

Moments

1 (2 marks)

$$\text{Moment} = \text{force} \times \text{perpendicular distance} = 20 \times 0.6 \\ = 12 \text{ N m}$$

M1

A1

Answer: 12 N m

2 marks

2 (3 marks)

$$\text{Moment} = 45 \times 0.30 = 13.5 \text{ N m}$$

M1 A1

$$F \times 0.25 = 30, \text{ so } F = 120 \text{ N}$$

B1

Answer: 13.5 N m; 120 N

3 marks

3 (2 marks)

$$\text{Moments about the pivot: } 30g \times 1.5 = 25g \times d$$

M1

$$d = \frac{45}{25} = 1.80$$

A1

Answer: d = 1.80

2 marks

4 (2 marks)

$$\text{Clockwise moment} = 10 \times 2 = 20 \text{ N m; anticlockwise} = 8 \times 1.5 = 12 \text{ N m}$$

M1

$$\text{Resultant} = 20 - 12 = 8 \text{ N m clockwise}$$

A1

Answer: 8 N m clockwise

2 marks

5 (2 marks)

$$\text{Moment} = 4g \times 0.5$$

M1

$$= 19.6 \text{ N m}$$

A1

Answer: 19.6 N m

2 marks

6 (2 marks)

The weight acts at the midpoint, 1 m from the pivot

B1

$$\text{Moment} = 30 \times 1 = 30 \text{ N m}$$

A1

Answer: 30 N m

2 marks

7 (4 marks)

$$\text{Moments about P: } 40x = 60(3 - x)$$

M1 A1

$$100x = 180, \text{ so } x = 1.80$$

A1

$$\text{Vertical equilibrium: } R = 40 + 60 = 100 \text{ N}$$

B1

Answer: x = 1.80, R = 100 N

4 marks

8 (4 marks)

The weight of the rod acts at its midpoint, 1 m from P on the B side

B1

$$\text{Moments about P: } M \times 2 = 12 \times 1$$

M1 A1

$$M = 6$$

A1

Answer: M = 6

4 marks

9 (4 marks)

$$\text{Component of the force perpendicular to the door} = 50 \sin 60^\circ$$

M1 A1

$$\text{Moment} = 50 \sin 60^\circ \times 0.8 = 34.6 \text{ N m}$$

M1 A1

Answer: 34.6 N m

4 marks

10 (6 marks)

$$\text{Moments about C: } T_D \times 3.5 = 400 \times 2 + 250 \times 5 - 150 \times 1$$

M1 A2

$$T_D = \frac{1900}{3.5} = 543 \text{ N}$$

A1

$$\text{Resolving vertically: } T_C + T_D = 150 + 400 + 250 = 800$$

M1

$$T_C = 257 \text{ N}$$

A1

Answer: T_C = 257 N, T_D = 543 N

6 marks

11 (3 marks)

$$\text{Moment} = F \times 0.4 \times \sin 40^\circ$$

M1 A1

$$F = \frac{12}{0.4 \sin 40^\circ} = 46.7 \text{ N}$$

A1

Answer: 46.7 N

3 marks

12 (5 marks)6 N: perpendicular distance from A is 1.5 m, moment = $6 \times 1.5 = 9 \text{ N m}$ clockwise

M1 A1

10 N: perpendicular distance from A is 1.5 m, moment = $10 \times 1.5 = 15 \text{ N m}$ clockwise

M1

4 N: moment = $4 \times 1.5 = 6 \text{ N m}$ anticlockwise

A1

Total = $9 + 15 - 6 = 18 \text{ N m}$ clockwise

A1

Answer: 18 N m clockwise

5 marks

13 (3 marks)Moments about the hinge B: $T \times 4 = 8g \times 2$

M1 A1

$$T = \frac{8 \times 9.8 \times 2}{4} = 39.2 \text{ N}$$

A1

Answer: 39.2 N

3 marks

14 (5 marks)Moments about C: $R_D \times 3 = 100 \times 1.5 + 40 \times 2.5$

M1 A1

$$R_D = \frac{250}{3} = 83.33 \text{ N}$$

A1

Vertical equilibrium: $R_C + R_D = 140$

M1

$$R_C = 56.67 \text{ N}$$

A1

Answer: $R_C = 56.67 \text{ N}$, $R_D = 83.33 \text{ N}$

5 marks

15 (5 marks)Let the centre of mass be $x \text{ m}$ from P on the side of B

B1

Moments about P: $80g \times 1.5 = 55g \times 2 + 20g \times x$

M1 A1

$$120 = 110 + 20x, \text{ so } x = 0.500$$

A1

The centre of mass is 0.500 m from P on the side of B

A1

Answer: 0.500 m from P towards B

5 marks

16 (5 marks)Moments about A (the string has perpendicular component $T \sin 30^\circ$ to the rod): $T \sin 30^\circ \times 3 =$

M1 A1 A1

$$20g \times 1.5 + 10g \times 3$$

$$1.5T = 30g + 30g = 60g, \text{ so } T = 40g$$

M1

$$T = 40 \times 9.8 = 392 \text{ N}$$

A1

Answer: 392 N

5 marks

17 (4 marks)

The beam is about to tip about D when the reaction at C becomes zero

B1

Moments about D: $120 \times 2 = P \times 1$

M1 A1

$$P = 240 \text{ N}$$

A1

Answer: 240 N

4 marks

18 (5 marks)

The plank is about to tip about C when the reaction at A is zero

B1

Moments about C: $20g \times (4 - 2.5) = 60g \times (x - 4)$

M1 A1

$$30 = 60(x - 4), \text{ so } x = 4.50$$

A1

He can walk 4.50 m from A

A1

Answer: 4.50 m from A

5 marks

19 (6 marks)

Moments about C (taking anticlockwise as positive): $R_D \times 5 + 15g \times 1 - 40g \times 3 - 25g \times 4 = 0$

M1 A1 A1

$$R_D = \frac{g(120 + 100 - 15)}{5} = 402 \text{ N}$$

A1

$$R_C = 80g - R_D = 382 \text{ N}$$

M1 A1

Answer: $R_C = 382 \text{ N}$, $R_D = 402 \text{ N}$

6 marks

20 (4 marks)

Vertical equilibrium check: $70 + 28 = 98 = 10g$

B1

Moments about A: $28 \times 4 = 98 \times x$

M1 A1

$$x = \frac{112}{98} = 1.14 \text{ m}$$

A1

Answer: 1.14 m

4 marks

21 (6 marks)

$R_P + R_Q = 150$ and $R_Q = 3R_P$, so $R_P = 37.5 \text{ N}$ and $R_Q = 112.5 \text{ N}$

M1 A1

Moments about P: $R_Q(d - 1) = 150 \times (3 - 1)$

M1 A1

$$112.5(d - 1) = 300, \text{ so } d - 1 = \frac{8}{3}$$

M1

$$d = \frac{11}{3} = 3.667$$

A1

Answer: $d = 11/3 = 3.667$

6 marks

22 (6 marks)

(a) Moments about the axle: $F \times 1.2 = 600 \times 0.4$

M1 A1

$$F = 200 \text{ N}$$

A1

Answer: 200 N

3 marks

(b) Vertical equilibrium: $R + F = 600$

M1

$$R = 400 \text{ N}$$

A1

Answer: 400 N

2 marks

(c) The distance 0.4 m decreases, so the moment of the load about the axle is smaller, and a smaller F is needed to balance it

E1

Answer: smaller moment of load

1 mark

23 (9 marks)

(a) Moments about C: $R_A \times 1.5 + 12g \times 0.5 = 30g \times 2.5$

M1 A2

$$R_A = \frac{(75 - 6)g}{1.5} = 451 \text{ N, acting downwards}$$

A1

Vertically: $R_C = R_A + 12g + 30g = 862 \text{ N}$, acting upwards

M1 A1

Answer: $R_A = 451 \text{ N down}$, $R_C = 862 \text{ N up}$

6 marks

(b) The rod is about to tip about C: $M \times 2.5 = 12 \times 0.5$ (taking g out of both sides)

M1 A1

$$M = 2.40$$

A1

Answer: $M = 2.40$

3 marks

24 (9 marks)

(a) Moments about A: $P \times 2a \cos \theta = W \times a \sin \theta$

M1 A1 A1

$$P = \frac{W \sin \theta}{2 \cos \theta} = \frac{1}{2} W \tan \theta$$

A1

Answer: shown

4 marks

(b) $P = 30 \tan 35^\circ = 21.0 \text{ N}$

M1 A1

Hinge: horizontal component P, vertical component 60

B1

$$R = \sqrt{21.01^2 + 60^2} = 63.6 \text{ N}$$

M1 A1

Answer: $P = 21.0 \text{ N}$, $R = 63.6 \text{ N}$

5 marks

25 (10 marks)

(a) The weight acts at the midpoint, with perpendicular distance $2 \cos 40^\circ$ from A

