

Vector Notation and Operations

- 1 Given that $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, find $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$. [2]

- 2 (a) Write $5\mathbf{i} - 2\mathbf{j}$ as a column vector.
(b) Hence, or otherwise, find $3(5\mathbf{i} - 2\mathbf{j}) - (\mathbf{i} + 4\mathbf{j})$ in the form $x\mathbf{i} + y\mathbf{j}$. [3]

- 3 ABCD is a rectangle with $\mathbf{AB} = \mathbf{a}$ and $\mathbf{AD} = \mathbf{d}$. Write down, in terms of $\mathbf{a}$ and $\mathbf{d}$, the vectors $\mathbf{BC}$, $\mathbf{DC}$, $\mathbf{BD}$ and $\mathbf{CA}$. [4]

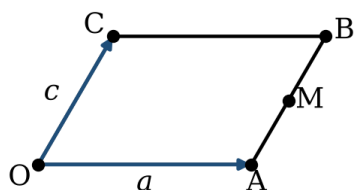
- 4 Given that $(x + 2)\mathbf{i} + (3 - y)\mathbf{j} = 5\mathbf{i} - 2\mathbf{j}$, find the values of x and y . [2]

- 5 Find the vector that must be added to $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ to give $\begin{pmatrix} -1 \\ 7 \end{pmatrix}$. [2]

- 6 Four vectors are $\mathbf{p} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
State which of the other three vectors are parallel to $\mathbf{p}$, giving the scalar multiple in each case. [3]

- 7 The vectors $\mathbf{p} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{q} = k\mathbf{i} - 6\mathbf{j}$ are parallel. Find the value of k . [2]

- 8 OABC is a parallelogram. The position vectors of A and C relative to O are $\mathbf{a}$ and $\mathbf{c}$.
The point M is the midpoint of AB.
Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, the vectors



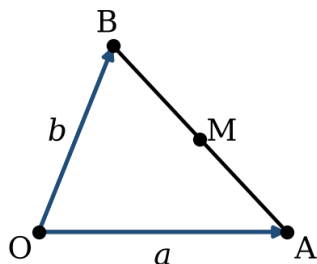
- (a) $\mathbf{OB}$, [1]
(b) $\mathbf{AC}$, [1]
(c) $\mathbf{OM}$. [2]

- 9 Given that $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, find the vector $\mathbf{x}$ such that $2\mathbf{x} - \mathbf{a} = 3(\mathbf{x} + \mathbf{b})$. [3]

- 10 Given $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$, find the scalars m and n such that $\mathbf{c} = m\mathbf{a} + n\mathbf{b}$. [4]

- 11 In triangle ABC, $\mathbf{AB} = \mathbf{p}$ and $\mathbf{BC} = \mathbf{q}$. Find, in terms of $\mathbf{p}$ and $\mathbf{q}$,
(a) $\mathbf{AC}$, (b) $\mathbf{CA}$, (c) $\mathbf{AB} - \mathbf{CB}$. [4]

- 12** In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. M is the midpoint of AB. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vectors $\mathbf{AB}$ and $\mathbf{OM}$.



[4]

- 13** A boat is rowed with displacement $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ km and then $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ km, where $\mathbf{i}$ is east and $\mathbf{j}$ is north. Find the resultant displacement, and the displacement needed to return the boat to its starting point. [3]

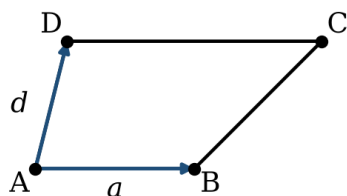
- 14** The vectors $\mathbf{p} = (x^2 - 1)\mathbf{i} + 6\mathbf{j}$ and $\mathbf{q} = (x + 1)\mathbf{i} + 3\mathbf{j}$ are parallel. Find the possible values of x . [4]

- 15** A drone leaves a base O and flies three straight legs. Displacements are in metres, with $\mathbf{i}$ east and $\mathbf{j}$ north. Leg 1 is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, leg 2 is $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ and leg 3 is the vector $\mathbf{d}$. After leg 3 the drone is at the point with position vector $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$.

- (a) Find $\mathbf{d}$. [2]
 (b) The drone now flies straight back to O. Find the displacement of this fourth leg. [2]

- 16** The vectors $\mathbf{a}$ and $\mathbf{b}$ are not parallel. Given that $(p + q)\mathbf{a} + (2p - q)\mathbf{b} = 5\mathbf{a} + \mathbf{b}$, find p and q . [4]

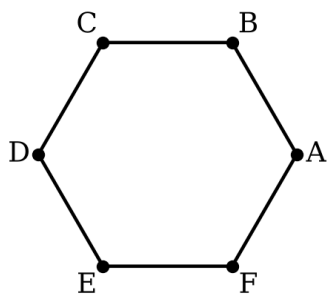
- 17** ABCD is a trapezium with $\mathbf{AB} = \mathbf{a}$, $\mathbf{AD} = \mathbf{d}$ and $\mathbf{DC} = \frac{3}{2}\mathbf{a}$. Find, in terms of $\mathbf{a}$ and $\mathbf{d}$, (a) $\mathbf{AC}$, (b) $\mathbf{BD}$, (c) $\mathbf{BC}$.



[5]

- 18** Three forces $\mathbf{F}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ N, $\mathbf{F}_2 = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ N and $\mathbf{F}_3$ act on a particle which is in equilibrium, so the resultant force is zero. Find $\mathbf{F}_3$, and find the value of t if $\mathbf{F}_1 + t\mathbf{F}_2$ is parallel to the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [5]

- 19** ABCDEF is a regular hexagon with $\mathbf{AB} = \mathbf{a}$ and $\mathbf{BC} = \mathbf{b}$. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, (a) $\mathbf{CD}$, (b) $\mathbf{AD}$, (c) $\mathbf{AE}$.



[7]

- 20** Points A, B and C have position vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 18 \end{pmatrix}$ relative to an origin O. Show that A, B and C are collinear and find the ratio AB : BC.

[4]

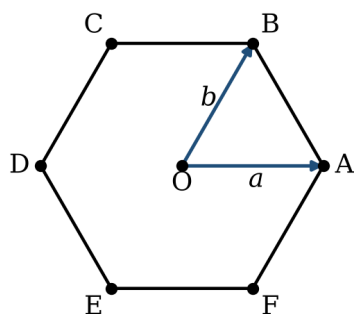
- 21** The vectors $\mathbf{a}$ and $\mathbf{b}$ are not parallel and neither is the zero vector.
- (a) Given that $(\lambda + 2\mu - 3)\mathbf{a} + (2\lambda - \mu + 4)\mathbf{b} = \mathbf{0}$, explain why $\lambda + 2\mu - 3 = 0$ and $2\lambda - \mu + 4 = 0$.
- (b) Hence find λ and μ .
- (c) Given that $(k + 1)\mathbf{a} + 6\mathbf{b}$ is parallel to $2\mathbf{a} + k\mathbf{b}$, find the possible values of k .

