

Sine Rule and Cosine Rule

1 (2 marks)

$$\frac{AC}{\sin 65^\circ} = \frac{7.2}{\sin 40^\circ}$$

M1

$$AC = \frac{7.2 \sin 65^\circ}{\sin 40^\circ} = 10.2 \text{ cm}$$

A1

Answer: AC = 10.2 cm

2 marks

2 (3 marks)

$$BC^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 50^\circ$$

M1

$$BC^2 = 47.58$$

A1

$$BC = 6.90 \text{ cm}$$

A1

Answer: BC = 6.90 cm

3 marks

3 (3 marks)

$$\frac{\sin B}{9} = \frac{\sin 35^\circ}{7}$$

M1

$$\sin B = \frac{9 \sin 35^\circ}{7} = 0.7375$$

M1

$$\text{angle ABC} = 47.5^\circ \text{ (acute)}$$

A1

Answer: ABC = 47.5^\circ

3 marks

4 (2 marks)

$$\text{Area} = \frac{1}{2} \times 8 \times 11 \times \sin 42^\circ$$

M1

$$= 29.4 \text{ cm}^2$$

A1

Answer: 29.4 cm²

2 marks

5 (3 marks)

$$\text{Angle R} = 180^\circ - 55^\circ - 80^\circ = 45^\circ$$

B1

$$\frac{PR}{\sin 80^\circ} = \frac{12}{\sin 45^\circ}$$

M1

$$PR = \frac{12 \sin 80^\circ}{\sin 45^\circ} = 16.7 \text{ cm}$$

A1

Answer: PR = 16.7 cm

3 marks

6 (2 marks)

$$\text{Angle C} = 180^\circ - 52^\circ - 71^\circ = 57^\circ$$

M1

$$\frac{BC}{\sin 52^\circ} = \frac{12}{\sin 57^\circ}, \text{ so } BC = 11.3 \text{ cm}$$

A1

Answer: BC = 11.3 cm

2 marks

7 (3 marks)

$$\cos ACB = \frac{9^2 + 6^2 - 13^2}{2 \times 9 \times 6}$$

M1 A1

$$\cos ACB = \frac{-52}{108} = -\frac{13}{27}$$

$$\text{angle ACB} = 118.8^\circ$$

A1

Answer: angle ACB = 118.8^\circ

3 marks

8 (5 marks)

$$\text{(a) Area} = \frac{1}{2} \times 9 \times 12 \times \sin 38^\circ$$

M1

$$= 33.2 \text{ cm}^2$$

A1

Answer: 33.2 cm²

2 marks

$$\text{(b) } QR^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times \cos 38^\circ$$

M1

$$QR^2 = 54.79$$

A1

$$QR = 7.40 \text{ cm}$$

Answer: 7.40 cm

A1

3 marks

9 (4 marks)

$$\cos C = \frac{9^2 + 12^2 - 15^2}{2 \times 9 \times 12} = \frac{0}{216} = 0$$

M1

So the angle opposite the 15 cm side is 90°

A1

$$\text{Area} = \frac{1}{2} \times 9 \times 12 \sin 90^\circ = 54 \text{ cm}^2$$

M1 A1

Answer: 54 cm²

4 marks

10 (3 marks)

$$\frac{1}{2} \times 10 \times 8 \times \sin A = 30$$

M1

$$\sin A = \frac{30}{40} = 0.75$$

A1

$$A = 48.6^\circ \text{ (acute)}$$

A1

Answer: BAC = 48.6°

3 marks

11 (4 marks)

The shorter diagonal lies opposite the 65° angle

B1

$$d^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 65^\circ$$

M1

$$d^2 = 71.36, d = 8.45 \text{ cm}$$

A1 A1

Answer: 8.45 cm

4 marks

12 (4 marks)

$$\frac{\sin B}{14} = \frac{\sin 30^\circ}{10}, \text{ so } \sin B = 0.700$$

M1 A1

$$\text{Obtuse: } B = 180^\circ - 44.4^\circ = 135.6^\circ$$

A1

$$C = 180^\circ - 30^\circ - 135.6^\circ = 14.4^\circ$$

A1

Answer: B = 135.6°, C = 14.4°

4 marks

13 (7 marks)

(a) Bearing of P from R is 250° , so angle $PRQ = 250^\circ - 150^\circ = 100^\circ$

B1

$$PQ^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \times \cos 100^\circ = 262.5$$

M1

$$PQ = 16.2 \text{ km}$$

A1

Answer: PQ = 16.2 km

3 marks

$$(b) \frac{\sin QPR}{9} = \frac{\sin 100^\circ}{16.20}$$

M1

$$\text{angle } QPR = 33.2^\circ$$

A1

$$\text{Bearing of Q from P} = 070^\circ + 33.2^\circ = 103.2^\circ$$

M1

$$\text{Bearing of P from Q} = 103.2^\circ + 180^\circ = 283.2^\circ, \text{ so } 283^\circ \text{ to the nearest degree}$$

A1

Answer: 283°

4 marks

14 (7 marks)

$$(a) \frac{\sin ACB}{10} = \frac{\sin 40^\circ}{7}$$

M1

$$\sin ACB = 0.9183$$

A1

$$\text{acute: } ACB = 66.7^\circ$$

A1

$$\text{obtuse: } ACB = 180^\circ - 66.7^\circ = 113.3^\circ$$

B1

Answer: 66.7° or 113.3°

4 marks

(b) Angle $BAC = 180^\circ - 40^\circ - 66.7^\circ = 73.3^\circ$ (larger when ACB is acute)

M1

$$\text{Area} = \frac{1}{2} \times 10 \times 7 \times \sin 73.3^\circ$$

M1

$$= 33.5 \text{ cm}^2$$

A1

Answer: 33.5 cm²

3 marks

15 (8 marks)

$$(a) \frac{\sin B}{12} = \frac{\sin 40^\circ}{6}, \text{ so } \sin B = 1.286$$

M1 A1

This is greater than 1, which is impossible, so no triangle exists

E1

Answer: sin B > 1, impossible

3 marks

$$(b) \text{ Need } \sin B \leq 1, \text{ so } a \geq 12 \sin 40^\circ$$

M1

$$a = 7.71 \text{ cm}$$

A1

Answer: 7.71 cm

2 marks

$$(c) \sin B = \frac{12 \sin 40^\circ}{9} = 0.8571$$

M1

$$B = 59.0^\circ \text{ or } 180^\circ - 59.0^\circ = 121.0^\circ$$

A1 A1

Answer: 59.0° or 121.0°

3 marks

16 (5 marks)

$$\text{Angle ATB} = 180^\circ - 62^\circ - 48^\circ = 70^\circ$$

B1

$$\frac{AT}{\sin 48^\circ} = \frac{80}{\sin 70^\circ}, \text{ so } AT = 63.27 \text{ m}$$

M1 A1

$$\text{Width} = AT \sin 62^\circ = 55.9 \text{ m}$$

M1 A1

Answer: Width = 56 m

5 marks

17 (7 marks)

$$(a) \text{ Area} = \frac{1}{2} \times 10 \times AB \times \sin 120^\circ = 15\sqrt{3}$$

M1

$$5 AB \times \frac{\sqrt{3}}{2} = 15\sqrt{3}, \text{ so } AB = 6 \text{ cm}$$

A1

Answer: shown

2 marks

$$(b) BC^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos 120^\circ$$

M1

$$= 136 + 60 = 196, \text{ so } BC = 14 \text{ cm}$$

A1

Answer: BC = 14 cm

2 marks

(c) The smallest angle is opposite the shortest side AB, so it is angle ACB

B1

$$\frac{\sin ACB}{6} = \frac{\sin 120^\circ}{14}$$

M1

$$\sin ACB = 0.3712, \text{ so angle ACB} = 21.8^\circ$$

A1

Answer: 21.8°

3 marks

18 (6 marks)

$$\text{Bearing of A from B is } 280^\circ, \text{ so angle ABC} = 360^\circ - 280^\circ + 25^\circ = 105^\circ$$

M1 A1

$$AC^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 105^\circ, \text{ so } AC = 12.0 \text{ km}$$

M1 A1

$$\frac{\sin A}{6} = \frac{\sin 105^\circ}{12.04}, \text{ angle BAC} = 28.8^\circ$$

M1

$$\text{Bearing of C from A} = 100^\circ - 28.8^\circ = 71^\circ$$

A1

Answer: AC = 12.0 km, bearing 071°

6 marks

19 (5 marks)

$$(a) 49 = x^2 + (x + 3)^2 - 2x(x + 3) \cos 60^\circ$$

M1

$$49 = x^2 + x^2 + 6x + 9 - x(x + 3)$$

M1

$$49 = x^2 + 3x + 9, \text{ so } x^2 + 3x - 40 = 0$$

A1

Answer: shown

3 marks

$$(b) (x + 8)(x - 5) = 0, x = 5 \text{ (} x > 0 \text{)}$$

M1

$$\text{Sides 5, 8, 7: perimeter} = 20 \text{ cm}$$

A1

Answer: 20 cm

2 marks

20 (8 marks)

$$(a) \frac{BC}{\sin 45^\circ} = \frac{6}{\sin 60^\circ}$$

M1

$$BC = 6 \times \frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{3}} = \frac{6\sqrt{2}}{\sqrt{3}} = 2\sqrt{6} \text{ cm}$$

M1 A1

Answer: shown

3 marks

(b) Angle B = $180^\circ - 45^\circ - 60^\circ = 75^\circ$

B1

$$\text{Area} = \frac{1}{2} \times 6 \times 2\sqrt{6} \times \sin 75^\circ = 6\sqrt{6} \times \frac{\sqrt{6} + \sqrt{2}}{4}$$

M1 A1

$$= \frac{36 + 6\sqrt{12}}{4} = \frac{36 + 12\sqrt{3}}{4}$$

M1

$$= 9 + 3\sqrt{3}$$

A1

Answer: shown

5 marks

21 (9 marks)

(a) $AC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 65^\circ$

M1 A1

$$AC^2 = 110.6, \text{ so } AC = 10.5 \text{ cm}$$

A1

Answer: AC = 10.5 cm

3 marks

(b) $\cos ADC = \frac{9^2 + 7^2 - AC^2}{2 \times 9 \times 7}$

M1 A1

$$\text{angle ADC} = 81.2^\circ$$

A1

Answer: 81.2°

3 marks

(c) Area of ABC = $\frac{1}{2} \times 8 \times 11 \times \sin 65^\circ = 39.88$

M1

$$\text{Area of ACD} = \frac{1}{2} \times 9 \times 7 \times \sin 81.2^\circ = 31.13$$

M1

$$\text{Total area} = 71.0 \text{ cm}^2$$

A1

Answer: 71.0 cm²

3 marks

22 (6 marks)

(a) The angle of elevation at B is an exterior angle of triangle ATB, so it equals angle TAB + angle ATB

M1

$$41^\circ = 28^\circ + \text{angle ATB, so angle ATB} = 13^\circ$$

E1

Answer: angle ATB = 13°

2 marks

(b) $\frac{TB}{\sin 28^\circ} = \frac{50}{\sin 13^\circ}$

M1

$$TB = 104.3 \text{ m}$$

A1

$$\text{Height} = TB \sin 41^\circ$$

M1

$$= 68.46 \text{ m, so } 68 \text{ m to the nearest metre}$$

A1

Answer: 68 m

4 marks

23 (10 marks)

(a) $BC^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos 80^\circ$

M1

$$BC^2 = 108.1, BC = 10.4 \text{ cm}$$

A1 A1

Answer: BC = 10.4 cm

3 marks

(b) $\cos B = \frac{9^2 + 10.40^2 - 7^2}{2 \times 9 \times 10.40}$

M1

$$\text{angle ABC} = 41.5^\circ$$

A1 A1

Answer: ABC = 41.5°

3 marks

(c) $BM = \frac{1}{2} BC = 5.199 \text{ cm}$

B1

$$AM^2 = 9^2 + 5.199^2 - 2 \times 9 \times 5.199 \times \cos 41.5^\circ$$

M1 A1

$$AM = 6.16 \text{ cm}$$

A1

Answer: AM = 6.16 cm

4 marks

24 (7 marks)

(a) Largest angle is opposite the longest side (15 cm)

B1

$$\cos \theta = \frac{13^2 + 14^2 - 15^2}{2 \times 13 \times 14} = \frac{140}{364} = \frac{5}{13}$$

M1 A1

$$\theta = 67.4^\circ$$

A1

Answer: 67.4°

4 marks

(b) $2R = \frac{15}{\sin \theta}, \sin \theta = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$

M1

$$2R = \frac{15 \times 13}{12} = 16.25$$

M1

$$R = 8.125 \text{ cm} = 8.13 \text{ cm (3 s.f.)}$$

A1

Answer: R = 8.13 cm

3 marks

25 (6 marks)

$$AC^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 70^\circ = 61.64$$

M1 A1

Opposite angles sum to 180° , so angle ADC = 110°

B1

$$AC^2 = 6^2 + CD^2 - 12 CD \cos 110^\circ$$

M1

$$CD^2 + 4.10 CD - 25.6 = 0$$

A1

$$CD = 3.41 \text{ cm (positive root)}$$

A1

Answer: CD = 3.41 cm

6 marks

26 (5 marks)

$$(a) \cos A = \frac{7^2 + 6^2 - 12^2}{2 \times 7 \times 6} = -\frac{59}{84}$$

M1 A1

$$A = 134.6^\circ$$

A1

Answer: BAC = 134.6°

3 marks

(b) $\sin A = \sin(180^\circ - A)$, so the sine rule gives an acute angle even when A is obtuse

E1

The cosine rule gives a unique angle (cos is negative for obtuse angles), so it is safer

E1

Answer: sine rule hides obtuse angles

2 marks

27 (8 marks)

$$(a) C = (b \cos A, b \sin A)$$

B1

$$a^2 = BC^2 = (b \cos A - c)^2 + (b \sin A - 0)^2$$

M1

$$= b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$$

A1

$$= b^2(\cos^2 A + \sin^2 A) + c^2 - 2bc \cos A = b^2 + c^2 - 2bc \cos A, \text{ using } \cos^2 A + \sin^2 A = 1$$

E1

Answer: proof

4 marks

$$(b) \text{ Comparing with the cosine rule: } -2bc \cos A = bc, \text{ so } \cos A = -\frac{1}{2}$$

M1

$$A = 120^\circ$$

A1

Answer: A = 120°

2 marks

$$(c) \text{ Area} = \frac{1}{2} \times 5 \times 8 \times \sin 120^\circ$$

M1

$$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

A1

Answer: $10\sqrt{3}$

2 marks

28 (12 marks)

$$(a) \text{ The largest angle is opposite the longest side, so } (x+4)^2 = x^2 + (x+2)^2 - 2x(x+2)\cos 120^\circ$$

M1 A1

$$\cos 120^\circ = -\frac{1}{2}, \text{ so } (x+4)^2 = x^2 + (x+2)^2 + x(x+2)$$

M1

$$x^2 + 8x + 16 = 3x^2 + 6x + 4, \text{ so } 2x^2 - 2x - 12 = 0, \text{ hence } x^2 - x - 6 = 0$$

A1

Answer: proof

4 marks

$$(b) (x-3)(x+2) = 0$$

M1

$$x = 3 \text{ (} x = -2 \text{ is rejected as a length must be positive)}$$

A1

Sides are 3 cm, 5 cm and 7 cm

B1

Answer: 3 cm, 5 cm, 7 cm

3 marks

$$(c) \text{ Area} = \frac{1}{2} \times 3 \times 5 \times \sin 120^\circ$$

M1

$$= \frac{15\sqrt{3}}{4} \text{ cm}^2$$

A1

Answer: $\frac{15\sqrt{3}}{4} \text{ cm}^2$

2 marks

(d) The smallest angle is opposite the side 3 cm; call it θ

B1

$$\frac{\sin \theta}{3} = \frac{\sin 120^\circ}{7}$$

M1

$$\sin \theta = \frac{3 \times (\sqrt{3})}{2/7} = \frac{3\sqrt{3}}{14}$$

A1

Answer: proof

3 marks

29 (7 marks)

$$(a) \text{ Area ABC} = \frac{1}{2}bc \sin 2\theta$$

B1

$$\text{Area ABD} + \text{area ACD} = \frac{1}{2}c \cdot AD \sin \theta + \frac{1}{2}b \cdot AD \sin \theta$$

M1 A1

$$\frac{1}{2}bc(2 \sin \theta \cos \theta) = \frac{1}{2}AD(b + c) \sin \theta$$

M1

$$\text{Divide by } \frac{1}{2} \sin \theta: 2bc \cos \theta = AD(b + c), \text{ so } AD = \frac{2bc \cos \theta}{b + c}$$

A1

Answer: shown

5 marks

$$(b) AD = \frac{2 \times 6 \times 10 \cos 30^\circ}{16}$$

M1

$$= \frac{120 \times \sqrt{3}}{2/16} = \frac{15\sqrt{3}}{4} \text{ cm}$$

A1

$$\text{Answer: } \frac{15\sqrt{3}}{4} \text{ cm}$$

2 marks

30 (8 marks)

$$(a) \frac{a}{\sin 2B} = \frac{b}{\sin B}$$

M1

$$a = \frac{b \sin 2B}{\sin B} = \frac{2b \sin B \cos B}{\sin B} = 2b \cos B$$

M1 A1

Answer: shown

3 marks

$$(b) \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \text{ so } a = 2b \cdot \frac{a^2 + c^2 - b^2}{2ac}$$

M1

$$a^2c = b(a^2 + c^2 - b^2), \text{ so } a^2(c - b) = b(c^2 - b^2) = b(c - b)(c + b)$$

M1 A1

$$c \neq b, \text{ so } a^2 = b(b + c)$$

E1

Answer: shown

4 marks

$$(c) a^2 = 4 \times 9 = 36, \text{ so } a = 6$$

B1

Answer: a = 6

1 mark

Trigonometric Graphs**1 (3 marks)**

$$\text{Maximum} = 3 + 2 = 5$$

B1

$$\text{Minimum} = -3 + 2 = -1$$

B1

$$\text{Period of } \cos 4x = 360^\circ \div 4 = 90^\circ$$

B1

Answer: (a) max 5, min -1; (b) 90°

3 marks

2 (3 marks)

Cosine shape starting at the maximum (0°, 1), one full period

B1

Minimum at (180°, -1) and maximum at (360°, 1)

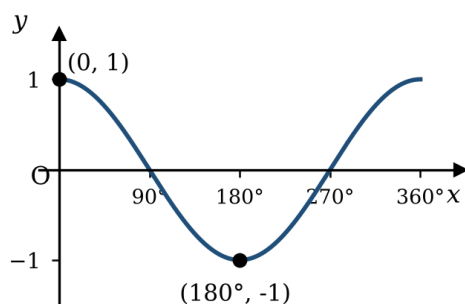
B1

Crosses the x-axis at (90°, 0) and (270°, 0)

B1

Answer: see sketch

3 marks



3 (3 marks)

(a) $360^\circ \div \frac{1}{2} = 720^\circ$

B1

(b) $\tan$ has period 180° , so $180^\circ \div 3 = 60^\circ$

B1

(c) $360^\circ \div 4 = 90^\circ$

B1

Answer: (a) 720° , (b) 60° , (c) 90°

3 marks

4 (4 marks)

The sine curve translated 3 units up

B1

Maximum $(90^\circ, 4)$, minimum $(270^\circ, 2)$

B1

y-intercept $(0, 3)$

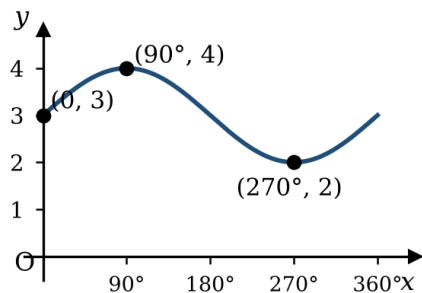
B1

The least value of y is 2, which is above the x-axis, so there are no x-intercepts

E1

Answer: see sketch

4 marks

**5 (3 marks)**

(a) $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \frac{1}{2}$

B1

(b) $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$

B1

(c) $\cos 60^\circ = \sin 30^\circ = \frac{1}{2}$

B1

Answer: (a) $\frac{1}{2}$, (b) $-\frac{1}{2}$, (c) $\frac{1}{2}$

3 marks

6 (2 marks)Translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ (3 units up)

B1

Maximum = 4, minimum = 2

B1

Answer: translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$; max 4, min 2

2 marks

7 (3 marks)Maximum value of y is a , so $a = 3$

B1

The first maximum is a quarter of a period from the start, so the period is $4 \times 45^\circ = 180^\circ$

M1

$$b = \frac{360^\circ}{180^\circ} = 2$$

A1

Answer: $a = 3$, $b = 2$

3 marks

8 (4 marks)

(a) Translation

B1

by the vector $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$ (30° to the right)

B1

(b) Stretch parallel to the x-axis with scale factor $\frac{1}{3}$

B1

and stretch parallel to the y-axis with scale factor 2

B1

Answer: (a) translation $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$; (b) stretch x by $\frac{1}{3}$, y by 2

4 marks

9 (3 marks)

