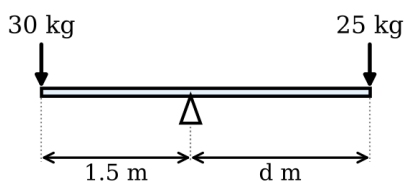


Moments

- 1 Find the moment about O of a force of 20 N whose line of action is at a perpendicular distance of 0.6 m from O. [2]

- 2 A spanner is used to tighten a nut. A force of 45 N is applied at right angles to the spanner at a distance of 0.30 m from the centre of the nut. Find the moment of the force about the centre of the nut. What force, applied at 0.25 m from the centre at right angles, would produce a moment of 30 N m? [3]

- 3 A light plank is balanced horizontally on a pivot. A child of mass 30 kg sits 1.5 m from the pivot on one side and a child of mass 25 kg sits d m from the pivot on the other side, as shown. Find d .



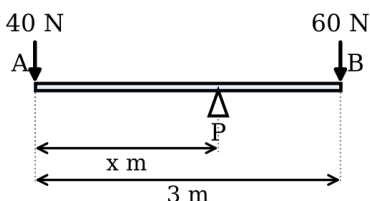
[2]

- 4 A force of 10 N acts at a perpendicular distance of 2 m from a point P, tending to turn a body clockwise about P. A second force of 8 N acts at a perpendicular distance of 1.5 m from P, tending to turn the body anticlockwise. Find the resultant moment about P, stating its sense. [2]

- 5 A light rod of length 0.5 m is freely pivoted at one end and held horizontal. A particle of mass 4 kg is attached to the other end. Take $g = 9.8 \text{ m s}^{-2}$. Find the moment of the weight of the particle about the pivot. [2]

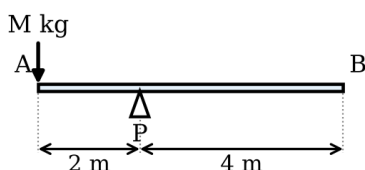
- 6 A uniform rod of weight 30 N and length 2 m is pivoted at one end and held in a horizontal position. Find the moment of the weight of the rod about the pivot. [2]

- 7 A light rod AB of length 3 m has a force of 40 N acting downwards at A and a force of 60 N acting downwards at B. The rod balances horizontally on a pivot at P, where $AP = x$ m, as shown. Find x and the magnitude of the force exerted by the pivot on the rod.



[4]

- 8 A uniform rod AB of length 6 m and mass 12 kg rests horizontally on a pivot at P, where $AP = 2$ m. A particle of mass M kg is attached at A, and the rod is in equilibrium, as shown. Find M .

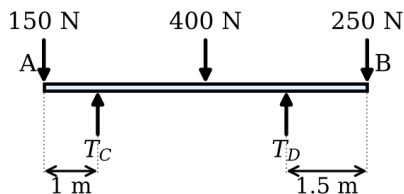


[4]

- 9 A door is hinged along one edge. A force of 50 N acts on the handle, which is 0.8 m from the hinge, in a direction making an angle of 60° with the plane of the door. Find the moment of the force about the hinge. [4]

- 10** A uniform girder AB of length 6 m and weight 400 N carries a load of weight 150 N attached at A and a load of weight 250 N attached at B. The girder hangs in a horizontal position from two light vertical cables, attached at the points C and D, where AC = 1 m and DB = 1.5 m, as shown.

Find the tension in each cable.

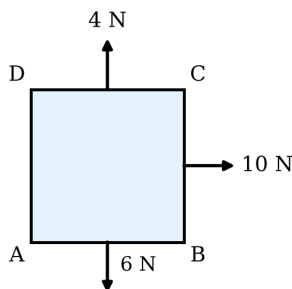


[6]

- 11** A force of magnitude F N acts at a point 0.4 m from a pivot at an angle of 40° to the line joining the pivot to the point of application. The force has a moment of 12 N m about the pivot. Find F .

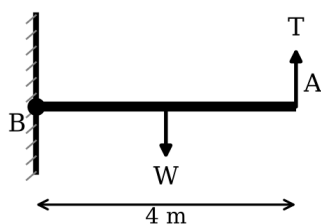
[3]

- 12** ABCD is a square lamina of side 3 m. Forces of 6 N, 10 N and 4 N act at the midpoints of AB, BC and CD respectively, as shown: the 6 N force acts downwards perpendicular to AB, the 10 N force acts perpendicular to BC away from the lamina, and the 4 N force acts upwards perpendicular to CD. Find the sum of the moments of the three forces about A, stating the sense.



[5]

- 13** A uniform rod AB of length 4 m and mass 8 kg is smoothly hinged at B to a wall. The rod is held horizontally by a light vertical string attached to A, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the tension T in the string.

