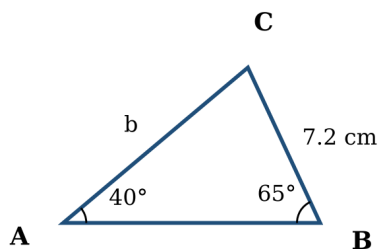


Sine Rule and Cosine Rule

- 1 In triangle ABC, angle A = 40° , angle B = 65° and BC = 7.2 cm.
Use the sine rule to find the length of AC, giving your answer to 3 significant figures.



[2]

- 2 In triangle ABC, AB = 9 cm, AC = 6 cm and angle BAC = 50° .
Find the length of BC, giving your answer to 3 significant figures.

[3]

- 3 In triangle ABC, angle A = 35° , BC = 7 cm and AC = 9 cm.
Find the acute angle ABC, to 1 decimal place. (Use the sine rule.)

[3]

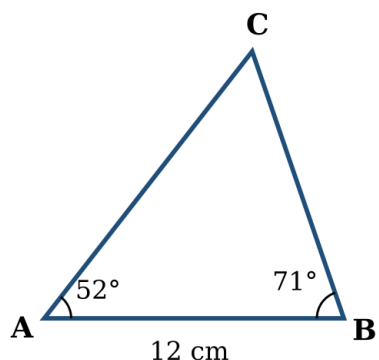
- 4 Triangle PQR has PQ = 8 cm, PR = 11 cm and angle QPR = 42° . Find its area, to 3 significant figures.

[2]

- 5 In triangle PQR, angle P = 55° , angle Q = 80° and PQ = 12 cm.
Find the length of PR, giving your answer to 3 significant figures.

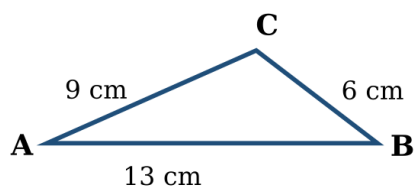
[3]

- 6 In the triangle shown, find the length of BC, to 3 significant figures.



[2]

- 7 The sides of triangle ABC are AB = 13 cm, BC = 6 cm and AC = 9 cm.
Find the size of angle ACB, giving your answer to 1 decimal place.



[3]

- 8 Triangle PQR has PQ = 9 cm, PR = 12 cm and angle QPR = 38° .

(a) Find the area of the triangle, to 3 significant figures.

[2]

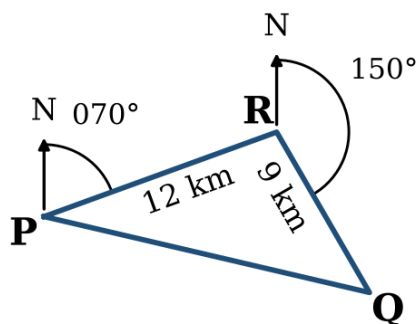
(b) Find the length of QR, to 3 significant figures.

[3]

- 9 A triangle has sides 9 cm, 12 cm and 15 cm. Show that it is right-angled and find its area.

[4]

- 10** Triangle ABC has area 30 cm^2 , $AB = 10 \text{ cm}$ and $AC = 8 \text{ cm}$, and angle BAC is acute. Find angle BAC, to 1 decimal place. [3]
- 11** A parallelogram has sides 6 cm and 9 cm and one angle of 65° . The shorter diagonal splits it into two congruent triangles. Find the length of the shorter diagonal, to 3 significant figures. [4]
- 12** In triangle ABC, angle $A = 30^\circ$, $BC = 10 \text{ cm}$ and $AC = 14 \text{ cm}$, and angle ABC is obtuse. Find angle ABC and angle ACB, to 1 decimal place. [4]
- 13** A ship leaves port P and sails 12 km on a bearing of 070° to a buoy R. It then sails 9 km on a bearing of 150° to a lighthouse Q.



- (a) Show that angle $PRQ = 100^\circ$ and find the distance PQ, giving your answer to 3 significant figures. [3]
- (b) Find the bearing of P from Q, to the nearest degree. [4]
- 14** In triangle ABC, $AB = 10 \text{ cm}$, $AC = 7 \text{ cm}$ and angle $ABC = 40^\circ$. The diagram shows that there are two possible positions, C and C', for the third vertex.
-
- (a) Find the two possible sizes of angle ACB, to 1 decimal place. [4]
- (b) Find the greater of the two possible areas of the triangle, to 3 significant figures. [4]
- 15** In triangle ABC, angle $A = 40^\circ$, $a = BC$ and $b = AC = 12 \text{ cm}$.
- (a) Show that no triangle exists when $a = 6 \text{ cm}$. [3]
- (b) Find the least value of a for which a triangle exists, to 3 significant figures. [2]
- (c) Given instead that $a = 9 \text{ cm}$, find the two possible sizes of angle B, to 1 decimal place. [3]
- 16** A surveyor wants the width of a river. She marks points A and B on one bank, 80 m apart, and sights a tree T on the opposite bank. Angle $TAB = 62^\circ$ and angle $TBA = 48^\circ$. Find the shortest distance from T to the line AB (the width of the river), to the nearest metre. [5]

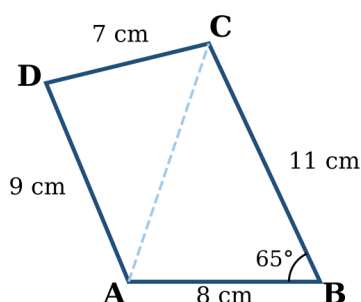
- 17** In triangle ABC, $AC = 10$ cm, angle $BAC = 120^\circ$ and the area of the triangle is $15\sqrt{3}$ cm².
 (a) Show that $AB = 6$ cm. [2]
 (b) Find the length of BC. [2]
 (c) Find the smallest angle of the triangle, to 1 decimal place. [3]

- 18** A boat sails 9 km from A to B on a bearing of 100° . It then sails 6 km from B to C on a bearing of 025° .
 Find the distance AC and the bearing of C from A. Give the distance to 3 significant figures and the bearing to the nearest degree. [6]

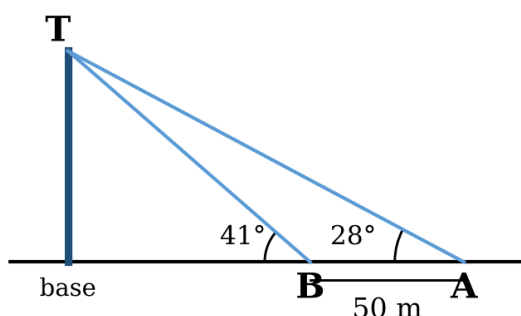
- 19** In triangle ABC, $AB = x$ cm, $AC = (x + 3)$ cm, angle $BAC = 60^\circ$ and $BC = 7$ cm.
 (a) Show that $x^2 + 3x - 40 = 0$. [3]
 (b) Hence find the perimeter of the triangle. [2]

- 20** In triangle ABC, $AB = 6$ cm, angle $A = 45^\circ$ and angle $C = 60^\circ$. Do not use a calculator in this question.
 You may use the fact that $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$.
 (a) Show that $BC = 2\sqrt{6}$ cm. [3]
 (b) Show that the area of the triangle is $(9 + 3\sqrt{3})$ cm². [5]

- 21** In the quadrilateral ABCD, $AB = 8$ cm, $BC = 11$ cm, $CD = 7$ cm, $DA = 9$ cm and angle $ABC = 65^\circ$.



- (a) Find the length of AC, to 3 significant figures. [3]
 (b) Find the size of angle ADC, to 1 decimal place. [3]
 (c) Find the area of the quadrilateral, to 3 significant figures. [3]
- 22** A vertical tower stands on level ground. Points A and B are on the ground, in a straight line with the foot of the tower, with B between A and the tower and $AB = 50$ m. From A the angle of elevation of the top T of the tower is 28° and from B it is 41° .



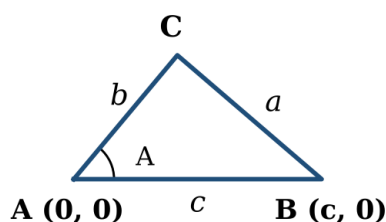
- (a) Show that angle $ATB = 13^\circ$. [2]
 (b) Find the height of the tower, to the nearest metre. [4]
- 23** In triangle ABC, $AB = 9$ cm, $AC = 7$ cm and angle $BAC = 80^\circ$. M is the midpoint of BC.
 (a) Find BC, to 3 significant figures. [3]
 (b) Find angle ABC, to 1 decimal place. [3]
 (c) Find the length AM, to 3 significant figures. [4]

- 24** A triangle has sides 13 cm, 14 cm and 15 cm. The sine rule states that $\frac{a}{\sin A} = 2R$, where R is the radius of the circle through the three vertices (circumcircle).
- (a) Find the largest angle of the triangle, to 1 decimal place. [4]
- (b) Hence find the radius of the circumcircle, to 3 significant figures. [3]

- 25** ABCD is a cyclic quadrilateral with $AB = 5$ cm, $BC = 8$ cm, $AD = 6$ cm and angle $ABC = 70^\circ$. Find the length CD , to 3 significant figures. [6]

- 26** In triangle ABC, $AB = 7$ cm, $BC = 12$ cm and $AC = 6$ cm. A student uses the sine rule to find angle BAC and gets an acute angle.
- (a) Find angle BAC using the cosine rule, to 1 decimal place. [3]
- (b) Explain why the sine rule alone can give a wrong answer here, and which rule is safer. [2]

- 27** Triangle ABC has sides $BC = a$, $CA = b$ and $AB = c$. The triangle is placed on coordinate axes with A at the origin O and B at $(c, 0)$, and angle $BAC = A$, as shown.



- (a) Write down the coordinates of C in terms of b and A , and use the distance formula for BC to prove that $a^2 = b^2 + c^2 - 2bc \cos A$. [4]
- (b) A different triangle has $a^2 = b^2 + c^2 + bc$. Find the size of angle A. [2]
- (c) In the triangle from part (b), $b = 5$ and $c = 8$. Find the exact area of the triangle. [2]
- 28** The sides of a triangle have lengths x cm, $(x + 2)$ cm and $(x + 4)$ cm, where $x > 0$. The largest angle of the triangle is 120° .
- (a) Show that $x^2 - x - 6 = 0$. [4]
- (b) Solve the equation and hence state the three side lengths. [3]
- (c) Find the exact area of the triangle. [2]
- (d) Show that the sine of the smallest angle is $\frac{3\sqrt{3}}{14}$. [3]
- 29** In triangle ABC, the bisector of angle A meets BC at D. $AB = c$, $AC = b$ and angle $BAC = 2\theta$.
- (a) By considering the areas of triangles ABD, ACD and ABC, show that $AD = \frac{2bc \cos \theta}{b + c}$. [5]
- (b) Given $b = 6$, $c = 10$ and $2\theta = 60^\circ$, find AD exactly. [2]
- 30** A triangle ABC has angle $A = 2 \times$ angle B, with $BC = a$, $CA = b$ and $AB = c$. This question proves that $a^2 = b(b + c)$.
- (a) Use the sine rule and $\sin 2B = 2 \sin B \cos B$ to show that $a = 2b \cos B$. [3]
- (b) Use the cosine rule for angle B to show that $a^2 = b(b + c)$. [4]
- (c) Hence find a when $b = 4$ and $c = 5$. [1]

Trigonometric Graphs

- 1** (a) Write down the maximum and minimum values of $y = 3 \sin x + 2$. [3]
- (b) State the period of the function $y = \cos 4x$, in degrees. [1]

2 Sketch the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$. Mark clearly the coordinates of the maximum and minimum points and the points where the curve meets the x-axis. [3]

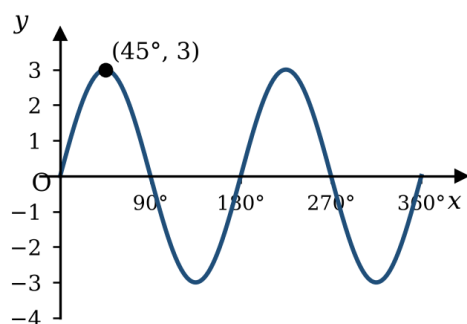
3 State the period, in degrees, of (a) $y = \sin(\frac{x}{2})$, (b) $y = \tan 3x$, (c) $y = 2 \cos 4x$. [3]

4 Sketch the graph of $y = 3 + \sin x$ for $0^\circ \leq x \leq 360^\circ$. Give the coordinates of its maximum point, its minimum point and its y-intercept, and explain why the curve does not meet the x-axis. [4]

5 Given that $\sin 30^\circ = \frac{1}{2}$, write down the exact values of (a) $\sin 150^\circ$, (b) $\sin 210^\circ$, (c) $\cos 60^\circ$. [3]

6 The graph of $y = \sin x$ is transformed to $y = \sin x + 3$. Describe the transformation, and state the maximum and minimum values of $y = \sin x + 3$. [2]

7 The diagram shows part of the curve with equation $y = a \sin(bx)$, where a and b are positive constants. The curve has a maximum point at $(45^\circ, 3)$. Find the values of a and b .

