

Basic Probability

- 1** A bag contains 5 red counters, 3 blue counters and 2 green counters. One counter is taken at random.
- (a) Find the probability that the counter is blue. [1]
- (b) Find the probability that the counter is not green. [1]
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- 2** A fair six-sided die is rolled once.
- (a) Find the probability that the score is a prime number. [1]
- (b) Find the probability that the score is a prime number or an even number. [2]
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- 3** A fair 12-sided die has faces numbered 1 to 12. It is rolled once.
- (a) Find the probability of scoring a prime number. [1]
- (b) Find the probability of scoring a multiple of 4. [1]
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- 4** The probability that a train arrives late is 0.18.
- (a) Find the probability that the train does not arrive late. [1]
- (b) Out of 250 journeys, estimate how many times the train arrives late. [1]
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- 5** A card is chosen at random from a standard pack of 52 playing cards.
- (a) Find the probability that it is a heart. [1]
- (b) Find the probability that it is a king. [1]
- (c) Find the probability that it is a heart or a king. [2]
-
- 6** Two fair coins are tossed. Write down all the possible outcomes, and hence find the probability of obtaining exactly one head. [2]
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- 7** The table shows the main activity of the 80 members of a leisure centre, split by age group. One member is chosen at random.

	Gym	Swimming	Classes	Total
Under 30	18	9	13	40
30 and over	12	14	14	40
Total	30	23	27	80

- (a) Find the probability that the member's main activity is the gym. [1]
- (b) Find the probability that the member is aged 30 and over and mainly swims. [1]
- (c) Find the probability that the member is under 30 or mainly attends classes. [2]
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- 8** Spinner X has four equally likely sectors numbered 1, 2, 3, 4. Spinner Y has three equally likely sectors numbered 1, 2, 3. Both spinners are spun and the two scores are multiplied together.

×	1	2	3
1			
2			
3			
4			

- (a) Complete the sample space diagram to show all possible products. [1]
- (b) Find the probability that the product is greater than 6. [1]
- (c) Find the probability that the product is even. [1]
- (d) Find the probability that the product is a multiple of 3 or is greater than 6. [2]

- 9** Two fair six-sided dice are rolled and their scores are added.
Draw a sample space diagram for the total score, and use it to find the probability that the total is
- (a) equal to 8, [2]
 (b) at least 10, [2]
 (c) a prime number. [2]

- 10** A die is thought to be biased. It is thrown 200 times and the scores are recorded in the table.

Score	1	2	3	4	5	6
Frequency	28	31	35	30	36	40

- (a) Estimate the probability of scoring a 6 with this die. [1]
 (b) Estimate how many 6s would be obtained in 500 throws. [1]
 (c) Does the table provide strong evidence that the die is biased? Give a reason. [2]

- 11** A and B are mutually exclusive events with $P(A) = 0.3$ and $P(B) = 0.45$.

- (a) Find $P(A \cup B)$. [1]
 (b) Find the probability that neither A nor B occurs. [1]
 (c) A third event C is mutually exclusive with both A and B, and $P(C) = 0.2$. Find $P(A \cup B \cup C)$. [1]

- 12** One letter is chosen at random from the word STATISTICS. Find the probability that it is

- (a) a T, [1]
 (b) a vowel, [1]
 (c) an S or an I. [2]

- 13** A fair coin is tossed three times. Find the probability of obtaining at least two heads. [3]

- 14** A and B are events with $P(A) = 0.35$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.

- (a) Find $P(A \cap B)$. [2]
 (b) State, with a reason, whether A and B are mutually exclusive. [1]
 (c) Find $P(A' \cap B')$. [1]
 (d) Find the probability that exactly one of A and B occurs. [2]

- 15** A biased five-sector spinner is spun. The probability distribution of the score is shown in the table.

Score	1	2	3	4	5
P(score)	0.1	x	2x	3x	0.15

- (a) Find the value of x. [2]
 (b) Find the probability that the score is at least 3. [2]
 (c) The spinner is spun 200 times. Estimate how many times the score will be 4. [1]

- 16** Two fair six-sided dice are rolled. The positive difference between the scores (larger minus smaller, or 0 if equal) is recorded.

By drawing a suitable sample space diagram, or otherwise, find the probability that

- (a) the difference is exactly 2, [2]
 (b) the difference is at least 3, [2]
 (c) the product of the scores is greater than 20. [2]

- 17** A bag contains 4 red counters and 5 blue counters. Three counters are taken at random, without replacement.

Find the probability that all three counters are the same colour. [4]

- 18** A three-digit number is chosen at random from all the three-digit numbers 100, 101, ..., 999. Find the probability that the number
- (a) is a multiple of 7, [2]
 - (b) has three different digits, [2]
 - (c) is a palindrome (reads the same forwards and backwards). [2]

- 19** The discrete random variable X takes the values 1, 2, 3, 4, 5 with the probabilities shown, where k is a constant.

x	1	2	3	4	5
$P(X = x)$	$5k$	$4k$	$3k$	$2k$	k

- (a) Find the value of k . [2]
- (b) Find $P(X \geq 3)$. [1]
- (c) Find $P(X \text{ is odd})$. [1]

- 20** A bag contains n counters. The probability that a counter taken at random is red is $\frac{3}{8}$. Four more red counters are added to the bag, and the probability of taking a red counter is then $\frac{1}{2}$. Find the value of n . [4]

- 21** Ten cards are numbered 1 to 10. Two cards are taken at random, one after the other, without replacement.
- (a) Find the probability that both cards show even numbers. [2]
 - (b) Find the probability that the sum of the two numbers is a multiple of 3. [3]
 - (c) Find the probability that the sum of the two numbers is 11 and the first card is larger than the second. [2]

- 22** A bag contains n counters, of which 4 are blue and the rest are white. Two counters are taken at random without replacement. The probability that both counters are blue is $\frac{2}{15}$.
- (a) Show that $n^2 - n - 90 = 0$. [3]
 - (b) Hence find the value of n . [2]
 - (c) Find the probability that exactly one of the two counters is blue. [3]

- 23** Two fair six-sided dice are rolled and X is the larger of the two scores (X = the common score if they are equal).
- (a) Show that $P(X = k) = \frac{2k - 1}{36}$ for $k = 1, 2, \dots, 6$. [3]
 - (b) Find $P(X \geq 5)$. [2]
 - (c) Find the probability that the larger score is even. [2]

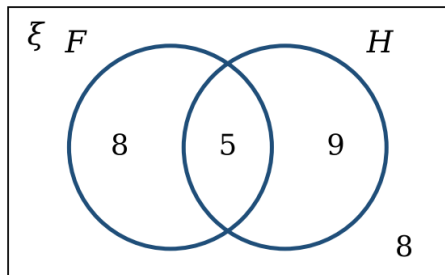
- 24** Assume that every person is equally likely to have been born on any of the 365 days of the year (ignore 29 February), independently of other people.
- n people are chosen at random.
- (a) Show that for $n = 3$ the probability that all three have different birthdays is $\frac{365 \times 364 \times 363}{365^3}$. [2]
 - (b) Find the probability that, among 3 people, at least two share a birthday. Give your answer to 3 significant figures. [2]
 - (c) Show that the probability of at least two people sharing a birthday exceeds 0.5 when $n = 23$ but not when $n = 22$. [4]

- 25** A fair coin is tossed repeatedly until the first head appears. Find the probability that
- (a) the first head appears on the 4th toss, [2]
 - (b) at most 5 tosses are needed, [2]
 - (c) an even number of tosses is needed. [4]

