

Linear Momentum

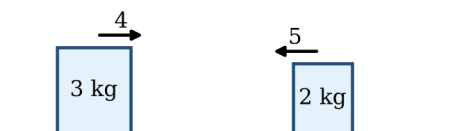
1 Find the momentum of a ball of mass 0.4 kg moving at 15 m s^{-1} . [2]

2 A ball of mass 0.3 kg moves to the left at 5 m s^{-1} . Taking right as the positive direction, find its momentum. [2]

3 A bullet of mass 0.05 kg travels at 300 m s^{-1} and a runner of mass 70 kg runs at 8 m s^{-1} . Find the momentum of each and say which is greater. [2]

4 A particle of mass 150 g has momentum 18 kg m s^{-1} . Find its speed. [2]

5 Two particles move along a straight line as shown (speeds in m s^{-1}). Taking right as the positive direction, find the total momentum of the two particles.



[3]

6 A ball of mass 0.5 kg speeds up from 6 m s^{-1} to 10 m s^{-1} in the same direction. Find the change in its momentum. [2]

7 A ball of mass 0.2 kg hits a wall at 12 m s^{-1} and rebounds along the same line at 8 m s^{-1} . Find the magnitude of the change in momentum. [3]

8 A particle of mass 2 kg has velocity $(3\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$. Find its momentum vector and its magnitude. [3]

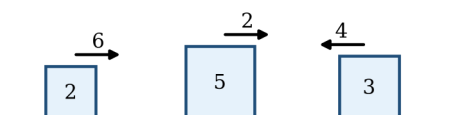
9 A particle of mass 0.5 kg has momentum $(6\mathbf{i} + 8\mathbf{j}) \text{ kg m s}^{-1}$. Find its speed and the angle its direction makes with $\mathbf{i}$. [4]

10 A stone of mass 3 kg is dropped from rest from a height of 20 m. Find its momentum just before it hits the ground (use $g = 9.8 \text{ m s}^{-2}$). [4]

11 Particle A has mass 4 kg and speed 6 m s^{-1} . Particle B has mass 1.5 kg and the same momentum as A. Find the speed of B. [2]

12 A constant force of 12 N acts on a particle of mass 2 kg, initially at rest, for 5 seconds. Find the momentum gained and the final speed. [3]

13 Three particles of masses 2 kg, 5 kg and 3 kg move along a line with the speeds (in m s^{-1}) and directions shown. Find the total momentum, taking right as positive.



[3]

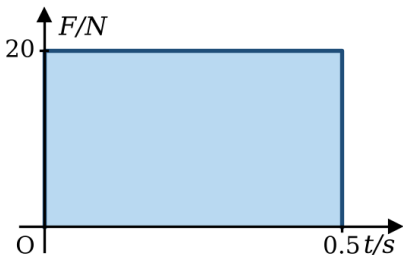
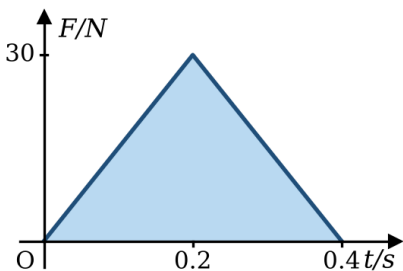
14 A particle of mass 2 kg has kinetic energy 25 J. By using $E = \frac{1}{2}mv^2$ and $p = mv$ to show that $p^2 = 2mE$, find the momentum of the particle. [4]

