

Basic Probability

1 (2 marks)

(a) $P(\text{blue}) = \frac{3}{10} = \frac{3}{10}$

B1

Answer: $\frac{3}{10}$

1 mark

(b) $P(\text{not green}) = \frac{8}{10} = \frac{4}{5}$

B1

Answer: $\frac{4}{5}$

1 mark

2 (3 marks)

(a) Prime scores: 2, 3, 5, so $P = \frac{3}{6} = \frac{1}{2}$

B1

Answer: $\frac{1}{2}$

1 mark

(b) Scores that are prime or even: 2, 3, 4, 5, 6 (five outcomes)

M1

$P = \frac{5}{6}$

A1

Answer: $\frac{5}{6}$

2 marks

3 (2 marks)

(a) Primes: 2, 3, 5, 7, 11, so $P = \frac{5}{12}$

B1

Answer: $\frac{5}{12}$

1 mark

(b) 4, 8, 12, so $P = \frac{3}{12} = \frac{1}{4}$

B1

Answer: $\frac{1}{4}$

1 mark

4 (2 marks)

(a) $1 - 0.18 = 0.82$

B1

Answer: 0.82

1 mark

(b) $250 \times 0.18 = 45$

B1

Answer: 45

1 mark

5 (4 marks)

(a) $P = \frac{13}{52} = \frac{1}{4}$

B1

Answer: $\frac{1}{4}$

1 mark

(b) $P = \frac{4}{52} = \frac{1}{13}$

B1

Answer: $\frac{1}{13}$

1 mark

(c) 13 hearts + 4 kings - 1 (king of hearts counted twice) = 16 cards

M1

$P = \frac{16}{52} = \frac{4}{13}$

A1

Answer: $\frac{4}{13}$

2 marks

6 (2 marks)

Outcomes: HH, HT, TH, TT (equally likely)

B1

Exactly one head: HT, TH, so $P = \frac{2}{4} = \frac{1}{2}$

B1

Answer: $\frac{1}{2}$

2 marks

7 (4 marks)

$$(a) P = \frac{30}{80} = \frac{3}{8}$$

B1

$$\text{Answer: } \frac{3}{8}$$

1 mark

$$(b) P = \frac{14}{80} = \frac{7}{40}$$

B1

$$\text{Answer: } \frac{7}{40}$$

1 mark

(c) Under 30: 40 members; 30 and over who attend classes: 14

M1

$$P = \frac{(40 + 14)}{80} = \frac{27}{40}$$

A1

$$\text{Answer: } \frac{27}{40}$$

2 marks**8 (5 marks)**

(a) Each cell is the product of the row and column scores (12 equally likely outcomes)

B1**Answer: See table****1 mark**(b) Products greater than 6: 8, 9, 12, so $P = \frac{3}{12} = \frac{1}{4}$ **B1**

$$\text{Answer: } \frac{1}{4}$$

1 mark(c) Even products: 2, 2, 4, 4, 6, 6, 8, 12, so $P = \frac{8}{12} = \frac{2}{3}$ **B1**

$$\text{Answer: } \frac{2}{3}$$

1 mark

(d) Outcomes that are multiples of 3 or greater than 6: 3, 3, 6, 6, 8, 9, 12

M1

$$P = \frac{7}{12}$$

A1

$$\text{Answer: } \frac{7}{12}$$

2 marks

×	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9
4	4	8	12

9 (6 marks)

(a) Total 8 occurs for (2,6), (3,5), (4,4), (5,3), (6,2): 5 of the 36 outcomes

M1

$$P = \frac{5}{36}$$

A1

$$\text{Answer: } \frac{5}{36}$$

2 marks(b) Totals 10, 11, 12 occur $3 + 2 + 1 = 6$ times**M1**

$$P = \frac{6}{36} = \frac{1}{6}$$

A1

$$\text{Answer: } \frac{1}{6}$$

2 marks

(c) Primes: 2 (1 way), 3 (2), 5 (4), 7 (6), 11 (2) = 15 outcomes

M1

$$P = \frac{15}{36} = \frac{5}{12}$$

A1

$$\text{Answer: } \frac{5}{12}$$

2 marks

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

10 (4 marks)

(a) Relative frequency = $\frac{40}{200} = 0.2$

B1

Answer: 0.2

1 mark

(b) $500 \times 0.2 = 100$

B1

Answer: 100

1 mark

(c) For a fair die the relative frequency of a 6 would be $\frac{1}{6} \approx 0.167$, and 0.2 is close to this

E1

All six frequencies are close to the fair value 33.3, so there is no strong evidence of bias

E1

Answer: No strong evidence

2 marks

11 (3 marks)

(a) Mutually exclusive, so $P(A \cup B) = 0.3 + 0.45 = 0.75$

B1

Answer: 0.75

1 mark

(b) $1 - 0.75 = 0.25$

B1

Answer: 0.25

1 mark

(c) $0.75 + 0.2 = 0.95$

B1

Answer: 0.95

1 mark

12 (4 marks)

(a) There are 10 letters and 3 are T: $P = \frac{3}{10}$

B1

Answer: $\frac{3}{10}$

1 mark

(b) Vowels: A, I, I, so $P = \frac{3}{10}$

B1

Answer: $\frac{3}{10}$

1 mark

(c) S appears 3 times and I appears 2 times (no letter is both)

M1

$P = \frac{5}{10} = \frac{1}{2}$

A1

Answer: $\frac{1}{2}$

2 marks

13 (3 marks)

Outcomes: $2^3 = 8$ equally likely

B1

At least two heads: HHH, HHT, HTH, THH

M1

$P = \frac{4}{8} = \frac{1}{2}$

A1

Answer: $\frac{1}{2}$

3 marks

14 (6 marks)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

M1

$P(A \cap B) = 0.35 + 0.4 - 0.6 = 0.15$

A1

Answer: 0.15

2 marks

(b) $P(A \cap B) = 0.15 \neq 0$, so the events can occur together and are not mutually exclusive

E1

Answer: Not mutually exclusive

1 mark

(c) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.6 = 0.4$

Answer: 0.4

A1

1 mark

(d) $P(\text{exactly one}) = P(A \cup B) - P(A \cap B)$

$= 0.6 - 0.15 = 0.45$

Answer: 0.45

M1

A1

2 marks

15 (5 marks)

(a) Probabilities sum to 1: $0.1 + x + 2x + 3x + 0.15 = 1$

$6x = 0.75$, so $x = 0.125$

Answer: 0.125

M1

A1

2 marks

(b) $P(3) + P(4) + P(5) = 0.25 + 0.375 + 0.15$

$= 0.775$

Answer: 0.775

M1

A1

2 marks

(c) $200 \times 0.375 = 75$

Answer: 75

B1

1 mark

16 (6 marks)

(a) Difference of 2: (1,3), (2,4), (3,5), (4,6) and the reverse orders, 8 outcomes

M1

$P = \frac{8}{36} = \frac{2}{9}$

A1

Answer: $\frac{2}{9}$

2 marks

(b) Differences 3, 4, 5 occur $6 + 4 + 2 = 12$ times

M1

$P = \frac{12}{36} = \frac{1}{3}$

A1

Answer: $\frac{1}{3}$

2 marks

(c) Products above 20: 4×6 , 6×4 , 5×5 , 5×6 , 6×5 , 6×6 , so 6 outcomes

M1

$P = \frac{6}{36} = \frac{1}{6}$

A1

Answer: $\frac{1}{6}$

2 marks

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

17 (4 marks)

$P(RRR) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$

M1 A1

$P(BBB) = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

A1

$P(\text{same colour}) = \frac{1}{21} + \frac{5}{42} = \frac{1}{6}$

A1

Answer: $\frac{1}{6}$

4 marks

18 (6 marks)

(a) Multiples of 7 from 105 to 994: $\frac{994 - 105}{7} + 1 = 128$ M1

$$P = \frac{128}{900} = \frac{32}{225}$$

A1

Answer: $\frac{32}{225}$ 2 marks

(b) First digit 9 ways, second 9 ways (not the first), third 8 ways: 648 M1

$$P = \frac{648}{900} = \frac{18}{25}$$

A1

Answer: $\frac{18}{25}$ 2 marks

(c) First digit 9 ways, middle digit 10 ways, last digit fixed: 90 M1

$$P = \frac{90}{900} = \frac{1}{10}$$

A1

Answer: $\frac{1}{10}$ 2 marks

19 (4 marks)

(a) $5k + 4k + 3k + 2k + k = 1$ M1

$$15k = 1, \text{ so } k = \frac{1}{15}$$

A1

Answer: $\frac{1}{15}$ 2 marks

(b) $\frac{3 + 2 + 1}{15} = \frac{6}{15} = \frac{2}{5}$ A1

Answer: $\frac{2}{5}$ 1 mark

(c) $\frac{5 + 3 + 1}{15} = \frac{9}{15} = \frac{3}{5}$ A1

Answer: $\frac{3}{5}$ 1 mark

20 (4 marks)

Originally $\frac{3}{8}n$ red counters, so now $(\frac{3}{8}n + 4)$ red out of $(n + 4)$ M1

$$\frac{\frac{3}{8}n + 4}{n + 4} = \frac{1}{2}$$

A1

$$\frac{3}{8}n + 4 = \frac{1}{2}n + 2, \text{ so } \frac{1}{8}n = 2$$

M1

$$n = 16$$

A1

Answer: 16 4 marks

21 (7 marks)

(a) $P = \frac{5}{10} \times \frac{4}{9}$ M1

$$= \frac{2}{9}$$

A1

Answer: $\frac{2}{9}$ 2 marks

(b) Numbers leaving remainder 0 on division by 3: 3, 6, 9 (3 numbers); remainder 1: 1, 4, 7, 10 (4 numbers); remainder 2: 2, 5, 8 (3 numbers) B1

Sum is a multiple of 3 if both are from the first group (3 pairs) or one from each of the other M1

groups ($4 \times 3 = 12$ pairs): 15 pairs out of $\frac{10 \times 9}{2} = 45$

$$P = \frac{15}{45} = \frac{1}{3}$$

A1

Answer: $\frac{1}{3}$ 3 marks

(c) Ordered pairs (first, second): (10,1), (9,2), (8,3), (7,4), (6,5); there are $10 \times 9 = 90$ equally likely ordered pairs M1

$$P = \frac{5}{90} = \frac{1}{18}$$

A1

Answer: $\frac{1}{18}$

2 marks

22 (8 marks)

(a) $P(\text{both blue}) = \frac{4}{n} \times \frac{3}{(n-1)}$

M1

$$\frac{12}{n(n-1)} = \frac{2}{15}, \text{ so } 180 = 2n(n-1)$$

M1

$$n^2 - n - 90 = 0$$

A1

Answer: $n^2 - n - 90 = 0$

3 marks

(b) $(n-10)(n+9) = 0$

M1

n must be positive, so $n = 10$

A1

Answer: $n = 10$

2 marks

(c) $P(\text{blue then white}) = \frac{4}{10} \times \frac{6}{9}$

M1

Two orders are possible, so $P = 2 \times \frac{4}{10} \times \frac{6}{9}$

M1

$$= \frac{8}{15}$$

A1

Answer: $\frac{8}{15}$

3 marks

23 (7 marks)

(a) $X = k$ means both scores are at most k , but not both at most $(k-1)$

M1

Number of outcomes: $k^2 - (k-1)^2 = 2k - 1$

M1

Out of 36 equally likely outcomes, so $P(X = k) = \frac{2k-1}{36}$

A1

Answer: shown

3 marks

(b) $P(X = 5) + P(X = 6) = \frac{9}{36} + \frac{11}{36}$

M1

$$= \frac{20}{36} = \frac{5}{9}$$

A1

Answer: $\frac{5}{9}$

2 marks

(c) $P(X = 2) + P(X = 4) + P(X = 6) = \frac{3+7+11}{36}$

M1

$$= \frac{21}{36} = \frac{7}{12}$$

A1

Answer: $\frac{7}{12}$

2 marks

max	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

24 (8 marks)

(a) First person: any day. Second person must avoid one day: $\frac{364}{365}$

M1

Third person must avoid two days: $\frac{363}{365}$, so the product is $\frac{365 \times 364 \times 363}{365^3}$

A1

Answer: shown

2 marks

$$(b) P = 1 - \frac{365 \times 364 \times 363}{365^3} \quad \text{M1}$$

$$= 0.0082 \quad \text{A1}$$

Answer: 0.0082 2 marks

$$(c) P(\text{all different}) \text{ for } n \text{ people} = \frac{365 \times 364 \times \dots \times (365 - n + 1)}{365^n} \quad \text{M1}$$

$$\text{From a calculator, for } n = 22: P(\text{at least two share}) = 0.4757 \quad \text{B1}$$

$$\text{For } n = 23: P(\text{at least two share}) = 0.5073 \quad \text{B1}$$

The first is below 0.5 and the second is above 0.5, so 23 is the smallest such n E1

Answer: $n = 23$ 4 marks

25 (8 marks)

$$(a) \text{ Tail, tail, tail, head: } \left(\frac{1}{2}\right)^4 \quad \text{M1}$$

$$= \frac{1}{16} \quad \text{A1}$$

Answer: $\frac{1}{16}$ 2 marks

$$(b) P = 1 - P(5 \text{ tails in a row}) = 1 - \left(\frac{1}{2}\right)^5 \quad \text{M1}$$

$$= \frac{31}{32} \quad \text{A1}$$

Answer: $\frac{31}{32}$ 2 marks

$$(c) P(\text{first head on toss } 2k) = \left(\frac{1}{2}\right)^{2k} \quad \text{M1}$$

$$\text{Series } \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \text{ is geometric, } a = \frac{1}{4}, r = \frac{1}{4} \quad \text{A1}$$

$$S = \frac{a}{1 - r} = \frac{1}{4} \div \frac{3}{4} \quad \text{M1}$$

$$= \frac{1}{3} \quad \text{A1}$$

Answer: $\frac{1}{3}$ 4 marks

26 (7 marks)

(a) If three letters are correct, the fourth envelope and letter are the only ones left, so the fourth letter is also correct E1

Answer: Impossible 1 mark

(b) Choose which 2 are correct: $C(4, 2) = 6$ M1

The other two must be swapped: 1 way, so 6 favourable arrangements out of $4! = 24$ M1

$$P = \frac{6}{24} = \frac{1}{4} \quad \text{A1}$$

Answer: $\frac{1}{4}$ 3 marks

(c) The cases (none, one, two, three, four correct) are exhaustive and exclusive M1

$$P(\text{none}) = 1 - \frac{1}{3} - \frac{1}{4} - 0 - \frac{1}{24} \quad \text{M1}$$

$$= \frac{9}{24} = \frac{3}{8} \quad \text{A1}$$

Answer: $\frac{3}{8}$ 3 marks

27 (9 marks)

(a) The events "six on roll 1", "six on roll 2" and so on are not mutually exclusive (more than one six is possible), so their probabilities cannot simply be added E1

The same method would give a probability of 1 for 6 rolls, which is clearly not certain E1