[1]

[4]

[4]

- 22** ABCDEF is a regular hexagon with centre O. $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.



- (a) Explain why $\mathbf{OC} = \mathbf{b} - \mathbf{a}$.
- (b) Find $\mathbf{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
- (c) Find $\mathbf{FD}$. What does your answer tell you about AC and FD?
- (d) Show that $\mathbf{FC} = 2\mathbf{AB}$.

[2]

[2]

[3]

[2]

- 23** A student writes: "In triangle ABC, $\mathbf{AB} + \mathbf{BC} + \mathbf{CA} = 2\mathbf{AC}$, because $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$ and $\mathbf{CA} = \mathbf{AC}$." Explain the two errors, and state the correct value of the sum.

[3]

- 24** Given $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, find the values of k for which $k\mathbf{a} + \mathbf{b}$ is parallel to $\mathbf{a} + k\mathbf{b}$.

[6]

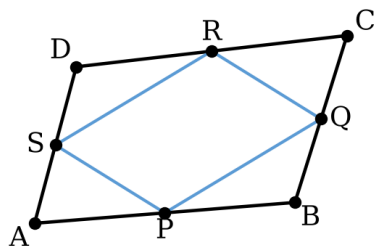
- 25** In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. The point P is on OA with $OP = \frac{1}{3}OA$ and Q is the midpoint of AB. The line PQ is extended to meet the line OB extended at R. Find $\mathbf{OR}$ in terms of $\mathbf{b}$, and describe the position of R relative to O and B.

[6]

- 26** Prove that the diagonals of a parallelogram bisect each other.

[5]

- 27 ABCD is any quadrilateral with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ relative to an origin O. The points P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.

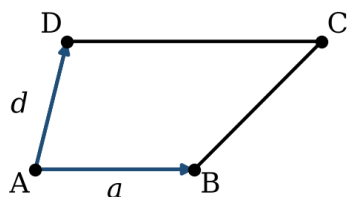


- (a) Prove that PQRS is a parallelogram. [6]
 (b) Given that the diagonals AC and BD of ABCD are equal in length, show that PQRS is a rhombus. [3]

- 28 Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$. A student is asked to find λ such that $\mathbf{a} + \lambda\mathbf{b}$ is parallel to $\mathbf{i} + 2\mathbf{j}$. The student writes: " $\mathbf{a} + \lambda\mathbf{b} = (2 + \lambda)\mathbf{i} + (1 - 3\lambda)\mathbf{j}$. For this to equal $\mathbf{i} + 2\mathbf{j}$, $2 + \lambda = 1$ and $1 - 3\lambda = 2$, so $\lambda = -1$ and $\lambda = -\frac{1}{3}$. There is no solution."

- (a) Explain the error in the student's argument. [2]
 (b) Find the correct value of λ . [4]
 (c) Hence write $\mathbf{a} + \lambda\mathbf{b}$ in the form $k(\mathbf{i} + 2\mathbf{j})$ for this value of λ . [2]

- 29 ABCD is a trapezium with $\mathbf{AB} = \mathbf{a}$, $\mathbf{AD} = \mathbf{d}$ and $\mathbf{DC} = \lambda\mathbf{a}$, where $\lambda > 0$. M and N are the midpoints of AD and BC respectively.



- (a) Find the vector $\mathbf{MN}$ in terms of $\mathbf{a}$ and λ . [5]
 (b) Hence prove that MN is parallel to AB and DC, and that MN is equal to half of the sum of the lengths of the parallel sides. [3]

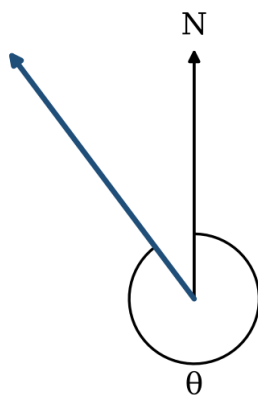
- 30 The points A, B and C are not collinear. The point P satisfies $2\mathbf{PA} + \mathbf{PB} + 3\mathbf{PC} = \mathbf{0}$.

- (a) Show that $\mathbf{AP} = \frac{1}{6}\mathbf{AB} + \frac{1}{2}\mathbf{AC}$. [4]
 (b) The point D lies on BC with $BD : DC = 3 : 1$. Show that A, P and D are collinear, and find the ratio AP : PD. [5]

Magnitude and Direction

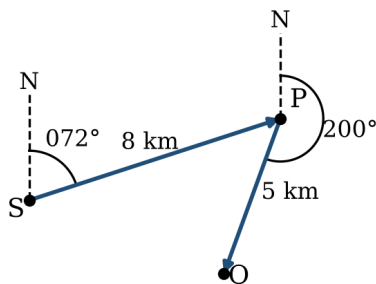
- 1 Find the magnitude of the vector $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$. [2]
 2 Find a unit vector in the direction of $\begin{pmatrix} 8 \\ -15 \end{pmatrix}$. [2]
 3 A force of 6 N acts due east and a force of 8 N acts due north on a particle. Write each force as a column vector and find the magnitude of the resultant force. [3]
 4 Given that $\mathbf{a} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -11 \end{pmatrix}$, find (a) $|\mathbf{a} - \mathbf{b}|$, (b) $|\mathbf{a}| + |\mathbf{b}|$ in the form $p\sqrt{2} + q\sqrt{5}$. [4]

- 5 Find the distance between the points A(1, 2) and B(7, 10). [2]
- 6 A particle has velocity $\begin{pmatrix} -5 \\ 12 \end{pmatrix} \text{ m s}^{-1}$. Find its speed. [2]
- 7 The vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$. Find the angle between $\mathbf{v}$ and the positive x-axis, measured anticlockwise, giving your answer in degrees to 1 decimal place. [3]
- 8 Given that $|3\mathbf{i} + k\mathbf{j}| = 7$, find the possible values of k , giving your answers as exact values. [3]
- 9 Find the vector of magnitude 20 that has the same direction as $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. [3]
- 10 Find the exact angle that the vector $\begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix}$ makes with the positive x-axis. [3]
- 11 A force of magnitude 8 N acts at 30° above the positive x-axis. Write the force as a column vector, giving exact components. [3]
- 12 Relative to an origin O, the point A has coordinates $(-4, 1)$ and the point B is such that $\mathbf{AB} = 7\mathbf{i} - 5\mathbf{j}$. Find the distance of B from O. [3]
- 13 The points P(0, 0), Q(3, 4) and R(-4, 3) are given. Show that triangle PQR is isosceles and right-angled, and find the length QR as a surd. [4]
- 14 A ship moves with constant velocity $\begin{pmatrix} -6 \\ 8 \end{pmatrix} \text{ km h}^{-1}$, where $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ a unit vector due north.



