$\text{(b) Area} = \frac{1}{2} \times 5 \times \sin 60^\circ$$

M1

$$= \frac{5\sqrt{3}}{4}$$

A1

$$\text{Answer: } \frac{5\sqrt{3}}{4}$$

2 marks

29 (6 marks)

A rhombus has all sides equal, so $|OA| = |OC|$: $p^2 + q^2 = r^2 + s^2$ (1) B1

$$\mathbf{OB} = \mathbf{OA} + \mathbf{OC} = \begin{pmatrix} p + r \\ q + s \end{pmatrix} \quad \text{M1}$$

$$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = \begin{pmatrix} r - p \\ s - q \end{pmatrix} \quad \text{M1}$$

$$\text{Gradient of OB} \times \text{gradient of AC} = \frac{q+s}{p+r} \times \frac{s-q}{r-p} = \frac{s^2 - q^2}{r^2 - p^2} \quad \text{M1 A1}$$

From (1), $s^2 - q^2 = -(r^2 - p^2)$, so the product of the gradients is -1 : OB is perpendicular to AC. E1

Answer: Proved 6 marks

30 (10 marks)

$$\text{(a) Ship: } \begin{pmatrix} 16 \\ 4 \end{pmatrix} + t \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 16 - 4t \\ 4 + t \end{pmatrix} \quad \text{B1}$$

The patrol boat travels 5t km, so at interception $|\text{position of ship}| = 5t$ M1

$$(16 - 4t)^2 + (4 + t)^2 = 25t^2 \quad \text{A1}$$

$$256 - 128t + 16t^2 + 16 + 8t + t^2 = 25t^2 \quad \text{M1}$$

$$8t^2 + 120t - 272 = 0, \text{ so } t^2 + 15t - 34 = 0 \quad \text{A1}$$

Answer: Shown 5 marks

$$\text{(b) } (t + 17)(t - 2) = 0, \text{ so } t = 2 \ (t > 0) \quad \text{M1 A1}$$

$$\text{Interception point} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \text{ km} \quad \text{M1}$$

$$\text{Velocity of boat} = \frac{1}{2} \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ km h}^{-1} \quad \text{A1}$$

$$\text{Bearing} = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ km h}^{-1}, \text{ bearing } 53.1^\circ \quad \text{5 marks}$$

Position Vectors**1 (2 marks)**

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} \quad \text{M1}$$

$$= \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} 5 \\ -4 \end{pmatrix} \quad \text{2 marks}$$

2 (2 marks)

$$\text{Midpoint} = \frac{1}{2}(\mathbf{OA} + \mathbf{OB}) \quad \text{M1}$$

$$= \frac{1}{2}(4\mathbf{i} + 3\mathbf{j}) = 2\mathbf{i} + \frac{3}{2}\mathbf{j} \quad \text{A1}$$

$$\text{Answer: } 2\mathbf{i} + \frac{3}{2}\mathbf{j} \quad \text{2 marks}$$

3 (3 marks)

$$\mathbf{AB} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \quad \text{M1 A1}$$

$$\mathbf{OM} = \frac{1}{2}\left(\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 7 \\ 5 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \text{B1}$$

$$\text{Answer: } \mathbf{AB} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \mathbf{OM} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad \text{3 marks}$$

4 (2 marks)

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \quad \text{B1}$$

$$\mathbf{BA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \quad \text{B1}$$

$$\text{Answer: } \begin{pmatrix} -6 \\ -2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 2 \end{pmatrix} \quad 2 \text{ marks}$$

5 (2 marks)

$$\text{New position} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} -1 \\ 9 \end{pmatrix} \text{ m} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} -1 \\ 9 \end{pmatrix} \text{ m} \quad 2 \text{ marks}$$

6 (3 marks)

$$\mathbf{AB} = (1 - 3)\mathbf{i} + (4 + 2)\mathbf{j} = -2\mathbf{i} + 6\mathbf{j} \quad \text{M1 A1}$$

$$AB = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10} \quad \text{A1}$$

$$\text{Answer: } \mathbf{AB} = -2\mathbf{i} + 6\mathbf{j}, AB = 2\sqrt{10} \quad 3 \text{ marks}$$

7 (3 marks)

$$\mathbf{AB} = \begin{pmatrix} 10 \\ -5 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{AP} = \frac{2}{5}\mathbf{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{OP} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad 3 \text{ marks}$$

8 (4 marks)

$$\text{(a) } \mathbf{DC} = \mathbf{AB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \text{ so } \mathbf{OD} = \mathbf{OC} - \mathbf{AB} \quad \text{M1}$$

$$= \begin{pmatrix} 8 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad 2 \text{ marks}$$

$$\text{(b) Midpoint of AC} = \frac{1}{2}\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} \frac{9}{2} \\ \frac{9}{2} \end{pmatrix} \quad \text{B1}$$

$$\text{Midpoint of BD} = \frac{1}{2}\left(\begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix}\right) = \begin{pmatrix} \frac{9}{2} \\ \frac{9}{2} \end{pmatrix}, \text{ the same point} \quad \text{B1}$$

$$\text{Answer: Both midpoints are } \left(\frac{9}{2}, \frac{9}{2}\right) \quad 2 \text{ marks}$$

9 (4 marks)

$$\mathbf{AB} = \begin{pmatrix} 12 \\ -4 \end{pmatrix}, \text{ and } \mathbf{AM} = \frac{3}{4}\mathbf{AB} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$$

M1

$$\mathbf{OM} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

A1

$$\mathbf{OQ} = 2\mathbf{OM} - \mathbf{OP}$$

M1

$$= \begin{pmatrix} 12 \\ -2 \end{pmatrix} - \begin{pmatrix} 10 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

4 marks

10 (4 marks)

$$\mathbf{BC} = \mathbf{AC} - \mathbf{AB} = \begin{pmatrix} -9 \\ 12 \end{pmatrix}$$

B1

$$\mathbf{BD} = \frac{2}{3}\mathbf{BC} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

M1

$$\mathbf{AD} = \mathbf{AB} + \mathbf{BD} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

A1

$$AD = 6$$

A1

$$\text{Answer: } \mathbf{AD} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}, AD = 6$$

4 marks

11 (3 marks)

$$\mathbf{OM} = \frac{1}{2}(\mathbf{OA} + \mathbf{OB})$$

M1

$$\mathbf{OB} = 2\mathbf{OM} - \mathbf{OA} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

M1

$$= \begin{pmatrix} 7 \\ -1 \end{pmatrix}, \text{ so B is } (7, -1)$$

A1

$$\text{Answer: } \mathbf{B}(7, -1)$$

3 marks

12 (4 marks)

$$\mathbf{PQ} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

M1

$$\mathbf{QR} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

A1

PQ and QR are equal vectors, so parallel with the common point Q: P, Q, R are collinear

E1

PQ : QR = 1 : 1 (Q is the midpoint of PR)

B1

Answer: Collinear, PQ : QR = 1 : 1

4 marks

13 (4 marks)

$$t = 3: \mathbf{r} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

B1

On the y-axis the x component is 0: $6 - 3t = 0$

M1

$$t = 2 \text{ s}$$

A1

$$(\text{position } \begin{pmatrix} 0 \\ 5 \end{pmatrix})$$

B1

$$\text{Answer: } \begin{pmatrix} -3 \\ 7 \end{pmatrix}; t = 2 \text{ s}$$

4 marks

14 (5 marks)

$$(a) \mathbf{AB} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \text{ and } \mathbf{AC} = \begin{pmatrix} 9 \\ k+2 \end{pmatrix}$$

M1 A1

A, B, C collinear, so $\mathbf{AC} = \lambda \mathbf{AB}$: $9 = 3\lambda$, so $\lambda = 3$

M1

$$k + 2 = 3\lambda = 9, \text{ so } k = 7$$

A1

Answer: $k = 7$

4 marks

$$(b) \mathbf{BC} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} = 2\mathbf{AB}, \text{ so } \mathbf{AB} : \mathbf{BC} = 1 : 2$$

B1

Answer: $1 : 2$

1 mark

15 (7 marks)

$$(a) \mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 + 3t \\ 1 + 4t \end{pmatrix}$$

B1

$$\text{Answer: } \begin{pmatrix} 2 + 3t \\ 1 + 4t \end{pmatrix}$$

1 mark

$$(b) 2 + 3t = 20, \text{ so } t = 6 \text{ s}$$

M1

$$\mathbf{r} = \begin{pmatrix} 20 \\ 25 \end{pmatrix}$$

A1

$$\text{Answer: } t = 6 \text{ s, } \begin{pmatrix} 20 \\ 25 \end{pmatrix}$$

2 marks

$$(c) 2 + 3t = 11 \text{ gives } t = 3$$

M1

At $t = 3$, $1 + 4t = 13$, so the drone passes through (11, 13) after 3 s.