- 15** A particle of mass 2 kg moves so that its velocity at time t seconds is $(3t^2\mathbf{i} + 4t\mathbf{j}) \text{ m s}^{-1}$.
(a) Find the momentum at $t = 2$. [2]
(b) Find the magnitude of the force acting at $t = 2$. [3]
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- 16** Two particles, A of mass $4m$ and B of mass m , have the same kinetic energy. Find the ratio of their momenta, $p_A : p_B$. [4]
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- 17** A ball of mass 0.4 kg is hit so that its velocity changes from $8\mathbf{i} \text{ m s}^{-1}$ to $(-3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Find the change in momentum as a vector, and its magnitude. [5]
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- 18** A car of mass 900 kg travels at 30 m s^{-1} and a lorry of mass 3000 kg travels at 9 m s^{-1} .
(a) Which has the greater momentum, and by how much? [3]
(b) Find the ratio of the kinetic energy of the car to that of the lorry. [3]
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- 19** A ball of mass 0.25 kg is dropped from a height of 0.9 m onto a floor and rebounds to a height of 0.4 m. Taking $g = 9.8 \text{ m s}^{-2}$, find the magnitude of the change in its momentum in the impact. [6]
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- 20** A lorry of mass 8 tonnes travels at 72 km h^{-1} . Find its momentum in kg m s^{-1} . [4]
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- 21** A particle of mass 0.5 kg has velocity $(4t\mathbf{i} + (t^2 - 4)\mathbf{j}) \text{ m s}^{-1}$ at time t seconds, for $t \geq 0$.
(a) Find the time at which the momentum is in the direction of $\mathbf{i}$. [2]
(b) Find the time at which the speed of the particle is least, and the least momentum. [5]
(c) Find the magnitude of the force on the particle at $t = 2$. [3]
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- 22** Three particles of masses 3 kg, 4 kg and m kg move along a straight line with speeds 5, 2 and 6 m s^{-1} in the directions shown.
The total momentum of the system is zero. Find m , and find the total kinetic energy of the three particles.
- The diagram shows three rectangular blocks representing particles on a horizontal line. The first block is labeled '3' and has an arrow above it pointing right with the number '5'. The second block is labeled '4' and has an arrow above it pointing right with the number '2'. The third block is labeled 'm' and has an arrow above it pointing left with the number '6'.
- [5]
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- 23** A particle of mass 4 kg moves in a straight line, always in the same direction. Its kinetic energy increases from 50 J to 200 J.
(a) Find the momentum of the particle before and after the increase. [3]
(b) Find the change in momentum, and show that the speed has doubled. [3]
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- 24** A water jet of 0.8 kg per second hits a vertical wall at 15 m s^{-1} . Find the force exerted on the wall (a) if the water stops dead, (b) if the water rebounds at 5 m s^{-1} . [4]
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- 25** A machine gun fires 10 bullets per second, each of mass 0.02 kg, at a muzzle speed of 400 m s^{-1} . Find the average force needed to hold the gun in place, ignoring the momentum of the gases. [4]
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- 26** The speed of a particle of constant mass increases so that its momentum increases by 20%. Find the percentage increase in its kinetic energy. [4]
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- 27** A particle of mass m and speed v has momentum p and kinetic energy E .
- Show that $E = \frac{p^2}{2m}$. [2]
 - Two particles have the same momentum, with masses 2 kg and 8 kg. Find the ratio of their kinetic energies and say which has more. [4]
 - A particle's momentum is doubled and its mass is halved. Find the factor by which its speed changes and by which its KE changes. [4]
- 28** The graph shows the momentum p (in kg m s^{-1}) of a particle of mass 2 kg moving in a straight line against time t (in seconds). The graph is made of straight line segments.
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- Find the force acting on the particle in each of the three stages. [3]
 - Find the velocity at $t = 4$ and at $t = 10$. [3]
 - Find the displacement of the particle from $t = 0$ to $t = 7$, and find the time at which the particle is instantaneously at rest. [5]
- 29** Particle A of mass 2 kg has velocity $(4\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ and particle B of mass 3 kg has velocity $(-2\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$.
- Find the total momentum of A and B, and its magnitude. [4]
 - Find the velocity of a single particle of mass 5 kg with the same total momentum. [2]
 - A third particle C of mass 0.5 kg is added so that the total momentum of the three particles is zero. Find the velocity of C. [2]
 - Find the speed of C and the angle its velocity makes with the negative $\mathbf{i}$ direction. [4]
- 30** Sand falls onto a horizontal conveyor belt at a constant rate of 5 kg per second. The belt moves at a constant speed of 2 m s^{-1} and the sand is carried at the speed of the belt.
- Find the force, F , needed to keep the belt at constant speed, and the power developed by this force. [4]
 - Find the rate at which the kinetic energy of the sand is increasing. [3]
 - Explain the difference between your answers to (a) and (b). [2]

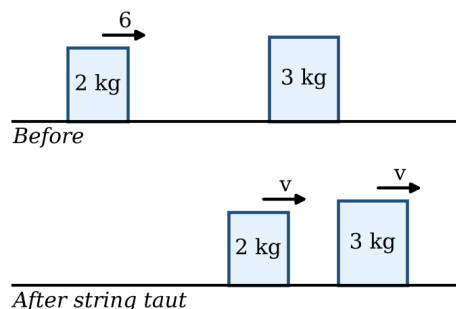
Impulse

- A constant force of 8 N acts on a particle for 3 seconds. Find the magnitude of the impulse of the force. [2]
- A ball of mass 0.5 kg is struck so that its speed changes from 4 m s^{-1} to 10 m s^{-1} in the same direction. Find the impulse. [2]
- A force of 15 N produces an impulse of 45 N s. Find the time for which the force acts. [2]
- An impulse of 12 N s gives a particle, initially at rest, a speed of 6 m s^{-1} . Find the mass of the particle. [2]
- A cricket ball of mass 0.16 kg moving at 25 m s^{-1} is caught and brought to rest. Find the magnitude of the impulse on the ball. [2]