B1

Moments about A: $T \times 4 = 80 \times 2 \cos 40^\circ$

M1 A1

$T = 40 \cos 40^\circ = 30.6 \text{ N}$

A1

Answer: 30.6 N

4 marks

(b) The string BC makes an angle 40° with the horizontal (it is perpendicular to the rod):
horizontal component of $T = T \sin 40^\circ = 19.70 \text{ N}$ towards the wall; vertical component $= T \cos 40^\circ = 23.47 \text{ N}$ upwards

M1 A1

Hinge: $H = 19.70 \text{ N}$ (away from the wall), $V = 80 - 23.47 = 56.53 \text{ N}$

M1 A1

$R = \sqrt{19.7^2 + 56.5^2} = 59.9 \text{ N}$

M1 A1

Answer: 59.9 N

6 marks

26 (8 marks)

(a) First: $2 \times 4 - 1 \times 3 = 5 \text{ N m}$

M1 A1

Second: $4 \times 5 - (-1)(-2) = 18 \text{ N m}$; third: $(-1)(-6) - 3 \times 0 = 6 \text{ N m}$

A1 A1

Answer: 5, 18, 6 N m anticlockwise

4 marks

(b) Total $= 5 + 18 + 6 = 29 \text{ N m}$ anticlockwise

M1 A1

Resultant force $= (3 - 2)\mathbf{i} + (4 + 5 - 6)\mathbf{j} = (\mathbf{i} + 3\mathbf{j}) \text{ N}$, magnitude $\sqrt{10} = 3.16 \text{ N}$

M1 A1

Answer: 29 N m anticlockwise; 3.16 N

4 marks

27 (12 marks)

(a) Moments about A: $T \sin \alpha \times 2a = W \times a$

M1 A1 A1

$$T = \frac{W}{2 \sin \alpha}$$

A1

Answer: shown

4 marks

(b) Horizontal: $H = T \cos \alpha = \frac{W \cos \alpha}{2 \sin \alpha} = \frac{W}{2 \tan \alpha}$

M1 A1

Vertical: $V = W - T \sin \alpha = W - \frac{1}{2}W = \frac{1}{2}W$

M1 A1

Answer: $H = W/(2 \tan \alpha)$, $V = W/2$

4 marks

(c) $T \leq 2W$: $\frac{W}{2 \sin \alpha} \leq 2W$, so $\sin \alpha \geq \frac{1}{4}$

M1 A1

$\alpha \geq 14.5^\circ$

A1

Answer: $\alpha \geq 14.5^\circ$

3 marks

(d) The vertical forces are W , V and $T \sin \alpha$, and $T \sin \alpha = \frac{1}{2}W$ from the moment equation

E1

whatever α is, so $V = W - \frac{1}{2}W = \frac{1}{2}W$

Answer: $T \sin \alpha = W/2$ always

1 mark

28 (9 marks)

(a) The weight of the plank acts at the midpoint, 0.6 m from P on the side of B

B1

Moments about P: $40 \times 1.4 = 24 \times 0.6 + 20x$

M1 A1

$56 = 14.4 + 20x$, so $x = 2.08$

A1

Answer: $x = 2.08$

4 marks

(b) Moment of the plank and child about P on the side of B $= 14.4 + 20 \times 2.6 = 66.4 \text{ g-units}$, which exceeds the man's 56

M1

So the end B goes down

A1

Answer: B goes down

2 marks

(c) $56 = 14.4 + m \times 2.6$

M1 A1

$$m = \frac{41.6}{2.6} = 16$$

A1

Answer: $m = 16$

3 marks

29 (11 marks)

(a) The weight of the plank acts at its midpoint, a distance $2 - x$ from P on the side of B

B1

Moments about P for the first situation: $20x = M(2 - x) + 10(4 - x)$

M1 A1

Second situation: $30x = M(2 - x) + 16(4 - x)$

B1

Answer: shown; $30x = M(2 - x) + 16(4 - x)$

4 marks

(b) Subtracting the first from the second: $10x = 6(4 - x)$

M1

$16x = 24$, so $x = 1.5$

A1

$20 \times 1.5 = M \times 0.5 + 10 \times 2.5$, so $30 = 0.5M + 25$ and $M = 10$

M1 A1

Answer: $x = 1.5$, $M = 10$

4 marks

(c) Moments about P: $40 \times 1.5 = 10 \times 0.5 + 25y$

M1 A1

$y = \frac{55}{25} = 2.20$ m

A1

Answer: 2.20 m

3 marks

30 (14 marks)

(a) Moments about C: $R_D \times 4 = 120 \times 2 + 300(x - 1)$

M1 A1

$R_D = 60 + 75(x - 1) = 75x - 15$

A1

$R_C = 420 - R_D = (435 - 75x)$ N

M1 A1

Answer: $R_C = 435 - 75x$

5 marks

(b) Contact is maintained at D when $R_D \geq 0$: $75x \geq 15$, so $x \geq 0.2$

M1 A1

Contact is maintained at C when $R_C \geq 0$: $75x \leq 435$, so $x \leq 5.8$

M1 A1

$0.2 \leq x \leq 5.8$

A1

Answer: $0.2 \leq x \leq 5.8$

5 marks

(c) At A ($x = 0$) R_D would be negative: the beam would tip about C, with end B rising

E1

Load Q at B: moments about C with the man at A: $R_D \times 4 = 240 + 300(0 - 1) + Q \times 5 \geq 0$, so

M1 A1

$5Q \geq 60$

$Q \geq 12$ N

A1

Answer: Beam tips about C; $Q \geq 12$ N

4 marks

Equilibrium of Rigid Bodies

1 (2 marks)

The vector sum of the forces is zero (the components of the forces in any two perpendicular directions each sum to zero)

B1

The sum of the moments of the forces about any point is zero

B1

Answer: Resultant force zero and resultant moment zero

2 marks

2 (3 marks)

Resultant = $6 + 10 = 16$ N

B1

Moments about A: $16x = 10 \times 4$, so $x = 2.50$ m

M1 A1

Answer: 16 N, 2.50 m from A

3 marks

3 (3 marks)

By symmetry the two reactions are equal

B1

$2R = 40 + 20$, so $R = 30$ N

M1 A1

Answer: 30 N at each support

3 marks

4 (3 marks)

Moments about A: $T \times AB = 50 \times \frac{1}{2}AB$, so $T = 25$ N

M1 A1

Vertically: $V + 25 = 50$, so $V = 25$ N

A1

Answer: $T = 25$ N, $V = 25$ N

3 marks

5 (3 marks)

$$\text{Moments about A: } T_C \times 3 = 40 \times 2$$

M1 A1

$$T_C = 26.67 \text{ N, so } T_A = 40 - 26.67 = 13.33 \text{ N}$$

A1

$$\text{Answer: } T_A = 13.33 \text{ N, } T_C = 26.67 \text{ N}$$

3 marks

6 (3 marks)

$$\text{Moments about B: } R_A \times 4 = 200 \times 2 + 600 \times 1$$

M1 A1

$$R_A = 250 \text{ N, so } R_B = 800 - 250 = 550 \text{ N}$$

A1

$$\text{Answer: } R_A = 250 \text{ N, } R_B = 550 \text{ N}$$

3 marks

7 (6 marks)

The reaction N at the smooth peg is perpendicular to the rod

B1

$$\text{Moments about A: } N \times 5 = 12g \times 3.5 \cos 55^\circ$$

M1 A1

$$N = 47.2 \text{ N}$$

A1

$$\text{Horizontal: } F = N \sin 55^\circ = 38.7 \text{ N}$$

M1

$$\text{Vertical: } R + N \cos 55^\circ = 12g, \text{ so } R = 90.5 \text{ N}$$

A1

$$\text{Answer: } N = 47.2 \text{ N, } R = 90.5 \text{ N, } F = 38.7 \text{ N}$$

6 marks

8 (6 marks)

$$\text{Moments about A: } P \sin 50^\circ \times 2 = 30 \times 1$$

M1 A1

$$P = \frac{15}{\sin 50^\circ} = 19.6 \text{ N}$$

A1

$$\text{Horizontal: } H = P \cos 50^\circ = 12.6 \text{ N (away from the wall)}$$

M1 A1

$$\text{Vertical: } V = 30 - P \sin 50^\circ = 15 \text{ N}$$

A1

$$\text{Answer: } P = 19.6 \text{ N, } H = 12.6 \text{ N, } V = 15 \text{ N}$$

6 marks

9 (4 marks)

$$\text{Vertical equilibrium: } T = 40 + 20 = 60 \text{ N}$$

B1

$$\text{Moments about A: } 60x = 40 \times 1 + 20 \times 2$$

M1 A1

$$x = \frac{80}{60} = 1.333$$

A1

$$\text{Answer: } x = 4/3 = 1.333, T = 60 \text{ N}$$

4 marks

10 (5 marks)