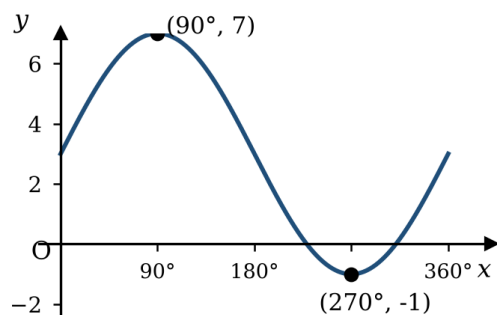


[3]

8 Describe fully the single transformation which maps the graph of
(a) $y = \sin x$ onto the graph of $y = \sin(x - 30^\circ)$,
(b) $y = \cos x$ onto the graph of $y = 2\cos 3x$ (two stretches are needed). [4]

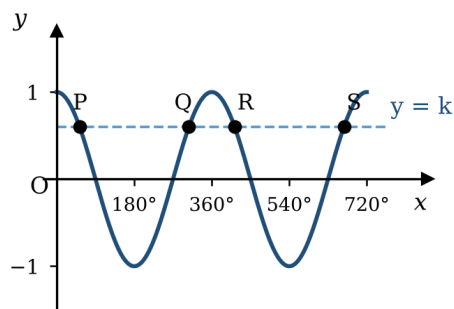
9 Sketch the graph of $y = \cos 2x$ for $0^\circ \leq x \leq 360^\circ$, marking the coordinates of the turning points. [3]

10 The diagram shows the graph of $y = a + b \sin x$ for $0^\circ \leq x \leq 360^\circ$. The maximum is at $(90^\circ, 7)$ and the minimum is at $(270^\circ, -1)$. Find a and b .



[3]

- 11** The diagram shows the graph of $y = \cos x$ for $0^\circ \leq x \leq 720^\circ$ and the line $y = k$, where $0 < k < 1$. The line meets the curve at the points P, Q, R and S, in order of increasing x . The x -coordinate of P is α° .



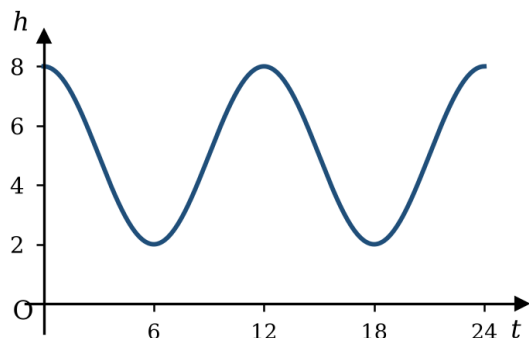
- (a) Write down, in terms of α , the x -coordinates of Q, R and S. [4]
 (b) The line $y = -k$ meets the curve at the points T and U with x between 0° and 360° .
 Write down, in terms of α , their x -coordinates. [3]

- 12** Sketch the graph of $y = 3 \cos x$ for $0 \leq x \leq 2\pi$ (x in radians), giving the coordinates of the turning points and the values of x where the curve meets the x -axis. [3]

- 13** Solve $\cos x = 0.5$ for $0^\circ \leq x \leq 720^\circ$. [3]

- 14** Sketch the graph of $y = 2 \sin(x + 60^\circ)$ for $0^\circ \leq x \leq 360^\circ$.
 Give the exact coordinates of the point where the curve crosses the y -axis, the coordinates of the maximum and minimum points, and the values of x where the curve crosses the x -axis. [5]

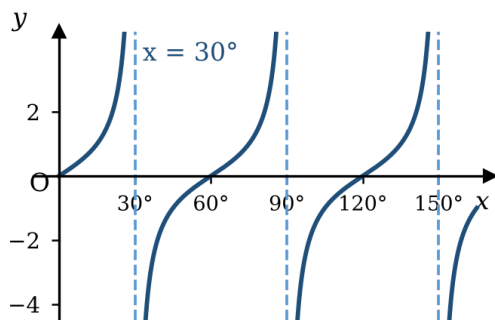
- 15** The depth of water, h metres, in a harbour t hours after midnight is modelled by $h = 5 + 3 \cos(30t)^\circ$, for $0 \leq t \leq 24$. Part of the graph is shown.



- (a) Write down the maximum and minimum depths, and the time between successive high tides. [2]
 (b) Find the times, in the first 12 hours, when the depth is 6.5 m. [3]
 (c) A boat needs a depth of at least 6.5 m to enter the harbour. For how many hours in the first 12 hours can it enter? [2]

- 16** The diagram shows part of the graph of $y = \tan(bx)$, where b is a positive constant. The first vertical asymptote for $x > 0$ is $x = 30^\circ$.

- (a) Find the value of b .
 (b) State the period of the function and write down the next two asymptotes after $x = 30^\circ$.



[4]

- 17** The mean monthly temperature, T °C, in a town is modelled by $T = 14 - 7\cos(30t)^\circ$, where t is the number of months after 1 January ($0 \leq t \leq 12$).

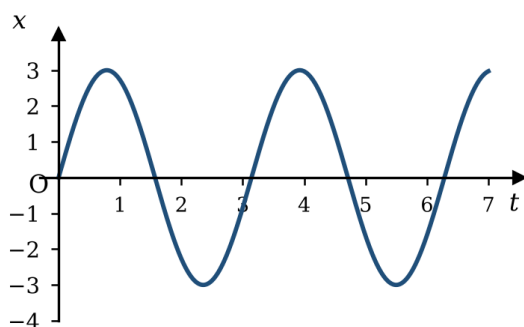
- (a) State the highest and lowest temperatures predicted by the model, and the month in which the highest occurs.
 (b) Find the values of t for which $T = 17.5$.

[3]

[3]

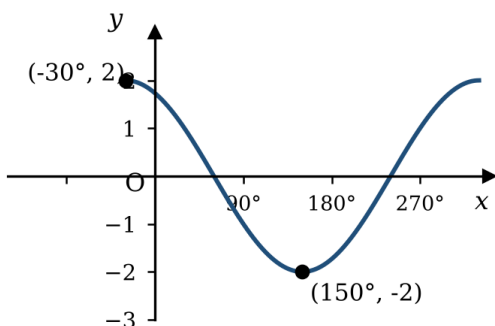
- 18** A mass on a spring has displacement x cm from its rest position at time t seconds given by $x = 3 \sin 2t$, where t is in radians. Part of the graph is shown.

- (a) Find the period of the motion.
 (b) Find the first time at which the displacement is 2 cm, to 2 decimal places.



[3]

- 19** The diagram shows the graph of $y = a \cos(x + b)^\circ$, where $a > 0$ and $0 < b < 90$. The maximum point is $(-30^\circ, 2)$ and a minimum point is $(150^\circ, -2)$. Find a and b .



[4]

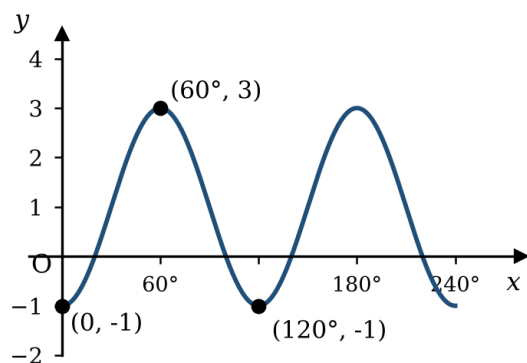
- 20** Use the graph of $y = \cos x$ to answer these questions, for $0^\circ \leq x \leq 360^\circ$.

- (a) Solve $\cos x > 0.5$.
 (b) Solve $\sin x \leq -0.5$.

[3]

[2]

- 21** The diagram shows part of the curve with equation $y = a \cos(bx) + c$, where a , b and c are constants and $b > 0$. The curve has a minimum point at $(0^\circ, -1)$ and a maximum point at $(60^\circ, 3)$.



- (a) Find the values of a , b and c . [5]
 (b) Solve $1 - 2\cos 3x = 2$ for $0^\circ \leq x \leq 180^\circ$. [4]

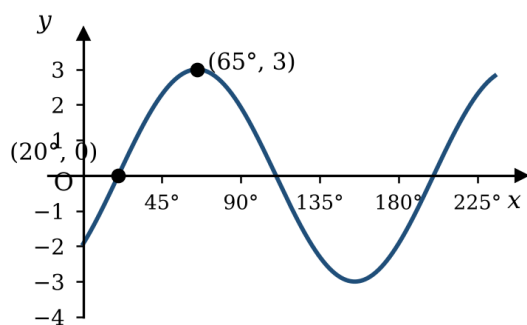
- 22** The function f is defined by $f(x) = \tan(x - 45^\circ)$ for $0^\circ \leq x \leq 360^\circ$, $x \neq 135^\circ, 315^\circ$.

- (a) Sketch the graph of $y = f(x)$, showing the equations of the asymptotes and the coordinates of every point where the curve meets the axes. [5]
 (b) Solve $f(x) = \sqrt{3}$ for $0^\circ \leq x \leq 360^\circ$. [3]

- 23** The number of hours of daylight, H , in a northern city on day t of the year is modelled by $H = 12 + 4 \sin\left(\frac{360(t - 80)}{365}\right)^\circ$, for $0 \leq t \leq 365$.

- (a) Find the maximum and minimum number of hours of daylight, and the day on which the maximum occurs, to the nearest day. [4]
 (b) Find the number of days in the year for which there are more than 14 hours of daylight, to the nearest day. [5]

- 24** The diagram shows the graph of $y = a \sin(bx - c)^\circ$, where a , b and c are positive constants and $0 < c < 90$. The curve crosses the x -axis, going upwards, at $(20^\circ, 0)$ and has its first maximum at $(65^\circ, 3)$. Find a , b and c .



[6]

- 25** The function f is defined by $f(x) = 2 \tan\left(\frac{x}{2}\right)$, for $0^\circ \leq x \leq 720^\circ$, $x \neq 180^\circ, 540^\circ$.