Period is $360^\circ \div 2 = 180^\circ$, so two complete cycles

Maxima at $(0^\circ, 1)$, $(180^\circ, 1)$, $(360^\circ, 1)$

Minima at $(90^\circ, -1)$ and $(270^\circ, -1)$

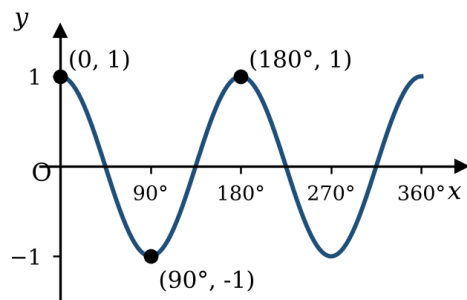
Answer: see sketch

B1

B1

B1

3 marks

**10 (3 marks)**

$$a = \frac{7 + (-1)}{2} = 3$$

M1 A1

$$b = \frac{7 - (-1)}{2} = 4$$

A1

Answer: a = 3, b = 4

3 marks

11 (7 marks)

(a) The curve is symmetrical about $x = 360^\circ$, and has period 360°

B1

Q: $360 - \alpha$

B1

R: $360 + \alpha$, S: $720 - \alpha$

B1 B1

Answer: Q: $360 - \alpha$; R: $360 + \alpha$; S: $720 - \alpha$

4 marks

(b) $\cos(180^\circ - \alpha) = -\cos \alpha$

M1

T: $180 - \alpha$, U: $180 + \alpha$

A1 A1

Answer: T: $180 - \alpha$; U: $180 + \alpha$

3 marks

12 (3 marks)

Maximum at $(0, 3)$ and $(2\pi, 3)$, minimum at $(\pi, -3)$

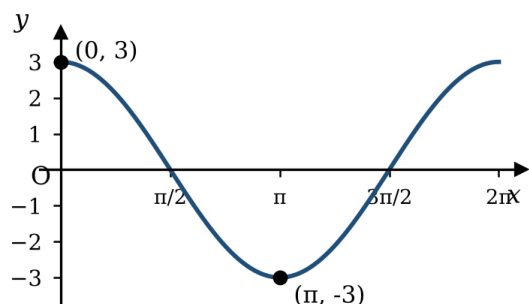
B1 B1

Crosses the x-axis where $\cos x = 0$: $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

B1

Answer: see sketch

3 marks

**13 (3 marks)**

$\cos x = 0.5$: $x = 60^\circ$ or 300° in the first cycle

M1 A1

Add 360° : $x = 420^\circ$ or 660°

A1

Answer: $x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

3 marks

14 (5 marks)

Sine curve translated 60° to the left, amplitude 2

B1

When $x = 0$: $y = 2 \sin 60^\circ = \sqrt{3}$, so $(0, \sqrt{3})$

B1

Maximum when $x + 60^\circ = 90^\circ$: $(30^\circ, 2)$; minimum when $x + 60^\circ = 270^\circ$: $(210^\circ, -2)$

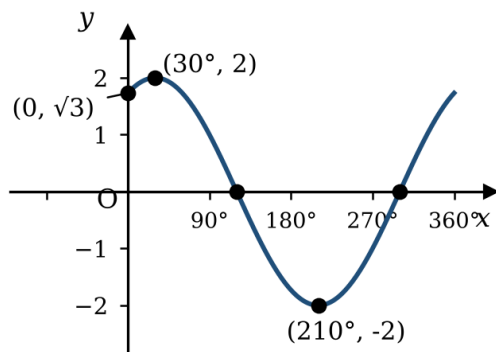
B1 B1

$y = 0$ when $x + 60^\circ = 180^\circ$ or 360° , so $x = 120^\circ$ and $x = 300^\circ$

B1

Answer: $(0, \sqrt{3})$; max $(30^\circ, 2)$; min $(210^\circ, -2)$; $x = 120^\circ, 300^\circ$

5 marks

**15 (7 marks)**

(a) Max = $5 + 3 = 8$ m, min = $5 - 3 = 2$ m

B1

Period = $360 \div 30 = 12$ hours

B1

Answer: 8 m, 2 m, 12 hours

2 marks

(b) $5 + 3\cos(30t)^\circ = 6.5$, so $\cos(30t)^\circ = \frac{1}{2}$

M1

$30t = 60$ or 300

A1

$t = 2$ or $t = 10$, that is 02:00 and 10:00

A1

Answer: $t = 2$ and $t = 10$

3 marks

(c) Depth is at least 6.5 m for $0 \leq t \leq 2$ and for $10 \leq t \leq 12$

M1

Total time = $2 + 2 = 4$ hours

A1

Answer: 4 hours

2 marks

16 (4 marks)

Asymptote of $\tan \theta$ at $\theta = 90^\circ$, so $30b = 90$

M1

$b = 3$

A1

Period = $180^\circ \div 3 = 60^\circ$

B1

Next asymptotes $x = 90^\circ$ and $x = 150^\circ$

B1

Answer: $b = 3$; period 60° ; $x = 90^\circ, 150^\circ$

4 marks

17 (6 marks)

(a) Highest $14 + 7 = 21^\circ\text{C}$, lowest $14 - 7 = 7^\circ\text{C}$

B1 B1

Highest when $30t = 180$, $t = 6$ (July)

B1

Answer: $21^\circ\text{C}, 7^\circ\text{C}; \text{July}$

3 marks

(b) $\cos(30t)^\circ = -0.5$

M1

$30t = 120$ or 240 , so $t = 4$ or $t = 8$

A1 A1

Answer: $t = 4, 8$

3 marks

18 (3 marks)

Period = $\frac{2\pi}{2} = \pi = 3.14$ s (3 s.f.)

B1

$\sin 2t = \frac{2}{3}$, so $2t = 0.7297$

M1

$t = 0.36$ s

A1

Answer: (a) π s; (b) 0.36 s

3 marks

19 (4 marks)

Amplitude $a = 2$

B1

Maximum when $x + b = 0$: $-30 + b = 0$, so $b = 30$

M1 A1

Check: minimum at $x + 30 = 180$, $x = 150$

B1

Answer: $a = 2, b = 30$

4 marks

20 (5 marks)

(a) $\cos x = 0.5$ at $x = 60^\circ$ and 300°

B1

Above 0.5 near the maximum: $0^\circ \leq x < 60^\circ$ and $300^\circ < x \leq 360^\circ$

B1 B1

Answer: $0^\circ \leq x < 60^\circ$ or $300^\circ < x \leq 360^\circ$

3 marks

(b) $\sin x = -0.5$ at $x = 210^\circ$ and 330°

B1

The curve is below -0.5 between them: $210^\circ \leq x \leq 330^\circ$

Answer: $210^\circ \leq x \leq 330^\circ$

B1

2 marks

21 (9 marks)

(a) Amplitude: $|a| = \frac{3 - (-1)}{2} = 2$, and the curve starts at a minimum so $a = -2$

M1 A1

Midline $c = \frac{3 + (-1)}{2} = 1$

B1

Half a period is 60° , so the period is 120° and $b = \frac{360}{120} = 3$

M1 A1

Answer: $a = -2$, $b = 3$, $c = 1$

5 marks

(b) $\cos 3x = -\frac{1}{2}$

M1

$3x = 120^\circ, 240^\circ, 480^\circ$ (range is $0^\circ \leq 3x \leq 540^\circ$)

M1 A1

$x = 40^\circ, 80^\circ, 160^\circ$

A1

Answer: $x = 40^\circ, 80^\circ, 160^\circ$

4 marks

22 (8 marks)

(a) $\tan$ curve translated 45° to the right of $y = \tan x$

B1

Asymptotes $x = 135^\circ$ and $x = 315^\circ$

B1 B1

Crosses the x -axis at $(45^\circ, 0)$ and $(225^\circ, 0)$

B1

y -intercept: $\tan(-45^\circ) = -1$, so $(0, -1)$

B1

Answer: see sketch

5 marks

(b) $\tan(x - 45^\circ) = \sqrt{3}$, so $x - 45^\circ = 60^\circ$ or 240°

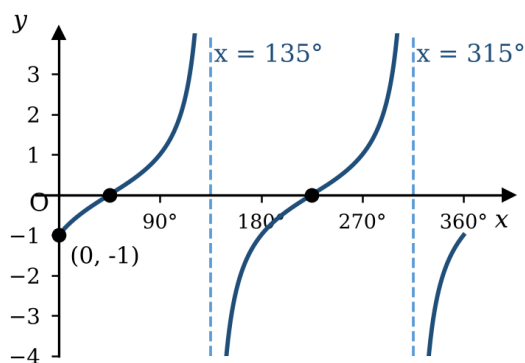
M1 A1

$x = 105^\circ$ or 285°

A1

Answer: $x = 105^\circ, 285^\circ$

3 marks



23 (9 marks)

(a) Max 16, min 8

B1 B1

$\sin = 1$: $\frac{360(t - 80)}{365} = 90$, $t - 80 = 91.25$, $t = 171.25$, so day 171

M1 A1

Answer: 16 h, 8 h; day 171

4 marks

(b) $H > 14$ means $\sin\left(\frac{360(t - 80)}{365}\right) > \frac{1}{2}$

M1

$30^\circ < \frac{360(t - 80)}{365} < 150^\circ$

M1 A1

$80 + 30.4 < t < 80 + 152.1$, so about $110.4 < t < 232.1$

A1

Number of days ≈ 122

A1

Answer: 122 days

5 marks

24 (6 marks)

$a = 3$

B1

Quarter of a period = $65 - 20 = 45^\circ$, so period 180°

M1

$b = \frac{360}{180} = 2$

A1

Rising zero: $2 \times 20 - c = 0$, so $c = 40$

M1 A1

Check: $2(65) - 40 = 90$, a maximum

B1

Answer: $a = 3$, $b = 2$, $c = 40$

6 marks

25 (8 marks)

(a) Period = $180^\circ \div \frac{1}{2} = 360^\circ$

B1

Asymptotes where $\frac{x}{2} = 90^\circ$ or 270° : $x = 180^\circ$ and $x = 540^\circ$

B1 B1

Crosses the x-axis at $0^\circ, 360^\circ, 720^\circ$; each branch increases

B1 B1

Answer: period 360° ; $x = 180^\circ, 540^\circ$

5 marks

(b) $\tan\left(\frac{x}{2}\right) = 1$

M1

$\frac{x}{2} = 45^\circ$ or 225°

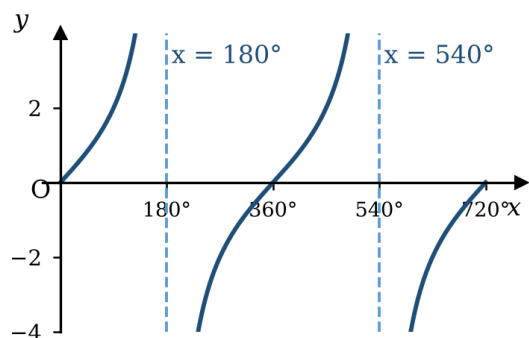
A1

$x = 90^\circ$ or 450°

A1

Answer: $x = 90^\circ, 450^\circ$

3 marks

**26 (9 marks)**

(a) Stretch factor 3 in the x-direction: $\sin\left(\frac{x}{3}\right)$

M1

Then y-stretch and translation: $g(x) = 2 \sin\left(\frac{x}{3}\right) - 1$

A1

Answer: $g(x) = 2 \sin\left(\frac{x}{3}\right) - 1$

2 marks

(b) $g(0) = -1$, so the curve starts at $(0, -1)$

B1

Maximum when $\frac{x}{3} = 90^\circ$: $(270^\circ, 1)$

B1

$g(x) = 0$ when $\sin\left(\frac{x}{3}\right) = \frac{1}{2}$, so $\frac{x}{3} = 30^\circ$ or 150° : $(90^\circ, 0)$ and $(450^\circ, 0)$

M1 A1

The curve ends at $(540^\circ, -1)$

B1

Answer: see sketch

5 marks

(c) The curve is symmetrical about $x = 270^\circ$

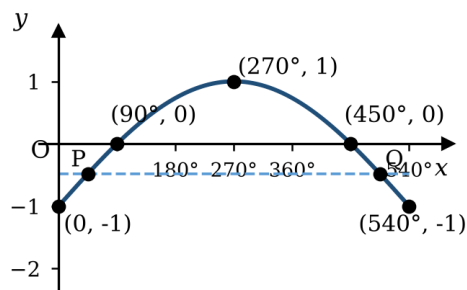
M1

Q has x-coordinate $540^\circ - 45^\circ = 495^\circ$, and the same y-coordinate $k = 2 \sin 15^\circ - 1 = -0.482$

A1

Answer: Q = $(495^\circ, -0.482)$

2 marks

**27 (9 marks)**

(a) $y = \sin x$: one period, maximum $(90^\circ, 1)$, minimum $(270^\circ, -1)$

B1

$y = \sin 2x$: two periods, crossing the axis at $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$, amplitude 1

B1 B1

Answer: see sketch

3 marks

(b) The curves meet at 5 points (including the ends of the interval)

B1

Answer: 5

1 mark

(c) $\sin x = 2 \sin x \cos x$, so $\sin x (1 - 2 \cos x) = 0$ (do not divide by $\sin x$)

M1

$$\sin x = 0: x = 0^\circ, 180^\circ, 360^\circ$$

A1

$$\cos x = \frac{1}{2}: x = 60^\circ, 300^\circ$$

M1 A1

$$\text{Answer: } x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$$

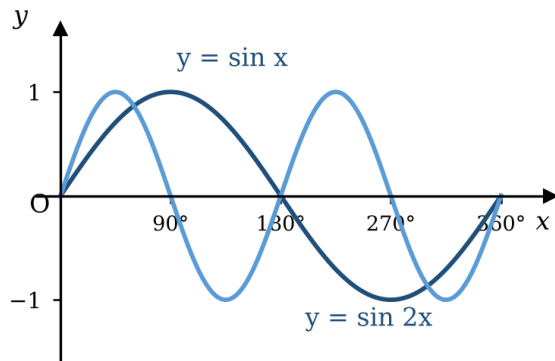
4 marks

$$(d) x = 60^\circ, y = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

B1

$$\text{Answer: } \frac{\sqrt{3}}{2}$$

1 mark

**28 (10 marks)**

$$(a) t = 0: h = 12 - 10 = 2 \text{ m}$$

B1

$\cos(18t)^\circ$ repeats when $18t$ increases by 360 , i.e. every $t = 20 \text{ s}$

B1

$$\text{Answer: } 2 \text{ m; period } 20 \text{ s}$$

2 marks

$$(b) 12 - 10\cos(18t)^\circ = 17, \text{ so } \cos(18t)^\circ = -\frac{1}{2}$$

M1

$$18t = 120 \text{ (first solution)}$$

A1

$$t = \frac{20}{3} = 6.67 \text{ s (3 s.f.)}$$

A1

$$\text{Answer: } t = 6.67 \text{ s}$$

3 marks

$$(c) h \geq 17 \text{ when } \cos(18t)^\circ \leq -\frac{1}{2}, \text{ so } 120 \leq 18t \leq 240$$

M1 A1

$$\frac{20}{3} \leq t \leq \frac{40}{3}, \text{ a time of } \frac{20}{3} = 6.67 \text{ s}$$

A1

$$\text{Answer: } 6.67 \text{ s}$$

3 marks

$$(d) \sin(\theta - 90^\circ) = \sin \theta \cos 90^\circ - \cos \theta \sin 90^\circ = -\cos \theta$$

M1

$$\text{so } 12 + 10 \sin(18t - 90)^\circ = 12 - 10 \cos(18t)^\circ$$

E1

$$\text{Answer: shown}$$

2 marks

29 (9 marks)

$$(a) \cos x = \sin(x + 90^\circ)$$

B1

$$\text{Translation by } \begin{pmatrix} -90 \\ 0 \end{pmatrix}$$

B1

$$\text{Answer: translation } \begin{pmatrix} -90 \\ 0 \end{pmatrix}$$

2 marks

$$(b) \text{ Divide by } \cos x \text{ (} \cos x \neq 0 \text{ at a solution): } \tan x = 1$$

M1

$$x = 45^\circ \text{ or } 225^\circ$$

A1

$$\tan x \text{ has period } 180^\circ \text{ so exactly two solutions in } 360^\circ$$

E1

$$\text{Answer: } x = 45^\circ, 225^\circ$$

3 marks

$$(c) \text{ Solutions at } 45^\circ + 180^\circ n$$

M1

$$\text{Largest below } 1000: n = 5 \text{ gives } 945^\circ; n = 4 \text{ gives } 765^\circ, \text{ so } 945^\circ \text{ is largest and } 765^\circ \text{ the second}$$

M1

$$\text{Number: } n = 0 \text{ to } 5, \text{ so } 6 \text{ solutions; sum} = 945^\circ + 765^\circ = 1710^\circ$$

A1 A1

$$\text{Answer: } 6 \text{ solutions; } 1710^\circ$$

4 marks

30 (12 marks)**(a)** Greatest 5.5 m, least 2.5 m**B1 B1**

$$\text{Period} = \frac{2\pi}{(\pi/6)} = 12 \text{ hours}$$

B1**Answer: 5.5 m, 2.5 m, 12 h****3 marks****(b)** $4 + 1.5 \sin(\frac{\pi t}{6}) \geq 4.75$, so $\sin(\frac{\pi t}{6}) \geq 0.5$ **M1**

$$\frac{\pi t}{6} \text{ between } \frac{\pi}{6} \text{ and } \frac{5\pi}{6}$$

M1 A1

$$1 \leq t \leq 5$$

A1**Answer: $1 \leq t \leq 5$, that is 07:00 to 11:00****4 marks****(c)** $\sin(\frac{\pi t}{6}) < -\frac{2}{3}$: reference angle 0.7297**M1**In one 12 hour cycle, $\frac{\pi t}{6}$ in $(\pi + 0.7297, 2\pi - 0.7297)$ **M1 A1**

$$\text{Length per cycle} = \frac{6}{\pi}(\pi - 2 \times 0.7297) = 3.213 \text{ h}$$

M1

Two cycles: 6.425 h = 386 minutes

A1**Answer: 386 minutes****5 marks****Radians, Arc Length and Sector Area****1 (2 marks)**

$$72^\circ = 72 \times \frac{\pi}{180} = \frac{2\pi}{5} \text{ radians}$$

B1

$$\frac{3\pi}{8} \times \frac{180}{\pi} = 67.5^\circ$$

B1**Answer: (a) $\frac{2\pi}{5}$; (b) 67.5°** **2 marks****2 (3 marks)**

$$\text{(a) } s = r\theta = 6 \times 0.8 = 4.8 \text{ cm}$$

B1**Answer: 4.8 cm****1 mark**

$$\text{(b) } A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 0.8$$

M1

$$= 14.4 \text{ cm}^2$$

A1**Answer: 14.4 cm^2** **2 marks****3 (5 marks)**

$$\text{(a) } 75^\circ = 75 \times \frac{\pi}{180}$$

M1

$$= \frac{5\pi}{12} \text{ radians}$$

A1**Answer: $\frac{5\pi}{12}$ radians****2 marks**

$$\text{(b) Arc length} = 8 \times \frac{5\pi}{12} = \frac{10\pi}{3} \text{ cm}$$

B1

$$\text{Area} = \frac{1}{2} \times 8^2 \times \frac{5\pi}{12} = \frac{40\pi}{3} \text{ cm}^2$$

M1 A1**Answer: arc $\frac{10\pi}{3}$ cm; area $\frac{40\pi}{3} \text{ cm}^2$** **3 marks****4 (2 marks)**

$$\text{(a) } 1.2 \times \frac{180}{\pi} = 68.8^\circ$$

B1

$$\text{(b) } \frac{5\pi}{6} \times \frac{180}{\pi} = 150^\circ$$

B1**Answer: (a) 68.8° , (b) 150°** **2 marks**

5 (4 marks)

(a) $\frac{1}{2}$

B1

(b) $\frac{1}{2}$

B1

(c) 1

B1

(d) $\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

B1

Answer: $\frac{1}{2}, \frac{1}{2}, 1, \frac{\sqrt{3}}{2}$

4 marks

6 (4 marks)

Perimeter = $12 + 6\theta$

B1

Area = $\frac{1}{2} \times 6^2 \times \theta = 18\theta$

B1

$12 + 6\theta = 18\theta$

M1

$12\theta = 12$, so $\theta = 1$

A1

Answer: $\theta = 1$

4 marks

7 (3 marks)

Arc length = $9 \times \frac{2\pi}{3} = 6\pi$ cm

M1 A1

Perimeter = $2 \times 9 + 6\pi = 18 + 6\pi$ cm

A1

Answer: $(18 + 6\pi)$ cm

3 marks

8 (4 marks)

(a) $\frac{1}{2}r^2(1.25) = 40$, so $r^2 = 64$

M1

$r = 8$ cm

A1

(b) Arc length = $8 \times 1.25 = 10$ cm, so perimeter = $2 \times 8 + 10 = 26$ cm

M1

A1

Answer: (a) 8 cm; (b) 26 cm

4 marks

9 (5 marks)

(a) Arc = $r\theta$ and perimeter = $2r + r\theta$

B1

$r\theta = \frac{3}{5}(2r + r\theta)$

M1

$5\theta = 6 + 3\theta$, so $\theta = 3$

A1

Answer: shown

3 marks

(b) $\frac{1}{2}r^2 \times 3 = 54$

M1

$r^2 = 36$, so $r = 6$ cm

A1

Answer: $r = 6$ cm

2 marks

10 (3 marks)