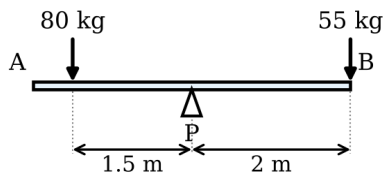


[3]

- 14** A uniform rod AB of length 5 m and weight 100 N rests horizontally on two supports C and D, where AC = 1 m and DB = 1 m. A particle of weight 40 N is placed on the rod at a point 3.5 m from A. Find the reactions at C and D.

[5]

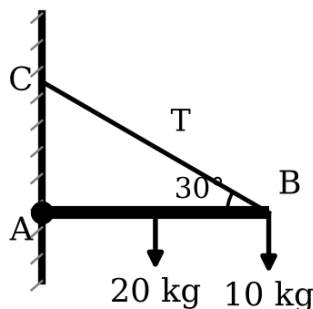
- 15** A non-uniform plank AB of length 4 m and mass 20 kg is supported at its midpoint P. A man of mass 80 kg stands on the plank 1.5 m from P on the side of A, and a child of mass 55 kg sits at B. The plank rests horizontally in equilibrium, as shown. Find the distance of the centre of mass of the plank from P, and state on which side of P it lies.



[5]

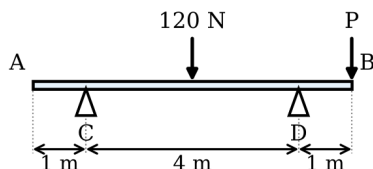
- 16** A uniform rod AB of mass 20 kg and length 3 m is smoothly hinged at A to a vertical wall. A particle of mass 10 kg is attached to the rod at B. The rod is held horizontal by a light string BC, where C is a point on the wall above A, and angle ABC = 30° , as shown. Take $g = 9.8 \text{ m s}^{-2}$.

Find the tension in the string.



[5]

- 17** A uniform beam AB of length 6 m and weight 120 N rests horizontally on two supports at C and D, where AC = 1 m and DB = 1 m. A load of weight P N is hung from B, as shown. Find the greatest value of P for which the beam remains in equilibrium without tipping.

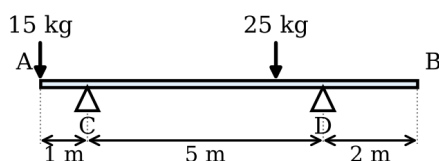


[4]

- 18** A uniform plank AB of length 5 m and mass 20 kg rests on two supports, one at A and one at a point C where AC = 4 m. A man of mass 60 kg walks along the plank from A towards B. Find the greatest distance from A that he can walk without the plank tipping.

[5]

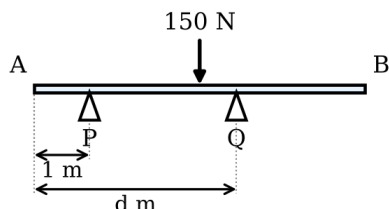
- 19** A uniform beam AB of length 8 m and mass 40 kg rests horizontally on supports at C and D, where AC = 1 m and DB = 2 m. A particle of mass 15 kg is attached at A and a particle of mass 25 kg is attached at the point 3 m from B, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the reactions at the supports.



[6]

- 20** A non-uniform rod AB of length 4 m and mass 10 kg hangs horizontally in equilibrium, suspended by two light vertical strings attached at A and B. The tension in the string at A is 70 N and the tension in the string at B is 28 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the distance of the centre of mass of the rod from A. [4]

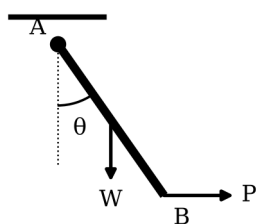
- 21** A uniform beam AB of length 6 m and weight 150 N rests horizontally on two smooth supports at P and Q, where AP = 1 m and AQ = d m with $d > 1$, as shown. The magnitude of the reaction at Q is three times the magnitude of the reaction at P. Find d. [6]



- 22** A wheelbarrow carries a load of weight 600 N. The weight of the wheelbarrow and load acts at a point 0.4 m horizontally from the axle of the wheel. The gardener lifts the handles with a vertical force F at a horizontal distance of 1.2 m from the axle. [3]
 (a) Taking moments about the axle, find F. [3]
 (b) Find the force exerted by the ground on the wheel. [2]
 (c) Explain why the wheelbarrow is easier to lift if the load is moved closer to the wheel. [1]

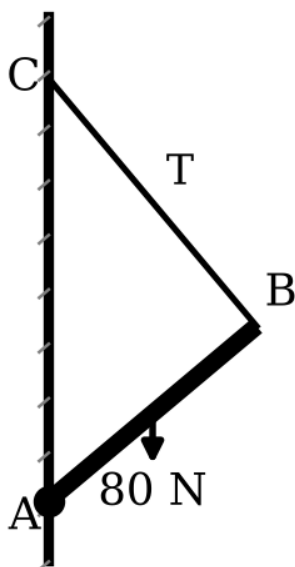
- 23** A uniform rod AB of length 4 m and mass 12 kg is held in a horizontal position by two smooth pegs, one at A and one at a point C of the rod, where AC = 1.5 m. The peg at A is above the rod and the peg at C is below the rod. A box of mass 30 kg is attached to the rod at B. Take $g = 9.8 \text{ m s}^{-2}$. [6]
 (a) Find the magnitude of the force exerted on the rod by each peg, and state the direction of each. [6]
 (b) The peg at A is removed and the box is replaced by a box of mass M kg at B. Find the greatest value of M for which the rod can rest horizontally on the peg at C alone. [3]

