

Linear Momentum

1 (2 marks)

$$p = mv = 0.4 \times 15$$

M1

$$= 6 \text{ kg m s}^{-1}$$

A1

Answer: 6 kg m s⁻¹

2 marks

2 (2 marks)

$$p = mv = 0.3 \times (-5)$$

M1

$$= -1.5 \text{ kg m s}^{-1} \text{ (that is } 1.5 \text{ kg m s}^{-1} \text{ to the left)}$$

A1

Answer: -1.5 kg m s⁻¹

2 marks

3 (2 marks)

$$\text{Bullet: } 0.05 \times 300 = 15 \text{ kg m s}^{-1}$$

M1

$$\text{Runner: } 70 \times 8 = 560 \text{ kg m s}^{-1}, \text{ so the runner has the greater momentum}$$

A1

Answer: bullet 15 kg m s⁻¹, runner 560 kg m s⁻¹; the runner

2 marks

4 (2 marks)

$$m = 0.15 \text{ kg}, v = \frac{p}{m} = \frac{18}{0.15}$$

M1

$$= 120 \text{ m s}^{-1}$$

A1

Answer: 120 m s⁻¹

2 marks

5 (3 marks)

$$\text{Momentum of A} = 3 \times 4 = 12$$

M1

$$\text{Momentum of B} = 2 \times (-5) = -10, \text{ total} = 12 - 10 = 2 \text{ kg m s}^{-1} \text{ to the right}$$

M1 A1

Answer: 2 kg m s⁻¹ to the right

3 marks

6 (2 marks)

$$\text{Change} = m(v - u) = 0.5(10 - 6)$$

M1

$$= 2 \text{ kg m s}^{-1}$$

A1

Answer: 2 kg m s⁻¹

2 marks

7 (3 marks)

$$\text{Take the direction away from the wall as positive: } u = -12, v = 8$$

B1

$$\text{Change} = 0.2(8 - (-12)) = 0.2 \times 20$$

M1

$$= 4 \text{ kg m s}^{-1}$$

A1

Answer: 4 kg m s⁻¹

3 marks

8 (3 marks)

$$p = 2(3\mathbf{i} - 4\mathbf{j}) = (6\mathbf{i} - 8\mathbf{j}) \text{ kg m s}^{-1}$$

M1 A1

$$|p| = \sqrt{6^2 + 8^2} = 10 \text{ kg m s}^{-1}$$

A1

Answer: (6i - 8j), 10 kg m s⁻¹

3 marks

9 (4 marks)

$$v = \frac{p}{m} = 2(6\mathbf{i} + 8\mathbf{j}) = 12\mathbf{i} + 16\mathbf{j}, \text{ speed} = \sqrt{144 + 256} = 20 \text{ m s}^{-1}$$

M1 A1

$$\tan \theta = \frac{16}{12}, \theta = 53.1^\circ$$

M1 A1

Answer: 20 m s⁻¹ at 53.1° to i

4 marks

10 (4 marks)

$$v^2 = u^2 + 2as = 0 + 2(9.8)(20) = 392, v = 19.80 \text{ m s}^{-1}$$

M1 A1

$$p = 3 \times 19.80 = 59.4 \text{ kg m s}^{-1}$$

M1 A1

Answer: 59.4 kg m s⁻¹

4 marks

11 (2 marks)

$$\text{Momentum of A} = 4 \times 6 = 24$$

M1

$$1.5v = 24 \text{ so } v = 16 \text{ m s}^{-1}$$

A1

Answer: 16 m s⁻¹

2 marks

12 (3 marks)

$$\text{Rate of change of momentum} = \text{force, so momentum} = 12 \times 5 = 60 \text{ kg m s}^{-1}$$

M1 A1

$$v = \frac{60}{2} = 30 \text{ m s}^{-1}$$

A1

Answer: 60 kg m s⁻¹, 30 m s⁻¹

3 marks

13 (3 marks)

$$\text{Total} = 2(6) + 5(2) + 3(-4)$$

M1 A1

$$= 12 + 10 - 12 = 10 \text{ kg m s}^{-1} \text{ to the right}$$

A1

Answer: 10 kg m s⁻¹ to the right

3 marks

14 (4 marks)

$$E = \frac{1}{2}mv^2 \text{ so } 2mE = m^2v^2 = (mv)^2 = p^2$$

M1 A1

$$p = \sqrt{2 \times 2 \times 25} = 10 \text{ kg m s}^{-1}$$

M1 A1

Answer: 10 kg m s⁻¹

4 marks

15 (5 marks)

$$\text{(a) } p = 2(3t^2\mathbf{i} + 4t\mathbf{j}); \text{ at } t = 2, p = 2(12\mathbf{i} + 8\mathbf{j})$$

M1

$$= (24\mathbf{i} + 16\mathbf{j}) \text{ kg m s}^{-1}$$

A1

Answer: (24i + 16j) kg m s⁻¹

2 marks

$$\text{(b) } F = \frac{dp}{dt} = 2(6t\mathbf{i} + 4\mathbf{j}); \text{ at } t = 2, F = 2(12\mathbf{i} + 4\mathbf{j}) = 24\mathbf{i} + 8\mathbf{j}$$

M1 A1

$$|F| = \sqrt{24^2 + 8^2} = 25.30 \text{ N}$$

A1

Answer: 25.30 N

3 marks

16 (4 marks)

$$p = \sqrt{2mE} \text{ for each particle, with the same } E$$

M1 A1

$$p_A : p_B = \sqrt{2(4m)E} : \sqrt{2mE} = \sqrt{4} : 1$$

M1

$$= 2 : 1$$

A1

Answer: 2 : 1

4 marks

17 (5 marks)

$$\text{Initial } p = 0.4(8\mathbf{i}) = 3.2\mathbf{i}; \text{ final } p = 0.4(-3\mathbf{i} + 4\mathbf{j}) = -1.2\mathbf{i} + 1.6\mathbf{j}$$

M1 A1

$$\text{Change} = -4.4\mathbf{i} + 1.6\mathbf{j} \text{ kg m s}^{-1}$$

M1 A1

$$\text{Magnitude} = \sqrt{4.4^2 + 1.6^2} = 4.682 \text{ kg m s}^{-1}$$

A1

Answer: (-4.4i + 1.6j) kg m s⁻¹, 4.682 kg m s⁻¹

5 marks

18 (6 marks)

$$\text{(a) Car: } 900 \times 30 = 27000; \text{ lorry: } 3000 \times 9 = 27000$$

M1 A1

They have equal momentum

A1

Answer: Equal, 27000 kg m s⁻¹

3 marks

$$\text{(b) } KE_{\text{car}} = \frac{1}{2}(900)(30)^2 = 405000, KE_{\text{lorry}} = \frac{1}{2}(3000)(9)^2 = 121500$$

M1 A1

$$\text{Ratio} = 405000 : 121500 = 10 : 3$$

A1

Answer: 10 : 3

3 marks

19 (6 marks)

$$\text{Speed before impact: } v^2 = 2(9.8)(0.9) = 17.64, v = 4.2 \text{ m s}^{-1}$$

M1 A1

$$\text{Speed after impact: } u^2 = 2(9.8)(0.4) = 7.84, u = 2.8 \text{ m s}^{-1}$$

M1 A1

$$\text{Change} = 0.25(2.8 + 4.2) = 1.75 \text{ kg m s}^{-1} \text{ upwards}$$

M1 A1

$$\text{Answer: } 1.75 \text{ kg m s}^{-1}$$

6 marks

20 (4 marks)

$$8 \text{ tonnes} = 8000 \text{ kg}$$

B1

$$72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} = 20 \text{ m s}^{-1}$$

M1 A1

$$p = 8000 \times 20 = 160000 \text{ kg m s}^{-1}$$

A1

$$\text{Answer: } 160000 \text{ kg m s}^{-1}$$

4 marks

21 (10 marks)

$$\text{(a) Momentum parallel to } \mathbf{i} \text{ when the } \mathbf{j} \text{ component is zero: } t^2 - 4 = 0$$

M1

$$t = 2 \text{ s}$$

A1

$$\text{Answer: } t = 2$$

2 marks

$$\text{(b) Speed}^2 = 16t^2 + (t^2 - 4)^2 = t^4 + 8t^2 + 16 = (t^2 + 4)^2$$

M1 A1

$$\text{Speed} = t^2 + 4, \text{ least at } t = 0, \text{ where speed} = 4$$

M1 A1

$$\text{Least momentum} = 0.5 \times 4 = 2 \text{ kg m s}^{-1}$$

A1

$$\text{Answer: } t = 0, 2 \text{ kg m s}^{-1}$$

5 marks

$$\text{(c) } F = \frac{dp}{dt} = 0.5(4\mathbf{i} + 2t\mathbf{j}); \text{ at } t = 2 \text{ } F = 2\mathbf{i} + 2\mathbf{j}$$