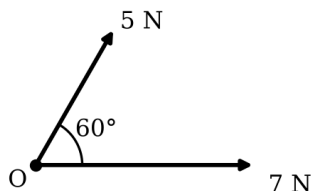
- (a) Find the speed of the ship. [2]
- (b) Find the bearing on which the ship is travelling, to 1 decimal place. [3]

- 15** A hiker walks 8 km on a bearing of 072° from a start point S to a point P, and then 5 km on a bearing of 200° to a point Q. Take $\mathbf{i}$ as a unit vector due east and $\mathbf{j}$ as a unit vector due north.



- (a) Write the displacements $\mathbf{SP}$ and $\mathbf{PQ}$ as column vectors, giving the components to 3 significant figures. [3]
 (b) Find the distance SQ and the bearing of Q from S. [4]

- 16** Two forces of magnitude 7 N and 5 N act on a particle at O. The angle between them is 60° , as shown. Find the magnitude of the resultant force and the angle it makes with the 7 N force, giving answers to 3 significant figures.



[5]

- 17** The vectors $\mathbf{OA}$ and $\mathbf{OB}$ have $|\mathbf{OA}| = 3$ and $|\mathbf{OB}| = 5$, and the angle AOB is 60° . The vector $\mathbf{OP} = 2\mathbf{OA} - \mathbf{OB}$. Find $|\mathbf{OP}|$ and the angle between $\mathbf{OP}$ and $\mathbf{OA}$, to 1 decimal place. [5]

- 18** The points A(1, 1), B(5, 2) and C(2, 6) are the vertices of a triangle. Find the angle ABC to 1 decimal place, and the area of the triangle. [5]

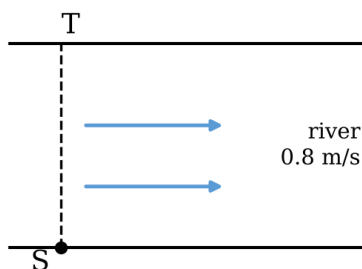
- 19** Find the two unit vectors that are perpendicular to $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$. [4]

- 20** The vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ are drawn from the same point O. Use the cosine rule in the triangle formed by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ to find the angle between $\mathbf{a}$ and $\mathbf{b}$, to 1 decimal place. [5]

- 21** $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and λ is a real number. Show that $|\mathbf{a} + \lambda\mathbf{b}|^2 = 5\lambda^2 + 10\lambda + 10$, and hence find the value of λ for which $|\mathbf{a} + \lambda\mathbf{b}|$ is least. State this least value. [6]

- 22** The vector $\mathbf{v} = \begin{pmatrix} k \\ k+1 \end{pmatrix}$ has magnitude 5.
 (a) Find the two possible values of k , and write down the two possible vectors. [4]
 (b) Find the angle between these two vectors, to 1 decimal place. [4]

- 23** A river 60 m wide flows due east at 0.8 m s^{-1} between two parallel straight banks. A swimmer, who can swim at 1.5 m s^{-1} in still water, starts from a point S on the south bank. T is the point directly opposite S on the north bank. Take $\mathbf{i}$ as east and $\mathbf{j}$ as north.

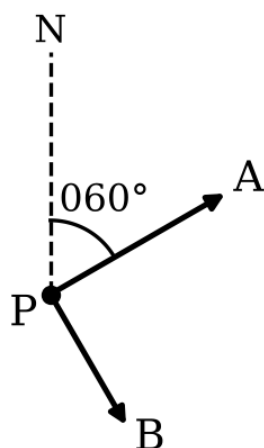


- The swimmer heads due north. Find the velocity of the swimmer relative to the bank, her speed, and the bearing of her path, to 1 decimal place. [4]
- Find how far downstream of T she reaches the north bank. [3]
- She now wishes to reach T. Find the bearing on which she should head, to 1 decimal place. [3]

- 24** A drone moves so that its position vector relative to the base O of a mast, in metres at time t seconds, is $\mathbf{r} = \begin{pmatrix} 1 + 3t \\ 2 - 4t \end{pmatrix}$.

- Find the initial distance of the drone from O and its speed. [3]
- Show that $|\mathbf{r}|^2 = 25t^2 - 10t + 5$, and hence find the least distance of the drone from O and the time at which it occurs. [5]

- 25** Two ships A and B leave a port P at the same time. A sails at 12 km h^{-1} on a bearing of 060° and B sails at 9 km h^{-1} on a bearing of 150° .



- Find the distance between the ships after 2 hours. [3]
- Find the bearing of B from A, to 1 decimal place. [4]

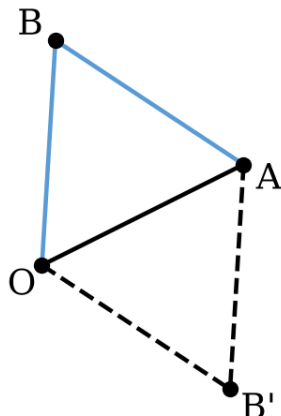
- 26** A force $\mathbf{F}$ of magnitude 26 N acts at an angle θ to the positive x-axis, where $\tan \theta = -\frac{5}{12}$ and

$90^\circ < \theta < 180^\circ$. A second force $\mathbf{G} = \begin{pmatrix} 34 \\ -20 \end{pmatrix}$ N also acts.

- Find $\mathbf{F}$ as a column vector. (b) Find the magnitude and direction of the resultant of $\mathbf{F}$ and $\mathbf{G}$, giving the direction as an angle below the positive x-axis. [6]

- 27 An aircraft flies at an airspeed of 200 km h^{-1} on a heading of θ (a bearing). The wind has velocity $\begin{pmatrix} -30 \\ 40 \end{pmatrix} \text{ km h}^{-1}$, where $\mathbf{i}$ is due east and $\mathbf{j}$ due north. The velocity of the aircraft over the ground is the sum of its velocity through the air and the wind velocity.
- The pilot wants to fly due north over the ground. Find θ , to 1 decimal place. [3]
 - Find the speed of the aircraft over the ground on this heading, to 3 significant figures. [2]
 - Find the time to complete a journey of 600 km over the ground, giving your answer in hours and minutes to the nearest minute. [2]
 - The pilot instead flies on a heading of 000° . Find the speed and bearing of the aircraft over the ground, to 1 decimal place. [4]

- 28 OAB is an equilateral triangle with $\mathbf{OA} = 2\mathbf{i} + \mathbf{j}$. The point B is at either of the two possible positions B or B' shown in the diagram.