- 6** A particle of mass 2 kg is moving at 3 m s^{-1} when it receives an impulse of 10 N s in the direction of motion. Find its new speed. [2]
- 7** A ball of mass 0.3 kg hits a wall at 10 m s^{-1} and rebounds directly at 6 m s^{-1} . Find the magnitude of the impulse exerted by the wall on the ball. [3]
- 8** A particle of mass 2 kg has velocity $(3\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$ when it receives an impulse of $(4\mathbf{i} - 6\mathbf{j}) \text{ N s}$. Find its new velocity. [3]
- 9** A footballer kicks a stationary ball of mass 0.45 kg , which leaves his boot at 22 m s^{-1} . The contact lasts 0.01 s . Find the average force exerted on the ball. [4]
- 10** A hammer of mass 0.8 kg strikes a nail at 5 m s^{-1} and is brought to rest in 0.004 s . Find the average force exerted by the nail on the hammer. [4]
- 11** The graph shows the force acting on a particle of mass 2.5 kg , initially at rest, over 0.5 seconds. Find the impulse of the force and the speed of the particle at the end. [5]
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- The graph shows a constant force of 20 N acting on a particle for a duration of 0.5 s . The area under the graph is a rectangle with a height of 20 and a width of 0.5 .
- 12** A ball of mass 0.2 kg is moving with velocity $12\mathbf{i} \text{ m s}^{-1}$ when it is struck and receives an impulse of $(-4\mathbf{i} + 3\mathbf{j}) \text{ N s}$. [3]
(a) Find the velocity of the ball after the impact and its speed. [3]
(b) Find the gain in kinetic energy of the ball. [3]
- 13** A particle of mass 1.5 kg moving with velocity $(4\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ receives an impulse of $(p\mathbf{i} - 9\mathbf{j}) \text{ N s}$, where p is a constant. Immediately afterwards its velocity is $(10\mathbf{i} + q\mathbf{j}) \text{ m s}^{-1}$. Find the values of p and q . [4]
- 14** A force of $(6t + 2) \text{ N}$ acts on a particle of mass 3 kg for $0 \leq t \leq 3$, where t is in seconds. The particle has speed 1 m s^{-1} at $t = 0$ in the direction of the force. Find the impulse of the force and the speed at $t = 3$. [5]
- 15** A bat strikes a stationary ball of mass 0.15 kg . The force exerted by the bat varies with time as shown in the graph. [3]
(a) Find the impulse on the ball. [3]
(b) Find the speed with which the ball leaves the bat and the average force during contact. [4]
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- The graph shows a triangular force profile. The force starts at 0 N at $t = 0$, reaches a maximum of 30 N at $t = 0.2 \text{ s}$, and returns to 0 N at $t = 0.4 \text{ s}$. The area under the graph is a triangle with a base of 0.4 and a height of 30 .

- 16** A ball of mass 0.4 kg is moving with velocity $5\mathbf{j} \text{ m s}^{-1}$ when it is struck and receives an impulse of $(-3\mathbf{i} + 4\mathbf{j}) \text{ N s}$. Find the speed of the ball after the impact and the angle its direction makes with $\mathbf{j}$. [6]

- 17** Particles A (2 kg) and B (3 kg) lie on a smooth horizontal table joined by a light inextensible string, initially slack. A is projected directly away from B at 6 m s^{-1} while B is at rest. When the string becomes taut the particles move on with a common speed v . Find v and the magnitude of the impulse in the string. [5]



- 18** A particle of mass 2 kg moving with velocity $(3\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$ receives an impulse of $(k\mathbf{i} + 2k\mathbf{j}) \text{ N s}$, where k is a constant. Immediately afterwards the speed of the particle is $\sqrt{65} \text{ m s}^{-1}$. Determine the possible values of k . [5]

- 19** A particle P of mass 0.4 kg is moving with velocity $(2\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$ when it receives an impulse $\mathbf{I} \text{ N s}$. Immediately afterwards P has velocity $(8\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$. [3]
 (a) Find $\mathbf{I}$ and its magnitude. [3]
 (b) Find the angle between the direction of $\mathbf{I}$ and the original direction of motion of P. [3]

- 20** A particle of mass 0.5 kg moves in a straight line on a smooth horizontal surface under the action of a single force of magnitude $(6t - 8) \text{ N}$ in the direction of motion, where t is the time in seconds. [6]
 When $t = 1$ the speed of the particle is 5 m s^{-1} . Find the value of t at which the speed is 21 m s^{-1} .

- 21** A force of $20e^{-2t} \text{ N}$ acts on a particle of mass 4 kg , initially at rest, from $t = 0$. Find the speed at time t , and show that the speed approaches a limit as t becomes large. State the limit. [6]

- 22** A smooth ball of mass 0.5 kg moving at 12 m s^{-1} strikes a smooth vertical wall at an angle of 30° to the wall and rebounds at the same speed, the angle with the wall again 30° . Find the magnitude and direction of the impulse exerted by the wall on the ball. [6]

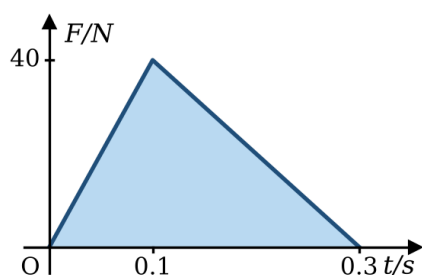
- 23** A ball of mass 0.2 kg is moving horizontally with speed 20 m s^{-1} when it is struck by a bat. Immediately afterwards it has speed 30 m s^{-1} and its path has been deflected through 60° from its original direction. The ball is modelled as a particle. [5]
 (a) Find the magnitude of the impulse exerted by the bat on the ball. [3]
 (b) Find the angle between the impulse and the original direction of motion of the ball. [3]