- 26** Four letters, each addressed to a different person, are put at random into four envelopes addressed to those people, one letter in each envelope.
- Explain why it is impossible for exactly three letters to be in the correct envelope. [1]
 - Find the probability that exactly two letters are in the correct envelope. [3]
 - Given that the probability that exactly one letter is correct is $\frac{1}{3}$ and that all four are correct is $\frac{1}{24}$, find the probability that no letter is correct. [3]
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- 27** A fair six-sided die is rolled 4 times. Amir says: "The probability of getting at least one six is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$."
- Explain why Amir's method is wrong. [2]
 - Find the probability of getting at least one six in the 4 rolls. [3]
 - Find the smallest number of rolls needed so that the probability of getting at least one six exceeds 0.95. [4]
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- 28** Aisha and Ben take turns to roll a fair six-sided die, with Aisha rolling first. The first person to roll a six wins the game.
- Find the probability that Ben wins on his first roll. [2]
 - Find the probability that Aisha wins on her second roll. [2]
 - Show that the probability that Aisha wins the game is $\frac{6}{11}$. [5]
 - Hence find the probability that Ben wins, stating the assumption you make. [1]
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- 29** In a dice game a player rolls two fair six-sided dice and adds the scores. If the total is 7 or 11 the player wins immediately. If it is 2, 3 or 12 the player loses immediately. Otherwise the total becomes the "point" and the player keeps rolling the two dice until either the point is rolled again (the player wins) or a total of 7 is rolled (the player loses).
- Find the probability that the player wins on the first roll. [2]
 - The point is 4. Show that the probability of eventually rolling a 4 before a 7 is $\frac{1}{3}$. [4]
 - Using the same method for each possible point, show that the probability that the player wins the game is $\frac{244}{495}$. [6]
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- 30** A fair coin is tossed repeatedly, producing a sequence of heads (H) and tails (T). Alex wins the game if the sequence HH appears (as two consecutive tosses) before the sequence TH does. Beth wins if TH appears first.
- Find the probability that the first two tosses are HH. [1]
 - Explain why, if the first two tosses are not HH, then Beth wins. [4]
 - Hence write down the probability that Beth wins. [1]
 - The players bet on the game: Beth pays Alex £x if Alex wins, and Alex pays Beth £1 if Beth wins. Find x so that the game is fair to both players. [2]

Venn Diagrams

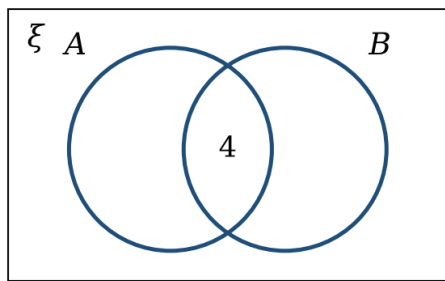
- 1 30 students were asked whether they play football (F) or hockey (H). The Venn diagram shows the numbers of students in each region. One of the 30 students is chosen at random.



- (a) Find the probability that the student plays both sports. [1]
 (b) Find the probability that the student plays football. [1]
 (c) Find the probability that the student plays neither sport. [1]

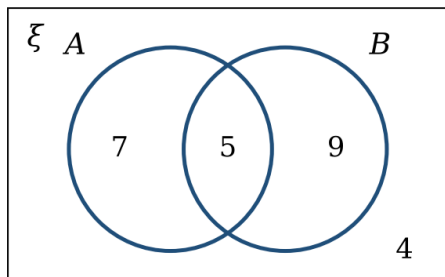
- 2 ξ is the set of students in a class, with $n(\xi) = 25$. Set A has $n(A) = 12$ and set B has $n(B) = 9$, and $n(A \cap B) = 4$.

Copy the Venn diagram and complete it to show the number of elements in each region.



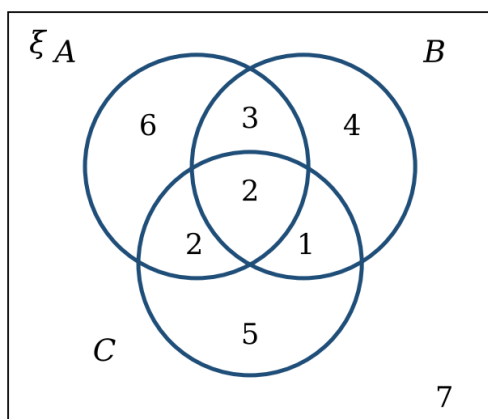
[3]

- 3 The Venn diagram shows the numbers of elements in each region, with $n(\xi) = 25$.



- (a) Find $n(A \cup B)$. [1]
 (b) Find $n(A')$. [1]
 (c) An element is chosen at random. Find the probability that it is in A but not in B. [1]
- 4 $\xi = 1, 2, 3, \dots, 12$, A = even numbers and B = multiples of 3.
 (a) Draw a Venn diagram showing ξ , A and B , with every element written in the correct region. [3]
 (b) A number is chosen at random from ξ . Find $P(A \cup B)$. [1]
- 5 A and B are events with $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$.
 (a) Draw a Venn diagram showing the probability of each of the four regions. [3]
 (b) Find $P(A \cup B)$. [1]
 (c) Find $P(A' \cap B)$. [1]

- 6** The Venn diagram shows the numbers of elements in each region for three sets A, B and C, with $n(\xi) = 30$. One element is chosen at random from ξ .

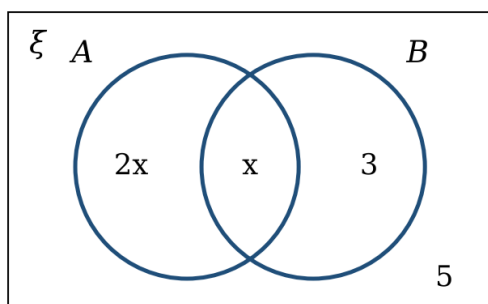


- (a) Find the probability that the element is in all three sets. [1]
 (b) Find $P(A \cap B)$. [2]
 (c) Find the probability that the element is in exactly one of the three sets. [2]

- 7** In a survey of 60 people, 25 drink tea (T), 30 drink coffee (C) and 10 drink both.
 (a) Draw a Venn diagram to show this information. [3]
 (b) Find the probability that a person chosen at random drinks exactly one of tea and coffee. [2]

- 8** A and B are events with $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(A' \cup B') = 0.7$.
 (a) Find $P(A \cap B)$. [2]
 (b) Find $P(B)$. [2]

- 9** The Venn diagram shows the numbers of elements in each region, in terms of x . $n(\xi) = 38$.

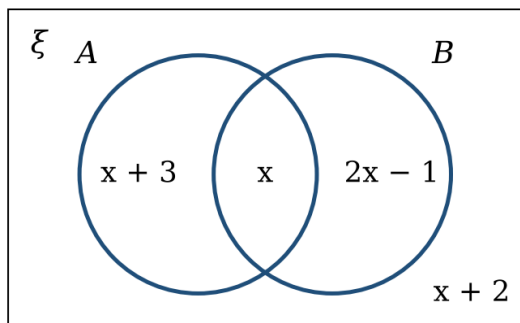


- (a) Find the value of x . [2]
 (b) An element is chosen at random. Find the probability that it is in A. [1]
- 10** ξ is a set with 40 elements. $n(A) = 12$, $n(B) = 20$ and $A \subset B$ (every element of A is also in B).
 (a) Draw a Venn diagram to show the number of elements in each region. [2]
 (b) Find $n(A \cup B)$ and $P(A' \cap B)$. [2]
- 11** A and B are events with $P(A) = 0.65$ and $P(B) = 0.7$. The events are exhaustive, that is $P(A \cup B) = 1$.
 (a) Find $P(A \cap B)$. [2]
 (b) Find $P(A \cap B')$ and $P(A' \cap B)$. [2]
 (c) Explain why $A' \cap B'$ is impossible. [1]

- 12** 50 people were asked which of tea (T), coffee (C) and juice (J) they drink. 4 people drink all three. 9 drink tea and coffee, 7 drink coffee and juice and 8 drink tea and juice (these numbers include those who drink all three). 22 people drink tea, 20 drink coffee and 18 drink juice.

- (a) Draw a Venn diagram showing the number of people in each of the seven regions inside the circles. [4]
 (b) Find how many of the 50 people drink none of the three. [2]
 (c) Find the probability that a person chosen at random drinks exactly two of the three drinks. [1]

- 13** In a survey of 44 students, some study art (A) and some study biology (B). The Venn diagram shows the numbers in each region, in terms of x .
 The number studying art only is $x + 3$, the number studying both is x , the number studying biology only is $2x - 1$ and the number studying neither is $x + 2$.