M1 A1

$$|F| = \sqrt{8} = 2.83 \text{ N}$$

A1

$$\text{Answer: } 2.83 \text{ N}$$

3 marks

22 (5 marks)

$$\text{Take right as positive: } 3(5) + 4(2) - 6m = 0$$

M1 A1

$$6m = 23 \text{ so } m = \frac{23}{6} = 3.833$$

A1

$$KE = \frac{1}{2}(3)(25) + \frac{1}{2}(4)(4) + \frac{1}{2}(3.833)(36)$$

M1

$$= 37.5 + 8 + 69.00 = 114.5 \text{ J}$$

A1

$$\text{Answer: } m = \frac{23}{6}, KE = 114.5 \text{ J}$$

5 marks

23 (6 marks)

$$\text{(a) } E = \frac{p^2}{2m} \text{ so } p = \sqrt{2mE}$$

M1

$$\text{Before: } p = \sqrt{2 \times 4 \times 50} = 20 \text{ kg m s}^{-1}$$

A1

$$\text{After: } p = \sqrt{2 \times 4 \times 200} = 40 \text{ kg m s}^{-1}$$

A1

$$\text{Answer: } 20 \text{ kg m s}^{-1} \text{ and } 40 \text{ kg m s}^{-1}$$

3 marks

$$\text{(b) Change} = 40 - 20 = 20 \text{ kg m s}^{-1}$$

A1

$$\text{Speed} = \frac{p}{m}; \text{ from } 5 \text{ m s}^{-1} \text{ to } 10 \text{ m s}^{-1}, \text{ so the speed has doubled}$$

M1 A1

$$\text{Answer: } 20 \text{ kg m s}^{-1}; \text{ speed } 5 \text{ to } 10 \text{ m s}^{-1}$$

3 marks

24 (4 marks)

$$\text{(a) Rate of change of momentum} = 0.8 \times 15$$

M1

$$\text{Force} = 12 \text{ N}$$

A1

$$\text{(b) Rate} = 0.8(15 + 5)$$

M1

$$\text{Force} = 16 \text{ N}$$

A1

$$\text{Answer: (a) } 12 \text{ N, (b) } 16 \text{ N}$$

4 marks

25 (4 marks)

Momentum given to bullets per second = $10 \times 0.02 \times 400$

M1 A1

= 80 kg m s^{-1} per s, so the average force is 80 N

M1 A1

Answer: 80 N

4 marks

26 (4 marks)

$p' = 1.2p$ with the same mass, so $v' = 1.2v$

M1 A1

$$KE' = \frac{1}{2}m(1.2v)^2 = 1.44 \times KE$$

M1

Increase = 44%

A1

Answer: 44%

4 marks

27 (10 marks)

(a) $p = mv$ so $v = \frac{p}{m}$

M1

$$E = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

A1

Answer: shown

2 marks

(b) $E \propto \frac{1}{m}$ for equal p

M1

$$E_2 : E_8 = \frac{1}{2} : \frac{1}{8} = 4 : 1$$

M1 A1

The lighter particle has four times as much kinetic energy

A1

Answer: 4 : 1, the 2 kg particle

4 marks

(c) $v = \frac{p}{m}$: new speed = $2p \div \left(\frac{1}{2}m\right) = 4\frac{p}{m} = 4v$, factor 4

M1 A1

$$E = \frac{p^2}{2m}$$
: new $E = \frac{(2p)^2}{m} = \frac{4p^2}{m} = 8 \times \frac{p^2}{2m}$, factor 8

M1 A1

Answer: speed $\times 4$, KE $\times 8$

4 marks

28 (11 marks)

(a) Force = gradient of the p - t graph

M1

0 to 4 s: $\frac{20}{4} = 5 \text{ N}$; 4 to 7 s: 0 N

A1

7 to 10 s: $\frac{-10 - 20}{3} = -10 \text{ N}$

A1

Answer: 5 N, 0 N, -10 N

3 marks

(b) $v = \frac{p}{m}$: at $t = 4$, $v = \frac{20}{2} = 10 \text{ m s}^{-1}$

M1 A1

At $t = 10$, $v = \frac{-10}{2} = -5 \text{ m s}^{-1}$

A1

Answer: 10 and -5 m s⁻¹

3 marks

(c) Velocity-time shape: 0 to 4 rising from 0 to 10; 4 to 7 constant 10

B1

$$\text{Distance} = \frac{1}{2}(4)(10) + 3(10) = 50 \text{ m}$$

M1 A1

$p = 0$ when $20 - 10(t - 7) = 0$, so $t = 9 \text{ s}$

M1 A1

Answer: 50 m, $t = 9$

5 marks

29 (12 marks)

(a) $p_A = 8\mathbf{i} - 2\mathbf{j}$, $p_B = -6\mathbf{i} + 9\mathbf{j}$

M1 A1

Total = $2\mathbf{i} + 7\mathbf{j} \text{ kg m s}^{-1}$, magnitude $\sqrt{4 + 49} = \sqrt{53} = 7.280$

M1 A1

Answer: $(2\mathbf{i} + 7\mathbf{j}) \text{ kg m s}^{-1}$, 7.280

4 marks

(b) $v = \frac{1}{5}(2\mathbf{i} + 7\mathbf{j}) = (0.4\mathbf{i} + 1.4\mathbf{j}) \text{ m s}^{-1}$

M1 A1

Answer: $(0.4\mathbf{i} + 1.4\mathbf{j}) \text{ m s}^{-1}$

2 marks

(c) $p_C = -(2\mathbf{i} + 7\mathbf{j})$

M1

$$v_C = \frac{-(2\mathbf{i} + 7\mathbf{j})}{0.5} = (-4\mathbf{i} - 14\mathbf{j}) \text{ m s}^{-1}$$

A1

Answer: $(-4\mathbf{i} - 14\mathbf{j}) \text{ m s}^{-1}$

2 marks

$$(d) \text{ Speed} = \sqrt{16 + 196} = 14.56 \text{ m s}^{-1}$$

M1 A1

$$\tan \theta = \frac{14}{4}, \theta = 74.1^\circ$$

M1 A1

Answer: 14.56 m s⁻¹, 74.1°

4 marks

30 (9 marks)

$$(a) F = \text{rate of change of momentum} = 5 \times 2 = 10 \text{ N}$$

M1 A1

$$\text{Power} = Fv = 10 \times 2 = 20 \text{ W}$$

M1 A1

Answer: 10 N, 20 W

4 marks

$$(b) \text{ Each second 5 kg gains speed } 2 \text{ m s}^{-1}; \text{ gain} = \frac{1}{2}(5)(2)^2$$

M1 A1

$$= 10 \text{ J per second, that is } 10 \text{ W}$$

A1

Answer: 10 W

3 marks

(c) Half the work is converted to KE; the rest is lost as heat and sound because the sand slips on the belt before reaching belt speed

E1 E1

Answer: Half the power is dissipated

2 marks

Impulse

1 (2 marks)

$$I = Ft = 8 \times 3$$

M1

$$= 24 \text{ N s}$$

A1

Answer: 24 N s

2 marks

2 (2 marks)

$$I = mv - mu = 0.5(10 - 4)$$

M1

$$= 3 \text{ N s}$$

A1

Answer: 3 N s

2 marks

3 (2 marks)

$$I = Ft \text{ so } t = \frac{45}{15}$$

M1

$$= 3 \text{ s}$$

A1

Answer: 3 s

2 marks

4 (2 marks)

$$I = mv - 0 \text{ so } m = \frac{12}{6}$$

M1

$$= 2 \text{ kg}$$

A1

Answer: 2 kg

2 marks

5 (2 marks)

$$I = 0.16 \times 25 - 0$$

M1

$$= 4 \text{ N s}$$

A1

Answer: 4 N s

2 marks

6 (2 marks)

$$10 = 2(v - 3)$$

M1

$$v = 8 \text{ m s}^{-1}$$

A1

Answer: 8 m s⁻¹

2 marks

7 (3 marks)

Take the direction away from the wall as positive: $u = -10$, $v = 6$

B1

$$I = 0.3(6 - (-10)) = 0.3 \times 16$$

M1

$$= 4.8 \text{ N s}$$

A1

Answer: 4.8 N s

3 marks

8 (3 marks)

$$I = m(\mathbf{v} - \mathbf{u}): 4\mathbf{i} - 6\mathbf{j} = 2(\mathbf{v} - 3\mathbf{i} - \mathbf{j})$$