Toppling occurs about the lower far edge when the reaction acts through that edge: $P \times 1.0 = 20g \times 0.3$

M1 A1

$$P = 58.8 \text{ N}$$

A1

$$\text{Sliding would need } P = \mu R = 0.5 \times 20g = 98 \text{ N}$$

M1

$$58.8 < 98, \text{ so the block topples first}$$

A1

$$\text{Answer: Topples first, at } P = 58.8 \text{ N}$$

5 marks

11 (4 marks)

(a) Each force has a moment $12 \times 0.25 = 3 \text{ N m}$, in the same sense

M1

$$\text{Total} = 6 \text{ N m}$$

A1

$$\text{Answer: } 6 \text{ N m}$$

2 marks

(b) Moments about A: the force at A contributes 0; the force at B contributes $12 \times 0.5 = 6 \text{ N m}$, the same

M1

A1

$$\text{Answer: } 6 \text{ N m}$$

2 marks

12 (6 marks)

The centre of the rod is 1 m from A, so it is 0.6 m from C on the same side as A

B1

$$\text{Moments about C: } F \times 1.6 + 60 \times 0.6 = 600 \times 0.4$$

M1 A1

$$F = \frac{240 - 36}{1.6} = 127 \text{ N}$$

A1

$$\text{Pivot force} = F + 60 + 600 = 788 \text{ N}$$

M1 A1

$$\text{Answer: } F = 127 \text{ N, pivot force } 788 \text{ N}$$

6 marks

13 (4 marks)

Moments about A: $20 \times 3 = W \times 1.2$

M1 A1

$W = 50 \text{ N}$

A1

$V + 20 = 50$, so $V = 30 \text{ N}$

B1

Answer: $W = 50 \text{ N}$, $V = 30 \text{ N}$

4 marks

14 (7 marks)

(a) Moments about the lower far edge: $P \times 0.9 = 25g \times 0.4$

M1 A1

$P = 109 \text{ N}$

A1

Answer: 109 N

3 marks

(b) Sliding occurs when $P = 0.4 \times 25g = 98 \text{ N}$

M1

$98 < 109$, so the block slides first

A1

Answer: It slides first

2 marks

(c) Toppling first needs $\mu \times 25g \geq P_{\text{topple}}$, that is $\mu \geq \frac{0.4}{0.9}$

M1

$\mu \geq 0.444$

A1

Answer: $\mu \geq 0.444$

2 marks

15 (7 marks)

$AC = \frac{1.0}{\sin 40^\circ} = 1.556 \text{ m}$

B1

Moments about A (the peg reaction N is perpendicular to the rod): $N \times AC = 80 \times 1.5 \cos 40^\circ$, so

M1 A1

$N = 59.1 \text{ N}$

Vertically: $R + N \cos 40^\circ = 80$, so $R = 34.7 \text{ N}$

M1 A1

Horizontally: $T = N \sin 40^\circ = 38.0 \text{ N}$

M1 A1

Answer: $T = 38.0 \text{ N}$, $R = 34.7 \text{ N}$, $N = 59.1 \text{ N}$

7 marks

16 (7 marks)

Moments about A: $45 \sin 40^\circ \times 3 = 60x$

M1 A1

$x = 1.45 \text{ m}$

A1

$H = 45 \cos 40^\circ = 34.47 \text{ N}$; $V = 60 - 45 \sin 40^\circ = 31.07 \text{ N}$

M1 A1

$R = \sqrt{34.5^2 + 31.1^2} = 46.4 \text{ N}$

M1 A1

Answer: $x = 1.45 \text{ m}$; $R = 46.4 \text{ N}$

7 marks

17 (5 marks)

(a) Moments about A, with the rod of length L: $P \times L \cos \theta = 30 \times \frac{L}{2} \sin \theta + 10 \times L \sin \theta$

M1 A1

$P \cos \theta = 25 \sin \theta$, so $\tan \theta = \frac{P}{25}$

A1

Answer: shown

3 marks

(b) $\tan \theta = \frac{15}{25} = 0.6$

M1

$\theta = 31.0^\circ$

A1

Answer: $\theta = 31.0^\circ$

2 marks

18 (9 marks)

(a) Moments about A: $T \sin 40^\circ \times 2 = 80 \times 1$

M1 A1

$T = 62.2 \text{ N}$

A1

Answer: $T = 62.2 \text{ N}$

3 marks

(b) Horizontal: $N = T \cos 40^\circ = 47.7 \text{ N}$

M1 A1

Vertical: $F + T \sin 40^\circ = 80$, so $F = 40 \text{ N}$

M1 A1

$\mu \geq \frac{F}{N} = 0.839$

M1 A1

Answer: $\mu \geq 0.839$

6 marks

19 (8 marks)

Moments about A: $T \times 6 \sin 60^\circ = 200g \times 3 \cos 60^\circ + 500g \times 6 \cos 60^\circ$

M1 A1 A1

$$T = \frac{g(300 + 1500)}{6 \tan 60^\circ} = 3390 \text{ N}$$

A1

Horizontal: $H = T = 3395 \text{ N}$; vertical: $V = 700g = 6860 \text{ N}$

M1 A1

$$R = \sqrt{3390^2 + 6860^2} = 7650 \text{ N}$$

M1 A1

Answer: $T = 3390 \text{ N}$; $R = 7650 \text{ N}$

8 marks

20 (6 marks)

Moments about B: $H_A \times 1.2 = 150 \times 1$

M1 A1

$H_A = 125 \text{ N}$, horizontal

A1

Hinge B: the horizontal component has the same magnitude, 125 N, in the opposite direction and the vertical component is 150 N

M1

B1

$$\text{Magnitude at B} = \sqrt{125^2 + 150^2} = 195 \text{ N}$$

A1

Answer: A: 125 N horizontal; B: 125 N horizontal and 150 N vertical (magnitude 195 N)

6 marks

21 (9 marks)

(a) Toppling about the lower far edge: $P \times h = W \times 0.3$, so $P = \frac{0.3W}{h}$

M1 A1

Sliding: $P = \mu R = 0.4W$ ($R = W$)

M1 A1

Answer: shown

4 marks

(b) Sliding occurs first if $0.4W < \frac{0.3W}{h}$, that is $h < \frac{0.3}{0.4}$

M1 A1

$h < 0.75$ (with $h \leq 1.0$)

A1

Answer: $h < 0.75$

3 marks

(c) Toppling first: $\mu W \geq \frac{0.3W}{1.0}$

M1

$\mu \geq 0.3$

A1

Answer: $\mu \geq 0.3$

2 marks

22 (11 marks)

(a) $BC = 5 \text{ m}$, so $\sin \angle ABC = \frac{3}{5}$

B1

Moments about A: $T \times 4 \times \frac{3}{5} = 60 \times 2$

M1 A1

$T = 50 \text{ N}$

A1

Answer: 50 N

4 marks

(b) Horizontal: $H = T \cos \angle ABC = 50 \times \frac{4}{5} = 40 \text{ N}$

M1

Vertical: $V = 60 - 50 \times \frac{3}{5} = 30 \text{ N}$

M1

$R = \sqrt{40^2 + 30^2} = 50 \text{ N}$, at $\tan^{-1}(\frac{30}{40}) = 36.9^\circ$ above the horizontal (away from the wall)

A1 A1

Answer: 50 N at 36.9° above the horizontal

4 marks

(c) The weight acts through the midpoint M of AB, 2 m from A. The string BC, extended, meets the vertical line through M at a point D with $MD = 2 \times \frac{3}{4} = 1.5 \text{ m}$ above the rod

M1 A1

The hinge force has gradient $\frac{30}{40} = \frac{3}{4}$, so at 2 m from A it is at height 1.5 m: it also passes through D

A1

Answer: concurrent at D

3 marks

23 (8 marks)

(a) Moments about A: $P \times a \cos \theta = W \times a \sin \theta + w \times 2a \sin \theta$

M1 A1 A1

$$P \cos \theta = (W + 2w) \sin \theta, \text{ so } \tan \theta = \frac{P}{W + 2w}$$

A1

Answer: shown

4 marks

(b) $\tan \theta = \frac{18}{64}$, so $\theta = 15.7^\circ$

M1 A1

Hinge: horizontal component 18 N (opposing P), vertical component $W + w = 52$ N

M1

$$R = \sqrt{18^2 + 52^2} = 55.0 \text{ N}$$

A1

Answer: $\theta = 15.7^\circ$, $R = 55.0$ N

4 marks

24 (8 marks)

The peg reaction N is perpendicular to the rod; horizontally: $T = N \sin 50^\circ$, so $N = \frac{40}{\sin 50^\circ} =$