Answer: Events are not mutually exclusive 2 marks

(b) $P(\text{at least one six}) = 1 - P(\text{no sixes})$ M1

$$P(\text{no sixes}) = \left(\frac{5}{6}\right)^4 = \frac{625}{1296} \quad \text{M1}$$

$$P = \frac{671}{1296} = 0.518 \text{ (3 s.f.)} \quad \text{A1}$$

Answer: $\frac{671}{1296} = 0.518$ 3 marks

(c) Need $1 - \left(\frac{5}{6}\right)^n > 0.95$, so $\left(\frac{5}{6}\right)^n < 0.05$ M1

$n \log \frac{5}{6} < \log 0.05$, and $\log \frac{5}{6}$ is negative M1

$$n > \frac{\log 0.05}{\log (5/6)} = 16.43 \quad \text{A1}$$

Smallest integer $n = 17$ A1

Answer: $n = 17$ 4 marks

28 (10 marks)

(a) Aisha fails then Ben succeeds: $\frac{5}{6} \times \frac{1}{6}$ M1

$$= \frac{5}{36} \quad \text{A1}$$

Answer: $\frac{5}{36}$ 2 marks

(b) Aisha fails, Ben fails, Aisha succeeds: $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$ M1

$$= \frac{25}{216} \quad \text{A1}$$

Answer: $\frac{25}{216}$ 2 marks

(c) Aisha wins on her n th roll if all earlier rolls ($n - 1$ by her and $n - 1$ by Ben) fail: $P = \left(\frac{5}{6}\right)^{n-1} \times \frac{1}{6}$ B1

$$\left(\frac{5}{6}\right)^{n-1} \times \frac{1}{6}$$

These probabilities form a geometric series with first term $a = \frac{1}{6}$ M1

and common ratio $r = \frac{25}{36}$, where $|r| < 1$ A1

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{6} \div \frac{11}{36} \quad \text{M1}$$

$$= \frac{6}{11} \quad \text{A1}$$

Answer: $\frac{6}{11}$ 5 marks

(d) $P(\text{Ben wins}) = 1 - \frac{6}{11} = \frac{5}{11}$, assuming the game must end (a six is eventually rolled) B1

Answer: $\frac{5}{11}$ 1 mark

29 (12 marks)

(a) Totals 7 or 11: $6 + 2 = 8$ outcomes out of 36 M1

$$P = \frac{8}{36} = \frac{2}{9} \quad \text{A1}$$

Answer: $\frac{2}{9}$ 2 marks

(b) $P(4) = \frac{3}{36}$, $P(7) = \frac{6}{36}$, so $P(\text{neither}) = \frac{27}{36}$ B1

Win on the n th further roll: $\left(\frac{27}{36}\right)^{n-1} \times \frac{3}{36}$ M1

Geometric series with $a = \frac{3}{36}$, $r = \frac{27}{36}$: $\text{sum} = \frac{\frac{3}{36}}{1 - \frac{27}{36}} = \frac{3}{9}$ M1

$$= \frac{1}{3} \quad \text{A1}$$

Answer: $\frac{1}{3}$

4 marks

(c) For a point with probability p of being rolled, $P(\text{win} | \text{point}) = \frac{p}{p + \frac{6}{36}}$

M1

Points 4 and 10: $P(\text{point}) = \frac{3}{36}$, $P(\text{win} | \text{point}) = \frac{3}{9}$; contribution $\frac{1}{36}$ each

A1

Points 5 and 9: $P(\text{point}) = \frac{4}{36}$, $P(\text{win} | \text{point}) = \frac{4}{10}$; contribution $\frac{2}{45}$ each

A1

Points 6 and 8: $P(\text{point}) = \frac{5}{36}$, $P(\text{win} | \text{point}) = \frac{5}{11}$; contribution $\frac{25}{396}$ each

A1

$$P(\text{win}) = \frac{2}{9} + 2\left(\frac{1}{36} + \frac{2}{45} + \frac{25}{396}\right)$$

M1

$$= \frac{244}{495}$$

A1

Answer: $\frac{244}{495}$

6 marks

30 (8 marks)

(a) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

B1

Answer: $\frac{1}{4}$

1 mark

(b) If the first two tosses are not HH, then either the sequence begins with T, or it begins with HT
If it begins with T: the first head to appear is directly preceded by a tail, so TH occurs at that toss, and no HH has occurred before it (there was no earlier head)

E1

E1

If it begins with HT: no HH can occur until another head appears, and that next head is preceded by a tail, so TH occurs first

E1

These cases cover every sequence that does not begin HH, so Beth wins unless the sequence begins HH

E1

Answer: Beth wins

4 marks

(c) $1 - \frac{1}{4} = \frac{3}{4}$

B1

Answer: $\frac{3}{4}$

1 mark

(d) Alex's expected gain = $\frac{1}{4}x - \frac{3}{4} \times 1$

M1

Fair game: expected gain 0, so $x = 3$

A1

Answer: £3

2 marks

Venn Diagrams**1 (3 marks)**

(a) $P = \frac{5}{30} = \frac{1}{6}$

B1

Answer: $\frac{1}{6}$

1 mark

(b) Football: $8 + 5 = 13$ students, so $P = \frac{13}{30}$

B1

Answer: $\frac{13}{30}$

1 mark

(c) $P = \frac{8}{30} = \frac{4}{15}$

B1

Answer: $\frac{4}{15}$

1 mark

2 (3 marks)

Only A: $12 - 4 = 8$

B1

Only B: $9 - 4 = 5$

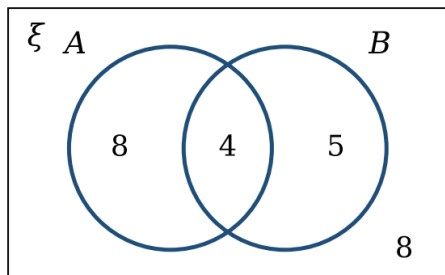
B1

Neither: $25 - (8 + 4 + 5) = 8$

B1

Answer: 8, 5 and 8 in the regions A only, B only and outside

3 marks



3 (3 marks)

(a) $7 + 5 + 9 = 21$

B1

Answer: 21

1 mark

(b) $9 + 4 = 13$

B1

Answer: 13

1 mark

(c) $P = \frac{7}{25}$

B1

Answer: $\frac{7}{25}$

1 mark

4 (4 marks)

(a) $A \cap B = 6, 12$

B1

A only = 2, 4, 8, 10 and B only = 3, 9

B1

Outside both = 1, 5, 7, 11

B1

Answer: See diagram

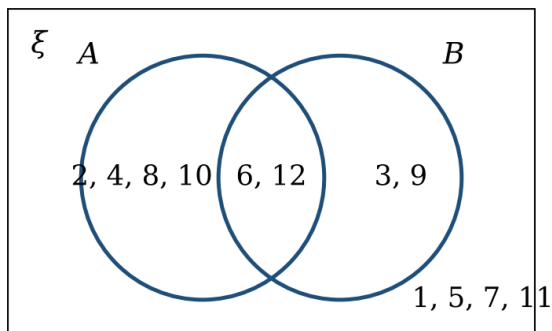
3 marks

(b) 8 elements in $A \cup B$, so $P = \frac{8}{12} = \frac{2}{3}$

B1

Answer: $\frac{2}{3}$

1 mark



5 (5 marks)

(a) $P(\text{A only}) = 0.5 - 0.1 = 0.4$

M1

$P(\text{B only}) = 0.3 - 0.1 = 0.2$

A1

$P(\text{neither}) = 1 - (0.4 + 0.1 + 0.2) = 0.3$

A1

Answer: 0.4, 0.1, 0.2, 0.3

3 marks

(b) $0.4 + 0.1 + 0.2 = 0.7$

A1

Answer: 0.7

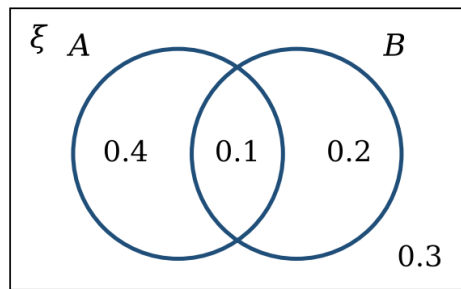
1 mark

(c) The part of B outside A: 0.2

A1

Answer: 0.2

1 mark



6 (5 marks)

(a) $P = \frac{2}{30} = \frac{1}{15}$

B1

Answer: $\frac{1}{15}$

1 mark

(b) Elements in both A and B: $3 + 2 = 5$

M1

$P = \frac{5}{30} = \frac{1}{6}$

A1

Answer: $\frac{1}{6}$

2 marks

(c) $6 + 4 + 5 = 15$ elements

M1

$P = \frac{15}{30} = \frac{1}{2}$

A1

Answer: $\frac{1}{2}$

2 marks

7 (5 marks)

(a) Both: 10; tea only: $25 - 10 = 15$; coffee only: $30 - 10 = 20$

M1 A1

Neither: $60 - (15 + 10 + 20) = 15$

A1

Answer: 15, 10, 20, 15

3 marks

(b) $15 + 20 = 35$ people

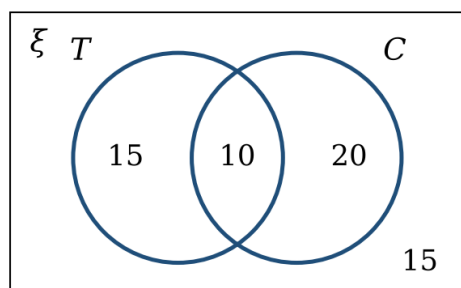
M1

$P = \frac{35}{60} = \frac{7}{12}$

A1

Answer: $\frac{7}{12}$

2 marks



8 (4 marks)

(a) $A' \cup B' = (A \cap B)'$, so $P(A \cap B) = 1 - 0.7 = 0.3$

M1

A1

Answer: 0.3

2 marks

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so $0.8 = 0.5 + P(B) - 0.3$

M1

$P(B) = 0.6$

A1

Answer: 0.6

2 marks

9 (3 marks)

(a) $2x + x + 3 + 5 = 38$

M1

$3x = 30$, so $x = 10$

A1

Answer: $x = 10$

2 marks

(b) A has $2x + x = 30$ elements, so $P = \frac{30}{38} = \frac{15}{19}$

B1

Answer: $\frac{15}{19}$

1 mark

10 (4 marks)

(a) $A \subset B$ so $A \cap B = A$ has 12 elements and A only has 0

B1

B only: $20 - 12 = 8$ and outside: $40 - 20 = 20$

B1

Answer: 0, 12, 8, 20

2 marks

(b) $n(A \cup B) = 20$

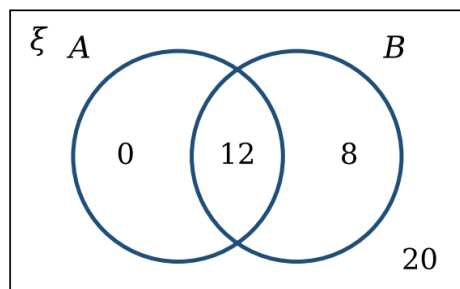
B1

$$P(A' \cap B) = \frac{8}{40} = \frac{1}{5}$$

B1

Answer: 20 and $\frac{1}{5}$

2 marks



11 (5 marks)

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

M1

$$= 0.65 + 0.7 - 1 = 0.35$$

A1

Answer: 0.35

2 marks

(b) $P(A \cap B') = 0.65 - 0.35 = 0.3$

B1

$$P(A' \cap B) = 0.7 - 0.35 = 0.35$$

B1

Answer: 0.3 and 0.35

2 marks

(c) The events are exhaustive so $P(A' \cap B') = 1 - P(A \cup B) = 0$

E1

Answer: $P(A' \cap B') = 0$

1 mark

12 (7 marks)

(a) Exactly two drinks: T and C only $9 - 4 = 5$, C and J only $7 - 4 = 3$, T and J only $8 - 4 = 4$

M1 A1

T only: $22 - 5 - 4 - 4 = 9$

C only: $20 - 5 - 3 - 4 = 8$; J only: $18 - 3 - 4 - 4 = 7$

M1 A1

Answer: 9, 8, 7, 5, 4, 3, 4

4 marks

(b) Inside the circles: $9 + 8 + 7 + 5 + 4 + 3 + 4 = 40$

M1

None: $50 - 40 = 10$

A1

Answer: 10

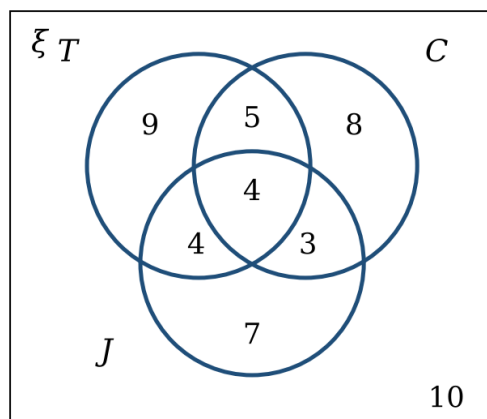
2 marks

$$(c) P = \frac{5 + 4 + 3}{50} = \frac{12}{50} = \frac{6}{25}$$

A1

Answer: $\frac{6}{25}$

1 mark



13 (5 marks)

(a) $(x + 3) + x + (2x - 1) + (x + 2) = 44$

M1

$$5x + 4 = 44$$

M1

$$x = 8$$

A1

Answer: $x = 8$

3 marks

(b) Both 8 and biology only 15: $P = \frac{23}{44}$

A1

Answer: $\frac{23}{44}$

1 mark

(c) $P = \frac{11 + 15}{44} = \frac{26}{44} = \frac{13}{22}$

A1

Answer: $\frac{13}{22}$

1 mark

14 (6 marks)

(a) A, B, C all: 0.05; A and B only: $0.2 - 0.05 = 0.15$; A and C only: 0.10; B and C only: 0.05

M1 A1

A only: $0.5 - 0.15 - 0.10 - 0.05 = 0.2$; B only: $0.4 - 0.15 - 0.05 - 0.05 = 0.15$; C only: $0.35 - 0.10 - 0.05 - 0.05 = 0.15$

M1 A1

Answer: See diagram

4 marks

(b) $1 - (0.2 + 0.15 + 0.15 + 0.15 + 0.10 + 0.05 + 0.05) = 0.15$

A1

Answer: 0.15

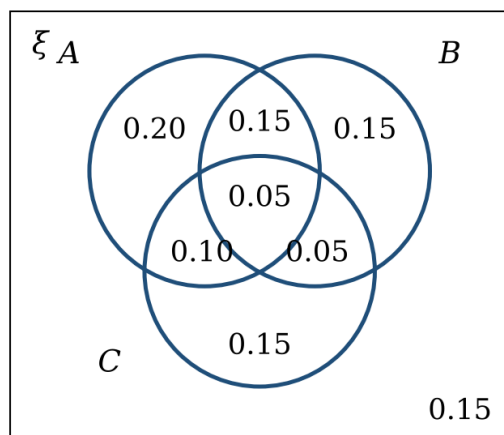
1 mark

(c) $0.15 + 0.10 + 0.05 = 0.30$

A1

Answer: 0.3

1 mark

**15 (6 marks)**

(a) A only + (A and C only) = $5 + 2 = 7$

B1

Answer: 7

1 mark

(b) Inside B and C but not A: 6

B1

Answer: 6

1 mark

(c) In A or C but outside B: A only 5, A and C only 2, C only 4

M1

$$5 + 2 + 4 = 11$$

A1

Answer: 11

2 marks

(d) $3 + 2 + 6 = 11$ elements

M1

$$P = \frac{11}{36}$$

A1

Answer: $\frac{11}{36}$

2 marks

16 (6 marks)