M1

$$\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{i} - 3\mathbf{j}$$

M1

$$= (5\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$$

A1

$$\text{Answer: } (5\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$$

3 marks

9 (4 marks)

$$I = 0.45 \times 22 = 9.9 \text{ N s}$$

M1 A1

$$F = \frac{I}{t} = \frac{9.9}{0.01} = 990 \text{ N}$$

M1 A1

$$\text{Answer: } 990 \text{ N}$$

4 marks

10 (4 marks)

$$I = 0.8 \times 5 = 4 \text{ N s}$$

M1 A1

$$F = \frac{4}{0.004} = 1000 \text{ N}$$

M1 A1

$$\text{Answer: } 1000 \text{ N}$$

4 marks

11 (5 marks)

$$\text{Impulse} = \text{area under the graph} = 20 \times 0.5$$

M1 A1

$$= 10 \text{ N s}$$

A1

$$10 = 2.5v \text{ so } v = 4 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } 10 \text{ N s, } 4 \text{ m s}^{-1}$$

5 marks

12 (6 marks)

$$\text{(a) Impulse} = m(\mathbf{v} - \mathbf{u}) \text{ so } 0.2\mathbf{v} = (-4\mathbf{i} + 3\mathbf{j}) + 0.2(12\mathbf{i})$$

M1

$$0.2\mathbf{v} = -1.6\mathbf{i} + 3\mathbf{j}, \text{ so } \mathbf{v} = (-8\mathbf{i} + 15\mathbf{j}) \text{ m s}^{-1}$$

A1

$$\text{Speed} = \sqrt{64 + 225} = 17 \text{ m s}^{-1}$$

A1

$$\text{Answer: } (-8\mathbf{i} + 15\mathbf{j}) \text{ m s}^{-1}, 17 \text{ m s}^{-1}$$

3 marks

$$\text{(b) KE before} = \frac{1}{2}(0.2)(12)^2 = 14.4 \text{ J, KE after} = \frac{1}{2}(0.2)(17)^2 = 28.9 \text{ J}$$

M1 A1

$$\text{Gain} = 14.5 \text{ J}$$

A1

$$\text{Answer: } 14.5 \text{ J}$$

3 marks

13 (4 marks)

$$\text{Impulse} = m(\mathbf{v} - \mathbf{u})$$

M1

$$\mathbf{i}: p = 1.5(10 - 4) = 9$$

A1

$$\mathbf{j}: -9 = 1.5(q - 2)$$

M1

$$q - 2 = -6 \text{ so } q = -4$$

A1

$$\text{Answer: } p = 9, q = -4$$

4 marks

14 (5 marks)

$$I = \int_0^3 (6t + 2) \, dt = [3t^2 + 2t]_0^3$$

M1 A1

$$= 27 + 6 = 33 \text{ N s}$$

A1

$$33 = 3(v - 1) \text{ so } v = 12 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } 33 \text{ N s, } 12 \text{ m s}^{-1}$$

5 marks

15 (7 marks)

$$\text{(a) Impulse} = \text{area of triangle} = \frac{1}{2} \times 0.4 \times 30$$

M1 A1

$$= 6 \text{ N s}$$

A1

$$\text{Answer: } 6 \text{ N s}$$

3 marks

$$\text{(b) } 6 = 0.15v \text{ so } v = 40 \text{ m s}^{-1}$$

M1 A1

$$\text{Average force} = \frac{6}{0.4} = 15 \text{ N}$$

M1 A1

$$\text{Answer: } 40 \text{ m s}^{-1}, 15 \text{ N}$$

4 marks

16 (6 marks)

$$0.4(\mathbf{v} - 5\mathbf{j}) = -3\mathbf{i} + 4\mathbf{j} \quad \text{M1}$$

$$\mathbf{v} = 5\mathbf{j} + (-7.5\mathbf{i} + 10\mathbf{j}) = -7.5\mathbf{i} + 15\mathbf{j} \quad \text{M1 A1}$$

$$\text{Speed} = \sqrt{7.5^2 + 15^2} = 16.77 \text{ m s}^{-1} \quad \text{M1 A1}$$

$$\tan \theta = \frac{7.5}{15} \text{ so } \theta = 26.6^\circ \quad \text{A1}$$

Answer: 16.77 m s⁻¹, 26.6° from j

6 marks

17 (5 marks)

Conservation of momentum along the line: $2(6) = (2 + 3)v$ M1 A1

$$v = 2.4 \text{ m s}^{-1} \quad \text{A1}$$

$$\text{Impulse on B: } I = 3v = 7.2 \text{ N s} \quad \text{M1 A1}$$

Answer: v = 2.4 m s⁻¹, I = 7.2 N s

5 marks

18 (5 marks)

$$\text{Impulse} = m(\mathbf{v} - \mathbf{u}) \text{ so } \mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + \frac{1}{2}(\mathbf{k}\mathbf{i} + 2\mathbf{k}\mathbf{j}) \quad \text{M1}$$

$$\mathbf{v} = \left(3 + \frac{k}{2}\right)\mathbf{i} + (k - 4)\mathbf{j} \text{ m s}^{-1} \quad \text{A1}$$

$$\left(3 + \frac{k}{2}\right)^2 + (k - 4)^2 = 65 \quad \text{M1}$$

$$\frac{5}{4}k^2 - 5k + 25 = 65 \text{ so } k^2 - 4k - 32 = 0 \quad \text{A1}$$

$$(k - 8)(k + 4) = 0 \text{ so } k = 8 \text{ or } k = -4 \quad \text{A1}$$

Answer: k = 8 or k = -4

5 marks

19 (6 marks)

$$\text{(a) } \mathbf{I} = 0.4[(8\mathbf{i} - 3\mathbf{j}) - (2\mathbf{i} + 5\mathbf{j})] \quad \text{M1}$$

$$= (2.4\mathbf{i} - 3.2\mathbf{j}) \text{ N s} \quad \text{A1}$$

$$|\mathbf{I}| = \sqrt{2.4^2 + 3.2^2} = 4 \text{ N s} \quad \text{A1}$$

Answer: (2.4i - 3.2j) N s, 4 N s

3 marks

$$\text{(b) } \cos \varphi = (\mathbf{I} \cdot \mathbf{u}) \div (|\mathbf{I}||\mathbf{u}|) = (2.4 \times 2 - 3.2 \times 5) \div (4\sqrt{29}) \quad \text{M1 A1}$$

$$= -11.2 \div 21.54 \text{ so } \varphi = 121.3^\circ \quad \text{A1}$$

Answer: 121.3°

3 marks

20 (6 marks)

$$\text{Impulse from } t = 1 \text{ to } t: \int_1^t (6s - 8) \, ds = [3s^2 - 8s]_1^t = 3t^2 - 8t + 5 \quad \text{M1 A1}$$

$$0.5v - 0.5(5) = 3t^2 - 8t + 5, \text{ so } v = 6t^2 - 16t + 15 \quad \text{M1 A1}$$

$$6t^2 - 16t + 15 = 21 \text{ so } 3t^2 - 8t - 3 = 0 \quad \text{M1}$$

$$(3t + 1)(t - 3) = 0 \text{ and } t > 1, \text{ so } t = 3 \quad \text{A1}$$

Answer: t = 3

6 marks

21 (6 marks)

$$\text{Impulse in time } T = \int_0^T 20e^{-2t} \, dt = [-10e^{-2t}]_0^T = 10(1 - e^{-2T}) \quad \text{M1 A1}$$

$$4v = 10(1 - e^{-2T}), \text{ so } v = 2.5(1 - e^{-2t}) \quad \text{M1 A1}$$

$$\text{As } t \rightarrow \infty, e^{-2t} \rightarrow 0 \quad \text{M1}$$

$$\text{Limit} = 2.5 \text{ m s}^{-1} \quad \text{A1}$$

Answer: v = 2.5(1 - e^{-2t}), limit 2.5 m s⁻¹

6 marks

22 (6 marks)

The wall is smooth so the component parallel to the wall is unchanged B1

Component perpendicular to wall: $12 \sin 30^\circ = 6 \text{ m s}^{-1}$, reversed by the impact M1 A1

$$I = 0.5(6 + 6) = 6 \text{ N s} \quad \text{M1 A1}$$

Direction: perpendicular to the wall, away from it A1

Answer: 6 N s, perpendicular to the wall

6 marks

23 (8 marks)

(a) Take **i** along the original direction: initial velocity $20\mathbf{i}$, final velocity $(30\cos 60^\circ)\mathbf{i} + (30\sin 60^\circ)\mathbf{j}$
 $= (15\mathbf{i} + 15\sqrt{3}\mathbf{j})$

M1 A1

Change in velocity $= (-5\mathbf{i} + 15\sqrt{3}\mathbf{j})$, with magnitude $\sqrt{25 + 675} = \sqrt{700}$