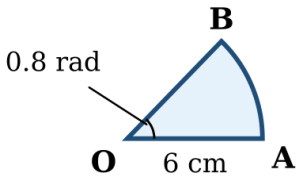
- (a) Write down the period of f and the equations of the asymptotes, and sketch the graph. [5]
 (b) Solve $f(x) = 2$ for $0^\circ \leq x \leq 720^\circ$. [3]

- 26** The graph of $y = \sin x$ is transformed by a stretch of scale factor 3 parallel to the x -axis, then a stretch of scale factor 2 parallel to the y -axis, then a translation of 1 unit down. The new graph is $y = g(x)$.

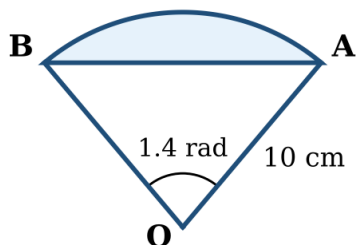
- (a) Write down an expression for $g(x)$. [2]
 (b) Sketch the graph of $y = g(x)$ for $0^\circ \leq x \leq 540^\circ$, showing the coordinates of the maximum point and of every point where the curve meets the axes. [5]
 (c) The line $y = k$, where $-1 < k < 1$, meets the curve at P and Q . The x -coordinate of P is 45° . Find the coordinates of Q . [2]

- 27** Consider the equation $\sin x = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$.
- On the same axes, sketch the graphs of $y = \sin x$ and $y = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$. [3]
 - State the number of solutions of the equation in this interval. [1]
 - Use $\sin 2x = 2 \sin x \cos x$ to solve the equation algebraically. [4]
 - Find the exact y-coordinate of the intersection in $0^\circ < x < 90^\circ$. [1]
-
- 28** A big wheel has radius 10 m and its centre is 12 m above the ground. A rider boards at the lowest point when $t = 0$, where t is the time in seconds. The wheel makes one revolution in 20 seconds. The rider's height above the ground, h metres, is modelled by $h = 12 - 10\cos(18t)^\circ$.
- Find the rider's height at the start and show that the model gives a period of 20 s. [2]
 - Find the first time the rider is 17 m above the ground. [3]
 - Find the time in one revolution for which the rider is at least 17 m above the ground. [3]
 - A student writes the model as $h = 12 + 10 \sin(18t - 90)^\circ$. Show that this is the same model. [2]
-
- 29** Consider the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ for $0^\circ \leq x \leq 360^\circ$.
- Describe the single transformation that maps the graph of $y = \sin x$ onto the graph of $y = \cos x$. [2]
 - Hence, or otherwise, find the exact solutions of $\sin x = \cos x$ in the given interval and explain why there are exactly two. [3]
 - Find the number of solutions of $\sin x = \cos x$ in $0^\circ \leq x \leq 1000^\circ$ and the sum of the two largest. [4]
-
- 30** The depth of water, d metres, at the end of a jetty t hours after 06:00 is modelled by $d = 4 + 1.5 \sin(\frac{\pi t}{6})$, where t is in radians for the sine function and $0 \leq t \leq 24$.
- Find the greatest and least depths, and the time between two successive high tides. [3]
 - A boat needs at least 4.75 m of water. Find the first time interval (in hours after 06:00, to 2 decimal places) during which the boat can use the jetty. [4]
 - Find the total time in the 24 hours during which the depth is below 3 m, to the nearest minute. [5]

Radians, Arc Length and Sector Area

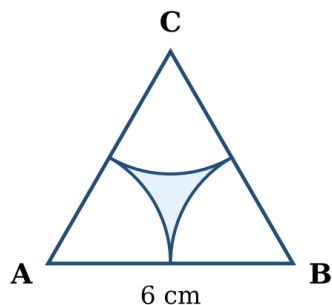
- 1** (a) Convert 72° to radians, giving your answer as a multiple of π .
(b) Convert $\frac{3\pi}{8}$ radians to degrees. [2]
-
- 2** The diagram shows a sector OAB of a circle with centre O and radius 6 cm. Angle AOB = 0.8 radians.
- 
- Find the arc length AB. [1]
 - Find the area of the sector. [2]
-
- 3** A sector of a circle has radius 8 cm and angle 75° .
- Write 75° in radians, as a multiple of π . [2]
 - Find the exact arc length and the exact area of the sector. [3]
-
- 4** Convert (a) 1.2 radians to degrees, to 1 decimal place, and (b) $\frac{5\pi}{6}$ radians to degrees. [2]

- 5 Write down the exact values of (a) $\sin(\frac{\pi}{6})$, (b) $\cos(\frac{\pi}{3})$, (c) $\tan(\frac{\pi}{4})$, (d) $\sin(\frac{2\pi}{3})$. [4]
- 6 A sector of a circle has radius 6 cm and angle θ radians. The perimeter of the sector is numerically equal to its area. Find the value of θ . [4]
- 7 A sector of a circle has radius 9 cm and angle $\frac{2\pi}{3}$ radians. Find the exact perimeter of the sector. [3]
- 8 A sector of a circle has area 40 cm^2 and angle 1.25 radians.
(a) Find the radius of the circle.
(b) Find the perimeter of the sector. [4]
- 9 A sector of a circle has radius r cm and angle θ radians. The length of the arc is $\frac{3}{5}$ of the perimeter of the sector.
(a) Show that $\theta = 3$. [3]
(b) Given also that the area of the sector is 54 cm^2 , find r . [2]
- 10 A sector of radius 9 cm has perimeter 30 cm. Find the angle of the sector, in radians, and its area. [3]
- 11 O is the centre of a circle of radius 7 cm. A chord AB subtends an angle of 2.2 radians at O. Find the length of the chord AB, to 3 significant figures. [3]
- 12 The minute hand of a clock is 12 cm long. Find the distance moved by its tip in 25 minutes, giving an exact answer. [3]
- 13 The diagram shows a circle, centre O and radius 10 cm. The chord AB subtends an angle of 1.4 radians at O. The minor segment between the chord AB and the arc AB is shaded. (Use your calculator in radian mode.)



- (a) Find the area of triangle OAB. [2]
(b) Find the area of the shaded segment. [3]

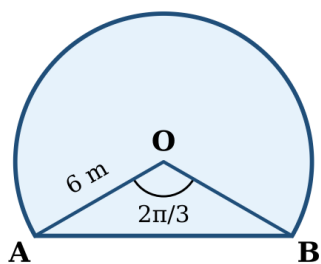
- 14** ABC is an equilateral triangle of side 6 cm. Three sectors, each of radius 3 cm and centred at A, B and C, are removed from the triangle, leaving the shaded region shown.



- (a) Show that the area of the triangle is $9\sqrt{3} \text{ cm}^2$. [2]
 (b) Find the exact area of the shaded region. [3]
 (c) Find the exact perimeter of the shaded region. [2]

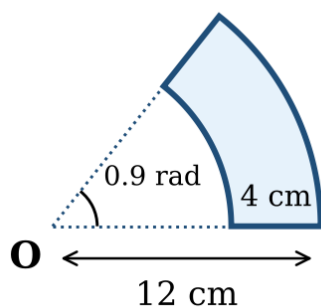
- 15** A chord AB of a circle with centre O and radius 6 cm subtends an angle of $\frac{\pi}{3}$ at O.
 Show that the area of the minor segment cut off by AB is $(6\pi - 9\sqrt{3}) \text{ cm}^2$, and find its value to 3 significant figures. [5]

- 16** The diagram shows the cross-section of a tunnel. It is the major segment of a circle with centre O and radius 6 m, cut off by the chord AB, where angle $AOB = \frac{2\pi}{3}$ radians.



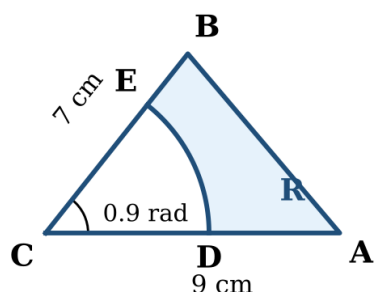
- (a) Find the exact length of the chord AB. [2]
 (b) Show that the area of the cross-section is $(24\pi + 9\sqrt{3}) \text{ m}^2$. [4]
 (c) Find the exact perimeter of the cross-section. [3]
- 17** A pendulum has length 80 cm and swings through an angle of 0.35 radians from one extreme position to the other.
 Find (a) the distance travelled by the bob in one swing, (b) the area swept out by the pendulum in one swing. [4]
- 18** A bicycle wheel has radius 35 cm. Find the distance travelled by the bicycle when the wheel turns through 40 radians (without slipping), giving your answer in metres to 3 significant figures.
 Find also how many complete revolutions this is, to the nearest whole number. [4]

- 19** The diagram shows the region between two concentric arcs of radii 8 cm and 12 cm, with the angle at the common centre O equal to 0.9 radians.



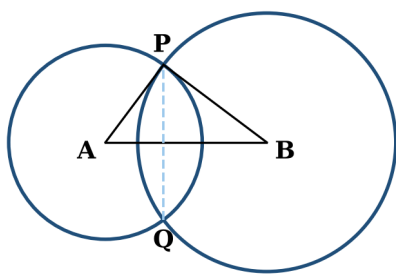
- (a) Find the area of the shaded region. [3]
 (b) Find the perimeter of the shaded region. [3]

- 20** The diagram shows a triangle ABC in which CA = 9 cm, CB = 7 cm and angle ACB = 0.9 radians. An arc of a circle with centre C and radius 5 cm meets CA at D and CB at E. The shaded region R is bounded by DA, AB, BE and the arc ED.



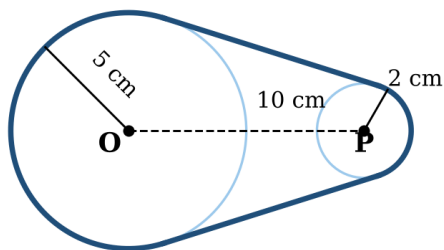
- (a) Find the length of AB, to 3 significant figures. [2]
 (b) Find the area of the triangle ABC, to 3 significant figures. [2]
 (c) Find the area of R, to 3 significant figures. [3]
 (d) Find the perimeter of R, to 3 significant figures. [4]

- 21** Circle C_1 has centre A and radius 6 cm. Circle C_2 has centre B and radius 8 cm. AB = 10 cm and the circles intersect at P and Q, as shown.



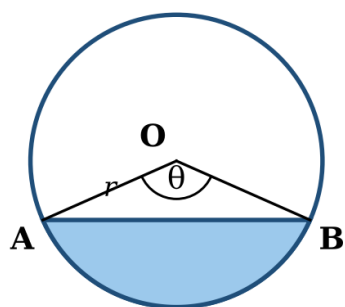
- (a) Show that angle PAB = 0.927 radians, correct to 3 decimal places. [3]
 (b) Find the area of the region common to both circles, to 3 significant figures. [4]

- 22** A belt passes tightly round two circular pulleys with centres O and P and radii 5 cm and 2 cm. $OP = 10$ cm. The belt does not cross itself.