M1 A1

52.22 N

Moments about A: $N \times AC = 100 \times 2 \times \cos 50^\circ$

M1 A1

$$AC = \frac{128.6}{52.22} = 2.462 \text{ m}$$

A1

Height of peg = $AC \sin 50^\circ = 1.89$ m

A1

Vertically: $R = 100 - N \cos 50^\circ = 100 - 33.56 = 66.44$ N, which is greater than 50

M1 A1

Answer: $AC = 2.46$ m, height 1.89 m

8 marks

25 (8 marks)

(a) $CD = \sqrt{1.8^2 + 2.4^2} = 3$ m, and the perpendicular distance from A to CD is $\frac{1.8 \times 2.4}{3} = 1.44$ m

M1 A1

Moments about A: $T \times 1.44 = 45 \times 1.2$

M1

$T = 37.5$ N

A1

Answer: 37.5 N

4 marks

(b) Components of T: horizontal $T \times \frac{1.8}{3} = 22.5$ N (towards the wall); vertical $T \times \frac{2.4}{3} = 30$ N

M1 A1

(upwards)

$H = 22.5$ N away from the wall; $V = 45 - 30 = 15$ N

M1 A1

Answer: $H = 22.5$ N, $V = 15$ N

4 marks

26 (6 marks)

Moments about A: $T \times 6 \sin 60^\circ = 200g \times 3 \cos 60^\circ + mg \times 6 \cos 60^\circ$

M1 A1 A1

With $T = 5000$: $5000 \times 6 \tan 60^\circ = g(600 + 6m)$

M1

$$\frac{51\,961.5}{9.8} = 600 + 6m, \text{ so } 6m = 4702$$

A1

$m = 784$ kg

A1

Answer: 784 kg

6 marks

27 (15 marks)

(a) Since C is on the rod at height h, $AC = \frac{h}{\sin \theta}$

B1

Moments about A: $N \times \frac{h}{\sin \theta} = W \times a \cos \theta$

M1 A1

$$N = \frac{Wa \sin \theta \cos \theta}{h}$$

A1

Answer: shown

4 marks

(b) Horizontally: $F = N \sin \theta$ (the peg reaction has horizontal component $N \sin \theta$)

M1 A1

Vertically: $R + N \cos \theta = W$, so $R = W - N \cos \theta$

M1 A1

Answer: $F = N \sin \theta$, $R = W - N \cos \theta$

4 marks

$$(c) N = \frac{100 \times 1.5 \times \sin 60^\circ \cos 60^\circ}{1.2} = 54.1 \text{ N}$$

M1 A1

$$F = 54.13 \times \sin 60^\circ = 46.88 \text{ N}, R = 100 - 54.13 \times \cos 60^\circ = 72.94 \text{ N}$$

M1 A1

$$\mu \geq \frac{F}{R} = 0.643$$

M1 A1

Answer: $\mu \geq 0.643$

6 marks

(d) As $\theta \rightarrow 90^\circ$, $\cos \theta \rightarrow 0$ so $N \rightarrow 0$ and $F \rightarrow 0$: the rod is almost vertical and the peg hardly needs to push, so very little friction is needed

E1

Answer: friction required tends to zero

1 mark

28 (13 marks)**(a)** The vertical component of P acts through the pivot edge, so it has no moment about it**B1**Moments about the bottom left edge: $P \cos 30^\circ \times 1.2 = 200 \times 0.25$ **M1 A1** $P = 48.1 \text{ N}$ **A1****Answer: 48.1 N****4 marks****(b)** Vertically: $R = 200 - P \sin 30^\circ$ **M1**Limiting friction: $P \cos 30^\circ = 0.4(200 - P \sin 30^\circ)$ **M1 A1** $P(\cos 30^\circ + 0.4 \sin 30^\circ) = 80$, so $P = 75.0 \text{ N}$ **A1****Answer: 75.0 N****4 marks****(c)** $48.1 < 75.0$, so the block topples before it slides**E1****Answer: It topples first****1 mark****(d)** Toppling first requires $\frac{200 \times 0.25}{1.2 \cos 30^\circ} \leq \frac{200\mu}{\cos 30^\circ + \mu \sin 30^\circ}$ **M1**Rearranging: $0.25(\cos 30^\circ + \mu \sin 30^\circ) \leq 1.2\mu \cos 30^\circ$, so $\mu(1.2 \cos 30^\circ - 0.25 \sin 30^\circ) \geq 0.25 \cos 30^\circ$ **M1 A1** $\mu \geq \frac{0.25 \cos 30^\circ}{1.2 \cos 30^\circ - 0.25 \sin 30^\circ} = 0.237$ **A1****Answer: $\mu \geq 0.237$** **4 marks****29 (14 marks)****(a)** Moments about A: $T \times 2a \sin \theta = W \times a \cos \theta$ **M1 A1 A1** $T = \frac{W \cos \theta}{2 \sin \theta} = \frac{W}{2 \tan \theta}$ **A1****Answer: shown****4 marks****(b)** Horizontal: $H = T = \frac{W}{2 \tan \theta}$ (directed away from the wall)**M1**Vertical: $V = W$ **B1** $\tan \varphi = \frac{V}{H} = 2 \tan \theta$ **A1****Answer: shown****3 marks****(c)** $T = \frac{40}{\tan 50^\circ} = 33.6 \text{ N}$ **M1 A1** $R = \sqrt{33.6^2 + 80^2} = 86.8 \text{ N}$ **M1 A1****Answer: $T = 33.6 \text{ N}$, $R = 86.8 \text{ N}$** **4 marks****(d)** $T \leq W$: $\frac{1}{2 \tan \theta} \leq 1$, so $\tan \theta \geq \frac{1}{2}$ **M1 A1** $\theta \geq 26.6^\circ$ (and $\theta < 90^\circ$)**A1****Answer: $\theta \geq 26.6^\circ$** **3 marks****30 (15 marks)****(a)** $OA = OP = a$, so the base angles of triangle OAP are equal, and angle OPA = θ (the rod makes angle θ with the horizontal rim OP)**M1 A1** $AP = 2a \cos \theta$ **A1**The reaction at A acts along AO, and angle OAP = θ , so AO makes angle $\theta + \theta = 2\theta$ with the horizontal**M1 A1****Answer: shown****5 marks****(b)** The weight acts at the midpoint, $1.5a$ from A along the rod, and the reaction S at P is perpendicular to the rod**B1**Moments about A: $S \times 2a \cos \theta = W \times 1.5a \cos \theta$, so $S = \frac{3W}{4}$ **M1 A1**Resolving horizontally: $R \cos 2\theta = S \sin \theta$; vertically: $R \sin 2\theta + S \cos \theta = W$ **M1**Eliminating R: $S(\sin \theta \sin 2\theta + \cos \theta \cos 2\theta)/\cos 2\theta = W$, i.e. $S \cos \theta = W \cos 2\theta$ **M1 A1** $\frac{3}{4} \cos \theta = 2 \cos^2 \theta - 1$, so $8 \cos^2 \theta - 3 \cos \theta - 4 = 0$ **A1****Answer: shown****7 marks****(c)** $\cos \theta = \frac{3 + \sqrt{9 + 128}}{16} = 0.9190$ **M1 A1** $\theta = 23.2^\circ$ **A1**

Answer: $\theta = 23.2^\circ$ **3 marks**

Centres of Mass

1 (3 marks)

Total mass = 10 kg

B1

$$10x_G = 2 \times 1 + 3 \times 4 + 5 \times 6 = 44, \text{ so } x_G = 4.40 \text{ m}$$

M1 A1**Answer: 4.40 m****3 marks**

2 (4 marks)

$$x_G = \frac{2 \times 1 + 3 \times 4 + 5 \times 2}{10} = 2.40$$

M1 A1

$$y_G = \frac{2 \times 2 + 3 \times 0 + 5 \times 6}{10} = 3.40$$

M1 A1**Answer: (2.40, 3.40)****4 marks**

3 (3 marks)

Taking moments about the free end: $5x = 4 \times 1 + 1 \times 2$ **M1 A1**

$$x = 1.20 \text{ m}$$

A1**Answer: 1.20 m from the free end****3 marks**

4 (2 marks)

The centre of mass is one third of the way from each side towards the opposite vertex

B1

$$\text{Distance from OB} = \frac{1}{3} \times 6 = 2 \text{ m; distance from OA} = \frac{1}{3} \times 9 = 3 \text{ m}$$

A1**Answer: 2 m from OB, 3 m from OA****2 marks**

5 (4 marks)

Areas: 18 cm^2 and 10 cm^2 , centres 4.5 cm and 1 cm from the left edge**B1**

$$28x = 18 \times 4.5 + 10 \times 1$$

M1 A1

$$x = 3.25 \text{ cm}$$

A1**Answer: 3.25 cm****4 marks**

6 (4 marks)

Masses are proportional to lengths: AB (0.6) at distance 0, and each of the other two sides (0.6) at distance 0.3