M1 A1

$|I| = 0.2\sqrt{700} = 5.29 \text{ N s}$

A1

Answer: 5.29 N s

5 marks

(b) The impulse is parallel to the change in velocity: $\tan \alpha = 15\sqrt{3} \div (-5)$

M1

The **i** component is negative, so $\alpha = 180^\circ - \arctan(3\sqrt{3}) = 100.9^\circ$

M1 A1

Answer: 100.9°

3 marks

24 (6 marks)

$v^2 = 2(9.8)(0.9) = 17.64$ so $v = 4.2 \text{ m s}^{-1}$

M1 A1

Impulsive tension acts equally on both: for A, $-I = 2(w - 4.2)$; for B, $I = 3w$

M1 A1

$-3w = 2w - 8.4$ so $w = 1.68 \text{ m s}^{-1}$

M1 A1

Answer: 4.2 m s⁻¹, 1.68 m s⁻¹

6 marks

25 (5 marks)

$I = \int_0^2 (6t\mathbf{i} - 4t\mathbf{j}) dt = [3t^2\mathbf{i} - 4t\mathbf{j}]_0^2$

M1 A1

$= (12\mathbf{i} - 8\mathbf{j}) \text{ N s}$

A1

$2(\mathbf{v} - \mathbf{i}) = 12\mathbf{i} - 8\mathbf{j}$

M1

$\mathbf{v} = (7\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$

A1

Answer: (12i - 8j) N s, (7i - 4j) m s⁻¹

5 marks

26 (4 marks)

(a) The velocity changes direction, so with away from the wall as positive $u = -6$ and $v = +4$ (or vice versa)

E1

The student used speeds, not velocities, so the change in velocity is $4 + 6 = 10$ not $4 - 6$

E1

Answer: Directions ignored

2 marks

(b) $I = 0.5(4 - (-6)) = 0.5 \times 10$

M1

$= 5 \text{ N s}$

A1

Answer: 5 N s

2 marks

27 (13 marks)

(a) Impulse $= \frac{1}{2} \times 0.3 \times 40 = 6 \text{ N s}$

M1 A1

$0.2v = 6$ so $v = 30 \text{ m s}^{-1}$

M1 A1

Answer: 6 N s, 30 m s⁻¹

4 marks

(b) Impulse so far $= \frac{1}{2} \times 0.1 \times 40 = 2 \text{ N s}$

M1 A1

$0.2v = 2$ so $v = 10 \text{ m s}^{-1}$

A1

Answer: 10 m s⁻¹

3 marks

(c) Impulse needed $= 0.2 \times 20 = 4 \text{ N s}$

M1

Impulse after 0.1 s is 2; remaining force $F = 40 - 200(t - 0.1)$ on $0.1 \leq t \leq 0.3$

M1 A1

Impulse from 0.1 to $t = 40s - 100s^2$ ($s = t - 0.1$) $= 2$

M1

$100s^2 - 40s + 2 = 0$ so $s = 0.0586$, $t = 0.159 \text{ s}$

A1

Answer: t = 0.159 s

5 marks

(d) Average $= \frac{6}{0.3} = 20 \text{ N}$

B1

Answer: 20 N

1 mark

28 (13 marks)

$$(a) I = [6t^2 - t^3]_0^4 = 96 - 64$$

M1 A1

$$= 32 \text{ N s}$$

A1

$$1.5(v - 2) = 32 \text{ so } v = 23.33 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } 32 \text{ N s, } 23.33 \text{ m s}^{-1}$$

5 marks

$$(b) F' = 12 - 6t = 0 \text{ so } t = 2$$

M1 A1

$$F(2) = 24 - 12 = 12 \text{ N}$$

A1

$$\text{Answer: } 12 \text{ N}$$

3 marks

$$(c) 1.5(10 - 2) = 12 = 6t^2 - t^3$$

M1 A1

$$\text{So } t^3 - 6t^2 + 12 = 0$$

A1

$$\text{Solving numerically } (t^3 - 6t^2 + 12 = 0 \text{ changes sign between 1 and 2}); t = 1.66 \text{ s}$$

M1 A1

$$\text{Answer: } t = 1.66 \text{ s}$$

5 marks

29 (15 marks)

(a) B moves along a line perpendicular to AB, at distance a from A. The string is taut when B is 2a from A

B1

$$\text{If B has travelled a distance } d: d^2 + a^2 = (2a)^2, \text{ so } d = \sqrt{3}a$$

M1 A1

$$\cos \theta = \frac{a}{2a} = \frac{1}{2}, \text{ so the angle between the string and AB is } 60^\circ$$

M1 A1

Answer: shown

5 marks

(b) The angle between B's velocity and the string is 30° , so B's component along the string is $10\cos 30^\circ = 5\sqrt{3} \text{ m s}^{-1}$ and its component perpendicular is 5 m s^{-1}

M1 A1

$$\text{A and B then have the same speed } w \text{ along the string: } 2(5\sqrt{3}) = (3 + 2)w, \text{ so } w = 2\sqrt{3} \text{ m s}^{-1}$$

M1 A1

$$\text{Impulse on A} = 3w, \text{ so } J = 6\sqrt{3} = 10.39 \text{ N s}$$

A1

$$\text{Answer: } 6\sqrt{3} = 10.39 \text{ N s}$$

5 marks

$$(c) \text{ KE before} = \frac{1}{2}(2)(10)^2 = 100 \text{ J}$$

B1

$$\text{KE after} = \frac{1}{2}(3)(2\sqrt{3})^2 + \frac{1}{2}(2)[(2\sqrt{3})^2 + 5^2] = 18 + 37 = 55 \text{ J}$$

M1 A1 A1

$$\text{Loss} = 45 \text{ J}$$

A1

$$\text{Answer: } 45 \text{ J}$$

5 marks

30 (11 marks)

$$(a) \int_0^T F \, dt = \int_0^T m \frac{dv}{dt} \, dt$$

M1 A1

$$= m[v]_0^T = mv - mu$$

M1 A1

Answer: shown

4 marks

$$(b) \text{ Impulse} = \int_0^t 0.6s \, ds = 0.3t^2$$

M1 A1

$$3v = 0.3t^2 \text{ so } v = 0.1t^2$$

A1

Answer: shown

3 marks

$$(c) 0.1t^2 = 10 \text{ so } t = 10 \text{ s}$$

M1 A1

$$s = \int_0^{10} 0.1t^2 \, dt = \left[\frac{0.1t^3}{3} \right]_0^{10} = \frac{100}{3} = 33.3 \text{ m}$$

M1 A1

$$\text{Answer: } 10 \text{ s, } 33.3 \text{ m}$$

4 marks

Conservation of Momentum**1 (2 marks)**

$$3(4) + 1(0) = (3 + 1)v$$

M1

$$v = 3 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 3 \text{ m s}^{-1}$$

2 marks

2 (2 marks)

$$\text{Momentum conserved: } 0 = 0.01(400) + 2v$$

M1

$$v = -2, \text{ so the recoil speed is } 2 \text{ m s}^{-1}$$

A1

Answer: 2 m s^{-1}

2 marks

3 (2 marks)

$$2(5) + 3(-5) = 5v$$

M1

$$v = -1: \text{ speed } 1 \text{ m s}^{-1} \text{ in the direction of the } 3 \text{ kg particle's original motion}$$

A1

Answer: 1 m s^{-1} towards the 3 kg particle's original direction

2 marks

4 (2 marks)

$$4(6) = (4 + m)4$$

M1

$$4 + m = 6 \text{ so } m = 2$$

A1

Answer: $m = 2$

2 marks

5 (2 marks)

$$0 = 3(8) + 60v$$

M1

$$v = -0.4, \text{ so the skater recoils at } 0.4 \text{ m s}^{-1}$$

A1

Answer: 0.4 m s^{-1}

2 marks

6 (2 marks)

$$0.02(300) = (0.02 + 2.98)v$$

M1

$$v = 2 \text{ m s}^{-1}$$

A1

Answer: 2 m s^{-1}

2 marks

7 (3 marks)

$$\text{Total momentum is zero throughout: } 0 = 1.5m - 5(6)$$

M1 A1

$$m = \frac{30}{1.5} = 20$$

A1

Answer: $m = 20$

3 marks

8 (4 marks)

$$2(6) + 3(1) = 2(1.5) + 3v$$

M1 A1

$$15 - 3 = 3v \text{ so } v = 4 \text{ m s}^{-1}$$

M1 A1

Answer: 4 m s^{-1}

4 marks

9 (3 marks)

$$0 = 2(9) + 3v$$

M1 A1

$$v = -6: \text{ speed } 6 \text{ m s}^{-1} \text{ in the opposite direction}$$

A1

Answer: 6 m s^{-1} in the opposite direction

3 marks

10 (3 marks)

$$0 = 60(3) + 120v$$

M1 A1

$$v = -1.5, \text{ so the boat moves at } 1.5 \text{ m s}^{-1} \text{ in the opposite direction}$$