- 24** A particle A of mass 2 kg is attached by a light inextensible string, passing over a smooth pulley at the edge of a smooth horizontal table, to a particle B of mass 3 kg on the table. The string is slack when A is released from rest at the edge, and falls vertically. The string becomes taut after A has fallen 0.9 m. Find the speed of A just before the string becomes taut, and the speed of both particles immediately afterwards. Use $g = 9.8 \text{ m s}^{-2}$. [6]

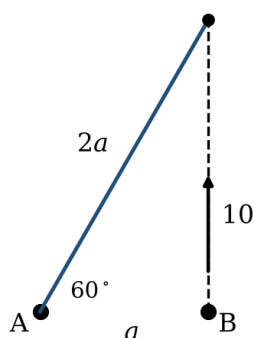
- 25** A particle of mass 2 kg has velocity $\mathbf{i} \text{ m s}^{-1}$ at time $t = 0$ and is acted on by a force $(6t\mathbf{i} - 4\mathbf{j})$ N for $0 \leq t \leq 2$. Find the impulse of the force and the velocity at $t = 2$. [5]

- 26** A ball of mass 0.5 kg hits a wall at 6 m s^{-1} and rebounds at 4 m s^{-1} . A student writes: "Impulse = $m(v - u) = 0.5(4 - 6) = -1$, so the impulse is 1 N s." [2]
 (a) Explain the error in the student's working. [2]
 (b) Find the correct impulse. [2]

- 27** A stationary ball of mass 0.2 kg is struck by a racket. The force on the ball increases uniformly from zero to 40 N in 0.1 s, then decreases uniformly to zero over a further 0.2 s, as shown.



- (a) Find the total impulse and the speed with which the ball leaves the racket. [4]
 (b) Find the speed of the ball at $t = 0.1$ s. [3]
 (c) Find the time at which the ball's speed is 20 m s^{-1} . [5]
 (d) State the average force over the contact time. [1]
- 28** A force $F = (12t - 3t^2)$ N acts on a particle of mass 1.5 kg for $0 \leq t \leq 4$. At $t = 0$ the particle has speed 2 m s^{-1} in the direction of the force. [5]
 (a) Find the impulse of the force over the 4 seconds and the final speed. [5]
 (b) Find the greatest value of the force. [3]
 (c) Show that the speed is 10 m s^{-1} when $t^3 - 6t^2 + 12 = 0$, and find the first time this happens. [5]
- 29** Two particles A and B, of masses 3 kg and 2 kg, are joined by a light inextensible string of length $2a$. They lie at rest on a smooth horizontal table with $AB = a$ and the string slack. B is projected with speed 10 m s^{-1} in a direction perpendicular to AB, away from the line AB.

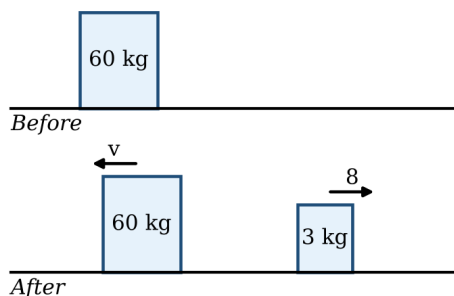


- (a) Show that the string becomes taut when B has travelled a distance $\sqrt{3}a$, and that the string then makes an angle of 60° with AB. [5]
 (b) Find the magnitude of the impulsive tension in the string when it becomes taut. [5]
 (c) Find the kinetic energy lost when the string becomes taut. [5]

- 30** A force $F(t)$ acts on a particle of constant mass m along a straight line, changing its velocity from u at $t = 0$ to v at $t = T$.
- (a) By writing $F = m \frac{dv}{dt}$, prove that $\int_0^T F dt = mv - mu$. [4]
- (b) A particle of mass 3 kg starts from rest and is acted on by a force $F = 0.6t$ N. Show that its speed after t seconds is $0.1t^2 \text{ m s}^{-1}$. [3]
- (c) Find the time for the speed to reach 10 m s^{-1} and the distance travelled in that time. [4]

Conservation of Momentum

- 1** A particle of mass 3 kg moving at 4 m s^{-1} collides with a particle of mass 1 kg at rest, and they coalesce. Find their common speed. [2]
- 2** A gun of mass 2 kg fires a bullet of mass 0.01 kg at 400 m s^{-1} . Find the recoil speed of the gun, which is initially at rest. [2]
- 3** Particles of masses 2 kg and 3 kg move towards each other at 5 m s^{-1} each and coalesce on impact. Find the velocity of the combined particle, in the direction of the 2 kg particle's original motion. [2]
- 4** A particle of mass 4 kg moving at 6 m s^{-1} collides with a stationary particle of mass m kg and they coalesce, moving on at 4 m s^{-1} . Find m . [2]
- 5** A skater of mass 60 kg stands at rest on smooth ice and throws a ball of mass 3 kg horizontally at 8 m s^{-1} . Find the speed at which the skater recoils.