Arc = $30 - 2 \times 9 = 12$ cm

M1

$\theta = \frac{12}{9} = \frac{4}{3}$ rad

A1

Area = $\frac{1}{2} \times 9 \times 12 = 54$ cm²

A1

Answer: $\frac{4}{3}$ rad; 54 cm²

3 marks

11 (3 marks)

Triangle OAB is isosceles; $AB^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \cos 2.2$

M1

$AB^2 = 155.7$

A1

$AB = 12.5$ cm

A1

Answer: $AB = 12.5$ cm

3 marks

12 (3 marks)

$$\text{Angle} = \frac{25}{60} \times 2\pi = \frac{5\pi}{6} \text{ rad}$$

M1

$$s = r\theta = 12 \times \frac{5\pi}{6} = 10\pi \text{ cm}$$

M1 A1

Answer: $10\pi \text{ cm}$

3 marks

13 (5 marks)

$$\text{(a) Area} = \frac{1}{2} \times 10^2 \times \sin 1.4$$

M1

$$= 49.27 \text{ cm}^2$$

A1

Answer: 49.3 cm^2

2 marks

$$\text{(b) Sector area} = \frac{1}{2} \times 10^2 \times 1.4 = 70 \text{ cm}^2$$

M1

$$\text{Segment} = \text{sector} - \text{triangle}$$

M1

$$= 70 - 49.27 = 20.7 \text{ cm}^2$$

A1

Answer: 20.7 cm^2

3 marks

14 (7 marks)

$$\text{(a) Area} = \frac{1}{2} \times 6 \times 6 \times \sin 60^\circ$$

M1

$$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$$

A1

Answer: shown

2 marks

$$\text{(b) Each angle is } \frac{\pi}{3}, \text{ so one sector has area } \frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{3\pi}{2}$$

M1 A1

$$\text{Shaded area} = 9\sqrt{3} - 3 \times \frac{3\pi}{2} = 9\sqrt{3} - \frac{9\pi}{2} \text{ cm}^2$$

A1

Answer: $(9\sqrt{3} - \frac{9\pi}{2}) \text{ cm}^2$

3 marks

(c) The sectors touch at the midpoints of the sides, so the boundary is three arcs

M1

$$\text{Each arc} = 3 \times \frac{\pi}{3} = \pi, \text{ so perimeter} = 3\pi \text{ cm}$$

A1

Answer: $3\pi \text{ cm}$

2 marks

15 (5 marks)

$$\text{Sector area} = \frac{1}{2} \times 36 \times \frac{\pi}{3} = 6\pi$$

M1 A1

$$\text{Triangle OAB} = \frac{1}{2} \times 36 \times \sin \frac{\pi}{3} = 9\sqrt{3}$$

M1 A1

$$\text{Segment} = 6\pi - 9\sqrt{3} = 3.26 \text{ cm}^2$$

A1

Answer: 3.26 cm^2

5 marks

16 (9 marks)

$$\text{(a) Angle AOB} = \frac{2\pi}{3}, \text{ so } AB = 2 \times 6 \times \sin \frac{\pi}{3}$$

M1

$$= 6\sqrt{3} \text{ m}$$

A1

Answer: $6\sqrt{3} \text{ m}$

2 marks

$$\text{(b) Area of circle} = 36\pi$$

B1

$$\text{Minor segment} = \frac{1}{2} \times 6^2 \times \left(\frac{2\pi}{3} - \sin \frac{2\pi}{3} \right) = 12\pi - 9\sqrt{3}$$

M1 A1

$$\text{Area} = 36\pi - (12\pi - 9\sqrt{3}) = 24\pi + 9\sqrt{3}$$

A1

Answer: shown

4 marks

$$\text{(c) Major arc} = 6 \times \left(2\pi - \frac{2\pi}{3} \right) = 8\pi$$

M1 A1

$$\text{Perimeter} = (8\pi + 6\sqrt{3}) \text{ m}$$

A1

Answer: $(8\pi + 6\sqrt{3}) \text{ m}$

3 marks

17 (4 marks)

$$(a) s = r\theta = 80 \times 0.35 = 28 \text{ cm}$$

M1 A1

$$(b) A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 6400 \times 0.35 = 1120 \text{ cm}^2$$

M1 A1

Answer: (a) 28 cm; (b) 1120 cm²

4 marks

18 (4 marks)

$$\text{Distance} = r\theta = 0.35 \times 40$$

M1

$$= 14.0 \text{ m}$$

A1

$$\text{Revolutions} = \frac{40}{2\pi} = 6.366$$

M1

$$\approx 6 \text{ revolutions}$$

A1

Answer: 14.0 m; 6 revolutions

4 marks

19 (6 marks)

$$(a) \text{Area} = \frac{1}{2} \times 0.9 \times (12^2 - 8^2)$$

M1 A1

$$= 36.0 \text{ cm}^2$$

A1

Answer: 36.0 cm²

3 marks

(b) Two straight edges of $12 - 8 = 4 \text{ cm}$ each

B1

$$\text{Arcs: } 12 \times 0.9 + 8 \times 0.9 = 18$$

M1

$$\text{Perimeter} = 8 + 18 = 26.0 \text{ cm}$$

A1

Answer: 26 cm

3 marks

20 (11 marks)

$$(a) AB^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos 0.9$$

M1

$$= 51.68, \text{ so } AB = 7.19 \text{ cm}$$

A1

Answer: AB = 7.19 cm

2 marks

$$(b) \text{Area} = \frac{1}{2} \times 9 \times 7 \times \sin 0.9$$

M1

$$= 24.7 \text{ cm}^2$$

A1

Answer: 24.7 cm²

2 marks

$$(c) \text{Sector CDE} = \frac{1}{2} \times 5^2 \times 0.9 = 11.25 \text{ cm}^2$$

M1 A1

$$\text{Area of R} = 24.67 - 11.25 = 13.4 \text{ cm}^2$$

A1

Answer: 13.4 cm²

3 marks

$$(d) DA = 9 - 5 = 4 \text{ and } BE = 7 - 5 = 2$$

B1

$$\text{Arc ED} = 5 \times 0.9 = 4.5$$

B1

$$\text{Perimeter} = 4 + 7.189 + 2 + 4.5 = 17.7 \text{ cm}$$

M1 A1

Answer: 17.7 cm

4 marks

21 (9 marks)

$$(a) \cos PAB = \frac{6^2 + 10^2 - 8^2}{2 \times 6 \times 10}$$

M1

$$= \frac{72}{120} = \frac{3}{5}$$

A1

$$\text{angle PAB} = \cos^{-1}(0.6) = 0.927 \text{ rad}$$

A1

Answer: shown

3 marks

$$(b) 6^2 + 8^2 = 10^2, \text{ so angle APB} = \frac{\pi}{2} \text{ and angle PBA} = \frac{\pi}{2} - 0.9273 = 0.6435$$

M1 A1

$$\text{Segment in } C_1: \text{angle PAQ} = 2 \times 0.9273, \text{ area} = \frac{1}{2} \times 6^2(2 \times 0.9273 - \sin 1.8546) = 16.10$$

M1 A1

$$\text{Segment in } C_2: \text{angle PBQ} = 2 \times 0.6435, \text{ area} = \frac{1}{2} \times 8^2(2 \times 0.6435 - \sin 1.2870) = 10.46$$

M1

$$\text{Common area} = 16.10 + 10.46 = 26.6 \text{ cm}^2$$

A1

Answer: 26.6 cm²

6 marks

22 (8 marks)

(a) The straight section is a common tangent. A line drawn parallel to it from P to the radius of the large pulley forms a right-angled triangle with hypotenuse 10 and one side $5 - 2 = 3$

M1

$$\text{Length} = \sqrt{10^2 - 3^2} = \sqrt{91}$$

A1

Answer: shown

3 marks

(b) The radius to the point of contact makes an angle with OP whose cosine is $\frac{3}{10}$, so this angle is

M1 A1

$$\frac{\pi}{2} - \alpha, \text{ where } \sin \alpha = 0.3$$

The angle at O subtended by the arc is $2\pi - 2(\frac{\pi}{2} - \alpha) = \pi + 2\alpha$

E1

Answer: shown

3 marks

(c) $\alpha = \sin^{-1}(0.3) = 0.3047 \text{ rad}$

B1

Arc on large pulley = $5(\pi + 2\alpha) = 18.75$; arc on small pulley = $2(\pi - 2\alpha) = 5.064$

M1

Length = $2\sqrt{91} + 18.75 + 5.064 = 42.9 \text{ cm}$

A1

Answer: 42.9 cm

3 marks

23 (8 marks)

(a) Half of the angle: $\sin(\frac{\theta}{2}) = \frac{4}{5}$

M1

$$\frac{\theta}{2} = 0.9273, \theta = 1.855 \text{ rad}$$

A1 A1

Answer: $\theta = 1.855 \text{ rad}$

3 marks

(b) Sector = $\frac{1}{2} \times 25 \times 1.855 = 23.18$

M1

Triangle = $\frac{1}{2} \times 25 \times \sin \theta = 12$ (since $\sin \theta = \frac{24}{25}$)

M1

Segment = $23.18 - 12 = 11.2 \text{ cm}^2$

A1

Answer: 11.2 cm²

3 marks

(c) Arc = $5\theta = 9.273$

M1

Perimeter = $9.273 + 8 = 17.3 \text{ cm}$

A1

Answer: 17.3 cm

2 marks

24 (7 marks)

(a) Arc length = $15 \times \frac{4\pi}{3} = 20\pi$

M1 A1

This is the base circumference: $2\pi r = 20\pi$, so $r = 10$

A1

Answer: shown

3 marks

(b) Slant height 15, so $h = \sqrt{15^2 - 10^2} = \sqrt{125} = 5\sqrt{5}$

M1 A1

$$V = \frac{1}{3}\pi \times 10^2 \times 5\sqrt{5} = \frac{500\sqrt{5}\pi}{3} \text{ cm}^3$$

M1 A1

Answer: $\frac{500\sqrt{5}\pi}{3} \text{ cm}^3$

4 marks

25 (7 marks)

(a) $\frac{1}{2}r^2\theta = 32$ and $2r + r\theta = 24$

B1

$r\theta = 24 - 2r$, so $\frac{1}{2}r(24 - 2r) = 32$

M1

$12r - r^2 = 32$, so $r^2 - 12r + 32 = 0$

A1

Answer: shown

3 marks

(b) $(r - 4)(r - 8) = 0$

M1

$r = 4$: $\theta = \frac{24 - 8}{4} = 4$

A1

$r = 8$: $\theta = \frac{24 - 16}{8} = 1$

A1

Answer: $r = 4, \theta = 4$ or $r = 8, \theta = 1$

3 marks

(c) θ must be less than $2\pi \approx 6.28$; both 4 and 1 satisfy this

E1

Answer: both $\theta < 2\pi$ **1 mark****26 (4 marks)**The formula $\frac{1}{2}r^2\theta$ needs θ in radians**E1**

$$40^\circ = 40 \times \frac{\pi}{180} = \frac{2\pi}{9} \text{ rad}$$

M1

$$\text{Area} = \frac{1}{2} \times 36 \times \frac{2\pi}{9} = 4\pi$$

M1

$$= 12.6 \text{ cm}^2$$

A1**Answer: 12.6 cm²****4 marks****27 (10 marks)****(a)** Perimeter: $2r + r\theta = 20$, so $r\theta = 20 - 2r$ **M1**

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}r(r\theta)$$

M1

$$= \frac{1}{2}r(20 - 2r) = 10r - r^2$$

A1**Answer: shown****3 marks****(b)** $A = 25 - (r - 5)^2$ **M1 A1**Maximum area is 25 cm^2 when $r = 5$ **A1**

$$\theta = \frac{20 - 2(5)}{5} = 2 \text{ radians}$$

A1**Answer: 25 cm², $\theta = 2$** **4 marks****(c)** Arc length = $5 \times 2 = 10 \text{ cm}$ becomes the base circumference: $2\pi R = 10$, so $R = \frac{5}{\pi}$ **M1**Slant height = 5 cm **B1**

$$h = \sqrt{5^2 - \left(\frac{5}{\pi}\right)^2} = 4.74 \text{ cm}$$

A1**Answer: 4.74 cm****3 marks****28 (10 marks)****(a)** A sector with angle $\frac{\pi}{2}$ has area one quarter of the circle**M1**

but the segment is the sector minus triangle OAB, so it would be smaller than a quarter of the circle

E1**Answer: segment is smaller than the sector****2 marks****(b)** Segment area = $\frac{1}{2}r^2(\theta - \sin \theta)$ **M1**This equals $\frac{1}{4}\pi r^2$ **M1**

$$\text{so } \theta - \sin \theta = \frac{\pi}{2}$$

A1**Answer: shown****3 marks****(c)** $\theta_2 = 2.1693$, $\theta_3 = 2.3970$, $\theta_4 = 2.2485$ **M1 A1**The iteration converges to $\theta = 2.310$ (3 d.p.)**A1****Answer: $\theta = 2.310$** **3 marks****(d)** $AB = 2r \sin\left(\frac{\theta}{2}\right)$ **M1**

$$= 1.83r$$

A1**Answer: $AB = 1.83r$** **2 marks****29 (9 marks)****(a)** The centre C of the small circle is on the bisector of angle AOB, at distance d from O**B1**The perpendicular from C to OA has length $r = d \sin \frac{\pi}{6} = \frac{d}{2}$, so $d = 2r$ **M1 A1**The circle touches the arc: $d + r = R$, so $R = 3r$ **M1 A1****Answer: shown****5 marks**

$$(b) \text{ Sector} = \frac{1}{2}R^2\frac{\pi}{3} = \frac{\pi R^2}{6} \quad \text{M1}$$

$$\text{Circle} = \pi r^2 = \pi\left(\frac{R}{3}\right)^2 = \frac{\pi R^2}{9} \quad \text{M1}$$

$$\text{Difference} = \frac{\pi R^2}{6} - \frac{\pi R^2}{9} = \frac{\pi R^2}{18} \quad \text{A1}$$

$$\text{Answer: } \frac{\pi R^2}{18} \quad \text{3 marks}$$

$$(c) \frac{\pi R^2}{18} \div \frac{\pi R^2}{6} = \frac{1}{3} \quad \text{A1}$$

$$\text{Answer: } \frac{1}{3} \quad \text{1 mark}$$

30 (9 marks)

$$(a) \text{ Segment} = \text{sector} - \text{triangle} \quad \text{M1}$$

$$\text{Sector} = \frac{1}{2}r^2\theta, \text{ triangle} = \frac{1}{2}r^2 \sin \theta \quad \text{M1 A1}$$

$$\text{Answer: shown} \quad \text{3 marks}$$

$$(b) S \approx \frac{1}{2}r^2(\theta - \theta + \frac{\theta^3}{6}) \quad \text{M1}$$

$$= \frac{r^2\theta^3}{12} \quad \text{A1}$$

$$\text{Answer: shown} \quad \text{2 marks}$$

$$(c) S = 50(0.1 - \sin 0.1) = 0.008329 \quad \text{M1 A1}$$

$$\text{Approximation} = \frac{100 \times 0.001}{12} = 0.008333 \quad \text{A1}$$

$$\text{Percentage error} = 0.050\% \quad \text{A1}$$

$$\text{Answer: } S = 0.008329, \text{ approx } 0.008333, \text{ error } 0.050\% \quad \text{4 marks}$$

Trigonometric Identities

1 (2 marks)

$$3(\sin^2 x + \cos^2 x) = 3 \quad \text{B1}$$

$$\tan x \cos x = \frac{\sin x}{\cos x} \times \cos x = \sin x \quad \text{B1}$$

$$\text{Answer: (a) 3; (b) } \sin x \quad \text{2 marks}$$

2 (3 marks)

$$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{25}{169} = \frac{144}{169} \quad \text{M1 A1}$$

$$\theta \text{ acute so } \sin \theta = \frac{12}{13} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12}{5} \quad \text{A1}$$

$$\text{Answer: } \sin \theta = \frac{12}{13}, \tan \theta = \frac{12}{5} \quad \text{3 marks}$$

3 (2 marks)

$$(a) 4(1 - \cos^2 \theta) = 4 \sin^2 \theta \quad \text{B1}$$

$$(b) \sin x \div \frac{\sin x}{\cos x} = \cos x \quad \text{B1}$$

$$\text{Answer: (a) } 4 \sin^2 \theta, (b) \cos x \quad \text{2 marks}$$

4 (2 marks)

$$\text{Use a right-angled triangle with opposite 3 and adjacent 4; hypotenuse} = \sqrt{9 + 16} = 5 \quad \text{M1}$$

$$\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5} \quad \text{A1}$$

$$\text{Answer: } \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5} \quad \text{2 marks}$$

5 (2 marks)

$$\cos^2 x = 1 - \sin^2 x$$

M1

So the expression is $1 + 2 \sin^2 x$, $a = 2$

A1

Answer: $a = 2$

2 marks

6 (2 marks)

$$\text{Expand: } \sin^2 x + 2 \sin x \cos x + \cos^2 x - 2 \sin x \cos x$$

M1

$$= \sin^2 x + \cos^2 x = 1$$

A1

Answer: 1

2 marks

7 (3 marks)

$$1 - \cos^2 x = \sin^2 x$$

M1

$$\frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x}$$

M1

$$= \tan x$$

A1

Answer: shown

3 marks

8 (3 marks)

$$\cos^2 \theta = 1 - \frac{144}{169} = \frac{25}{169}$$

M1

$$\theta \text{ is obtuse so } \cos \theta \text{ is negative: } \cos \theta = -\frac{5}{13}$$

A1

$$\tan \theta = \frac{12}{13} \div \left(-\frac{5}{13}\right) = -\frac{12}{5}$$

A1

$$\text{Answer: } \cos \theta = -\frac{5}{13}, \tan \theta = -\frac{12}{5}$$

3 marks

9 (2 marks)

$$\text{LHS} = \cos^2 x - (1 - \cos^2 x)$$

M1

$$= 2 \cos^2 x - 1 = \text{RHS}$$

A1

Answer: proved

2 marks

10 (3 marks)

$$\text{LHS} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

M1

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

M1

$$= \frac{1}{\sin x \cos x}$$

A1

Answer: proved

3 marks

11 (3 marks)

Multiply numerator and denominator of LHS by $(1 + \sin x)$

M1

$$\frac{(1 - \sin x)(1 + \sin x)}{\cos x (1 + \sin x)} = \frac{1 - \sin^2 x}{\cos x (1 + \sin x)}$$

M1

$$= \frac{\cos^2 x}{\cos x (1 + \sin x)} = \frac{\cos x}{1 + \sin x}$$

A1

Answer: shown

3 marks

12 (3 marks)

$$\sin^2 \theta = 1 - \frac{4}{9} = \frac{5}{9}$$

M1

$$\sin \theta \text{ is negative in the third quadrant: } \sin \theta = -\frac{\sqrt{5}}{3}$$

A1

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{5}}{2}$$

A1

$$\text{Answer: } \sin \theta = -\frac{\sqrt{5}}{3}, \tan \theta = \frac{\sqrt{5}}{2}$$

3 marks

13 (3 marks)Divide numerator and denominator by $\cos \theta$

M1

$$\frac{\tan \theta + 2}{3 \tan \theta - 1} = \frac{2 + 2}{6 - 1}$$

M1

$$= \frac{4}{5}$$

A1

Answer: $\frac{4}{5}$

3 marks

14 (4 marks)

$$\text{LHS} = \frac{\sin^2 x + (1 + \cos x)^2}{(1 + \cos x) \sin x}$$

M1 A1

$$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{(1 + \cos x) \sin x} = \frac{2 + 2\cos x}{(1 + \cos x) \sin x}$$

M1

$$= \frac{2(1 + \cos x)}{(1 + \cos x) \sin x} = \frac{2}{\sin x} = \text{RHS}$$

A1

Answer: proved

4 marks

15 (6 marks)(a) $4 = 4(\sin^2 \theta + \cos^2 \theta)$, so $3 \sin^2 \theta + 5 \cos^2 \theta = 4 \sin^2 \theta + 4 \cos^2 \theta$

M1

$$\cos^2 \theta = \sin^2 \theta$$

A1

Divide by $\cos^2 \theta$: $\tan^2 \theta = 1$

M1

$$\tan \theta = 1 \text{ or } \tan \theta = -1$$

A1

Answer: $\tan \theta = \pm 1$

4 marks

(b) $3(1 - \cos^2 \theta) + 5 \cos^2 \theta = 3 + 2\cos^2 \theta$

M1

Least value is 3 (when $\cos \theta = 0$)