A1

Answer: 1.5 m s^{-1}

3 marks

11 (6 marks)

$$\text{B: impulse} = \text{change in momentum, so } 6 = 3v_B, v_B = 2 \text{ m s}^{-1}$$

M1 A1

$$\text{A: } 2(5) - 6 = 2v_A, \text{ so } v_A = 2 \text{ m s}^{-1}$$

M1 A1

$$\text{KE before} = \frac{1}{2}(2)(25) = 25 \text{ J, KE after} = \frac{1}{2}(2)(4) + \frac{1}{2}(3)(4) = 10 \text{ J}$$

M1

$$\text{Loss} = 15 \text{ J}$$

A1

Answer: $v_A = 2 \text{ m s}^{-1}$, $v_B = 2 \text{ m s}^{-1}$, loss 15 J

6 marks

12 (3 marks)Taking right as positive: $5(4) + 3(-2) = 8v$

M1 A1

 $v = 1.75 \text{ m s}^{-1}$ to the right

A1

Answer: 1.75 m s^{-1} to the right

3 marks

13 (4 marks) $0 = 5(100) + 500v$ so the recoil speed is 1 m s^{-1}

M1 A1

 $\text{KE} = \frac{1}{2}(5)(100)^2 + \frac{1}{2}(500)(1)^2 = 25000 + 250$

M1

 $= 25250 \text{ J}$

A1

Answer: 1 m s^{-1} , 25250 J

4 marks

14 (6 marks)Let the speed of A be v , so B has speed $2v$. Conservation of momentum: $4(9) = 4v + 2(2v)$

M1 A1

 $8v = 36$ so $v = 4.5$, and the speeds are 4.5 m s^{-1} and 9 m s^{-1}

A1

 $\text{KE before} = \frac{1}{2}(4)(81) = 162 \text{ J}$

B1

 $\text{KE after} = \frac{1}{2}(4)(4.5)^2 + \frac{1}{2}(2)(9)^2 = 40.5 + 81 = 121.5 \text{ J}$

M1

Loss = 40.5 J

A1

Answer: 4.5 m s^{-1} and 9 m s^{-1} , loss 40.5 J

6 marks

15 (5 marks)A and B: $2(6) = 3v_1$ so $v_1 = 4 \text{ m s}^{-1}$

M1 A1

AB then catches C: $3(4) + 3(1) = 6v_2$

M1 A1

 $v_2 = 2.5 \text{ m s}^{-1}$

A1

Answer: 2.5 m s^{-1}

5 marks

16 (5 marks)Speed of block just after impact: $\frac{1}{2}v^2 = 9.8(0.2)$

M1

 $v = \sqrt{3.92} = 1.98 \text{ m s}^{-1}$

A1

 $0.01u = 2v$

M1 A1

 $u = 396 \text{ m s}^{-1}$ (3 s.f.)

A1

Answer: 396 m s^{-1}

5 marks

17 (6 marks) $0.05(200) = 5v$ so $v = 2 \text{ m s}^{-1}$

M1 A1

Deceleration = $\mu g = 0.4 \times 9.8 = 3.92 \text{ m s}^{-2}$

M1 A1

 $0 = 2^2 - 2(3.92)s$ so $s = 0.510 \text{ m}$

M1 A1

Answer: 2 m s^{-1} , 0.510 m

6 marks

18 (5 marks) $3(4) = 3(1) + 3m$

M1 A1

 $m = 3$

A1

 $\text{KE before} = \frac{1}{2}(3)(16) = 24 \text{ J}$; after = $1.5 + 13.5 = 15 \text{ J}$

M1

Loss = 9 J

A1

Answer: $m = 3$, loss 9 J

5 marks

19 (5 marks)

$$\text{Total momentum after} = 0: 1(6\mathbf{i}) + 2(-2\mathbf{i} + 3\mathbf{j}) + 3\mathbf{v} = 0$$

M1 A1

$$(2\mathbf{i} + 6\mathbf{j}) + 3\mathbf{v} = 0$$

M1

$$\mathbf{v} = \left(-\frac{2}{3}\mathbf{i} - 2\mathbf{j}\right) \text{ m s}^{-1}$$

A1

$$\text{Speed} = \sqrt{\frac{4}{9} + 4} = 2.11 \text{ m s}^{-1}$$

A1

$$\text{Answer: } \left(-\frac{2}{3}\mathbf{i} - 2\mathbf{j}\right) \text{ m s}^{-1}, 2.11 \text{ m s}^{-1}$$

5 marks

20 (5 marks)

$$\text{Let speeds be } v_1 \text{ and } v_2: 50v_1 = 70v_2 \text{ and } v_1 + v_2 = 6$$

M1 A1

$$v_1 = \frac{7}{5}v_2 \text{ so } \frac{12}{5}v_2 = 6$$

M1

$$v_2 = 2.5 \text{ m s}^{-1} \text{ and } v_1 = 3.5 \text{ m s}^{-1}$$

A1 A1

$$\text{Answer: } 3.5 \text{ m s}^{-1} \text{ (50 kg), } 2.5 \text{ m s}^{-1} \text{ (70 kg)}$$

5 marks

21 (6 marks)

$$0.02(400) = 0.02(100) + 2v$$

M1 A1

$$v = 3 \text{ m s}^{-1}$$

A1

$$\text{KE before} = \frac{1}{2}(0.02)(400)^2 = 1600 \text{ J}$$

M1

$$\text{KE after} = 100 + \frac{1}{2}(2)(9) = 109 \text{ J, so loss} = 1491 \text{ J}$$

M1 A1

$$\text{Answer: } 3 \text{ m s}^{-1}, 1491 \text{ J}$$

6 marks

22 (10 marks)

$$\text{(a) Momentum: } 3(8) = 3a + 5b, \text{ so } 3a + 5b = 24$$

M1 A1

$$\text{KE after} = 96 - 50.4 = 45.6 = \frac{3}{2}a^2 + \frac{5}{2}b^2$$

M1

$$a = \frac{24 - 5b}{3}; \frac{3}{2}\left(\frac{24 - 5b}{3}\right)^2 = \frac{(24 - 5b)^2}{6}; \text{ so } (24 - 5b)^2 + 15b^2 = 273.6$$

M1

$$40b^2 - 240b + 302.4 = 0, \text{ that is } 20b^2 - 120b + 151.2 = 0$$

A1

$$\text{Answer: shown}$$

5 marks

$$\text{(b) } b = \frac{120 \pm \sqrt{14400 - 12096}}{40} = 4.2 \text{ or } 1.8$$

M1 A1

$$b = 1.8 \text{ gives } a = 5 > b, \text{ so A would have passed through B, which is impossible}$$

M1 E1

$$b = 4.2 \text{ and } a = \frac{24 - 21}{3} = 1$$

A1

$$\text{Answer: } a = 1, b = 4.2$$

5 marks

23 (6 marks)

$$\text{(a) } 0 = 4(-5) + 84v_1$$

M1 A1

$$v_1 = \frac{5}{21} = 0.238 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 0.238 \text{ m s}^{-1}$$

3 marks

$$\text{(b) } 84v_1 = 20 = 4(-5) + 80v_2$$

M1 A1

$$v_2 = \frac{40}{80} = 0.5 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 0.5 \text{ m s}^{-1}$$

3 marks

24 (5 marks)

$$\text{Horizontal momentum is conserved: } 500(4) = 600v$$

M1 A1

$$v = 3.333 \text{ m s}^{-1}$$

A1

$$\text{KE before} = \frac{1}{2}(500)(16) = 4000 \text{ J; KE after} = \frac{1}{2}(600)\left(\frac{10}{3}\right)^2 = 3333 \text{ J}$$

M1

$$\text{Percentage lost} = 16.7\%$$

A1

$$\text{Answer: } 3.333 \text{ m s}^{-1}, 16.7\%$$

5 marks

25 (6 marks)

Total momentum = $6\mathbf{i} + 6\mathbf{j}$

M1 A1

$\mathbf{v} = \frac{1}{5}(6\mathbf{i} + 6\mathbf{j}) = (1.2\mathbf{i} + 1.2\mathbf{j}) \text{ m s}^{-1}$, speed = $1.2\sqrt{2} = 1.70 \text{ m s}^{-1}$

M1 A1

KE before = $\frac{1}{2}(2)(9) + \frac{1}{2}(3)(4) = 15 \text{ J}$; after = $\frac{1}{2}(5)(2.88) = 7.2 \text{ J}$