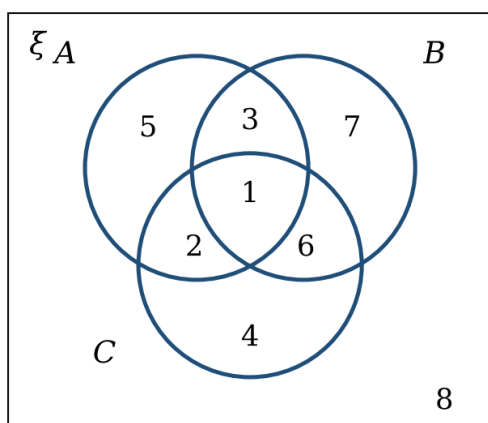


- (a) Find the value of x . [3]
 (b) Find the probability that a student chosen at random studies biology. [1]
 (c) Find the probability that a student chosen at random studies exactly one of the two subjects. [1]

- 14** A, B and C are events with $P(A) = 0.5$, $P(B) = 0.4$, $P(C) = 0.35$, $P(A \cap B) = 0.2$, $P(A \cap C) = 0.15$, $P(B \cap C) = 0.1$ and $P(A \cap B \cap C) = 0.05$.

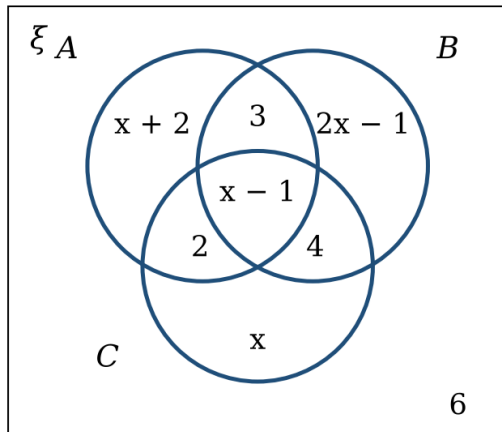
- (a) Draw a Venn diagram showing the probability of each region. [4]
 (b) Find the probability that none of the three events occurs. [1]
 (c) Find the probability that exactly two of the events occur. [1]

- 15** The Venn diagram shows the numbers of elements in the regions formed by sets A, B and C, with $n(\xi) = 36$.

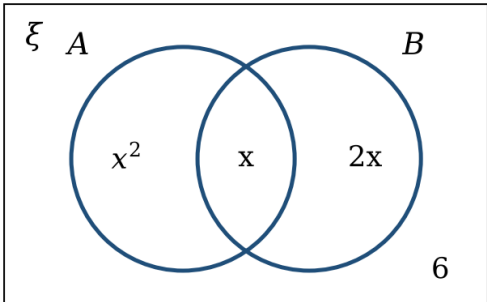


- (a) Find $n(A \cap B')$. [1]
 (b) Find $n(A' \cap B \cap C)$. [1]
 (c) Find $n((A \cup C) \cap B')$. [2]
 (d) An element is chosen at random. Find the probability that it belongs to exactly two of the three sets. [2]

- 16** 40 people were asked which of three sports (A, B and C) they play. The Venn diagram shows the numbers in each region, in terms of x .



- (a) Show that $x = 5$. [3]
 (b) Find the probability that a person plays exactly one of the three sports. [2]
 (c) Find the probability that a person plays sport A. [1]
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- 17** A and B are events with $P(A \cup B) = 0.8$, $P(A' \cap B) = 0.3$ and $P(A \cap B') = 0.2$.
 (a) Find $P(A \cap B)$. [2]
 (b) Find $P(A)$ and $P(B)$. [1]
 (c) Determine whether A and B are independent. [2]
-
- 18** $\xi = 1, 2, 3, \dots, 30$. A is the set of multiples of 2, B is the set of multiples of 3 and C is the set of multiples of 5.
 A number is chosen at random from ξ .
 (a) Draw a Venn diagram showing the number of elements in each region. [4]
 (b) Find the probability that the number is a multiple of exactly two of 2, 3 and 5. [1]
 (c) Find the probability that the number is a multiple of 2 but not of 5. [1]
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- 19** In a year group of 60 students, 35 study French (F) and 28 study Spanish (S). Every student who studies neither language is in a group of between 5 and 8 students inclusive. Let b be the number of students who study both languages.
 (a) By considering $n(F \cup S)$, show that $b = 3 + k$, where k is the number of students who study neither language. [3]
 (b) Find the least and the greatest possible values of b . [2]
 (c) In the case where b takes its greatest value, find the probability that a student chosen at random studies exactly one of the two languages. [2]
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- 20** A, B and C are three events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(C) = 0.3$. Also $P(A \cap B) = 0.2$, $P(B \cap C) = 0.1$, $P(A \cap C) = 0.15$ and $P(A \cup B \cup C) = 0.8$.
 (a) Find $P(A \cap B \cap C)$. [3]
 (b) Draw a Venn diagram showing the probability of each of the eight regions. [3]
 (c) Find the probability that exactly two of the three events occur. [2]
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- 21** 100 people were asked about three charities A, B and C. 10 people support none of them and 50 support exactly one. $n(A) = 60$, $n(B) = 44$ and $n(C) = 34$.
 (a) Let t people support all three and let m people support exactly two. Write down two equations connecting t and m . [3]
 (b) Hence find t and m . [2]
 (c) Find the probability that a person chosen at random supports at least two of the charities. [2]

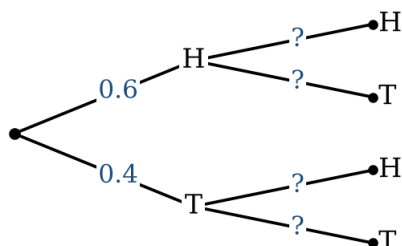
- 22** A and B are events such that $P(A) = 2P(B)$, $P(A \cap B) = 0.14$ and $P(A \cup B) = 0.76$.
- Find $P(A)$ and $P(B)$. [3]
 - Determine whether A and B are independent. [2]
 - Find $P(A' \cap B')$. [1]
-
- 23** A and B are events.
- By using a Venn diagram, or otherwise, show that $P(\text{exactly one of A and B occurs}) = P(A) + P(B) - 2P(A \cap B)$. [3]
 - Given that $P(A) = 0.5$, $P(B) = 0.4$ and $P(\text{exactly one of A and B occurs}) = 0.6$, find $P(A \cap B)$. [2]
 - Find the probability that neither event occurs, and state whether A and B are independent. [3]
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- 24** The Venn diagram shows the numbers of elements in each region of ξ , with $n(\xi) = 34$.
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- Show that $x^2 + 3x - 28 = 0$. [2]
 - Solve the equation and state the value of x , giving a reason. [3]
 - Find the probability that a randomly chosen element is in exactly one of A and B. [1]
-
- 25** A and B are events with $P(A \cup B') = 0.85$, $P(A' \cup B) = 0.7$ and $P(A' \cup B') = 0.9$.
- Find $P(A \cap B)$, $P(A \cap B')$ and $P(A' \cap B)$. [4]
 - Draw a fully labelled Venn diagram and find $P(A' \cap B')$. [3]
-
- 26** A and B are events with $P(A) = 0.7$ and $P(B) = 0.5$.
- Show that A and B cannot be mutually exclusive. [2]
 - Find the least possible value of $P(A \cap B)$. [3]
 - Find the greatest possible value of $P(A \cap B)$, giving a reason. [2]
 - Hence find the range of possible values of $P(A' \cap B')$. [3]
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- 27** For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Prove that $P(A \cap B) \geq P(A) + P(B) - 1$. [3]
 - Events A, B and C have $P(A) = 0.9$, $P(B) = 0.85$ and $P(C) = 0.8$. Show that $P(A \cap B \cap C) \geq 0.55$. [4]
 - Show that it is possible for $P(A \cap B \cap C)$ to equal 0.55. [3]
-
- 28** A, B and C are events.
- Show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$. [5]
 - $P(A) = P(B) = P(C) = 0.4$, $P(A \cap B) = P(A \cap C) = P(B \cap C) = 0.15$ and $P(A \cup B \cup C) = 0.8$. Find $P(A \cap B \cap C)$. [2]
 - For the events in part (b), find the probability that exactly one of the three events occurs. [3]
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- 29** A and B are independent events, that is $P(A \cap B) = P(A)P(B)$.
- Show that A' and B' are also independent. [4]
 - Given that $P(A) = 0.4$ and $P(A \cup B) = 0.88$, find $P(B)$. [3]
 - For these events, find the probability that exactly one of A and B occurs. [2]

- 30** A, B and C are events with $P(A) = 0.3$ and $P(C) = 0.4$. A and C are mutually exclusive, A and B are independent, and $P(A \cup B) = 0.58$.