B1

$$1.8y = 0.6 \times 0 + 1.2 \times 0.3$$

M1 A1

$$y = 0.200 \text{ m}$$

A1**Answer: 0.2 m from AB****4 marks**

7 (5 marks)

By symmetry the centre of mass is on the vertical axis of symmetry, 4 cm from the left-hand edge

B1

Areas 16 and 10; centres 6 cm and 2.5 cm above the bottom

B1

$$26y = 16 \times 6 + 10 \times 2.5$$

M1 A1

$$y = 4.654 \text{ cm}$$

A1**Answer: 4.654 cm above the bottom; 4 cm from the left****5 marks**

8 (6 marks)

Use the full rectangle (area 96, centre (6, 4)) and the removed square (area 16, centre (10, 2))

B1

$$80x = 96 \times 6 - 16 \times 10$$

M1 A1

$$x = 5.20 \text{ cm}$$

A1

$$80y = 96 \times 4 - 16 \times 2, \text{ so } y = 4.40 \text{ cm}$$

M1 A1**Answer: 5.20 cm along AB, 4.40 cm up AD****6 marks**

9 (5 marks)

Areas: rectangle 24, triangle 9. Centres at heights 2 and $4 + \frac{1}{3} \times 3 = 5$

B1 B1

$$33y = 24 \times 2 + 9 \times 5$$

M1 A1

$$y = 2.818 \text{ cm}$$

A1

Answer: 2.818 cm

5 marks

10 (7 marks)

Rectangle: area 60 at (5, 3); disc: area $\frac{9\pi}{4} = 7.069$ at (3, 2)

B1 B1

$$(60 - 7.069)x = 60 \times 5 - 7.069 \times 3$$

M1 A1

$$x = 5.267 \text{ cm}$$

A1

$$(60 - 7.069)y = 60 \times 3 - 7.069 \times 2, \text{ so } y = 3.134 \text{ cm}$$

M1 A1

Answer: 5.267 cm from the left edge, 3.134 cm from the bottom edge

7 marks

11 (6 marks)

Total mass = 20 kg

B1

$$x: 20 \times 2 = 2 \times 1 + 3 \times 5 + 5 \times (-3) + 10x_D, \text{ so } 40 = 2 + 10x_D$$

M1 A1

$$x_D = 3.80$$

A1

$$y: 20 \times 1 = 2 \times 4 + 3 \times (-2) + 5 \times 0 + 10y_D, \text{ so } y_D = 1.80$$

M1 A1

Answer: D = (3.80, 1.80)

6 marks

12 (6 marks)

Areas: rectangle 40, semicircle $\frac{1}{2}\pi \times 4^2 = 25.13$

B1

Centre of the semicircle is $\frac{4 \times 4}{3\pi} = 1.698$ cm beyond the rectangle, so at height 6.698 cm

M1 A1

$$(40 + 25.13)y = 40 \times 2.5 + 25.13 \times 6.698$$

M1 A1

$$y = 4.12 \text{ cm}$$

A1

Answer: 4.12 cm

6 marks

13 (5 marks)

Areas (in units of π): disc 36, hole 4, remaining 32

B1

Take moments about O: $32x = 36 \times 0 - 4 \times 3$

M1 A1

$$x = -0.375 \text{ cm}$$

A1

so the centre of mass is 0.375 cm from O on the opposite side of O to C

A1

Answer: 0.375 cm from O, on the side opposite to the hole

5 marks

14 (8 marks)

(a) Areas 20 and 12; centres at heights 7 and 3

B1

$$32y = 20 \times 7 + 12 \times 3 = 176$$

M1 A1

$$y = 5.5 \text{ cm}$$

A1

Answer: 5.5 cm

4 marks

(b) The centre of mass G is 5 cm to the right of A and $(8 - 5.5) = 2.5$ cm below the top edge

B1 B1

The line AG is vertical, so $\tan \theta = \frac{2.5}{5}$

M1

$$\theta = 26.57^\circ$$

A1

Answer: 26.6°

4 marks

15 (7 marks)

Square (area 36) has centre (3, 3); the triangle (area 9, vertices (6, 0), (6, 6), (9, 0)) has centroid (7, 2)

B1 B1

$$45x = 36 \times 3 + 9 \times 7, \text{ so } x = 3.80$$

M1 A1

$$45y = 36 \times 3 + 9 \times 2, \text{ so } y = 2.80$$

A1

G is 3.80 cm to the right of D and 3.20 cm below DC; $\tan \theta = \frac{3.20}{3.80}$, so $\theta = 40.1^\circ$

M1 A1

Answer: 40.1°

7 marks

16 (6 marks)

Areas: large $\frac{1}{2}\pi(3a)^2 = \frac{9\pi}{2}a^2$, small $\frac{1}{2}\pi(2a)^2 = 2\pi a^2$

B1 B1

Centres: $\frac{4 \times 3a}{3\pi} = \frac{4a}{\pi}$ and $\frac{4 \times 2a}{3\pi} = \frac{8a}{3\pi}$ from O

B1

$$\left(\frac{9\pi}{2} - 2\pi\right)a^2 y = \frac{9\pi}{2}a^2 \times \frac{4a}{\pi} - 2\pi a^2 \times \frac{8a}{3\pi}$$

M1 A1

$$\frac{5\pi}{2}y = 18a - \frac{16a}{3} = \frac{38a}{3}, \text{ so } y = \frac{76a}{15\pi} = 1.613a$$

A1

Answer: 1.613a from O

6 marks

17 (4 marks)

Total mass = 12 kg; the rod's centre of mass is 2 m from A

B1

Taking moments about A: $12 \times AP = 6 \times 2 + 4 \times 4 = 28$

M1 A1

$AP = 2.333$ m

A1

Answer: 2.333 m

4 marks

18 (6 marks)

(a) The cylinder is about to topple when the vertical through the centre of mass passes through the lowest point of the base

M1

The centre of mass is at height 0.5 m, so $\tan \theta = \frac{0.3}{0.5} = 0.6$

M1 A1

$$\theta = 31.0^\circ \approx 31^\circ$$

A1

Answer: $\theta = 31.0^\circ$

4 marks

(b) Slipping occurs when $\tan \theta = \mu = 0.5$, that is $\theta = 26.6^\circ$

M1

$26.6^\circ < 31.0^\circ$, so the cylinder slips first

A1

Answer: It slips first, at 26.6°

2 marks

19 (8 marks)

(a) BC = 5 cm, total length 12 cm. Masses are proportional to lengths

B1

From AC: $12x = 4 \times 2 + 3 \times 0 + 5 \times 2 = 18$, so $x = 1.50$ cm

M1 A1

From AB: $12y = 4 \times 0 + 3 \times 1.5 + 5 \times 1.5 = 12$, so $y = 1$ cm

M1 A1

Answer: 1.50 cm from AC, 1 cm from AB

5 marks

(b) G is 1.50 cm from A along AB, so $(4 - 1.50) = 2.50$ cm from B horizontally and 1 cm above AB

B1

$$\tan \theta = \frac{1}{2.50}$$

M1

$$\theta = 21.8^\circ$$

A1

Answer: 21.8°

3 marks

20 (8 marks)

(a) Triangle: area 54, centre 3 cm above AB; square: area 4, centre 1 cm above AB

B1 B1

$$50y = 54 \times 3 - 4 \times 1$$

M1 A1

$$y = 3.160 \text{ cm}$$

A1

Answer: 3.160 cm

5 marks

(b) G is 3.160 cm above AB and 6 cm from A along AB (by symmetry)

B1

$$AG \text{ is vertical, so } \tan(\text{angle}) = \frac{3.160}{6}$$

M1

$$\text{Angle} = 27.8^\circ$$

A1

Answer: 27.8°

3 marks

21 (8 marks)

(a) Hole: area $\frac{1}{2}\pi \times 3^2 = 14.14$; centre at distance $\frac{4 \times 3}{3\pi} = 1.273$ cm from the edge, so height

B1 B1

$$8.727 \text{ cm}$$

$$(100 - 14.14)y = 100 \times 5 - 14.14 \times 8.727$$

M1 A1

$$y = 4.39 \text{ cm}$$

A1

Answer: 4.39 cm

5 marks

(b) By symmetry G is 5 cm from the vertical edge through A; $\tan \theta = \frac{4.386}{5}$

M1 A1

$$\theta = 41.3^\circ$$

A1

Answer: 41.3°

3 marks

22 (10 marks)

(a) Moments about AD: $(96 - x^2)d = 96 \times 6 - x^2(12 - \frac{x}{2})$

M1 A1 A1

$$\text{so } d = \frac{576 - 12x^2 + 0.5x^3}{96 - x^2}$$

A1

Answer: shown

4 marks

(b) $5.2(96 - x^2) = 576 - 12x^2 + 0.5x^3$, so $0.5x^3 - 6.8x^2 + 76.8 = 0$

M1

$$x^3 - 13.6x^2 + 153.6 = 0$$

A1

$$x = 4: 64 - 217.6 + 153.6 = 0$$

A1

Answer: shown

3 marks

(c) Factorising: $(x - 4)(x^2 - 9.6x - 38.4) = 0$

M1

$x^2 - 9.6x - 38.4 = 0$ has roots $x = 12.64$ and $x = -3.038$, neither in the range