A1

Answer: $3 + 2\cos^2 \theta$; least value 3

2 marks

16 (4 marks)

$$\text{LHS} = \frac{(1 - \cos x) + (1 + \cos x)}{(1 + \cos x)(1 - \cos x)}$$

M1 A1

$$= \frac{2}{1 - \cos^2 x}$$

M1

$$= \frac{2}{\sin^2 x}$$

A1

Answer: proved

4 marks

17 (4 marks)

$$\tan^2 x - \sin^2 x = \tan^2 x - \tan^2 x \cos^2 x$$

M1 A1

$$= \tan^2 x (1 - \cos^2 x)$$

M1

$$= \tan^2 x \sin^2 x$$

A1

Answer: proved

4 marks

18 (6 marks)

$$(a) \cos \theta = \frac{x}{3}, \sin \theta = \frac{y}{2}$$

M1

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

M1 A1

$$\text{Answer: } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

3 marks

$$(b) \sin \theta = \frac{x-2}{4}, \cos \theta = \frac{y-1}{4}$$

M1

$$(x-2)^2 + (y-1)^2 = 16, \text{ a circle of radius 4}$$

M1 A1

Answer: $(x-2)^2 + (y-1)^2 = 16$

3 marks

19 (5 marks)

$$\begin{aligned} \text{(a)} f(x) &= \frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x} \\ &= \frac{\cos^2 x}{\sin x} \end{aligned}$$

M1

A1

Answer: shown

2 marks

$$\text{(b)} \cos^2 x \geq 0 \text{ and, for } 0^\circ < x < 180^\circ, \sin x > 0$$

M1

$$\text{so } \frac{\cos^2 x}{\sin x} \geq 0$$

A1

$$f(x) = 0 \text{ only when } \cos x = 0, \text{ that is } x = 90^\circ$$

B1

Answer: $f(x) \geq 0$; $x = 90^\circ$

3 marks

20 (7 marks)

$$\text{(a)} \cos^2 x = 1 - \sin^2 x$$

M1

$$f(x) = 2 - 2 \sin^2 x - 3 \sin x + 1 = 3 - 3 \sin x - 2 \sin^2 x$$

A1

Answer: shown

2 marks

$$\text{(b)} \text{ Let } s = \sin x: f = 3 - 3s - 2s^2 = \frac{33}{8} - 2\left(s + \frac{3}{4}\right)^2$$

M1 A1

$$\text{Greatest } \frac{33}{8} \text{ at } s = -\frac{3}{4}, \text{ which is possible}$$

A1

$$\text{Least at } s = 1 \text{ (furthest from } -\frac{3}{4}\text{): } 3 - 3 - 2 = -2$$

M1 A1

Answer: greatest $\frac{33}{8}$, least -2

5 marks

21 (8 marks)

$$\begin{aligned} \text{(a)} \text{ Expanding: } \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta \\ = 2(\sin^2 \theta + \cos^2 \theta) = 2 \end{aligned}$$

M1

A1

Answer: proved

2 marks

$$\text{(b)} \text{ Squaring the given equation: } 1 + 2 \sin \theta \cos \theta = \frac{1}{4}, \text{ so } \sin \theta \cos \theta = -\frac{3}{8}$$

M1 A1

$$\text{From (a): } (\sin \theta - \cos \theta)^2 = 2 - \frac{1}{4}$$

M1

$$= \frac{7}{4}$$

A1

Answer: $-\frac{3}{8}$ and $\frac{7}{4}$

4 marks

$$\text{(c)} \sin^3 \theta + \cos^3 \theta = (\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)$$

M1

$$= \frac{1}{2}\left(1 + \frac{3}{8}\right) = \frac{11}{16}$$

A1

Answer: $\frac{11}{16}$

2 marks

22 (9 marks)

$$\text{(a)} \sqrt{a^2 + b^2} \text{ is not } a + b: \text{ square roots cannot be taken term by term}$$

E1

$$\text{Counter-example } x = 45^\circ: \sin x + \cos x = \sqrt{2} \neq 1$$

B1

Answer: root of a sum is not the sum of roots; e.g. $x = 45^\circ$

2 marks

$$\text{(b)} 1 + \tan^2 x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

M1 A1

$$1 - \tan^2 x = \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$

M1

$$\text{Dividing: } \frac{(\cos^2 x - \sin^2 x)}{\cos^2 x} \times \cos^2 x = \cos^2 x - \sin^2 x$$

A1

Answer: proved

4 marks

$$\text{(c)} \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x}$$

M1

$$= \frac{\sin^2 x}{\cos x}$$

A1

$$= \sin x \times \frac{\sin x}{\cos x} = \sin x \tan x$$

A1

Answer: proved

3 marks

23 (8 marks)(a) Difference of two squares: $(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

M1 A1

$$= \sin^2 x - \cos^2 x$$

A1

Answer: proved

3 marks

(b) $\sin^2 x - (1 - \sin^2 x) = \frac{1}{2}$, so $\sin^2 x = \frac{3}{4}$

M1 A1

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

M1

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

A1 A1

Answer: $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

5 marks

24 (5 marks)

$$(a) \cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{x^2}{9}$$

M1 A1

$$y = 4 + 2 - \frac{2x^2}{9} = 6 - \frac{2x^2}{9}$$

A1

Answer: shown

3 marks

(b) $-1 \leq \sin \theta \leq 1$ gives $-3 \leq x \leq 3$

B1

$$0 \leq \cos^2 \theta \leq 1 \text{ gives } 4 \leq y \leq 6$$

B1

Answer: $-3 \leq x \leq 3, 4 \leq y \leq 6$

2 marks

25 (7 marks)(a) $\sin^3 x + \cos^3 x$ is a sum of cubes $= (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$

M1 A1

$$= (\sin x + \cos x)(1 - \sin x \cos x)$$

A1

Answer: proved

3 marks

(b) Square: $1 + 2 \sin x \cos x = \frac{36}{25}$, so $\sin x \cos x = \frac{11}{50}$

M1 A1

$$\sin^3 x + \cos^3 x = \frac{6}{5} \left(1 - \frac{11}{50}\right) = \frac{117}{125}$$

M1 A1

Answer: $\frac{11}{50}; \frac{117}{125}$

4 marks

26 (8 marks)

$$(a) \tan^2 x = \sec^2 x - 1$$

M1

$$3 \sec^2 x - 3 + 5 \sec x + 1 = 0$$

M1

$$3 \sec^2 x + 5 \sec x - 2 = 0$$

A1

Answer: shown

3 marks

(b) $(3 \sec x - 1)(\sec x + 2) = 0$, so $\sec x = \frac{1}{3}$ or $\sec x = -2$

M1 A1

 $\sec x = \frac{1}{3}$ is impossible, since $|\sec x| \geq 1$ for all x

E1

$$\cos x = -\frac{1}{2}, \text{ so } x = 120^\circ \text{ or } 240^\circ$$

M1 A1

Answer: $x = 120^\circ, 240^\circ$

5 marks

27 (9 marks)

$$(a) \text{ LHS} = \frac{(1 + \sin x) - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$$

M1 A1

$$= \frac{2 \sin x}{1 - \sin^2 x}$$

M1

$$= \frac{2 \sin x}{\cos^2 x}$$

A1

Answer: proved

4 marks

(b) $\frac{2 \sin x}{\cos^2 x} = 4 \sin x$, so $2 \sin x = 4 \sin x \cos^2 x$

M1

$$2 \sin x(1 - 2\cos^2 x) = 0 \text{ (do not divide by } \sin x)$$

M1

$$\sin x = 0: x = 0^\circ, 180^\circ, 360^\circ$$

A1

$$\cos^2 x = \frac{1}{2}, \text{ so } \cos x = \pm \frac{1}{\sqrt{2}}$$

A1

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ \text{ (} \cos x \neq 0 \text{ at all of these)}$$

A1

$$\text{Answer: } x = 0^\circ, 45^\circ, 135^\circ, 180^\circ, 225^\circ, 315^\circ, 360^\circ$$

5 marks

28 (10 marks)

$$(a) x - 2 = 3 \sin \theta \text{ and } y - 1 = -3 \cos \theta$$

M1

$$(x - 2)^2 + (y - 1)^2 = 9 \sin^2 \theta + 9 \cos^2 \theta$$

M1 A1

$$= 9, \text{ a circle with centre } (2, 1) \text{ and radius } 3$$

A1

$$\text{Answer: } (x - 2)^2 + (y - 1)^2 = 9$$

4 marks

$$(b) x^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$$

M1

$$= 1 + 2y$$

A1

$$\text{so } 2y = x^2 - 1$$

A1

$$\text{Answer: } 2y = x^2 - 1$$

3 marks

$$(c) (\sin \theta - \cos \theta)^2 \geq 0, \text{ so } 1 - 2 \sin \theta \cos \theta \geq 0$$

M1

$$\text{hence } y = \sin \theta \cos \theta \leq \frac{1}{2}$$

A1

$$x^2 = 1 + 2y \leq 2, \text{ so greatest } x \text{ is } \sqrt{2}$$

A1

$$\text{Answer: } y \leq \frac{1}{2}; \text{ greatest } x \text{ is } \sqrt{2}$$

3 marks

29 (12 marks)

$$(a) \text{ Let } p = \sin^2 x, q = \cos^2 x, p + q = 1$$

B1

$$p^3 + q^3 = (p + q)^3 - 3pq(p + q)$$

M1 A1

$$= 1 - 3pq = 1 - 3 \sin^2 x \cos^2 x$$

A1

$$\text{Answer: shown}$$

4 marks

$$(b) 1 - 3 \sin^2 x(1 - \sin^2 x) = \frac{7}{16}$$

M1

$$3 \sin^2 x - 3 \sin^4 x = \frac{9}{16}, \text{ so } 48 \sin^4 x - 48 \sin^2 x + 9 = 0$$

M1

$$\text{Divide by 3: } 16 \sin^4 x - 16 \sin^2 x + 3 = 0$$

A1

$$\text{Answer: shown}$$

3 marks

$$(c) (4 \sin^2 x - 1)(4 \sin^2 x - 3) = 0$$

M1 A1

$$\sin x = \pm \frac{1}{2} \text{ or } \sin x = \pm \frac{\sqrt{3}}{2}$$

A1

$$x = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$$

A1 A1

$$\text{Answer: } x = 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ$$

5 marks

30 (11 marks)

$$(a) \operatorname{cosec} \theta - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$$

M1

$$= \frac{\cos^2 \theta}{\sin \theta}; \text{ similarly } \sec \theta - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta}$$

A1 A1

$$\text{Answer: shown}$$

3 marks

$$(b) a^2 b^2 = \frac{\cos^4 \theta}{\sin^2 \theta} \times \frac{\sin^4 \theta}{\cos^2 \theta} = \sin^2 \theta \cos^2 \theta$$

M1 A1

$$a^2 + b^2 = \frac{\cos^6 \theta + \sin^6 \theta}{\sin^2 \theta \cos^2 \theta}$$

M1

$$= \frac{1 - 3 \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}, \text{ using } \sin^6 + \cos^6 \equiv 1 - 3 \sin^2 \cos^2$$

A1

$$\text{Answer: shown}$$

4 marks

$$(c) \text{ Let } k = \sin^2 \theta \cos^2 \theta: a^2 b^2 = k$$

B1

$$a^2 + b^2 + 3 = \frac{1 - 3k}{k} + 3 = \frac{1}{k}$$

M1 A1

$$\text{So } a^2b^2(a^2 + b^2 + 3) = k \times \frac{1}{k} = 1$$

A1

Answer: proved

4 marks

Trigonometric Equations

1 (2 marks)

$$x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

M1

$$\text{Also } x = 180^\circ - 30^\circ = 150^\circ$$

A1

Answer: x = 30°, 150°

2 marks

2 (3 marks)

$$x = \cos^{-1}(-0.4) = 113.6^\circ$$

M1 A1

By symmetry of the graph, the other solution is $360^\circ - 113.6^\circ = 246.4^\circ$

A1

Answer: x = 113.6°, 246.4°

3 marks

3 (2 marks)

$$x = \tan^{-1}1 = 45^\circ$$

M1

tan has period 180° , so $x = 45^\circ + 180^\circ = 225^\circ$

A1

Answer: x = 45°, 225°

2 marks

4 (3 marks)

$$\sin^{-1}(-0.3) = -17.5^\circ; \text{ reference angle } 17.5^\circ$$

M1

Third and fourth quadrants: $x = 180^\circ + 17.5^\circ$ and $360^\circ - 17.5^\circ$

M1

$$x = 197.5, 342.5^\circ$$

A1

Answer: x = 197.5, 342.5°

3 marks

5 (3 marks)

$$\text{Let } \theta = 2x, 0^\circ \leq \theta \leq 720^\circ: \theta = 60^\circ, 120^\circ, 420^\circ, 480^\circ$$

M1 A1

$$x = 30^\circ, 60^\circ, 210^\circ, 240^\circ$$

A1

Answer: x = 30°, 60°, 210°, 240°

3 marks

6 (3 marks)

$$\tan x = \frac{5}{3}$$

M1

$$x = 59.0^\circ \text{ and } 239.0^\circ$$

A1 A1

Answer: x = 59.0, 239.0°

3 marks

7 (4 marks)

$$\text{Let } \theta = 2x, \text{ with } 0^\circ \leq \theta \leq 720^\circ$$

M1

$$\sin \theta = 0.6: \theta = 36.9^\circ, 143.1^\circ, 396.9^\circ, 503.1^\circ$$

M1 A1

$$x = \theta \div 2 = 18.4, 71.6, 198.4, 251.6^\circ$$

A1

Answer: x = 18.4, 71.6, 198.4, 251.6°

4 marks

8 (3 marks)

$$x - 30^\circ = \tan^{-1}2 = 63.4^\circ$$

M1

$$x - 30^\circ = 63.4^\circ \text{ or } 243.4^\circ$$

A1

$$x = 93.4, 273.4^\circ$$

A1

Answer: x = 93.4, 273.4°

3 marks

9 (4 marks)

$$20^\circ \leq x + 20^\circ \leq 380^\circ$$

B1

$$x + 20^\circ = 120^\circ \text{ or } 240^\circ$$

M1 A1

$$x = 100^\circ \text{ or } 220^\circ$$

A1

Answer: x = 100°, 220°

4 marks

10 (4 marks)

$$\text{Either } \cos x = \frac{1}{2}: x = 60^\circ \text{ or } 300^\circ$$

M1 A1

$$\text{Or } \tan x = -1: x = 135^\circ \text{ or } 315^\circ$$

M1 A1

$$\text{Answer: } x = 60^\circ, 135^\circ, 300^\circ, 315^\circ$$

4 marks

11 (3 marks)

$$\sin x = \frac{1}{2}, \text{ so } x = \frac{\pi}{6}$$

M1 A1

$$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

A1

$$\text{Answer: } x = \frac{\pi}{6}, \frac{5\pi}{6}$$

3 marks

12 (4 marks)

$$\cos x = \pm \frac{1}{2}$$

M1

$$\cos x = \frac{1}{2}: 60^\circ, 300^\circ; \cos x = -\frac{1}{2}: 120^\circ, 240^\circ$$

A1 A1

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

A1

$$\text{Answer: } x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

4 marks

13 (4 marks)

$$0^\circ \leq 2x \leq 360^\circ$$

B1

$$2x = 135^\circ \text{ or } 315^\circ$$

M1 A1

$$x = 67.5^\circ \text{ or } 157.5^\circ$$

A1

$$\text{Answer: } x = 67.5^\circ, 157.5^\circ$$

4 marks

14 (5 marks)

$$\text{Factorise: } (2 \sin x - 1)(\sin x + 1) = 0$$

M1 A1

$$\sin x = \frac{1}{2}: x = 30^\circ, 150^\circ$$

M1 A1

$$\sin x = -1: x = 270^\circ$$

A1

$$\text{Answer: } x = 30^\circ, 150^\circ, 270^\circ$$

5 marks

15 (6 marks)

$$\text{Use } \cos^2 x = 1 - \sin^2 x: 2 - 2 \sin^2 x + 3 \sin x = 3$$

M1

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

A1

$$(2 \sin x - 1)(\sin x - 1) = 0$$

M1

$$\sin x = \frac{1}{2}: x = 30^\circ, 150^\circ$$

A1 A1

$$\sin x = 1: x = 90^\circ$$

A1

$$\text{Answer: } x = 30^\circ, 90^\circ, 150^\circ$$

6 marks

16 (5 marks)

$$(\tan x + 3)(\tan x - 2) = 0$$

M1 A1

$$\tan x = 2: x = 63.4^\circ, 243.4^\circ$$

A1

$$\tan x = -3: x = 108.4^\circ, 288.4^\circ$$

M1 A1

$$\text{Answer: } x = 63.4, 108.4, 243.4, 288.4^\circ$$

5 marks

17 (5 marks)

$$\cos(2x - \frac{\pi}{3}) = \frac{1}{2}, \text{ and } -\frac{\pi}{3} \leq 2x - \frac{\pi}{3} \leq \frac{5\pi}{3}$$

M1 A1

$$2x - \frac{\pi}{3} = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$$

M1 A1

$$x = 0, \frac{\pi}{3}, \pi$$

A1

$$\text{Answer: } x = 0, \frac{\pi}{3}, \pi$$

5 marks

18 (4 marks)

$$2 + 1.5 \sin(0.8t) = 3.2, \text{ so } \sin(0.8t) = 0.8$$

M1 A1

$$0.8t = 0.9273 \text{ or } \pi - 0.9273 = 2.214$$

M1

$$t = 1.16 \text{ s and } 2.77 \text{ s}$$

A1

Answer: t = 1.16, 2.77

4 marks

19 (5 marks)

$$3 \cos x = 2(1 - \cos^2 x), \text{ so } 2 \cos^2 x + 3 \cos x - 2 = 0$$

M1 A1

$$(2 \cos x - 1)(\cos x + 2) = 0$$

M1

$$\cos x = -2 \text{ is impossible since } \cos x \geq -1$$

E1

$$\cos x = \frac{1}{2}: x = 60^\circ, 300^\circ$$

A1

Answer: x = 60°, 300°

5 marks

20 (7 marks)

$$(a) \cos(2x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$$

M1

$$\text{Let } \theta = 2x + \frac{\pi}{4}, \text{ so } \frac{\pi}{4} \leq \theta \leq \frac{17\pi}{4}$$

M1

$$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$$

M1 A1

$$2x = \theta - \frac{\pi}{4}, \text{ so } x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$$

A1

$$\text{Answer: } x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$$

5 marks

(b) The graph has period π , so in 4π there are four periods

M1

Each period gives 2 solutions, so 8 solutions

A1

Answer: 8

2 marks

21 (7 marks)

$$(a) \frac{\sin x}{\cos x} = 2 \sin x, \text{ so } \sin x = 2 \sin x \cos x (\cos x \neq 0)$$

M1

$$\sin x(1 - 2\cos x) = 0$$

M1

$$\sin x = 0: x = 0^\circ, 180^\circ, 360^\circ$$

A1

$$\cos x = \frac{1}{2}: x = 60^\circ, 300^\circ$$

M1 A1

Answer: x = 0°, 60°, 180°, 300°, 360°

5 marks

(b) Dividing by $\sin x$ assumes $\sin x \neq 0$, but $\sin x = 0$ is a possible solution

E1

The solutions $x = 0^\circ, 180^\circ$ and 360° are lost

B1

Answer: solutions with $\sin x = 0$ are lost: 0°, 180°, 360°

2 marks

22 (7 marks)

$$(a) 3 \frac{\sin x}{\cos x} = 8 \cos x, \text{ so } 3 \sin x = 8 \cos^2 x$$

M1 A1

$$3 \sin x = 8(1 - \sin^2 x), \text{ so } 8 \sin^2 x + 3 \sin x - 8 = 0$$

A1

Answer: shown

3 marks

$$(b) \sin x = \frac{-3 \pm \sqrt{9 + 256}}{16}$$

M1

$$\sin x = 0.8299 \text{ (the other root } -1.205 \text{ is rejected)}$$

A1

$$x = 56.1^\circ \text{ or } 123.9^\circ$$

M1 A1

Answer: x = 56.1°, 123.9°

4 marks

23 (6 marks)

$$\text{Range: } -30^\circ \leq 3x - 30^\circ \leq 510^\circ$$

M1 A1

$$3x - 30^\circ = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

M1 A1

$$3x = 60^\circ, 180^\circ, 420^\circ, 540^\circ$$

A1

$$x = 20^\circ, 60^\circ, 140^\circ, 180^\circ$$

A1

Answer: x = 20°, 60°, 140°, 180°

6 marks

24 (6 marks)Case 1: $2x - 30^\circ = x + 45^\circ + 360^\circ n$, so $x = 75^\circ + 360^\circ n$

M1 A1

Case 2: $2x - 30^\circ = -(x + 45^\circ) + 360^\circ n$, so $3x = -15^\circ + 360^\circ n$

M1 A1

 $x = -5^\circ + 120^\circ n$ gives $x = 115^\circ, 235^\circ, 355^\circ$

M1

 $x = 75^\circ, 115^\circ, 235^\circ, 355^\circ$

A1

Answer: $x = 75^\circ, 115^\circ, 235^\circ, 355^\circ$

6 marks

25 (6 marks)

(a) Two (sine is positive in two quadrants)

B1

Answer: 2

1 mark

(b) Three cycles in the interval, 2 per cycle

M1

6 solutions

A1

Answer: 6

2 marks

(c) $0^\circ \leq \frac{x}{2} \leq 180^\circ$, which is one period of tan

M1

1 solution

A1

Answer: 1

2 marks

(d) Only $x = 180^\circ$

B1

Answer: 1

1 mark

26 (9 marks)(a) $\sin^2 x = (1.5 - 2 \cos x)^2 = 2.25 - 6 \cos x + 4 \cos^2 x$

M1

 $1 - \cos^2 x = 2.25 - 6 \cos x + 4 \cos^2 x$

M1

 $5 \cos^2 x - 6 \cos x + 1.25 = 0$, so $20 \cos^2 x - 24 \cos x + 5 = 0$

A1

Answer: shown

3 marks

(b) $\cos x = \frac{24 \pm \sqrt{176}}{40} = 0.9317$ or 0.2683

M1 A1

Candidates: $x = 21.3^\circ, 74.4^\circ$ (from the first) and $x = 285.6^\circ, 338.7^\circ$ (from the second)