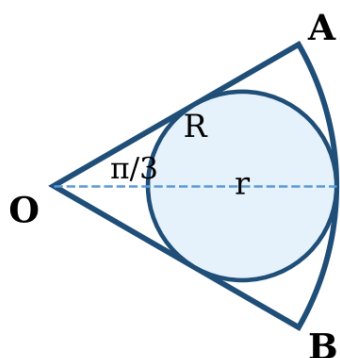
- (a) Show that each straight section of the belt has length $\sqrt{91}$ cm. [2]
- (b) Show that the belt is in contact with the large pulley for an arc subtending an angle of $\pi + 2\alpha$ at O, where $\sin \alpha = 0.3$. [3]
- (c) Find the total length of the belt, to 3 significant figures. [3]
-
- 23** A chord AB of length 8 cm is drawn in a circle with centre O and radius 5 cm.
- (a) Find the angle AOB in radians, to 4 significant figures. [3]
- (b) Find the area of the minor segment. [3]
- (c) Find the perimeter of the minor segment. [2]
-
- 24** A sector of a circle of radius 15 cm has angle $\frac{4\pi}{3}$. Its straight edges are joined together to form a cone.
- (a) Show that the base radius of the cone is 10 cm. [3]
- (b) Find the exact volume of the cone. [4]
-
- 25** A sector of a circle has radius r cm and angle θ radians. Its area is 32 cm^2 and its perimeter is 24 cm.
- (a) Show that $r^2 - 12r + 32 = 0$. [3]
- (b) Hence find the two possible sectors, giving r and θ for each. [3]
- (c) Explain why both are possible sectors. [1]
-
- 26** A student calculates the area of a sector of radius 6 cm and angle 40° as $\frac{1}{2} \times 6^2 \times 40 = 720 \text{ cm}^2$. Explain the error and find the correct area, to 3 significant figures. [4]
-
- 27** A sector of a circle has radius r cm and angle θ radians. The perimeter of the sector is 20 cm.
- (a) Show that the area $A \text{ cm}^2$ of the sector is given by $A = 10r - r^2$. [3]
- (b) By completing the square, find the maximum value of A and the value of θ for which it occurs. [4]
- (c) The sector with the maximum area is cut out and its straight edges are joined to form a hollow cone. Find the perpendicular height of the cone, to 3 significant figures. [3]

- 28** The diagram shows a circle with centre O and radius r . The chord AB divides the circle into two parts. Angle $AOB = \theta$ radians, where $0 < \theta < \pi$. The area of the shaded minor segment is exactly one quarter of the area of the circle.



- Explain why θ is not equal to $\frac{\pi}{2}$. [2]
- Show that $\theta - \sin \theta = \frac{\pi}{2}$. [3]
- Use the iterative formula $\theta_{n+1} = \frac{\pi}{2} + \sin \theta_n$ with $\theta_1 = 2.5$ to find θ_2 , θ_3 and θ_4 , and hence the value of θ correct to 3 decimal places. [3]
- Find the length of the chord AB in terms of r , to 3 significant figures. [2]

- 29** The diagram shows a sector OAB of a circle of radius R , with angle $AOB = \frac{\pi}{3}$. A smaller circle of radius r lies inside the sector and touches both straight edges OA and OB and the arc AB .



- Show that $R = 3r$. [5]
- Find, in terms of R , the exact area of the part of the sector outside the small circle. [3]
- Find the fraction of the sector's area that is shaded. [1]

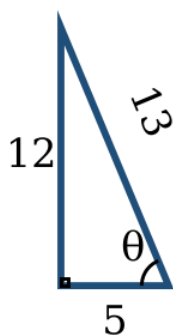
- 30** A chord of a circle of radius r subtends a small angle θ radians at the centre. The area of the minor segment is S .

- Show that $S = \frac{1}{2}r^2(\theta - \sin \theta)$. [3]
- Using $\sin \theta \approx \theta - \frac{\theta^3}{6}$ for small θ , show that $S \approx \frac{r^2\theta^3}{12}$. [2]
- For $r = 10$ cm and $\theta = 0.1$, find the exact-formula value of S and the approximation to 4 significant figures, and the percentage error of the approximation. [4]

Trigonometric Identities

- 1** Simplify (a) $3 \sin^2 x + 3 \cos^2 x$, (b) $\tan x \cos x$. [2]

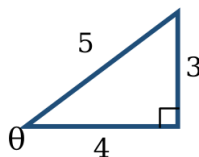
- 2 The angle θ is acute and $\cos \theta = \frac{5}{13}$.
Find the exact values of $\sin \theta$ and $\tan \theta$.



[3]

- 3 Simplify (a) $4 - 4 \cos^2 \theta$, (b) $\frac{\sin x}{\tan x}$. [2]

- 4 The angle θ is acute and $\tan \theta = \frac{3}{4}$.
Find the exact values of $\sin \theta$ and $\cos \theta$.



[2]

- 5 Write $\cos^2 x + 3 \sin^2 x$ in the form $1 + a \sin^2 x$, stating the value of a . [2]

- 6 Simplify $(\sin x + \cos x)^2 - 2 \sin x \cos x$. [2]

- 7 Show that $\frac{1 - \cos^2 x}{\sin x \cos x} \equiv \tan x$. [3]

- 8 The angle θ is obtuse and $\sin \theta = \frac{12}{13}$.
Find the exact values of $\cos \theta$ and $\tan \theta$. [3]

- 9 Prove that $\cos^2 x - \sin^2 x \equiv 2 \cos^2 x - 1$. [2]

- 10 Prove that $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [3]

- 11 Show that $\frac{1 - \sin x}{\cos x} \equiv \frac{\cos x}{1 + \sin x}$. [3]

- 12 The angle θ is in the third quadrant ($180^\circ < \theta < 270^\circ$) and $\cos \theta = -\frac{2}{3}$.
Find the exact values of $\sin \theta$ and $\tan \theta$. [3]

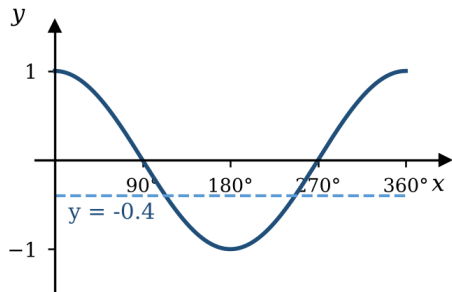
- 13 Given that $\tan \theta = 2$, find the value of $\frac{\sin \theta + 2 \cos \theta}{3 \sin \theta - \cos \theta}$. [3]

- 14 Prove that $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} \equiv \frac{2}{\sin x}$. [4]

- 15** The angle θ is not a multiple of 90° .
- (a) Given that $3 \sin^2 \theta + 5 \cos^2 \theta = 4$, show that $\tan^2 \theta = 1$ and hence find the possible values of $\tan \theta$. [4]
- (b) Write $3 \sin^2 \theta + 5 \cos^2 \theta$ in the form $a + b \cos^2 \theta$, where a and b are integers, and state its least possible value. [2]
-
- 16** Prove that $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} \equiv \frac{2}{\sin^2 x}$. [4]
-
- 17** Prove that $\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x$. [4]
-
- 18** Eliminate θ to find a Cartesian equation for each curve.
- (a) $x = 3 \cos \theta$, $y = 2 \sin \theta$ [3]
- (b) $x = 2 + 4 \sin \theta$, $y = 1 + 4 \cos \theta$ [3]
-
- 19** The function f is defined by $f(x) = \operatorname{cosec} x - \sin x$ for $0^\circ < x < 180^\circ$.
- (a) Show that $f(x) \equiv \frac{\cos^2 x}{\sin x}$. [2]
- (b) Hence show that $f(x) \geq 0$ for every x in the domain, and state the value of x for which $f(x) = 0$. [3]
-
- 20** $f(x) = 2 \cos^2 x - 3 \sin x + 1$.
- (a) Show that $f(x) = 3 - 3 \sin x - 2 \sin^2 x$. [2]
- (b) Find the greatest and least values of $f(x)$. [5]
-
- 21** It is given that $\sin \theta + \cos \theta = \frac{1}{2}$.
- (a) Prove that $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 \equiv 2$. [2]
- (b) Find the value of $\sin \theta \cos \theta$, and of $(\sin \theta - \cos \theta)^2$. [4]
- (c) Find the value of $\sin^3 \theta + \cos^3 \theta$. [2]
-
- 22** A student writes: " $\sin^2 x + \cos^2 x = 1$, so, taking square roots, $\sin x + \cos x = 1$."
- (a) Explain the error and give a value of x which shows the statement is false. [2]
- (b) Prove that $\frac{1 - \tan^2 x}{1 + \tan^2 x} \equiv \cos^2 x - \sin^2 x$. [4]
- (c) Prove that $\frac{1}{\cos x} - \cos x \equiv \sin x \tan x$. [3]
-
- 23** The angle x is such that $0^\circ \leq x \leq 360^\circ$.
- (a) Prove that $\sin^4 x - \cos^4 x \equiv \sin^2 x - \cos^2 x$. [3]
- (b) Hence solve $\sin^4 x - \cos^4 x = \frac{1}{2}$. [5]
-
- 24** The point P has coordinates $x = 3 \sin \theta$, $y = 4 + 2 \cos^2 \theta$.
- (a) Show that all points P lie on a curve with equation $y = 6 - \frac{2x^2}{9}$. [3]
- (b) State the range of values of x and the range of values of y . [2]
-
- 25** Sums of cubes appear in trigonometric identities.
- (a) Prove that $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. [3]
- (b) Given that $\sin x + \cos x = \frac{6}{5}$, find $\sin x \cos x$ and hence $\sin^3 x + \cos^3 x$. [4]
-

- 26** Consider the equation $3 \tan^2 x + 5 \sec x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.
- (a) Use the identity $1 + \tan^2 x \equiv \sec^2 x$ to show that $3 \sec^2 x + 5 \sec x - 2 = 0$. [3]
- (b) Hence solve the equation, explaining why one of the roots of the quadratic in $\sec x$ gives no solutions. [5]
-
- 27** The function f is defined by $f(x) = \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x}$ for x not an odd multiple of 90° .
- (a) Prove that $\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} \equiv \frac{2 \sin x}{\cos^2 x}$. [4]
- (b) Hence solve $\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 4 \sin x$ for $0^\circ \leq x \leq 360^\circ$. [5]
-
- 28** Eliminate θ from each pair of equations to find a Cartesian equation.
- (a) $x = 2 + 3 \sin \theta$, $y = 1 - 3 \cos \theta$. Show that the points lie on a circle and state its centre and radius. [4]
- (b) $x = \sin \theta + \cos \theta$, $y = \sin \theta \cos \theta$. Show that $2y = x^2 - 1$. [3]
- (c) For the curve in part (b), show that $y \leq \frac{1}{2}$, and hence find the greatest possible value of x . [3]
-
- 29** Consider the equation $\sin^6 x + \cos^6 x = \frac{7}{16}$ for $0^\circ \leq x \leq 360^\circ$.
- (a) Show that $\sin^6 x + \cos^6 x \equiv 1 - 3 \sin^2 x \cos^2 x$. [4]
- (b) Hence show that the equation can be written as $16 \sin^4 x - 16 \sin^2 x + 3 = 0$. [3]
- (c) Solve the equation. [5]
-
- 30** The angle θ satisfies $\operatorname{cosec} \theta - \sin \theta = a$ and $\sec \theta - \cos \theta = b$, where a and b are non-zero.
- (a) Show that $a = \frac{\cos^2 \theta}{\sin \theta}$ and $b = \frac{\sin^2 \theta}{\cos \theta}$. [3]
- (b) Hence show that $a^2 b^2 = \sin^2 \theta \cos^2 \theta$ and $a^2 + b^2 = \frac{1 - 3 \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$. [4]
- (c) Prove that $a^2 b^2 (a^2 + b^2 + 3) = 1$. [4]

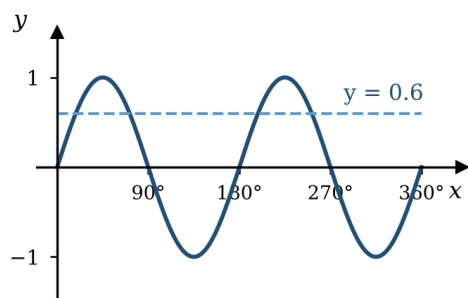
Trigonometric Equations

- 1** Solve $\sin x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$. [2]
-
- 2** The diagram shows the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ and the line $y = -0.4$. Solve $\cos x = -0.4$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place.
- 
- [3]
-
- 3** Solve $\tan x = 1$ for $0^\circ \leq x \leq 360^\circ$. [2]
-
- 4** Solve $\sin x = -0.3$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [3]

5 Solve $\sin 2x = \frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$. [3]

6 Solve $3 \tan x - 5 = 0$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [3]

7 The diagram shows the graph of $y = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$ and the line $y = 0.6$.
Solve $\sin 2x = 0.6$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place.