A1

Answer: $t = 3 \text{ s}$

2 marks

$$(d) \text{Speed} = \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-1}$$

M1

$$\text{Distance} = 5 \times 6 = 30 \text{ m}$$

A1

Answer: 30 m

2 marks

16 (5 marks)

$$\mathbf{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

B1

$$\mathbf{OP} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

M1 A1

$$\mathbf{BQ} = \frac{1}{3} \mathbf{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

M1

$$\mathbf{OQ} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{OP} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{OQ} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

5 marks

17 (5 marks)

$$\mathbf{BA} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}, \mathbf{BC} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

M1

$$|\mathbf{BA}| = \sqrt{9 + 16} = 5 \text{ and } |\mathbf{BC}| = \sqrt{16 + 9} = 5, \text{ so isosceles}$$

A1

$$\text{Gradient BA} = \frac{4}{3}, \text{ gradient BC} = -\frac{3}{4}; \text{ product} = -1, \text{ so BA is perpendicular to BC}$$

M1 A1

$$\text{Area} = \frac{1}{2} \times 5 \times 5 = 12.5$$

A1

Answer: Shown; area 12.5

5 marks

18 (6 marks)

$$\mathbf{AB} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}, \text{ so } \mathbf{OR} = \begin{pmatrix} 1 + 6\mu \\ 7 - 8\mu \end{pmatrix} \quad \text{B1}$$

$$|\mathbf{OR}|^2 = (1 + 6\mu)^2 + (7 - 8\mu)^2 = 1 + 12\mu + 36\mu^2 + 49 - 112\mu + 64\mu^2$$

$$= 100\mu^2 - 100\mu + 50 \quad \text{M1}$$

$$= 100\left(\mu - \frac{1}{2}\right)^2 + 25 \quad \text{A1}$$

$$\text{Least when } \mu = \frac{1}{2}: \mathbf{R} = (4, 3) \quad \text{M1}$$

$$\text{Least distance} = \sqrt{25} = 5 \quad \text{A1}$$

Answer: R(4, 3), distance 5

6 marks

19 (5 marks)

$$\mathbf{r}_P = \begin{pmatrix} -2 + 2t \\ 3 - t \end{pmatrix}, \mathbf{r}_Q = \begin{pmatrix} 6 - t \\ -1 + t \end{pmatrix} \quad \text{B1}$$

$$\mathbf{PQ} = \begin{pmatrix} 8 - 3t \\ -4 + 2t \end{pmatrix} \quad \text{M1}$$

Collision needs $8 - 3t = 0$ and $-4 + 2t = 0$, i.e. $t = \frac{8}{3}$ and $t = 2$: not the same time, so no collision M1 A1

At $t = 2$: $\mathbf{PQ} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, so the distance is 2 m A1

Answer: No collision; 2 m

5 marks

20 (4 marks)

$$\mathbf{AP} : \mathbf{PB} = 2 : 3 \text{ means } \mathbf{AP} = \frac{2}{5}\mathbf{AB} \quad \text{B1}$$

$$\mathbf{OP} = \mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a}) = \frac{3\mathbf{a} + 2\mathbf{b}}{5} \quad \text{M1 A1}$$

The student's weights are the wrong way round: the point is nearer to A, so A gets the larger weight.

$$\text{Check: } \mathbf{P} = \frac{1}{5}\left(0 + 2\begin{pmatrix} 10 \\ 5 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \text{ which is } 2/5 \text{ of the way from A to B} \quad \text{E1}$$

$$\text{Answer: } \mathbf{OP} = \frac{3\mathbf{a} + 2\mathbf{b}}{5} \quad \text{4 marks}$$

21 (6 marks)

$$\text{(a) } \mathbf{OM} = \frac{1}{2}(\mathbf{b} + \mathbf{c}) \quad \text{B1}$$

$$\text{Answer: } \frac{1}{2}(\mathbf{b} + \mathbf{c}) \quad \text{1 mark}$$

$$\text{(b) } \mathbf{AM} = \mathbf{OM} - \mathbf{a} = \frac{1}{2}(\mathbf{b} + \mathbf{c}) - \mathbf{a} \quad \text{M1}$$

$$\mathbf{OG} = \mathbf{a} + \frac{2}{3}\mathbf{AM} = \mathbf{a} + \frac{1}{3}(\mathbf{b} + \mathbf{c}) - \frac{2}{3}\mathbf{a} \quad \text{M1}$$

$$= \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c}) \quad \text{A1}$$

Answer: Shown

3 marks

(c) The expression for $\mathbf{OG}$ is symmetrical in $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, so the same working on the medians from B and C gives the same point.

Answer: Symmetry

1 mark

$$\text{(d) } \mathbf{G} = \frac{1}{3}\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix} 12 \\ 9 \end{pmatrix} = (4, 3) \quad \text{B1}$$

Answer: (4, 3)

1 mark

22 (7 marks)

(a) $\mathbf{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{BC} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, $\mathbf{AC} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

M1

$$AB^2 = 20, BC^2 = 20, AC^2 = 40$$

M1 A1

$AB = BC$, so isosceles; $AB^2 + BC^2 = 40 = AC^2$, so the right angle is at B (converse of Pythagoras)

B1

Answer: Right angle at B, $AB = BC$

4 marks

(b) $\mathbf{OD} = \mathbf{OA} + \mathbf{BC}$ ($AD = BC$)

M1

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}, \text{ so D is } (-1, 5)$$

A1

Answer: D(-1, 5)

2 marks

(c) Area = $AB^2 = 20$

B1

Answer: 20

1 mark

23 (6 marks)

(a) $\mathbf{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ and $\mathbf{ON} = \frac{1}{2}(\mathbf{c} + \mathbf{d})$

B1

$$\mathbf{MN} = \mathbf{ON} - \mathbf{OM} = \frac{1}{2}(\mathbf{c} + \mathbf{d} - \mathbf{a} - \mathbf{b})$$

M1

$$= \frac{1}{2}((\mathbf{d} - \mathbf{a}) + (\mathbf{c} - \mathbf{b})) = \frac{1}{2}(\mathbf{AD} + \mathbf{BC})$$

A1

Answer: $\mathbf{MN} = \frac{1}{2}(\mathbf{AD} + \mathbf{BC})$

3 marks

(b) $\mathbf{AD} + \mathbf{BC} = 2\mathbf{MN}$, so $(\lambda^2 - 5\lambda + 8)\mathbf{MN} = 2\mathbf{MN}$

M1

As $\mathbf{MN}$ is not the zero vector, $\lambda^2 - 5\lambda + 8 = 2$

M1

$$(\lambda - 2)(\lambda - 3) = 0, \text{ so } \lambda = 2 \text{ or } \lambda = 3$$

A1

Answer: $\lambda = 2$ or $\lambda = 3$

3 marks

24 (6 marks)

(a) $\mathbf{r}_P = \begin{pmatrix} 2 + 3t \\ -1 + 2t \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} 26 - 3t \\ -5 + 3t \end{pmatrix}$

B1

Equate the i components: $2 + 3t = 26 - 3t$, so $t = 4$

M1

Equate the j components: $-1 + 2t = -5 + 3t$, so $t = 4$: the same time, so they collide

A1

Position vector = $\begin{pmatrix} 14 \\ 7 \end{pmatrix}$ m

A1

Answer: Collide at $t = 4$ s at $\begin{pmatrix} 14 \\ 7 \end{pmatrix}$ m

4 marks

(b) $t = 2$: $\mathbf{r}_P = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} 20 \\ 1 \end{pmatrix}$

M1

$$\text{Distance} = \sqrt{12^2 + 2^2} = \sqrt{148} = 2\sqrt{37} \approx 12.2 \text{ m}$$

A1

Answer: $2\sqrt{37} \approx 12.2$ m

2 marks

25 (7 marks)

(a) $\mathbf{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $|\mathbf{AB}| = 5$ and $|\mathbf{AD}| = \sqrt{16 + 9} = 5$: adjacent sides equal

M1 A1

$$\mathbf{D} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ 3 \end{pmatrix} = (-3, 4)$$

A1

$\mathbf{OC} = \mathbf{OB} + \mathbf{AD} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$, so C is (0, 8)

A1

Answer: C(0, 8), D(-3, 4)

4 marks

(b) $\mathbf{AC} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$, $\mathbf{BD} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$

M1

Gradient AC = -7, gradient BD = $\frac{1}{7}$

A1

Product of gradients = -1, so the diagonals are perpendicular

E1

Answer: Shown**3 marks****26 (7 marks)**

$$(a) \mathbf{AP} = \frac{\lambda}{\lambda + 1} \mathbf{AB} = \frac{\lambda}{1 + \lambda} (\mathbf{b} - \mathbf{a})$$