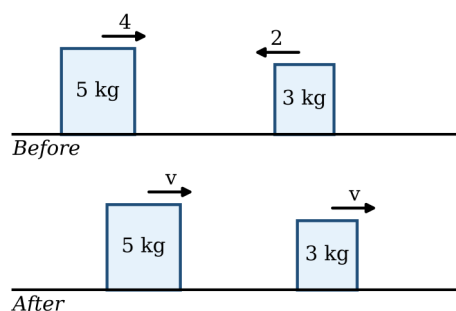


- [2]
- 6** A bullet of mass 0.02 kg travelling at 300 m s^{-1} embeds itself in a stationary block of mass 2.98 kg on a smooth surface. Find the speed of the block immediately afterwards. [2]
- 7** A child of mass m kg stands on a skateboard of mass 5 kg, initially at rest on smooth horizontal ground. The child jumps off horizontally and lands with horizontal speed 1.5 m s^{-1} relative to the ground, while the skateboard moves backwards at 6 m s^{-1} . Find m . [3]
- 8** A particle A of mass 2 kg moving at 6 m s^{-1} collides directly with a particle B of mass 3 kg moving at 1 m s^{-1} in the same direction. After the collision A moves at 1.5 m s^{-1} in its original direction. Find the speed of B after the collision. [4]
- 9** A shell of mass 5 kg, at rest, explodes into two parts of masses 2 kg and 3 kg. The 2 kg part moves off at 9 m s^{-1} . Find the velocity of the other part. [3]

- 10** A man of mass 60 kg jumps horizontally from a stationary boat of mass 120 kg with horizontal speed 3 m s^{-1} relative to the water. Find the speed with which the boat moves away, ignoring water resistance. [3]

- 11** A particle A of mass 2 kg moving at 5 m s^{-1} collides directly with a stationary particle B of mass 3 kg. The impulse that A exerts on B in the collision has magnitude 6 N s. Find the speeds of A and B immediately after the collision, and the kinetic energy lost. [6]

- 12** Two trucks of masses 5 kg and 3 kg collide as shown (speeds in m s^{-1}) and move on together with speed v . Find v and its direction.

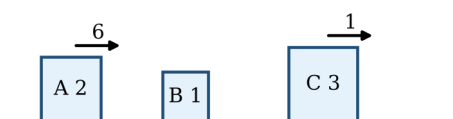


[3]

- 13** A cannon of mass 500 kg stands on smooth horizontal ground and fires a shell of mass 5 kg horizontally at 100 m s^{-1} relative to the ground. Find the recoil speed of the cannon and the total kinetic energy given to the system. [4]

- 14** A particle A of mass 4 kg moving at 9 m s^{-1} collides directly with a stationary particle B of mass 2 kg. After the collision both particles move in the original direction of A, and the speed of B is twice the speed of A. Find the speeds of A and B after the collision, and the kinetic energy lost. [6]

- 15** Three particles A, B and C of masses 2 kg, 1 kg and 3 kg lie on a smooth horizontal line. B is at rest, and A and C move in the same direction at 6 m s^{-1} and 1 m s^{-1} . A collides with B and they coalesce. The combined particle then collides with C and they all coalesce. Find the final speed.



[5]

- 16** A bullet of mass 0.01 kg is fired horizontally into a block of mass 1.99 kg hanging at rest on light inextensible strings. The bullet embeds in the block and the block rises 0.2 m. Find the speed of the bullet. Use $g = 9.8 \text{ m s}^{-2}$. [5]

- 17** A bullet of mass 0.05 kg travelling at 200 m s^{-1} embeds itself in a block of mass 4.95 kg at rest on a rough horizontal table. The coefficient of friction between the block and the table is 0.4. Find the speed of the block immediately after impact and the distance it slides before stopping. Use $g = 9.8 \text{ m s}^{-2}$. [6]

- 18** A particle A of mass 3 kg moving at 4 m s^{-1} collides with a stationary particle B of mass m kg. After the collision A moves at 1 m s^{-1} in the same direction and B moves at 3 m s^{-1} . Find m and the kinetic energy lost in the collision. [5]