(a) Sum of all regions: $(x + 2) + (2x - 1) + x + 3 + 2 + 4 + (x - 1) + 6 = 40$

M1

$$5x + 15 = 40$$

A1

$$x = 5$$

A1

Answer: $x = 5$

3 marks

(b) A only 7, B only 9, C only 5, total 21

M1

$$P = \frac{21}{40}$$

A1

Answer: $\frac{21}{40}$

2 marks

(c) A: $7 + 3 + 2 + 4 = 16$, so $P = \frac{16}{40} = \frac{2}{5}$

A1

Answer: $\frac{2}{5}$

1 mark

17 (5 marks)

(a) $A \cup B$ is made of three exclusive parts: $0.8 = 0.3 + 0.2 + P(A \cap B)$

M1

$$P(A \cap B) = 0.3$$

A1

Answer: 0.3

2 marks

(b) $P(A) = 0.2 + 0.3 = 0.5$ and $P(B) = 0.3 + 0.3 = 0.6$

B1

Answer: 0.5 and 0.6

1 mark

(c) $P(A) \times P(B) = 0.5 \times 0.6 = 0.3$

M1

This equals $P(A \cap B)$, so A and B are independent

E1

Answer: Independent

2 marks

18 (6 marks)

(a) $n(A) = 15$, $n(B) = 10$, $n(C) = 6$; $n(A \cap B) = 5$, $n(A \cap C) = 3$, $n(B \cap C) = 2$, $n(A \cap B \cap C) = 1$ (30)

M1 A1

Regions: AB only 4, AC only 2, BC only 1, A only $15 - 4 - 2 - 1 = 8$, B only 4, C only 2

M1

Neither: $30 - 22 = 8$

A1

Answer: See diagram

4 marks

(b) $4 + 2 + 1 = 7$, so $P = \frac{7}{30}$

A1

Answer: $\frac{7}{30}$

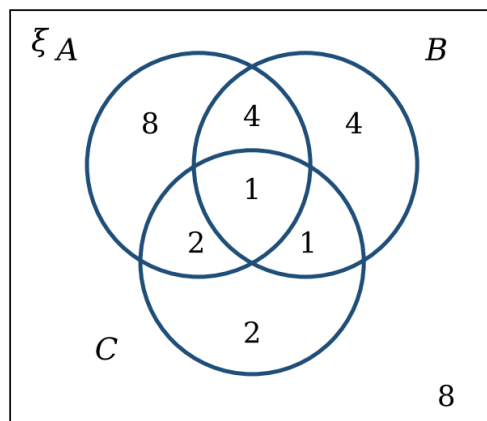
1 mark

(c) $8 + 4 = 12$, so $P = \frac{12}{30} = \frac{2}{5}$

A1

Answer: $\frac{2}{5}$

1 mark

**19 (7 marks)**

(a) $n(F \cup S) = n(F) + n(S) - n(F \cap S) = 35 + 28 - b = 63 - b$

M1

Also $n(F \cup S) = 60 - k$

M1

So $63 - b = 60 - k$, giving $b = 3 + k$

A1

Answer: $b = 3 + k$

3 marks

(b) $5 \leq k \leq 8$, so least $b = 8$

B1

greatest $b = 11$

B1

Answer: 8 and 11

2 marks

(c) French only $35 - 11 = 24$, Spanish only $28 - 11 = 17$, so exactly one language: 41 students

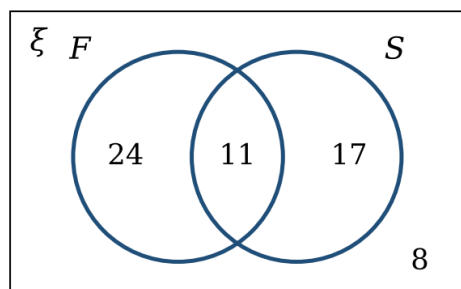
M1

$$P = \frac{41}{60}$$

A1

Answer: $\frac{41}{60}$

2 marks



20 (8 marks)

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

M1

$0.8 = 1.2 - 0.45 + P(A \cap B \cap C)$

M1

$P(A \cap B \cap C) = 0.05$

A1

Answer: 0.05

3 marks

(b) Two events only: A and B only 0.15, B and C only 0.05, A and C only 0.10

M1

A only 0.2, B only 0.15, C only 0.1

A1

Outside all three: $1 - 0.8 = 0.2$

A1

Answer: See diagram

3 marks

(c) $0.15 + 0.05 + 0.10$

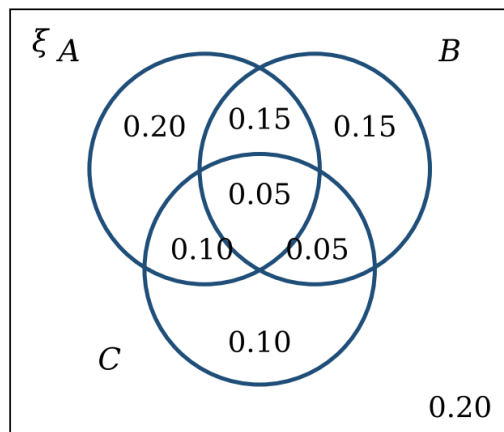
M1

$= 0.3$

A1

Answer: 0.3

2 marks



21 (7 marks)

(a) 90 people support at least one: $50 + m + t = 90$, so $m + t = 40$

M1 A1

Counting memberships: $60 + 44 + 34 = 138 = 50 + 2m + 3t$, so $2m + 3t = 88$

A1

Answer: $m + t = 40$, $2m + 3t = 88$

3 marks

(b) Subtract $2 \times (m + t = 40)$: $t = 88 - 80 = 8$

M1

$m = 32$

A1

Answer: $t = 8$, $m = 32$

2 marks

(c) $m + t = 40$

M1

$P = \frac{40}{100} = \frac{2}{5}$

A1

Answer: $\frac{2}{5}$

2 marks

22 (6 marks)

(a) Let $P(B) = b$ so $P(A) = 2b$: $2b + b - 0.14 = 0.76$

M1

$3b = 0.9$, so $b = 0.3$

A1

$P(B) = 0.3$ and $P(A) = 0.6$

A1

Answer: $P(A) = 0.6$, $P(B) = 0.3$

3 marks

(b) $P(A) \times P(B) = 0.6 \times 0.3 = 0.18$

M1

0.18 $\neq$ 0.14, so not independent	E1
Answer: Not independent	2 marks
(c) $1 - 0.76 = 0.24$	A1
Answer: 0.24	1 mark
23 (8 marks)	
(a) $P(A \text{ only}) = P(A) - P(A \cap B)$ and $P(B \text{ only}) = P(B) - P(A \cap B)$	M1 A1
The two regions are exclusive, so $P(\text{exactly one}) = P(A) + P(B) - 2P(A \cap B)$	E1
Answer: shown	3 marks
(b) $0.6 = 0.5 + 0.4 - 2P(A \cap B)$	M1
$P(A \cap B) = 0.15$	A1
Answer: 0.15	2 marks
(c) $P(A \cup B) = 0.5 + 0.4 - 0.15 = 0.75$, so $P(\text{neither}) = 0.25$	M1 A1
$P(A)P(B) = 0.2 \neq 0.15$, so not independent	B1
Answer: 0.25, not independent	3 marks
24 (6 marks)	
(a) $x^2 + x + 2x + 6 = 34$	M1
$x^2 + 3x - 28 = 0$	A1
Answer: shown	2 marks
(b) $(x + 7)(x - 4) = 0$	M1 A1
$x = -7$ is impossible as the number of elements cannot be negative, so $x = 4$	E1
Answer: $x = 4$	3 marks
(c) A only 16 and B only 8: $P = \frac{24}{34} = \frac{12}{17}$	A1
Answer: $\frac{12}{17}$	1 mark
25 (7 marks)	
(a) $A' \cap B$ is the complement of $A \cup B'$, so $P(A' \cap B) = 1 - 0.85 = 0.15$	M1 A1
$P(A \cap B') = 1 - P(A' \cup B) = 1 - 0.7 = 0.3$	A1
$P(A \cap B) = 1 - P(A' \cup B') = 1 - 0.9 = 0.1$	A1
Answer: 0.1, 0.3, 0.15	4 marks
(b) Regions: $A \cap B = 0.1$, A only = 0.3, B only = 0.15	B1
$P(A' \cap B') = 1 - (0.1 + 0.3 + 0.15)$	M1
$= 0.45$	A1
Answer: 0.45	3 marks
26 (10 marks)	
(a) If mutually exclusive, $P(A \cup B) = 0.7 + 0.5 = 1.2$	M1
A probability cannot exceed 1, so this is impossible	E1
Answer: Not mutually exclusive	2 marks
(b) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 1.2 - P(A \cup B)$	M1
$P(A \cup B) \leq 1$	M1
so $P(A \cap B) \geq 0.2$	A1
Answer: 0.2	3 marks
(c) $A \cap B$ is contained within B, so $P(A \cap B) \leq P(B)$	M1
greatest value is 0.5 (when B is a subset of A)	A1
Answer: 0.5	2 marks
(d) $P(A' \cap B') = 1 - P(A \cup B) = 1 - (1.2 - P(A \cap B))$	M1
$= P(A \cap B) - 0.2$	A1
so $0 \leq P(A' \cap B') \leq 0.3$	A1
Answer: 0 to 0.3	3 marks

27 (10 marks)

(a) Rearranging, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ M1

Since $P(A \cup B) \leq 1$, we have $-P(A \cup B) \geq -1$ M1

Therefore $P(A \cap B) \geq P(A) + P(B) - 1$ E1

Answer: Proof 3 marks

(b) Using (a): $P(A \cap B) \geq 0.9 + 0.85 - 1 = 0.75$ M1 A1

Apply (a) to the events $A \cap B$ and C : $P(A \cap B \cap C) \geq P(A \cap B) + P(C) - 1$ M1

$\geq 0.75 + 0.8 - 1 = 0.55$ A1

Answer: Shown 4 marks

(c) $P(A') = 0.1$, $P(B') = 0.15$, $P(C') = 0.2$ and $0.1 + 0.15 + 0.2 = 0.45 \leq 1$, so A' , B' and C' can be mutually exclusive B1

Then $P(A' \cup B' \cup C') = 0.45$ M1

$P(A \cap B \cap C) = 1 - 0.45 = 0.55$ A1

Answer: Shown 3 marks

28 (10 marks)

(a) Treat $A \cup B$ as one event: $P(A \cup B \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$ M1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ A1

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ B1

$P((A \cap C) \cup (B \cap C)) = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$, since $(A \cap C) \cap (B \cap C) = A \cap B \cap C$ M1

Substitute and collect terms to obtain the stated result A1

Answer: shown 5 marks

(b) $0.8 = 1.2 - 0.45 + P(A \cap B \cap C)$ M1

$P(A \cap B \cap C) = 0.05$ A1

Answer: 0.05 2 marks

(c) $P(A \text{ only}) = 0.4 - 0.15 - 0.15 + 0.05$ M1

$= 0.15$, and the same for B only and C only A1

$P(\text{exactly one}) = 3 \times 0.15 = 0.45$ A1

Answer: 0.45 3 marks

29 (9 marks)

(a) $P(A' \cap B') = 1 - P(A \cup B)$ M1

$= 1 - P(A) - P(B) + P(A)P(B)$ M1

$= (1 - P(A))(1 - P(B))$ A1

$= P(A')P(B')$, so A' and B' are independent E1

Answer: shown 4 marks

(b) $0.88 = 0.4 + P(B) - 0.4P(B)$ M1

$0.6P(B) = 0.48$ M1

$P(B) = 0.8$ A1

Answer: 0.8 3 marks

(c) $P(A \text{ only}) = 0.4 \times 0.2 = 0.08$ and $P(B \text{ only}) = 0.6 \times 0.8 = 0.48$ M1

$P = 0.56$ A1

Answer: 0.56 2 marks

30 (9 marks)

(a) Independence: $P(A \cap B) = 0.3 P(B)$ B1

$0.58 = 0.3 + P(B) - 0.3 P(B)$ M1

$0.7 P(B) = 0.28$ so $P(B) = 0.4$ A1

Answer: 0.4 3 marks

(b) $P(B \cap C) = P(B) + P(C) - P(B \cup C)$ M1

$= 0.4 + 0.4 - 0.7 = 0.1$ A1

Answer: 0.1 2 marks

(c) $P(A \cap B) = 0.3 \times 0.4 = 0.12$, so $A \text{ only} = 0.3 - 0.12 = 0.18$ M1 A1

$C \text{ only} = 0.4 - 0.1 = 0.3$ ($A \cap C$ is empty), so $B \text{ only} = 0.4 - 0.12 - 0.1 = 0.18$ M1

$P(\text{none}) = 1 - (0.18 + 0.12 + 0.1 + 0.3 + 0.18) = 0.12$ A1

Answer: 0.12 4 marks

Tree Diagrams

1 (2 marks)

(a) The tosses are independent, so each second-toss branch is 0.6 (H) or 0.4 (T)

Answer: 0.6, 0.4, 0.6, 0.4

B1

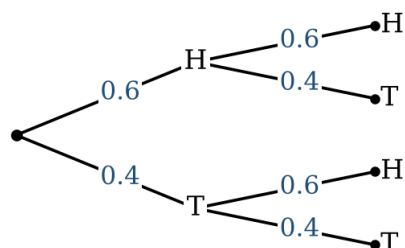
1 mark

(b) $0.6 \times 0.6 = 0.36$

Answer: 0.36

B1

1 mark



2 (3 marks)

(a) The probabilities on each pair of branches sum to 1: 0.7, 0.5 and 0.8

Answer: 0.7, 0.5, 0.8

B1 B1

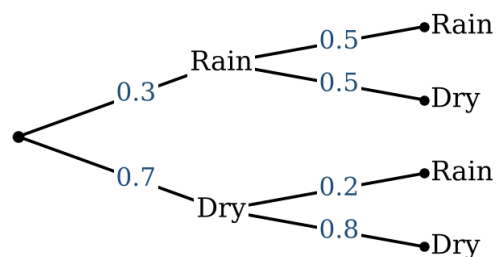
2 marks

(b) $0.3 \times 0.5 = 0.15$

Answer: 0.15

B1

1 mark



3 (3 marks)

(a) $P(\text{lose}) = 1 - 0.3 = 0.7$ on every unlabelled branch; 0.3 for the second game after a win or loss