M1

$$\mathbf{OP} = \mathbf{a} + \frac{\lambda}{1 + \lambda} (\mathbf{b} - \mathbf{a})$$

M1

$$= \frac{(1 + \lambda)\mathbf{a} + \lambda\mathbf{b} - \lambda\mathbf{a}}{1 + \lambda} = \frac{\mathbf{a} + \lambda\mathbf{b}}{1 + \lambda}$$

A1**Answer: Shown****3 marks**

$$(b) \mathbf{OP} = \begin{pmatrix} \frac{1 + 7\lambda}{1 + \lambda} \\ \frac{5 - \lambda}{1 + \lambda} \end{pmatrix}$$

M1

$$\text{On } y = x: 1 + 7\lambda = 5 - \lambda$$

M1

$$\lambda = \frac{1}{2}$$

A1

$$\mathbf{OP} = \begin{pmatrix} \frac{4.5}{1.5} \\ \frac{4.5}{1.5} \end{pmatrix} = (3, 3)$$

A1

$$\text{Answer: } \lambda = \frac{1}{2}, \mathbf{P}(3, 3)$$

4 marks**27 (11 marks)**

$$(a) \mathbf{r}_A = t \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 5t \\ 2t \end{pmatrix}$$

B1

$$\mathbf{r}_B = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 + 2t \\ 5 - 2t \end{pmatrix}$$

B1

$$\text{Answer: } \begin{pmatrix} 5t \\ 2t \end{pmatrix} \text{ and } \begin{pmatrix} 10 + 2t \\ 5 - 2t \end{pmatrix}$$

2 marks

$$(b) \mathbf{AB} = \mathbf{r}_B - \mathbf{r}_A = \begin{pmatrix} 10 - 3t \\ 5 - 4t \end{pmatrix}$$

M1

$$d^2 = (10 - 3t)^2 + (5 - 4t)^2$$

M1

$$= 100 - 60t + 9t^2 + 25 - 40t + 16t^2 = 25t^2 - 100t + 125$$

A1**Answer: Shown****3 marks**

$$(c) d^2 = 25(t - 2)^2 + 25 \text{ (completing the square)}$$

M1

$$\text{Least when } t = 2, \text{ i.e. } 2 \text{ pm}$$

A1

$$\text{Least distance} = \sqrt{25} = 5 \text{ km}$$

A1

$$\text{Answer: } 2 \text{ pm, } 5 \text{ km}$$

3 marks

$$(d) d^2 < 36: 25(t - 2)^2 + 25 < 36, \text{ so } (t - 2)^2 < \frac{11}{25}$$

M1

$$|t - 2| < \frac{\sqrt{11}}{5} = 0.6633$$

M1

$$\text{Duration} = 2 \times 0.6633 = 1.33 \text{ h} = 80 \text{ minutes}$$

A1

$$\text{Answer: } 80 \text{ minutes}$$

3 marks**28 (9 marks)**

$$(a) \text{ In a parallelogram } \mathbf{AB} = \mathbf{DC}, \text{ so } \mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{d}$$

M1

$$\text{Rearranging, } \mathbf{a} + \mathbf{c} = \mathbf{b} + \mathbf{d}$$

A1

$$\text{Midpoint of AC} = \frac{1}{2}(\mathbf{a} + \mathbf{c}); \text{ midpoint of BD} = \frac{1}{2}(\mathbf{b} + \mathbf{d})$$

M1

$$\text{These are equal, so the diagonals meet at their common midpoint, i.e. they bisect each other.}$$

E1**Answer: Proved****4 marks**

$$(b) \text{ Equal midpoints: } \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \frac{1}{2}(\mathbf{b} + \mathbf{d}), \text{ so } \mathbf{a} + \mathbf{c} = \mathbf{b} + \mathbf{d}$$

M1

$$\text{Then } \mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{d}, \text{ i.e. } \mathbf{AB} = \mathbf{DC}$$

A1

$$\text{AB and DC are equal in length and parallel (and in the same direction), so ABCD is a}$$

E1

parallelogram.

Answer: Proved

3 marks

(c) $\mathbf{a} + \mathbf{c} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ and $\mathbf{b} + \mathbf{d} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, which are equal

M1

So ABCD is a parallelogram and the diagonals meet at $\frac{1}{2}(8, 2) = (4, 1)$

A1

Answer: (4, 1)

2 marks

29 (10 marks)

(a) $\mathbf{AB} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$, so $\mathbf{OP} = \begin{pmatrix} 1 + 8\mu \\ 2 + 6\mu \end{pmatrix}$

B1

$\mathbf{CP} = \begin{pmatrix} 2 + 8\mu \\ -11 + 6\mu \end{pmatrix}$

M1

$\mathbf{CP} \cdot \mathbf{AB} = 0: 8(2 + 8\mu) + 6(-11 + 6\mu) = 0$

M1

$16 + 64\mu - 66 + 36\mu = 0$, so $100\mu = 50$

M1 A1

$\mu = \frac{1}{2}$, so P is (5, 5)

A1

Answer: $\mu = \frac{1}{2}$, P(5, 5)

6 marks

(b) $\mathbf{CP} = \begin{vmatrix} 6 \\ -8 \end{vmatrix} = 10$

M1

$\mathbf{AB} = 10$

$\text{Area} = \frac{1}{2} \times \mathbf{AB} \times \mathbf{CP} = \frac{1}{2} \times 10 \times 10 = 50$

M1 A1

Answer: 50

3 marks

(c) P is the midpoint of AB ($\mu = \frac{1}{2}$) and CP is perpendicular to AB, so CP is the perpendicular bisector of AB and $\mathbf{CA} = \mathbf{CB}$.

E1

Answer: $\mathbf{CA} = \mathbf{CB}$

1 mark

30 (9 marks)

(a) AC: $\mathbf{r} = s \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

B1

BD: $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + u \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

B1

Answer: $\mathbf{r} = s \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + u \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

2 marks

(b) At X: $8s = 6 - 4u$ and $4s = 6u$

M1

From the second, $s = \frac{3u}{2}$; substitute: $12u = 6 - 4u$, so $u = \frac{3}{8}$

M1 A1

$\mathbf{OX} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \frac{3}{8} \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ \frac{9}{4} \end{pmatrix}$

A1

Answer: $\begin{pmatrix} \frac{9}{2} \\ \frac{9}{4} \end{pmatrix}$

4 marks

(c) $s = \frac{3u}{2} = \frac{9}{16}$, so $\mathbf{AX} = \frac{9}{16}\mathbf{AC}$

M1

$\mathbf{AX} : \mathbf{XC} = 9 : 7$

A1

$u = \frac{3}{8}$, so $\mathbf{BX} : \mathbf{XD} = 3 : 5$

A1

Answer: $\mathbf{AX} : \mathbf{XC} = 9 : 7$, $\mathbf{BX} : \mathbf{XD} = 3 : 5$

3 marks

Vector Geometry

1 (2 marks)

$$\mathbf{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \text{ and } \mathbf{CD} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

M1

$\mathbf{AB} = 2\mathbf{CD}$, so AB is parallel to CD (and twice as long)

A1

Answer: $\mathbf{AB} = 2\mathbf{CD}$

2 marks

2 (3 marks)

$$\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$$

M1

$$= \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

A1

$\mathbf{AB} = \mathbf{b} - \mathbf{a}$, so $\mathbf{PQ} = \frac{1}{2}\mathbf{AB}$: parallel to AB and half its length

E1

Answer: $\mathbf{PQ} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$; parallel to AB, half the length

3 marks

3 (2 marks)

$$\lambda = 2: \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

B1

$$\lambda = -1: \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

B1

Answer: (7, 4) and (-2, 1)

2 marks

4 (3 marks)

$$\text{Direction } \mathbf{AB} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

M1 A1

$$\mathbf{r} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{r} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

3 marks

5 (2 marks)

$$7 = 1 + 2\lambda \text{ gives } \lambda = 3$$

M1

Then $y = 5 - 2(3) = -1$, so yes the point lies on the line.

A1

Answer: Yes ($\lambda = 3$)

2 marks

6 (2 marks)

$$\begin{pmatrix} -3 \\ 6 \end{pmatrix} = -\frac{3}{2} \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

M1

So the first and second lines are parallel.