- 24** A uniform rod AB of weight W and length 2a is smoothly hinged at A. A horizontal force of magnitude P is applied at B, and the rod rests in equilibrium inclined at an angle θ to the downward vertical through A, as shown. [4]



- (a) Show that $P = \frac{1}{2}W \tan \theta$. [4]
 (b) Given that $W = 60 \text{ N}$ and $\theta = 35^\circ$, find P and the magnitude of the force exerted by the hinge on the rod. [5]

- 25** A uniform rod AB of weight 80 N and length 4 m is smoothly hinged at A to a vertical wall. The rod is held in equilibrium at 40° above the horizontal by a light string BC, which is perpendicular to the rod, where C is a point on the wall above A, as shown.



- (a) Find the tension T in the string. [4]
 (b) Find the magnitude of the force exerted by the hinge on the rod. [6]

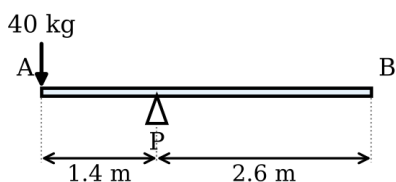
- 26** Forces act on a lamina in the x - y plane, where distances are in metres and the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions of the x - and y -axes. A force $(3\mathbf{i} + 4\mathbf{j})$ N acts at the point $(2, 1)$, a force $(-2\mathbf{i} + 5\mathbf{j})$ N acts at the point $(4, -1)$ and a force $(-6\mathbf{j})$ N acts at the point $(-1, 3)$. The moment of a force with components (X, Y) acting at the point (x, y) about the origin O is $xY - yX$ anticlockwise.

- (a) Find the moment about O of each of the three forces. [4]
 (b) Find the total moment about O and the magnitude of the resultant force. [4]

- 27** A uniform rod AB of weight W and length $2a$ is smoothly hinged at A to a vertical wall. The rod is held in a horizontal position by a light string BC, attached to the rod at B and to a point C on the wall vertically above A, with angle $ABC = \alpha$.

- (a) Show that the tension in the string is $T = \frac{W}{2 \sin \alpha}$. [4]
 (b) Find, in terms of W and α , the horizontal and vertical components of the force exerted by the hinge on the rod. [4]
 (c) The string will break if T exceeds $2W$. Find the least angle α for which the string does not break. [3]
 (d) Explain why the vertical component of the hinge force is independent of α . [1]

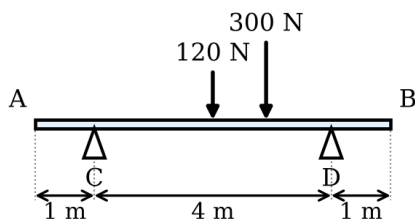
- 28** A uniform plank AB of length 4 m and mass 24 kg rests on a pivot at P, where $AP = 1.4$ m. A man of mass 40 kg stands at A, as shown. A child of mass 20 kg stands on the plank on the side of P towards B, at a distance x m from P.



- (a) Find the value of x for which the plank is in equilibrium in a horizontal position. [4]
 (b) State, with a reason, which end of the plank goes down if the child stands at B instead. [2]
 (c) The child is replaced by a particle of mass m kg at B. Find the value of m for which the plank balances horizontally. [3]

- 29** A uniform plank AB has length 4 m and unknown mass M kg. It rests on a pivot at P, where $AP = x$ m and $x < 2$. When a child of mass 20 kg sits at A, a child of mass 10 kg sitting at B balances the plank horizontally. When a child of mass 30 kg sits at A, a child of mass 16 kg sitting at B is needed to balance the plank.
- (a) Show that $20x = M(2 - x) + 10(4 - x)$ and write down a second equation involving x and M . [4]
- (b) Solve the equations to find x and M . [4]
- (c) A man of mass 40 kg sits at A. Find how far from P, on the side of B, a child of mass 25 kg must sit for the plank to balance. [3]

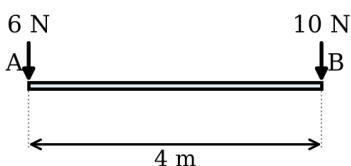
- 30** A uniform beam AB of length 6 m and weight 120 N rests horizontally on two supports at C and D, where $AC = 1$ m and $DB = 1$ m. A man of weight 300 N stands on the beam at a point X, where $AX = x$ m, as shown.