Answer: See diagram

B1

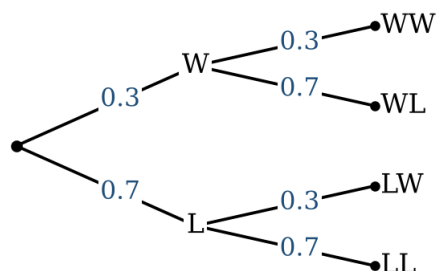
1 mark

(b) $P(WL) + P(LW) = 0.3 \times 0.7 + 0.7 \times 0.3 = 0.42$

M1

A1

2 marks



4 (2 marks)

Replacement, so the draws are independent with $P(\text{red}) = \frac{3}{5}$ each time

M1

$$P(RR) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

A1

Answer: $\frac{9}{25}$

2 marks

5 (3 marks)

(a) $0.1 \times 0.1 = 0.01$

M1 A1

(b) $0.9 \times 0.9 = 0.81$

A1

Answer: (a) 0.01 (b) 0.81

3 marks

6 (3 marks)

Branches: coin H or T (each $\frac{1}{2}$), then die even or odd (each $\frac{1}{2}$)

B1

$$P(\text{H and even}) = \frac{1}{2} \times \frac{1}{2}$$

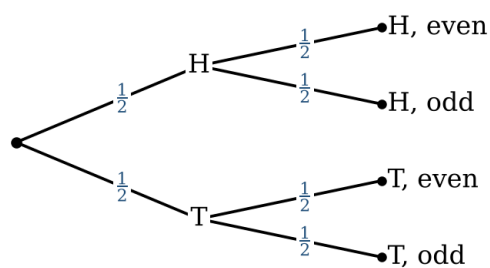
M1

$$= \frac{1}{4}$$

A1

Answer: $\frac{1}{4}$

3 marks

**7 (4 marks)**

(a) First counter: $\frac{4}{10}$ red, $\frac{6}{10}$ blue

B1

Second counter: after red $\frac{3}{9}$, $\frac{6}{9}$; after blue $\frac{4}{9}$, $\frac{5}{9}$

B1

Answer: See tree diagram

2 marks

(b) $P = \frac{4}{10} \times \frac{3}{9} + \frac{6}{10} \times \frac{5}{9}$

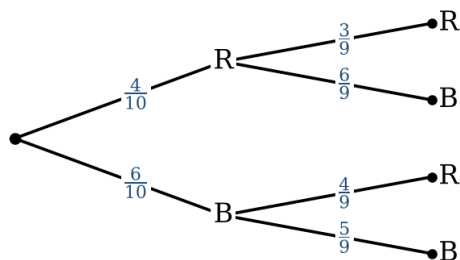
M1

$$= \frac{12}{90} + \frac{30}{90} = \frac{7}{15}$$

A1

Answer: $\frac{7}{15}$

2 marks

**8 (4 marks)**

(a) $P = 0.4 \times 0.05$

M1

$$= 0.02$$

A1

Answer: 0.02

2 marks

$$(b) P(\text{defective}) = 0.6 \times 0.02 + 0.4 \times 0.05 \\ = 0.012 + 0.02 = 0.032$$

M1

A1

Answer: 0.032

2 marks

9 (3 marks)

$$P(\text{rain, rain}) = 0.4 \times 0.7 = 0.28$$

M1

$$P(\text{dry, dry}) = 0.6 \times 0.8 = 0.48$$

M1

$$P = 0.28 + 0.48 = 0.76$$

A1

Answer: 0.76

3 marks

10 (3 marks)Three arrangements HHT, HTH, THH, each with probability $0.3 \times 0.3 \times 0.7$

M1 A1

$$P = 3 \times 0.063 = 0.189$$

A1

Answer: 0.189

3 marks

11 (3 marks)First roll 6 with probability $\frac{1}{6}$; if it is not 6 the game stops

B1

A second 6 follows with probability $\frac{1}{6}$, but the first 6 is already enough

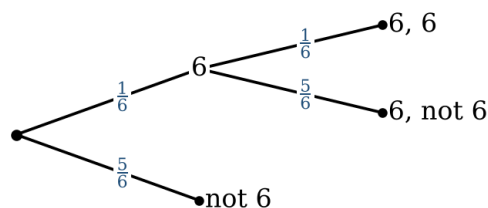
M1

$$P(\text{at least one 6}) = P(\text{first roll is 6}) = \frac{1}{6}$$

A1

Answer: $\frac{1}{6}$

3 marks

**12 (6 marks)**

$$(a) P = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

M1

$$= \frac{5}{28}$$

A1

Answer: $\frac{5}{28}$

2 marks

(b) RRG, RGR and GRR each have the same probability $\frac{5}{28}$ (the numerators 5, 4, 3 appear in a different order, the denominators 8, 7, 6 do not change)

M1

$$P = 3 \times \frac{5}{28} = \frac{15}{28}$$

A1

Answer: $\frac{15}{28}$

2 marks

$$(c) P(\text{no green}) = P(\text{RRR}) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$$

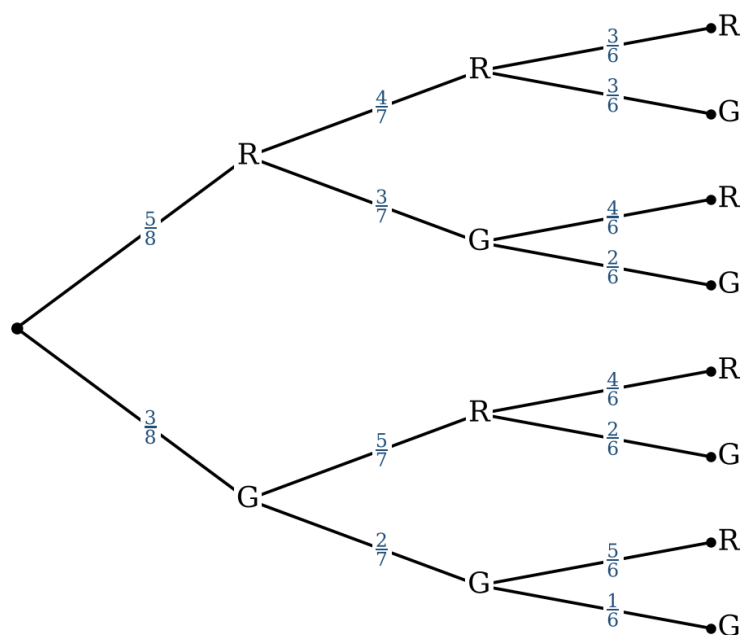
M1

$$P = 1 - \frac{5}{28} = \frac{23}{28}$$

A1

Answer: $\frac{23}{28}$

2 marks

**13 (6 marks)**

(a) $1 - 0.8 = 0.2$ and $1 - 0.6 = 0.4$

Answer: 0.2 and 0.4

(b) $P = 0.7 \times 0.2 + 0.3 \times 0.6$

$= 0.14 + 0.18 = 0.32$

Answer: 0.32

(c) $P(\text{scores second}) = 0.7 \times 0.8 + 0.3 \times 0.6 = 0.56 + 0.18 = 0.74$

$P(\text{first} \mid \text{second}) = \frac{0.56}{0.74} = \frac{28}{37}$

Answer: $\frac{28}{37}$

B1

1 mark

M1

A1

2 marks

M1 A1

A1

3 marks

14 (6 marks)

(a) $P(D) = 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.05$

$= 0.01 + 0.009 + 0.01 = 0.029$

Answer: 0.029

(b) Two orders, each 0.029×0.971

$P = 2 \times 0.029 \times 0.971 = 0.0563$

Answer: 0.0563

M1 A1

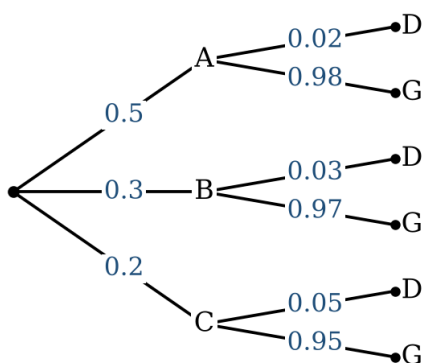
A1

3 marks

M1 A1

A1

3 marks



15 (4 marks)

Probability of the order R, B, G is $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{24}{504} = \frac{1}{21}$

M1 A1

There are $3! = 6$ orders, each with the same probability

M1

$$P = \frac{6}{21} = \frac{2}{7}$$

A1

Answer: $\frac{2}{7}$

4 marks

16 (4 marks)

Win via first serve: $0.65 \times 0.7 = 0.455$

M1 A1

Win via second serve: $0.35 \times 0.9 \times 0.5 = 0.1575$

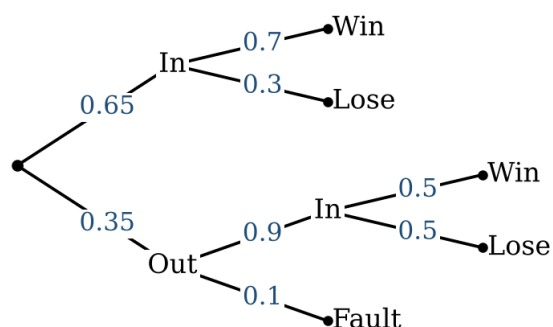
M1

$P = 0.6125$

A1

Answer: 0.6125

4 marks

**17 (6 marks)**

(a) $P(\text{win after win}) = 0.5 \times 0.6 = 0.30$

M1

$P(\text{win after draw or loss}) = (0.3 + 0.2) \times 0.3 = 0.15$

M1

$P = 0.45$

A1

Answer: 0.45

3 marks

(b) $P(\text{lose first}) = 0.2$

B1

$P(\text{not lose first, lose second}) = 0.5 \times 0.1 + 0.3 \times 0.4 = 0.17$

M1

$P = 0.2 + 0.17 = 0.37$

A1

Answer: 0.37

3 marks

18 (4 marks)

Exactly one six in the first 4 rolls, then a six on the 5th

M1

One six in 4 rolls: $4 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^3$

M1 A1

Multiply by $\frac{1}{6}$: $\frac{125}{1944} \approx 0.0643$

A1

Answer: $\frac{125}{1944} = 0.0643$

4 marks

19 (8 marks)

(a) $P(\text{both red}) = \frac{6}{(n+6)} \times \frac{5}{(n+5)} = \frac{30}{(n+6)(n+5)}$

M1

$P(\text{both yellow}) = \frac{n}{(n+6)} \times \frac{n-1}{n+5}$

M1

$\frac{30 + n(n-1)}{(n+6)(n+5)} = \frac{7}{15}$, so $15(n^2 - n + 30) = 7(n^2 + 11n + 30)$

M1

$8n^2 - 92n + 240 = 0$, which gives $2n^2 - 23n + 60 = 0$

A1

Answer: $2n^2 - 23n + 60 = 0$

4 marks

(b) $(2n - 15)(n - 4) = 0$

M1

$n = 4$, since n must be a whole number ($n = \frac{15}{2}$ is rejected)

A1

Answer: $n = 4$

2 marks

$$(c) P = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$$

M1

$$= \frac{1}{30}$$

A1

$$\text{Answer: } \frac{1}{30}$$

2 marks

20 (8 marks)

$$(a) P = P(RR) + P(DR) = 0.3 \times 0.6 + 0.7 \times 0.2$$

$$= 0.18 + 0.14 = 0.32$$

M1

A1

$$\text{Answer: } 0.32$$

2 marks

$$(b) P = 0.3 \times 0.6 \times 0.6$$

$$= 0.108$$

M1

A1

$$\text{Answer: } 0.108$$

2 marks

$$(c) \text{RRD: } 0.3 \times 0.6 \times 0.4 = 0.072$$

M1

$$\text{RDR: } 0.3 \times 0.4 \times 0.2 = 0.024$$

M1

$$\text{DRR: } 0.7 \times 0.2 \times 0.6 = 0.084$$

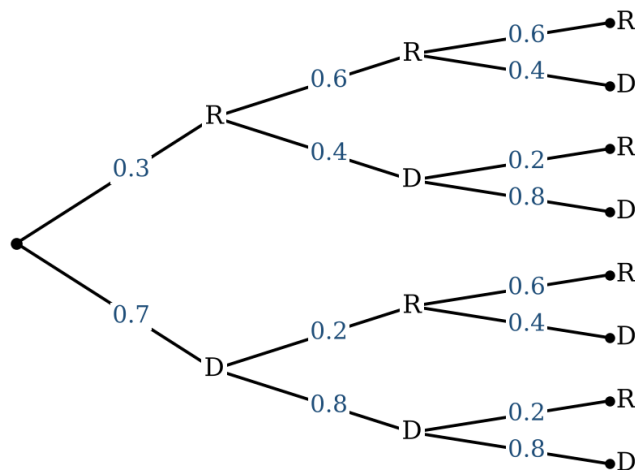
M1

$$\text{Total} = 0.18$$

A1

$$\text{Answer: } 0.18$$

4 marks

**21 (6 marks)**

$$(a) P(\text{first red}) = \frac{a}{a+b}$$

B1

$$P(\text{second red}) = \frac{a}{a+b} \times \frac{a-1}{a+b-1} + \frac{b}{a+b} \times \frac{a}{a+b-1}$$

$$= \frac{a(a-1) + ab}{(a+b)(a+b-1)} = \frac{a(a+b-1)}{(a+b)(a+b-1)} = \frac{a}{a+b}$$

M1 A1

$$\text{Answer: shown}$$

4 marks

$$(b) \frac{7}{12}$$

B1

By symmetry, each position in the sequence is equally likely to be any of the 12 counters

E1

$$\text{Answer: } \frac{7}{12}$$

2 marks

22 (8 marks)

$$(a) \text{Win, win: } 0.6 \times 0.6 = 0.36$$

B1

$$\text{Three games with Ann winning the third: the first two are split in either order, } 2 \times 0.6 \times 0.4 \times 0.6$$

$$= 0.288$$

M1

A1

$$\text{Answer: } 0.288$$

3 marks

$$(b) 0.36 + 0.288$$

M1

$$= 0.648$$

A1

$$\text{Answer: } 0.648$$

2 marks

$$(c) \text{The first two games must be split: } 2 \times 0.6 \times 0.4 = 0.48$$

M1 A1

The third game then decides the series, so $P = 0.48$

Answer: 0.48

A1

3 marks

23 (8 marks)

(a) After each red draw the bag has one more red and one more counter

M1

$$\frac{3}{5} \times \frac{4}{6} \times \frac{5}{7} = \frac{2}{7}$$

A1

Answer: $\frac{2}{7}$

2 marks

(b) $P(RR) = \frac{3}{5} \times \frac{4}{6} = \frac{2}{5}$

M1

$$P(BR) = \frac{2}{5} \times \frac{3}{6} = \frac{1}{5}$$

M1

$$P(\text{second red}) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

A1

Answer: shown

3 marks

(c) $P(BBB) = \frac{2}{5} \times \frac{3}{6} \times \frac{4}{7} = \frac{4}{35}$

M1 A1

$$P = \frac{2}{7} + \frac{4}{35} = \frac{14}{35} = \frac{2}{5}$$

A1

Answer: $\frac{2}{5}$

3 marks

24 (8 marks)