A1

Answer: First and second

2 marks

7 (5 marks)

$$\text{(a) } \mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

B1

$$\text{Answer: } \mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

1 mark

$$\text{(b) } x: 2 + 3\lambda = 11 \text{ gives } \lambda = 3$$

M1

$$y: 1 - 2(3) = -5 \text{ (checks), so B lies on l}$$

A1

Answer: $\lambda = 3$ for both components

2 marks

$$\text{(c) } y = 0: 1 - 2\lambda = 0, \text{ so } \lambda = \frac{1}{2}$$

M1

$$x = 2 + \frac{3}{2} = \frac{7}{2}, \text{ so the point is } \left(\frac{7}{2}, 0\right)$$

A1

Answer: $\left(\frac{7}{2}, 0\right)$

2 marks

8 (4 marks)

$$(a) \mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \mathbf{c} + \frac{2}{3}\mathbf{a}$$

B1

$$\text{Answer: } \mathbf{c} + \frac{2}{3}\mathbf{a}$$

1 mark

$$(b) \mathbf{AB} = \mathbf{OB} - \mathbf{a} = \mathbf{c} - \frac{1}{3}\mathbf{a}$$

B1

$$\text{Answer: } \mathbf{c} - \frac{1}{3}\mathbf{a}$$

1 mark

$$(c) \mathbf{AC} = \mathbf{c} - \mathbf{a}, \text{ so } \mathbf{AM} = \frac{1}{3}(\mathbf{c} - \mathbf{a})$$

M1

$$\mathbf{OM} = \mathbf{a} + \frac{1}{3}(\mathbf{c} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$$

A1

$$\text{Answer: } \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$$

2 marks

9 (3 marks)

$$x = 2 + 3\lambda, y = -1 + 4\lambda$$

B1

$$\lambda = \frac{x-2}{3} = \frac{y+1}{4}$$

M1

$$4x - 8 = 3y + 3, \text{ so } 4x - 3y = 11$$

A1

$$\text{Answer: } 4x - 3y = 11$$

3 marks

10 (2 marks)

The point $(0, -3)$ is on the line and the gradient 2 gives direction $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

M1

$$\mathbf{r} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{r} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

2 marks

11 (4 marks)

$$1 + \lambda = 5 - \mu \text{ and } \lambda = -2 + 2\mu$$

M1

$$\text{Substituting: } 1 - 2 + 2\mu = 5 - \mu, \text{ so } 3\mu = 6, \mu = 2$$

M1 A1

$$\text{Position vector} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

4 marks

12 (4 marks)

$$\begin{pmatrix} -4 \\ 6 \end{pmatrix} = -2 \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \text{ so the directions are parallel}$$

B1

$$\text{Is } (1, 2) \text{ on the second line? } 1 = -4\mu \text{ gives } \mu = -\frac{1}{4}$$

M1

$$\text{then } y = 5 + 6(-\frac{1}{4}) = \frac{7}{2} \neq 2$$

A1

The point is not on the second line, so the lines are distinct.

E1

Answer: Parallel, distinct

4 marks

13 (4 marks)

$$\text{x-axis: } y = 0, \text{ so } 1 + 3\lambda = 0, \lambda = -\frac{1}{3}$$

M1

$$x = 2 - \frac{1}{3} = \frac{5}{3}, \text{ point } (\frac{5}{3}, 0)$$

A1

$$\text{y-axis: } x = 0, \text{ so } 2 + \lambda = 0, \lambda = -2$$

M1

$$y = 1 - 6 = -5, \text{ point } (0, -5)$$

A1

$$\text{Answer: } (\frac{5}{3}, 0) \text{ and } (0, -5)$$

4 marks

14 (6 marks)

$$(a) \mathbf{CB} = 3\mathbf{OA} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \begin{pmatrix} 15 \\ -6 \end{pmatrix} \quad \text{A1}$$

$$\mathbf{AC} = \begin{pmatrix} 6 \\ -9 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -10 \end{pmatrix}, \text{ so } \mathbf{AD} = \frac{1}{4}\mathbf{AC} \quad \text{M1}$$

$$\mathbf{OD} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 3 \\ -10 \end{pmatrix} = \begin{pmatrix} \frac{15}{4} \\ -\frac{3}{2} \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \mathbf{OB} = \begin{pmatrix} 15 \\ -6 \end{pmatrix}, \mathbf{OD} = \begin{pmatrix} \frac{15}{4} \\ -\frac{3}{2} \end{pmatrix} \quad \text{4 marks}$$

$$(b) \mathbf{OB} = \begin{pmatrix} 15 \\ -6 \end{pmatrix} = 4\begin{pmatrix} \frac{15}{4} \\ -\frac{3}{2} \end{pmatrix} = 4\mathbf{OD}, \text{ so O, D and B are collinear} \quad \text{M1}$$

$$\mathbf{OD} : \mathbf{DB} = 1 : 3 \quad \text{A1}$$

$$\text{Answer: Collinear, } \mathbf{OD} : \mathbf{DB} = 1 : 3 \quad \text{2 marks}$$

15 (6 marks)

$$(a) \mathbf{OB} = \mathbf{a} + \mathbf{c} \quad \text{B1}$$

$$\mathbf{OM} = \mathbf{a} + \frac{1}{2}\mathbf{c} \quad \text{B1}$$

$$\text{Answer: } \mathbf{OB} = \mathbf{a} + \mathbf{c}, \mathbf{OM} = \mathbf{a} + \frac{1}{2}\mathbf{c} \quad \text{2 marks}$$

$$(b) \mathbf{CX} = \mathbf{OX} - \mathbf{c} = \frac{2}{3}(\mathbf{a} + \mathbf{c}) - \mathbf{c} = \frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{c} \quad \text{M1}$$

$$\mathbf{CM} = \mathbf{OM} - \mathbf{c} = \mathbf{a} - \frac{1}{2}\mathbf{c} \quad \text{M1}$$

$$\mathbf{CM} = \frac{3}{2}\mathbf{CX}, \text{ so CX and CM are parallel with a common point C: collinear} \quad \text{E1}$$

$$\text{Answer: Shown} \quad \text{3 marks}$$

$$(c) \mathbf{CM} = \frac{3}{2}\mathbf{CX}, \text{ so } \mathbf{CX} : \mathbf{CM} = 2 : 3, \text{ hence } \mathbf{CX} : \mathbf{XM} = 2 : 1 \quad \text{B1}$$

$$\text{Answer: } 2 : 1 \quad \text{1 mark}$$

16 (4 marks)

$$\cos \theta = \frac{|\mathbf{a} \cdot \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|} \text{ where } \mathbf{a} \cdot \mathbf{b} = 1 \times 3 + 2 \times (-1) = 1 \quad \text{M1 A1}$$

$$|\mathbf{a}| = \sqrt{5}, |\mathbf{b}| = \sqrt{10} \quad \text{B1}$$

$$\cos \theta = \frac{1}{\sqrt{50}}, \text{ so } \theta = 81.9^\circ \quad \text{A1}$$

$$\text{Answer: } 81.9^\circ \quad \text{4 marks}$$

17 (6 marks)

$$\text{A point Q on l: } \begin{pmatrix} 1 + 3\lambda \\ 1 + 4\lambda \end{pmatrix}; \mathbf{PQ} = \begin{pmatrix} 3\lambda - 11 \\ 4\lambda + 2 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{PQ} \cdot \text{direction} = 0: 3(3\lambda - 11) + 4(4\lambda + 2) = 0 \quad \text{M1}$$

$$25\lambda - 25 = 0, \text{ so } \lambda = 1 \quad \text{M1 A1}$$

$$\text{Foot Q} = (4, 5) \quad \text{A1}$$

$$\mathbf{PQ} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}, \text{ distance} = 10 \quad \text{A1}$$

$$\text{Answer: Foot (4, 5), distance 10} \quad \text{6 marks}$$

18 (5 marks)

Paths: P: $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, Q: $\mathbf{r} = \begin{pmatrix} 13 \\ 3 \end{pmatrix} + s \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

B1

$$1 + 2t = 13 - 3s \text{ and } 2 + t = 3 + s$$

M1

$$t = 1 + s; 1 + 2 + 2s = 13 - 3s, \text{ so } s = 2, t = 3$$

M1 A1

The paths meet at (7, 5), but P is there at $t = 3$ and Q at $t = 2$, different times, so no collision.