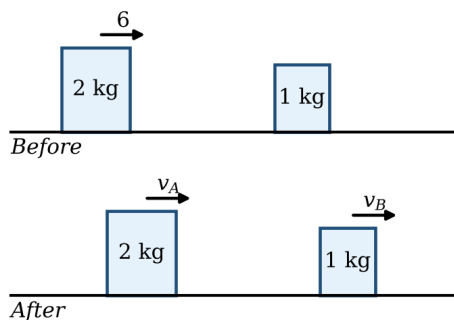
- 19** A shell of mass 6 kg, at rest, explodes into three parts. Part A has mass 1 kg and velocity $6\mathbf{i}$ m s⁻¹, part B has mass 2 kg and velocity $(-2\mathbf{i} + 3\mathbf{j})$ m s⁻¹, and part C has mass 3 kg. Find the velocity of C and its speed. [5]
- 20** Two skaters of masses 50 kg and 70 kg stand at rest on smooth ice and push each other apart. Immediately afterwards they are moving apart with a relative speed of 6 m s⁻¹. Find the speed of each skater. [5]
- 21** A bullet of mass 0.02 kg travelling horizontally at 400 m s⁻¹ passes through a block of mass 2 kg, at rest on a smooth horizontal surface, and emerges at 100 m s⁻¹. Find the speed of the block after the bullet has passed through, and the kinetic energy lost. [6]
- 22** A particle A of mass 3 kg moving at 8 m s⁻¹ collides directly with a stationary particle B of mass 5 kg. After the collision A has velocity a and B has velocity b (in the original direction of A), and 50.4 J of kinetic energy is lost.
- (a) Show that $3a + 5b = 24$ and that $20b^2 - 120b + 151.2 = 0$. [5]
- (b) Hence find a and b , explaining why one root must be rejected. [5]
- 23** A man, his trolley and two parcels of mass 4 kg each have a total mass of 88 kg and are at rest on a smooth horizontal track. The man throws the two parcels, one after the other, both horizontally backwards with speed 5 m s⁻¹ relative to the ground.
- (a) Find the speed of the trolley and man after the first parcel is thrown. [3]
- (b) Find the final speed after the second parcel is thrown. [3]
- 24** A truck of mass 500 kg moves at 4 m s⁻¹ on a smooth horizontal track. 100 kg of sand is dropped vertically onto it and stays at rest relative to the truck. Find the new speed of the truck and the percentage of the kinetic energy that is lost. [5]
- 25** Particle A of mass 2 kg moves with velocity $3\mathbf{i}$ m s⁻¹ and particle B of mass 3 kg moves with velocity $2\mathbf{j}$ m s⁻¹. They collide and coalesce. Find the velocity of the combined particle, its speed, and the loss of kinetic energy. [6]
- 26** Two particles A and B, of masses 2 kg and 3 kg, are in contact and at rest on a rough horizontal surface. An internal explosion drives them apart along a straight line. A moves off with speed 6 m s⁻¹. The coefficient of friction between each particle and the surface is 0.5. Use $g = 9.8$ m s⁻².
- (a) Find the initial speed of B. [3]
- (b) Find the distance between A and B when both have come to rest. [6]
- 27** A particle of mass m_1 moving at speed u_1 collides with a particle of mass m_2 moving at u_2 in the same direction ($u_1 > u_2$) and they coalesce.
- (a) Show that the common velocity is $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$. [3]
- (b) Show that the kinetic energy lost is $\frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$. [5]
- (c) Use this to find the kinetic energy lost when particles of mass 4 kg and 6 kg moving at 9 m s⁻¹ and 4 m s⁻¹ coalesce. [3]
- (d) Explain why no kinetic energy is lost when $u_1 = u_2$. [1]

- 28** Particle A has mass m and speed u and collides with a particle B of mass m at rest; they coalesce and then strike a particle C of mass M at rest, coalescing with it. The motion is along a straight line on a smooth horizontal plane.
- Find the speed of AB after the first collision and the final speed in terms of m , M and u . [4]
 - The final speed is $\frac{u}{5}$. Show that $M = 3m$. [3]
 - Find the fraction of the original kinetic energy remaining at the end. [3]
 - Find the fraction of the original kinetic energy lost in each of the two collisions. [4]
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- 29** A pile driver consists of a hammer of mass 500 kg that falls freely from rest through 2.5 m onto the top of a pile of mass 1500 kg, which stands vertically in the ground. The hammer and pile move on together after the impact. Use $g = 9.8 \text{ m s}^{-2}$.
- Find the speed of the hammer just before impact. [2]
 - Find the speed of the hammer and pile immediately after impact. [3]
 - The pile moves 0.1 m into the ground before coming to rest. Find the constant resistance of the ground. [5]
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- 30** A rocket of mass 1000 kg is travelling at 400 m s^{-1} in a straight line. An explosive charge separates a stage of mass 200 kg, which moves backwards relative to the remaining 800 kg at 150 m s^{-1} .
- Find the speeds of the rocket and the stage after separation. [5]
 - Find the impulse on the stage and the kinetic energy gained. [4]
 - Explain why the kinetic energy increases though momentum is conserved. [1]

Collisions and Restitution

- Two spheres approach each other with a relative speed of 6 m s^{-1} and separate with a relative speed of 3 m s^{-1} . Find the coefficient of restitution. [2]
- A ball hits a horizontal floor at 6 m s^{-1} and rebounds at 4.2 m s^{-1} . Find the coefficient of restitution between the ball and the floor. [2]
- A sphere hits a smooth fixed wall at right angles at 8 m s^{-1} . The coefficient of restitution is 0.5. Find the speed of rebound. [2]
- Two spheres collide directly with a speed of approach of 10 m s^{-1} . The coefficient of restitution is 0.4. Find the speed of separation. [2]
- A sphere of mass 2 kg hits a wall at 5 m s^{-1} and rebounds at 3 m s^{-1} . Find the coefficient of restitution and the kinetic energy lost. [3]
- A ball is dropped from rest from a height of 4.9 m onto a horizontal floor, with coefficient of restitution 0.5. Find the speed with which it leaves the floor. Use $g = 9.8 \text{ m s}^{-2}$. [3]

- 7 A sphere A of mass 2 kg moving at 6 m s^{-1} collides directly with a stationary sphere B of mass 1 kg. The coefficient of restitution is 0.5. Find the speeds of A and B after the collision.



[5]

- 8 A sphere A of mass 3 kg moving at 5 m s^{-1} collides directly with a sphere B of mass 2 kg moving towards it at 4 m s^{-1} . The coefficient of restitution is 0.5. Find the velocities of A and B after the collision (take the initial direction of A as positive).