M1

Loss = 7.8 J

A1

Answer: $(1.2\mathbf{i} + 1.2\mathbf{j}) \text{ m s}^{-1}$, 1.70 m s^{-1} , 7.8 J

6 marks

26 (9 marks)

(a) Momentum conserved: $0 = 2(6) - 3v_B$

M1 A1

$v_B = 4 \text{ m s}^{-1}$

A1

Answer: 4 m s^{-1}

3 marks

(b) Friction: $F = \mu R = 0.5 \text{ mg}$, so the deceleration is $0.5 \times 9.8 = 4.9 \text{ m s}^{-2}$ for each particle

M1 A1

A: $0 = 6^2 - 2(4.9)s_A$, so $s_A = 3.673 \text{ m}$

M1 A1

B: $0 = 4^2 - 2(4.9)s_B$, so $s_B = 1.633 \text{ m}$

A1

Distance apart = $s_A + s_B = 5.31 \text{ m}$

A1

Answer: 5.31 m

6 marks

27 (12 marks)

(a) Conservation of momentum: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

M1 A1

Divide by $(m_1 + m_2)$

A1

Answer: shown

3 marks

(b) Loss = $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{(m_1u_1 + m_2u_2)^2}{2(m_1 + m_2)}$

M1 A1

Common denominator $2(m_1 + m_2)$: numerator = $(m_1u_1^2 + m_2u_2^2)(m_1 + m_2) - (m_1u_1 + m_2u_2)^2$

M1

$= m_1m_2u_1^2 + m_1m_2u_2^2 - 2m_1m_2u_1u_2 = m_1m_2(u_1 - u_2)^2$

M1 A1

Answer: shown

5 marks

(c) Loss = $\frac{4 \times 6}{2(10)}(9 - 4)^2 = \frac{24}{20} \times 25$

M1 A1

$= 30 \text{ J}$

A1

Answer: 30 J

3 marks

(d) No collision occurs: the particles move together at the same speed already

E1

Answer: $(u_1 - u_2)^2 = 0$

1 mark

28 (14 marks)

(a) $\mu u = 2mv_1$ so $v_1 = \frac{\mu}{2}$

M1 A1

$2m(u/2) = (2m + M)v_2$

M1

$v_2 = \frac{\mu u}{2m + M}$

A1

Answer: $\frac{\mu}{2}$, $\frac{\mu u}{2m + M}$

4 marks

(b) $\frac{\mu u}{2m + M} = \frac{u}{5}$ so $5m = 2m + M$

M1 A1

$M = 3m$

A1

Answer: M = 3m

3 marks

(c) $\text{KE}_{\text{final}} = \frac{1}{2}(5m)\left(\frac{u}{5}\right)^2 = \frac{\mu u^2}{10}$

M1 A1

$\text{KE}_{\text{initial}} = \frac{1}{2}\mu u^2$, fraction = $\frac{1}{5}$

A1

Answer: $\frac{1}{5}$

3 marks

(d) After the first: $\frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{\mu u^2}{4}$, fraction retained = $\frac{1}{2}$, lost = $\frac{1}{2}$

M1 A1

Second collision loses $\frac{1}{2} - \frac{1}{5} = \frac{3}{10}$ of the original

M1 A1

Answer: $\frac{1}{2}$ and $\frac{3}{10}$

4 marks

29 (10 marks)

(a) $v^2 = 2(9.8)(2.5) = 49$

M1

$v = 7 \text{ m s}^{-1}$

A1

Answer: 7 m s^{-1}

2 marks

(b) $500(7) = 2000w$

M1 A1

$w = 1.75 \text{ m s}^{-1}$

A1

Answer: 1.75 m s^{-1}

3 marks

(c) Work-energy: loss of KE plus loss of PE = work done against the resistance

M1

$\frac{1}{2}(2000)(1.75)^2 + 2000(9.8)(0.1) = R(0.1)$

M1 A1 A1

$3062.5 + 1960 = 0.1R$ so $R = 50225 \text{ N}$

A1

Answer: 50225 N

5 marks

30 (10 marks)

(a) Let the rocket have velocity V ; the stage has $V - 150$

B1

$1000(400) = 800V + 200(V - 150)$

M1 A1

$1000V = 400000 + 30000$, $V = 430.0 \text{ m s}^{-1}$, stage 280.0 m s^{-1}

M1 A1

Answer: rocket 430.0 m s^{-1} , stage 280.0 m s^{-1}

5 marks

(b) Impulse on stage = $200(280.0 - 400) = -24000 \text{ N s}$

M1 A1

KE before = 80000000 J , after = 81800000 J

M1

Gain = 1800000 J

A1

Answer: -24000 N s , gain 1800000 J

4 marks

(c) Chemical energy of the charge is converted into kinetic energy, so the total kinetic energy is not conserved

E1

Answer: Chemical energy released

1 mark

Collisions and Restitution

1 (2 marks)

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$

M1

$= \frac{3}{6} = 0.5$

A1

Answer: 0.5

2 marks

2 (2 marks)

$e = \frac{4.2}{6}$

M1

$= 0.7$

A1

Answer: 0.7

2 marks

3 (2 marks)

Speed after = $e \times$ speed before = 0.5×8

M1

$= 4 \text{ m s}^{-1}$

A1

Answer: 4 m s^{-1}

2 marks

4 (2 marks)

Speed of separation = $e \times$ speed of approach

M1

$= 0.4 \times 10 = 4 \text{ m s}^{-1}$

A1

Answer: 4 m s^{-1}

2 marks

5 (3 marks)

$$e = \frac{3}{5} = 0.6$$

B1

$$\text{KE lost} = \frac{1}{2}(2)(25 - 9)$$

M1

$$= 16 \text{ J}$$

A1

Answer: $e = 0.6$, 16 J

3 marks

6 (3 marks)

$$\text{Speed on impact: } v^2 = 2(9.8)(4.9) = 96.04, v = 9.8 \text{ m s}^{-1}$$

M1 A1

$$\text{Rebound speed} = 0.5 \times 9.8 = 4.9 \text{ m s}^{-1}$$

A1

Answer: 4.9 m s^{-1}

3 marks

7 (5 marks)

$$\text{Momentum: } 2(6) = 2v_A + v_B$$

M1 A1

$$\text{Restitution: } v_B - v_A = 0.5 \times 6 = 3$$

M1 A1

$$3v_A = 12 - 3 \text{ so } v_A = 3 \text{ m s}^{-1}, v_B = 6 \text{ m s}^{-1}$$

A1

Answer: A: 3 m s^{-1} , B: 6 m s^{-1}

5 marks

8 (6 marks)

$$\text{Momentum: } 3(5) + 2(-4) = 3v_A + 2v_B, \text{ so } 3v_A + 2v_B = 7$$

M1 A1

$$\text{Restitution: } v_B - v_A = 0.5(5 + 4) = 4.5$$

M1 A1

$$5v_A = 7 - 9 \text{ so } v_A = -0.4 \text{ m s}^{-1}, v_B = 4.1 \text{ m s}^{-1}$$

M1 A1

Answer: A: -0.4 , B: 4.1 (in m s^{-1})

6 marks

9 (4 marks)

$$\text{Momentum: } 2(5) + 2(1) = 2v_A + 2v_B, \text{ so } v_A + v_B = 6$$

M1 A1

$$\text{Restitution: } v_B - v_A = 1 \times (5 - 1) = 4$$

M1

$$v_A = 1 \text{ m s}^{-1}, v_B = 5 \text{ m s}^{-1}: \text{the spheres exchange velocities}$$

A1

Answer: A: 1, B: 5 (in m s^{-1})

4 marks

10 (4 marks)

$$\text{Momentum: } 2(6) = 2(-1) + 4v_B$$

M1

$$v_B = 3.5 \text{ m s}^{-1}$$

A1

$$e = \frac{3.5 + 1}{6}$$

M1

$$= 0.75$$

A1

Answer: 3.5 m s^{-1} , 0.75

4 marks

11 (6 marks)

$$\text{Momentum: } 2m(4) + m(1) = 2mv_A + mv_B, \text{ so } 2v_A + v_B = 9$$

M1 A1

$$\text{Restitution: } v_B - v_A = \frac{2}{3}(4 - 1) = 2$$

M1 A1

$$3v_A = 7 \text{ so } v_A = \frac{7}{3} \text{ m s}^{-1} \text{ and } v_B = \frac{13}{3} \text{ m s}^{-1}$$