- (a) Show that the reaction at D is $R_D = (75x - 15)$ N, and find an expression for the reaction at C. [5]
- (b) Find the range of values of x for which the beam remains in equilibrium resting on both supports. [5]
- (c) Explain what happens to the beam if the man stands at A, and state the minimum weight of an additional load that could be placed at B to prevent this. [4]

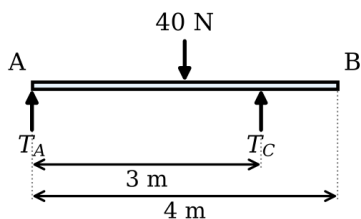
Equilibrium of Rigid Bodies

- 1** State the two conditions that must be satisfied for a rigid body acted on by a set of coplanar forces to be in equilibrium. [2]
- 2** Two parallel forces of 6 N and 10 N act in the same direction, perpendicular to a light rod AB of length 4 m, at A and B respectively, as shown. Find the magnitude of the resultant force and the distance of its line of action from A. [3]



- 3** A uniform rod of weight 40 N and length 2 m rests horizontally on two supports, one at each end. A particle of weight 20 N is placed at the midpoint of the rod. Find the reaction at each support. [3]
- 4** A uniform rod AB of weight 50 N is hinged at A and held horizontal by a vertical string attached at B. Find the tension in the string and the vertical component of the force exerted by the hinge on the rod. [3]

- 5 A uniform rod AB of length 4 m and weight 40 N is supported in a horizontal position by two vertical strings, one at A and one at C, where AC = 3 m, as shown. Find the tension in each string.

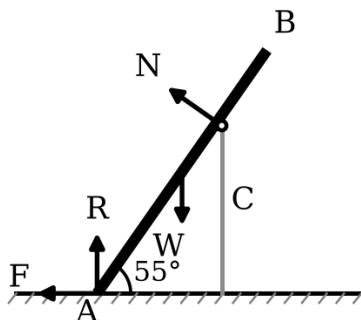


[3]

- 6 A uniform plank of weight 200 N and length 4 m lies across a stream, resting horizontally on the two banks at its ends A and B. A man of weight 600 N stands on the plank 1 m from B. Find the reaction at each end.

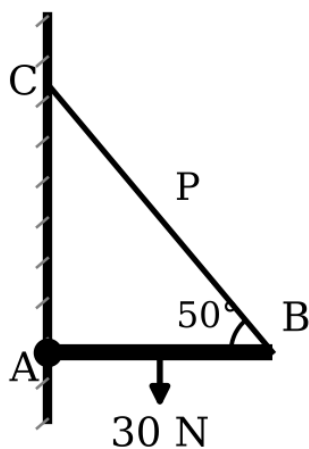
[3]

- 7 A uniform rod AB of length 7 m and mass 12 kg rests in equilibrium with the end A on rough horizontal ground and a point C on a smooth peg, where AC = 5 m. The rod is inclined at 55° to the horizontal, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the magnitude of the reaction at the peg, and the normal reaction and the friction force at A.



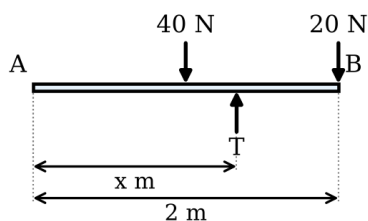
[6]

- 8 A uniform rod AB of length 2 m and weight 30 N is smoothly hinged at A to a vertical wall. It is held in a horizontal position by a light string BC attached to the wall above A, where the string makes an angle of 50° with the rod, as shown. Find the tension P in the string, and the horizontal and vertical components of the force exerted by the hinge on the rod.



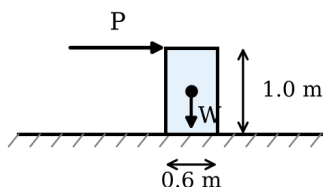
[6]

- 9** A uniform rod AB of length 2 m and weight 40 N has a particle of weight 20 N attached at B. The loaded rod hangs horizontally from a single vertical string attached to the rod at C, where $AC = x$ m, as shown. Find x and the tension in the string.



[4]

- 10** A uniform rectangular block of mass 20 kg stands on a rough horizontal floor. The base of the block is 0.6 m wide and its height is 1.0 m. A horizontal force P is applied to the top edge of one vertical face, perpendicular to that face, as shown. The coefficient of friction between the block and the floor is 0.5. Take $g = 9.8 \text{ m s}^{-2}$. Show that the block topples before it slides as P is increased, and find the value of P at which it begins to topple.