M1 A1

Answer: x = 4 only

3 marks

23 (7 marks)

(a) Toppling about O occurs when the vertical through G passes through O

M1

$$\tan \theta = \frac{x}{y} = \frac{2.929}{3.429} = 0.8542$$

M1 A1

$$\theta = 40.5^\circ$$

A1

Answer: 40.5°

4 marks

(b) Slipping occurs when $\tan \theta = \mu = 0.7$, $\theta = 35.0^\circ$

M1 A1

$35.0^\circ < 40.5^\circ$, so it slips first

A1

Answer: slips first, at 35.0°

3 marks

24 (7 marks)

(a) Lengths: arc πr , diameter $2r$

B1

$$(\pi r + 2r)y = \pi r \times \frac{2r}{\pi} + 2r \times 0$$

M1 A1

$$y = \frac{2r^2}{r(\pi + 2)} = \frac{2r}{\pi + 2}$$

A1

Answer: shown

4 marks

(b) $y = \frac{2 \times 0.5}{\pi + 2} = 0.1945$ m; G is $r = 0.5$ m horizontally from A

B1

$$\tan \theta = \frac{0.1945}{0.5}$$

M1

$$\theta = 21.3^\circ$$

A1

Answer: 21.3°

3 marks

25 (8 marks)

(a) Centres of the rods: (3, 0) and (6, 2) taking A as the origin

B1

$5x = 3 \times 3 + 2 \times 6 = 21$, so $x = 4.20$ m from A, which is 1.80 m from BC

M1 A1

$5y = 3 \times 0 + 2 \times 2$, so $y = 0.800$ m from AB

M1 A1

Answer: 1.80 m from BC, 0.800 m from AB

5 marks

(b) G is 1.80 m horizontally from C and $(4 - 0.800) = 3.20$ m below C

B1

$$\tan \theta = \frac{1.80}{3.20}$$

M1

$$\theta = 29.4^\circ$$

A1

Answer: 29.4°

3 marks

26 (12 marks)**(a)** Total mass = 3.2 kg; lamina acts at (10, 7.5), particle at (20, 15)**B1**

$$3.2x = 2.4 \times 10 + 0.8 \times 20, \text{ so } x = 12.50 \text{ cm from AD}$$

M1 A1

$$3.2y = 2.4 \times 7.5 + 0.8 \times 15, \text{ so } y = 9.375 \text{ cm from AB}$$

M1 A1**Answer: 12.50 cm from AD, 9.375 cm from AB****5 marks****(b)** Moments about A: $T_B \times 20 = 3.2g \times 12.50$ **M1 A1**

$$T_B = 19.6 \text{ N}$$

A1

$$T_A = 3.2g - T_B = 11.8 \text{ N}$$

M1 A1**Answer: $T_A = 11.8 \text{ N}$, $T_B = 19.6 \text{ N}$** **5 marks****(c)** G is vertically below A, so $\tan(\text{angle}) = \frac{y}{x} = \frac{9.375}{12.50}$ **M1**

$$\text{Angle} = 36.9^\circ$$

A1**Answer: 36.9°** **2 marks****27 (11 marks)****(a)** Areas: rectangle 60 at height 2.5; triangle 6k at height $5 + \frac{k}{3}$ **B1 B1**

$$(60 + 6k)y = 60 \times 2.5 + 6k(5 + \frac{k}{3})$$

M1 A1

$$= 150 + 30k + 2k^2, \text{ giving the result}$$

A1**Answer: shown****5 marks****(b)** Put $y = 5$: $5(60 + 6k) = 150 + 30k + 2k^2$ **M1**

$$300 + 30k = 150 + 30k + 2k^2, \text{ so } k^2 = 75$$

A1

$$k = 5\sqrt{3} = 8.660$$

A1**Answer: $k = 5\sqrt{3}$** **3 marks****(c)** $y = \frac{150 + 180 + 72}{96} = 4.188 \text{ cm}$; by symmetry G is 6 cm from the vertical through A**M1 A1**

$$\tan \theta = \frac{4.188}{6}, \text{ so } \theta = 34.9^\circ$$

A1**Answer: 34.9°** **3 marks****28 (12 marks)****(a)** Moments about CD: rectangle area $2rh$ at distance $\frac{h}{2}$ below CD; semicircle area $\frac{1}{2}\pi r^2$ at $\frac{4r}{3\pi}$ above CD**B1 B1**

$$\text{For the centre of mass to lie on CD: } 2rh \times \frac{h}{2} = \frac{1}{2}\pi r^2 \times \frac{4r}{3\pi}$$

M1 A1

$$rh^2 = \frac{2}{3}r^3, \text{ so } h = r\sqrt{2/3}$$

A1**Answer: shown****5 marks****(b)** Rectangle: area $2r^2$, centre at height $\frac{r}{2}$. Semicircle: area $\frac{\pi}{2}r^2$, centre at height $r + \frac{4r}{3\pi}$ **B1 B1**

$$\text{With } r = 1: (2 + \frac{\pi}{2})y = 2 \times 0.5 + \frac{\pi}{2}(1 + \frac{4}{3\pi})$$

M1 A1

$$y = 0.9067 \text{ r}$$

A1

$$\tan \theta = \frac{y}{r} = 0.9067, \text{ so } \theta = 42.2^\circ$$

M1 A1**Answer: 42.2°** **7 marks****29 (8 marks)****(a)** Areas (in units of π): disc $4a^2$, hole a^2 , remaining $3a^2$ **B1**

$$\text{Moments about O: } 3a^2x = 4a^2 \times 0 - a^2 \times a$$

M1 A1

$$x = -\frac{a}{3}, \text{ so the distance from O is } \frac{a}{3}, \text{ on the side away from C}$$

A1**Answer: shown****4 marks****(b)** G is $\frac{a}{3}$ from O perpendicular to OP, and $OP = 2a$ **B1**

Taking P as the point of suspension, the line PG is vertical, so $\tan \theta = \frac{a}{3/2a} = \frac{1}{6}$

M1 A1

$$\theta = 9.46^\circ$$

A1

Answer: 9.46°

4 marks

30 (13 marks)

(a) $12h = 10 \times 0.4 + 2 \times 0.8 = 5.6$

M1 A1

$$h = 0.4667 \text{ m}$$

A1

Answer: 0.4667 m

3 marks

(b) The vertical through G passes through the lowest point of the base

M1

$$\tan \theta = \frac{0.3}{0.4667} = 0.6429$$

M1 A1

$$\theta = 32.7^\circ$$

A1

Answer: 32.7°

4 marks

(c) At the point of toppling, equilibrium requires $F = 12g \sin \theta \leq \mu R = \mu \times 12g \cos \theta$

M1 A1

$$\mu \geq \tan \theta = 0.643$$

A1

Answer: $\mu \geq 0.643$

3 marks

(d) Slipping when $\tan \theta = 0.55$, $\theta = 28.8^\circ$

M1 A1

$28.8^\circ < 32.7^\circ$, so the cylinder slides down the plane without toppling

E1

Answer: 28.8° ; it slides

3 marks

Rods, Beams and Ladders

1 (3 marks)

Moments about A: $R_C \times 5 = 120 \times 3$

M1 A1

$$R_C = 72 \text{ N, so } R_A = 120 - 72 = 48 \text{ N}$$

A1

Answer: $R_A = 48 \text{ N}$, $R_C = 72 \text{ N}$

3 marks

2 (5 marks)

Moments about B: $R_A \times 4 = 200 \times 2 + 700 \times 3$

M1 A1 A1

$$R_A = 625 \text{ N}$$

A1

$$R_B = 900 - 625 = 275 \text{ N}$$

A1

Answer: $R_A = 625 \text{ N}$, $R_B = 275 \text{ N}$

5 marks

3 (4 marks)

W is the weight of the ladder, acting vertically downwards through its midpoint

B1

R is the normal reaction of the wall on the ladder, horizontal and directed away from the wall

B1

N is the normal reaction of the ground, vertically upwards

B1

F is the friction force at the foot, horizontal and directed towards the wall (preventing the foot from slipping away)

B1

Answer: W down; R horizontal away from the wall; N up; F horizontal towards the wall

4 marks

4 (4 marks)

Moments about the foot: $R \times L \sin 60^\circ = 200 \times \frac{L}{2} \cos 60^\circ$

M1 A1

$$R = \frac{100}{\tan 60^\circ} = 57.7 \text{ N}$$

A1

$$\text{Horizontally, } T = R = 57.7 \text{ N}$$

B1

Answer: $T = 57.7 \text{ N}$

4 marks

5 (4 marks)