M1 A1

Answer: $v_A = \frac{7}{3}$, $v_B = \frac{13}{3}$

6 marks

12 (4 marks)

$$\text{Rebound speed} = 0.6 \times 10 = 6 \text{ m s}^{-1}$$

M1 A1

$$I = 0.5(6 + 10)$$

M1

$$= 8 \text{ N s}$$

A1

Answer: 6 m s^{-1} , 8 N s

4 marks

13 (3 marks)

$$KE \propto \text{speed}^2, \text{ so } (e u)^2 = \frac{1}{4}u^2$$

M1 A1

$$e = \frac{1}{2}$$

A1

$$\text{Answer: } \frac{1}{2}$$

3 marks

14 (8 marks)

$$\text{Momentum: } 3(4) + 5(-2) = 3v_A + 5v_B, \text{ so } 3v_A + 5v_B = 2$$

M1 A1

$$\text{Restitution: } v_B - v_A = \frac{5}{6}(4 + 2) = 5$$

M1 A1

$$8v_A = 2 - 25 \text{ so } v_A = -\frac{23}{8} = -2.875 \text{ m s}^{-1}, v_B = \frac{17}{8} = 2.125 \text{ m s}^{-1}$$

M1 A1

$$\text{KE before} = 24 + 10 = 34 \text{ J; after} = \frac{1}{2}(3)(-\frac{23}{8})^2 + \frac{1}{2}(5)(\frac{17}{8})^2 = 23.69 \text{ J}$$

M1

$$\text{Loss} = 10.3 \text{ J}$$

A1

$$\text{Answer: } v_A = -\frac{23}{8}, v_B = \frac{17}{8}, \text{ loss } 10.3 \text{ J}$$

8 marks

15 (10 marks)

(a) Let the speed be u . Parallel to the wall the component is $u \cos \theta$ before and after

B1

Perpendicular to the wall the component is $u \sin \theta$ before and $e u \sin \theta$ after (reversed)

B1

Directions at right angles, so the dot product of the velocities is zero: $u^2 \cos^2 \theta - e u^2 \sin^2 \theta = 0$

M1 A1

$$\tan^2 \theta = \frac{1}{e}$$

A1

Answer: shown

5 marks

(b) $\tan \theta = \sqrt{2}$, so $\theta = 54.7^\circ$

M1 A1

$$\cos^2 \theta = \frac{1}{3}, \sin^2 \theta = \frac{2}{3}$$

M1

$$\text{Speed after} = u \sqrt{\cos^2 \theta + e^2 \sin^2 \theta} = u \sqrt{1/3 + 1/6} = u \sqrt{0.5} = 0.707u$$

M1 A1

Answer: $\theta = 54.7^\circ$, speed $0.707u$

5 marks

16 (7 marks)

$$\text{Momentum: } 2(5) + 3(-1) = 2v_A + 3v_B, \text{ so } 2v_A + 3v_B = 7$$

M1 A1

$$\text{Restitution: } v_B - v_A = 0.4(5 + 1) = 2.4$$

M1 A1

$$5v_B = 7 + 4.8 \text{ so } v_B = 2.36 \text{ m s}^{-1}$$

A1

$$\text{Impulse on B} = 3(2.36 - (-1)) = 10.08 \text{ N s}$$

M1 A1

Answer: 2.36 m s^{-1} , 10.08 N s

7 marks

17 (10 marks)

(a) Momentum: $2(6) = 2v_P + v_Q$

M1

$$\text{Restitution: } v_Q - v_P = 0.5(6) = 3$$

M1

$$3v_P + 3 = 12, \text{ so } v_P = 3 \text{ m s}^{-1} \text{ and } v_Q = 6 \text{ m s}^{-1}$$

A1 A1

Answer: $v_P = 3$, $v_Q = 6 \text{ (m s}^{-1}\text{)}$

4 marks

(b) Q reaches the wall after $\frac{2.4}{6} = 0.4 \text{ s}$, when P has moved $3(0.4) = 1.2 \text{ m}$

M1 A1

$$\text{Q rebounds at } 0.5 \times 6 = 3 \text{ m s}^{-1}, \text{ towards P, and the gap is } 2.4 - 1.2 = 1.2 \text{ m}$$

M1 A1

$$\text{The two now approach at } 3 + 3 = 6 \text{ m s}^{-1}, \text{ so they meet after } \frac{1.2}{6} = 0.2 \text{ s}$$

M1

$$\text{Total time} = 0.4 + 0.2 = 0.6 \text{ s after the first collision}$$

A1

Answer: 0.6 s

6 marks

18 (5 marks)

Momentum: $2(5) = 2(1) + 2m$

M1 A1

$m = 4$

A1

$e = \frac{2-1}{5} = 0.2$

M1 A1

Answer: $m = 4$, $e = 0.2$

5 marks

19 (5 marks)

Speed after wall A = $0.8u$

B1

Speed after wall B = $0.5(0.8u) = 0.4u$

M1 A1

KE fraction retained = $(0.4)^2 = 0.16$

M1

Fraction lost = 0.84

A1

Answer: $0.4u$, 0.84

5 marks

20 (6 marks)Take A's original direction as positive: A has velocity u , B has velocity $-u$ before, and B has velocity $+\frac{u}{2}$ after

B1

Momentum: $2mu - 3mu = 2mv_A + 3m(\frac{u}{2})$

M1 A1

$-u = 2v_A + 1.5u$ so $v_A = -1.25u$: A rebounds with speed $1.25u$

A1

$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{0.5u + 1.25u}{2u} = \frac{7}{8}$

M1 A1

Answer: A: $1.25u$ (reversed), $e = \frac{7}{8}$

6 marks

21 (14 marks)

(a) Momentum: $4u = v_A + 3v_B$; restitution: $v_B - v_A = \frac{1}{2}(4u) = 2u$

M1 A1

$4u = v_A + 3v_B$ and $v_A = v_B - 2u$, so $4v_B = 6u$

M1

$v_B = 1.5u$ (towards the wall), $v_A = -0.5u$

A1

Answer: $v_A = -0.5u$, $v_B = 1.5u$

4 marks

(b) Rebound speed of B = $\frac{1}{2}(1.5u) = 0.75u$ away from the wall

M1 A1

A moves away from the wall at $0.5u$ and B follows at $0.75u > 0.5u$, so B catches A

M1 E1

Answer: $0.75u$; B catches A

4 marks

(c) Take direction away from the wall as positive: A at $0.5u$, B at $0.75u$

B1

Momentum: $3m(0.75u) + m(0.5u) = 2.75mu = mv_A' + 3mv_B'$; restitution: $v_A' - v_B' = \frac{1}{2}(0.25u)$

M1 A1

$4v_B' = 2.75u - 0.125u$ so $v_B' = 0.6562u$, $v_A' = 0.7812u$

M1 A1

A now moves away from the wall faster than B, ahead of it, so no more collisions

E1

Answer: B: $0.656u$, A: $0.781u$ away from the wall

6 marks

22 (11 marks)(a) Resolve along and perpendicular to the plane. Along the plane the velocity component is $6 \sin \theta$ before and after (smooth plane)

B1

Perpendicular to the plane it is $6 \cos \theta$ towards the plane before and $\frac{4}{9}(6 \cos \theta)$ away from it after

B1

The ball moves horizontally after the impact, so the vertical component of its velocity is zero: $-6 \sin^2 \theta + \frac{4}{9}(6) \cos^2 \theta = 0$

M1 A1

$\tan^2 \theta = \frac{4}{9}$ so $\tan \theta = \frac{2}{3}$ and $\theta = 33.69^\circ$

A1

Answer: $\theta = 33.69^\circ$

5 marks

(b) Horizontal speed = $6 \sin \theta \cos \theta + \frac{4}{9}(6) \sin \theta \cos \theta = 6 \sin \theta \cos \theta (\frac{13}{9})$

M1

$\sin \theta \cos \theta = \frac{2 \times 3}{13} = \frac{6}{13}$, so speed = $6(\frac{6}{13})(\frac{13}{9}) = 4 \text{ m s}^{-1}$

M1 A1

Answer: 4 m s^{-1}

3 marks

(c) The impulse is perpendicular to the plane: $I = m(6\cos\theta)(1 + \frac{4}{9})$

M1 A1

$$= 0.5 \times 6 \times (3 \div \sqrt{13}) \times \frac{13}{9} = \sqrt{13} = 3.61 \text{ N s}$$

A1

Answer: $\sqrt{13} = 3.61 \text{ N s}$

3 marks

23 (9 marks)