(a) Exactly one: $p(1 - p) + (1 - p)p = 2p(1 - p) = 0.42$

M1 A1

$$p^2 - p + 0.21 = 0$$

M1

$(p - 0.3)(p - 0.7) = 0$, and $p < 0.5$ so $p = 0.3$

A1

Answer: $p = 0.3$

4 marks

(b) $1 - (1 - 0.3)^2$

M1

$$= 0.51$$

A1

Answer: 0.51

2 marks

(c) $P(\text{both}) = 0.09$ and $P(\text{at least one}) = 0.51$

M1

$$P = \frac{0.09}{0.51} = \frac{3}{17}$$

A1

Answer: $\frac{3}{17}$

2 marks

25 (6 marks)

(a) Amy wins 2 of the first 3 sets and then the 4th: 3 ways

M1

$$3 \times 0.6^2 \times 0.4 \times 0.6$$

M1

$$= 0.2592$$

A1

Answer: 0.2592

3 marks

(b) After four sets the score is 2 to 2: 6 arrangements

M1

$$6 \times 0.6^2 \times 0.4^2$$

M1

$$= 0.3456$$

A1

Answer: 0.3456

3 marks

26 (7 marks)

(a) $P(\text{different}) = \frac{r}{9} \times \frac{9-r}{8} + \frac{9-r}{9} \times \frac{r}{8}$

M1

$$\frac{2r(9-r)}{72} = \frac{5}{9} \text{ so } r(9-r) = 20$$

M1

$$r^2 - 9r + 20 = 0$$

A1

Answer: shown

3 marks

(b) $(r-4)(r-5) = 0$

M1

$$r < 5 \text{ so } r = 4$$

A1

Answer: $r = 4$

2 marks

(c) $\frac{4}{9} \times \frac{3}{8}$

M1

$$= \frac{1}{6}$$

A1

Answer: $\frac{1}{6}$

2 marks

27 (10 marks)

(a) Exactly two heads can happen in 3 ways (HHT, HTH, THH), each with probability $p^2(1 - p)$

M1

$$3p^2(1 - p) = 0.288, \text{ so } p^2(1 - p) = 0.096$$

M1

$$p^2 - p^3 = 0.096, \text{ hence } p^3 - p^2 + 0.096 = 0$$

A1

Answer: Shown

3 marks

(b) $0.4^3 - 0.4^2 + 0.096 = 0.064 - 0.16 + 0.096 = 0$

B1

Factor theorem: $(p - 0.4)(p^2 - 0.6p - 0.24) = 0$

M1

$$p = \frac{0.6 \pm \sqrt{0.36 + 0.96}}{2} = 0.3 \pm \sqrt{0.33}$$

A1

$p = 0.875$ (3 d.p.) is the only other root between 0 and 1 ($0.3 - \sqrt{0.33}$ is negative)

A1

Answer: $p = 0.4$ or $p = 0.875$

4 marks

(c) Biased towards tails means $p < 0.5$, so $p = 0.4$

E1

$$P(\text{at least one head}) = 1 - P(\text{TTT}) = 1 - 0.6^3$$

M1

$$= 0.784$$

A1

Answer: 0.784

3 marks

28 (10 marks)

(a) $P = \frac{2}{6} = \frac{1}{3}$

B1

Answer: $\frac{1}{3}$

1 mark

(b) Priya draws white, then Quentin draws black: $\frac{4}{6} \times \frac{2}{5}$

M1

$$= \frac{4}{15}$$

A1

Answer: $\frac{4}{15}$

2 marks

(c) Priya wins on draw 1, 3 or 5 (the game must end by draw 5 since there are only 4 white counters)

M1

Draw 1: $\frac{2}{6} = \frac{1}{3}$

B1

Draw 3: $\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{1}{5}$

M1 A1

Draw 5: $\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{15}$; total $\frac{5}{15} + \frac{3}{15} + \frac{1}{15} = \frac{9}{15} = \frac{3}{5}$

A1

Answer: $\frac{3}{5}$

5 marks

(d) $P(\text{Quentin wins}) = 1 - \frac{3}{5} = \frac{2}{5}$

B1

This is less than $\frac{1}{2}$, so the game favours the player who draws first

E1

Answer: $\frac{2}{5}$, not fair

2 marks

29 (10 marks)

(a) From £1: win (0.6) leads to £2, lose (0.4) leads to £0 and $p_0 = 0$, so $p_1 = 0.6p_2 + 0.4 \times 0$

M1 E1

From £2: $p_2 = 0.6p_3 + 0.4p_1 = 0.6 + 0.4p_1$

A1

Answer: shown

3 marks

(b) $p_1 = 0.6(0.6 + 0.4p_1)$

M1

$$p_1 = 0.36 + 0.24p_1$$

A1

$$0.76p_1 = 0.36$$

M1

$$p_1 = \frac{9}{19}$$

A1

Answer: $\frac{9}{19}$

4 marks

(c) $p_1 = 0.5p_2$ and $p_2 = 0.5 + 0.5p_1$ give $p_1 = 0.25 + 0.25p_1$

M1

$$p_1 = \frac{1}{3}$$

A1

 $\frac{9}{19} \approx 0.474 > \frac{1}{3}$: the bias towards winning increases her chance of success

E1

Answer: $\frac{1}{3}$

3 marks

30 (8 marks)(a) Car at 1: host can open 2 or 3, each with probability $\frac{1}{2}$

B1

Car at 2: host must open 3 (probability 1); car at 3: host must open 2 (probability 1)

B1

Each car position has probability $\frac{1}{3}$

B1

Answer: See diagram

3 marks

(b) The contestant wins by sticking exactly when the car is at door 1

M1

$$P = \frac{1}{3}$$

A1

Answer: $\frac{1}{3}$

2 marks

(c) $P(\text{win by switching}) = P(\text{car at 2}) + P(\text{car at 3}) = \frac{1}{3} + \frac{1}{3}$

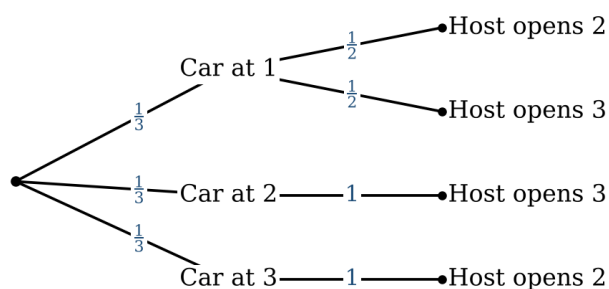
M1 A1

 $= \frac{2}{3}$: the host's choice is forced when the car is not behind door 1, so he gives away information, and switching is better

E1

Answer: $\frac{2}{3}$

3 marks

**Independent Events****1 (3 marks)**(a) $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.5 = 0.15$

B1

Answer: 0.15

1 mark

(b) $P(A \cup B) = 0.3 + 0.5 - 0.15$

M1

 $= 0.65$

A1

Answer: 0.65

2 marks

2 (3 marks)(a) $P(A) \times P(B) = 0.4 \times 0.5 = 0.2$

M1

This equals $P(A \cap B)$, so A and B are independent

E1

Answer: Independent

2 marks

(b) $P(A) \times P(C) = 0.12 \neq 0.15 = P(A \cap C)$, so not independent

E1

Answer: Not independent

1 mark

3 (2 marks)

$$P(A) \times P(B) = 0.5 \times 0.2 = 0.1$$

M1

This equals $P(A \cap B)$, so A and B are independent

E1

Answer: Independent

2 marks

4 (2 marks)

The coin and die are independent, so $P = \frac{1}{2} \times \frac{1}{6}$

M1

$$= \frac{1}{12}$$

A1

Answer: $\frac{1}{12}$

2 marks

5 (2 marks)

Independent games: $P = 0.3 \times 0.3 \times 0.3$

M1

$$= 0.027$$

A1

Answer: 0.027

2 marks

6 (4 marks)

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.84 = 0.6 + b - 0.6b$

M1

$$0.24 = 0.4b$$

M1

$$b = 0.6$$

A1

Answer: $P(B) = 0.6$

3 marks

(b) $P(A')P(B') = 0.4 \times 0.4 = 0.16$ (also $1 - 0.84$)

A1

Answer: 0.16

1 mark

7 (5 marks)

(a) $0.8 \times 0.7 = 0.56$

B1

Answer: 0.56

1 mark

(b) $P(\text{neither scores}) = 0.2 \times 0.3 = 0.06$

M1

$$P(\text{at least one}) = 1 - 0.06 = 0.94$$

A1

Answer: 0.94

2 marks

(c) $0.8 \times 0.3 + 0.2 \times 0.7$

M1

$$= 0.24 + 0.14 = 0.38$$

A1

Answer: 0.38

2 marks

8 (3 marks)

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

M1

$$0.65 = 0.3 + P(B) - 0.3P(B), \text{ so } 0.7P(B) = 0.35$$

M1

$$P(B) = 0.5$$

A1

Answer: 0.5

3 marks

9 (5 marks)

(a) $P(X \text{ hits, } Y \text{ misses}) = 0.7 \times 0.4 = 0.28$

M1

$$P(X \text{ misses, } Y \text{ hits}) = 0.3 \times 0.6 = 0.18$$

M1

$$P = 0.28 + 0.18 = 0.46$$

A1

Answer: 0.46

3 marks

(b) $1 - P(\text{both miss}) = 1 - 0.3 \times 0.4$

M1

$$= 0.88$$

A1

Answer: 0.88

2 marks

10 (5 marks)

(a) $0.2 + 0.5 = 0.7$

B1

Answer: 0.7

1 mark

(b) $P(A \cap B) = 0.2 \times 0.5 = 0.1$

M1

$$P(A \cup B) = 0.2 + 0.5 - 0.1 = 0.6$$

A1

Answer: 0.6

2 marks

(c) Mutually exclusive means $P(A \cap B) = 0$

E1

Independent means $P(A \cap B) = 0.2 \times 0.5 = 0.1 \neq 0$, a contradiction

E1

Answer: Contradiction**2 marks****11 (4 marks)**Parallel: $P(\text{works}) = 1 - P(\text{both fail})$

M1

$$= 1 - 0.1 \times 0.1 = 0.99$$

A1

Series: $P(\text{works}) = 0.9 \times 0.9$

M1

$$= 0.81$$

A1

Answer: 0.99 and 0.81**4 marks****12 (3 marks)**

$$P(F) = \frac{60}{100} \text{ and } P(T) = \frac{50}{100}$$

B1

$$P(F \cap T) = \frac{30}{100} = 0.3 \text{ and } P(F) \times P(T) = 0.6 \times 0.5 = 0.3$$

M1

These are equal, so F and T are independent

E1

Answer: Independent**3 marks****13 (7 marks)**

$$(a) P(A \cap B) = 0.5 \times 0.4 = 0.2$$

M1

$$P(A \text{ only}) = 0.5 - 0.2 = 0.3 \text{ and } P(B \text{ only}) = 0.4 - 0.2 = 0.2$$

A1

$$P(\text{neither}) = 1 - (0.3 + 0.2 + 0.2) = 0.3$$

A1

Answer: 0.3, 0.2, 0.2, 0.3**3 marks**

$$(b) P(A' \cup B') = 1 - P(A \cap B)$$

M1

$$= 1 - 0.2 = 0.8$$

A1

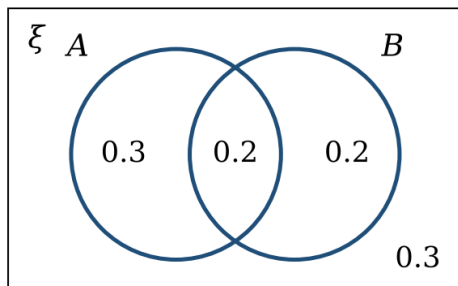
Answer: 0.8**2 marks**

$$(c) P(A \cap B') = 0.3 \text{ from the diagram}$$

B1

$$P(A) \times P(B') = 0.5 \times 0.6 = 0.3, \text{ which equals } P(A \cap B')$$

E1

Answer: Shown**2 marks****14 (5 marks)**

$$(a) \text{ Both A and B must work: } 0.9 \times 0.8$$

M1

$$= 0.72$$

A1

Answer: 0.72**2 marks**

$$(b) \text{ System fails only if both paths fail: } P(\text{upper fails}) = 1 - 0.72 = 0.28 \text{ and } P(C \text{ fails}) = 0.3$$

M1

$$P(\text{system fails}) = 0.28 \times 0.3 = 0.084$$

M1

$$P(\text{system works}) = 1 - 0.084 = 0.916$$

A1

Answer: 0.916**3 marks****15 (5 marks)**

$$P(A \cup B) = p + 2p - p \times 2p$$

M1

$$3p - 2p^2 = 0.52, \text{ so } 2p^2 - 3p + 0.52 = 0$$

A1

$$\text{Multiply by 50: } 100p^2 - 150p + 26 = 0, \text{ so } (5p - 1)(20p - 26) = 0$$

M1

$$p = 0.2 \text{ or } p = 1.3$$

A1

p must be a probability, so p = 0.2

E1

Answer: p = 0.2**5 marks**

16 (6 marks)

$$(a) P = 0.5 \times 0.6 \times 0.8 = 0.24$$

M1

A1

Answer: 0.24

2 marks

$$(b) 1 - 0.24 = 0.76$$

A1

Answer: 0.76

1 mark

$$(c) A \text{ and } B \text{ only: } 0.5 \times 0.4 \times 0.8 = 0.16; A \text{ and } C \text{ only: } 0.5 \times 0.6 \times 0.2 = 0.06$$

M1 A1

$$B \text{ and } C \text{ only: } 0.5 \times 0.4 \times 0.2 = 0.04, \text{ so } P = 0.26$$

A1

Answer: 0.26

3 marks

17 (5 marks)

$$\text{Total outcomes } 2^4 = 16$$

B1

$$A: \text{first two the same (HH or TT) then any two tosses: } 2 \times 4 = 8, \text{ so } P(A) = \frac{1}{2}$$

M1

$$B: \text{half of all outcomes have an even number of heads: } P(B) = \frac{8}{16} = \frac{1}{2}$$

M1

$$A \cap B: \text{HH then an even number of heads in the last two (HH, TT): 2; TT then an even number of heads in the last two: 2; total 4, so } P(A \cap B) = \frac{4}{16} = \frac{1}{4}$$

M1

$$P(A)P(B) = \frac{1}{4} = P(A \cap B), \text{ so independent}$$

A1

Answer: Independent

5 marks

18 (6 marks)

$$(a) P(A) = \frac{6}{36} = \frac{1}{6} \text{ and } P(B) = \frac{6}{36} = \frac{1}{6}$$

B1

$$A \cap B \text{ is only } (3, 4), \text{ so } P(A \cap B) = \frac{1}{36}$$

M1

$$P(A)P(B) = \frac{1}{36} = P(A \cap B), \text{ so independent}$$

A1

Answer: Independent

3 marks

$$(b) P(C) = \frac{5}{36}, P(B) = \frac{1}{6}, \text{ so } P(B)P(C) = \frac{5}{216}$$