E1

Answer: Paths meet at (7, 5) but at different times

5 marks

19 (5 marks)

(a) Parallel: $\begin{pmatrix} k \\ 6 \end{pmatrix} = c \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, so $c = 2$ and $k = 4$

M1 A1

(b) Perpendicular: scalar product of the directions is zero

M1

$$2k + 18 = 0$$

M1

$$k = -9$$

A1

Answer: (a) $k = 4$ (b) $k = -9$

5 marks

20 (6 marks)

A vector perpendicular to $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (scalar product $2 - 2 = 0$)

M1 A1

$$\mathbf{l}_2: \mathbf{r} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

A1

$$1 + 2\lambda = 5 + \mu \text{ and } -\lambda = 3 + 2\mu$$

M1

$$\lambda = -3 - 2\mu; 1 - 6 - 4\mu = 5 + \mu, \text{ so } \mu = -2$$

M1

$$\text{Meeting point } \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{r} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \end{pmatrix}; (3, -1)$$

6 marks

21 (9 marks)

(a) $\mathbf{OP} = \frac{1}{3}\mathbf{a}$

B1

$$\mathbf{OQ} = \frac{2}{3}\mathbf{b}$$

B1

Answer: $\mathbf{OP} = \frac{1}{3}\mathbf{a}, \mathbf{OQ} = \frac{2}{3}\mathbf{b}$

2 marks

(b) On AQ: $\mathbf{OX} = \mathbf{a} + \mu(\mathbf{OQ} - \mathbf{a}) = (1 - \mu)\mathbf{a} + \frac{2}{3}\mu\mathbf{b}$

M1

On BP: $\mathbf{OX} = \mathbf{b} + \lambda(\mathbf{OP} - \mathbf{b}) = \frac{1}{3}\lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$

M1

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$: $1 - \mu = \frac{1}{3}\lambda$ and $\frac{2}{3}\mu = 1 - \lambda$

M1

Solving: $\lambda = \frac{3}{7}, \mu = \frac{6}{7}$

A1

$$\mathbf{OX} = \frac{1}{7}\mathbf{a} + \frac{4}{7}\mathbf{b}$$

A1

Answer: $\frac{1}{7}\mathbf{a} + \frac{4}{7}\mathbf{b}$

5 marks

(c) $\mathbf{BX} = \lambda\mathbf{BP}$ with $\lambda = \frac{3}{7}$

M1

$$\mathbf{BX} : \mathbf{XP} = 3 : 4$$

A1

Answer: 3 : 4

2 marks

22 (9 marks)

(a) $2 + \lambda = 10 - 3\mu$ (1) and $1 + 2\lambda = 3 + \mu$ (2)

M1 A1

From (2): $\mu = 2\lambda - 2$. Substituting in (1): $2 + \lambda = 10 - 6\lambda + 6$, so $7\lambda = 14$, $\lambda = 2$

M1

$$\mathbf{OP} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \text{ so P is } (4, 5)$$

A1

Answer: P(4, 5)

4 marks

(b) $l_1: 1 + 2\lambda = 0, \lambda = -\frac{1}{2}, x = 2 - \frac{1}{2} = \frac{3}{2}$, giving $(\frac{3}{2}, 0)$

B1

$l_2: 3 + \mu = 0, \mu = -3, x = 10 + 9 = 19$, giving $(19, 0)$

B1

Answer: $(\frac{3}{2}, 0)$ and $(19, 0)$

2 marks

(c) Base = $19 - \frac{3}{2} = \frac{35}{2}$

M1

Height = y-coordinate of P = 5

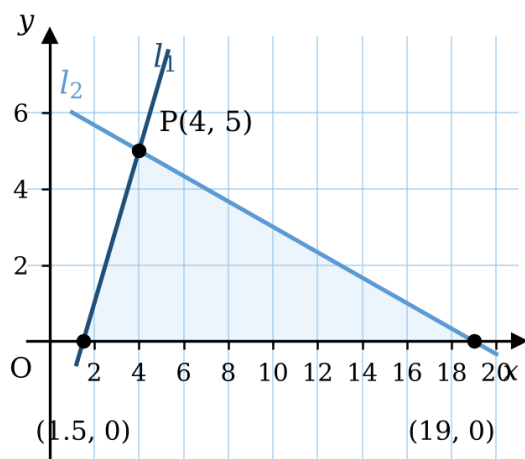
A1

Area = $\frac{1}{2} \times \frac{35}{2} \times 5 = \frac{175}{4} = 43.75 \text{ cm}^2$

A1

Answer: $\frac{175}{4} \text{ cm}^2$

3 marks



23 (7 marks)

(a) $|a| = 5$ and $|b| = 13$

B1

Unit vectors: $\begin{pmatrix} 3 \\ 5 \\ 4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 13 \\ 5 \\ 13 \end{pmatrix}$

B1

Answer: $\begin{pmatrix} 3 \\ 5 \\ 4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 13 \\ 5 \\ 13 \end{pmatrix}$

2 marks

(b) The sum of two unit vectors lies along the bisector (it is a diagonal of a rhombus)

M1

$$\begin{pmatrix} 3 \\ 5 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 12 \\ 13 \\ 5 \\ 13 \end{pmatrix} = \begin{pmatrix} 15 \\ 18 \\ 9 \\ 18 \end{pmatrix} = \frac{9}{65} \begin{pmatrix} 9 \\ 18 \\ 7 \\ 18 \end{pmatrix}$$

A1

$l: r = \lambda \begin{pmatrix} 9 \\ 7 \end{pmatrix}$

A1

Answer: $r = \lambda \begin{pmatrix} 9 \\ 7 \end{pmatrix}$

3 marks

(c) $9\lambda = 27$, so $\lambda = 3$

M1

P is the point $(27, 21)$

A1

Answer: P(27, 21)

2 marks

24 (6 marks)

$$\text{Point on l: } \begin{pmatrix} \lambda \\ 1 + \lambda \end{pmatrix}; \mathbf{PQ} = \begin{pmatrix} \lambda - 2 \\ \lambda - 6 \end{pmatrix} \quad \text{M1}$$

$$\text{Perpendicular to the direction (1, 1): } (\lambda - 2) + (\lambda - 6) = 0 \quad \text{M1}$$

$$\lambda = 4, \text{ so } Q = (4, 5) \quad \text{A1 A1}$$

$$Q \text{ is the midpoint of } PP': \mathbf{OP}' = 2 \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \text{ so } P' \text{ is } (6, 3) \quad \text{A1}$$

$$\text{Answer: } Q(4, 5), P'(6, 3) \quad \text{6 marks}$$

25 (5 marks)

$$|\mathbf{a}| = |\mathbf{p}| = \text{radius } r \quad \text{B1}$$

$$\mathbf{AP} = \mathbf{p} - \mathbf{a}, \mathbf{BP} = \mathbf{p} + \mathbf{a} \quad \text{M1 A1}$$

$$\mathbf{AP} \cdot \mathbf{BP} = (\mathbf{p} - \mathbf{a}) \cdot (\mathbf{p} + \mathbf{a}) = |\mathbf{p}|^2 - |\mathbf{a}|^2 \quad \text{M1}$$

$$= r^2 - r^2 = 0, \text{ so } AP \text{ is perpendicular to } BP: \text{ angle } APB = 90^\circ. \quad \text{E1}$$

$$\text{Answer: Proved} \quad \text{5 marks}$$

26 (7 marks)

$$\text{(a) } \mathbf{DM} = \frac{1}{2}\mathbf{DC} = \frac{1}{2}\mathbf{a} \quad \text{M1}$$

$$\mathbf{AM} = \mathbf{AD} + \mathbf{DM} = \mathbf{d} + \frac{1}{2}\mathbf{a} \quad \text{A1}$$

$$\text{Answer: } \mathbf{d} + \frac{1}{2}\mathbf{a} \quad \text{2 marks}$$

$$\text{(b) E on AM: } \mathbf{AE} = \lambda(\mathbf{d} + \frac{1}{2}\mathbf{a}) \quad \text{B1}$$

$$\text{E on BC: } \mathbf{AE} = \mathbf{a} + \mu\mathbf{d} \quad \text{B1}$$

$$\text{Equate coefficients: } \frac{\lambda}{2} = 1 \text{ and } \lambda = \mu \quad \text{M1}$$

$$\text{So } \lambda = \mu = 2 \text{ and } \mathbf{BE} = 2\mathbf{d} \quad \text{A1}$$

$$\mathbf{BC} = \mathbf{d}, \text{ so } BE = 2BC: C \text{ is the midpoint of } BE. \quad \text{E1}$$

$$\text{Answer: Shown} \quad \text{5 marks}$$

27 (11 marks)

$$\text{(a) } \mathbf{OM} = \mathbf{a} + \frac{1}{2}\mathbf{c}, \mathbf{AN} = \mathbf{ON} - \mathbf{a} = \mathbf{c} + \frac{1}{2}\mathbf{a} - \mathbf{a} = \mathbf{c} - \frac{1}{2}\mathbf{a} \quad \text{B1 B1}$$

$$\text{On OM: } \mathbf{OX} = s(\mathbf{a} + \frac{1}{2}\mathbf{c}) \quad \text{M1}$$