(a) Momentum: $6 = v_A + 3v_B$; restitution: $v_B - v_A = 6e$

M1 A1

$$4v_A = 6 - 18e$$

M1

$$v_A = 1.5 - 4.5e$$

A1

Answer: shown

4 marks

(b) A reverses when $v_A < 0$: $1.5 - 4.5e < 0$

M1

$$e > \frac{1}{3}, \text{ so } \frac{1}{3} < e \leq 1$$

A1

Answer: $\frac{1}{3} < e \leq 1$

2 marks

(c) $v_B = v_A + 6e = 1.5 + 1.5e$

M1 A1

$$\text{For } 0 \leq e \leq 1, 1.5 \leq v_B \leq 3 \text{ m s}^{-1}$$

A1

Answer: $1.5 \leq v_B \leq 3$

3 marks

24 (9 marks)

(a) Momentum: $mu = mv_A + 3mv_B$

M1

Restitution: $v_B - v_A = eu$

M1

$$4v_B = u(1 + e) \text{ so } v_B = \frac{u(1 + e)}{4}, \text{ and } v_A = \frac{u(1 - 3e)}{4}$$

A1 A1

$$\textbf{Answer: } v_A = \frac{u(1 - 3e)}{4}, v_B = \frac{u(1 + e)}{4}$$

4 marks

(b) After the wall B has velocity $-\frac{1}{3} \times \frac{u(1 + e)}{4} = -\frac{u(1 + e)}{12}$

M1 A1

A and B collide again only if A is moving towards B faster than B moves away: $v_A > -\frac{u(1 + e)}{12}$

M1

$$3(1 - 3e) > -(1 + e)$$

M1

$$4 > 8e \text{ so } e < \frac{1}{2}$$

A1

Answer: shown

5 marks

25 (7 marks)

Component parallel to wall unchanged: $10 \cos 60^\circ = 5 \text{ m s}^{-1}$

M1 A1

Component perpendicular before = $10 \sin 60^\circ = 8.660$; after = $0.5 \times 8.660 = 4.330$

M1 A1

$$\text{Speed} = \sqrt{25 + 18.75} = 6.614 \text{ m s}^{-1}$$

M1 A1

$$\tan \theta = \frac{4.330}{5} \text{ so } \theta = 40.9^\circ$$

A1

Answer: 6.614 m s^{-1} , 40.9° to the wall

7 marks

26 (9 marks)

(a) Along the line of centres, A's component is $16\cos 60^\circ = 8 \text{ m s}^{-1}$; perpendicular to it, $16\sin 60^\circ$

B1

$$= 8\sqrt{3} \text{ m s}^{-1}$$

Momentum along the line: $8m = mv_A + mv_B$; restitution: $v_B - v_A = 0.5(8) = 4$

M1 A1

$v_B = 6 \text{ m s}^{-1}$ (along the line of centres), and $v_A = 2 \text{ m s}^{-1}$ along the line

A1

Answer: 6 m s^{-1}

4 marks

(b) The component of A perpendicular to the line of centres is unchanged: $8\sqrt{3} \text{ m s}^{-1}$

B1

$$\text{Speed of A} = \sqrt{2^2 + (8\sqrt{3})^2} = \sqrt{196} = 14 \text{ m s}^{-1}$$

M1 A1

$$\tan \beta = 8\sqrt{3} \div 2 = 4\sqrt{3}, \text{ so } \beta = 81.8^\circ \text{ to the line of centres}$$

M1 A1

Answer: 14 m s^{-1} at 81.8° to the line of centres

5 marks

27 (13 marks)**(a)** Momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$; restitution: $v_2 - v_1 = e(u_1 - u_2)$ M1 A1Substitute $v_1 = v_2 - e(u_1 - u_2)$: $(m_1 + m_2)v_2 = m_1u_1 + m_2u_2 + em_1(u_1 - u_2)$ M1 A1Divide by $(m_1 + m_2)$ A1**Answer: shown** 5 marks**(b)** Use the centre of mass: the relative KE before is $\frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$, since the KE of the centre of mass motion is unchanged M1 A1After the collision the relative speed is $e(u_1 - u_2)$, so relative KE is e^2 times as much M1 A1

$$\text{Loss} = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$$
A1

Answer: shown 5 marks

$$\text{(c) Loss} = \frac{4 \times 2}{2 \times 6}(1 - 0.25)(6)^2 = \frac{2}{3} \times 0.75 \times 36$$
M1 A1

= 18 J, which agrees with $51 \text{ J} - 33 \text{ J}$ A1**Answer: 18 J** 3 marks**28 (17 marks)****(a)** Speed on first impact $V = \sqrt{2gh}$, time $t_0 = \frac{V}{g}$ M1 A1After the n -th impact the speed is $e^n V$, so the time up and down is $\frac{2e^n V}{g} = 2e^n t_0$ M1 A1**Answer: shown** 4 marks**(b)** Total = $t_0 + 2t_0(e + e^2 + \dots)$ M1 A1

$$= t_0 + 2t_0 \frac{e}{1 - e} = t_0 \frac{1 - e + 2e}{1 - e} = t_0 \frac{1 + e}{1 - e}$$
M1 A1

Answer: shown 4 marks**(c)** Rebound height after the n -th impact = $e^{2n}h$ M1 A1

$$\text{Total} = h + 2h(e^2 + e^4 + \dots) = h + 2h \frac{e^2}{1 - e^2}$$
M1

$$= h \frac{1 + e^2}{1 - e^2}$$
A1

Answer: shown 4 marks**(d)** $t_0 = \sqrt{2(1.8)/9.8} = 0.6061 \text{ s}$ B1

$$\text{Time} = 0.6061 \times \frac{1.8}{0.2} = 5.45 \text{ s}$$
M1 A1

$$\text{Distance} = 1.8 \times \frac{1.64}{0.36} = 8.20 \text{ m}$$
M1 A1

Answer: 5.45 s, 8.20 m 5 marks**29 (13 marks)****(a)** Momentum: $3mu = 3mv_P + mv_Q$ M1Restitution: $v_Q - v_P = \frac{1}{2}u$ M1

$$4v_P = 3u - \frac{1}{2}u \text{ so } v_P = \frac{5u}{8}, \text{ and } v_Q = \frac{9u}{8}$$
A1 A1

Answer: $v_P = \frac{5u}{8}$, $v_Q = \frac{9u}{8}$ 4 marks**(b)** Q and R: $m(\frac{9u}{8}) = mw_Q + kmw_R$, and $w_R - w_Q = \frac{1}{2}(\frac{9u}{8})$ M1 A1

$$(1 + k)w_Q = \frac{9u}{8} - \frac{k}{2}(\frac{9u}{8}), \text{ so } w_Q = \frac{9u(2 - k)}{16(1 + k)}$$
M1 A1

P catches Q again only if $w_Q < \frac{5u}{8}$ M1

$$9(2 - k) < 10(1 + k) \text{ so } 8 < 19k, \text{ that is } k > \frac{8}{19}$$
A1

Answer: shown 6 marks

$$(c) w_Q = \frac{9u(2-1)}{16(2)} = \frac{9u}{32}$$

M1 A1

$$w_R = w_Q + \frac{1}{2}\left(\frac{9u}{8}\right) = \frac{9u}{32} + \frac{9u}{16} = \frac{27u}{32}$$

A1

$$\text{Answer: Q: } \frac{9u}{32}, \text{ R: } \frac{27u}{32}$$

3 marks

30 (17 marks)

$$(a) \text{ Momentum: } 12 = 2v_A + v_B; \text{ restitution: } v_B - v_A = 3$$

M1 A1

$$3v_A = 9, v_A = 3 \text{ m s}^{-1}, v_B = 6 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: A: 3, B: 6 m s}^{-1}$$

4 marks

$$(b) \text{ Speed after wall} = 0.4 \times 6 = 2.4 \text{ m s}^{-1} \text{ away from the wall}$$

M1 A1

A moves towards the wall at 3 m s^{-1} ; B moves away at 2.4 m s^{-1} , so they approach

M1 E1

$$\text{Answer: } 2.4 \text{ m s}^{-1}; \text{ A and B approach}$$

4 marks

$$(c) \text{ Before: A at } +3, \text{ B at } -2.4$$

B1

$$\text{Momentum: } 2(3) + 1(-2.4) = 3.6 = 2v_A + v_B; \text{ restitution: } v_B - v_A = 0.5(5.4) = 2.7$$

M1 A1

$$3v_A = 0.9; v_A = 0.3 \text{ m s}^{-1}, v_B = 3 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: A: 0.3, B: 3 (towards wall)}$$

5 marks

$$(d) \text{ B rebounds at } 0.4 \times 3 = 1.2 \text{ m s}^{-1} \text{ away from the wall}$$

M1 A1

A moves towards the wall at 0.3 m s^{-1} ; they approach with relative speed 1.5 m s^{-1} , so a third collision occurs

M1 E1

$$\text{Answer: relative approach speed } 1.5 \text{ m s}^{-1}$$

4 marks