M1

Squaring can introduce false roots, so test in $\sin x + 2 \cos x = 1.5$

M1

 21.3° and 285.6° give 2.23 and -0.43 , so are rejected

A1

 $x = 74.4^\circ$ or 338.7°

A1

Answer: $x = 74.4^\circ, 338.7^\circ$

6 marks

27 (11 marks)(a) Use $\sin^2 x = 1 - \cos^2 x$: $6 - 6 \cos^2 x + \cos x = 4$

M1 A1

Rearranging: $6 \cos^2 x - \cos x - 2 = 0$

A1

Answer: shown

3 marks

(b) $(3 \cos x - 2)(2 \cos x + 1) = 0$

M1 A1

 $\cos x = \frac{2}{3}$: $x = 48.2^\circ, 311.8^\circ$

M1 A1

 $\cos x = -\frac{1}{2}$: $x = 120^\circ, 240^\circ$

A1

Answer: $x = 48.2^\circ, 120^\circ, 240^\circ, 311.8^\circ$

5 marks

(c) $6 \sin^2 x + \cos x = 6 - 6 \cos^2 x + \cos x$

M1

 $= 6 + \frac{1}{24} - 6(\cos x - \frac{1}{12})^2$

M1

Greatest value $\frac{145}{24} = 6.04$ (3 s.f.) when $\cos x = \frac{1}{12}$

A1

Answer: $\frac{145}{24}$ when $\cos x = \frac{1}{12}$

3 marks

28 (10 marks)(a) Divide every term by $\cos^2 x$

M1

 $2 \tan^2 x - 3 \tan x + 1 = 0$

A1

Answer: shown

2 marks

(b) $(2 \tan x - 1)(\tan x - 1) = 0$

M1

$$\tan x = \frac{1}{2}: x = 26.6^\circ, 206.6^\circ$$

M1 A1

$$\tan x = 1: x = 45^\circ, 225^\circ$$

A1

If $\cos x = 0$ then $\sin^2 x = 1$ and the left side equals 2, not 0

E1

Answer: $x = 26.6^\circ, 45^\circ, 206.6^\circ, 225^\circ$

5 marks

(c) Let $x = 2y$, with $0^\circ \leq x \leq 360^\circ$; the solutions are those found in (b)

M1

$$2y = 26.6^\circ, 45^\circ, 206.6^\circ, 225^\circ$$

A1

$$y = 13.3^\circ, 22.5^\circ, 103.3^\circ, 112.5^\circ$$

A1

Answer: $y = 13.3^\circ, 22.5^\circ, 103.3^\circ, 112.5^\circ$

3 marks

29 (10 marks)

(a) $2 \sin^2 30^\circ + p \sin 30^\circ - 1 = 0$

M1

$$2 \times \frac{1}{4} + \frac{p}{2} - 1 = 0, \text{ so } p = 1$$

A1

Answer: $p = 1$

2 marks

(b) $2 \sin^2 x + \sin x - 1 = (2 \sin x - 1)(\sin x + 1) = 0$

M1 A1

$$\sin x = \frac{1}{2}: x = 30^\circ, 150^\circ$$

A1

$$\sin x = -1: x = 270^\circ$$

A1

Answer: $x = 30^\circ, 150^\circ, 270^\circ$

4 marks

(c) $2(\sin x + \frac{1}{4})^2 - \frac{9}{8}$

M1 A1

Least value $-\frac{9}{8}$, when $\sin x = -\frac{1}{4}$

A1

$$x = 194.5^\circ \text{ (and } 345.5^\circ)$$

A1

Answer: $-\frac{9}{8}$ at $x = 194.5^\circ$

4 marks

30 (9 marks)

(a) $-\frac{\pi}{4} \leq 2x - \frac{\pi}{4} \leq \frac{15\pi}{4}$

M1 A1

$$2x - \frac{\pi}{4} = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$$

M1 A1

$$2x = \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{31\pi}{12}, \frac{43\pi}{12}, \text{ so } x = \frac{7\pi}{24}, \frac{19\pi}{24}, \frac{31\pi}{24}, \frac{43\pi}{24}$$

A1

Answer: $x = \frac{7\pi}{24}, \frac{19\pi}{24}, \frac{31\pi}{24}, \frac{43\pi}{24}$

5 marks

(b) $\text{Sum} = \frac{7\pi + 19\pi + 31\pi + 43\pi}{24} = \frac{100\pi}{24}$

M1

$$= \frac{25\pi}{6}$$

A1

Answer: shown

2 marks

(c) $2x - \frac{\pi}{4} = \frac{\pi}{3} + n\pi$

M1

$$x = \frac{7\pi}{24} + \frac{n\pi}{2}$$

A1

Answer: $x = \frac{7\pi}{24} + \frac{n\pi}{2}$

2 marks

Reciprocal and Inverse Trigonometric Functions

1 (3 marks)

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

B1

$$\operatorname{cosec} 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

B1

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$$

B1

Answer: (a) 2; (b) $\sqrt{2}$; (c) $\sqrt{3}$

3 marks

2 (3 marks)

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

B1

$$\cos^{-1}(-1) = \pi$$

B1

$$\tan^{-1}(1) = \frac{\pi}{4}$$

B1

Answer: (a) $\frac{\pi}{6}$; (b) π ; (c) $\frac{\pi}{4}$

3 marks

3 (3 marks)

$$(a) \sec 120^\circ = \frac{1}{\cos 120^\circ} = \frac{1}{-1/2} = -2$$

B1

$$(b) \cot 45^\circ = \frac{1}{\tan 45^\circ} = 1$$

B1

$$(c) \operatorname{cosec} 210^\circ = \frac{1}{\sin 210^\circ} = -2$$

B1

Answer: (a) -2 , (b) 1 , (c) -2

3 marks

4 (3 marks)

$$(a) \frac{1}{\tan 70^\circ} = 0.364$$

B1

$$(b) \frac{1}{\cos 25^\circ} = 1.10$$

B1

$$(c) \frac{1}{\sin 100^\circ} = 1.02$$

B1

Answer: 0.364, 1.10, 1.02

3 marks

5 (3 marks)

$$(a) -\frac{\pi}{6}$$

B1

$$(b) \frac{\pi}{6}$$

B1

$$(c) -\frac{\pi}{4}$$

B1

Answer: $-\frac{\pi}{6}$, $\frac{\pi}{6}$, $-\frac{\pi}{4}$

3 marks

6 (2 marks)

$$|\cos x| \leq 1, \text{ so } |\sec x| \geq 1: y \leq -1 \text{ or } y \geq 1$$

B1

$$\cos x = -1 \text{ at } x = 180^\circ$$

B1

Answer: $y \leq -1$ or $y \geq 1$; $x = 180^\circ$

2 marks

7 (3 marks)

$$(a) \operatorname{cosec} x \text{ is undefined where } \sin x = 0: x = 0^\circ, 180^\circ, 360^\circ$$

B1

Answer: $x = 0^\circ, 180^\circ, 360^\circ$

1 mark

$$(b) \text{ Turning points where } \sin x = \pm 1: (90^\circ, 1) \text{ and } (270^\circ, -1)$$

B1

$$\text{Range: } y \leq -1 \text{ or } y \geq 1$$

B1

Answer: $(90^\circ, 1), (270^\circ, -1); y \leq -1$ or $y \geq 1$

2 marks

8 (3 marks)

$$\frac{1}{\cos x} = 2, \text{ so } \cos x = \frac{1}{2}$$

M1

$$x = 60^\circ$$

A1

$$x = 360^\circ - 60^\circ = 300^\circ$$

A1

Answer: $x = 60^\circ, 300^\circ$

3 marks

9 (3 marks)

$$\tan x = \frac{1}{2}$$

M1

$$x = 26.6^\circ$$

A1

$$x = 180^\circ + 26.6^\circ = 206.6^\circ$$

A1

Answer: $x = 26.6^\circ, 206.6^\circ$

3 marks

10 (4 marks)

$$(a) \frac{1}{\sin x} \times \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$$

M1 A1

$$(b) \text{numerator} = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

M1

$$\text{Divide by } \sin x: \frac{\sin x}{\cos x} = \tan x$$

A1

Answer: (a) $\sec x$, (b) $\tan x$

4 marks

11 (3 marks)

$$\text{LHS} = \frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x}$$

M1

$$= \frac{\cos^2 x}{\sin x}$$

A1

$$= \cos x \times \frac{\cos x}{\sin x} = \cos x \cot x$$

A1

Answer: proved

3 marks

12 (4 marks)

$$\sec x = \frac{1}{\cos x}; \text{ asymptotes where } \cos x = 0: x = 90^\circ, 270^\circ$$

B1 B1

Turning points at $(0^\circ, 1)$, $(180^\circ, -1)$, $(360^\circ, 1)$

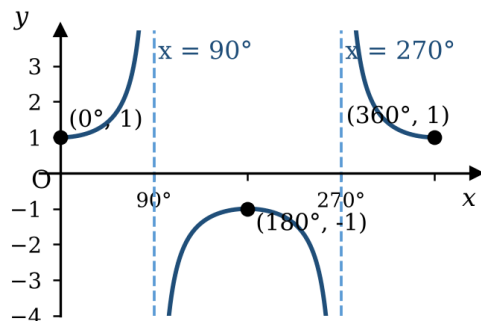
B1

Branches never enter $-1 < y < 1$

B1

Answer: see sketch

4 marks

**13 (9 marks)****(a)** Translation by 1 unit to the left

B1

Stretch of scale factor 2 in the y-direction

B1

Translation by π units upwards

B1

Answer: translate 1 left; stretch factor 2 in y; translate π up

3 marks

(b) Domain: $-1 \leq x + 1 \leq 1$, so $-2 \leq x \leq 0$

B1

End points $(-2, 2(-\frac{\pi}{2}) + \pi) = (-2, 0)$ and $(0, 2\pi)$

B1 B1

Passes through $(-1, \pi)$, with the arcsin shape

B1

Answer: see sketch

4 marks

$$(c) \arcsin(x + 1) = \frac{\pi}{4}$$

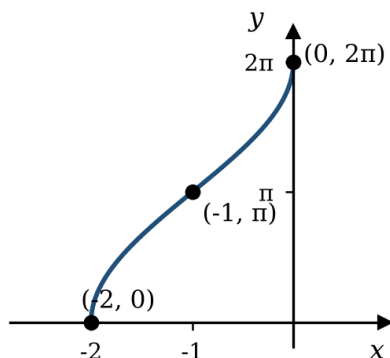
M1

$$x + 1 = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \text{ so } x = \frac{\sqrt{2}}{2} - 1$$

A1

$$\text{Answer: } x = \frac{\sqrt{2}}{2} - 1$$

2 marks

**14 (4 marks)**

Shape: decreasing branches between the asymptotes

Asymptotes $x = 0^\circ, 180^\circ, 360^\circ$ (where $\tan x = 0$)

$\cot x = 0$ when $\tan x$ is undefined: $(90^\circ, 0)$ and $(270^\circ, 0)$

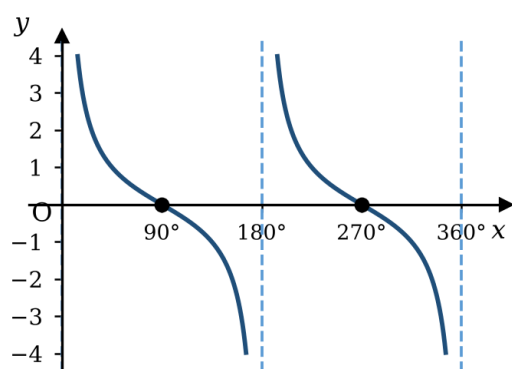
Answer: see sketch

B1

B1

B1 B1

4 marks

**15 (6 marks)**

$$(a) \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

M1 A1

$$= \frac{1}{\sin x \cos x} = \operatorname{cosec} x \sec x$$

A1

Answer: proved

3 marks

$$(b) \text{ Let } t = \tan x: t + \frac{1}{t} = \frac{5}{2}, \text{ so } 2t^2 - 5t + 2 = 0, (2t - 1)(t - 2) = 0$$

M1

$$\tan x = \frac{1}{2} \text{ or } 2$$

A1

$$x = 26.6^\circ \text{ or } 63.4^\circ$$

A1

Answer: $x = 26.6^\circ, 63.4^\circ$

3 marks

16 (5 marks)

$$\sec^2 x = 1 + \tan^2 x, \text{ so } 2 \tan^2 x - 5 \tan x - 3 = 0$$

M1 A1

$$(2 \tan x + 1)(\tan x - 3) = 0$$

M1

$$\tan x = 3: x = 71.6^\circ, 251.6^\circ$$

A1

$$\tan x = -\frac{1}{2}: x = 153.4^\circ, 333.4^\circ$$

A1

Answer: $x = 71.6, 153.4, 251.6, 333.4^\circ$

5 marks

17 (7 marks)

$$(a) \tan(A - B) = \frac{4 - 3}{1 + 4 \times 3}$$

M1

$$= \frac{1}{13}$$

A1

Answer: $\frac{1}{13}$

2 marks

$$(b) \tan(A + B) = \frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}} \quad \text{M1}$$

$$= \frac{7}{10} \div \frac{9}{10} = \frac{7}{9} \quad \text{A1}$$

Answer: $\frac{7}{9}$ 2 marks

$$(c) \frac{3x - x}{1 + 3x^2} = \frac{1}{2} \quad \text{M1}$$

$$4x = 1 + 3x^2, \text{ so } 3x^2 - 4x + 1 = 0 \quad \text{M1}$$

$$(3x - 1)(x - 1) = 0, \text{ so } x = \frac{1}{3} \text{ or } x = 1 \quad \text{A1}$$

Answer: $x = \frac{1}{3} \text{ or } x = 1$ 3 marks

18 (4 marks)

$$(a) 3x = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \quad \text{M1}$$

$$x = -\frac{1}{6} \quad \text{A1}$$

Answer: $x = -\frac{1}{6}$ 2 marks

$$(b) x - 1 = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \text{M1}$$

$$x = 1 + \frac{\sqrt{2}}{2} \quad \text{A1}$$

Answer: $x = 1 + \frac{\sqrt{2}}{2}$ 2 marks

19 (5 marks)

$$(a) \text{ As } x \rightarrow \pm\infty, \tan^{-1}x \rightarrow \pm\frac{\pi}{2} \quad \text{B1}$$

$$y = \frac{\pi}{2} \text{ and } y = -\frac{\pi}{2} \quad \text{B1}$$

Answer: $y = \pm\frac{\pi}{2}$ 2 marks

$$(b) \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}, \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}, \text{ sum} = 0 \quad \text{M1 A1}$$

The graph has rotational symmetry about O (it is an odd function) E1

Answer: 0 3 marks

20 (4 marks)

$$\text{LHS} = \frac{1 + \sin x}{\cos x} \quad \text{M1}$$

$$= \frac{(1 + \sin x)(1 - \sin x)}{\cos x (1 - \sin x)} \quad \text{M1}$$

$$= \frac{1 - \sin^2 x}{\cos x (1 - \sin x)} = \frac{\cos x}{1 - \sin x} \quad \text{A1 A1}$$

Answer: proved 4 marks

21 (7 marks)

$$\text{Use } \cot^2 x = \operatorname{cosec}^2 x - 1: 2 \operatorname{cosec}^2 x - 2 = \operatorname{cosec} x + 1 \quad \text{M1 A1}$$

$$2 \operatorname{cosec}^2 x - \operatorname{cosec} x - 3 = 0 \quad \text{A1}$$

$$(2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 1) = 0 \quad \text{M1}$$

$$\operatorname{cosec} x = \frac{3}{2}: \sin x = \frac{2}{3}, x = 41.8^\circ, 138.2^\circ \quad \text{M1 A1}$$

$$\operatorname{cosec} x = -1: \sin x = -1, x = 270^\circ \quad \text{A1}$$

Answer: $x = 41.8^\circ, 138.2^\circ, 270^\circ$ 7 marks

22 (8 marks)**(a)** Reflection of the principal part of $y = \sin x$ in the line $y = x$: increasing, through the origin B1Domain $-1 \leq x \leq 1$, range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ B1End points $(-1, -\frac{\pi}{2})$ and $(1, \frac{\pi}{2})$ B1**Answer: see sketch** 3 marks

(b) $2x - 1 = \sin \frac{\pi}{6} = \frac{1}{2}$ M1

$$x = \frac{3}{4}$$
 A1

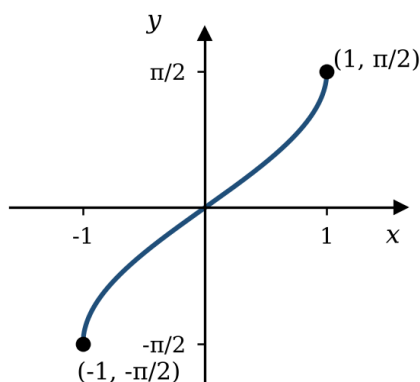
Answer: $x = \frac{3}{4}$ 2 marks

(c) Let $\theta = \sin^{-1} \frac{5}{13}$, so $\sin \theta = \frac{5}{13}$ and θ is acute B1

$$\cos \theta = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$$
 M1

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{5}{12}$$
 A1

Answer: $\frac{12}{13}$ and $\frac{5}{12}$ 3 marks

**23 (9 marks)****(a)** Let $y = 1 + 3 \sin 2x$, so $\sin 2x = \frac{y-1}{3}$ M1

$$2x = \arcsin\left(\frac{y-1}{3}\right), \text{ so } x = \frac{1}{2} \arcsin\left(\frac{y-1}{3}\right)$$
 M1

$$f^{-1}(x) = \frac{1}{2} \arcsin\left(\frac{x-1}{3}\right)$$
 A1

Answer: $f^{-1}(x) = \frac{1}{2} \arcsin\left(\frac{x-1}{3}\right)$ 3 marks

(b) Domain of f^{-1} = range of f : $-2 \leq x \leq 4$ B1

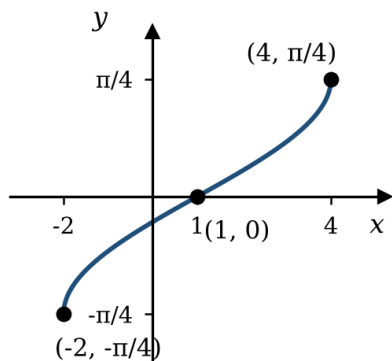
Range of f^{-1} = domain of f : $-\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$ B1

End points $(-2, -\frac{\pi}{4})$ and $(4, \frac{\pi}{4})$ B1Crosses the axis at $(1, 0)$, with the arcsin shape B1**Answer: see sketch** 4 marks

(c) $\frac{1}{2} \arcsin\left(\frac{x-1}{3}\right) = \frac{\pi}{12}$, so $\frac{x-1}{3} = \sin \frac{\pi}{6} = \frac{1}{2}$ M1

$$x = \frac{5}{2}$$
 A1

Answer: $x = \frac{5}{2}$ 2 marks

**24 (7 marks)**

(a) $\sin \frac{5\pi}{6} = \frac{1}{2}$

M1

$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$; the range of $\sin^{-1}$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and $\frac{5\pi}{6}$ lies outside it

A1 E1

Answer: $\frac{\pi}{6}$

3 marks

(b) $\tan \frac{3\pi}{4} = -1$

M1

$\tan^{-1}(-1) = -\frac{\pi}{4}$

A1

Answer: $-\frac{\pi}{4}$

2 marks

(c) Let $\cos \theta = \frac{3}{5}$ with θ in $[0, \pi]$, so $\sin \theta \geq 0$

M1

$\sin \theta = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

A1

Answer: $\frac{4}{5}$

2 marks

25 (7 marks)

(a) $\sec x = \frac{1}{2}$ means $\cos x = 2$, which is impossible

E1

Answer: $\cos x = 2$ impossible

1 mark

(b) Solutions exist when $|\cos x| = \frac{1}{|k|} \leq 1$, so $|k| \geq 1$

M1 A1

Answer: $k \leq -1$ or $k \geq 1$

2 marks

(c) $k = 2$: $\cos x = \frac{1}{2}$, $x = 60^\circ, 300^\circ$

B1 B1

$k = -2$: $\cos x = -\frac{1}{2}$, $x = 120^\circ, 240^\circ$

B1 B1

Answer: $k = 2$: $60^\circ, 300^\circ$; $k = -2$: $120^\circ, 240^\circ$

4 marks

26 (7 marks)

(a) Period 180° ; asymptotes where $\sin 2x = 0$: $x = 0^\circ, 90^\circ, 180^\circ$