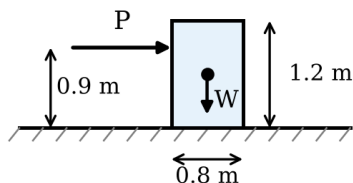
[5]

- 11** Two equal and opposite parallel forces of magnitude 12 N act at the ends of a light rod AB of length 0.5 m, each perpendicular to the rod. Such a pair of forces is called a couple.
- (a) Find the total moment of the two forces about the midpoint of the rod. [2]
- (b) Show that the moment of the couple is the same about the end A. [2]

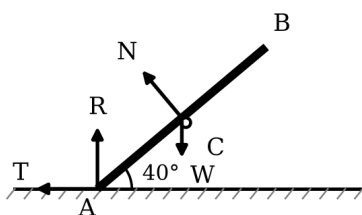
- 12** A uniform rod AB of length 2 m and weight 60 N is used as a lever. It rests on a smooth pivot at C, where $AC = 1.6$ m. A box of weight 600 N is hung from B and a vertical downward force of magnitude F N is applied at A, so that the rod rests in equilibrium in a horizontal position. Find F and the magnitude of the force exerted by the pivot on the rod. [6]

- 13** A non-uniform rod AB of length 3 m is hinged at A and held horizontal by a vertical string at B, which has tension 20 N. The centre of mass of the rod is 1.2 m from A. Find the weight of the rod and the vertical component of the hinge force. [4]

- 14** A uniform rectangular block of mass 25 kg stands on a rough horizontal floor. The base is 0.8 m wide and the block is 1.2 m high. A horizontal force P is applied to a vertical face, perpendicular to it, at a height of 0.9 m above the floor, as shown. The force is gradually increased from zero. Take $g = 9.8 \text{ m s}^{-2}$.

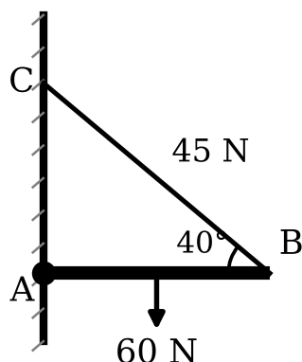


- Find the value of P at which the block is about to topple (assuming it does not slide first). [3]
 - The coefficient of friction between the block and the floor is 0.4. State whether the block slides or topples first, giving a reason. [2]
 - Find the least value of the coefficient of friction for which the block topples before it slides. [2]
-
- 15** A uniform rod AB of length 3 m and weight 80 N rests with its end A on smooth horizontal ground and touches a smooth peg C at a height of 1.0 m above the ground. The rod is inclined at 40° to the horizontal. It is held in equilibrium by a horizontal string attached to A , as shown. Find the tension in the string and the normal reactions at the ground and the peg.



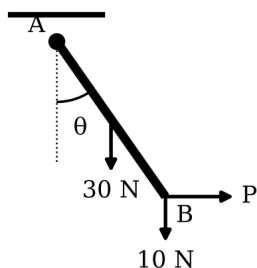
[7]

- 16** A non-uniform rod AB of length 3 m and weight 60 N is smoothly hinged at A to a vertical wall. It is held in a horizontal position by a light string BC attached to the wall above A . The string makes an angle of 40° with the rod and its tension is 45 N, as shown. Find the distance of the centre of mass of the rod from A , and the magnitude of the force exerted by the hinge on the rod.



[7]

- 17** A uniform rod AB of weight 30 N is smoothly hinged at A and hangs freely. A particle of weight 10 N is attached at B. A horizontal force of magnitude P N is applied at B, and the rod rests in equilibrium inclined at an angle θ to the downward vertical, as shown.

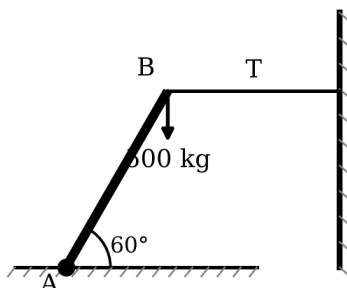


- (a) Show that $\tan \theta = \frac{P}{25}$. [3]
 (b) Find θ when $P = 15$. [2]

- 18** A uniform rod AB of length 2 m and weight 80 N has its end A in contact with a rough vertical wall. The rod is held in equilibrium in a horizontal position by a light string BC, where C is a point of the wall vertically above A and angle $ABC = 40^\circ$.

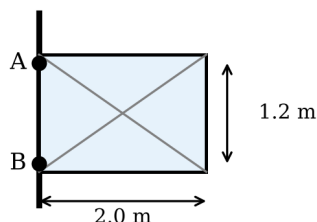
- (a) Find the tension in the string. [3]
 (b) Find the least possible value of the coefficient of friction between the rod and the wall. [6]

- 19** A uniform boom AB of mass 200 kg and length 6 m is smoothly hinged at A to horizontal ground. It is held at 60° to the horizontal by a horizontal cable attached to B and to a vertical wall. A load of mass 500 kg hangs from B, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the tension in the cable and the magnitude of the force exerted by the hinge on the boom.



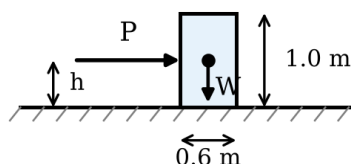
[8]

- 20** A uniform rectangular gate of weight 150 N and width 2 m is hung from a vertical post by two hinges A and B, where A is vertically above B and $AB = 1.2 \text{ m}$, as shown. The upper hinge exerts only a horizontal force on the gate. Find the force exerted on the gate by each hinge.