$$\text{On AN: } \mathbf{OX} = \mathbf{a} + t(\mathbf{c} - \frac{1}{2}\mathbf{a}) = (1 - \frac{1}{2}t)\mathbf{a} + t\mathbf{c} \quad \text{M1}$$

$$\text{Equate coefficients: } s = 1 - \frac{1}{2}t \text{ and } \frac{1}{2}s = t; \text{ so } s = 2t \text{ and } 2t = 1 - \frac{1}{2}t, \text{ giving } t = \frac{2}{5}, s = \frac{4}{5} \quad \text{M1}$$

$$\mathbf{OX} = \frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} \quad \text{A1}$$

$$\text{Answer: } \frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} \quad \text{6 marks}$$

$$\text{(b) } t = \frac{2}{5}, \text{ so } AX = \frac{2}{5}AN \text{ and } AX : XN = 2 : 3 \quad \text{B1}$$

$$\text{Answer: } 2 : 3 \quad \text{1 mark}$$

$$\text{(c) } \mathbf{CX} = \frac{4}{5}\mathbf{a} - \frac{3}{5}\mathbf{c}; \mathbf{OR} = \mathbf{c} + k\mathbf{CX} = \frac{4k}{5}\mathbf{a} + (1 - \frac{3k}{5})\mathbf{c} \quad \text{M1}$$

$$R \text{ lies on } AB, \text{ where } \mathbf{OR} = \mathbf{a} + \mu\mathbf{c} \quad \text{M1}$$

$$\text{Coefficient of } \mathbf{a}: \frac{4k}{5} = 1, \text{ so } k = \frac{5}{4} \quad \text{A1}$$

$$\mu = 1 - \frac{3}{4} = \frac{1}{4}, \text{ so } \mathbf{AR} = \frac{1}{4}\mathbf{AB} \text{ and } AR : RB = 1 : 3 \quad \text{A1}$$

$$\text{Answer: } 1 : 3 \quad \text{4 marks}$$

28 (8 marks)

(a) $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, so the lines are parallel M1

Is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ on l_1 ? $1 + 2\lambda = 3$ gives $\lambda = 1$, and then $2 - \lambda = 1$ (checks) M1

Parallel lines sharing a point are the same line: l_1 and l_2 coincide. E1

Answer: The lines coincide 3 marks

(b) l_3 is parallel to l_1 since $(-4, 2) = -2(2, -1)$ B1

The point of l_1 with $y = 0$: $2 - \lambda = 0$, $\lambda = 2$, $x = 1 + 4 = 5$ M1

So $(k, 0) = (5, 0)$ and $k = 5$ A1

Answer: $k = 5$ 3 marks

(c) They are always parallel, as the directions are multiples of each other E1

If $k \neq 5$ then $(k, 0)$ is not on l_1 ; two parallel lines sharing any point must coincide, so they share none. E1

Answer: Parallel, distinct 2 marks

29 (11 marks)

(a) $\mathbf{r} = \begin{pmatrix} 2 + 3t \\ -1 + 4t \end{pmatrix}$ B1

Speed = $\sqrt{9 + 16} = 5$ units per minute B1

Answer: 5 units per minute 2 marks

(b) $\mathbf{CD} = \begin{pmatrix} 2 + 3t - 12 \\ -1 + 4t - 4 \end{pmatrix} = \begin{pmatrix} 3t - 10 \\ 4t - 5 \end{pmatrix}$ M1

$CD^2 = 9t^2 - 60t + 100 + 16t^2 - 40t + 25$ M1

$= 25t^2 - 100t + 125 = 25(t - 2)^2 + 25$ A1

Least when $t = 2$, $CD^2 = 25$ M1

Least distance = 5 units A1

Answer: 5 units (500 m) 5 marks

(c) Inside when $CD^2 < 50$ B1

$25(t - 2)^2 + 25 < 50$, so $(t - 2)^2 < 1$ M1

$1 < t < 3$ A1

Time inside = 2 minutes A1

Answer: 2 minutes 4 marks

30 (9 marks)

(a) $\mathbf{OP} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$, $\mathbf{OR} = \frac{1}{2}(\mathbf{c} + \mathbf{d})$ B1

Midpoint of PR = $\frac{1}{2}(\mathbf{OP} + \mathbf{OR}) = \frac{1}{4}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$ M1 A1

$\mathbf{OQ} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$, $\mathbf{OS} = \frac{1}{2}(\mathbf{d} + \mathbf{a})$ gives the same midpoint $\frac{1}{4}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$ A1

Answer: Shown 4 marks

(b) The diagonals PR and QS bisect each other (common midpoint), so PQRS is a parallelogram. E1

The diagonals meet at the point with position vector $\frac{1}{4}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$, the average of the four vertices. B1

Answer: Shown 2 marks

(c) For a parallelogram $\mathbf{OB} + \mathbf{OD} = \mathbf{OA} + \mathbf{OC}$, i.e. $\mathbf{b} + \mathbf{d} = \mathbf{a} + \mathbf{c}$ M1

So $\frac{1}{4}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}) = \frac{1}{2}(\mathbf{a} + \mathbf{c})$ A1

which is the midpoint of AC (and similarly of BD), the intersection of the diagonals. E1

Answer: Shown 3 marks

Three-Dimensional Vectors

1 (2 marks)

$$\text{Magnitude} = \sqrt{2^2 + (-3)^2 + 6^2}$$

$$= \sqrt{49} = 7$$

Answer: 7

M1

A1

2 marks

2 (3 marks)

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$= \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

$$|\mathbf{AB}| = \sqrt{9 + 16 + 4} = \sqrt{29}$$

$$\text{Answer: } \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}, \sqrt{29}$$

M1

A1

A1

3 marks

3 (2 marks)

$$\mathbf{OP} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$$

$$3\mathbf{i} - 4\mathbf{k} = 3\mathbf{i} + 0\mathbf{j} - 4\mathbf{k} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

$$\text{Answer: } 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}; \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

B1

B1

2 marks

4 (3 marks)

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 7 \end{pmatrix}, \text{ so B is } (1, 1, 7)$$

$$OB = \sqrt{1 + 1 + 49} = \sqrt{51}$$

$$\text{Answer: } \mathbf{B}(1, 1, 7), \mathbf{OB} = \sqrt{51}$$

M1

A1

A1

3 marks

5 (2 marks)

$$\text{Midpoint} = \left(\frac{1+7}{2}, \frac{-2+4}{2}, \frac{5-3}{2} \right)$$

$$= (4, 1, 1)$$

Answer: (4, 1, 1)

M1

A1

2 marks

6 (2 marks)

$$\begin{pmatrix} -6 \\ 9 \\ -3 \end{pmatrix} = -3 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

Each component is multiplied by the same scalar, -3 , so they are parallel.

Answer: Parallel (scalar -3)

M1

E1

2 marks

7 (3 marks)

$$\mathbf{PQ} = \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{PR} = \frac{1}{3}\mathbf{PQ} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} \quad \text{M1}$$

$$\mathbf{OR} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \text{ so R is } (3, 0, 1) \quad \text{A1}$$

Answer: R(3, 0, 1) 3 marks

8 (4 marks)

$$\text{(a) } \mathbf{OF} = \mathbf{OA} + \mathbf{OC} + \mathbf{OD} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \quad \text{B1}$$

$$\text{Answer: } \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \quad \text{1 mark}$$

$$\text{(b) } \mathbf{AG} = \mathbf{OG} - \mathbf{OA} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix} \quad \text{M1}$$

$$|\mathbf{AG}| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7 \quad \text{A1}$$

$$\text{Answer: } \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix} \quad \text{2 marks}$$

$$\text{(c) } \mathbf{OE} = \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}, \mathbf{OF} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}, \text{ so } \mathbf{OM} = \begin{pmatrix} 6 \\ \frac{3}{2} \\ 2 \end{pmatrix} \quad \text{B1}$$

$$\text{Answer: } \begin{pmatrix} 6 \\ \frac{3}{2} \\ 2 \end{pmatrix} \quad \text{1 mark}$$

9 (3 marks)

$$|(2, -3, 6)| = \sqrt{4 + 9 + 36} = 7 \quad \text{B1}$$

$$\text{Unit vector} = \frac{1}{7} \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \quad \text{M1}$$

$$\text{Vector of magnitude 14} = 2 \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 12 \end{pmatrix} \quad \text{A1}$$

$$\text{Answer: } \frac{1}{7} \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}; \begin{pmatrix} 4 \\ -6 \\ 12 \end{pmatrix} \quad \text{3 marks}$$