B1 B1

Minimum value of the upper branch is 3 at $2x = 90^\circ$: $(45^\circ, 3)$

B1

Maximum of the lower branch is -3 at $2x = 270^\circ$: $(135^\circ, -3)$

B1

Answer: see sketch

4 marks

(b) $3 \operatorname{cosec} 2x = 6$, so $\sin 2x = \frac{1}{2}$

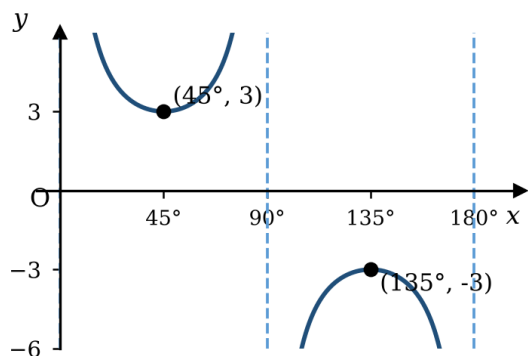
M1

$2x = 30^\circ$ or 150° , $x = 15^\circ$ or 75°

A1 A1

Answer: $x = 15^\circ, 75^\circ$

3 marks



27 (10 marks)

(a) Expand: $\sec^2 x - \tan^2 x$

M1

Use $1 + \tan^2 x = \sec^2 x$

M1

so $\sec^2 x - \tan^2 x = 1$

A1

Answer: shown

3 marks

(b) From (a): $\sec x - \tan x = \frac{1}{2}$

M1 A1

Adding the equations: $2 \sec x = \frac{5}{2}$, so $\sec x = \frac{5}{4}$

M1

$\cos x = \frac{4}{5}$ and $\tan x = \sec x - \frac{1}{2} = \frac{3}{4}$, which is positive, so x is in the first quadrant

A1

$x = 36.9^\circ$ (the solution 323.1° has $\tan x < 0$ and is rejected)

A1

Answer: $x = 36.9^\circ$

5 marks

(c) If $\sec x + \tan x = 0$, then the product in (a) would be 0

M1

but the product is always 1, which is a contradiction

E1

Answer: product is always 1

2 marks

28 (12 marks)

(a) Let $\theta = \tan^{-1} x$, so $\tan \theta = x$ and $0 < \theta < \frac{\pi}{2}$

M1

Then $\tan(\frac{\pi}{2} - \theta) = \cot \theta = \frac{1}{x}$

M1 A1

and $\frac{\pi}{2} - \theta$ is in the range of $\tan^{-1}$, so $\tan^{-1}(\frac{1}{x}) = \frac{\pi}{2} - \theta$, giving the result

E1

Answer: proved

4 marks

(b) Let $A = \tan^{-1} 2$, $B = \tan^{-1} 3$. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

M1

$= \frac{2 + 3}{1 - 6} = -1$

A1

$A + B$ is between $\frac{\pi}{2}$ and π (as $A, B > \frac{\pi}{4}$)

M1

so $A + B = \frac{3\pi}{4}$

A1

Answer: proved

4 marks

(c) Take $\tan$ of both sides: $\frac{x + 2x}{1 - 2x^2} = 1$

M1

$3x = 1 - 2x^2$, so $2x^2 + 3x - 1 = 0$

A1

$x = \frac{-3 \pm \sqrt{17}}{4}$

M1

$x > 0$, so $x = \frac{-3 + \sqrt{17}}{4}$

A1

Answer: $x = \frac{-3 + \sqrt{17}}{4}$

4 marks

29 (12 marks)

(a) Let $y = \sin^{-1}x$, so $x = \sin y$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

B1

$$x = \sin y = \cos\left(\frac{\pi}{2} - y\right)$$

M1

$\frac{\pi}{2} - y$ lies in $[0, \pi]$, the range of $\cos^{-1}$, so $\cos^{-1}x = \frac{\pi}{2} - y$

M1 E1

$$\text{Hence } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

A1

Answer: proved

5 marks

(b) $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

M1

$\sin^{-1}x = \pi - 2 \sin^{-1}x$, so $\sin^{-1}x = \frac{\pi}{3}$

M1 A1

$$x = \frac{\sqrt{3}}{2}$$

A1

Answer: $x = \frac{\sqrt{3}}{2}$

4 marks

(c) $2 \sin^{-1}x + \frac{\pi}{2} = \frac{5\pi}{6}$

M1

$$\sin^{-1}x = \frac{\pi}{6}, x = \frac{1}{2}$$

A1 A1

Answer: $x = \frac{1}{2}$

3 marks

30 (10 marks)

(a) Let $\theta = \arctan x$, so $\tan \theta = x$

B1

$$\text{Then } \sin \theta = \frac{x}{\sqrt{1+x^2}} \text{ and } \cos \theta = \frac{1}{\sqrt{1+x^2}}$$

M1 A1

$$\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2x}{1+x^2}$$

A1

Answer: shown

4 marks

(b) $\frac{2x}{1+x^2} = \frac{24}{25}$, so $24x^2 - 50x + 24 = 0$

M1

$$12x^2 - 25x + 12 = 0, (3x - 4)(4x - 3) = 0$$

M1

$$x = \frac{4}{3} \text{ or } x = \frac{3}{4}$$

A1

Answer: $x = \frac{4}{3}$ or $x = \frac{3}{4}$

3 marks

(c) $1 - \frac{2x}{1+x^2} = \frac{(1-x)^2}{1+x^2} \geq 0$

M1 A1

Equality when $x = 1$

B1

Answer: $x = 1$

3 marks

Compound and Double Angle Formulae**1 (3 marks)**

$$\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

M1

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

M1

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

A1

Answer: shown

3 marks

2 (3 marks)

$$\cos \theta = \frac{4}{5} \text{ (3, 4, 5 triangle)} \quad \text{B1}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25} \quad \text{M1}$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta = 1 - \frac{18}{25} = \frac{7}{25} \quad \text{A1}$$

$$\text{Answer: } \sin 2\theta = \frac{24}{25}, \cos 2\theta = \frac{7}{25} \quad \text{3 marks}$$

3 (2 marks)

$$\cos 2x = 1 - 2 \sin^2 x \quad \text{M1}$$

$$= 1 - \frac{2}{9} = \frac{7}{9} \quad \text{A1}$$

$$\text{Answer: } \frac{7}{9} \quad \text{2 marks}$$

4 (3 marks)

$$\text{(a) } 2 \sin 20^\circ \cos 20^\circ = \sin 40^\circ \quad \text{B1}$$

$$\text{(b) } \cos^2 A - \sin^2 A = \cos 2A, \text{ so } = \cos 30^\circ \quad \text{M1}$$

$$= \frac{\sqrt{3}}{2} \quad \text{A1}$$

$$\text{Answer: (a) } \sin 40^\circ, \text{ (b) } \frac{\sqrt{3}}{2} \quad \text{3 marks}$$

5 (2 marks)

$$\sin x \cos 30^\circ + \cos x \sin 30^\circ \quad \text{M1}$$

$$= \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \quad \text{A1}$$

$$\text{Answer: } \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \quad \text{2 marks}$$

6 (3 marks)

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \quad \text{M1}$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{M1 A1}$$

$$\text{Answer: shown} \quad \text{3 marks}$$

7 (5 marks)

$$\text{(a) } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{2 + \frac{1}{3}}{1 - \frac{2}{3}} \quad \text{M1}$$

$$= \frac{7}{3} \div \frac{1}{3} = 7 \quad \text{A1}$$

$$\text{(b) } \tan(A - B) = \frac{2 - \frac{1}{3}}{1 + \frac{2}{3}} = \frac{5}{3} \div \frac{5}{3} = 1 \quad \text{M1 A1}$$

$$A - B \text{ is between } -90^\circ \text{ and } 90^\circ \text{ and } \tan(A - B) = 1, \text{ so } A - B = 45^\circ \quad \text{A1}$$

$$\text{Answer: (a) } 7; \text{ (b) } \tan(A - B) = 1, \text{ so } A - B = 45^\circ \quad \text{5 marks}$$

8 (5 marks)

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{12}{13} \quad \text{B1}$$

$$\text{(a) } \cos(A + B) = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{48 - 15}{65} = \frac{33}{65} \quad \text{M1 A1}$$

$$\text{(b) } \sin(A - B) = \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13} = \frac{36 - 20}{65} = \frac{16}{65} \quad \text{M1 A1}$$

$$\text{Answer: (a) } \frac{33}{65}, \text{ (b) } \frac{16}{65} \quad \text{5 marks}$$

9 (3 marks)

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \quad \text{B1}$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \quad \text{B1}$$

$$\text{Adding: } 2 \sin x \cos y \quad \text{A1}$$

Answer: proved

3 marks

10 (4 marks)

$$2 \sin x \cos x - \cos x = 0 \quad \text{M1}$$

$$\cos x (2 \sin x - 1) = 0 \quad \text{M1}$$

$$\cos x = 0: x = 90^\circ; \sin x = \frac{1}{2}: x = 30^\circ, 150^\circ \quad \text{A1 A1}$$

Answer: $x = 30^\circ, 90^\circ, 150^\circ$

4 marks

11 (3 marks)

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta) \quad \text{M1 A1}$$

$$\text{Cancel } (\cos\theta - \sin\theta): \cos\theta + \sin\theta \quad \text{A1}$$

Answer: shown

3 marks

12 (5 marks)

$$\text{The left-hand side is } 4 \sin(2x + x) = 4 \sin 3x \quad \text{M1 A1}$$

$$\sin 3x = \frac{3}{4}, \text{ and } 0^\circ \leq 3x \leq 540^\circ \quad \text{M1}$$

$$3x = 48.59^\circ, 131.41^\circ, 408.59^\circ, 491.41^\circ \quad \text{A1}$$

$$x = 16.2^\circ, 43.8^\circ, 136.2^\circ, 163.8^\circ \quad \text{A1}$$

Answer: $x = 16.2^\circ, 43.8^\circ, 136.2^\circ, 163.8^\circ$

5 marks

13 (6 marks)

$$\text{(a) } \cos\theta = -\frac{40}{41} \text{ (}\theta \text{ is obtuse) and } \sin\varphi = -\frac{4}{5} \text{ (third quadrant)} \quad \text{B1 B1}$$

$$\cos(\theta + \varphi) = \cos\theta \cos\varphi - \sin\theta \sin\varphi = \left(-\frac{40}{41}\right)\left(-\frac{3}{5}\right) - \left(\frac{9}{41}\right)\left(-\frac{4}{5}\right) \quad \text{M1}$$

$$= \frac{120 + 36}{205} = \frac{156}{205} \quad \text{A1}$$

Answer: $\frac{156}{205}$

4 marks

$$\text{(b) } \sin(\theta - \varphi) = \sin\theta \cos\varphi - \cos\theta \sin\varphi = \left(\frac{9}{41}\right)\left(-\frac{3}{5}\right) - \left(-\frac{40}{41}\right)\left(-\frac{4}{5}\right) \quad \text{M1}$$

$$= \frac{-27 - 160}{205} = -\frac{187}{205} \quad \text{A1}$$

Answer: $-\frac{187}{205}$

2 marks

14 (5 marks)

$$\text{Use } \cos 2x = 2 \cos^2 x - 1: 2 \cos^2 x - 1 = 3 \cos x + 1 \quad \text{M1}$$

$$2 \cos^2 x - 3 \cos x - 2 = 0 \quad \text{A1}$$

$$(2 \cos x + 1)(\cos x - 2) = 0 \quad \text{M1}$$

$$\cos x = 2 \text{ is impossible; } \cos x = -\frac{1}{2} \quad \text{A1}$$

$$x = 120^\circ, 240^\circ \quad \text{A1}$$

Answer: $x = 120^\circ, 240^\circ$

5 marks

15 (7 marks)

$$\text{(a) } \sin 3x = \sin 2x \cos x + \cos 2x \sin x \quad \text{M1}$$

$$= 2 \sin x \cos^2 x + (1 - 2 \sin^2 x) \sin x \quad \text{M1 A1}$$

$$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x \quad \text{M1}$$

$$= 3 \sin x - 4 \sin^3 x \quad \text{A1}$$

Answer: proved

5 marks

$$\text{(b) LHS} = \sin 90^\circ = 1 \quad \text{B1}$$

$$\text{RHS} = 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{1}{2} = 1$$

B1

Answer: both sides equal 1

2 marks

16 (4 marks)

$$\sin 2x = 2 \sin x \cos x$$

B1

$$1 + \cos 2x = 1 + 2 \cos^2 x - 1 = 2 \cos^2 x$$

M1 A1

$$\frac{2 \sin x \cos x}{2 \cos^2 x} = \tan x$$

A1

Answer: proved

4 marks

17 (4 marks)

$$\frac{\tan x + 1}{1 - \tan x} = 3$$

M1

$$\tan x + 1 = 3 - 3 \tan x, \text{ so } \tan x = \frac{1}{2}$$

M1 A1

$$x = 26.6^\circ, 206.6^\circ$$

A1

Answer: $x = 26.6^\circ, 206.6^\circ$

4 marks

18 (4 marks)

$$\text{The perpendicular from A bisects BC, so } \sin \theta = \frac{6}{10} = \frac{3}{5}$$

M1 A1

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

M1

$$= 1 - \frac{18}{25} = \frac{7}{25}$$

A1

$$\text{Answer: } \sin \theta = \frac{3}{5}; \cos 2\theta = \frac{7}{25}$$

4 marks

19 (7 marks)

$$\text{(a) } \sin \theta \cos \alpha + \cos \theta \sin \alpha = 2 \sin \theta$$

M1

$$\text{Divide by } \cos \theta: \tan \theta \cos \alpha + \sin \alpha = 2 \tan \theta$$

M1

$$\tan \theta (2 - \cos \alpha) = \sin \alpha$$

M1

$$\tan \theta = \frac{\sin \alpha}{2 - \cos \alpha}$$

A1

Answer: shown

4 marks

$$\text{(b) } \tan \theta = \frac{\sin 60^\circ}{2 - \cos 60^\circ} = \frac{\sqrt{3}}{2/3/2} = \frac{1}{\sqrt{3}}$$

M1 A1

$$\theta = 30^\circ \text{ or } 210^\circ$$

A1

Answer: $\theta = 30^\circ, 210^\circ$

3 marks

20 (9 marks)

$$\text{(a) } \tan 55^\circ = \tan(45^\circ + 10^\circ) = \frac{\tan 45^\circ + \tan 10^\circ}{1 - \tan 45^\circ \tan 10^\circ}$$

M1

$$= \frac{1+t}{1-t}$$

A1

$$\text{Answer: } \frac{1+t}{1-t}$$

2 marks

$$\text{(b) } \tan 35^\circ = \tan(45^\circ - 10^\circ) = \frac{1-t}{1+t}$$

M1 A1

$$\tan 55^\circ \tan 35^\circ = \frac{1+t}{1-t} \times \frac{1-t}{1+t} = 1$$

A1

$$\text{Answer: } \frac{1-t}{1+t}; \text{ shown}$$

3 marks

$$\text{(c) } 3 \cos \theta \cos 10^\circ - 3 \sin \theta \sin 10^\circ = 2 \sin \theta \cos 10^\circ - 2 \cos \theta \sin 10^\circ$$

M1

$$\text{Divide by } \cos \theta \cos 10^\circ: 3 - 3t \tan \theta = 2 \tan \theta - 2t$$

M1 A1

$$3 + 2t = \tan \theta (2 + 3t), \text{ so } \tan \theta = \frac{3+2t}{2+3t}$$

A1

Answer: shown

4 marks

21 (9 marks)

(a) $\sin x \cos 30^\circ + \cos x \sin 30^\circ = 2 \cos x$

M1

$$\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = 2 \cos x, \text{ so } \frac{\sqrt{3}}{2} \sin x = \frac{3}{2} \cos x$$

M1 A1

$$\tan x = \sqrt{3}$$

A1

$$x = 60^\circ, 240^\circ$$

A1

Answer: $x = 60^\circ, 240^\circ$

5 marks

(b) $\cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$

M1

$$= 2 \cos A \cos B$$

A1

Answer: proved

2 marks

(c) Take $A = 45^\circ$, $B = 30^\circ$: $\cos 75^\circ + \cos 15^\circ = 2 \cos 45^\circ \cos 30^\circ$

M1

$$= 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$$

A1

Answer: shown

2 marks

22 (8 marks)

(a) $\frac{2 \tan x}{1 - \tan^2 x} = 3 \tan x$

M1

$$2 \tan x = 3 \tan x(1 - \tan^2 x) = 3 \tan x - 3 \tan^3 x$$

M1

$$3 \tan^3 x - \tan x = 0, \text{ so } \tan x (3 \tan^2 x - 1) = 0$$

A1

Answer: shown

3 marks

(b) $\tan x = 0$: $x = 0^\circ, 180^\circ$

B1

$$\tan^2 x = \frac{1}{3}, \text{ so } \tan x = \pm \frac{1}{\sqrt{3}}$$

M1

$$x = 30^\circ, 150^\circ$$

A1

Answer: $x = 0^\circ, 30^\circ, 150^\circ, 180^\circ$

3 marks

(c) When $x = 45^\circ$, $\tan 2x = \tan 90^\circ$, which is undefined

M1

so the equation has no meaning there (the left side cannot equal 3)

E1

Answer: $\tan 90^\circ$ is undefined

2 marks

23 (6 marks)

(a) $1 - \cos 2x = 2 \sin^2 x$, $\sin 2x = 2 \sin x \cos x$

M1 A1

$$\frac{2 \sin^2 x}{2 \sin x \cos x} = \tan x$$

A1

Answer: proved

3 marks

(b) $x = 22.5^\circ$: $\tan 22.5^\circ = \frac{1 - \cos 45^\circ}{\sin 45^\circ} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

M1 A1

$$= \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1$$

A1

Answer: shown

3 marks

24 (6 marks)

(a) $2 \sin x \cos x = \sin 2x$

M1

$$4 \sin x \cos x \cos 2x = 2 \sin 2x \cos 2x = \sin 4x$$

M1 A1

Answer: shown

3 marks

(b) $\sin 4x = \frac{1}{2}$, $0^\circ \leq 4x \leq 720^\circ$

M1

$$4x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

A1

$$x = 7.5^\circ, 37.5^\circ, 97.5^\circ, 127.5^\circ$$

A1

Answer: $x = 7.5^\circ, 37.5^\circ, 97.5^\circ, 127.5^\circ$

3 marks

25 (5 marks)

(a) Take $A = B = 90^\circ$: $\sin 180^\circ = 0$ but $\sin 90^\circ + \sin 90^\circ = 2$

M1 A1

Answer: false, $A = B = 90^\circ$

2 marks

(b) Take $x = 0^\circ$: $\cos 0^\circ = 1$ and $2 \cos 0^\circ = 2$

M1 A1

Answer: false, $x = 0^\circ$ **2 marks****(c) True: this is the double angle formula for sine****B1****Answer: true****1 mark****26 (7 marks)**

(a) $\cos 2x = 1 - 2 \sin^2 x$

B1

$1 - 2 \sin^2 x + 5 \sin x + 2 = 0$, so $2 \sin^2 x - 5 \sin x - 3 = 0$

M1 A1**Answer: shown****3 marks**

(b) $(2 \sin x + 1)(\sin x - 3) = 0$

M1

$\sin x = -\frac{1}{2}$ ($\sin x = 3$ rejected)

A1

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

A1 A1

Answer: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

4 marks**27 (12 marks)**

(a) $\cos 4x = 2 \cos^2 2x - 1$

M1

$= 2(2 \cos^2 x - 1)^2 - 1$

M1

$= 8 \cos^4 x - 8 \cos^2 x + 2 - 1 = 8 \cos^4 x - 8 \cos^2 x + 1$

A1**Answer: proved****3 marks****(b) The equation is $\cos 4x = 0$** **M1**

$4x = 90^\circ, 270^\circ, 450^\circ, 630^\circ, 810^\circ, 990^\circ, 1170^\circ, 1350^\circ$

M1

$x = 22.5^\circ, 67.5^\circ, 112.5^\circ, 157.5^\circ, 202.5^\circ, 247.5^\circ, 292.5^\circ, 337.5^\circ$

A1

Answer: $x = 22.5^\circ, 67.5^\circ, 112.5^\circ, 157.5^\circ, 202.5^\circ, 247.5^\circ, 292.5^\circ, 337.5^\circ$

3 marks

(c) $\frac{\sqrt{2}}{2} = 2 \cos^2 22.5^\circ - 1$

M1

$\cos^2 22.5^\circ = \frac{2 + \sqrt{2}}{4}$

A1

$\cos 22.5^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}$ (positive as 22.5° is acute)

A1

Answer: $\frac{\sqrt{2 + \sqrt{2}}}{2}$

3 marks

(d) $u = \frac{8 \pm \sqrt{64 - 32}}{16} = \frac{2 \pm \sqrt{2}}{4}$

M1

From (b), $u = \cos^2 x$ takes the values $\cos^2 22.5^\circ = \frac{2 + \sqrt{2}}{4}$ and $\cos^2 67.5^\circ$

M1

so $\cos^2 67.5^\circ = \frac{2 - \sqrt{2}}{4}$ and $\cos 67.5^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2}$

A1**Answer: shown****3 marks****28 (10 marks)**

(a) $A + B = \pi - C$, so $\tan(A + B) = -\tan C$

M1

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

M1

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$

A1

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

A1**Answer: proved****4 marks**

(b) $2 + 3 + \tan C = 6 \tan C$

M1

$\tan C = 1$

A1

Answer: $\tan C = 1$

2 marks

(c) $\tan A = 2$ gives $\sin A = \frac{2}{\sqrt{5}}$; $\tan B = 3$ gives $\sin B = \frac{3}{\sqrt{10}}$; $\tan C = 1$ gives $\sin C = \frac{1}{\sqrt{2}}$