[4]

8 Solve $\tan(x - 30^\circ) = 2$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [3]

9 Solve $\cos(x + 20^\circ) = -0.5$ for $0^\circ \leq x \leq 360^\circ$. [4]

10 Solve $(2 \cos x - 1)(\tan x + 1) = 0$ for $0^\circ \leq x \leq 360^\circ$. [4]

11 Solve $2 \sin x = 1$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π . [3]

12 Solve $\cos^2 x = \frac{1}{4}$ for $0^\circ \leq x \leq 360^\circ$. [4]

13 Solve $\tan 2x = -1$ for $0^\circ \leq x \leq 180^\circ$. [4]

14 Solve $2 \sin^2 x + \sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. [5]

15 Solve $2 \cos^2 x + 3 \sin x = 3$ for $0^\circ \leq x \leq 360^\circ$. [6]

16 Solve $\tan^2 x + \tan x - 6 = 0$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [5]

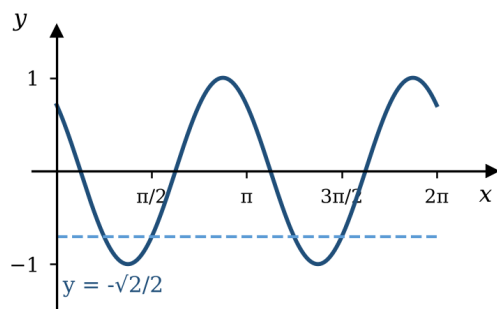
17 Solve $2 \cos(2x - \frac{\pi}{3}) = 1$ for $0 \leq x \leq \pi$, giving exact answers. [5]

18 The height of a swing seat above the ground is $h = 2 + 1.5 \sin(0.8t)$ metres, where t is time in seconds and the sine is of an angle in radians.
Find the first two times at which the seat is 3.2 m above the ground, to 2 decimal places. [4]

19 Solve $3 \cos x = 2 \sin^2 x$ for $0^\circ \leq x \leq 360^\circ$, explaining why one root of the quadratic is rejected. [5]

20

The diagram shows the curve $y = \cos(2x + \frac{\pi}{4})$ for $0 \leq x \leq 2\pi$ and the line $y = -\frac{\sqrt{2}}{2}$.



- (a) Solve $2\cos(2x + \frac{\pi}{4}) + \sqrt{2} = 0$ for $0 \leq x \leq 2\pi$, giving your answers as multiples of π . [5]
- (b) Write down the number of solutions of the same equation for $0 \leq x \leq 4\pi$. [2]

21 Consider the equation $\tan x = 2 \sin x$.

- (a) Solve $\tan x = 2 \sin x$ for $0^\circ \leq x \leq 360^\circ$. [5]
- (b) A student divides both sides of $\sin x = 2 \sin x \cos x$ by $\sin x$ to get $\cos x = \frac{1}{2}$. Explain what is wrong with this and state which solutions are lost. [2]

22 Consider the equation $3 \tan x = 8 \cos x$ for $0^\circ \leq x \leq 360^\circ$.

- (a) Show that the equation can be written as $8 \sin^2 x + 3 \sin x - 8 = 0$. [3]
- (b) Hence solve the equation, giving your answers to 1 decimal place. [4]

23 Solve $\sin(3x - 30^\circ) = 0.5$ for $0^\circ \leq x \leq 180^\circ$. [6]

24 Solve $\cos(2x - 30^\circ) = \cos(x + 45^\circ)$ for $0^\circ \leq x \leq 360^\circ$. [6]

25 Find the number of solutions in the interval $0^\circ \leq x \leq 360^\circ$ of each equation.

- (a) $\sin x = 0.8$ [1]
- (b) $\sin 3x = 0.8$ [2]
- (c) $\tan(\frac{x}{2}) = 5$ [2]
- (d) $\cos x = -1$ [1]

26 Consider the equation $\sin x + 2 \cos x = 1.5$ for $0^\circ \leq x \leq 360^\circ$.

- (a) By writing $\sin x = 1.5 - 2 \cos x$ and squaring, show that $20 \cos^2 x - 24 \cos x + 5 = 0$. [3]
- (b) Hence solve $\sin x + 2 \cos x = 1.5$, giving your answers to 1 decimal place. Check each value in the original equation. [6]

27 Consider the equation $6 \sin^2 x + \cos x = 4$ for $0^\circ \leq x \leq 360^\circ$.

- (a) Show that the equation can be written as $6 \cos^2 x - \cos x - 2 = 0$. [3]
- (b) Hence solve the equation, giving your answers to 1 decimal place where necessary. [5]
- (c) Find the greatest possible value of $6 \sin^2 x + \cos x$, and the values of $\cos x$ at which it occurs. [3]

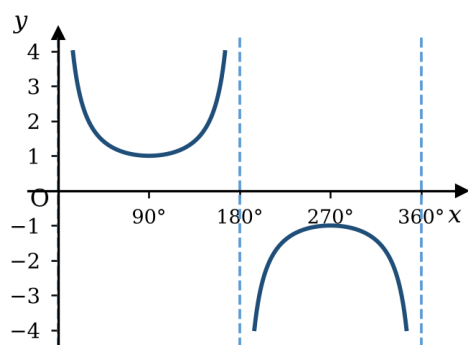
28 Consider the equation $2 \sin^2 x - 3 \sin x \cos x + \cos^2 x = 0$.

- (a) Show that, for $\cos x \neq 0$, the equation can be written as $2 \tan^2 x - 3 \tan x + 1 = 0$. [2]
- (b) Solve the equation for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. Explain why $\cos x = 0$ cannot be a solution. [5]
- (c) Hence solve $2 \sin^2 2y - 3 \sin 2y \cos 2y + \cos^2 2y = 0$ for $0^\circ \leq y \leq 180^\circ$. [3]

- 29** The equation $2 \sin^2 x + p \sin x - 1 = 0$ has $x = 30^\circ$ as one solution, where p is a constant.
- (a) Find the value of p . [2]
- (b) Solve the equation for $0^\circ \leq x \leq 360^\circ$. [4]
- (c) Find the least value of $2 \sin^2 x + \sin x - 1$ and a value of x at which it occurs, to 1 decimal place. [4]
- 30** Consider the equation $\tan(2x - \frac{\pi}{4}) = \sqrt{3}$ for $0 \leq x \leq 2\pi$.
- (a) Find all the solutions, as multiples of π . [5]
- (b) Show that the sum of the solutions is $\frac{25\pi}{6}$. [2]
- (c) Write down the general solution of the equation in x , where n is an integer. [2]

Reciprocal and Inverse Trigonometric Functions

- 1** Write down the exact values of (a) $\sec 60^\circ$, (b) $\operatorname{cosec} 45^\circ$, (c) $\cot 30^\circ$. [3]
- 2** Giving your answers in radians as multiples of π , find (a) $\sin^{-1}(\frac{1}{2})$, (b) $\cos^{-1}(-1)$, (c) $\tan^{-1}(1)$. [3]
- 3** Find the exact values of (a) $\sec 120^\circ$, (b) $\cot 45^\circ$, (c) $\operatorname{cosec} 210^\circ$. [3]
- 4** Use a calculator to find, to 3 significant figures, (a) $\cot 70^\circ$, (b) $\sec 25^\circ$, (c) $\operatorname{cosec} 100^\circ$. [3]
- 5** Write down, in radians and as multiples of π , (a) $\sin^{-1}(-\frac{1}{2})$, (b) $\cos^{-1}(\frac{\sqrt{3}}{2})$, (c) $\tan^{-1}(-1)$. [3]
- 6** State the range of values of $y = \sec x$, and the value of x between 0° and 360° at which $\sec x = -1$. [2]
- 7** The diagram shows the graph of $y = \operatorname{cosec} x$ for $0^\circ < x < 360^\circ$, $x \neq 180^\circ$.



- (a) Write down the equations of the vertical asymptotes shown. [1]
- (b) Write down the coordinates of the turning points and state the range of $y = \operatorname{cosec} x$. [2]
- 8** Solve $\sec x = 2$ for $0^\circ \leq x \leq 360^\circ$. [3]
- 9** Solve $\cot x = 2$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [3]
- 10** Simplify (a) $\operatorname{cosec} x \tan x$, (b) $\frac{\sec x - \cos x}{\sin x}$. [4]
- 11** Prove that $\operatorname{cosec} x - \sin x \equiv \cos x \cot x$. [3]
- 12** Sketch the graph of $y = \sec x$ for $0^\circ \leq x \leq 360^\circ$, showing the asymptotes and the coordinates of the turning points. [4]

- 13** The curve C has equation $y = 2 \arcsin(x + 1) + \pi$, where $\arcsin$ takes values in the interval from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.
- (a) Describe fully the three transformations which map the graph of $y = \arcsin x$ onto C , in the order they are applied. [3]
- (b) Sketch C , stating the coordinates of its end points and of the point where $x = -1$. [4]
- (c) Solve $2 \arcsin(x + 1) + \pi = \frac{3\pi}{2}$. [2]

- 14** Sketch the graph of $y = \cot x$ for $0^\circ < x < 360^\circ$. Show the equations of the asymptotes and the coordinates of the points where the curve crosses the x -axis. [4]

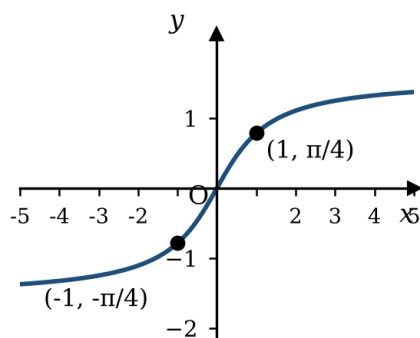
- 15** The angle x is such that $\sin x \neq 0$ and $\cos x \neq 0$.
- (a) Prove that $\tan x + \cot x \equiv \operatorname{cosec} x \sec x$. [3]
- (b) Hence, or otherwise, solve $\tan x + \cot x = \frac{5}{2}$ for $0^\circ \leq x \leq 180^\circ$, giving your answers to 1 decimal place. [3]

- 16** Solve $2 \sec^2 x - 5 \tan x - 5 = 0$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [5]

- 17** Giving all answers as exact values, use $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ to answer the following.
- (a) Find $\tan(\arctan 4 - \arctan 3)$ as a fraction. [2]
- (b) Find $\tan(\arctan \frac{1}{2} + \arctan \frac{1}{5})$ as a fraction. [2]
- (c) Solve $\tan(\arctan 3x - \arctan x) = \frac{1}{2}$. [3]

- 18** Solve, giving exact answers.
- (a) $\cos^{-1}(3x) = \frac{2\pi}{3}$ [2]
- (b) $\sin^{-1}(x - 1) = \frac{\pi}{4}$ [2]

- 19** The diagram shows the graph of $y = \tan^{-1}x$.