- (a) Find $P(B)$. [3]
 (b) Given also that $P(B \cup C) = 0.7$, find $P(B \cap C)$. [2]
 (c) Find the probability that none of A, B and C occurs. [4]

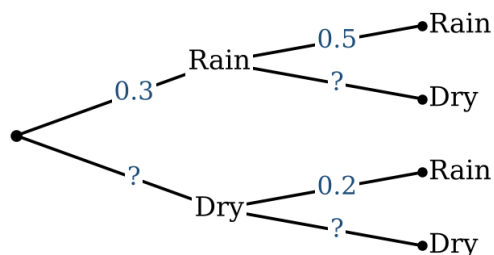
Tree Diagrams

- 1** A biased coin lands on heads with probability 0.6. It is tossed twice. The incomplete tree diagram shows the possible outcomes.



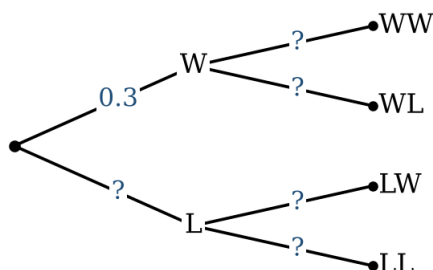
- (a) Complete the tree diagram by filling in the four missing probabilities. [1]
 (b) Find the probability of two heads. [1]

- 2** The probability that it rains on Monday is 0.3. If it rains on Monday, the probability that it rains on Tuesday is 0.5. If it is dry on Monday, the probability that it rains on Tuesday is 0.2.



- (a) Complete the tree diagram. [2]
 (b) Find the probability that it rains on both days. [1]

- 3** A player has probability 0.3 of winning each game of a computer game, independently. She plays two games. The tree diagram shows the outcomes (W = win, L = lose).



- (a) Complete the tree diagram. [1]
 (b) Find the probability that she wins exactly one of the two games. [2]

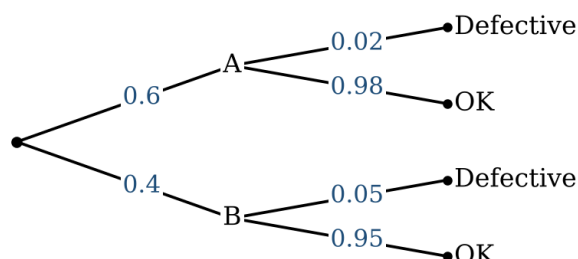
- 4** A bag contains 3 red counters and 2 blue counters. A counter is taken at random, its colour is noted and it is replaced. This is done twice. Find the probability that both counters are red. [2]

- 5** The probability that Ravi's bus is late on any day is 0.1, independently from day to day. Find the probability that over two days the bus is
(a) late on both days, (b) on time on both days. [3]

- 6** A fair coin is tossed and then a fair die is rolled. Draw a tree diagram, and find the probability that the coin shows a head and the die shows an even number. [3]

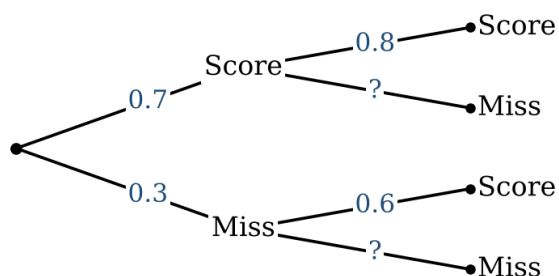
- 7** A bag contains 4 red counters and 6 blue counters. Two counters are taken at random from the bag, one after the other, without replacement.
(a) Draw a tree diagram to show this information, including all the probabilities. [2]
(b) Find the probability that the two counters are the same colour. [2]

- 8** A factory makes an item on two machines, A and B. Machine A makes 60% of the items and machine B makes the rest. 2% of the items from A are defective and 5% of the items from B are defective. The tree diagram shows this information. An item is chosen at random.



- (a) Find the probability that the item was made by machine B and is defective. [2]
(b) Find the probability that the item is defective. [2]
- 9** The probability that it rains on a Saturday is 0.4. If it rains on Saturday, the probability that it rains on Sunday is 0.7. If it does not rain on Saturday, the probability that it rains on Sunday is 0.2.
Find the probability that the weather (rain or no rain) is the same on both days. [3]
- 10** A biased coin shows heads with probability 0.3. It is tossed three times. Find the probability of obtaining exactly two heads. [3]
- 11** A fair die is rolled. If the score is 6 it is rolled a second time; otherwise the game stops. Draw a tree diagram, and find the probability that at least one 6 is scored in the game. [3]
- 12** A bag contains 5 red counters and 3 green counters. Three counters are taken at random from the bag, one at a time, without replacement.
(a) Find the probability that the counters are red, red, green in that order. [2]
(b) Find the probability that exactly two of the three counters are red. [2]
(c) Find the probability that at least one of the counters is green. [2]

- 13** A basketball player takes two free throws. The probability that she scores the first is 0.7. If she scores the first, the probability that she scores the second is 0.8. If she misses the first, the probability that she scores the second is 0.6.



- (a) Find the two missing probabilities on the tree diagram. [1]
 (b) Find the probability that she scores exactly one of the two throws. [2]
 (c) Given that she scores with the second throw, find the probability that she also scored with the first. [3]
-
- 14** A shop buys 50% of its light bulbs from supplier A, 30% from supplier B and 20% from supplier C. The proportions of defective bulbs are 2% from A, 3% from B and 5% from C.
- (a) Find the probability that a bulb chosen at random is defective. [3]
 (b) Two bulbs are chosen independently at random. Find the probability that exactly one is defective, giving your answer to 3 significant figures. [3]
-
- 15** A bag contains 4 red, 3 blue and 2 green counters. Three counters are taken at random without replacement. Find the probability that one counter of each colour is taken. [4]
-
- 16** A tennis player's first serve is in with probability 0.65, and she then wins the point with probability 0.7. If the first serve is out, she takes a second serve which is in with probability 0.9, and she then wins the point with probability 0.5. If the second serve is also out she loses the point.
 Find the probability that she wins the point. [4]
-
- 17** A football team plays two matches. In the first match it wins with probability 0.5, draws with probability 0.3 and loses with probability 0.2. After a win, the probabilities of win, draw, loss in the next match are 0.6, 0.3, 0.1. After a draw or a loss, they are 0.3, 0.3, 0.4.
- (a) Find the probability that the team wins the second match. [3]
 (b) Find the probability that the team loses at least one of the two matches. [3]
-
- 18** A fair six-sided die is rolled repeatedly. Find the probability that the second six is obtained on the 5th roll. [4]
-
- 19** A bag contains 6 red counters and n yellow counters. Two counters are taken at random without replacement. The probability that the two counters are the same colour is $\frac{7}{15}$.
- (a) Show that $2n^2 - 23n + 60 = 0$. [4]
 (b) Hence find the value of n . [2]
 (c) Using this value of n , three counters are taken at random without replacement. Find the probability that all three are yellow. [2]
-
- 20** The probability that it rains on a particular Monday is 0.3. On any later day, the probability that it rains is 0.6 if it rained the day before and 0.2 if it was dry the day before. Let R denote rain and D denote dry. Consider Monday, Tuesday and Wednesday.
- (a) Find the probability that it rains on Tuesday. [2]
 (b) Find the probability that it rains on all three days. [2]
 (c) Find the probability that it rains on exactly two of the three days. [4]