[6]

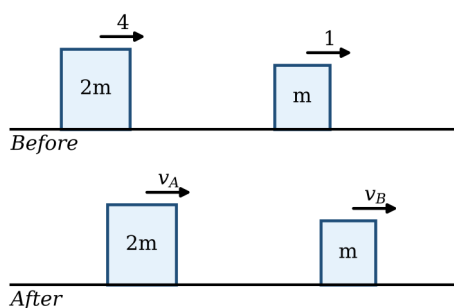
- 9 Two identical smooth spheres of mass 2 kg collide directly. Sphere A moves at 5 m s^{-1} and sphere B at 1 m s^{-1} in the same direction. The collision is perfectly elastic ($e = 1$). Find the velocities after the collision.

[4]

- 10 A sphere A of mass 2 kg moving at 6 m s^{-1} collides directly with a sphere B of mass 4 kg at rest. After the collision A moves at 1 m s^{-1} in the opposite direction. Find the speed of B after the collision and the coefficient of restitution.

[4]

- 11 Two spheres of masses $2m$ and m collide as shown (speeds in m s^{-1}), with coefficient of restitution $\frac{2}{3}$. Find the speeds v_A and v_B after the collision.



[6]

- 12 A ball of mass 0.5 kg hits a smooth vertical wall at 10 m s^{-1} . The coefficient of restitution is 0.6. Find the speed of rebound and the magnitude of the impulse on the ball.

[4]

- 13 A sphere hits a smooth wall at right angles and rebounds with one quarter of its kinetic energy. Find the coefficient of restitution.

[3]

- 14 A smooth sphere A of mass 3 kg moving at 4 m s^{-1} collides directly with a smooth sphere B of mass 5 kg moving in the opposite direction at 2 m s^{-1} . The coefficient of restitution is $\frac{5}{6}$.

(a) Find the velocities of A and B after the collision.

(b) Find the kinetic energy lost.

[8]

- 15** A smooth ball moving on a smooth horizontal floor strikes a smooth vertical wall with its path at an angle θ to the wall, where $0^\circ < \theta < 90^\circ$. The coefficient of restitution between the ball and the wall is e . After the impact the direction of motion of the ball has been turned through exactly 90° .

(a) Show that $\tan^2 \theta = \frac{1}{e}$. [5]

(b) For $e = \frac{1}{2}$, find θ to the nearest 0.1° , and find the speed of the ball after the impact in terms of u . [5]

- 16** A smooth sphere A of mass 2 kg moving at 5 m s^{-1} collides directly with a smooth sphere B of mass 3 kg moving towards A at 1 m s^{-1} . The coefficient of restitution is 0.4. Find the speed of B after the collision and the magnitude of the impulse A exerts on B. [7]

- 17** A smooth sphere P of mass 2 kg moving at 6 m s^{-1} on a smooth horizontal surface collides directly with a stationary smooth sphere Q of mass 1 kg. The coefficient of restitution between P and Q is 0.5. Q then moves towards a smooth vertical wall, which is perpendicular to the line of motion and 2.4 m from the point of the collision, and rebounds. The coefficient of restitution between Q and the wall is 0.5.

(a) Find the speeds of P and Q immediately after their collision. [4]

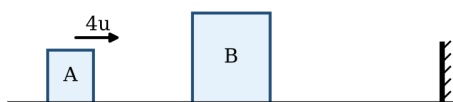
(b) Show that P and Q collide again, and find the time after the first collision at which this happens. [6]

- 18** A particle of mass 2 kg moving at 5 m s^{-1} collides directly with a stationary particle of mass m kg. After the collision the particles move in the same direction at 1 m s^{-1} and 2 m s^{-1} respectively. Find m and the coefficient of restitution. [5]

- 19** A particle moves along the line joining two parallel smooth vertical walls, A and B. It hits wall A at speed u , rebounds, hits wall B and rebounds again. The coefficients of restitution for the walls A and B are 0.8 and 0.5 respectively. Find the speed after the second rebound in terms of u , and the fraction of the original kinetic energy which is lost. [5]

- 20** A smooth sphere A of mass $2m$ and a smooth sphere B of mass $3m$ are moving towards each other on a smooth horizontal plane, each with speed u , and collide directly. After the collision the direction of motion of B is reversed and its speed is halved. Find the speed of A after the collision, and the coefficient of restitution between the spheres. [6]

- 21** A smooth sphere A of mass m moving at $4u$ hits a stationary smooth sphere B of mass $3m$, which lies between A and a smooth vertical wall, with line of centres perpendicular to the wall. The coefficient of restitution between A and B is $\frac{1}{2}$ and between B and the wall is $\frac{1}{2}$.