- (a) Write down the equations of the horizontal asymptotes. [2]
- (b) Find the exact value of $\tan^{-1}(\sqrt{3}) + \tan^{-1}(-\sqrt{3})$ and explain the result from the graph. [3]

- 20** Prove that $\sec x + \tan x \equiv \frac{\cos x}{1 - \sin x}$. [4]

- 21** Solve $2 \cot^2 x = \operatorname{cosec} x + 1$ for $0^\circ \leq x \leq 360^\circ$, giving non-exact answers to 1 decimal place. [7]

- 22** The principal value of $\sin^{-1}x$ lies in the interval from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.
- (a) Sketch the graph of $y = \sin^{-1}x$, stating its domain and range and the coordinates of its end points. [3]
- (b) Solve $\sin^{-1}(2x - 1) = \frac{\pi}{6}$. [2]
- (c) Find the exact values of $\cos(\sin^{-1}\frac{5}{13})$ and $\tan(\sin^{-1}\frac{5}{13})$. [3]
-
- 23** The function f is defined by $f(x) = 1 + 3 \sin 2x$ for $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$.
- (a) Find an expression for $f^{-1}(x)$. [3]
- (b) State the domain and range of f^{-1} , and sketch its graph, showing the coordinates of its end points and of the point where it crosses the x -axis. [4]
- (c) Solve $f^{-1}(x) = \frac{\pi}{12}$. [2]
-
- 24** Principal values of the inverse functions are used. Find the exact value of
- (a) $\sin^{-1}(\sin \frac{5\pi}{6})$, and explain why it is not $\frac{5\pi}{6}$. [3]
- (b) $\tan^{-1}(\tan \frac{3\pi}{4})$. [2]
- (c) $\sin(\cos^{-1}\frac{3}{5})$. [2]
-
- 25** Consider the equation $\sec x = k$ for $0^\circ \leq x \leq 360^\circ$.
- (a) Explain why there are no solutions when $k = \frac{1}{2}$. [1]
- (b) State the set of values of k for which the equation has solutions. [2]
- (c) Find x when $k = 2$ and when $k = -2$, giving all solutions in the interval. [4]
-
- 26** The function f is defined by $f(x) = 3 \operatorname{cosec} 2x$ for $0^\circ < x < 180^\circ$, $x \neq 90^\circ$.
- (a) Sketch the graph of $y = f(x)$, showing the asymptotes and the coordinates of the turning points. [4]
- (b) Solve $f(x) = 6$. [3]
-
- 27** The angle x is not an odd multiple of 90° .
- (a) Show that $(\sec x - \tan x)(\sec x + \tan x) \equiv 1$. [3]
- (b) Hence solve $\sec x + \tan x = 2$ for $0^\circ \leq x \leq 360^\circ$, giving your answer to 1 decimal place. [5]
- (c) Explain why the equation $\sec x + \tan x = 0$ has no solutions. [2]
-
- 28** Use the principal values of the inverse trigonometric functions.
- (a) Prove that $\tan^{-1}x + \tan^{-1}(\frac{1}{x}) = \frac{\pi}{2}$ for $x > 0$. [4]
- (b) Show that $\tan^{-1}2 + \tan^{-1}3 = \frac{3\pi}{4}$. [4]
- (c) Solve $\tan^{-1}x + \tan^{-1}(2x) = \frac{\pi}{4}$ for $x > 0$, giving your answer in exact form. [4]
-
- 29** This question is about inverse sine and inverse cosine, using principal values.
- (a) Prove that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for $-1 \leq x \leq 1$. [5]
- (b) Hence solve $\sin^{-1}x = 2 \cos^{-1}x$, giving an exact answer. [4]
- (c) Solve $3 \sin^{-1}x + \cos^{-1}x = \frac{5\pi}{6}$. [3]
-

30 Use the principal value of $\arctan x$.

(a) By drawing a right-angled triangle, or otherwise, show that $\sin(2 \arctan x) \equiv \frac{2x}{1+x^2}$. [4]

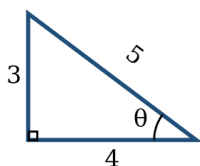
(b) Hence solve $\sin(2 \arctan x) = \frac{24}{25}$. [3]

(c) Show that $\sin(2 \arctan x) \leq 1$ for all x , and state the value of x for which it equals 1. [3]

Compound and Double Angle Formulae

1 By writing $75^\circ = 45^\circ + 30^\circ$, show that $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$. [3]

2 The angle θ is acute and $\sin \theta = \frac{3}{5}$.
Find the exact values of $\sin 2\theta$ and $\cos 2\theta$.



[3]

3 Given that $\sin x = \frac{1}{3}$, find the exact value of $\cos 2x$. [2]

4 Write as a single trigonometric ratio, and hence find the exact value where asked: (a) $2 \sin 20^\circ \cos 20^\circ$, (b) $\cos^2 15^\circ - \sin^2 15^\circ$. [3]

5 Expand $\sin(x + 30^\circ)$ in terms of $\sin x$ and $\cos x$, giving exact coefficients. [2]

6 By writing $15^\circ = 45^\circ - 30^\circ$, show that $\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$. [3]

7 A and B are acute angles with $\tan A = 2$ and $\tan B = \frac{1}{3}$.
(a) Find the value of $\tan(A + B)$.
(b) Find $\tan(A - B)$ and hence show that $A - B = 45^\circ$. [5]

8 A and B are acute angles with $\cos A = \frac{4}{5}$ and $\sin B = \frac{5}{13}$.
Find the exact values of (a) $\cos(A + B)$, (b) $\sin(A - B)$. [5]

9 Prove that $\sin(x + y) + \sin(x - y) \equiv 2 \sin x \cos y$. [3]

10 Solve $\sin 2x = \cos x$ for $0^\circ \leq x \leq 180^\circ$. [4]

11 Show that $\frac{\cos 2\theta}{\cos \theta - \sin \theta} \equiv \cos \theta + \sin \theta$. [3]

12 Solve $4 \sin 2x \cos x + 4 \cos 2x \sin x = 3$ for $0^\circ \leq x \leq 180^\circ$, giving your answers to 1 decimal place. [5]

- 13** The angle θ is obtuse with $\sin \theta = \frac{9}{41}$. The angle φ is in the third quadrant ($180^\circ < \varphi < 270^\circ$) with $\cos \varphi = -\frac{3}{5}$.
- (a) Find the exact value of $\cos(\theta + \varphi)$. [4]
 (b) Find the exact value of $\sin(\theta - \varphi)$. [2]

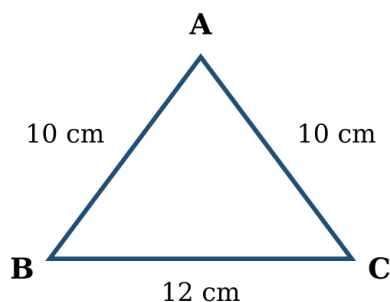
- 14** Solve $\cos 2x = 3 \cos x + 1$ for $0^\circ \leq x \leq 360^\circ$. [5]

- 15** The identity $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$ is to be proved.
- (a) By writing $3x = 2x + x$, prove that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [5]
 (b) Verify the result of part (a) when $x = 30^\circ$. [2]

- 16** Prove that $\frac{\sin 2x}{1 + \cos 2x} \equiv \tan x$. [4]

- 17** Solve $\tan(x + 45^\circ) = 3$ for $0^\circ \leq x \leq 360^\circ$, by first expanding the left-hand side. Give answers to 1 decimal place. [4]

- 18** In the isosceles triangle ABC, $AB = AC = 10$ cm and $BC = 12$ cm. Let angle $BAC = 2\theta$. Find $\sin \theta$, and use a double angle formula to find the exact value of $\cos 2\theta$.



[4]

- 19** This question is about the equation $\sin(\theta + \alpha) = 2 \sin \theta$, where α is a constant.
- (a) Show that $\tan \theta = \frac{\sin \alpha}{2 - \cos \alpha}$. [4]
 (b) Hence solve $\sin(\theta + 60^\circ) = 2 \sin \theta$ for $0^\circ \leq \theta < 360^\circ$. [3]

- 20** It is given that $\tan 10^\circ = t$.
- (a) Express $\tan 55^\circ$ in terms of t . [2]
 (b) Express $\tan 35^\circ$ in terms of t , and hence show that $\tan 55^\circ \tan 35^\circ = 1$. [3]
 (c) Given that $3 \cos(\theta + 10^\circ) = 2 \sin(\theta - 10^\circ)$, show that $\tan \theta = \frac{3 + 2t}{2 + 3t}$. [4]

- 21** This question is about compound angle formulae.
- (a) Solve $\sin(x + 30^\circ) = 2 \cos x$ for $0^\circ \leq x \leq 360^\circ$. [5]
 (b) Prove that $\cos(A + B) + \cos(A - B) \equiv 2 \cos A \cos B$. [2]
 (c) Hence show that $\cos 75^\circ + \cos 15^\circ = \frac{\sqrt{6}}{2}$. [2]

- 22** Consider the equation $\tan 2x = 3 \tan x$, where $\tan x$ and $\tan 2x$ are defined.
- (a) Show that the equation can be written as $\tan x (3 \tan^2 x - 1) = 0$. [3]
 (b) Hence solve the equation for $0^\circ \leq x \leq 180^\circ$. [3]
 (c) Explain why $x = 45^\circ$ is not a solution of $\tan 2x = 3 \tan x$. [2]

- 23** This question is about $\tan x$ in terms of double angles.
- (a) Prove that $\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x$. [3]
- (b) Hence show that $\tan 22.5^\circ = \sqrt{2} - 1$. [3]
-
- 24** Consider the expression $4 \sin x \cos x \cos 2x$.
- (a) Show that $4 \sin x \cos x \cos 2x \equiv \sin 4x$. [3]
- (b) Hence solve $4 \sin x \cos x \cos 2x = \frac{1}{2}$ for $0^\circ \leq x \leq 180^\circ$. [3]
-
- 25** For each statement, decide whether it is true for all values of the angles. If not, give a counterexample.
- (a) $\sin(A + B) = \sin A + \sin B$ [2]
- (b) $\cos 2x = 2 \cos x$ [2]
- (c) $\sin 2x = 2 \sin x \cos x$ [1]
-
- 26** Consider the equation $\cos 2x + 5 \sin x + 2 = 0$ for $0 \leq x \leq 2\pi$.
- (a) Show that the equation can be written as $2 \sin^2 x - 5 \sin x - 3 = 0$. [3]
- (b) Hence solve the equation, giving your answers as multiples of π . [4]
-
- 27** This question is about the identity $\cos 4x \equiv 8 \cos^4 x - 8 \cos^2 x + 1$.
- (a) Prove the identity. [3]
- (b) Hence solve $8 \cos^4 x - 8 \cos^2 x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]
- (c) Use $\cos 45^\circ = 2 \cos^2 22.5^\circ - 1$ to find the exact value of $\cos 22.5^\circ$. [3]
- (d) Solve the quadratic $8u^2 - 8u + 1 = 0$ and hence show that $\cos 67.5^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2}$. [3]
-
- 28** A, B and C are the angles of a triangle, none of which is a right angle.
- (a) Prove that $\tan A + \tan B + \tan C \equiv \tan A \tan B \tan C$. [4]
- (b) In a particular triangle $\tan A = 2$ and $\tan B = 3$. Find $\tan C$. [2]
- (c) For the triangle in part (b), find $\sin A$, $\sin B$ and $\sin C$ exactly, and hence find the ratio $a : b : c$ of the sides in the form $p : q : \sqrt{5}$. [4]
-
- 29** This question links triple angles to a cubic equation.
- (a) Prove that $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$. [4]
- (b) Hence show that $\cos 20^\circ$ is a root of $8c^3 - 6c - 1 = 0$. [2]
- (c) The other two roots are $\cos 100^\circ$ and $\cos 140^\circ$. Find the exact value of $\cos 20^\circ \cos 100^\circ \cos 140^\circ$. [2]
-
- 30** Let $c = \cos 36^\circ$. This question finds its exact value.
- (a) Explain why $\cos 108^\circ = -\cos 72^\circ$, and hence show that $4c^3 - 3c = -(2c^2 - 1)$. [4]
- (b) Show that $4c^3 + 2c^2 - 3c - 1 = (c + 1)(4c^2 - 2c - 1)$, and hence find the exact value of $\cos 36^\circ$. [4]
- (c) Hence show that $\cos 72^\circ = \frac{\sqrt{5} - 1}{4}$. [2]