- 21** A bag contains a red counters and b blue counters ($a \geq 1$, $b \geq 1$). Two counters are taken at random without replacement.
- (a) Show that the probability that the second counter is red is equal to the probability that the first counter is red. [4]
- (b) A bag contains 7 red and 5 blue counters. Three counters are taken without replacement. Write down, without calculating, the probability that the third counter is red, and give a reason. [2]
-
- 22** Ann and Bill play a series of games. Ann wins each game with probability 0.6, independently. The series ends as soon as one player has won two games.
- (a) Show that the probability that Ann wins the series in exactly two games is 0.36 and find the probability she wins in exactly three games. [3]
- (b) Hence find the probability that Ann wins the series. [2]
- (c) Find the probability that the series lasts exactly three games. [3]
-
- 23** A bag contains 3 red counters and 2 blue counters. A counter is taken at random. It is returned to the bag together with one extra counter of the same colour. This process is repeated for a second and then a third draw.
- (a) Find the probability that the first three counters are all red. [2]
- (b) Show that the probability that the second counter is red is $\frac{3}{5}$. [3]
- (c) Find the probability that all three counters are the same colour. [3]
-
- 24** Two independent events each occur with probability p , where $p < 0.5$. The probability that exactly one of the two events occurs is 0.42.
- (a) Show that $p^2 - p + 0.21 = 0$ and hence find p . [4]
- (b) Find the probability that at least one of the events occurs. [2]
- (c) Find the probability that both events occur given that at least one occurs, leaving your answer as a fraction. [2]
-
- 25** Amy and Ben play a match which ends as soon as one player has won three sets. Amy wins each set with probability 0.6, independently.
- (a) Find the probability that Amy wins the match in exactly four sets. [3]
- (b) Find the probability that the match lasts five sets. [3]
-
- 26** A bag contains 9 counters, of which r are red and the rest are blue, with $r < 5$. Two counters are taken at random without replacement. The probability that they are different colours is $\frac{5}{9}$.
- (a) Show that $r^2 - 9r + 20 = 0$. [3]
- (b) Find r . [2]
- (c) Find the probability that both counters are red. [2]
-
- 27** A biased coin lands on heads with probability p . The coin is tossed three times and the probability of obtaining exactly two heads is 0.288.
- (a) Show that $p^3 - p^2 + 0.096 = 0$. [3]
- (b) Show that $p = 0.4$ is a root and find the other values of p between 0 and 1 that satisfy the equation, giving them to 3 decimal places. [4]
- (c) The coin is known to be biased towards tails. Find the probability that at least one head is obtained in the three tosses. [3]

- 28** A bag contains 4 white counters and 2 black counters. Priya and Quentin take turns to draw a counter at random from the bag, without replacement, with Priya drawing first. The first person to draw a black counter wins the game.
- Write down the probability that Priya wins on her first draw. [1]
 - Find the probability that Quentin wins on his first draw. [2]
 - Show that the probability that Priya wins the game is $\frac{3}{5}$. [5]
 - Find the probability that Quentin wins, and comment on whether the game is fair. [2]
-
- 29** Zara starts with £1 and plays a game of repeated bets of £1. She wins each bet with probability 0.6 (gaining £1) and loses with probability 0.4 (losing £1), independently. She stops when she has £3 (success) or £0 (ruin). Let p_k be the probability that she eventually reaches £3 when she currently holds £k, so $p_0 = 0$ and $p_3 = 1$.
- Explain why $p_1 = 0.6p_2$ and show that $p_2 = 0.6 + 0.4p_1$. [3]
 - Hence find the probability, as a fraction in its lowest terms, that Zara reaches £3. [4]
 - Show that if the game were fair (win and lose both with probability 0.5) she would reach £3 with probability $\frac{1}{3}$, and comment on the effect of the bias. [3]
-
- 30** In a game show a car is behind one of three doors, chosen at random, and goats are behind the other two. A contestant picks door 1. The host, who knows where the car is, then opens one of the other doors to reveal a goat (choosing at random if he has a choice) and offers the contestant the chance to switch to the remaining closed door.
- Draw a tree diagram showing where the car is and which door the host opens. [3]
 - Show that the probability that the contestant wins the car by sticking with door 1 is $\frac{1}{3}$. [2]
 - Find the probability that the contestant wins by switching, and explain this result. [3]

Independent Events

- 1** A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.5$.
- Find $P(A \cap B)$. [1]
 - Find $P(A \cup B)$. [2]
-
- 2** $P(A) = 0.4$, $P(B) = 0.5$, $P(C) = 0.3$, $P(A \cap B) = 0.2$ and $P(A \cap C) = 0.15$.
- Show that A and B are independent. [2]
 - Determine whether A and C are independent. [1]
-
- 3** $P(A) = 0.5$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$. Determine whether A and B are independent. [2]
-
- 4** A fair coin is tossed and a fair six-sided die is rolled. Find the probability of obtaining a head and a 6. [2]
-
- 5** Sam wins each game of chess against his brother with probability 0.3, independently of other games. Find the probability that he wins all of the next three games. [2]
-
- 6** A and B are independent events with $P(A) = 0.6$ and $P(A \cup B) = 0.84$.
- Find $P(B)$. [3]
 - Find $P(A' \cap B')$. [1]
-
- 7** Two footballers take penalties independently. Jamal scores with probability 0.8 and Kofi scores with probability 0.7. Each takes one penalty.
- Find the probability that both score. [1]
 - Find the probability that at least one scores. [2]
 - Find the probability that exactly one scores. [2]

8 A and B are independent events with $P(A) = 0.3$ and $P(A \cup B) = 0.65$. Find $P(B)$. [3]

- 9** Two archers shoot at the same target independently. Archer X hits with probability 0.7 and archer Y hits with probability 0.6. Each fires one arrow.
- (a) Find the probability that exactly one arrow hits the target. [3]
- (b) Find the probability that at least one arrow hits. [2]

- 10** $P(A) = 0.2$ and $P(B) = 0.5$.
- (a) Find $P(A \cup B)$ if A and B are mutually exclusive. [1]
- (b) Find $P(A \cup B)$ if A and B are independent. [2]
- (c) Explain why A and B cannot be both mutually exclusive and independent. [2]

- 11** A machine contains two components in parallel, so that the machine works if at least one of them works. Each component works independently with probability 0.9. Find the probability that the machine works, and the probability that it would work if the two components were instead connected in series (both must work). [4]

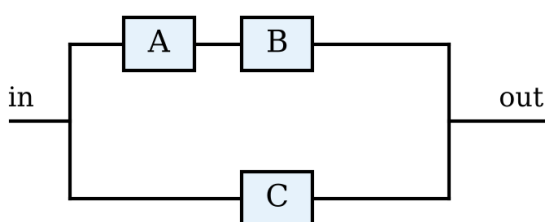
- 12** The table shows the preferred drink of 100 people. A person is chosen at random. Let F be the event that the person is female and T the event that the person prefers tea. Determine whether F and T are independent.

	Tea	Coffee	Other
Female	30	12	18
Male	20	8	12

[3]

- 13** A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.4$.
- (a) Draw a Venn diagram showing the probabilities of the four regions. [3]
- (b) Find $P(A' \cup B')$. [2]
- (c) Show that A and B' are independent. [2]

- 14** A system has three components A, B and C connected as shown. Current flows through the system if there is a working path from in to out. The components work independently with probabilities $P(A \text{ works}) = 0.9$, $P(B \text{ works}) = 0.8$ and $P(C \text{ works}) = 0.7$.