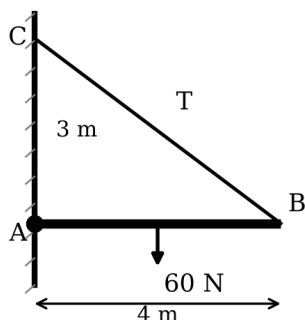
[6]

- 21** A uniform rectangular block of weight W stands on a rough horizontal floor with coefficient of friction $\mu = 0.4$. The block has a square base of width 0.6 m and height 1.0 m. A horizontal force P is applied to a vertical face of the block, perpendicular to it, at a height h m above the floor, as shown, and is gradually increased.



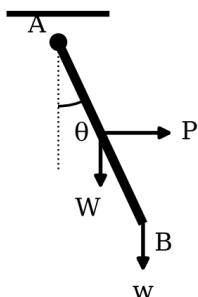
- (a) Show that the block topples (without sliding) when $P = \frac{0.3W}{h}$, and that it slides when $P = 0.4W$. [4]
 (b) Hence find the range of values of h for which the block slides before it topples. [3]
 (c) Find the least coefficient of friction needed for the block to topple before sliding when $h = 1.0$. [2]

- 22** A uniform rod AB of weight 60 N and length 4 m is smoothly hinged at A to a vertical wall. The rod is held horizontal by a light string BC, where C is on the wall 3 m vertically above A, as shown.



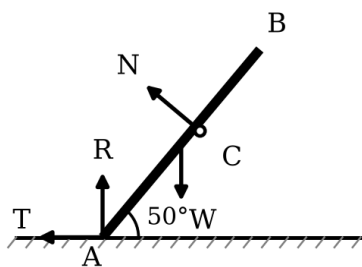
- (a) Find the tension in the string. [4]
 (b) Find the magnitude and direction of the force exerted by the hinge on the rod. [4]
 (c) Show that the three forces on the rod (weight, string tension and hinge force) act through a common point. [3]

- 23** A uniform rod AB of weight W and length $2a$ is smoothly hinged at A and hangs in equilibrium. A particle of weight w is attached to the rod at B. A horizontal force of magnitude P is applied to the midpoint of the rod, and the rod is inclined at an angle θ to the downward vertical, as shown.



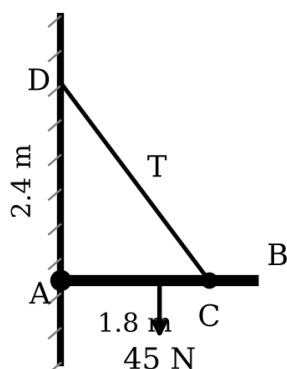
- (a) Show that $\tan \theta = \frac{P}{W + 2w}$. [4]
 (b) Given that $W = 40$ N, $w = 12$ N and $P = 18$ N, find θ and the magnitude of the force exerted by the hinge on the rod. [4]

- 24** A uniform rod AB of length 4 m and weight 100 N rests with its end A on smooth horizontal ground and a point C on a smooth peg. It is held at 50° to the horizontal by a horizontal string attached to A, the tension in the string being 40 N, as shown. Find the distance AC and the height of the peg above the ground, and show that the reaction of the ground at A is greater than 50 N.



[8]

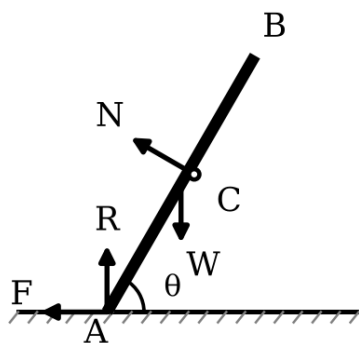
- 25** A uniform rod AB of weight 45 N and length 2.4 m is smoothly hinged at A to a vertical wall. The rod is held in a horizontal position by a light string attached to the point C of the rod, where $AC = 1.8$ m, and to the point D on the wall, which is 2.4 m vertically above A, as shown.



- (a) Find the tension in the string. [4]
 (b) Find the horizontal and vertical components of the force exerted by the hinge on the rod. [4]

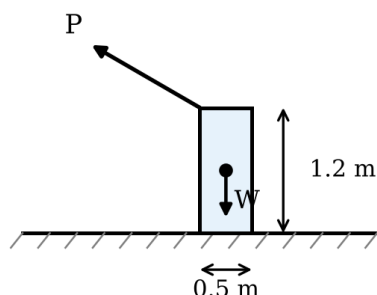
- 26** A uniform boom AB of mass 200 kg and length 6 m is smoothly hinged at A to horizontal ground and held at 60° to the horizontal by a horizontal cable attached to B. A load of mass m kg hangs from B. The cable will break if its tension exceeds 5000 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the greatest mass of load that can be hung from B, giving your answer to 3 significant figures. [6]

- 27 A uniform rod AB of length $2a$ and weight W rests in equilibrium with the end A on rough horizontal ground and a point C on a smooth peg at a height h above the ground. The rod is inclined at an angle θ to the horizontal, as shown. The coefficient of friction between the rod and the ground is μ .