R Formula

- 1** Write $5 \sin x + 12 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 1 decimal place. [3]
-
- 2** Express $\cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
-
- 3** Express $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 2 decimal places. [3]

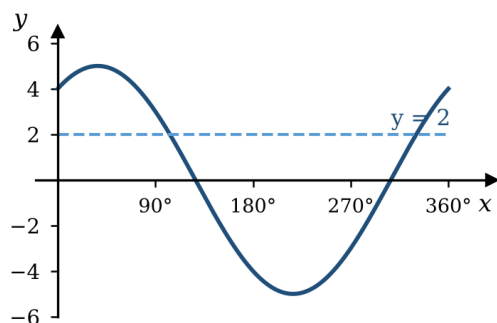
4 Express $\sqrt{3} \cos x + \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

5 Find the greatest value of $12 \sin x + 5 \cos x$. [2]

6 Given that $6 \sin x - 8 \cos x = 10 \sin(x - \alpha)$, find the smallest value of $7 + 6 \sin x - 8 \cos x$. [2]

7 (a) Express $3 \sin x - 2 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give R as a surd and α to 1 decimal place.
(b) Write down the greatest value of $3 \sin x - 2 \cos x$. [4]

8 The diagram shows the graph of $y = 3 \sin x + 4 \cos x$ for $0^\circ \leq x \leq 360^\circ$ and the line $y = 2$. Given that $3 \sin x + 4 \cos x \equiv 5 \sin(x + 53.13^\circ)$, solve $3 \sin x + 4 \cos x = 2$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place.



[3]

9 Solve $5 \sin x + 12 \cos x = 6$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [5]

10 Use x in radians.

(a) Express $\sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(b) Hence solve $\sin x - \cos x = 1$ for $0 \leq x \leq 2\pi$. [3]

11 Express $2 \sin 2x + 3 \cos 2x$ in the form $R \sin(2x + \alpha)$, $R > 0$, $0^\circ < \alpha < 90^\circ$. Hence state the greatest value and the smallest positive x at which it occurs, to 1 decimal place. [5]

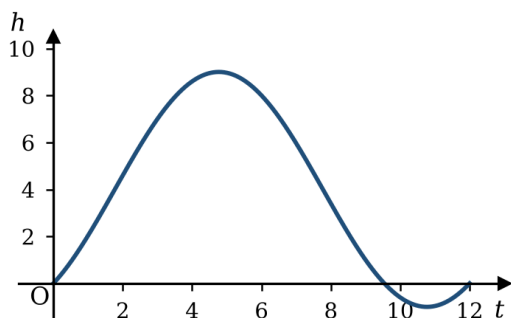
12 This question is about the function $8 \sin x + 6 \cos x$.

(a) Express $8 \sin x + 6 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 2 decimal places. [3]

(b) Find the greatest value of $8 \sin x + 6 \cos x$ and the smallest positive value of x at which it occurs. [2]

(c) Find the greatest value of $\frac{20}{8 \sin x + 6 \cos x + 15}$. [2]

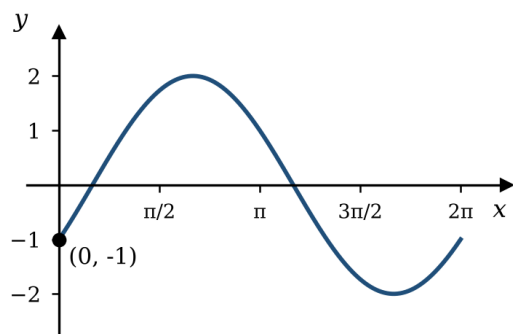
- 13** The height of the water, h metres, above a marker in a lock t hours after midnight is modelled by $h = 4 + 3 \sin(30t)^\circ - 4 \cos(30t)^\circ$, for $0 \leq t \leq 12$. Part of the graph is shown.



- (a) Express $3 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 2 decimal places. [3]
- (b) Find the maximum height and the first time at which it occurs, giving the time in hours to 2 decimal places. [2]
- (c) Find the first time at which the height is 6.5 m, to 2 decimal places. [2]
-
- 14** The temperature, T °C, in a greenhouse t hours after midnight is modelled by $T = 15 + 4 \sin(\frac{\pi t}{12}) - 3 \cos(\frac{\pi t}{12})$, for $0 \leq t < 24$, where the angles are in radians.
- (a) Express $4 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give α to 4 decimal places. [3]
- (b) Find the maximum temperature and the time, to the nearest minute, at which it first occurs. [3]
- (c) Find the times, to the nearest minute, at which $T = 17$. [4]
-
- 15** $f(x) = 7 + 5 \sin x - 12 \cos x$, for $0^\circ \leq x < 360^\circ$.
- (a) Express $5 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (b) Find the maximum and minimum values of $f(x)$ and the values of x at which they occur. [5]
-
- 16** Solve $2 \sin 2x + \cos 2x = 1$ for $0^\circ \leq x \leq 180^\circ$, giving your answers to 1 decimal place. [6]
-
- 17** Given that $3 \sin x + 4 \cos x \equiv 5 \sin(x + 53.13^\circ)$, find the number of solutions in $0^\circ \leq x \leq 360^\circ$ of $3 \sin x + 4 \cos x = k$ when
- (a) $k = 5$ [2]
- (b) $k = 4$ [1]
- (c) $k = 6$ [1]
- (d) $k = -5$ [1]
-
- 18** Sketch the graph of $y = 2 \sin x + 2\sqrt{3} \cos x$ for $0^\circ \leq x \leq 360^\circ$, after writing it in the form $R \sin(x + \alpha)$. Show the coordinates of the maximum and minimum points and of the y -intercept. [6]
-
- 19** This question is about the equation $3 \cot x + 2 = 3 \operatorname{cosec} x$ for $0^\circ < x < 360^\circ$.
- (a) Show that the equation can be written as $3 \cos x + 2 \sin x = 3$. [2]
- (b) Express $3 \cos x + 2 \sin x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 2 decimal places. [3]
- (c) Hence solve the equation $3 \cot x + 2 = 3 \operatorname{cosec} x$, giving your answer to 1 decimal place. Explain why one root of the R -form equation must be rejected. [5]

- 20** This question is about the expression $2 \cos x - \sin x$.
- (a) Express $2 \cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give α to 2 decimal places. [3]
- (b) Solve $2 \cos x - \sin x = 1$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to 1 decimal place. [3]
- (c) Find the greatest value of $\frac{1}{2 \cos x - \sin x + 3}$, giving your answer in the form $p + q\sqrt{5}$ where p and q are rational. [3]

- 21** The diagram shows the curve $y = \sqrt{3} \sin x - \cos x$ for $0 \leq x \leq 2\pi$.



- (a) Show that $\sqrt{3} \sin x - \cos x \equiv 2 \sin(x - \frac{\pi}{6})$. [3]
- (b) Hence solve $\sqrt{3} \sin x - \cos x = 1$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π . [3]
- (c) Find the number of solutions of $\sqrt{3} \sin x - \cos x = k$ for $0 \leq x \leq 2\pi$ when (i) $k = 2$, (ii) $k = 1.5$, (iii) $k = -1$. [3]

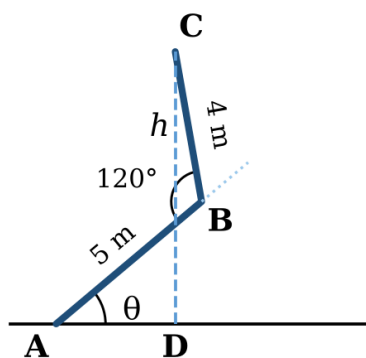
- 22** $f(x) = \frac{1}{6 + 3 \sin x + 4 \cos x}$, for $0^\circ \leq x \leq 360^\circ$.
- (a) Write $3 \sin x + 4 \cos x$ in the form $R \sin(x + \alpha)$ and hence find the greatest and least values of $f(x)$. [5]
- (b) Solve $f(x) = \frac{1}{3}$. [4]

- 23** It is given that $a \sin x + b \cos x \equiv 4 \sin(x + 60^\circ)$, where a and b are constants.
- (a) Find the exact values of a and b . [3]
- (b) Solve $a \sin x + b \cos x = 2$ for $0^\circ \leq x \leq 360^\circ$. [3]

- 24** A student writes: " $3 \sin x - 4 \cos x = 5 \sin(x + 53.13^\circ)$ ".
- (a) Show that this is wrong by substituting $x = 0^\circ$. [2]
- (b) Find the correct form $R \sin(x - \alpha)$ with $0^\circ < \alpha < 90^\circ$. [3]
- (c) Hence solve $3 \sin x - 4 \cos x = 4$ for $0^\circ \leq x \leq 360^\circ$. [3]

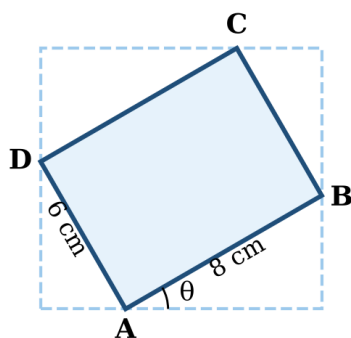
- 25** $g(x) = \sqrt{3} \sin x + \cos x$, for $0 \leq x \leq 2\pi$.
- (a) Express $g(x)$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]
- (b) Solve $g(x) = \sqrt{2}$, giving your answers as multiples of π . [3]

- 26** A rigid rod ABC has $AB = 5$ m, $BC = 4$ m and angle $ABC = 120^\circ$. The rod is hinged at A and can turn in a vertical plane. The angle between AB and the horizontal ground is θ , where $0^\circ < \theta < 180^\circ$, and h metres is the height of C above the ground.



- Show that $h = 7 \sin \theta + 2\sqrt{3} \cos \theta$. [4]
- Express h in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Hence find the greatest possible height of C and the value of θ at which it occurs. [6]
- Find the values of θ for which C is 6 m above the ground. [5]

- 27** A rectangle ABCD, with $AB = 8$ cm and $AD = 6$ cm, is placed so that AB makes an acute angle θ with the horizontal. It fits exactly inside the smallest possible horizontal rectangle (shown dashed) whose sides touch the rectangle ABCD, as in the diagram.