- (a) Find the probability that current flows through the upper path (through A and B). [2]
- (b) Find the probability that the system works. [3]

- 15** A and B are independent events with $P(A) = p$ and $P(B) = 2p$. Given that $P(A \cup B) = 0.52$, find p. [5]

- 16** Three independent events A, B and C have probabilities $P(A) = 0.5$, $P(B) = 0.4$ and $P(C) = 0.2$.
- (a) Find the probability that none of the events occurs. [2]
- (b) Find the probability that at least one of the events occurs. [1]
- (c) Find the probability that exactly two of the events occur. [3]

- 17** A fair coin is tossed four times. A is the event "the first two tosses show the same face" and B is the event "the total number of heads is even".
Show that A and B are independent. [5]
- 18** Two fair dice are rolled. A is the event that the sum is 7, B is the event that the first die shows 3 and C is the event that the sum is 8.
(a) Show that A and B are independent. [3]
(b) Show that B and C are not independent. [3]
- 19** A and B are independent events with $P(A) = P(B) = p$ and $P(A \cup B) = \frac{5}{9}$. Find the value of p. [4]
- 20** An archer fires arrows at a target. Each arrow independently hits the target with probability p. The probability that at least one of three arrows hits the target is 0.936.
(a) Find the value of p. [3]
(b) The archer fires n arrows. Find the smallest value of n for which the probability of at least one hit is greater than 0.9999. [3]
(c) Find the probability that the first hit comes with the third arrow. [2]
- 21** A and B are independent events.
(a) Show that A and B' are also independent. [4]
(b) Given that $P(A) = 0.6$ and $P(A \cap B') = 0.15$, find $P(B)$. [2]
- 22** A and B are independent events with $P(A) = x$, $P(B) = x + 0.2$ and $P(A \cap B) = 0.24$.
(a) Show that $x^2 + 0.2x - 0.24 = 0$ and hence find x. [4]
(b) Find $P(A \cup B)$ and $P(A' \cap B')$. [3]
- 23** A circuit has component A in series with a block of components that are connected in parallel. Every component works independently with probability 0.9. The circuit works if A works and at least one component in the parallel block works.
(a) The parallel block has two components. Find the probability that the circuit works. [3]
(b) A third component is added to the parallel block. Find the new probability that the circuit works. [2]
(c) Explain why the probability that the circuit works can never exceed 0.9, however many components are added in parallel. [2]
- 24** A taxi must pass three independent safety tests, brakes, tyres and lights, to be licensed. The probabilities of passing are 0.9, 0.95 and 0.8 respectively.
(a) Find the probability that a taxi passes all three tests. [2]
(b) Find the probability that a taxi fails exactly one of the tests. [3]
(c) Five taxis are tested independently. Find the probability that none of them is licensed, giving your answer to 3 significant figures. [2]
- 25** A, B and C are events with $P(A) = 0.4$, $P(B) = 0.5$ and $P(C) = 0.3$. A and B are independent, A and C are mutually exclusive and $P(B \cap C) = 0.1$.
(a) Find $P(A \cup B)$. [2]
(b) Find $P(A \cup B \cup C)$. [3]
- 26** Two fair coins are tossed. Let A be the event "the first coin shows heads", B the event "the second coin shows heads" and C the event "both coins show the same face".
(a) List the four equally likely outcomes and find $P(A)$, $P(B)$ and $P(C)$. [2]
(b) Show that A, B and C are independent in pairs. [3]
(c) Show that $P(A \cap B \cap C) \neq P(A) \times P(B) \times P(C)$. [2]
(d) Explain in context why the three events are not mutually independent. [2]

- 27 A and B are independent events, that is $P(A \cap B) = P(A)P(B)$.
- (a) Prove that A and B' are independent. [3]
- (b) Hence, or otherwise, prove that A' and B' are independent. [3]
- (c) X and Y are independent events with $P(X \cap Y) = 0.3$ and $P(X' \cap Y') = 0.2$. Given that $P(X) > P(Y)$, find $P(X)$ and $P(Y)$. [4]
-
- 28 Anna and Ben take turns to shoot at a target, Anna first. Anna hits with probability 0.3 and Ben hits with probability 0.4 on each shot, all shots independent. The first player to hit the target wins.
- (a) Find the probability that Anna wins on her second shot. [2]
- (b) Show that the probability that Anna wins the game is $\frac{15}{29}$. [5]
- (c) Hence find the probability that Ben wins. Anna's probability of hitting stays 0.3. Ben's probability of hitting is changed to q. Find q so that the game is fair to both players. [4]
-
- 29 Independent events $E_1, E_2, \dots, E_n$ have probabilities $p_1, p_2, \dots, p_n$.
- (a) Show that if two events are both independent and mutually exclusive, then at least one of them has probability 0. [3]
- (b) Show that the probability that at least one event occurs is $1 - (1 - p_1)(1 - p_2)\dots(1 - p_n)$. [2]
- (c) Given that $1 - x \leq e^{-x}$ for all x, show that the probability that at least one event occurs is at least $1 - e^{-(p_1 + p_2 + \dots + p_n)}$. [3]
- (d) 20 independent components each fail with probability 0.02. Find the exact probability that at least one fails, and the lower bound given by part (c), each to 3 significant figures. [3]
-
- 30 At a holiday camp, the probability that a child plays football only is 0.3 and the probability that a child plays rugby only is 0.2. It is thought that playing football and playing rugby are independent. Let x be the probability that a child plays both sports.
- (a) Show that $x^2 - 0.5x + 0.06 = 0$. [4]
- (b) Hence find the two possible values of x. [2]
- (c) Given that 60 children play both sports, find the two possible numbers of children who play neither sport. [4]

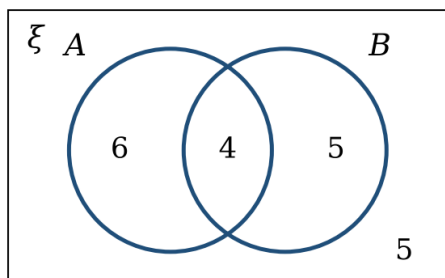
Conditional Probability

- 1 The table shows how 70 students travel to school. A student is chosen at random.

	Bus	Walk	Car	Total
Boys	12	15	8	35
Girls	18	7	10	35
Total	30	22	18	70

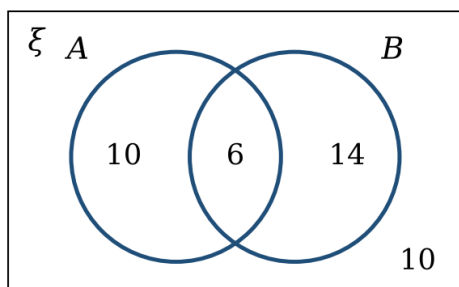
- (a) Given that the student is a girl, find the probability that she walks. [1]
- (b) Given that the student travels by bus, find the probability that the student is a boy. [1]
-
- 2 A and B are events with $P(A) = 0.5$, $P(B) = 0.25$ and $P(A \cap B) = 0.2$.
- (a) Find $P(B | A)$. [1]
- (b) Find $P(A | B)$. [1]
-
- 3 $P(A \cap B) = 0.12$, $P(A) = 0.5$ and $P(B) = 0.3$. Find $P(A | B)$ and $P(B | A)$. [2]
-
- 4 A fair six-sided die is rolled. Given that the score is greater than 2, find the probability that it is even. [2]

- 5 The Venn diagram shows the numbers of elements in each region, with $n(\xi) = 20$. An element is chosen at random.



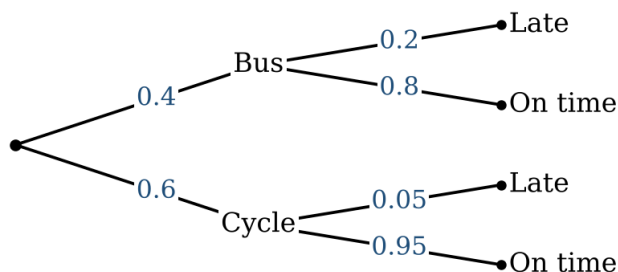
- (a) Find $P(B | A)$. [1]
 (b) Find $P(A | B')$. [1]

- 6 The Venn diagram shows the number of elements in each region of sets A and B, with $n(\xi) = 40$. An element is chosen at random.



- (a) Find $P(A | B)$. [1]
 (b) Find $P(B | A)$. [1]
 (c) Find $P(A | B')$. [2]

- 7 Tom travels to work by bus with probability 0.4 and otherwise cycles. The probability that he is late is 0.2 when he takes the bus and 0.05 when he cycles, as shown in the tree diagram.