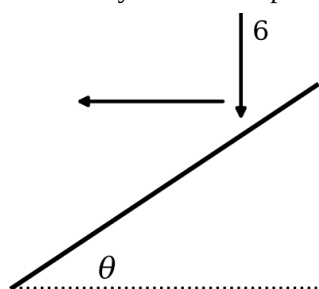


(a) Find the velocities of A and B after their first collision. [4]

(b) B hits the wall and rebounds. Find its speed and show that B and A collide again. [4]

(c) Find the velocities after the second collision, and show that there are no further collisions. [6]

- 22** A smooth ball of mass 0.5 kg falls vertically and strikes a smooth plane, inclined at an angle θ to the horizontal, with speed 6 m s^{-1} . The coefficient of restitution between the ball and the plane is $\frac{4}{9}$. Immediately after the impact the ball moves horizontally.



- (a) Show that $\tan^2 \theta = \frac{4}{9}$, and hence find θ . [5]
- (b) Find the speed of the ball after the impact. [3]
- (c) Find the magnitude of the impulse exerted by the plane on the ball. [3]
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- 23** A sphere A of mass 1 kg moving at 6 m s^{-1} collides directly with a stationary sphere B of mass 3 kg . The coefficient of restitution is e , with $0 \leq e \leq 1$.
- (a) Show that the velocity of A after the collision is $(1.5 - 4.5e) \text{ m s}^{-1}$. [4]
- (b) Find the range of values of e for which A reverses its direction. [2]
- (c) Find the range of the speed of B after the collision. [3]
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- 24** A smooth sphere A of mass m moving with speed u on a smooth horizontal plane collides directly with a stationary smooth sphere B of mass $3m$. The coefficient of restitution between A and B is e . After the collision B strikes a smooth vertical wall, perpendicular to its direction of motion, and rebounds. The coefficient of restitution between B and the wall is $\frac{1}{3}$.
- (a) Find, in terms of e and u , the velocities of A and B after their collision, taking the direction towards the wall as positive. [4]
- (b) Given that A and B collide again, show that $e < \frac{1}{2}$. [5]
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- 25** A smooth ball moving at 10 m s^{-1} strikes a smooth fixed vertical wall. Its direction of motion makes an angle of 60° with the wall. The coefficient of restitution is 0.5 . Find the speed of the ball after the impact and the angle its direction then makes with the wall. [7]
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- 26** Two identical smooth spheres A and B, each of mass m , lie on a smooth horizontal table. B is at rest. A moves with speed 16 m s^{-1} and collides with B when the direction of A's motion makes an angle of 60° with the line of centres at the instant of impact. The coefficient of restitution between the spheres is 0.5 .
- (a) Find the speed of B after the collision. [4]
- (b) Find the speed of A after the collision, and the angle its path then makes with the line of centres. [5]
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- 27** Sphere A of mass m_1 moving at u_1 collides directly with sphere B of mass m_2 moving at u_2 in the same direction ($u_1 > u_2$). The coefficient of restitution is e .
- (a) Show that the velocity of B after the collision is $\frac{m_1 u_1 + m_2 u_2 + e m_1 (u_1 - u_2)}{m_1 + m_2}$. [5]
- (b) Show that the kinetic energy lost is $\frac{m_1 m_2}{2(m_1 + m_2)} (1 - e^2) (u_1 - u_2)^2$. [5]
- (c) Use this to find the kinetic energy lost when $m_1 = 4$, $m_2 = 2$, $u_1 = 5$, $u_2 = -1$ and $e = 0.5$, and compare with a direct calculation. [3]

- 28** A ball is dropped from rest from height h above a horizontal floor, and the coefficient of restitution between ball and floor is e , with $0 < e < 1$. Let $t_0 = \sqrt{2h/g}$.
- (a) Show that the ball first reaches the floor with speed $\sqrt{2gh}$ and that the time between the n -th and $(n+1)$ -th impacts is $2e^n t_0$. [4]
- (b) Show that the total time for which the ball bounces is $t_0 \frac{1+e}{1-e}$. [4]
- (c) Show that the total distance travelled is $h \frac{1+e^2}{1-e^2}$. [4]
- (d) For $h = 1.8$ m and $e = 0.8$, find the total time and the total distance. Use $g = 9.8 \text{ m s}^{-2}$. [5]
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- 29** Three smooth spheres P, Q and R, of masses $3m$, m and km respectively, where k is a positive constant, lie at rest in a straight line, in that order, on a smooth horizontal plane. The coefficient of restitution between any two of the spheres is $\frac{1}{2}$. P is projected towards Q with speed u and collides directly with it.
- (a) Find, in terms of u , the speeds of P and Q immediately after the collision. [4]
- (b) Q then collides with R. Show that there is a further collision between P and Q if and only if $k > \frac{8}{19}$. [6]
- (c) For $k = 1$, find the speeds of Q and R after their collision. [3]
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- 30** Smooth spheres A (mass 2 kg) and B (mass 1 kg) lie on a smooth horizontal table, with B between A and a smooth vertical wall. B is at rest, A moves directly towards B at 6 m s^{-1} . The coefficient of restitution between the spheres is 0.5 and between B and the wall is 0.4.
- (a) Find the velocities of A and B just after the first collision. [4]
- (b) Find the velocity of B after it rebounds from the wall, and show that B and A collide again. [4]
- (c) Find the velocities after the second collision between A and B (take the direction towards the wall as positive). [5]
- (d) B moves towards the wall again and rebounds. Show that A and B collide for a third time. [4]
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