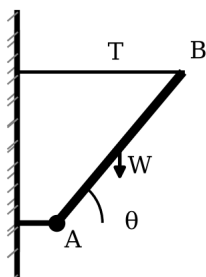
- Show that the reaction at the peg is $N = \frac{Wa \sin \theta \cos \theta}{h}$. [4]
- Show that the friction force at A is $F = N \sin \theta$ and find an expression for the normal reaction R. [4]
- Given that $a = 1.5$ m, $h = 1.2$ m, $W = 100$ N and $\theta = 60^\circ$, find the least value of μ for which equilibrium is possible. [6]
- Explain what happens to the friction required as the rod is made steeper (θ close to 90°). [1]

- 28 A uniform rectangular block of weight 200 N stands on a rough horizontal floor, with base width 0.5 m and height 1.2 m. A light string is attached to the top left-hand corner of the block and is pulled, away from the block, with tension P at an angle of 30° above the horizontal, as shown. The coefficient of friction between the block and the floor is 0.4. The tension P is gradually increased from zero.



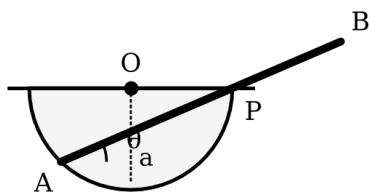
- Find the value of P at which the block is about to topple about its bottom left-hand edge. [4]
- Find the value of P at which the block is about to slide. [4]
- State, with a reason, whether the block topples or slides first. [1]
- Find the least value of the coefficient of friction for which the block topples before it slides. [4]

- 29** A uniform rod AB of weight W and length $2a$ is smoothly hinged at its lower end A to a vertical wall, with the rod pointing up and away from the wall, inclined at an angle θ to the horizontal. It is held in equilibrium by a light horizontal string attached to B and to the wall, as shown.



- Show that the tension in the string is $T = \frac{W}{2 \tan \theta}$. [4]
- Show that the force exerted by the hinge on the rod makes an angle ϕ with the horizontal, where $\tan \phi = 2 \tan \theta$. [3]
- When $W = 80 \text{ N}$ and $\theta = 50^\circ$, find the tension and the magnitude of the hinge force. [4]
- The string will break if its tension exceeds W . Find the range of values of θ for which the string does not break. [3]

- 30** A uniform rod AB of length $3a$ and weight W rests in equilibrium in a smooth hemispherical bowl of radius a and centre O, as shown. The rim of the bowl is horizontal. The end A of the rod is in contact with the inner surface of the bowl and a point P of the rod is in contact with the rim. The rod makes an angle θ with the horizontal.

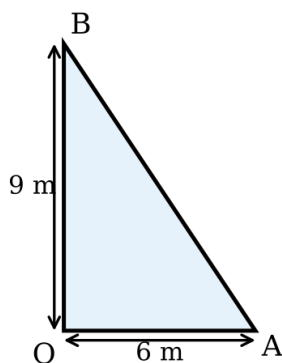


- By considering the isosceles triangle OAP, show that $AP = 2a \cos \theta$, and show that the reaction at A makes an angle 2θ with the horizontal. [5]
- By taking moments about A, show that the reaction at P is $S = \frac{3W}{4}$, and hence show that $8\cos^2\theta - 3\cos\theta - 4 = 0$. [7]
- Find θ . [3]

Centres of Mass

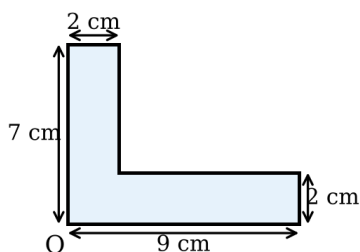
- Particles of mass 2 kg, 3 kg and 5 kg are placed at the points with x-coordinates 1 m, 4 m and 6 m on a straight line. Find the x-coordinate of the centre of mass of the particles. [3]
- Particles of mass 2 kg, 3 kg and 5 kg are placed at the points (1, 2), (4, 0) and (2, 6) respectively, where the units of length are metres. Find the coordinates of the centre of mass of the particles. [4]
- A uniform rod of mass 4 kg and length 2 m has a particle of mass 1 kg attached to one end. Find the distance of the centre of mass of the loaded rod from the other end. [3]

- 4 A uniform triangular lamina OAB has a right angle at O, with OA = 6 m and OB = 9 m, as shown. Find the distances of its centre of mass from OA and from OB.



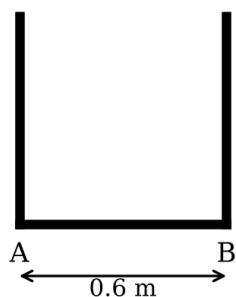
[2]

- 5 The diagram shows a uniform L-shaped lamina made from two rectangles, one measuring 9 cm by 2 cm and the other 2 cm by 5 cm. Find the distance of the centre of mass of the lamina from the left-hand edge (the edge of length 7 cm).