M1 A1

$$P(B \cap C) = P((3, 5)) = \frac{1}{36} = \frac{6}{216} \neq \frac{5}{216}, \text{ so not independent}$$

E1

Answer: Not independent

3 marks

19 (4 marks)

$$P(A \cup B) = 2p - p^2$$

M1

$$2p - p^2 = \frac{5}{9}, \text{ so } (1 - p)^2 = \frac{4}{9}$$

M1 A1

$$1 - p = \frac{2}{3}, \text{ so } p = \frac{1}{3}$$

A1

$$\text{Answer: } p = \frac{1}{3}$$

4 marks

20 (8 marks)

$$(a) P(\text{no hits in 3}) = (1 - p)^3, \text{ so } 1 - (1 - p)^3 = 0.936$$

M1

$$(1 - p)^3 = 0.064$$

M1

$$1 - p = 0.4, \text{ so } p = 0.6$$

A1

Answer: p = 0.6

3 marks

$$(b) 1 - 0.4^n > 0.9999, \text{ so } 0.4^n < 0.0001$$

M1

$$n > \frac{\log 0.0001}{\log 0.4} = 10.05$$

M1

$$n = 11$$

A1

Answer: n = 11

3 marks

$$(c) \text{Miss, miss, hit: } 0.4 \times 0.4 \times 0.6 = 0.096$$

M1

A1

Answer: 0.096

2 marks

21 (6 marks)

$$(a) P(A \cap B') = P(A) - P(A \cap B)$$

M1

$$= P(A) - P(A)P(B)$$

A1

$$= P(A)(1 - P(B)) = P(A)P(B')$$

M1

Hence A and B' are independent

E1

Answer: shown

4 marks

$$(b) P(A)P(B') = 0.15, \text{ so } P(B') = \frac{0.15}{0.6} = 0.25$$

M1

$$P(B) = 0.75$$

A1

Answer: 0.75

2 marks

22 (7 marks)

$$(a) x(x + 0.2) = 0.24$$

M1

$$x^2 + 0.2x - 0.24 = 0$$

A1

$$(x - 0.4)(x + 0.6) = 0$$

M1

$$x = 0.4 \text{ (} x = -0.6 \text{ is rejected as a probability cannot be negative)}$$

A1

Answer: x = 0.4

4 marks

$$(b) P(A) = 0.4, P(B) = 0.6, \text{ so } P(A \cup B) = 0.4 + 0.6 - 0.24 = 0.76$$

M1 A1

$$P(A' \cap B') = 1 - 0.76 = 0.24$$

A1

Answer: 0.76 and 0.24

3 marks

23 (7 marks)

$$(a) P(\text{block works}) = 1 - 0.1^2 = 0.99$$

M1 A1

$$P(\text{circuit works}) = 0.9 \times 0.99 = 0.891$$

A1

Answer: 0.891

3 marks

$$(b) P(\text{block works}) = 1 - 0.1^3 = 0.999$$

M1

$$P = 0.9 \times 0.999 = 0.8991$$

A1

Answer: 0.8991

2 marks

$$(c) \text{ The circuit needs A to work, so } P(\text{works}) \leq P(A \text{ works}) = 0.9$$

E1

More parallel components raise P(block works) towards 1 but can never increase P(A works)

E1

Answer: Limited by A

2 marks

24 (7 marks)

$$(a) 0.9 \times 0.95 \times 0.8$$

M1

$$= 0.684$$

A1

Answer: 0.684

2 marks

$$(b) \text{ Fails brakes only: } 0.1 \times 0.95 \times 0.8 = 0.076; \text{ tyres only: } 0.9 \times 0.05 \times 0.8 = 0.036$$

M1 A1

$$\text{Lights only: } 0.9 \times 0.95 \times 0.2 = 0.171, \text{ total } 0.283$$

A1

Answer: 0.283

3 marks

$$(c) P(\text{not licensed}) = 1 - 0.684 = 0.316$$

M1

$$0.316^5 = 0.00315$$

A1

Answer: 0.00315

2 marks

25 (5 marks)

$$(a) P(A \cap B) = 0.4 \times 0.5 = 0.2$$

M1

$$P(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$$

A1

Answer: 0.7

2 marks

(b) Only the part of C outside $A \cup B$ is new

M1

 $C \cap A = \emptyset$ so the part of C inside $A \cup B$ is $P(B \cap C) = 0.1$; new part $0.3 - 0.1 = 0.2$

M1

$$P(A \cup B \cup C) = 0.7 + 0.2 = 0.9$$

A1

Answer: 0.9

3 marks

26 (9 marks)**(a)** Outcomes: HH, HT, TH, TT

B1

 $A = \text{HH, HT, B} = \text{HH, TH, C} = \text{HH, TT}$, so $P(A) = P(B) = P(C) = \frac{1}{2}$

B1

Answer: each $\frac{1}{2}$

2 marks

(b) $A \cap B = \text{HH}$, $A \cap C = \text{HH}$, $B \cap C = \text{HH}$, each with probability $\frac{1}{4}$

M1

$$P(A)P(B) = P(A)P(C) = P(B)P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

M1

Each pair satisfies $P(X \cap Y) = P(X)P(Y)$

E1

Answer: Shown

3 marks

(c) $A \cap B \cap C = \text{HH}$, so $P = \frac{1}{4}$

M1

$$P(A)P(B)P(C) = \frac{1}{8} \neq \frac{1}{4}$$

A1

Answer: $\frac{1}{4} \neq \frac{1}{8}$

2 marks

(d) If A and B both occur (two heads) then C certainly occurs: $P(C | A \cap B) = 1$

E1

but $P(C) = \frac{1}{2}$, so knowing A and B together changes the probability of C

E1

Answer: $P(C | A \text{ and } B) = 1$

2 marks

27 (10 marks)**(a)** A is the union of the disjoint events $A \cap B$ and $A \cap B'$, so $P(A \cap B') = P(A) - P(A \cap B)$

M1

$$= P(A) - P(A)P(B) = P(A)(1 - P(B))$$

M1

 $= P(A)P(B')$, so A and B' are independent

E1

Answer: Proof

3 marks

(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A)P(B)$

M1 M1

$$= (1 - P(A))(1 - P(B)) = P(A')P(B')$$

E1

Answer: Proof

3 marks

(c) Let $P(X) = x$ and $P(Y) = y$: $xy = 0.3$ and $(1 - x)(1 - y) = 0.2$

M1

$$1 - (x + y) + xy = 0.2, \text{ so } x + y = 1.1$$

M1

 x and y are the roots of $t^2 - 1.1t + 0.3 = 0$, i.e. $(t - 0.6)(t - 0.5) = 0$

M1

 $P(X) > P(Y)$, so $P(X) = 0.6$ and $P(Y) = 0.5$

A1

Answer: $P(X) = 0.6, P(Y) = 0.5$

4 marks

28 (11 marks)**(a)** Anna misses, Ben misses, Anna hits: $0.7 \times 0.6 \times 0.3$

M1

$$= 0.126$$

A1

Answer: 0.126

2 marks

(b) Anna wins on her n th shot: $(0.7 \times 0.6)^{n-1} \times 0.3$

M1

Geometric series, first term $a = 0.3$ and common ratio $r = 0.42$

A1

$$S_{\infty} = \frac{0.3}{1 - 0.42}$$

M1

$$= \frac{0.3}{0.58}$$

A1

$$= \frac{15}{29}$$

A1

Answer: $\frac{15}{29}$

5 marks

(c) $P(\text{Ben wins}) = 1 - \frac{15}{29} = \frac{14}{29}$

B1

Anna wins with probability $\frac{0.3}{1 - 0.7(1 - q)}$

M1

$$\text{Fair: } \frac{0.3}{0.3 + 0.7q} = \frac{1}{2}, \text{ so } 0.6 = 0.3 + 0.7q$$

M1

$$q = \frac{3}{7}$$

A1

Answer: $\frac{14}{29}, q = \frac{3}{7}$

4 marks

29 (11 marks)

(a) Independent: $P(A \cap B) = P(A)P(B)$

M1

Mutually exclusive: $P(A \cap B) = 0$, so $P(A)P(B) = 0$

M1

A product is zero only if one factor is zero

E1

Answer: shown

3 marks

(b) $P(\text{none occurs}) = (1 - p_1)(1 - p_2)\dots(1 - p_n)$ by independence of the complements

M1

$P(\text{at least one}) = 1 - P(\text{none})$

A1

Answer: shown

2 marks

(c) Each factor satisfies $1 - p_i \leq e^{-p_i}$ and both sides are non-negative

M1

The product satisfies $\prod(1 - p_i) \leq e^{-p_1}e^{-p_2}\dots e^{-p_n} = e^{-\sum p_i}$

M1

Hence $1 - \prod(1 - p_i) \geq 1 - e^{-\sum p_i}$

A1

Answer: shown

3 marks

(d) Exact: $1 - 0.98^{20} = 0.332$

M1 A1

Bound: $1 - e^{-0.4} = 0.33$

A1

Answer: 0.332 and 0.33

3 marks

30 (10 marks)

(a) $P(\text{football}) = 0.3 + x$ and $P(\text{rugby}) = 0.2 + x$

B1

Independence: $x = (0.3 + x)(0.2 + x)$

M1

$$x = 0.06 + 0.5x + x^2$$

M1

$$x^2 - 0.5x + 0.06 = 0$$

A1

Answer: shown

4 marks

(b) $(x - 0.2)(x - 0.3) = 0$

M1

$x = 0.2$ or $x = 0.3$

A1

Answer: $x = 0.2$ or 0.3

2 marks

(c) If $x = 0.3$ the camp has $60 \div 0.3 = 200$ children, and $P(\text{neither}) = 1 - 0.3 - 0.2 - 0.3 = 0.2$

M1 A1

If $x = 0.2$ the camp has 300 children and $P(\text{neither}) = 1 - 0.3 - 0.2 - 0.2 = 0.3$

M1

Neither: 40 or 90

A1

Answer: 40 or 90

4 marks

Conditional Probability

1 (2 marks)

(a) $P = \frac{7}{35} = \frac{1}{5}$

B1

Answer: $\frac{1}{5}$

1 mark

(b) $P = \frac{12}{30} = \frac{2}{5}$

B1

Answer: $\frac{2}{5}$

1 mark

2 (2 marks)

(a) $P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.5} = 0.4$

B1

Answer: 0.4

1 mark

(b) $P(A | B) = \frac{0.2}{0.25} = 0.8$

B1

Answer: 0.8

1 mark

3 (2 marks)

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.3} = 0.4$$

B1

$$P(B | A) = \frac{0.12}{0.5} = 0.24$$

B1

Answer: 0.4 and 0.24

2 marks

4 (2 marks)

Scores greater than 2: 3, 4, 5, 6 (4 outcomes)

B1

Even among these: 4, 6, so $P = \frac{2}{4} = \frac{1}{2}$

B1

Answer: $\frac{1}{2}$

2 marks

5 (2 marks)

(a) $n(A) = 10$ and $n(A \cap B) = 4$: $\frac{4}{10} = \frac{2}{5}$

B1

Answer: $\frac{2}{5}$

1 mark

(b) $n(B') = 11$ and $n(A \cap B') = 6$: $\frac{6}{11}$

B1

Answer: $\frac{6}{11}$

1 mark

6 (4 marks)

(a) $n(B) = 20$, so $P = \frac{6}{20} = \frac{3}{10}$

B1

Answer: $\frac{3}{10}$

1 mark

(b) $n(A) = 16$, so $P = \frac{6}{16} = \frac{3}{8}$

B1

Answer: $\frac{3}{8}$

1 mark

(c) $n(B') = 10 + 10 = 20$

M1

$$P = \frac{10}{20} = \frac{1}{2}$$

A1

Answer: $\frac{1}{2}$

2 marks

7 (4 marks)

(a) $P(\text{late}) = 0.4 \times 0.2 + 0.6 \times 0.05$
 $= 0.08 + 0.03 = 0.11$

M1

A1

Answer: 0.11

2 marks

(b) $P(\text{cycle} | \text{late}) = \frac{0.6 \times 0.05}{0.11}$

M1

$$= \frac{0.03}{0.11} = \frac{3}{11}$$

A1

Answer: $\frac{3}{11}$

2 marks

8 (5 marks)

(a) $P(\text{win}) = 0.3 \times 0.4 + 0.7 \times 0.7$
 $= 0.61$

M1 A1

A1

Answer: 0.61

3 marks

(b) $P(\text{rain} | \text{win}) = \frac{0.3 \times 0.4}{0.61}$

M1

$$= \frac{12}{61}$$

A1

Answer: $\frac{12}{61}$

2 marks

9 (3 marks)

$$P(A \cap B) = P(A | B) \times P(B) = 0.5 \times 0.4 = 0.2$$

M1 A1

$$P(A | B) = P(A), \text{ so } A \text{ and } B \text{ are independent}$$

E1

Answer: 0.2, independent

3 marks

10 (4 marks)

$$(a) n(B) = 6 + 3 + 4 + 2 = 15 \text{ and } n(B \cap C) = 4 + 2 = 6$$

M1

$$P = \frac{6}{15} = \frac{2}{5}$$

A1

$$\text{Answer: } \frac{2}{5}$$

2 marks

$$(b) n(B \cap C) = 6 \text{ and } n(A \cap B \cap C) = 2$$

M1

$$P = \frac{2}{6} = \frac{1}{3}$$

A1

$$\text{Answer: } \frac{1}{3}$$

2 marks

11 (3 marks)Totals of at least 9: $4 + 3 + 2 + 1 = 10$ outcomes

M1

Of these, outcomes containing a 6: (3,6), (4,6), (5,6), (6,3), (6,4), (6,5), (6,6) = 7

M1

$$P = \frac{7}{10}$$

A1

$$\text{Answer: } \frac{7}{10}$$

3 marks

12 (5 marks)

$$(a) P(J \cap T) = P(J) \times P(T | J)$$

M1

$$= 0.6 \times 0.5 = 0.3$$

A1

Answer: 0.3

2 marks

$$(b) P(J \cap T') = 0.6 - 0.3 = 0.3$$

M1

$$P(T') = 0.6$$

B1

$$P(J | T') = \frac{0.3}{0.6} = 0.5$$

A1

Answer: 0.5

3 marks

13 (7 marks)

$$(a) \text{ Not members who are female: } 70 - 45 = 25; \text{ males: } 120 - 70 = 50, \text{ so male non-members } 50 - 30 = 20$$

M1

Totals: members 75, non-members 45

A1

Answer: See table

2 marks

$$(b) P = \frac{45}{75}$$

M1

$$= \frac{3}{5}$$

A1

$$\text{Answer: } \frac{3}{5}$$

2 marks

$$(c) P = \frac{20}{50} = \frac{2}{5}$$

A1

$$\text{Answer: } \frac{2}{5}$$

1 mark

$$(d) P(F) = \frac{70}{120} = \frac{7}{12} = 0.583 \text{ but } P(F | \text{member}) = \frac{3}{5} = 0.6$$