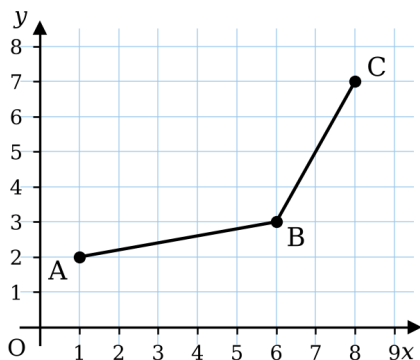


- Let $\mathbf{OB} = x\mathbf{i} + y\mathbf{j}$. By considering the lengths OB and AB, find the exact position vectors of the two possible positions of B. [8]
 - Find the exact area of triangle OAB. [2]
- 29 OABC is a rhombus with $\mathbf{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{OC} = \begin{pmatrix} r \\ s \end{pmatrix}$, where p, q, r and s are constants. Prove that the diagonals OB and AC are perpendicular. [6]

- 30 At time $t = 0$ a ship S is at the point with position vector $\begin{pmatrix} 16 \\ 4 \end{pmatrix} \text{ km}$ relative to a port O, and moves with constant velocity $\begin{pmatrix} -4 \\ 1 \end{pmatrix} \text{ km h}^{-1}$. At $t = 0$ a patrol boat leaves O and moves in a straight line at a constant speed of 5 km h^{-1} to intercept the ship. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ point east and north.
- Find the position vector of the ship at time t hours, and show that the patrol boat intercepts the ship when $t^2 + 15t - 34 = 0$. [5]
 - Solve this equation and hence find the velocity of the patrol boat and the bearing on which it travels, to 1 decimal place. [5]

Position Vectors

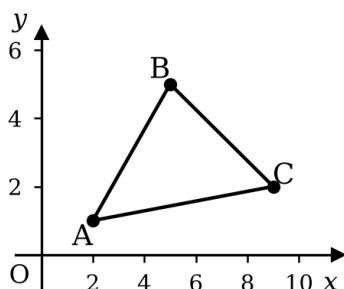
- The points A and B have coordinates (2, 5) and (7, 1). Find the vector $\mathbf{AB}$ as a column vector. [2]
- The position vectors of A and B relative to an origin O are $\mathbf{OA} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{OB} = \mathbf{i} + 4\mathbf{j}$. Find the position vector of the midpoint of AB. [2]
- The points A and B have coordinates (3, -1) and (7, 5). Find the vector $\mathbf{AB}$ and the position vector of the midpoint M of AB. [3]

- 4 Given $\mathbf{OA} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{OB} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$, find $\mathbf{AB}$ and $\mathbf{BA}$. [2]
-
- 5 A particle is at the point with position vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ m and then undergoes a displacement of $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ m. Find its new position vector. [2]
-
- 6 The point A has position vector $3\mathbf{i} - 2\mathbf{j}$ and the point B has position vector $\mathbf{i} + 4\mathbf{j}$. Find $\mathbf{AB}$ and the distance AB as a surd in its simplest form. [3]
-
- 7 A is the point $(-1, 4)$ and B is the point $(9, -1)$. The point P lies on AB such that $AP : PB = 2 : 3$. Find the position vector of P. [3]
-
- 8 The points A(1, 2), B(6, 3) and C(8, 7) are three vertices of the parallelogram ABCD, labelled in order.
- 
- (a) Find the position vector of D. [2]
 (b) Show that the midpoints of the diagonals AC and BD coincide. [2]
-
- 9 A(−3, 2) and B(9, −2) are two points. The point M lies on AB with $AM : MB = 3 : 1$. Given that M is the midpoint of PQ, where P is the point (10, 3), find the position vector of Q. [4]
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- 10 $\mathbf{AB} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} -3 \\ 10 \end{pmatrix}$. The point D lies on BC with $BD : DC = 2 : 1$. Find $\mathbf{AD}$ and hence the length AD. [4]
-
- 11 The midpoint of AB is M(3, 2) and A is the point (−1, 5). Find the coordinates of B. [3]
-
- 12 P(2, 3), Q(5, 9) and R(8, 15) are three points. Show that they are collinear and state the ratio PQ : QR. [4]
-
- 13 A particle moves so that its position vector at time t seconds is $\mathbf{r} = \begin{pmatrix} 6 - 3t \\ 1 + 2t \end{pmatrix}$ metres. Find its position when $t = 3$, and the time at which it crosses the y-axis. [4]
-
- 14 The points A(1, −2), B(4, 1) and C(10, k) are collinear.
 (a) Find the value of k. [4]
 (b) Find the ratio AB : BC. [1]
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- 15** A drone starts at the point A with position vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ m and moves with constant velocity $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ m s⁻¹, where **i** is east and **j** is north. Its position vector after *t* seconds is **r**.
- Write down **r** in terms of *t*. [1]
 - Find the time at which the drone is 20 m east of the origin, and its position vector then. [2]
 - Show that the drone passes through the point (11, 13) and find the time it takes to reach it. [2]
 - Find the distance the drone travels in the first 6 seconds. [2]

- 16** A(1, 2) and B(7, 5). The point P lies on AB with AP : PB = 2 : 1 and Q lies on AB extended beyond B with AB : BQ = 3 : 1. Find the position vectors of P and Q. [5]

- 17** The points A(2, 1), B(5, 5) and C(9, 2) are shown. Show that triangle ABC is isosceles and right-angled at B, and find its area.