The plank's own weight acts through the pivot and has no moment

B1

Moments about the pivot: $24 \times x = 30 \times 1.5$

M1 A1

$$x = 1.875 \text{ m}$$

A1

Answer: 1.875 m

4 marks

6 (4 marks)

Reaction = $6 + 9 = 15 \text{ N}$

B1

Moments about C: $6 \times x = 9 \times (1.2 - x)$

M1 A1

$15x = 10.8$, so $x = AC = 0.7200 \text{ m}$

A1

Answer: AC = 0.7200 m, reaction 15 N

4 marks

7 (7 marks)

Vertically: $N = 15g = 147 \text{ N}$

B1

Moments about A: $R \times 4 \sin 65^\circ = 15g \times 2 \cos 65^\circ$

M1 A1

$R = 34.27 \text{ N}$

A1

Horizontally: $F = R = 34.27 \text{ N}$

B1

$F \leq \mu N$, so $\mu \geq \frac{34.27}{147} = 0.233$

M1 A1

Answer: R = 34.3 N, N = 147 N, F = 34.3 N, $\mu \geq 0.233$

7 marks

8 (6 marks)

Moments about A: $R_C \times 6 = 50g \times 4 + 80g \times 4$

M1 A1 A1

$R_C = 849.3 \text{ N}$

A1

$R_A = 130g - R_C = 424.7 \text{ N}$

M1 A1

Answer: $R_A = 425 \text{ N}$, $R_C = 849 \text{ N}$

6 marks

9 (6 marks)Let the ladder have weight W and length $2a$ at angle θ . Moments about the foot: $R \times 2a \sin \theta = W \times a \cos \theta$

M1 A1

$R = \frac{W}{2 \tan \theta}$; $F = R$ and $N = W$

B1

$F \leq \mu N$; $\frac{1}{2 \tan \theta} \leq 0.4$, so $\tan \theta \geq 1.25$

M1 A1

$\theta \geq 51.3^\circ$

A1

Answer: 51.3°

6 marks

10 (7 marks)

Moments about the foot: $R \times 5 \sin 70^\circ = 20g \times 2.5 \cos 70^\circ + 70g \times 2 \cos 70^\circ$

M1 A1 A1

$R = \frac{\cos 70^\circ (490 + 1372)}{5 \sin 70^\circ} = 135.5 \text{ N}$

A1

Friction $F = R = 135.5 \text{ N}$

B1

$N = 90g = 882 \text{ N}$, so $\mu \geq \frac{135.5}{882} = 0.154$

M1 A1

Answer: F = 136 N, $\mu \geq 0.154$

7 marks

11 (4 marks)

When on the point of tipping, the reaction at C is zero

B1

Moments about D: $m g \times 2 = 40g \times 1$

M1 A1

$m = 20$

A1

Answer: m = 20

4 marks

12 (6 marks)

Moments about A: $T \sin 50^\circ \times 3 = 40g \times 1.5$

M1 A1

$T = 255.9 \text{ N}$

A1

$H = T \cos 50^\circ = 164.5 \text{ N}$, $V = 40g - T \sin 50^\circ = 196 \text{ N}$

M1 A1

$R = \sqrt{164^2 + 196^2} = 256 \text{ N}$

A1

Answer: T = 256 N, R = 256 N

6 marks

13 (5 marks)

$$R_A + R_B = 100g \text{ and } R_B = 2R_A, \text{ so } R_B = \frac{200}{3}g$$

M1 A1

$$\text{Moments about A: } R_B \times 5 = 30g \times 2.5 + 70g \times$$

M1 A1

$$\frac{1000}{3} = 75 + 70x, \text{ so } x = 3.690 \text{ m}$$

A1

Answer: 3.690 m from A

5 marks

14 (8 marks)

$$\text{Vertically: } N = 100g = 980 \text{ N}$$

B1

$$\text{Limiting friction: } F = 0.4 \times 980 = 392 \text{ N, and } F = R$$

M1 A1

$$\text{Moments about the foot: } R \times 6 \sin 60^\circ = 25g \times 3 \cos 60^\circ + 75g \times s \cos 60^\circ$$

M1 A1 A1

$$2036.9 = 367.5 + 367.5s, \text{ so } s = 4.54 \text{ m}$$

M1 A1

Answer: 4.54 m

8 marks

15 (7 marks)

$$\text{Moments about the foot: } R \times L \sin 60^\circ = 20g \times \frac{L}{2} \cos 60^\circ$$

M1 A1

$$R = \frac{98}{\tan 60^\circ} = 56.58 \text{ N}$$

A1

$$N = 20g = 196 \text{ N, so the greatest friction is } 0.2 \times 196 = 39.20 \text{ N}$$

B1 M1

$$\text{Horizontally, } R = F + P, \text{ so } P = 56.58 - 39.20 = 17.4 \text{ N}$$

M1 A1

Answer: P = 17.4 N

7 marks

16 (7 marks)

$$\text{Vertically: } N = 150$$

B1

$$\text{Moments about A: } R \times 5 \sin 65^\circ = 150 \times 2 \cos 65^\circ$$

M1 A1

$$R = 27.98 \text{ N}$$

A1

$$F = R, \text{ and } F \leq \mu N \text{ gives } \mu \geq \frac{27.98}{150}$$

M1 A1

$$\mu \geq 0.187$$

A1

Answer: $\mu \geq 0.187$

7 marks

17 (7 marks)

$$\text{The beam is about to tip about D when the reaction at C is zero: moments about D: } 200(x - 4) = 120 \times 1, \text{ with the load beyond D}$$

M1 A1

$$x = 4.6$$

A1

$$\text{The beam is about to tip about C when the reaction at D is zero: moments about C: } 200(2 - x) = 120 \times 1, \text{ with the load before C}$$

M1 A1

$$x = 1.4$$

A1

$$\text{So } 1.4 \leq x \leq 4.6$$

A1

Answer: $1.4 \leq x \leq 4.6$

7 marks

18 (7 marks)

$$\text{Man is at } s = \frac{10}{3} \text{ m from the foot}$$

B1

$$\text{Moments about the foot: } R \times 5 \sin 65^\circ = 20g \times 2.5 \cos 65^\circ + 60g \times \frac{10}{3} \cos 65^\circ$$

M1 A1 A1

$$R = 228.5 \text{ N}$$

A1

$$\text{Limiting friction: } R = \mu \times 80g, \text{ so } \mu = \frac{228.5}{784} = 0.291$$

M1 A1

Answer: $\mu = 0.291$

7 marks

19 (5 marks)

$$\text{Moments about A with the man at distance s: } T \sin 40^\circ \times 4 = 30g \times 2 + 80g \times s$$

M1 A1 A1

$$\text{With } T = 1200: 3085.4 = 588 + 784s$$

M1

$$s = 3.19 \text{ m (less than 4 m, so it is attainable)}$$

A1

Answer: 3.19 m

5 marks

20 (10 marks)

(a) The ground is smooth, so the tension in the rope equals the reaction R of the wall: $T = R$ B1

Moments about A: $R \times 5 \sin 60^\circ = 20g \times 2.5 \cos 60^\circ + T \times 3 \sin 60^\circ$ M1 A1 A1

$T \times 2 \sin 60^\circ = 245$, so $T = 141 \text{ N}$ M1 A1

Answer: $T = 141 \text{ N}$ 6 marks

(b) As before, $T(5 - AC) \sin 60^\circ = 20g \times 2.5 \cos 60^\circ = 245$, so $T(5 - AC) = \frac{245}{\sin 60^\circ}$ M1 A1

With $T = 200$: $5 - AC = 1.415$, so $AC = 3.59 \text{ m}$ M1 A1

Answer: $AC = 3.59 \text{ m}$ 4 marks

21 (11 marks)

(a) Ground: $F = 0.4N$. Wall: friction at B is $0.25R$ upwards B1

Horizontally: $R = F = 0.4N$ M1

Vertically: $N + 0.25R = W$, so $N + 0.1N = W$, $N = \frac{10W}{11}$ M1 A1

$R = 0.4 \times \frac{10W}{11} = \frac{4W}{11}$ A1

Answer: $N = 10W/11$ 5 marks

(b) Moments about A, ladder length $2a$: $R \times 2a \sin \theta + 0.25R \times 2a \cos \theta = W \times a \cos \theta$ M1 A1 A1

$\frac{4}{11} \times 2(\sin \theta + 0.25 \cos \theta) = \cos \theta$ M1

$\tan \theta = \frac{11}{8} - 0.25 = 1.125$ A1

$\theta = 48.4^\circ$ A1

Answer: $\theta = 48.4^\circ$ 6 marks

22 (11 marks)

(a) Moments about the foot: $R \times 2a \sin \theta = W \times a \cos \theta + w \times s \cos \theta$ M1 A1 A1