M1

These are not equal, so the events are not independent

E1

Answer: Not independent

2 marks

	Member	Not member	Total
Female	45	25	70
Male	30	20	50
Total	75	45	120

14 (6 marks)

(a) $P(\text{positive and disease}) = 0.02 \times 0.95 = 0.019$

M1

$P(\text{positive and no disease}) = 0.98 \times 0.04 = 0.0392$

M1

$P(\text{positive}) = 0.0582$

A1

Answer: 0.0582

3 marks

(b) $P(\text{disease} | \text{positive}) = \frac{P(\text{disease and positive})}{P(\text{positive})}$

M1

$= \frac{0.019}{0.0582}$

M1

$= 0.326 \text{ (3 s.f.)}$

A1

Answer: 0.326

3 marks

15 (5 marks)

(a) $P(D) = 0.45 \times 0.01 + 0.35 \times 0.02 + 0.2 \times 0.05$

M1 A1

$= 0.0045 + 0.007 + 0.01 = 0.0215$

A1

Answer: 0.0215

3 marks

(b) $P(Z | D) = \frac{0.2 \times 0.05}{0.0215}$

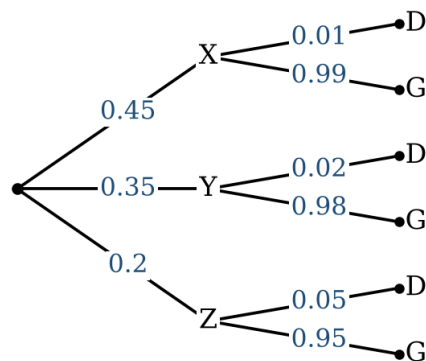
M1

$= \frac{20}{43}$

A1

Answer: $\frac{20}{43}$

2 marks

**16 (6 marks)**

(a) $P(A \cap B) = 0.5 + 0.4 - 0.7 = 0.2$

M1

$P(A | B) = \frac{0.2}{0.4} = 0.5$

A1

Answer: 0.5

2 marks

(b) $P(A' \cap B) = 0.4 - 0.2 = 0.2$ and $P(A') = 0.5$

M1

$P(B | A') = \frac{0.2}{0.5} = 0.4$

A1

Answer: 0.4

2 marks

(c) $P(A' \cap B') = 1 - 0.7 = 0.3$ and $P(B') = 0.6$

M1

$P(A' | B') = \frac{0.3}{0.6} = 0.5$

A1

Answer: 0.5

2 marks

17 (6 marks)

$$(a) P(\text{head}) = \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$$

M1 A1

$$P(\text{two-headed} | \text{head}) = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

A1

Answer: $\frac{2}{3}$

3 marks

$$(b) P(\text{two-headed} | \text{head}) = \frac{2}{3} \text{ and } P(\text{fair} | \text{head}) = \frac{1}{3}$$

B1

$$P(\text{head again}) = \frac{2}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} = \frac{5}{6}$$

M1

A1

Answer: $\frac{5}{6}$

3 marks

18 (4 marks)

$$P(\text{both mint}) = \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$$

M1 A1

$$P(\text{at least one mint}) = 1 - \frac{6}{10} \times \frac{5}{9} = \frac{2}{3}$$

M1

$$P = \frac{2}{15} \div \frac{2}{3} = \frac{1}{5}$$

A1

Answer: $\frac{1}{5}$

4 marks

19 (5 marks)

$$\text{Let } P(A \cap B) = x. \text{ Then } P(B) = \frac{x}{0.5} = 2x$$

M1 A1

$$P(A) = \frac{x}{0.25} = 4x$$

A1

$$P(A \cup B) = 4x + 2x - x = 5x = 0.8$$

M1

$$x = 0.16, \text{ so } P(A) = 0.64 \text{ and } P(B) = 0.32$$

A1

Answer: 0.16, 0.64, 0.32

5 marks

20 (6 marks)

$$(a) P(A \cup B) = 0.5 + 0.35 - 0.15 = 0.7$$

M1

$$A \subset A \cup B \text{ so } P(A | A \cup B) = \frac{0.5}{0.7}$$

M1

$$= \frac{5}{7}$$

A1

Answer: $\frac{5}{7}$

3 marks

$$(b) P(\text{exactly one}) = 0.35 + 0.2 = 0.55$$

M1

$$P(A \text{ only}) = 0.5 - 0.15 = 0.35$$

M1

$$\frac{0.35}{0.55} = \frac{7}{11}$$

A1

Answer: $\frac{7}{11}$

3 marks

21 (8 marks)

$$(a) P = P(RR) + P(\text{not } R \text{ then } R) = \frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{5}{11}$$

M1

$$= \frac{20 + 35}{132} = \frac{5}{12}$$

A1

Answer: $\frac{5}{12}$

2 marks

$$(b) P(\text{first } R | \text{second } R) = \frac{P(RR)}{P(\text{second } R)}$$

M1

$$= \frac{20}{132} \div \frac{55}{132}$$

M1

$$= \frac{4}{11}$$

A1

Answer: $\frac{4}{11}$

3 marks

(c) $P(\text{same colour}) = \frac{20 + 12 + 6}{132} = \frac{38}{132}$, so $P(\text{different}) = \frac{94}{132}$

M1

$$P(\text{different, one green}) = P(G \text{ then non-G}) + P(\text{non-G then G}) = 2 \times \frac{3}{12} \times \frac{9}{11} = \frac{54}{132}$$

M1

$$P = \frac{54}{94} = \frac{27}{47}$$

A1

Answer: $\frac{27}{47}$

3 marks

22 (8 marks)

(a) $P(A \cap B) = 0.5 \times 0.6 = 0.3$

M1

$$P(A \cap B') = 0.5 \times 0.3 = 0.15$$

M1

$$P(A) = 0.3 + 0.15 = 0.45$$

A1

Answer: 0.45

3 marks

(b) $P(B | A) = \frac{0.3}{0.45}$

M1

$$= \frac{2}{3}$$

A1

Answer: $\frac{2}{3}$

2 marks

(c) $P(A | B) = 0.6 \neq P(A) = 0.45$

E1

Answer: Not independent

1 mark

(d) Independence needs $P(A | B') = P(A | B)$

M1

$$\text{so } P(A | B') = 0.6$$

A1

Answer: 0.6

2 marks

23 (6 marks)

(a) Each order (RRB, RBR, BRR) has probability $\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$ (the same numerators in a different order)

M1 A1

$$P = 3 \times \frac{1}{6} = \frac{1}{2}$$

A1

Answer: shown

3 marks

(b) First red and exactly two red: RRB and RBR, probability $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

M1 A1

$$P = \frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$$

A1

Answer: $\frac{2}{3}$

3 marks

24 (8 marks)

(a) $P = 0.3 \times 0.6 + 0.7 \times 0.2$

M1

$$= 0.32$$

A1

Answer: 0.32

2 marks

(b) $P(\text{rain Mon} | \text{rain Tue}) = \frac{0.18}{0.32}$

M1

$$= \frac{9}{16}$$

A1

Answer: $\frac{9}{16}$

2 marks

(c) Rain Tue: $0.6 \times 0.6 = 0.36$ (Tue rain then Wed rain)

M1

$$\text{Dry Tue: } 0.4 \times 0.2 = 0.08$$

M1

$$P = \frac{0.36}{0.36 + 0.08}$$

M1

$$= \frac{9}{11}$$

A1

Answer: $\frac{9}{11}$

4 marks

25 (8 marks)

(a) Equally likely outcomes: BB, BG, GB, GG

B1

At least one boy: BB, BG, GB

M1

$$P(\text{BB} \mid \text{at least one boy}) = \frac{1}{3}$$

A1

Answer: $\frac{1}{3}$

3 marks

(b) Outcomes with the elder a boy: BB, BG

M1

$$P = \frac{1}{2}$$

A1

Answer: $\frac{1}{2}$

2 marks

(c) At least two girls: GGG, GGB, GBG, BGG (4 outcomes)

M1 A1

$$P(\text{GGG} \mid \text{at least two girls}) = \frac{1}{4}$$

A1

Answer: $\frac{1}{4}$

3 marks

26 (9 marks)

(a) $0.45 \times 0.6 + 0.35 \times 0.5 + 0.2 \times 0.3$

M1 M1

$$= 0.27 + 0.175 + 0.06 = 0.505$$

A1

Answer: 0.505

3 marks

(b) $P(\text{red and even}) = 0.06$

B1

$$P(\text{not red} \mid \text{even}) = 1 - \frac{0.06}{0.505}$$

M1

$$= \frac{89}{101} = 0.881$$

A1

Answer: $\frac{89}{101} = 0.881$

3 marks

(c) $P(\text{blue} \mid \text{even}) = \frac{0.27}{0.505} = \frac{54}{101}$

M1 A1

$$\text{Independent draws: } \left(\frac{54}{101}\right)^2 = \frac{2916}{10201} = 0.286$$

A1

Answer: $\frac{2916}{10201} = 0.286$

3 marks

27 (10 marks)

(a) $P(\text{red showing}) = \frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}$

M1

$$= \frac{1}{2}$$

A1

Answer: $\frac{1}{2}$

2 marks

(b) The underside is red only if card 1 was drawn: $P(\text{card 1 and red showing}) = \frac{1}{3} \times 1 = \frac{1}{3}$

M1

$$P(\text{underside red} \mid \text{red showing}) = \frac{1}{3} \div \frac{1}{2}$$

M1

$$= \frac{2}{3}$$

A1

Answer: $\frac{2}{3}$

3 marks

(c) Card 1 and card 3 are not equally likely given that red is showing

E1

Card 1 has two red sides so is twice as likely to show red as card 3: there are three equally likely red faces, two of which belong to card 1

E1

Answer: Cards are not equally likely

2 marks

(d) Number of occasions with red showing has a binomial distribution $B(3, \frac{1}{2})$

M1

$$P(2) + P(3) = 3 \times (\frac{1}{2})^3 + (\frac{1}{2})^3$$

M1

$$= \frac{4}{8} = \frac{1}{2}$$

A1

Answer: $\frac{1}{2}$

3 marks

28 (10 marks)

(a) $P(\text{positive}) = 0.01 \times 0.9 + 0.99 \times 0.05 = 0.009 + 0.0495 = 0.0585$

M1 A1

$$P(\text{disease} | \text{positive}) = \frac{0.009}{0.0585} = \frac{2}{13} = 0.154 \text{ (3 s.f.)}$$

A1

Answer: $\frac{2}{13} = 0.154$

3 marks

(b) $P(\text{both positive and disease}) = 0.01 \times 0.9^2 = 0.0081$

M1

$$P(\text{both positive and no disease}) = 0.99 \times 0.05^2 = 0.002475$$

M1

$$P(\text{disease} | \text{both positive}) = \frac{0.0081}{0.0081 + 0.002475}$$

M1

$$= 0.766 \text{ (3 s.f.)}$$

A1

Answer: 0.766

4 marks

(c) $P(A | B) > P(A)$ means $\frac{P(A \cap B)}{P(B)} > P(A)$

M1

Multiplying by $P(B) > 0$ and dividing by $P(A) > 0$: $\frac{P(A \cap B)}{P(A)} > P(B)$, that is $P(B | A) > P(B)$

M1

Every step is reversible, so the statements are equivalent

E1

Answer: Proof

3 marks

29 (11 marks)

(a) $P(WW | \text{urn 1}) = (\frac{3}{5})^2 = \frac{9}{25}$ and $P(WW | \text{urn 2}) = (\frac{1}{3})^2 = \frac{1}{9}$

M1 A1

$$P(\text{urn 1} | WW) = \frac{\frac{1}{2} \times \frac{9}{25}}{\frac{1}{2} \times \frac{9}{25} + \frac{1}{2} \times \frac{1}{9}}$$

M1

$$= \frac{81}{81 + 25} = \frac{81}{106}$$

A1

Answer: shown

4 marks

(b) $P(\text{urn 1} | W) = \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{1}{3}}$

M1

$$= \frac{9}{14}$$

A1

Answer: $\frac{9}{14}$

2 marks

(c) $P(\text{urn 1} | W) = \frac{9}{14}$, $P(\text{urn 2} | W) = \frac{5}{14}$

B1

$$P(\text{second W} | \text{first W}) = \frac{9}{14} \times \frac{3}{5} + \frac{5}{14} \times \frac{1}{3}$$

M1

$$= \frac{53}{105}$$

A1

Answer: $\frac{53}{105}$

3 marks

(d) Urn 1 has the higher proportion of white balls, so each white ball observed is more likely to come from urn 1 than urn 2

E1

Each white ball therefore increases the probability of urn 1: $\frac{1}{2}$ to $\frac{9}{14}$ to $\frac{81}{106}$

E1

Answer: Urn 1 more likely

2 marks

30 (10 marks)

$$(a) \frac{P(A \cap B)}{P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}, \text{ using } P(A \cap B') = P(A) - P(A \cap B) \quad \text{M1}$$

$$\text{Cross-multiply: } P(A \cap B)(1 - P(B)) = P(B)(P(A) - P(A \cap B)) \quad \text{M1}$$

$$P(A \cap B) - P(A \cap B)P(B) = P(A)P(B) - P(A \cap B)P(B) \quad \text{A1}$$

$$\text{So } P(A \cap B) = P(A)P(B) \quad \text{A1}$$

Answer: shown

4 marks

$$(b) P(A \cap B) > P(A)P(B) \text{ from } P(A | B) > P(A) \text{ with } P(B) > 0 \quad \text{M1}$$

$$\text{Divide by } P(A) > 0 \quad \text{M1}$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)} > P(B) \quad \text{A1}$$

Answer: shown

3 marks

$$(c) P(A) = 0.4 \times 0.7 + 0.6 \times 0.2 = 0.4 \quad \text{M1 A1}$$

$$P(B | A) = \frac{0.28}{0.4} = 0.7 \quad \text{A1}$$

Answer: 0.7

3 marks

Permutations and Combinations**1 (3 marks)**

$$(a) 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \quad \text{B1}$$

Answer: 720

1 mark

$$(b) {}^5P_2 = 5 \times 4 = 20 \quad \text{B1}$$

Answer: 20

1 mark

$$(c) {}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35 \quad \text{B1}$$

Answer: 35

1 mark

2 (2 marks)

$$(a) 5! = 120 \quad \text{B1}$$

Answer: 120

1 mark

$$(b) {}^8C_3 = 56 \quad \text{B1}$$

Answer: 56

1 mark

3 (2 marks)

$$(a) {}^8C_3 = 56 \quad \text{B1}$$

Answer: 56

1 mark

$$(b) {}^8P_4 = 8 \times 7 \times 6 \times 5 = 1680 \quad \text{B1}$$

Answer: 1680

1 mark

4 (2 marks)

$$(a) 10^4 = 10\,000 \quad \text{B1}$$

$$(b) 10 \times 9 \times 8 \times 7 = 5040 \quad \text{B1}$$

Answer: (a) 10 000 (b) 5040

2 marks

5 (2 marks)

$$\text{Multiply the choices: } 3 \times 4 \times 2 \quad \text{M1}$$

$$= 24 \quad \text{A1}$$

Answer: 24

2 marks

6 (4 marks)

$$(a) 6 \text{ letters with A repeated 3 times and N repeated twice} \quad \text{M1}$$

$$\frac{6!}{3! \times 2!} = 60 \quad \text{A1}$$

Answer: 60

2 marks

$$(b) 10 \text{ letters: S three times, T three times, I twice} \quad \text{M1}$$

$$\frac{10!}{3! \times 3! \times 2!} = 50\,400$$

Answer: 50 400

A1

2 marks

7 (4 marks)