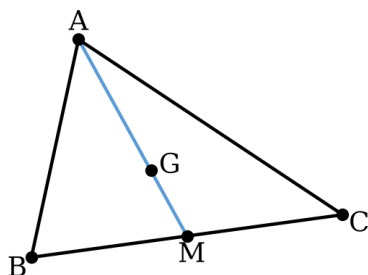
[5]

- 18** A(1, 7) and B(7, -1) are two points. The point R lies on AB with $\mathbf{OR} = \mathbf{OA} + \mu\mathbf{AB}$. Show that $|\mathbf{OR}|^2 = 100\mu^2 - 100\mu + 50$, and hence find the point of AB that is closest to the origin and its distance from O. [6]

- 19** At time *t* = 0 particle P is at $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ m and moves with velocity $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ m s⁻¹. Particle Q is at $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ m and moves with velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ m s⁻¹. Find the position vectors of P and Q at time *t*, show that the particles do not collide, and find the distance between them when *t* = 2. [5]

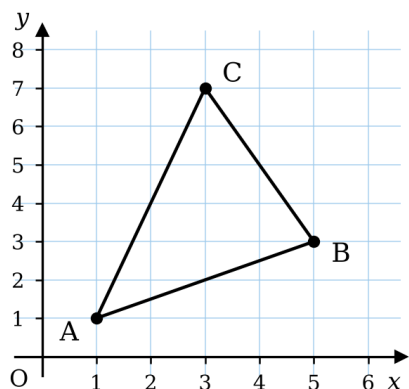
- 20** A student says: "The point P that divides AB in the ratio 2 : 3 has position vector $\frac{2\mathbf{a} + 3\mathbf{b}}{5}$." Explain the error, find the correct expression, and check it for A(0, 0), B(10, 5). [4]

- 21** A triangle ABC has vertices with position vectors **a**, **b** and **c** relative to an origin O. The point M is the midpoint of BC, and G is the point on AM such that AG : GM = 2 : 1.



- Write down **OM**. [1]
- Show that $\mathbf{OG} = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$. [3]
- Explain why G also lies on the other two medians of the triangle. [1]
- Find the coordinates of G when A, B, C are the points (1, 2), (7, -1) and (4, 8). [1]

- 22 The points A(1, 1), B(5, 3) and C(3, 7) are shown in the diagram.



- (a) Show that triangle ABC is right-angled and isosceles, stating which angle is the right angle. [4]
 (b) ABCD is a square. Find the coordinates of D. [2]
 (c) Find the area of the square. [1]

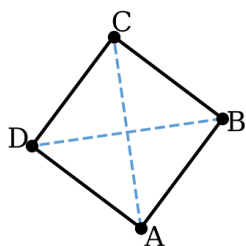
- 23 ABCD is a quadrilateral with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ relative to an origin O. The points M and N are the midpoints of AB and CD respectively.

- (a) Show that $\mathbf{MN} = \frac{1}{2}(\mathbf{AD} + \mathbf{BC})$. [3]
 (b) Given that $(\lambda^2 - 5\lambda + 8)\mathbf{MN} = \mathbf{AD} + \mathbf{BC}$, and that $\mathbf{MN}$ is not the zero vector, find the possible values of λ . [3]

- 24 A particle P is at the point with position vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ m at time $t = 0$ and moves with constant velocity $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ m s⁻¹. A particle Q is at the point with position vector $\begin{pmatrix} 26 \\ -5 \end{pmatrix}$ m at time $t = 0$ and moves with constant velocity $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ m s⁻¹.

- (a) Show that P and Q collide, and find the time and the position vector of the collision. [4]
 (b) Find the distance between P and Q when $t = 2$. [2]

- 25 ABCD is a rhombus with A(1, 1), B(4, 5) and $\mathbf{AD} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.



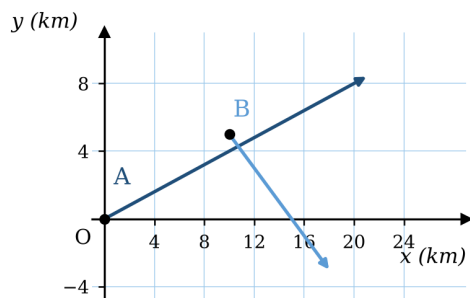
- (a) Show that ABCD is a rhombus (the diagram is not to scale) and find the coordinates of C and D. [4]
 (b) Show that the diagonals AC and BD are perpendicular. [3]

- 26 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$. The point P lies on AB with $AP : PB = \lambda : 1$.

- (a) Show that $\mathbf{OP} = \frac{\mathbf{a} + \lambda\mathbf{b}}{1 + \lambda}$. [3]
 (b) Given that A is (1, 5) and B is (7, -1), find the value of λ for which P lies on the line $y = x$, and state the coordinates of P. [4]

27

At noon ($t = 0$) ship A is at the origin O and moves with constant velocity $\begin{pmatrix} 5 \\ 2 \end{pmatrix} \text{ km h}^{-1}$. Ship B is at the point with position vector $\begin{pmatrix} 10 \\ 5 \end{pmatrix} \text{ km}$ and moves with constant velocity $\begin{pmatrix} 2 \\ -2 \end{pmatrix} \text{ km h}^{-1}$. Here $\mathbf{i}$ is a unit vector east and $\mathbf{j}$ a unit vector north, and t is the time in hours after noon.



- Write down the position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ of the ships at time t . [2]
- Show that the distance d km between the ships satisfies $d^2 = 25t^2 - 100t + 125$. [3]
- Hence find the time at which the ships are closest together and the least distance between them. [3]
- Each ship can see the other whenever they are less than 6 km apart. For how long (to the nearest minute) are the ships in sight of each other? [3]

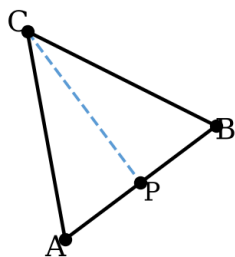
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ABCD is a quadrilateral with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ relative to an origin O.

- Prove that if ABCD is a parallelogram then its diagonals AC and BD bisect each other. [4]
- Prove the converse: if the diagonals of ABCD bisect each other then ABCD is a parallelogram. [3]
- Use the result in part (b) to show that A(1, -2), B(5, 0), C(7, 4) and D(3, 2) form a parallelogram, and find the coordinates of the point where its diagonals meet. [2]

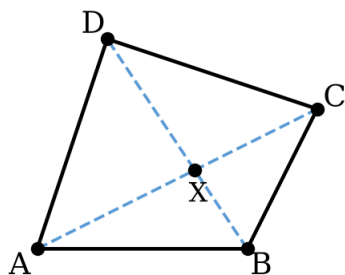
29

The points A(1, 2), B(9, 8) and C(-1, 13) form a triangle. The point P lies on AB and CP is perpendicular to AB, as shown.



- Using $\mathbf{OP} = \mathbf{OA} + \mu\mathbf{AB}$, and the fact that two perpendicular vectors have a scalar product of zero, find μ and the coordinates of P. [6]
- Find the area of triangle ABC. [3]
- Explain why triangle ABC is isosceles. [1]

- 30** ABCD is a quadrilateral with A at the origin and B(6, 0), C(8, 4), D(2, 6). The diagonals AC and BD intersect at X.