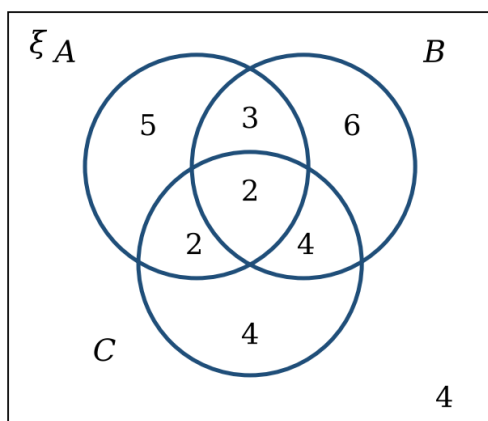
- (a) Find the probability that Tom is late. [2]
 (b) Given that Tom is late, find the probability that he cycled. [2]

- 8 The probability that it rains on the day of a match is 0.3. The home team wins with probability 0.4 if it rains and 0.7 if it does not rain.

- (a) Find the probability that the home team wins. [3]
 (b) Given that the home team wins, find the probability that it rained. [2]

- 9 $P(A | B) = 0.5$, $P(B) = 0.4$ and $P(A) = 0.5$. Find $P(A \cap B)$, and state what your answers show about A and B. [3]

- 10** The Venn diagram shows the numbers of elements in each region of three sets, with $n(\xi) = 30$. An element is chosen at random.



- (a) Find $P(C | B)$. [2]
 (b) Find $P(A | B \cap C)$. [2]

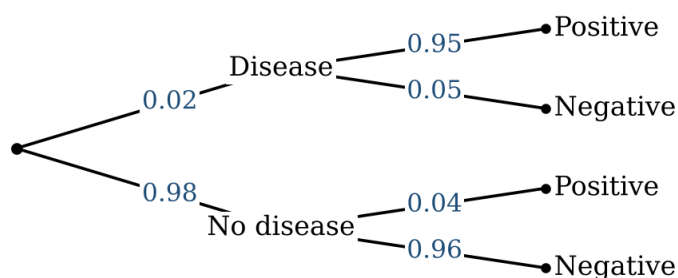
- 11** Two fair dice are rolled. Given that the total score is at least 9, find the probability that at least one of the dice shows a 6. [3]

- 12** On a working day the probability that James wears a tie is 0.4 and the probability that he wears a jacket is 0.6. If he wears a jacket, the probability that he wears a tie is 0.5.
 (a) Find the probability that he wears both a jacket and a tie. [2]
 (b) Given that he is not wearing a tie, find the probability that he is wearing a jacket. [2]

- 13** The incomplete table shows information about 120 adults and whether they are members of a gym.

	Member	Not member	Total
Female	45	?	70
Male	30	?	?
Total	?	?	120

- (a) Copy and complete the table. [2]
 (b) An adult is chosen at random from those who are members. Find the probability that the adult is female. [2]
 (c) Find the probability that a randomly chosen male is not a member. [1]
 (d) Determine whether being female and being a member are independent. [2]
- 14** In a population, 2% of people have a certain disease. A screening test gives a positive result for 95% of people who have the disease and for 4% of people who do not have the disease. A person is chosen at random and tested.



- (a) Find the probability that the test result is positive. [3]
 (b) Given that the test result is positive, find the probability that the person has the disease. Give your answer to 3 significant figures. [3]

- 15** A factory has three machines X, Y and Z, which produce 45%, 35% and 20% of its output respectively. The percentages of defective items from X, Y and Z are 1%, 2% and 5% respectively. An item is chosen at random from the output.
- (a) Find the probability that the item is defective. [3]
- (b) Given that the item is defective, find the probability that it came from machine Z. [2]
-
- 16** A and B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. Find
- (a) $P(A | B)$, [2]
- (b) $P(B | A')$, [2]
- (c) $P(A' | B')$. [2]
-
- 17** A box contains two coins. One is fair and the other has a head on both sides. A coin is chosen at random and tossed.
- (a) Given that the coin shows a head, show that the probability that the coin is the two-headed coin is $\frac{2}{3}$. [3]
- (b) The same coin is tossed again. Given that the first toss was a head, find the probability that the second toss is also a head. [3]
-
- 18** A bag contains 4 mint sweets and 6 fruit sweets. Two sweets are taken at random without replacement. Given that at least one of them is a mint, find the probability that both are mints. [4]
-
- 19** A and B are events with $P(A | B) = 0.5$, $P(B | A) = 0.25$ and $P(A \cup B) = 0.8$. Find $P(A \cap B)$, $P(A)$ and $P(B)$. [5]
-
- 20** A and B are events with $P(A) = 0.5$, $P(B) = 0.35$ and $P(A \cap B) = 0.15$.
- (a) Find $P(A | A \cup B)$. [3]
- (b) Find the probability that A occurs, given that exactly one of A and B occurs. [3]
-
- 21** A bag contains 5 red, 4 blue and 3 green counters. Two counters are taken at random, one after the other, without replacement.
- (a) Find the probability that the second counter is red. [2]
- (b) Given that the second counter is red, find the probability that the first counter was also red. [3]
- (c) Given that the two counters are different colours, find the probability that one of them is green. [3]
-
- 22** A and B are events with $P(B) = 0.5$, $P(A | B) = 0.6$ and $P(A | B') = 0.3$.
- (a) Find $P(A)$. [3]
- (b) Find $P(B | A)$. [2]
- (c) Show that A and B are not independent. [1]
- (d) The value of $P(A | B')$ is changed so that A and B become independent, with $P(B)$ and $P(A | B)$ unchanged. Find the new value of $P(A | B')$. [2]
-
- 23** A bag contains 6 red and 4 blue counters. Three counters are taken at random without replacement.
- (a) Show that the probability that exactly two of the counters are red is $\frac{1}{2}$. [3]
- (b) Given that exactly two of the counters are red, find the probability that the first counter taken was red. [3]
-
- 24** The probability of rain on Monday is 0.3. If it rains on a day, the probability that it rains the next day is 0.6. If it does not rain on a day, the probability that it rains the next day is 0.2.
- (a) Find the probability that it rains on Tuesday. [2]
- (b) Given that it rains on Tuesday, find the probability that it rained on Monday. [2]
- (c) Given that it rains on Monday and on Wednesday, find the probability that it rains on Tuesday. [4]

- 25** Assume that each child born is equally likely to be a boy or a girl, independently.
- A family has two children. Given that at least one is a boy, find the probability that both are boys. [3]
 - Given that the elder child is a boy, find the probability that both children are boys. [2]
 - A family has three children. Given that at least two are girls, find the probability that all three are girls. [3]
-
- 26** A bag contains blue, yellow and red discs, each showing a single digit. A disc is chosen at random: it is blue with probability 0.45, yellow with probability 0.35 and red with probability 0.2. The probability that a disc shows an even number is 0.6 for blue discs, 0.5 for yellow discs and 0.3 for red discs.
- Find the probability that a disc chosen at random shows an even number. [3]
 - Given that the disc shows an even number, find the probability that it is not red. [3]
 - Two discs are chosen at random, with replacement. Given that both show even numbers, find the probability that both are blue. [3]
-
- 27** Three cards are placed in a hat. Card 1 is red on both sides, card 2 is white on both sides, and card 3 is red on one side and white on the other. One card is drawn at random and placed on a table, and the side showing is chosen at random. The side showing is red.
- Show that the probability that the side showing is red is $\frac{1}{2}$. [2]
 - Find the probability that the side underneath is also red. [3]
 - Dylan says: "It must be card 1 or card 3, and these are equally likely, so the probability is $\frac{1}{2}$." Explain the error in Dylan's reasoning. [2]
 - The whole process is carried out three times, independently (the card is replaced each time). Find the probability that red is showing on at least two of the three occasions. [3]
-
- 28** A disease affects 1% of a population. A test gives a positive result for 90% of people who have the disease and for 5% of people who do not.
- A person is tested once and the result is positive. Find the probability that the person has the disease. [3]
 - The person is tested a second time with an independent repeat of the test (independent given whether the person has the disease) and again gets a positive result. Find the probability that the person has the disease given two positive results. Give your answer to 3 significant figures. [4]
 - For events A and B with non-zero probabilities, prove that $P(A | B) > P(A)$ if and only if $P(B | A) > P(B)$. [3]
-
- 29** Urn 1 contains 3 white and 2 black balls. Urn 2 contains 2 white and 4 black balls. A fair coin is tossed to choose an urn (heads for urn 1, tails for urn 2). Balls are then taken from the chosen urn, one at a time, with replacement.
- Two balls are taken and both are white. Show that the probability that urn 1 was chosen is $\frac{81}{106}$. [4]
 - One white ball has been taken. Find the probability that urn 1 was chosen. [2]
 - Given that the first ball taken is white, find the probability that the second ball is also white. [3]
 - Explain why observing white balls makes it more likely that urn 1 was chosen. [2]
-
- 30** A and B are events with $P(B)$ strictly between 0 and 1 and $P(A) > 0$.
- Show that if $P(A | B) = P(A | B')$, then A and B are independent. [4]
 - Show that if $P(A | B) > P(A)$, then $P(B | A) > P(B)$. [3]
 - $P(B) = 0.4$, $P(A | B) = 0.7$ and $P(A | B') = 0.2$. Find $P(B | A)$. [3]