10 (3 marks)

$$\text{F is } (4, 3, 12) \quad \text{B1}$$

$$\mathbf{OF} = \sqrt{4^2 + 3^2 + 12^2} \quad \text{M1}$$

$$= \sqrt{169} = 13 \quad \text{A1}$$

Answer: F(4, 3, 12), OF = 13 3 marks

11 (3 marks)

$$p^2 + 4p^2 + 4p^2 = 144$$

M1

$$9p^2 = 144, \text{ so } p^2 = 16$$

A1

$$p = \pm 4$$

A1

Answer: $p = \pm 4$

3 marks

12 (4 marks)

$$\mathbf{AB} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$$

B1

$$\mathbf{AD} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

B1

AB = 3AD, so AD is parallel to AB with the common point A: D lies on the line AB

M1

$$\mathbf{AD} = \frac{1}{3}\mathbf{AB}, \text{ so } \mathbf{AD} : \mathbf{DB} = 1 : 2$$

A1

Answer: $\mathbf{AB} = 3\mathbf{AD}$; $\mathbf{AD} : \mathbf{DB} = 1 : 2$

4 marks

13 (3 marks)

$$\mathbf{AD} = \mathbf{BC} = \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

M1 A1

$$\mathbf{OD} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$$

A1

Answer: $\mathbf{D}(3, 5, 2)$

3 marks

14 (6 marks)

$$\text{(a) } \mathbf{AB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

M1

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

A1

$$\text{Answer: } \mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

2 marks

$$\text{(b) } x: 1 + 3\lambda = 7 \text{ gives } \lambda = 2$$

M1

$$y: -2 + 4 = 2 \text{ (checks) and } z: 3 - 8 = -5 \text{ (checks), so C lies on l}$$

A1

Answer: $\lambda = 2$ satisfies all three

2 marks

$$\text{(c) } 3 - 4\lambda = 0, \text{ so } \lambda = \frac{3}{4}$$

M1

$$\text{Point: } (1 + \frac{9}{4}, -2 + \frac{3}{2}, 0) = (\frac{13}{4}, -\frac{1}{2}, 0)$$

A1

$$\text{Answer: } (\frac{13}{4}, -\frac{1}{2}, 0)$$

2 marks

15 (6 marks)

$$\text{(a) } \mathbf{a} \cdot \mathbf{b} = 2 - 4 - 16 = -18; |\mathbf{a}| = 3, |\mathbf{b}| = 9$$

M1 A1

$$\cos \theta = \frac{-18}{(3 \times 9)} = -\frac{2}{3}, \text{ so } \theta = 131.8^\circ$$

A1

Answer: 131.8°

3 marks

$$\text{(b) } (\mathbf{a} + \lambda \mathbf{b}) \cdot \mathbf{a} = 0$$

M1

$$|\mathbf{a}|^2 + \lambda \mathbf{a} \cdot \mathbf{b} = 9 - 18\lambda = 0$$

M1

$$\lambda = \frac{1}{2}$$

A1

Answer: $\lambda = \frac{1}{2}$

3 marks

16 (4 marks)

$$\mathbf{AB} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$$

B1

P between A and B: $\mathbf{AP} = \frac{2}{3}\mathbf{AB}$, so $\mathbf{OP} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

M1 A1

P beyond B: $\mathbf{AP} = 2\mathbf{AB}$, so $\mathbf{OP} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ 9 \end{pmatrix}$

A1

Answer: $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 7 \\ -4 \\ 9 \end{pmatrix}$

4 marks

17 (4 marks)

$$\mathbf{a} \cdot \mathbf{b} = 0: k \times k + k \times (-5) + 2 \times 3 = 0$$

M1 A1

$$k^2 - 5k + 6 = 0$$

A1

$$(k - 2)(k - 3) = 0, \text{ so } k = 2 \text{ or } k = 3$$

A1

Answer: $k = 2$ or $k = 3$

4 marks

18 (6 marks)

$$\mathbf{AB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \mathbf{AC} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

M1

$$|\mathbf{AB}| = \sqrt{4 + 4 + 1} = 3 \text{ and } |\mathbf{AC}| = \sqrt{1 + 4 + 4} = 3, \text{ so isosceles}$$

A1

$$\mathbf{AB} \cdot \mathbf{AC} = 2 - 4 + 2 = 0, \text{ so the angle at A is } 90^\circ$$

M1 A1

$$\text{Area} = \frac{1}{2} \times 3 \times 3$$

M1

$$= 4.5$$

A1

Answer: Shown; area 4.5

6 marks

19 (4 marks)

$$\mathbf{AB} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

M1

$$\mathbf{BC} = \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} = 2\mathbf{AB}$$

A1

AB and BC are parallel and share the point B, so A, B, C are collinear.

E1

$$\mathbf{AB} : \mathbf{BC} = 1 : 2$$

A1

Answer: Collinear, $\mathbf{AB} : \mathbf{BC} = 1 : 2$

4 marks

20 (4 marks)

$$4(\mathbf{a} - \mathbf{p}) + 3(\mathbf{b} - \mathbf{p}) = 5(\mathbf{c} - \mathbf{p})$$

M1

$$2\mathbf{p} = 4\mathbf{a} + 3\mathbf{b} - 5\mathbf{c}$$

M1

$$= \begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 9 \\ 12 \end{pmatrix} - \begin{pmatrix} 15 \\ 5 \\ 10 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix}$$

A1

$$\mathbf{p} = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$$

4 marks

21 (8 marks)

$$\text{(a) } \mathbf{AB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}, |\mathbf{AB}| = \sqrt{9 + 4 + 36} = 7$$

B1

$$\mathbf{AC} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, |\mathbf{AC}| = \sqrt{1 + 4 + 4} = 3$$

B1

$$\text{Answer: } \mathbf{AB} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \text{ (7), } \mathbf{AC} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \text{ (3)}$$

2 marks

$$\text{(b) } \mathbf{AB} \cdot \mathbf{AC} = -3 + 4 + 12 = 13$$

M1

$$\cos \theta = \frac{13}{(7 \times 3)} = \frac{13}{21}$$

M1

$$\theta = 51.8^\circ$$

A1

$$\text{Answer: } 51.8^\circ$$

3 marks

$$\text{(c) } \sin \theta = \sqrt{1 - (13/21)^2} = \frac{\sqrt{272}}{21} = \frac{4\sqrt{17}}{21}$$

M1 A1

$$\text{Area} = \frac{1}{2} \times 7 \times 3 \times \frac{4\sqrt{17}}{21} = 2\sqrt{17}$$

A1

$$\text{Answer: } 2\sqrt{17}$$

3 marks

22 (8 marks)

$$\text{(a) } x: 1 + \lambda = -1 + 2\mu \quad y: 2 - \lambda = -2 + \mu \quad z: 3 + 2\lambda = 13 - 3\mu$$

M1

$$\text{Adding the first two: } 3 = -3 + 3\mu, \text{ so } \mu = 2 \text{ and } \lambda = 2$$

M1 A1

$$z \text{ check: } 3 + 4 = 7 \text{ and } 13 - 6 = 7, \text{ so the third equation holds and the lines intersect}$$

B1

$$\text{Position vector} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 7 \end{pmatrix}$$

A1

$$\text{Answer: } \begin{pmatrix} 3 \\ 0 \\ 7 \end{pmatrix}$$

5 marks

$$\text{(b) } d_1 \cdot d_2 = 2 - 1 - 6 = -5; |d_1| = \sqrt{6}, |d_2| = \sqrt{14}$$

M1

$$\cos \varphi = \frac{5}{\sqrt{84}} \text{ (taking the acute angle)}$$

M1

$$\varphi = 56.9^\circ$$

A1

$$\text{Answer: } 56.9^\circ$$

3 marks

23 (6 marks)

$$AB : BC = 1 : 2, \text{ so } \mathbf{AB} = \frac{1}{3}\mathbf{AC}, \text{ giving } \mathbf{OB} = \mathbf{OA} + \frac{1}{3}\mathbf{AC} \quad \text{M1}$$

$$y: b = 4 + \frac{1}{3}(-2 - 4) = 2 \quad \text{M1 A1}$$

$$x: 2 = 5 + \frac{1}{3}(c - 5), \text{ so } c - 5 = -9 \text{ and } c = -4 \quad \text{M1 A1}$$

$$z: b - 3 = -1 = a + \frac{1}{3}(5 - a), \text{ so } 2a + 5 = -3 \text{ and } a = -4 \quad \text{A1}$$

$$\text{Answer: } \mathbf{a = -4, b = 2, c = -4} \quad \text{6 marks}$$

24 (7 marks)

$$\text{(a) } x: \lambda = 1; y: \lambda = \mu, \text{ so } \mu = 1 \quad \text{M1 A1}$$

$$z: \lambda = 0 + 2\mu \text{ gives } 1 = 2, \text{ a contradiction, so they do not meet} \quad \text{A1}$$