So $R = \frac{(Wa + ws) \cos \theta}{2a \sin \theta}$ A1

Vertically $N = W + w$, and equilibrium needs $R = F \leq \mu(W + w)$ M1

Hence $\tan \theta \geq \frac{Wa + ws}{2a\mu(W + w)}$ A1

Answer: shown 6 marks

(b) $\tan \theta \geq \frac{200 \times 2.5 + 600 \times 4}{2 \times 2.5 \times 0.3 \times 800} = \frac{2900}{1200}$ M1 A1

$\theta \geq 67.5^\circ$ A1

Answer: $\theta \geq 67.5^\circ$ 3 marks

(c) $s = 5$: $\tan \theta \geq \frac{500 + 3000}{1200} = 2.917$ M1

$\theta \geq 71.1^\circ$ A1

Answer: $\theta \geq 71.1^\circ$ 2 marks

23 (8 marks)

(a) $W = 90 + 150 = 240 \text{ N}$ B1

Moments about C: $150 \times 4 = 240 \times (x - 1)$ M1 A1

$x - 1 = 2.5$, so $x = 3.50 \text{ m}$ A1

Answer: 240 N , 3.50 m 4 marks

(b) The reaction at C is zero B1

Moments about D: $P \times 1 = 240 \times (5 - 3.5)$ M1 A1

$P = 360 \text{ N}$ A1

Answer: $P = 360 \text{ N}$ 4 marks

24 (8 marks)Friction at the wall $F_w = 0.3R$, acting upwards

B1

Moments about A: $R \times 5 \sin 65^\circ + 0.3R \times 5 \cos 65^\circ = 20g \times 2.5 \cos 65^\circ$

M1 A1 A1

 $R = 40.09 \text{ N}$

A1

Horizontally, $T = R = 40.1 \text{ N}$

A1

Vertically: $N + 0.3R = 20g$, so $N = 196 - 12.03 = 184 \text{ N}$

M1 A1

Answer: $T = 40.1 \text{ N}$, $N = 184 \text{ N}$

8 marks

25 (10 marks)(a) $N = 100g = 980 \text{ N}$

B1

Moments about the foot: $R \times 6 \sin 60^\circ = 25g \times 3 \cos 60^\circ + 75g \times 4 \cos 60^\circ$

M1 A1

 $F = R = 353.6 \text{ N}$

A1

Answer: $N = 980 \text{ N}$, $F = 354 \text{ N}$

4 marks

(b) $S = \sqrt{980^2 + 353.6^2} = 1042 \text{ N}$

M1 A1

 $\tan \lambda = \frac{F}{N} = \frac{354}{980}$, so $\lambda = 19.8^\circ$ to the vertical, tilted towards the wall

M1 A1

Answer: 1040 N at 19.8° to the vertical

4 marks

(c) $\mu \geq \frac{F}{N} = \tan \lambda$

M1

 $\mu \geq 0.361$

A1

Answer: $\mu \geq 0.361$

2 marks

26 (9 marks)(a) Moments about C: $R_D \times 3 = 30g \times 2$

M1 A1

 $R_D = 20g = 196 \text{ N}$

A1

Answer: 196 N

3 marks

(b) Each reaction is $45g$; moments about C: $45g \times 3 = 30g \times 2 + 60g(x - 1)$

M1 A1

 $135 = 60 + 60x - 60$, so $x = 2.25 \text{ m}$

A1

Answer: 2.25 m

3 marks

(c) Tipping about D when $R_C = 0$: moments about D: $60g(x - 4) = 30g \times 1$

M1 A1

 $x = 4.50 \text{ m}$

A1

Answer: 4.50 m

3 marks

27 (16 marks)(a) $AC = \frac{2.4}{\sin 60^\circ} = 2.771 \text{ m}$

B1

Moments about A: $N \times AC = 200 \times 4 \cos 60^\circ$, so $N = 144.3 \text{ N}$

M1 A1

 $F = N \sin 60^\circ = 125 \text{ N}$; $R = 200 - N \cos 60^\circ = 127.8 \text{ N}$

M1 A1

 $\mu \geq \frac{F}{R} = 0.978$

M1 A1

Answer: $N = 144 \text{ N}$, $F = 125 \text{ N}$, $R = 128 \text{ N}$, $\mu \geq 0.978$

7 marks

(b) In general $N = \frac{200 \times 4 \cos 60^\circ \sin 60^\circ}{h}$, $F = N \sin 60^\circ$, $R = 200 - N \cos 60^\circ$

M1 A1

In limiting equilibrium $N \sin 60^\circ = 0.4(200 - N \cos 60^\circ)$, so $N = \frac{80}{\sin 60^\circ + 0.4 \cos 60^\circ} = 75.05 \text{ N}$

M1 A1

 $h = \frac{200 \times 4 \times \cos 60^\circ \sin 60^\circ}{N} = 4.62 \text{ m}$

M1 A1

A larger h gives a smaller N and so less friction is needed, so $h \geq 4.62$

M1

Also $AC \leq 8$, so $h \leq 8 \sin 60^\circ = 6.93$; hence $4.62 \leq h \leq 6.93$

M1 A1

Answer: $4.62 \leq h \leq 6.93$

9 marks

28 (13 marks)(a) Horizontally: $R = \mu_1 N$. Vertically: $N + \mu_2 R = W$

M1 A1

 $R = \mu_1(W - \mu_2 R)$, so $R(1 + \mu_1 \mu_2) = \mu_1 W$

M1

 $R = \frac{\mu_1 W}{1 + \mu_1 \mu_2}$

A1

Answer: shown

4 marks

(b) Moments about the foot, ladder length $2a$: $R \times 2a \sin \theta + \mu_2 R \times 2a \cos \theta = W a \cos \theta$

M1 A1 A1

$$\tan \theta = \frac{W}{2R} - \mu_2$$

M1

$$= \frac{1 + \mu_1 \mu_2}{2\mu_1} - \mu_2 = \frac{1 - \mu_1 \mu_2}{2\mu_1}$$

A1

Answer: shown

5 marks

(c) $\tan \theta = \frac{1 - 0.2}{1.0} = 0.8$

M1

$$\theta = 38.7^\circ$$

A1

Answer: $\theta = 38.7^\circ$

2 marks

(d) $\mu_2 = 0$ gives $\tan \theta = \frac{1}{2\mu_1}$, the smooth wall result

B1

Friction at the wall provides an upward force that reduces the normal reaction needed at the ground, so less friction is required at the foot

E1

Answer: $\tan \theta = 1/(2\mu)$

2 marks

29 (12 marks)

(a) Horizontally the forces on the ladder are R (away from the wall), P (away from the wall) and F (towards the wall), so $F = R + P$

M1 A1

Moments about the foot: $R \times L \sin 60^\circ = 20g \times \frac{L}{2} \cos 60^\circ$

M1

$$R = \frac{98}{\tan 60^\circ} = 56.58 \text{ N}$$

A1

Answer: $R = 56.6 \text{ N}$

4 marks

(b) $N = 20g = 196 \text{ N}$, so $F \leq 0.5 \times 196 = 98 \text{ N}$

B1 M1

$R + P \leq 98$, so $P \leq 98 - 56.58 = 41.4 \text{ N}$

M1 A1

Answer: $P = 41.4 \text{ N}$

4 marks

(c) $P \geq 0$ requires $\mu W \geq R = \frac{W}{2 \tan \theta}$

M1 A1

$$\tan \theta \geq \frac{1}{2\mu} = 1, \text{ so } \theta \geq 45^\circ$$

M1 A1

Answer: $\theta = 45^\circ$

4 marks

30 (17 marks)

(a) Let N and R be the reactions of the ground and the wall. Horizontally: friction at $A = R$, so $0.5N = R$

M1 A1

Vertically: $N + 0.4R = 200$

M1 A1

$N + 0.2N = 200$, so $N = 167 \text{ N}$ and $R = 83.3 \text{ N}$

M1 A1

Answer: $N = 167 \text{ N}$, $R = 83.3 \text{ N}$

6 marks

(b) Friction at the wall $= 0.4R = 33.3 \text{ N}$

B1

Moments about A : $R \times 5 \sin \theta + 33.33 \times 5 \cos \theta = 200 \times 2 \cos \theta$

M1 A1 A1

$416.7 \tan \theta = 233.3$, so $\tan \theta = 0.5600$

M1

$$\theta = 29.2^\circ$$

A1

Answer: $\theta = 29.2^\circ$

6 marks

(c) Moments about A : $R' \times 5 \sin \theta = 200 \times 2 \cos \theta$, so $R' = 143 \text{ N}$

M1 A1

Horizontally $F = R'$, and $N' = 200$, so $\mu \geq \frac{142.9}{200} = 0.714$

M1 A1

This is greater than 0.5 , so friction at the wall has helped to hold the ladder

E1

Answer: $\mu \geq 0.714$; the wall friction contributes

5 marks