B1 B1

Sine rule: $a : b : c = \sin A : \sin B : \sin C = \frac{2}{\sqrt{5}} : \frac{3}{\sqrt{10}} : \frac{1}{\sqrt{2}}$

M1

Multiply by $\sqrt{10}$: $2\sqrt{2} : 3 : \sqrt{5}$

A1

Answer: $2\sqrt{2} : 3 : \sqrt{5}$

4 marks

29 (8 marks)

$$(a) \cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$$

M1

$$= (2 \cos^2 x - 1) \cos x - 2 \sin^2 x \cos x$$

M1 A1

$$= 2 \cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x = 4 \cos^3 x - 3 \cos x$$

A1

Answer: proved

4 marks

$$(b) \text{ Put } x = 20^\circ: \cos 60^\circ = \frac{1}{2} = 4c^3 - 3c$$

M1

$$8c^3 - 6c - 1 = 0$$

A1

Answer: shown

2 marks

$$(c) \text{ For a cubic with leading coefficient 8, the product of the roots is } -\frac{(-1)}{8}$$

M1

$$= \frac{1}{8}$$

A1

$$\text{Answer: } \frac{1}{8}$$

2 marks

30 (10 marks)

$$(a) 108^\circ = 180^\circ - 72^\circ, \text{ so } \cos 108^\circ = -\cos 72^\circ$$

E1

$$\cos 108^\circ = \cos(3 \times 36^\circ) = 4c^3 - 3c \text{ and } \cos 72^\circ = \cos(2 \times 36^\circ) = 2c^2 - 1$$

M1 A1

$$\text{So } 4c^3 - 3c = -(2c^2 - 1)$$

A1

Answer: shown

4 marks

$$(b) \text{ Expand } (c + 1)(4c^2 - 2c - 1) = 4c^3 - 2c^2 - c + 4c^2 - 2c - 1 = 4c^3 + 2c^2 - 3c - 1$$

M1

$$c = -1 \text{ is not possible } (\cos 36^\circ > 0), \text{ so } 4c^2 - 2c - 1 = 0, c = \frac{2 \pm \sqrt{4 + 16}}{8}$$

M1 A1

$$\cos 36^\circ > 0 \text{ so } c = \frac{1 + \sqrt{5}}{4}$$

A1

$$\text{Answer: } \cos 36^\circ = \frac{1 + \sqrt{5}}{4}$$

4 marks

$$(c) \cos 72^\circ = 2c^2 - 1$$

M1

$$= 2 \times \frac{6 + 2\sqrt{5}}{16} - 1 = \frac{3 + \sqrt{5}}{4} - 1 = \frac{\sqrt{5} - 1}{4}$$

A1

Answer: shown

2 marks

R Formula**1 (3 marks)**

$$R \cos \alpha = 5 \text{ and } R \sin \alpha = 12$$

M1

$$R = \sqrt{5^2 + 12^2} = 13$$

A1

$$\tan \alpha = \frac{12}{5}, \text{ so } \alpha = 67.4^\circ$$

A1

$$\text{Answer: } 13 \sin(x + 67.4^\circ)$$

3 marks

2 (3 marks)

$$R \cos(x + \alpha) = R \cos x \cos \alpha - R \sin x \sin \alpha$$

M1

$$R \cos \alpha = 1, R \sin \alpha = 1, \text{ so } R = \sqrt{2}$$

A1

$$\tan \alpha = 1, \text{ so } \alpha = 45^\circ; \cos x - \sin x = \sqrt{2} \cos(x + 45^\circ)$$

A1

$$\text{Answer: } \sqrt{2} \cos(x + 45^\circ)$$

3 marks

3 (3 marks)

$$R = \sqrt{3^2 + 4^2} = 5$$

B1

$$R \cos \alpha = 3, R \sin \alpha = 4, \text{ so } \tan \alpha = \frac{4}{3}$$

M1

$$\alpha = 53.13^\circ$$

A1

$$\text{Answer: } 5 \sin(x + 53.13^\circ)$$

3 marks

4 (3 marks)

$$R = \sqrt{3 + 1} = 2$$

B1

$$R \cos \alpha = \sqrt{3}, R \sin \alpha = 1, \text{ so } \tan \alpha = \frac{1}{\sqrt{3}}, \alpha = 30^\circ$$

M1 A1

Answer: $2 \cos(x - 30^\circ)$

3 marks

5 (2 marks)

$$R = \sqrt{12^2 + 5^2}$$

$$= 13$$

M1

A1

Answer: 13

2 marks

6 (2 marks)Smallest value of $6 \sin x - 8 \cos x$ is -10

M1

Smallest value = $7 - 10 = -3$

A1

Answer: -3

2 marks

7 (4 marks)

$$(a) R \sin(x - \alpha) = R \sin x \cos \alpha - R \cos x \sin \alpha$$

M1

$$R \cos \alpha = 3, R \sin \alpha = 2, \text{ so } R = \sqrt{13}$$

A1

$$\tan \alpha = \frac{2}{3}, \alpha = 33.7^\circ$$

A1

$$(b) \text{ Greatest value} = R = \sqrt{13}$$

B1

Answer: (a) $\sqrt{13} \sin(x - 33.7^\circ)$; (b) $\sqrt{13}$

4 marks

8 (3 marks)

$$5 \sin(x + 53.13^\circ) = 2, \text{ so } \sin(x + 53.13^\circ) = 0.4$$

M1

$$x + 53.13^\circ = 23.58^\circ \text{ or } 156.42^\circ \text{ (or } 383.58^\circ)$$

M1

$$x = 103.3^\circ \text{ or } 330.4^\circ$$

A1

Answer: $x = 103.3^\circ, 330.4^\circ$

3 marks

9 (5 marks)

$$5 \sin x + 12 \cos x = 13 \sin(x + 67.38^\circ)$$

M1 A1

$$\sin(x + 67.38^\circ) = \frac{6}{13}$$

M1

$$x + 67.38^\circ = 27.49^\circ \text{ or } 152.51^\circ$$

A1

$$x = 85.1^\circ \text{ (the first value gives } x < 0; \text{ add } 360^\circ \text{ to get } 320.1^\circ)$$

A1

Answer: $x = 85.1, 320.1^\circ$

5 marks

10 (6 marks)

$$(a) R = \sqrt{2}$$

B1

$$R \cos \alpha = 1, R \sin \alpha = 1, \text{ so } \alpha = \frac{\pi}{4}$$

M1 A1

$$\text{Answer: } \sqrt{2} \sin(x - \frac{\pi}{4})$$

3 marks

$$(b) \sqrt{2} \sin(x - \frac{\pi}{4}) = 1, \sin(x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$$

M1

$$x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$$

A1

$$x = \frac{\pi}{2}, \pi$$

A1

Answer: $x = \frac{\pi}{2}, \pi$

3 marks

11 (5 marks)

$$R = \sqrt{13}, \tan \alpha = \frac{3}{2}, \alpha = 56.31^\circ$$

M1 A1

$$\text{Greatest value} = \sqrt{13} = 3.61$$

B1

$$2x + 56.31^\circ = 90^\circ, \text{ so } x = 16.8^\circ$$

M1 A1

Answer: $\sqrt{13} \sin(2x + 56.31^\circ)$; max 3.61 at $x = 16.8^\circ$

5 marks

12 (7 marks)

(a) $R \cos \alpha = 8, R \sin \alpha = 6$

M1

$$R = \sqrt{64 + 36} = 10$$

A1

$$\tan \alpha = \frac{6}{8}, \alpha = 36.87^\circ$$

A1

Answer: 10 sin(x + 36.87°)

3 marks

(b) Greatest value is 10

B1

when $x + 36.87^\circ = 90^\circ$, so $x = 53.13^\circ$

B1

Answer: 10 when x = 53.13°

2 marks

(c) The denominator is least when $8 \sin x + 6 \cos x = -10$: $15 - 10 = 5$

M1

$$\text{Greatest value} = \frac{20}{5} = 4$$

A1

Answer: 4

2 marks

13 (7 marks)

(a) $R \cos \alpha = 3, R \sin \alpha = 4$

M1

$$R = 5$$

A1

$$\tan \alpha = \frac{4}{3}, \alpha = 53.13^\circ$$

A1

Answer: 5 sin(θ - 53.13°)

3 marks

(b) Maximum $h = 4 + 5 = 9$ m

B1

$$30t - 53.13 = 90, \text{ so } t = 4.77 \text{ hours}$$

B1

Answer: 9 m at t = 4.77

2 marks

(c) $4 + 5 \sin(30t - 53.13^\circ) = 6.5$, so $\sin(30t - 53.13^\circ) = 0.5$

M1

$$30t - 53.13 = 30, \text{ so } t = 2.77 \text{ hours}$$

A1

Answer: t = 2.77

2 marks

14 (10 marks)

(a) $R = \sqrt{4^2 + 3^2} = 5$

B1

$$\tan \alpha = \frac{3}{4}, \text{ so } \alpha = 0.6435$$

M1 A1

Answer: 5 sin(θ - 0.6435)

3 marks

(b) Maximum $T = 15 + 5 = 20$ °C

B1

$$\frac{\pi t}{12} - 0.6435 = \frac{\pi}{2}$$

M1

$$t = 8.458, \text{ which is } 08:27$$

A1

Answer: 20 °C at 08:27

3 marks

(c) $5 \sin(\frac{\pi t}{12} - 0.6435) = 2$, so $\sin(\frac{\pi t}{12} - 0.6435) = 0.4$

M1

$$\frac{\pi t}{12} - 0.6435 = 0.4115 \text{ or } 2.7301$$

M1 A1

$$t = 4.030 \text{ and } t = 12.89, \text{ that is } 04:02 \text{ and } 12:53$$

A1

Answer: 04:02 and 12:53

4 marks

15 (8 marks)

(a) $R = 13$

B1

$$\tan \alpha = \frac{12}{5}, \alpha = 67.38^\circ$$

M1 A1

Answer: 13 sin(x - 67.38°)

3 marks

(b) Maximum $7 + 13 = 20$ when $x - 67.38^\circ = 90^\circ$, $x = 157.38^\circ$

B1 M1 A1

Minimum $7 - 13 = -6$ when $x - 67.38^\circ = 270^\circ$, $x = 337.38^\circ$

B1 A1

Answer: max 20 at 157.38°; min -6 at 337.38°

5 marks

16 (6 marks)

$$2 \sin 2x + \cos 2x = \sqrt{5} \sin(2x + 26.57^\circ), \tan \alpha = \frac{1}{2}$$

M1 A1

$$\sin(2x + 26.57^\circ) = \frac{1}{\sqrt{5}} = 0.4472$$

M1

$$2x + 26.57^\circ = 26.57^\circ, 153.43^\circ, 386.57^\circ$$

M1 A1

$$2x = 0^\circ, 126.87^\circ, 360^\circ, \text{ so } x = 0^\circ, 63.4^\circ, 180^\circ$$

A1

Answer: x = 0°, 63.4°, 180°

6 marks

17 (5 marks)**(a)** Maximum value is 5, reached once

M1 A1

Answer: 1

2 marks

(b) The line $y = 4$ crosses the curve twice

B1

Answer: 2

1 mark

(c) $6 > 5$, the greatest value

B1

Answer: 0

1 mark

(d) The least value, reached once

B1

Answer: 1

1 mark

18 (6 marks)

$$R = \sqrt{4 + 12} = 4, \tan \alpha = \sqrt{3}, \alpha = 60^\circ, \text{ so } y = 4 \sin(x + 60^\circ)$$

M1 A1 A1

Maximum when $x + 60^\circ = 90^\circ$: $(30^\circ, 4)$; minimum when $x + 60^\circ = 270^\circ$: $(210^\circ, -4)$

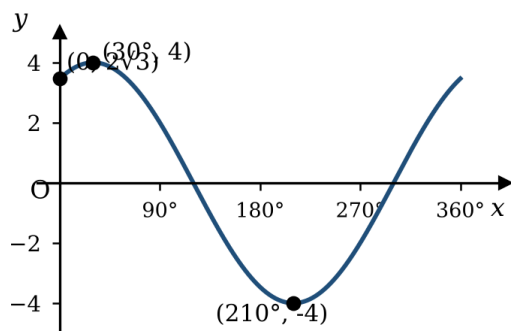
B1 B1

y-intercept: $4 \sin 60^\circ = 2\sqrt{3}$, so $(0, 2\sqrt{3})$

B1

Answer: see sketch

6 marks

**19 (10 marks)****(a)** Multiply both sides by $\sin x$ ($\sin x \neq 0$): $3 \cos x + 2 \sin x = 3$

M1 A1

Answer: shown

2 marks

$$\text{(b) } R = \sqrt{3^2 + 2^2} = \sqrt{13}$$

B1

$$\tan \alpha = \frac{3}{2}, \text{ so } \alpha = 56.31^\circ$$

M1 A1

$$\text{Answer: } \sqrt{13} \sin(x + 56.31^\circ)$$

3 marks

$$\text{(c) } \sin(x + 56.31^\circ) = \frac{3}{\sqrt{13}} = 0.8321$$

M1

$$x + 56.31^\circ = 56.31^\circ \text{ or } 123.69^\circ, \text{ so } x = 0^\circ \text{ or } x = 67.4^\circ$$

M1 A1

 $x = 0^\circ$ is rejected because $\cot x$ and $\operatorname{cosec} x$ are undefined there

E1

$$x = 67.4^\circ$$

A1

Answer: x = 67.4°

5 marks

20 (9 marks)**(a)** $R \cos \alpha = 2$, $R \sin \alpha = 1$

M1

$$R = \sqrt{5}$$

A1

$$\tan \alpha = \frac{1}{2}, \alpha = 26.57^\circ$$

A1

$$\text{Answer: } \sqrt{5} \cos(x + 26.57^\circ)$$

3 marks

$$(b) \sqrt{5} \cos(x + 26.57^\circ) = 1, \text{ so } \cos(x + 26.57^\circ) = \frac{1}{\sqrt{5}} = 0.4472$$

M1

$$x + 26.57^\circ = 63.43^\circ \text{ or } 296.57^\circ$$

A1

$$x = 36.9^\circ \text{ or } 270.0^\circ$$

A1

Answer: $x = 36.9^\circ, 270.0^\circ$

3 marks

$$(c) \text{ Denominator is least when } 2 \cos x - \sin x = -\sqrt{5}, \text{ giving } 3 - \sqrt{5}$$

M1

$$\text{Greatest value} = \frac{1}{3 - \sqrt{5}} = \frac{3 + \sqrt{5}}{(3 - \sqrt{5})(3 + \sqrt{5})}$$

M1

$$= \frac{3 + \sqrt{5}}{4} = \frac{3}{4} + \frac{1}{4}\sqrt{5}$$

A1

$$\text{Answer: } \frac{3}{4} + \frac{1}{4}\sqrt{5}$$

3 marks

21 (9 marks)

$$(a) 2 \sin(x - \frac{\pi}{6}) = 2 \sin x \cos \frac{\pi}{6} - 2 \cos x \sin \frac{\pi}{6}$$

M1

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \sin \frac{\pi}{6} = \frac{1}{2}$$

M1

$$= \sqrt{3} \sin x - \cos x$$

A1

Answer: shown

3 marks

$$(b) 2 \sin(x - \frac{\pi}{6}) = 1, \text{ so } \sin(x - \frac{\pi}{6}) = \frac{1}{2}$$

M1

$$x - \frac{\pi}{6} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

A1

$$x = \frac{\pi}{3} \text{ or } \pi$$

A1

$$\text{Answer: } x = \frac{\pi}{3}, \pi$$

3 marks

$$(c) (i) k = 2 \text{ is the maximum value: 1 solution } (x = \frac{2\pi}{3})$$

B1

$$(ii) 1.5 \text{ is between } -1 \text{ and } 2 \text{ on the rising part and falling part: 2 solutions}$$

B1

$$(iii) \text{ the line } y = -1 \text{ meets the curve at the y-intercept } (x = 0), \text{ at } x = \frac{4\pi}{3} \text{ and at the end point } x =$$

B1

$$2\pi: 3 \text{ solutions}$$

Answer: (i) 1; (ii) 2; (iii) 3

3 marks

22 (9 marks)

$$(a) 3 \sin x + 4 \cos x = 5 \sin(x + 53.13^\circ)$$

M1 A1

$$\text{Denominator ranges from } 6 - 5 = 1 \text{ to } 6 + 5 = 11$$

M1

$$\text{Greatest } f = \frac{1}{1} = 1, \text{ least } f = \frac{1}{11}$$

A1 A1

$$\text{Answer: greatest 1, least } \frac{1}{11}$$

5 marks

$$(b) 6 + 5 \sin(x + 53.13^\circ) = 3, \text{ so } \sin(x + 53.13^\circ) = -\frac{3}{5}$$

M1 A1

$$x + 53.13^\circ = 216.87^\circ \text{ or } 323.13^\circ$$

M1

$$x = 163.7^\circ \text{ or } 270.0^\circ$$

A1

$$\text{Answer: } x = 163.7^\circ, 270.0^\circ$$

4 marks

23 (6 marks)

$$(a) 4 \sin(x + 60^\circ) = 4 \sin x \cos 60^\circ + 4 \cos x \sin 60^\circ$$

M1

$$a = 4 \times \frac{1}{2} = 2$$

A1

$$b = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

A1

$$\text{Answer: } a = 2, b = 2\sqrt{3}$$

3 marks

$$(b) 4 \sin(x + 60^\circ) = 2, \sin(x + 60^\circ) = \frac{1}{2}$$

M1

$$x + 60^\circ = 150^\circ \text{ or } 390^\circ$$

A1

$$x = 90^\circ \text{ or } 330^\circ$$

Answer: $x = 90^\circ, 330^\circ$

A1

3 marks

24 (8 marks)

(a) $\text{LHS} = 0 - 4 = -4$

B1

$\text{RHS} = 5 \sin 53.13^\circ = 4$

B1

Answer: $-4 \neq 4$

2 marks

(b) $R = 5$

B1

$R \cos \alpha = 3, R \sin \alpha = 4, \text{ so } \alpha = 53.13^\circ$

M1

$3 \sin x - 4 \cos x = 5 \sin(x - 53.13^\circ)$

A1

Answer: $5 \sin(x - 53.13^\circ)$

3 marks

(c) $\sin(x - 53.13^\circ) = 0.8$

M1

$x - 53.13^\circ = 53.13^\circ \text{ or } 126.87^\circ$

A1

$x = 106.3^\circ \text{ or } 180.0^\circ$

A1

Answer: $x = 106.3^\circ, 180^\circ$

3 marks

25 (6 marks)

(a) $R = 2$

B1

$\tan \alpha = \frac{1}{\sqrt{3}}, \alpha = \frac{\pi}{6}$

M1 A1

Answer: $2 \sin(x + \frac{\pi}{6})$

3 marks

(b) $\sin(x + \frac{\pi}{6}) = \frac{\sqrt{2}}{2}$

M1

$x + \frac{\pi}{6} = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$

A1

$x = \frac{\pi}{12} \text{ or } \frac{7\pi}{12}$

A1

Answer: $x = \frac{\pi}{12}, \frac{7\pi}{12}$

3 marks

26 (15 marks)

(a) Extend AB beyond B: BC makes an angle of $180^\circ - 120^\circ = 60^\circ$ with this line, so BC makes angle $\theta + 60^\circ$ with the horizontal

M1 A1

$h = 5 \sin \theta + 4 \sin(\theta + 60^\circ)$

M1

$= 5 \sin \theta + 2 \sin \theta + 2\sqrt{3} \cos \theta = 7 \sin \theta + 2\sqrt{3} \cos \theta$

A1

Answer: shown

4 marks

(b) $R = \sqrt{49 + 12} = \sqrt{61}, \tan \alpha = \frac{2\sqrt{3}}{7}, \text{ so } \alpha = 26.33^\circ$

B1 M1 A1

Greatest $h = \sqrt{61} = 7.81 \text{ m}$

A1

when $\theta + 26.33^\circ = 90^\circ, \text{ so } \theta = 63.67^\circ$

M1 A1

Answer: $\sqrt{61} \text{ m} = 7.81 \text{ m}$ when $\theta = 63.67^\circ$

6 marks

(c) $\sqrt{61} \sin(\theta + 26.33^\circ) = 6$

M1

$\theta + 26.33^\circ = 50.19^\circ \text{ or } 129.81^\circ$

M1 A1

$\theta = 23.9^\circ \text{ or } 103.5^\circ$

A1 A1

Answer: $\theta = 23.9^\circ, 103.5^\circ$

5 marks

27 (11 marks)

(a) The horizontal extent of AB is $8 \cos \theta$

M1

The horizontal extent of AD is $6 \sin \theta$, so $W = 8 \cos \theta + 6 \sin \theta$

A1

Answer: shown

2 marks

(b) $R \cos \alpha = 8, R \sin \alpha = 6, R = 10, \alpha = 36.87^\circ$

M1 A1

Greatest width 10 cm when $\theta = \alpha = 36.87^\circ$

A1

Answer: 10 cm when $\theta = 36.87^\circ$

3 marks

(c) $10 \cos(\theta - 36.87^\circ) = 9, \text{ so } \cos(\theta - 36.87^\circ) = 0.9$