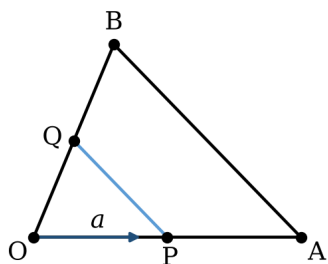


- Write down vector equations for AC and BD, using parameters s and u respectively. [2]
- Hence find the position vector of X. [4]
- Find the ratios AX : XC and BX : XD. [3]

Vector Geometry

- 1** The points A, B, C and D have coordinates A(1, 2), B(5, 4), C(0, -1) and D(2, 0). Show that AB is parallel to CD. [2]

- 2** In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. The points P and Q are the midpoints of OA and OB respectively. Find $\mathbf{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and describe the relationship between PQ and AB.



[3]

- 3** A line has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. Find the point on the line when $\lambda = 2$ and when $\lambda = -1$. [2]

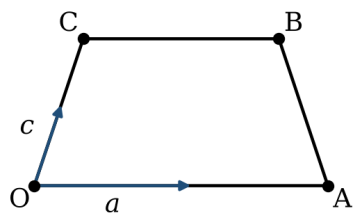
- 4** Find a vector equation of the line through the points A(1, 4) and B(3, 10). [3]

- 5** Does the point (7, -1) lie on the line $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \end{pmatrix}$? [2]

- 6** Three lines have direction vectors $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$. State which pair of lines are parallel. [2]

- 7** The line l passes through the point A(2, 1) and is parallel to the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
- Write down a vector equation of l . [1]
 - Show that the point B(11, -5) lies on l . [2]
 - Find the coordinates of the point where l crosses the x-axis. [2]

- 8 OABC is a trapezium in which CB is parallel to OA and $CB = \frac{2}{3}OA$. $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.



- Find $\mathbf{OB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [1]
- Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [1]
- The point M lies on AC with $AM : MC = 1 : 2$. Find $\mathbf{OM}$. [2]

- 9 Find the Cartesian equation of the line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, in the form $ax + by = c$. [3]

- 10 Write the line $y = 2x - 3$ in vector form. [2]

- 11 The lines $l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ meet at a point. Find its position vector. [4]

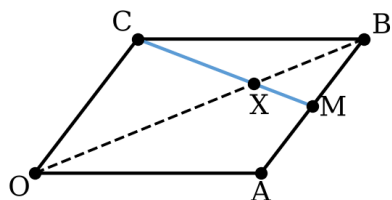
- 12 Show that the lines $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ are parallel but not the same line. [4]

- 13 The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Find the coordinates of the points where l crosses the x-axis and the y-axis. [4]

- 14 OABC is a trapezium in which CB is parallel to OA and $CB = 3OA$. Relative to the origin O, $\mathbf{OA} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{OC} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$. The point D lies on AC with $AD : DC = 1 : 3$.

- Find $\mathbf{OB}$ and $\mathbf{OD}$. [4]
- Show that O, D and B are collinear, and state the ratio $OD : DB$. [2]

- 15 OABC is a parallelogram with $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$. The point M is the midpoint of AB and X is the point on OB with $OX = \frac{2}{3}OB$.



- Find $\mathbf{OB}$ and $\mathbf{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]
- Show that C, X and M lie on a straight line. [3]
- Find the ratio $CX : XM$. [1]

- 16 Two lines have direction vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Find the acute angle between the lines, to 1 decimal place, using the scalar product. [4]

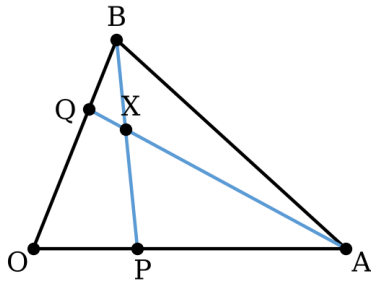
- 17 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find the foot of the perpendicular from $P(12, -1)$ to l , and the shortest distance from P to l . [6]

- 18 At time t , particle P is at $\begin{pmatrix} 1 + 2t \\ 2 + t \end{pmatrix}$ and particle Q is at $\begin{pmatrix} 13 - 3t \\ 3 + t \end{pmatrix}$, with $t \geq 0$. Show that the paths of P and Q cross, but that the particles do not collide. [5]

- 19 The lines $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 6 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ are given, where k is a constant. Find the value of k for which the lines are (a) parallel, (b) perpendicular. [5]

- 20 The line l_1 is $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find a vector equation of the line l_2 through $P(5, 3)$ that is perpendicular to l_1 , and the point where l_1 and l_2 meet. [6]

- 21 In triangle OAB , $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. The point P lies on OA with $OP : PA = 1 : 2$ and the point Q lies on OB with $OQ : QB = 2 : 1$. The lines AQ and BP intersect at X .

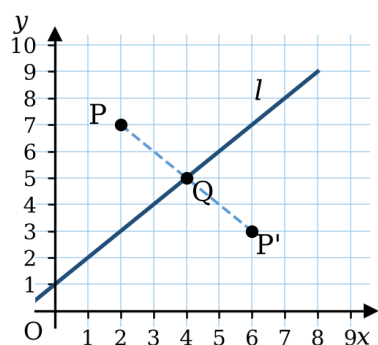


- (a) Write down $\mathbf{OP}$ and $\mathbf{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]
 (b) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]
 (c) Find the ratio $BX : XP$. [2]

- 22 Two lines have equations $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 10 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, where the units are centimetres.
 (a) Find the coordinates of the point P where the lines intersect. [4]
 (b) Find the coordinates of the points where l_1 and l_2 cross the x -axis. [2]
 (c) Hence find the area of the triangle bounded by l_1 , l_2 and the x -axis. [3]

- 23 The points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ relative to the origin O .
 (a) Find the unit vectors in the directions of $\mathbf{OA}$ and $\mathbf{OB}$. [2]
 (b) Hence find a vector equation of the line l which bisects the angle AOB . [3]
 (c) The point P lies on l and has x -coordinate 27. Find the coordinates of P . [2]

- 24 The line l has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The point P is $(2, 7)$. Find the foot Q of the perpendicular from P to l , and hence the coordinates of the reflection P' of P in l .