Permutations and Combinations

- 1 Evaluate
 - (a) $6!$ [1]
 - (b) ${}^5P_2$ [1]
 - (c) ${}^7C_3$ [1]
- 2 Find the number of ways of
 - (a) arranging 5 different books in a row on a shelf, [1]
 - (b) choosing a committee of 3 people from a group of 8 people. [1]
- 3 A student has 8 different library books.
 - (a) In how many ways can she choose 3 of them to take on holiday? [1]
 - (b) In how many ways can she choose 4 of them and put them in order on a shelf? [1]
- 4 A PIN consists of four digits chosen from 0 to 9. How many PINs are possible (a) if digits may be repeated, (b) if all four digits must be different? [2]
- 5 A restaurant menu has 3 starters, 4 main courses and 2 desserts. How many different three-course meals (one of each) can be chosen? [2]
- 6 Find the number of distinct arrangements of all the letters of each word.
 - (a) BANANA [2]
 - (b) STATISTICS [2]
- 7 A coach chooses a team of 5 players from a squad of 10.
 - (a) How many different teams are possible? [1]
 - (b) How many teams include the captain? [1]
 - (c) Two of the players, Ali and Ben, refuse to play in the same team. How many teams can be chosen? [2]
- 8 Six people are to be seated. Find the number of ways of seating them (a) in a row of six chairs, (b) around a circular table, where only the order relative to the others matters. [3]
- 9 A committee of 3 men and 2 women is to be chosen from 7 men and 5 women. In how many ways can this be done? [3]
- 10 Three-letter codes are made using the letters A, B, C, D, E, F.
 - (a) How many codes are possible if letters may be repeated? [1]
 - (b) How many codes have three different letters? [1]
 - (c) How many codes with three different letters begin with a vowel? [2]
- 11 A coach selects a team from 3 goalkeepers, 6 defenders, 5 midfielders and 4 strikers. The team must contain 1 goalkeeper, 3 defenders, 3 midfielders and 2 strikers. In how many ways can the team be chosen? [3]
- 12 An exam paper has Section A with 8 questions and Section B with 5 questions. A candidate must answer 5 questions from Section A and 3 questions from Section B.
 - (a) In how many ways can the candidate choose the questions? [2]
 - (b) A candidate who has not read the instructions chooses 8 of the 13 questions at random. Find the probability that the choice meets the rules. [2]

- 13** A committee of 4 people is chosen at random from 6 men and 5 women.
- (a) Show that the number of possible committees is 330. [1]
 - (b) Find the probability that the committee has exactly 2 women. [3]
 - (c) Find the probability that the committee contains at least one woman. [2]
-
- 14** Seven people, including Ann and Bob, stand in a row for a photograph.
- (a) Find the number of arrangements in which Ann and Bob are next to each other. [3]
 - (b) Find the number of arrangements in which Ann and Bob are not next to each other. [2]
 - (c) The seven people instead sit around a circular table. Find the number of different seating arrangements. [1]
-
- 15** Five-digit numbers are formed using five different digits from 0, 1, 2, 3, 4, 5 (a number cannot start with 0).
- (a) Show that there are 600 such numbers. [2]
 - (b) How many of these numbers are even? [3]
 - (c) How many of these numbers are greater than 30 000? [2]
-
- 16** The letters of the word ARRANGE are arranged in a row. Find the number of arrangements in which
- (a) there are no restrictions, [2]
 - (b) the two As are next to each other, [2]
 - (c) the first and last letters are both vowels. [3]
-
- 17** A hand of 5 cards is dealt from a standard pack of 52 cards. Find the probability that the hand contains
- (a) five hearts, [2]
 - (b) exactly two aces, [2]
 - (c) at least one ace. [2]
-
- 18** Four married couples are to be seated at a round table. Find the number of seatings in which
- (a) each husband sits next to his wife, [3]
 - (b) men and women sit alternately. [2]
-
- 19** Four boys and three girls stand in a row in a random order. Find the probability that no two girls are next to each other. [5]
-
- 20** Four-digit numbers are formed using four different digits chosen from 1, 2, 3, 4, 5, 6, 7. Find how many of these numbers are
- (a) even, [2]
 - (b) greater than 5000, [2]
 - (c) divisible by 5, [1]
 - (d) even and greater than 5000. [3]
-
- 21** Each person at a party shakes hands once with every other person. There were n people at the party and 45 handshakes took place.
- (a) Show that $n^2 - n - 90 = 0$ and hence find n . [4]
 - (b) Two of the people at the party are chosen at random. Find the probability that Ahmed is one of the two. [2]
 - (c) In a different group of m people, ${}^mC_3 = 4 \times {}^mC_2$. Find m . [4]
-
- 22** The letters of the word MATHEMATICS are arranged in a row.
- (a) Show that the number of arrangements is 4 989 600. [2]
 - (b) Find the probability that the two Ms are next to each other. [3]
 - (c) Find the probability that the first and last letters are both vowels. [4]

- 23** In a lottery, 6 numbers are drawn at random from 1 to 49 (without replacement). A player chooses 6 different numbers. Find the probability that the player's numbers match
- (a) all 6 drawn numbers, [2]
 - (b) exactly 4 of the drawn numbers, giving your answer to 3 significant figures, [3]
 - (c) at least 3 of the drawn numbers, giving your answer to 3 significant figures. [3]
-
- 24** Twelve people are to be divided into groups. Find the number of ways of dividing them into
- (a) three named teams A, B and C of 4 people each, [2]
 - (b) three unnamed groups of 4 people, [2]
 - (c) two unnamed teams of 6 people. [2]
-
- 25** A four-digit code is made from the digits 1, 2 and 3, and digits may be repeated. A code is chosen at random from all those possible. Find the probability that no digit is used more than twice in the code. [5]
-
- 26** Four adults and six children sit at random in a row of 10 seats. Find the probability that no two adults sit next to each other. [5]
-
- 27** In this question, ${}^nC_r$ denotes the number of ways of choosing r items from n .
- (a) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$ for $1 \leq r \leq n-1$. [4]
 - (b) A committee of 5 people is chosen from 7 men and 5 women. Find the number of committees with at least 2 men and at least 2 women. [3]
 - (c) Mr Ali (one of the men) and Ms Bell (one of the women) will not serve on the same committee. How many of the committees in part (b) are possible? [3]
-
- 28** For integers $1 \leq r \leq n-1$, consider the identity ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$.
- (a) Prove the identity algebraically. [4]
 - (b) Give a combinatorial explanation: choose r people from n people, focusing on one particular person Z. [3]
 - (c) Given that ${}^{10}C_4 = 210$ and ${}^{10}C_5 = 252$, find ${}^{11}C_5$. [2]
 - (d) By considering whether each of 4 people is included or not, show that ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4$. [2]
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- 29** Answer both parts.
- (a) Find the number of arrangements of the letters of the word BANANA in which no two As are next to each other. [4]
 - (b) Five letters, each addressed to a different person, are put at random into five addressed envelopes, one in each. Use the principle of inclusion and exclusion to show that the number of ways in which no letter is in the correct envelope is 44, and hence find the probability that no letter is correct. [6]
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- 30** Each packet of cereal contains one of four different toys, equally likely and independently. Maya wants to collect all four toys. Find the probability that she needs to buy more than 5 packets to complete the set. [6]
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