(a) ${}^{10}C_5 = 252$

Answer: 252

B1

1 mark

(b) Captain is fixed, choose the other 4 from 9: ${}^9C_4 = 126$

Answer: 126

B1

1 mark

(c) Teams with both Ali and Ben: choose 3 from the other 8, ${}^8C_3 = 56$

$$252 - 56 = 196$$

Answer: 196

M1

A1

2 marks

8 (3 marks)

(a) $6! = 720$

(b) Fix one person to remove rotations, then arrange the other five: $5!$

$$= 120$$

Answer: (a) 720 (b) 120

B1

M1

A1

3 marks

9 (3 marks)

Men: ${}^7C_3 = 35$

Women: ${}^5C_2 = 10$

Total $35 \times 10 = 350$

Answer: 350

B1

B1

B1

3 marks

10 (4 marks)

(a) $6^3 = 216$

Answer: 216

B1

1 mark

(b) $6 \times 5 \times 4 = 120$

Answer: 120

B1

1 mark

(c) First letter A or E: 2 ways

$$2 \times 5 \times 4 = 40$$

Answer: 40

M1

A1

2 marks

11 (3 marks)

Choices for each position: ${}^3C_1, {}^6C_3, {}^5C_3, {}^4C_2 = 3, 20, 10, 6$

$$3 \times 20 \times 10 \times 6 = 3600$$

Answer: 3600

M1 A1

A1

3 marks

12 (4 marks)

(a) ${}^8C_5 \times {}^5C_3 = 56 \times 10$

$$= 560$$

Answer: 560

M1

A1

2 marks

(b) Total selections ${}^{13}C_8 = 1287$

$$\frac{560}{1287} = 0.435$$

Answer: $\frac{560}{1287} = 0.435$

B1

A1

2 marks

13 (6 marks)

(a) ${}^{11}C_4 = 330$

Answer: 330

B1

1 mark

(b) Ways: ${}^5C_2 \times {}^6C_2 = 10 \times 15 = 150$

$$P = \frac{150}{330} = \frac{5}{11}$$

M1 A1

A1

Answer: $\frac{5}{11}$

3 marks

(c) $P(\text{no women}) = \frac{{}^6C_4}{330} = \frac{15}{330}$

M1

$$P = 1 - \frac{15}{330} = \frac{21}{22}$$

A1

Answer: $\frac{21}{22}$

2 marks

14 (6 marks)(a) Treat Ann and Bob as one block, so there are 6 objects: $6! = 720$ arrangements

M1 A1

Ann and Bob can be ordered in 2 ways within the block: $2 \times 720 = 1440$

A1

Answer: 1440

3 marks

(b) Total arrangements $7! = 5040$

M1

$$5040 - 1440 = 3600$$

A1

Answer: 3600

2 marks

(c) $(7 - 1)! = 720$

B1

Answer: 720

1 mark

15 (7 marks)

(a) First digit 5 ways (not 0), then 5, 4, 3, 2 ways for the others

M1

$$5 \times 5 \times 4 \times 3 \times 2 = 600$$

A1

Answer: 600

2 marks

(b) Last digit 0: the other four places from 5 digits, $5 \times 4 \times 3 \times 2 = 120$

M1

Last digit 2 or 4: 2 ways, first digit 4 ways (not 0, not the last), then 4, 3, 2: $2 \times 4 \times 4 \times 3 \times 2 = 192$

M1

$$\text{Total } 120 + 192 = 312$$

A1

Answer: 312

3 marks

(c) First digit 3, 4 or 5: 3 ways

M1

$$3 \times 5 \times 4 \times 3 \times 2 = 360$$

A1

Answer: 360

2 marks

16 (7 marks)(a) 7 letters with A twice and R twice: $\frac{7!}{2! \times 2!}$

M1

$$= 1260$$

A1

Answer: 1260

2 marks

(b) Treat AA as one block: 6 items with R twice, $\frac{6!}{2!}$

M1

$$= 360$$

A1

Answer: 360

2 marks

(c) Vowels are A, A, E; ends are (A, A), (A, E) or (E, A)

M1

(A, A): remaining R, R, N, G, E arranged in $\frac{5!}{2!} = 60$; (A, E) and (E, A): remaining A, R, R, N, G in 60 each

M1

$$\text{Total } 60 + 60 + 60 = 180$$

A1

Answer: 180

3 marks

17 (6 marks)(a) Number of hands: ${}^{52}C_5 = 2\,598\,960$; hands of five hearts: ${}^{13}C_5 = 1287$

M1

$$P = \frac{1287}{2\,598\,960} = 0.000495$$

A1

Answer: 0.000495

2 marks

(b) ${}^4C_2 \times {}^{48}C_3 = 6 \times 17\,296 = 103\,776$

M1

$$P = \frac{103\,776}{2\,598\,960} = 0.0399$$

A1

Answer: 0.0399

2 marks

$$(c) P = 1 - P(\text{no aces}) = 1 - \frac{{}^{48}C_5}{{}^{52}C_5}$$

= 0.341

Answer: 0.341

M1

A1

2 marks

18 (5 marks)

(a) Treat each couple as a block: 4 blocks round a table, $(4 - 1)! = 6$ arrangements

M1 A1

Each couple can swap places: $2^4 = 16$, so $6 \times 16 = 96$

A1

Answer: 96

3 marks

(b) Seat the 4 men round the table: $(4 - 1)! = 6$; then the women in the 4 gaps: $4! = 24$

M1

$6 \times 24 = 144$

A1

Answer: 144

2 marks

19 (5 marks)

Total arrangements $7! = 5040$

B1

Arrange the boys first: $4! = 24$ ways, creating 5 gaps

M1

Choose gaps for the girls in order: ${}^5P_3 = 60$

M1

Favourable $24 \times 60 = 1440$

A1

Probability $\frac{1440}{5040} = \frac{2}{7}$

A1

Answer: $\frac{2}{7}$

5 marks

20 (8 marks)

(a) Last digit 2, 4 or 6 (3 ways); then the other three places in $6 \times 5 \times 4$ ways

M1

$3 \times 120 = 360$

A1

Answer: 360

2 marks

(b) First digit 5, 6 or 7 (3 ways), then $6 \times 5 \times 4$ ways

M1

$3 \times 120 = 360$

A1

Answer: 360

2 marks

(c) Last digit must be 5; other places $6 \times 5 \times 4 = 120$

B1

Answer: 120

1 mark

(d) Split by first digit (it must be 5, 6 or 7), because 6 is also an even digit

M1

First digit 5 or 7: 2 ways; last digit 2, 4 or 6: 3 ways; middle two digits $5 \times 4 = 20$ ways: $2 \times 3 \times$

M1

$20 = 120$

First digit 6: last digit 2 or 4: 2 ways; middle 20 ways: 40. Total 160

A1

Answer: 160

3 marks

21 (10 marks)

(a) Number of handshakes $= {}^nC_2 = \frac{n(n-1)}{2}$

M1

$\frac{n(n-1)}{2} = 45$, so $n^2 - n - 90 = 0$

A1

$(n-10)(n+9) = 0$

M1

$n = 10$ since $n > 0$

A1

Answer: $n = 10$

4 marks

(b) Pairs including Ahmed: 9. Total pairs: 45

M1

$P = \frac{9}{45} = \frac{1}{5}$

A1

Answer: $\frac{1}{5}$

2 marks

(c) $\frac{{}^mC_3}{{}^mC_2} = \frac{m(m-1)(m-2)}{6} \times \frac{2}{m(m-1)}$

M1

$= \frac{m-2}{3}$

M1

$\frac{m-2}{3} = 4$

M1

$m = 14$

A1

Answer: m = 14**4 marks****22 (9 marks)****(a)** 11 letters with M, A, T each twice: $\frac{11!}{2! \times 2! \times 2!}$ **M1** $= 4\,989\,600$ **A1****Answer: shown****2 marks****(b)** Treat MM as one block: 10 items with A twice and T twice**M1**

$$\frac{10!}{2! \times 2!} = 907\,200$$

A1

$$P = \frac{907\,200}{4\,989\,600} = \frac{2}{11}$$

A1**Answer: $\frac{2}{11}$** **3 marks****(c)** Vowels: A, A, E, I; ends (A, A): remaining 9 letters give $\frac{9!}{2! \times 2!} = 90\,720$ **M1 A1**Ends (A, E), (E, A), (A, I), (I, A): $4 \times 90\,720$; ends (E, I), (I, E): $2 \times 45\,360$ **M1**

$$\text{Total } 544\,320, \text{ so } P = \frac{544\,320}{4\,989\,600} = \frac{6}{55}$$

A1**Answer: $\frac{6}{55}$** **4 marks****23 (8 marks)****(a)** Number of possible draws: ${}^{49}C_6 = 13\,983\,816$ **M1**

$$P = \frac{1}{13\,983\,816}$$

A1**Answer: $\frac{1}{13\,983\,816}$** **2 marks****(b)** Choose 4 of the 6 winning numbers and 2 of the 43 others: ${}^6C_4 \times {}^{43}C_2 = 15 \times 903 = 13\,545$ **M1 A1**

$$P = \frac{13\,545}{13\,983\,816} = 0.000969$$

A1**Answer: 0.000969****3 marks****(c)** Exactly 3: ${}^6C_3 \times {}^{43}C_3 = 20 \times 12\,341 = 246\,820$ **M1**At least 3: $246\,820 + 13\,545 + 258 + 1 = 260\,624$ **M1**

$$P = 0.0186$$

A1**Answer: 0.0186****3 marks****24 (6 marks)****(a)** Choose 4 for A, then 4 of the remaining 8 for B, the rest for C: ${}^{12}C_4 \times {}^8C_4$ **M1**

$$= 495 \times 70 = 34\,650$$

A1**Answer: 34 650****2 marks****(b)** The 3 teams can be named in $3! = 6$ ways, so divide by 6**M1**

$$\frac{34\,650}{6} = 5775$$

A1**Answer: 5775****2 marks****(c)** Choose 6 of 12 for one team: ${}^{12}C_6 = 924$ **M1**

Each division is counted twice, so 462

A1**Answer: 462****2 marks****25 (5 marks)**Total codes $3^4 = 81$ **B1**

All four digits the same: 3 codes

B1

Exactly three the same: 3 choices of digit, 2 choices of the other digit, 4 positions = 24

M1 A1

$$\text{Favourable } 81 - 3 - 24 = 54, \text{ so probability } \frac{54}{81} = \frac{2}{3}$$

A1**Answer: $\frac{2}{3}$** **5 marks**

26 (5 marks)

Seat the 6 children first, leaving 7 gaps (including the ends)

M1

Choose 4 of the 7 gaps for the adults: ${}^7C_4 = 35$

M1 A1

Choose 4 of the 10 seats for the adults at random: ${}^{10}C_4 = 210$

B1

Probability $\frac{35}{210} = \frac{1}{6}$

A1

Answer: $\frac{1}{6}$

5 marks

27 (10 marks)

$$(a) {}^{n-1}C_r + {}^{n-1}C_{r-1} = \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$$

M1

Common denominator $r!(n-r)!$: multiply the first fraction by $\frac{n-r}{n-r}$ and the second by $\frac{r}{r}$

M1

$$= \frac{(n-1)![(n-r) + r]}{r!(n-r)!}$$

M1

$$= \frac{n!}{r!(n-r)!} = {}^nC_r$$

E1

Answer: Proof

4 marks

(b) Either 2 men and 3 women, or 3 men and 2 women

M1

$${}^7C_2 \times {}^5C_3 + {}^7C_3 \times {}^5C_2 = 21 \times 10 + 35 \times 10$$

M1

$$= 560$$

A1

Answer: 560

3 marks

(c) Committees in (b) containing both: 2 men and 3 women: ${}^6C_1 \times {}^4C_2 = 36$; 3 men and 2 women:

M1 A1

$${}^6C_2 \times {}^4C_1 = 60$$

$$560 - 96 = 464$$

A1

Answer: 464

3 marks

28 (11 marks)

$$(a) {}^{n-1}C_{r-1} + {}^{n-1}C_r = \frac{(n-1)!}{(r-1)!(n-r)!} + \frac{(n-1)!}{r!(n-1-r)!}$$

M1

$$\text{Common denominator } r!(n-r)!: \frac{(n-1)![r + (n-r)]}{r!(n-r)!}$$

M1 A1

$$= \frac{n!}{r!(n-r)!} = {}^nC_r$$

A1

Answer: shown

4 marks

(b) Selections that include Z: choose the other $r-1$ from $n-1$, ${}^{n-1}C_{r-1}$

E1

Selections that exclude Z: choose r from the other $n-1$, ${}^{n-1}C_r$

E1

These cases are exclusive and exhaustive, so they add to ${}^nC_r$

E1

Answer: shown

3 marks

$$(c) {}^{11}C_5 = {}^{10}C_4 + {}^{10}C_5$$

M1

$$= 210 + 252 = 462$$

A1

Answer: 462

2 marks

(d) Each person is in or out, giving $2^4 = 16$ selections in total (of any size)

M1

Grouping the selections by size gives $1 + 4 + 6 + 4 + 1 = 16$

A1

Answer: shown

2 marks

29 (10 marks)(a) Arrange the other letters B, N, N first: $\frac{3!}{2!} = 3$ ways

M1 A1

This leaves 4 gaps (including the ends), in which the three As go, no two in the same gap: ${}^4C_3 = 4$

M1

$$3 \times 4 = 12$$

A1

Answer: 12

4 marks

(b) Total arrangements $5! = 120$

B1

Arrangements with a given k letters correct: $(5-k)!$, and there are ${}^5C_k$ choices of the k letters

M1

$$\text{Number with none correct} = 5! - {}^5C_1 4! + {}^5C_2 3! - {}^5C_3 2! + {}^5C_4 1! - {}^5C_5 0!$$

M1 A1

$$= 120 - 120 + 60 - 20 + 5 - 1 = 44$$

A1

$$P = \frac{44}{120} = \frac{11}{30}$$

A1

Answer: 44, $\frac{11}{30}$

6 marks

30 (6 marks)

Complete within 5 packets means all four toys appear in 5 packets

M1

Number of ways: one toy appears twice (4 choices of toy) and the 5 packets can be ordered in $\frac{5!}{2!}$

M1 A1

= 60 ways

$$4 \times 60 = 240 \text{ out of } 4^5 = 1024$$

M1

$$P(\text{complete in at most 5}) = \frac{240}{1024} = \frac{15}{64}$$

A1

$$P(\text{more than 5}) = 1 - \frac{15}{64} = \frac{49}{64}$$

A1

Answer: $\frac{49}{64}$

6 marks