M1

$\theta - 36.87^\circ = \pm 25.84^\circ$

A1

$\theta = 62.7^\circ \text{ or } 11.0^\circ$

A1

Answer: $\theta = 11.0^\circ, 62.7^\circ$ **3 marks****(d)** Height $H = 8 \sin \theta + 6 \cos \theta$ **B1**

$$WH = 64 \sin \theta \cos \theta + 48 \cos^2 \theta + 48 \sin^2 \theta + 36 \sin \theta \cos \theta = 48 + 100 \sin \theta \cos \theta = 48 + 50 \sin 2\theta$$

M1Greatest value 98 cm^2 (when $\theta = 45^\circ$)**A1****Answer: 98 cm^2** **3 marks****28 (12 marks)****(a)** $R \sin(x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$ **M1**Compare coefficients: $R \cos \alpha = a$ and $R \sin \alpha = b$ **M1**Squaring and adding: $R^2(\cos^2 \alpha + \sin^2 \alpha) = a^2 + b^2$, so $R = \sqrt{a^2 + b^2}$ **A1**and α satisfies $\tan \alpha = \frac{b}{a}$, with $\cos \alpha = \frac{a}{R}$ and $\sin \alpha = \frac{b}{R}$ **A1****Answer: proved****4 marks****(b)** $2 \sin x + p \cos x$ has greatest value $\sqrt{4 + p^2}$ and least value $-\sqrt{4 + p^2}$ **M1**Solutions exist if $\sqrt{4 + p^2} \geq 3$ **M1** $p^2 \geq 5$, so $p \leq -\sqrt{5}$ or $p \geq \sqrt{5}$ **A1****Answer: $p \leq -\sqrt{5}$ or $p \geq \sqrt{5}$** **3 marks****(c)** $\cos^2 x = \frac{1 + \cos 2x}{2}$ and $2 \sin x \cos x = \sin 2x$ **M1**

$$3 \cos^2 x + 4 \sin x \cos x = \frac{3}{2} + \frac{3}{2} \cos 2x + 2 \sin 2x$$

A1

$$\frac{3}{2} \cos 2x + 2 \sin 2x = R \sin(2x + \alpha) \text{ with } R = \sqrt{\frac{9}{4} + 4} = \frac{5}{2}$$

M1 A1

$$\text{Greatest value} = \frac{3}{2} + \frac{5}{2} = 4$$

A1**Answer: 4****5 marks****29 (9 marks)****(a)** Expand: $a^2 \sin^2 x + 2ab \sin x \cos x + b^2 \cos^2 x$ **M1**plus $a^2 \cos^2 x - 2ab \sin x \cos x + b^2 \sin^2 x$ **M1**

$$\text{Sum} = a^2(\sin^2 x + \cos^2 x) + b^2(\cos^2 x + \sin^2 x) = a^2 + b^2$$

A1**Answer: proved****3 marks****(b)** The second bracket squared is ≥ 0 , so $(a \sin x + b \cos x)^2 \leq a^2 + b^2$ **M1 E1**Equality when $a \cos x - b \sin x = 0$, i.e. $\tan x = \frac{a}{b}$ **A1****Answer: shown****3 marks****(c)** Greatest value $\sqrt{25 + 144} = 13$ **M1 A1**

$$\tan x = \frac{a}{b} = \frac{5}{12}$$

A1**Answer: 13 when $\tan x = \frac{5}{12}$** **3 marks****30 (12 marks)****(a)** $R = 5$, $\tan \alpha = \frac{4}{3}$ **M1**

$$\alpha = 53.13, \text{ so } h = 6 + 5 \sin(30t + 53.13^\circ)$$

A1**Answer: $5 \sin(30t + 53.13^\circ)$** **2 marks****(b)** Maximum depth 11 m**B1**

$$30t + 53.13 = 90, t = 1.229$$

M1 A1 $t = 1 \text{ h } 14 \text{ min}$, that is 01:14**A1****Answer: 11 m at 01:14****4 marks****(c)** $5 \sin(30t + 53.13^\circ) \geq 2$, so $\sin(30t + 53.13^\circ) \geq 0.4$ **M1 A1** $30t + 53.13^\circ$ between 23.58° and 156.42° (plus multiples of 360°)**M1**

$$\text{Each window lasts } \frac{180 - 2 \times 23.58}{30} = 4.428 \text{ hours}$$

M1 A1

The model has period 12 hours, so 24 hours contain exactly two periods: total = $2 \times 4.428 = 8.86$ hours

A1

Answer: 8.86 hours

6 marks

Small Angle Approximations

1 (3 marks)

$$\sin \theta \approx \theta$$

B1

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

B1

$$\tan \theta \approx \theta$$

B1

Answer: $\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{\theta^2}{2}$, $\tan \theta \approx \theta$

3 marks

2 (3 marks)

(a) 3θ

B1

(b) $1 - \frac{(2\theta)^2}{2} = 1 - 2\theta^2$

B1

(c) 5θ

B1

Answer: (a) 3θ , (b) $1 - 2\theta^2$, (c) 5θ

3 marks

3 (3 marks)

$$\cos 0.1 \approx 1 - \frac{0.1^2}{2} = 0.995$$

M1 A1

Calculator: $\cos 0.1 = 0.99500$, so the approximation is accurate to 4 decimal places

B1

Answer: 0.995 (calculator 0.99500)

3 marks

4 (2 marks)

$$4 \sin 2\theta \approx 8\theta \text{ and } 3 \tan \theta \approx 3\theta$$

M1

$$\text{Sum} \approx 11\theta$$

A1

Answer: 11θ

2 marks

5 (4 marks)

$$\text{Estimate} = 0.04$$

B1

$$\tan 0.04 = 0.040021$$

B1

$$\text{Percentage error} = \frac{0.040021 - 0.04}{0.040021} \times 100 = 0.053\%$$

M1 A1

Answer: 0.053%

4 marks

6 (2 marks)

$$1 - \cos \theta \approx 1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$$

M1

$$\text{So } \frac{1 - \cos \theta}{\theta^2} \approx \frac{\theta^2}{2\theta^2} = \frac{1}{2}$$

A1

Answer: shown

2 marks

7 (3 marks)

$$\sin \theta \tan \theta \approx \theta \times \theta = \theta^2$$

B1

$$\cos \theta \sin \theta \approx \left(1 - \frac{\theta^2}{2}\right)\theta$$

M1

$$= \theta - \frac{\theta^3}{2}$$

A1

Answer: shown

3 marks

8 (4 marks)

$$\cos 3\theta \approx 1 - \frac{(3\theta)^2}{2} = 1 - \frac{9\theta^2}{2}$$

M1 A1

$$2 \sin 2\theta \approx 4\theta$$

B1

$$\text{Sum} \approx 1 + 4\theta - \frac{9\theta^2}{2}$$

A1

Answer: shown

4 marks

9 (3 marks)

$$2^\circ = 2 \times \frac{\pi}{180} = \frac{\pi}{90} = 0.034907 \text{ rad}$$

M1

$$\sin 2^\circ \approx 0.03491$$

A1

Calculator: $\sin 2^\circ = 0.03490$, so the estimate is correct to 3 s.f.

B1

Answer: $\sin 2^\circ \approx 0.03491$

3 marks

10 (5 marks)

$$2(1 - \frac{\theta^2}{2}) + 3\theta \approx 2.29$$

M1 A1

$$\theta^2 - 3\theta + 0.29 = 0$$

A1

$$\theta = \frac{3 \pm \sqrt{9 - 1.16}}{2} = 0.1 \text{ or } 2.9; \text{ the small root is } \theta \approx 0.1 \text{ (sqrt} = 2.800)$$

M1 A1

Answer: $\theta \approx 0.1$

5 marks

11 (4 marks)

$$\text{Chord AB} = 2r \sin(\frac{\theta}{2})$$

M1 A1

$$\text{For small } \theta: \sin(\frac{\theta}{2}) \approx \frac{\theta}{2}$$

M1

$$\text{So } AB \approx 2r \times \frac{\theta}{2} = r\theta, \text{ which is the arc length}$$

A1

Answer: $AB \approx r\theta$

4 marks

12 (3 marks)

The diameter is approximately the arc of a circle of radius 384 000 km

M1

$$d \approx r\theta = 384\,000 \times 0.0087$$

M1

$$= 3340 \text{ km}$$

A1

Answer: 3340 km

3 marks

13 (5 marks)

$$\cos 2\theta \approx 1 - \frac{(2\theta)^2}{2} = 1 - 2\theta^2, \text{ so the numerator} \approx 4 - 2\theta^2$$

M1 A1

$$\sin \theta \approx \theta, \text{ so the denominator} \approx 1 + \theta$$

B1

$$(4 - 2\theta^2)(1 + \theta)^{-1} \approx 4(1 - \theta + \theta^2 \dots) - 2\theta^2$$

M1

$$\approx 4 - 4\theta, \text{ ignoring terms in } \theta^2 \text{ and higher}$$

A1

Answer: shown

5 marks

14 (4 marks)

$$4 \cos \theta \approx 4(1 - \frac{\theta^2}{2}) = 4 - 2\theta^2$$

M1 A1

$$3 \sin 2\theta \approx 6\theta$$

B1

$$\text{Sum} \approx 4 + 6\theta - 2\theta^2$$

A1

Answer: shown

4 marks

15 (5 marks)

$$1 - 2\theta^2 + 3\theta \approx 1.1$$

M1 A1

$$2\theta^2 - 3\theta + 0.1 = 0$$

A1

$$\theta = \frac{3 - \sqrt{9 - 0.8}}{4} = 0.0341 \text{ (the larger root is not small)}$$

M1 A1

Answer: $\theta \approx 0.0341$

5 marks

16 (4 marks)

$$\theta \sin 2\theta \approx 2\theta^2 \quad \text{M1}$$

$$1 - \cos 3\theta \approx \frac{9\theta^2}{2} \quad \text{M1 A1}$$

$$\text{Ratio} \approx \frac{\frac{2\theta^2}{\frac{9\theta^2}{2}}}{\frac{4}{9}} = \frac{4}{9} \quad \text{A1}$$

$$\text{Answer: } \frac{4}{9} \quad \text{4 marks}$$

17 (8 marks)

$$\text{(a) Approximation} = 1 - 0.02 = 0.98 \quad \text{B1}$$

$$\cos 0.2 = 0.980067 \quad \text{B1}$$

$$\text{Error} = \frac{0.980067 - 0.98}{0.980067} \times 100 = 0.0068\% \quad \text{M1 A1}$$

$$\text{Answer: } 0.0068\% \quad \text{4 marks}$$

$$\text{(b) Need } \frac{\theta - \sin \theta}{\sin \theta} < 0.01, \text{ i.e. } \frac{\theta}{\sin \theta} < 1.01 \quad \text{M1}$$

$$\text{Test } \theta = 0.24: \frac{\theta}{\sin \theta} = 1.0097, \text{ which is below } 1.01 \quad \text{M1}$$

$$\text{Test } \theta = 0.25: 1.0105 \text{ and } \theta = 0.26: 1.0114, \text{ both above } 1.01 \quad \text{A1}$$

$$\text{The boundary is at about } 0.244, \text{ so the largest value is } \theta = 0.24 \text{ to 2 d.p.} \quad \text{A1}$$

$$\text{Answer: } \theta = 0.24 \text{ (boundary at } 0.244) \quad \text{4 marks}$$

18 (4 marks)

$$\tan \theta = \frac{12}{800} = 0.015 \quad \text{M1}$$

$$\tan \theta \approx \theta, \text{ so } \theta \approx 0.015 \text{ rad} \quad \text{A1}$$

$$\theta \approx 0.015 \times \frac{180}{\pi} = 0.86^\circ \quad \text{M1 A1}$$

$$\text{Answer: } \theta \approx 0.015 \text{ rad} = 0.86^\circ \quad \text{4 marks}$$

19 (4 marks)

$$\text{(a) } \sin 3\theta \approx 3\theta \text{ and } 2 + \cos \theta \rightarrow 3 \quad \text{M1}$$

$$\frac{3\theta}{3\theta} = 1 \quad \text{A1}$$

$$\text{(b) } \tan 4\theta \approx 4\theta \text{ and } \cos \theta \rightarrow 1 \quad \text{M1}$$

$$\frac{4\theta}{2\theta} = 2 \quad \text{A1}$$

$$\text{Answer: (a) 1, (b) 2} \quad \text{4 marks}$$

20 (5 marks)

$$BC^2 = 80^2 + 80^2 - 2 \times 80 \times 80 \cos \theta = 12800(1 - \cos \theta) \quad \text{M1 A1}$$

$$1 - \cos \theta \approx \frac{\theta^2}{2}, \text{ so } BC^2 \approx 6400\theta^2, BC \approx 80\theta \quad \text{M1 A1}$$

$$\theta \approx \frac{2.4}{80} = 0.03 \text{ rad} \quad \text{A1}$$

$$\text{Answer: } \theta \approx 0.03 \quad \text{5 marks}$$

21 (9 marks)

$$\text{(a) Arc} = r\theta = 6371 \times 0.002 = 12.742 \text{ km} \quad \text{M1 A1}$$

$$\text{Chord} = 2r \sin\left(\frac{\theta}{2}\right) \approx 2 \times 6371 \times \frac{0.002}{2} = 12.742 \text{ km (to 5 s.f.)} \quad \text{M1 A1}$$

$$\text{Answer: both } 12.742 \text{ km to 5 s.f.} \quad \text{4 marks}$$

$$\text{(b) chord} \approx 2r\left(\frac{\theta}{2} - \frac{(\theta)^3}{2^3/6}\right) = r\theta - \frac{r\theta^3}{24} \quad \text{M1 A1}$$

$$\text{arc} - \text{chord} \approx \frac{r\theta^3}{24} \quad \text{A1}$$

$$= \frac{6371 \times 0.002^3}{24} = 0.00000212 \text{ km} = 2.1 \text{ mm}$$

M1 A1

Answer: 2.1 mm

5 marks

22 (6 marks)

$$3\left(1 - \frac{\theta^2}{2}\right) + 2\theta \approx 3.185$$

M1 A1

$$3 - \frac{3\theta^2}{2} + 2\theta = 3.185, \text{ so } 3\theta^2 - 4\theta + 0.37 = 0$$

M1 A1

$$\theta = \frac{4 - \sqrt{16 - 4.44}}{6} = 0.1000$$

M1

$$(\sqrt{11.56} = 3.4, \text{ so } \theta = 0.1; \text{ the other root is not small})$$

A1

Answer: $\theta \approx 0.1$

6 marks

23 (7 marks)**(a)** Bob's vertical depth below the pivot is $L \cos \theta$, so rise = $L - L \cos \theta = L(1 - \cos \theta)$

M1 A1

$$1 - \cos \theta \approx \frac{\theta^2}{2}, \text{ so rise} \approx \frac{L\theta^2}{2}$$

A1

Answer: shown

3 marks

$$\text{(b) Approximation} = \frac{1.5 \times 0.12^2}{2} = 0.0108 \text{ m}$$

B1

$$\text{Exact} = 1.5(1 - \cos 0.12) = 0.0108 \text{ m}$$

B1

$$\text{Error} = 0.12\%$$

M1 A1

Answer: approx 0.0108 m, exact 0.0108 m, error 0.12%

4 marks

24 (7 marks)**(a)** The approximation requires θ in radians; $30^\circ = \frac{\pi}{6} \text{ rad} \approx 0.524$

E1

A sine value cannot exceed 1

E1

Answer: θ must be in radians

2 marks

$$\text{(b) } \sin 30^\circ \approx \frac{\pi}{6} = 0.5236$$

M1 A1

$$\text{Error} = \frac{0.5236 - 0.5}{0.5} \times 100 = 4.7\%$$

M1 A1

Answer: 4.7%

4 marks

(c) The error is about 5%, so it is not accurate for angles as large as 30° ; it is only reliable for small angles

E1

Answer: not reliable

1 mark

25 (6 marks)

$$\text{(a) } h = \frac{\pi}{6} - 0.5 = 0.0236$$

B1

$$\sin\left(\frac{\pi}{6} - h\right) = \sin \frac{\pi}{6} \cos h - \cos \frac{\pi}{6} \sin h$$

M1

$$\cos h \approx 1 - \frac{h^2}{2} \text{ and } \sin h \approx h, \text{ which gives the result}$$

A1

Answer: $h = 0.0236$; shown

3 marks

$$\text{(b) } \frac{1}{2}\left(1 - \frac{0.02360^2}{2}\right) - \frac{\sqrt{3}}{2} \times 0.02360 = 0.4794$$

M1 A1

Calculator: $\sin 0.5 = 0.4794$, so the two agree to 4 decimal places

A1

Answer: 0.4794; 0.4794

3 marks

26 (5 marks)

$$\text{(a) } \tan \alpha = \frac{40}{d}$$

M1

$$\tan \alpha \approx \alpha, \text{ so } d \approx \frac{40}{\alpha}$$

A1

Answer: shown

2 marks

$$\text{(b) Initial distance} = 2000 + 1200 = 3200 \text{ m}$$

M1 A1

$$\alpha \approx \frac{40}{3200} = 0.0125 \text{ rad}$$

A1

Answer: 3200 m; 0.0125 rad

3 marks

27 (8 marks)

$$(a) \cos \theta \approx 1 - \frac{\theta^2}{2}, \sin \theta \approx \theta$$

M1

$$\text{Sum} \approx 1 + \theta - \frac{\theta^2}{2}$$

A1

Answer: shown

2 marks

$$(b) \frac{1}{1+u} \text{ with } u = \theta - \frac{\theta^2}{2}$$

M1

$$\approx 1 - u + u^2$$

M1

$$= 1 - \theta + \frac{\theta^2}{2} + \theta^2 + (\text{terms in } \theta^3, \theta^4)$$

A1

$$\approx 1 - \theta + \frac{3\theta^2}{2}$$

A1

Answer: shown

4 marks

$$(c) \frac{1}{\cos 0.05 + \sin 0.05} = 0.954$$

M1

$$1 - 0.05 + 1.5 \times 0.0025 = 0.954$$

A1

Answer: 0.954 $\approx$ 0.954

2 marks

28 (10 marks)

$$(a) \text{Triangle OAB} = \frac{1}{2} \times 1 \times 1 \times \sin \theta = \frac{1}{2} \sin \theta$$

B1

$$\text{Sector OAB} = \frac{1}{2} \times 1^2 \times \theta = \frac{1}{2} \theta$$

B1

$$\text{AT} = \tan \theta, \text{triangle OAT} = \frac{1}{2} \times 1 \times \tan \theta = \frac{1}{2} \tan \theta$$

M1 A1

$$\text{Answer: } \frac{1}{2} \sin \theta, \frac{1}{2} \theta, \frac{1}{2} \tan \theta$$

4 marks

(b) The triangle OAB lies inside the sector, which lies inside triangle OAT

E1

$$\text{So } \frac{1}{2} \sin \theta < \frac{1}{2} \theta < \frac{1}{2} \tan \theta$$

E1

Answer: shown

2 marks

$$(c) \text{From } \theta < \tan \theta: \theta \cos \theta < \sin \theta, \text{ so } \cos \theta < \frac{\sin \theta}{\theta}$$

M1 A1

$$\text{From } \sin \theta < \theta: \frac{\sin \theta}{\theta} < 1$$

A1

$$\text{As } \theta \rightarrow 0, \cos \theta \rightarrow 1, \text{ so } \frac{\sin \theta}{\theta} \text{ is squeezed to 1, which justifies } \sin \theta \approx \theta$$

E1

Answer: shown

4 marks

29 (10 marks)

$$(a) 1 - \frac{\theta^2}{2} = 2\theta, \text{ so } \theta^2 + 4\theta - 2 = 0$$

M1 A1

$$\theta = \frac{-4 \pm \sqrt{16+8}}{2} = -2 \pm \sqrt{6}$$

M1

$$\text{The positive root: } \theta = \sqrt{6} - 2 = 0.449$$

A1

Answer: $\theta \approx 0.449$

4 marks

$$(b) f(\theta) = \cos \theta - 2\theta: f(0.4) = 0.121 > 0$$

M1

$$f(0.5) = -0.122 < 0, \text{ sign change, } f \text{ continuous}$$

A1

Answer: sign change

2 marks

$$(c) \theta_2 = 0.45022, \theta_3 = 0.45017, \theta_4 = 0.45019$$

M1 A1

$$\text{Root} = 0.4502 \text{ to 4 d.p.}$$

A1

$$\text{Error in (a): } 0.4495 - 0.4502 \approx 0.0007$$

A1

Answer: root = 0.4502; error $\approx$ 0.0007

4 marks

30 (8 marks)

(a) $\sin(x + h) = \sin x \cos h + \cos x \sin h$

M1Subtract $\sin x$ and divide by h : $\sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right)$ **A1****Answer: shown****2 marks**

(b) $\frac{\cos h - 1}{h} \approx \frac{-\frac{h^2}{2}}{h} = -\frac{h}{2}$

M1 A1

$\frac{\sin h}{h} \approx 1$

B1

Expression $\approx \cos x - \frac{h}{2} \sin x$

A1**Answer: shown****4 marks**

(c) As $h \rightarrow 0$, $\frac{h}{2} \sin x \rightarrow 0$

M1So the derivative is $\cos x$ **A1****Answer: $\cos x$** **2 marks**