- Show that the width of the dashed rectangle is $W = 8 \cos \theta + 6 \sin \theta$. [2]
- Express W in the form $R \cos(\theta - \alpha)$, $R > 0$, $0^\circ < \alpha < 90^\circ$, and hence find the greatest possible width and the value of θ for which it occurs. [3]
- Find the two values of θ , between 0° and 90° , for which $W = 9$ cm. Give your answers to 1 decimal place. [3]
- Show that the area of the dashed rectangle is $48 + 50 \sin 2\theta \text{ cm}^2$, and find its greatest value. [3]

- 28** The R formula is used to analyse expressions of the form $a \sin x + b \cos x$.

- Prove that, for constants a and b which are not both zero, $a \sin x + b \cos x \equiv R \sin(x + \alpha)$, where $R = \sqrt{a^2 + b^2}$ and α is an angle you should define. [4]
- Find the set of values of p for which the equation $2 \sin x + p \cos x = 3$ has solutions. [3]
- Show that $3 \cos^2 x + 4 \sin x \cos x \equiv \frac{3}{2} + \frac{3}{2} \cos 2x + 2 \sin 2x$. [5]
 - Hence find the greatest value of $3 \cos^2 x + 4 \sin x \cos x$.

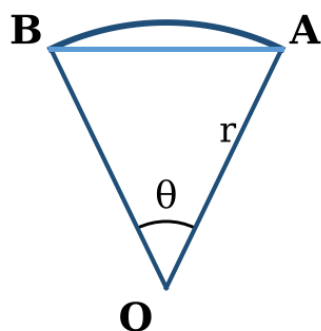
- 29** This question proves that $a \sin x + b \cos x \leq \sqrt{a^2 + b^2}$ for all x .

- Prove that $(a \sin x + b \cos x)^2 + (a \cos x - b \sin x)^2 \equiv a^2 + b^2$. [3]
- Deduce that $|a \sin x + b \cos x| \leq \sqrt{a^2 + b^2}$, with equality when $a \cos x = b \sin x$. [3]
- Use this to find the greatest value of $5 \sin x + 12 \cos x$ and the value of $\tan x$ at which it occurs. [3]

- 30** The depth of water, h metres, at a harbour t hours after midnight is modelled by $h = 6 + 3 \sin(30t)^\circ + 4 \cos(30t)^\circ$, for $0 \leq t \leq 24$.
- (a) Express $3 \sin(30t)^\circ + 4 \cos(30t)^\circ$ in the form $R \sin(30t + \alpha)^\circ$, with $R > 0$ and $0 < \alpha < 90$. [2]
- (b) Find the maximum depth and the first time, after midnight, at which it occurs, in hours and minutes to the nearest minute. [4]
- (c) A ship needs a depth of at least 8 m. Find the total time in the 24 hours for which the ship can use the harbour, to 2 decimal places. [6]

Small Angle Approximations

- 1** For small angles θ , measured in radians, write down approximations for $\sin \theta$, $\cos \theta$ and $\tan \theta$. [3]
- 2** θ is small and measured in radians. Write down approximations for (a) $\sin 3\theta$, (b) $\cos 2\theta$, (c) $\tan 5\theta$. [3]
- 3** Use a small angle approximation to estimate $\cos 0.1$, and compare it with the calculator value to 5 decimal places. [3]
- 4** For small θ (in radians), find an approximation of the form $k\theta$ for $4 \sin 2\theta + 3 \tan \theta$. [2]
- 5** Use $\tan \theta \approx \theta$ to estimate $\tan 0.04$. Find the percentage error compared with the calculator value, to 2 significant figures. [4]
- 6** Show that, for small θ (in radians), $\frac{1 - \cos \theta}{\theta^2} \approx \frac{1}{2}$. [2]
- 7** For small θ , show that $\sin \theta \tan \theta \approx \theta^2$ and $\cos \theta \sin \theta \approx \theta - \frac{\theta^3}{2}$. [3]
- 8** Show that, for small θ , $\cos 3\theta + 2 \sin 2\theta \approx 1 + 4\theta - \frac{9\theta^2}{2}$. [4]
- 9** Use a small angle approximation, after converting to radians, to estimate $\sin 2^\circ$. Give your answer to 4 significant figures and compare it with the calculator value. [3]
- 10** Given that θ is small and $2 \cos \theta + 3 \sin \theta = 2.29$, use small angle approximations to show that $\theta \approx 0.1$. [5]
- 11** The diagram shows a chord AB of a circle of radius r , centre O, with angle $AOB = \theta$, where θ is small and in radians. Show that the length of the chord is approximately $r\theta$.



[4]

- 12** The Moon subtends an angle of 0.0087 radians at an observer on Earth, and is 384 000 km away. Use a small angle approximation to estimate the Moon's diameter, to 3 significant figures. [3]

- 13** Show that, for small θ in radians, $\frac{3 + \cos 2\theta}{1 + \sin \theta} \approx 4 - 4\theta$, ignoring terms in θ^2 and higher. [5]

- 14** Show that, for small θ , $4 \cos \theta + 3 \sin 2\theta \approx 4 + 6\theta - 2\theta^2$. [4]

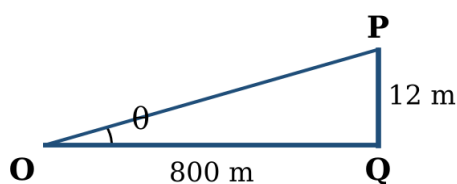
- 15** The small angle θ (in radians) satisfies $\cos 2\theta + 3 \sin \theta = 1.1$. Use small angle approximations to find an approximate value of θ , to 3 significant figures. [5]

- 16** Find the value that $\frac{\theta \sin 2\theta}{1 - \cos 3\theta}$ approaches as $\theta \rightarrow 0$. [4]

- 17** The approximation $\sin \theta \approx \theta$ is used for small θ in radians.

- (a) Find the percentage error in using $\cos \theta \approx 1 - \frac{\theta^2}{2}$ when $\theta = 0.2$, to 2 significant figures. [4]
 (b) Find the largest value of θ , to 2 decimal places, for which $\sin \theta \approx \theta$ has an error of less than 1% of the true value of $\sin \theta$. [4]

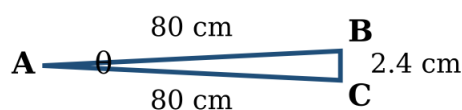
- 18** A vertical pole PQ of height 12 m stands on level ground. The angle of elevation of P from a point O, 800 m from Q, is θ radians. Use a small angle approximation to estimate θ , and convert it to degrees to 2 decimal places.



[4]

- 19** Find the values that the following approach as $\theta \rightarrow 0$: (a) $\frac{\sin 3\theta}{\theta(2 + \cos \theta)}$, (b) $\frac{\tan 4\theta}{2\theta \cos \theta}$. [4]

- 20** In the long thin triangle ABC, $AB = AC = 80$ cm, $BC = 2.4$ cm and angle $BAC = \theta$ radians. Use the cosine rule and a small angle approximation to show that $BC \approx 80\theta$, and hence estimate θ .



[5]

- 21** Two points A and B on the equator of the Earth (radius 6371 km) are separated by an angle of $\theta = 0.002$ radians at the centre.

- (a) Find the arc length AB along the surface, and the straight line chord AB. [4]
 (b) Using $\sin x \approx x - \frac{x^3}{6}$ for small x , show that arc - chord $\approx \frac{r\theta^3}{24}$ and evaluate it in millimetres, to 2 significant figures. [5]

- 22** The small angle θ (in radians) satisfies $3 \cos \theta + \tan 2\theta = 3.185$. Use small angle approximations to show that $3\theta^2 - 4\theta + 0.37 = 0$, and hence find θ to 2 significant figures. [6]

- 23** A pendulum bob hangs from a string of length $L = 1.5$ m. It is pulled aside until the string makes a small angle θ radians with the vertical.
- (a) Show that the bob has risen by $L(1 - \cos \theta)$ and that, for small θ , this is approximately $\frac{L\theta^2}{2}$. [3]
- (b) For $\theta = 0.12$ find the rise using the approximation and using the exact formula, in metres to 3 significant figures, and the percentage error. [4]

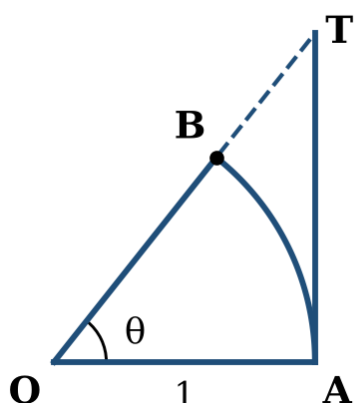
- 24** A student estimates $\sin 30^\circ$ using the small angle approximation $\sin \theta \approx \theta$ and writes $\sin 30^\circ \approx 30$.
- (a) Explain the student's error. [2]
- (b) Find the approximation of $\sin 30^\circ$ correctly, and its percentage error to 2 significant figures. [4]
- (c) Comment on whether the approximation is reliable at 30° . [1]

- 25** In this question, use $0.5 = \frac{\pi}{6} - h$.
- (a) Find h to 3 significant figures, and show that for small h , $\sin(0.5) \approx \frac{1}{2}(1 - \frac{h^2}{2}) - \frac{\sqrt{3}}{2}h$. [3]
- (b) Evaluate this approximation to 4 decimal places and compare it with the calculator value of $\sin 0.5$. [3]

- 26** A lighthouse of height 40 m is observed from a boat. The angle of elevation of the top is α radians. The boat then sails 1200 m directly towards the lighthouse and the new angle of elevation is 0.02 radians. Assume all angles are small.
- (a) Show that the boat is initially d metres from the lighthouse, where $d \approx \frac{40}{\alpha}$. [2]
- (b) After sailing, the distance is $40 \div 0.02 = 2000$ m. Find the initial distance and the initial angle α . [3]

- 27** For small θ in radians:
- (a) Show that $\cos \theta + \sin \theta \approx 1 + \theta - \frac{\theta^2}{2}$. [2]
- (b) Hence show that $\frac{1}{\cos \theta + \sin \theta} \approx 1 - \theta + \frac{3\theta^2}{2}$, ignoring terms in θ^3 and higher. [4]
- (c) Verify the result for $\theta = 0.05$, to 3 significant figures. [2]

- 28** The diagram shows a sector OAB of a unit circle with centre O and angle AOB = θ , where $0 < \theta < \frac{\pi}{2}$. The tangent at A meets OB extended at T.



- (a) Find, in terms of θ , the areas of triangle OAB, sector OAB and triangle OAT. [4]
- (b) Explain why $\sin \theta < \theta < \tan \theta$. [2]
- (c) Hence show that $\cos \theta < \frac{\sin \theta}{\theta} < 1$, and explain how this justifies $\sin \theta \approx \theta$ for small θ . [4]

29 Consider the equation $\cos \theta = 2\theta$.

- (a) Use $\cos \theta \approx 1 - \frac{\theta^2}{2}$ to show that an approximate solution satisfies $\theta^2 + 4\theta - 2 = 0$ and find it, to 3 significant figures. [4]
- (b) Show that the equation has a root between 0.4 and 0.5. [2]
- (c) Use the iteration $\theta_{n+1} = \frac{1}{2} \cos \theta_n$ with $\theta_1 = 0.45$ to find the root correct to 4 decimal places, and find the error in the approximation from part (a). [4]

30 This question differentiates $\sin x$ from first principles.

- (a) Show that $\frac{\sin(x+h) - \sin x}{h} = \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right)$. [2]
- (b) For small h (radians), use small angle approximations to show that this is approximately $\cos x - \frac{h}{2} \sin x$. [4]
- (c) Hence state the derivative of $\sin x$, and explain what happens to the extra term as $h \rightarrow 0$. [2]