(1, 1, 1) is not a multiple of (0, 1, 2), so the lines are not parallel. E1

Answer: Skew 4 marks

$$\text{(b) } \lambda = 1, \mu = 1 \text{ as before; } z: 1 = c + 2 \quad \text{M1}$$

$$c = -1 \quad \text{A1}$$

Intersection (1, 1, 1) A1

$$\text{Answer: } \mathbf{c = -1, \text{ point } (1, 1, 1)} \quad \text{3 marks}$$

25 (7 marks)

$$\text{(a) Centre of the base is } (3, 2, 0), \text{ so } V \text{ is } (3, 2, 5) \quad \text{B1}$$

$$\mathbf{VA} = \begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix} \quad \text{M1}$$

$$VA = \sqrt{9 + 4 + 25} = \sqrt{38} \quad \text{A1}$$

$$\text{Answer: } \mathbf{V(3, 2, 5), VA = \sqrt{38}} \quad \text{3 marks}$$

$$\text{(b) } \mathbf{VC} = \begin{pmatrix} -3 \\ 2 \\ -5 \end{pmatrix} \quad \text{B1}$$

$$VA \cdot VC = -9 - 4 + 25 = 12 \quad \text{M1}$$

$$\cos AVC = \frac{12}{38} = \frac{6}{19} \quad \text{A1}$$

$$AVC = 71.6^\circ \quad \text{A1}$$

$$\text{Answer: } \mathbf{71.6^\circ} \quad \text{4 marks}$$

26 (6 marks)

$$\text{Point on } l: Q = \begin{pmatrix} 1 + 2\lambda \\ 2 + \lambda \\ 3 + 2\lambda \end{pmatrix}, \mathbf{PQ} = \begin{pmatrix} 2\lambda - 4 \\ \lambda + 1 \\ 2\lambda - 1 \end{pmatrix} \quad \text{M1 A1}$$

$$\text{PQ perpendicular to } l: 2(2\lambda - 4) + (\lambda + 1) + 2(2\lambda - 1) = 0 \quad \text{M1}$$

$$9\lambda - 9 = 0, \text{ so } \lambda = 1 \quad \text{A1}$$

$$\text{Foot } Q = (3, 3, 5), \mathbf{PQ} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad \text{A1}$$

$$\text{Distance} = \sqrt{4 + 4 + 1} = 3 \quad \text{A1}$$

$$\text{Answer: } \mathbf{\text{Foot } (3, 3, 5), \text{ distance } 3} \quad \text{6 marks}$$

27 (11 marks)

(a) $x: 1 + \lambda = 3 + 2\mu$ $y: 2\lambda = 1 - \mu$ $z: 2 - \lambda = \mu$

M1

From $y: \mu = 1 - 2\lambda$. Into $x: 1 + \lambda = 5 - 4\lambda$, so $\lambda = \frac{4}{5}$, $\mu = -\frac{3}{5}$

M1 A1

$z: 2 - \frac{4}{5} = \frac{6}{5}$ but $\mu = -\frac{3}{5}$, so the equations are inconsistent: no intersection

M1

The directions $(1, 2, -1)$ and $(2, -1, 1)$ are not multiples of each other, so the lines are not parallel. Hence they are skew.

E1

Answer: Skew

5 marks

(b) $z: 2 - \frac{4}{5} = k - \frac{3}{5}$

M1

$k = \frac{6}{5} + \frac{3}{5} = \frac{9}{5}$

A1

Answer: $k = \frac{9}{5}$

2 marks

(c) $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \frac{4}{5} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

M1

$= \begin{pmatrix} \frac{9}{5} \\ \frac{8}{5} \\ \frac{6}{5} \end{pmatrix}$ (check with $l_2: \begin{pmatrix} 3 \\ 1 \\ \frac{9}{5} \end{pmatrix} - \frac{3}{5} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ gives the same point)

A1

Answer: $\begin{pmatrix} \frac{9}{5} \\ \frac{8}{5} \\ \frac{6}{5} \end{pmatrix}$

2 marks

(d) $\cos \varphi = \frac{|2 - 2 - 1|}{(\sqrt{6} \times \sqrt{6})} = \frac{1}{6}$

M1

$\varphi = 80.4^\circ$

A1

Answer: 80.4°

2 marks

28 (11 marks)

(a) $\mathbf{r} = \begin{pmatrix} -10 + 2t \\ -8 + 3t \\ -9 + 6t \end{pmatrix}$

B1

$d^2 = (2t - 10)^2 + (3t - 8)^2 + (6t - 9)^2$

M1

$= (4 + 9 + 36)t^2 - (40 + 48 + 108)t + (100 + 64 + 81) = 49t^2 - 196t + 245$

A1

Answer: Shown

3 marks

(b) $d^2 = 49(t - 2)^2 + 49$

M1 A1

Least distance $= \sqrt{49} = 7$ m, at $t = 2$ s

A1

Answer: 7 m at $t = 2$ s

3 marks

(c) $49(t - 2)^2 + 49 < 144$, so $(t - 2)^2 < \frac{95}{49}$

M1

$|t - 2| < \frac{\sqrt{95}}{7} = 1.392$

M1

$0.61 < t < 3.39$ (seconds)

A1

Answer: between $t = 0.61$ s and $t = 3.39$ s

3 marks

(d) At $t = 2$: $\mathbf{r} = \begin{pmatrix} -6 \\ -2 \\ 3 \end{pmatrix}$; velocity $= \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

M1

Scalar product $= -12 - 6 + 18 = 0$, so they are perpendicular

A1

Answer: Scalar product = 0

2 marks

29 (12 marks)

$$(a) \mathbf{PQ} = \begin{pmatrix} 12 + 2t - 4t \\ -3 + 3t - 2t \\ 49 - 3t - 40 + t \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} 12 - 2t \\ -3 + t \\ 9 - 2t \end{pmatrix} \quad \text{A1}$$

Answer: Shown

2 marks

$$(b) |\mathbf{PQ}|^2 = (12 - 2t)^2 + (t - 3)^2 + (9 - 2t)^2 \quad \text{M1}$$

$$= 144 - 48t + 4t^2 + t^2 - 6t + 9 + 81 - 36t + 4t^2 \quad \text{M1}$$

$$= 9t^2 - 90t + 234 \quad \text{A1}$$

$$= 9(t - 5)^2 + 9 \quad \text{M1}$$

$$\text{Least at } t = 5 \text{ s} \quad \text{A1}$$

$$\text{Least distance} = \sqrt{9} = 3 \text{ m} \quad \text{A1}$$

Answer: t = 5 s, least distance 3 m

6 marks

$$(c) \text{ Closer than 3.5 m when } 9(t - 5)^2 + 9 < 12.25 \quad \text{M1}$$

$$(t - 5)^2 < \frac{3.25}{9} = \frac{13}{36} \quad \text{A1}$$

$$|t - 5| < \frac{\sqrt{13}}{6} = 0.601 \quad \text{M1}$$

$$\text{Duration} = 2 \times 0.601 = 1.20 \text{ s} \quad \text{A1}$$

Answer: 1.20 s

4 marks

30 (10 marks)

$$(a) \text{ Midpoint of OF} = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \\ \frac{c}{2} \end{pmatrix} \quad \text{B1}$$

$$\mathbf{A} = (a, 0, 0) \text{ and } \mathbf{G} = (0, b, c), \text{ so the midpoint of AG} = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \\ \frac{c}{2} \end{pmatrix} \quad \text{M1}$$

The midpoints coincide, so OF and AG bisect each other.

A1

Answer: Shown

3 marks

$$(b) \mathbf{OF} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \mathbf{AG} = \begin{pmatrix} -a \\ b \\ c \end{pmatrix} \quad \text{M1}$$

$$\mathbf{BD} = \begin{pmatrix} -a \\ -b \\ c \end{pmatrix}, \mathbf{CE} = \begin{pmatrix} a \\ -b \\ c \end{pmatrix} \quad \text{A1}$$

Each has magnitude $\sqrt{a^2 + b^2 + c^2}$.

A1

Answer: Shown

3 marks

$$(c) \mathbf{OF} = \begin{pmatrix} 4 \\ 6 \\ 12 \end{pmatrix}, \mathbf{AG} = \begin{pmatrix} -4 \\ 6 \\ 12 \end{pmatrix} \quad \text{B1}$$

$$\mathbf{OF} \cdot \mathbf{AG} = -16 + 36 + 144 = 164 \quad \text{M1}$$

$$|\mathbf{OF}| = |\mathbf{AG}| = 14, \cos \theta = \frac{164}{196} = \frac{41}{49} \quad \text{A1}$$

$$\theta = 33.2^\circ \quad \text{A1}$$

Answer: 33.2°

4 marks