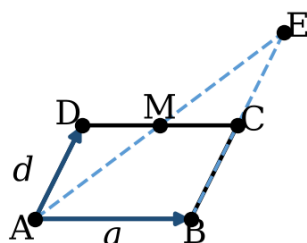


[6]

- 25 Prove that the angle in a semicircle is a right angle. Use a circle with centre O , diameter AB and a point P on the circumference, taking $\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = -\mathbf{a}$ and $\mathbf{OP} = \mathbf{p}$.

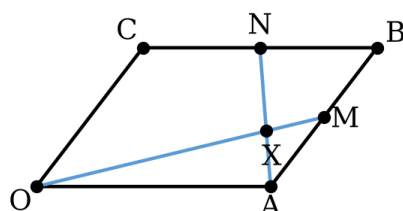
[5]

- 26 $ABCD$ is a parallelogram with $\mathbf{AB} = \mathbf{a}$ and $\mathbf{AD} = \mathbf{d}$. M is the midpoint of CD . The line AM meets the line BC extended at E .



- (a) Find $\mathbf{AM}$ in terms of $\mathbf{a}$ and $\mathbf{d}$. [2]
 (b) Show that C is the midpoint of BE . [5]

- 27 $OABC$ is a parallelogram with $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$. The point M is the midpoint of AB and the point N is the midpoint of BC . The lines OM and AN intersect at X .



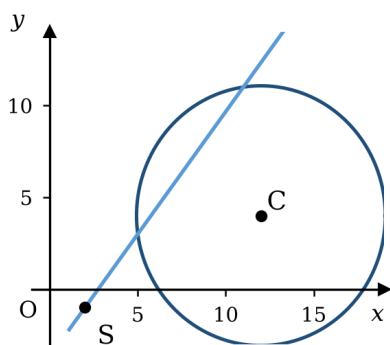
- (a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [6]
 (b) Find the ratio $AX : XN$. [1]
 (c) The line CX meets AB at the point R . Find the ratio $AR : RB$. [5]

- 28 The lines l_1 and l_2 have equations $l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \end{pmatrix}$.

- (a) A student says: "The direction vectors are parallel, so the lines never meet." Show that the student is wrong, and describe the relationship between the lines. [3]
 (b) The line l_3 has equation $\mathbf{r} = \begin{pmatrix} k \\ 0 \end{pmatrix} + \nu \begin{pmatrix} -4 \\ 2 \end{pmatrix}$. Find the value of k for which l_3 coincides with l_1 . [3]
 (c) Explain why, for any other value of k , l_1 and l_3 are parallel but have no point in common. [2]

29

A drone flies from the point $S(2, -1)$ with constant velocity $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ units per minute (lengths in hundreds of metres). A circular no-fly zone has its centre at $C(12, 4)$ and radius $5\sqrt{2}$.



- Write down the position vector of the drone after t minutes and find its speed. [2]
- Show that the distance CD from the centre to the drone satisfies $CD^2 = 25(t - 2)^2 + 25$, and find the least distance from C to the path. [5]
- Find the length of time for which the drone is inside the no-fly zone. [4]

30

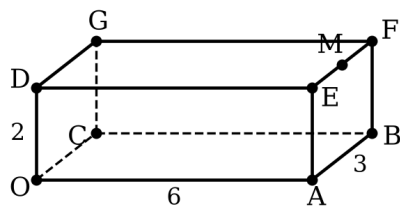
$ABCD$ is a quadrilateral with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ relative to an origin O . P , Q , R and S are the midpoints of AB , BC , CD and DA .

- Show that the line segments PR and QS have the same midpoint, with position vector $\frac{1}{4}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$. [4]
- Deduce that $PQRS$ is a parallelogram, whatever the shape of $ABCD$, and state where its diagonals meet. [2]
- Given that $ABCD$ is itself a parallelogram, show that this point is the intersection of its diagonals. [3]

Three-Dimensional Vectors

- Find the magnitude of the vector $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. [2]
- The points A and B have coordinates $(1, 2, 3)$ and $(4, -2, 5)$. Find the vector $\mathbf{AB}$ and its magnitude, leaving the magnitude as a surd. [3]
- Write the point $P(2, -3, 5)$ as a position vector in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and write the vector $3\mathbf{i} - 4\mathbf{k}$ as a column vector. [2]
- The point A has coordinates $(-1, 4, 2)$ and $\mathbf{AB} = 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$. Find the coordinates of B and the distance of B from the origin O , leaving your answer as a surd. [3]
- Find the coordinates of the midpoint of $P(1, -2, 5)$ and $Q(7, 4, -3)$. [2]
- Show that the vectors $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 9 \\ -3 \end{pmatrix}$ are parallel. [2]
- The points P and Q have coordinates $(1, 2, -1)$ and $(7, -4, 5)$. The point R lies on PQ such that $PR : RQ = 1 : 2$. Find the coordinates of R . [3]

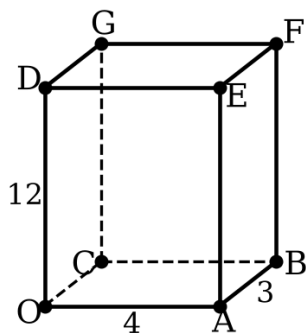
- 8 OABCDEFG is a cuboid. The position vectors of A, C and D relative to the origin O are $\mathbf{OA} = 6\mathbf{i}$, $\mathbf{OC} = 3\mathbf{j}$ and $\mathbf{OD} = 2\mathbf{k}$. The point M is the midpoint of EF.



- (a) Write $\mathbf{OF}$ as a column vector. [1]
 (b) Find $\mathbf{AG}$ and show that $|\mathbf{AG}| = 7$. [2]
 (c) Find $\mathbf{OM}$. [1]

- 9 Find a unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$, and the vector of magnitude 14 in that direction. [3]

- 10 OABCDEFG is a cuboid with $OA = 4$, $OC = 3$ and $OD = 12$ units, as shown (not to scale), with O at the origin and OA, OC, OD along the x, y and z axes. Write down the coordinates of F and find the length of the diagonal OF.



[3]

- 11 Given that $\left| \begin{pmatrix} p \\ 2p \\ -2p \end{pmatrix} \right| = 12$, find the possible values of p. [3]

- 12 Show that the point D(3, 1, 1) lies on the line through A(1, 2, -1) and B(7, -1, 5), and find the ratio AD : DB. [4]

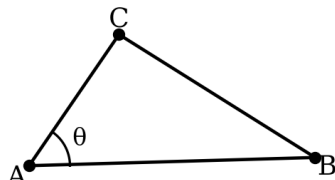
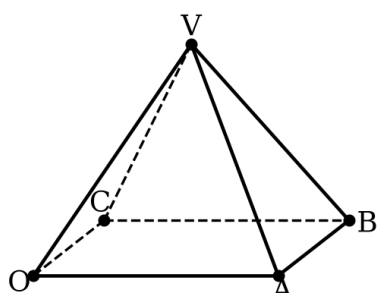
- 13 A(1, 2, 3), B(4, 2, 5) and C(6, 5, 4) are three vertices of the parallelogram ABCD, in that order. Find the coordinates of D. [3]

- 14 The line l passes through the points A(1, -2, 3) and B(4, 0, -1).
 (a) Find a vector equation of l. [2]
 (b) Show that C(7, 2, -5) lies on l. [2]
 (c) Find the coordinates of the point where l meets the plane $z = 0$. [2]

- 15 $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 4\mathbf{j} + 8\mathbf{k}$.
 (a) Find the angle between $\mathbf{a}$ and $\mathbf{b}$, to 1 decimal place. [3]
 (b) Find the value of λ for which $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{a}$. [3]

- 16 The points A(1, 2, -3) and B(4, -1, 3) are given. The point P lies on the line AB such that the distance AP is twice the distance PB. Find the two possible position vectors of P. [4]

- 17 The vectors $\mathbf{a} = \begin{pmatrix} k \\ k \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} k \\ -5 \\ 3 \end{pmatrix}$ are perpendicular. Find the possible values of k. [4]

- 18** Triangle ABC has vertices A(1, 1, 1), B(3, 3, 2) and C(2, -1, 3). Show that triangle ABC is isosceles with a right angle at A, and find its area. [6]
- 19** A(1, 2, 3), B(3, 5, 7) and C(7, 11, 15) are three points. Show that A, B and C are collinear and find the ratio AB : BC. [4]
- 20** The points A(2, -1, 0), B(-1, 3, 4) and C(3, 1, 2) are given, and the point P is such that $4\mathbf{PA} + 3\mathbf{PB} = 5\mathbf{PC}$. Find the position vector of P. [4]
- 21** The points A, B and C have coordinates A(2, 1, 0), B(5, 3, 6) and C(1, 3, 2). The angle BAC is θ .
- 
- (a) Find $\mathbf{AB}$ and $\mathbf{AC}$, and their magnitudes. [2]
 (b) Find θ , to 1 decimal place. [3]
 (c) Find the exact area of triangle ABC. [3]
- 22** Two lines have vector equations $l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 13 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.
- (a) Show that the lines intersect and find the position vector of the point of intersection. [5]
 (b) Find the acute angle between the lines, to 1 decimal place. [3]
- 23** The points A(5, 4, a), B(b, b, b - 3) and C(c, -2, 5) are collinear, where a, b and c are constants, and AB : BC = 1 : 2. Find the values of a, b and c. [6]
- 24** The lines $l_1: \mathbf{r} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ c \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ are given, where c is a constant.
- (a) Show that when $c = 0$ the lines do not intersect, and that they are not parallel. [4]
 (b) Find the value of c for which the lines intersect, and the point of intersection. [3]
- 25** A pyramid has a rectangular base OABC, where O is the origin, A is (6, 0, 0), B is (6, 4, 0) and C is (0, 4, 0). The vertex V is directly above the centre of the base, at height 5.
- 
- (a) Write down the coordinates of V and find the exact length of the edge VA. [3]
 (b) Find angle AVC, to 1 decimal place. [4]
- 26** The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. Find the shortest distance from the point P(5, 1, 4) to l, and the coordinates of the foot of the perpendicular. [6]

27

Two lines have vector equations $l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $l_2: \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ k \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, where k is a constant.

- Given that $k = 0$, show that l_1 and l_2 are skew. [5]
- Find the value of k for which the lines intersect. [2]
- For this value of k find the position vector of the point of intersection. [2]
- Find the acute angle between the lines, to 1 decimal place. [2]

28

A drone flies in a straight line at constant speed. Relative to the top T of a radio mast, taken as the origin,

the position vector of the drone t seconds after it is first observed is $\mathbf{r} = \begin{pmatrix} -10 \\ -8 \\ -9 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$, in metres, where $\mathbf{i}$, $\mathbf{j}$

are horizontal and $\mathbf{k}$ is vertically upwards.

- Show that the distance d metres of the drone from T satisfies $d^2 = 49t^2 - 196t + 245$. [3]
- Find the least distance between the drone and T , and the time at which it occurs. [3]
- The mast carries a transmitter that can only be received within 12 m of T . Find the times between which the drone is within this range, to 2 decimal places. [3]
- Show that at the time of closest approach the direction of motion is perpendicular to the position vector of the drone. [2]

29

Two drones fly with constant velocities. Relative to a control point O (with $\mathbf{i}$, $\mathbf{j}$ east and north, $\mathbf{k}$ vertically

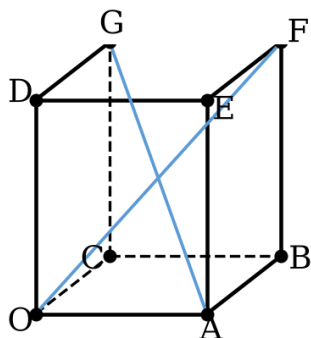
up, lengths in metres), drone P is initially at $\begin{pmatrix} 0 \\ 0 \\ 40 \end{pmatrix}$ and moves with velocity $\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ m s}^{-1}$. Drone Q is initially

at $\begin{pmatrix} 12 \\ -3 \\ 49 \end{pmatrix}$ and moves with velocity $\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix} \text{ m s}^{-1}$.

- Show that the vector from P to Q at time t seconds is $\begin{pmatrix} 12 - 2t \\ -3 + t \\ 9 - 2t \end{pmatrix}$. [2]
- Show that $|PQ|^2 = 9t^2 - 90t + 234$, and find the least distance between the drones and the time at which it occurs. [6]
- Safety rules require the drones always to stay at least 3.5 m apart. For how long are they closer than 3.5 m? [4]

30

OABCDEFG is a cuboid with $OA = a$, $OC = b$ and $OD = c$ along the x , y and z axes, so F is (a, b, c) . The space diagonals are OF , AG , BD and CE (diagram not to scale).



- Write down the position vectors of the midpoints of OF and AG , and deduce that these two diagonals bisect each other. [3]
- Show that all four space diagonals have length $\sqrt{a^2 + b^2 + c^2}$. [3]
- For $a = 4$, $b = 6$ and $c = 12$, find the acute angle between OF and AG , to 1 decimal place. [4]