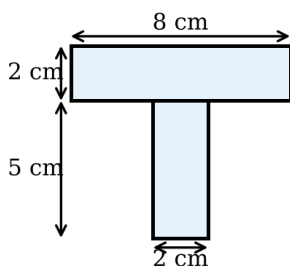
[4]

- 6 A uniform wire of length 1.8 m is bent into the shape of three sides of a square, as shown (each side is 0.6 m). Find the distance of the centre of mass of the wire from the side AB.



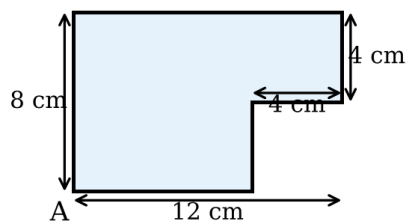
[4]

- 7 A uniform T-shaped lamina is made from two rectangles: the top bar measures 8 cm by 2 cm and the stem measures 2 cm by 5 cm, as shown. Find the distance of the centre of mass of the lamina from the bottom of the stem, and state its distance from the left-hand edge.



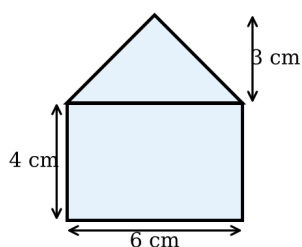
[5]

- 8** A uniform rectangular lamina measures 12 cm by 8 cm. A square of side 4 cm is cut from one corner, as shown. Find the distances of the centre of mass of the remaining lamina from the two edges meeting at A.



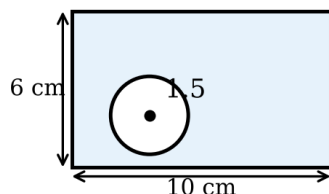
[6]

- 9** A uniform lamina is formed by joining an isosceles triangle, of base 6 cm and height 3 cm, to the top of a rectangle measuring 6 cm by 4 cm, as shown. Find the distance of the centre of mass of the lamina from the base of the rectangle.



[5]

- 10** A uniform rectangular lamina measures 10 cm by 6 cm. A circular disc of radius 1.5 cm is removed from it. The centre of the disc is 3 cm from the left-hand edge and 2 cm from the bottom edge, as shown. Find the distances of the centre of mass of the remaining lamina from the left-hand edge and from the bottom edge.

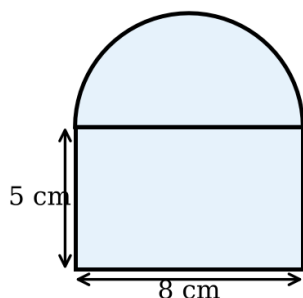


[7]

- 11** Particles A, B and C of masses 2 kg, 3 kg and 5 kg are placed at the points (1, 4), (5, -2) and (-3, 0) respectively, where the units of length are metres. A fourth particle D of mass 10 kg is placed so that the centre of mass of the four particles is at the point (2, 1). Find the coordinates of D.

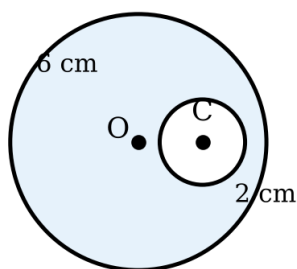
[6]

- 12** A uniform lamina consists of a rectangle measuring 8 cm by 5 cm with a semicircle of radius 4 cm attached along one of the 8 cm edges, as shown. The centre of mass of a uniform semicircular lamina of radius r is at a distance $\frac{4r}{3\pi}$ from the centre of its straight edge. Find the distance of the centre of mass of the whole lamina from the opposite edge of the rectangle.



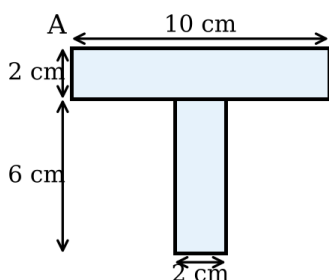
[6]

- 13** A uniform circular lamina has centre O and radius 6 cm. A circular hole of radius 2 cm is cut in the lamina with its centre C at a distance of 3 cm from O , as shown. Find the distance of the centre of mass of the remaining lamina from O , and state on which side of O it lies.



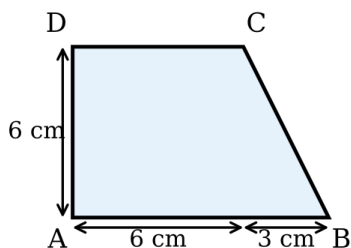
[5]

- 14** A uniform T-shaped lamina is made from a rectangle 10 cm by 2 cm and a rectangle 2 cm by 6 cm, as shown.



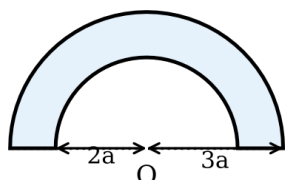
- (a) Show that the centre of mass is 5.5 cm from the bottom of the stem. [4]
 (b) The lamina is freely suspended from the corner A and hangs in equilibrium. Find the angle that the top edge makes with the vertical. [4]

- 15** A uniform lamina ABCD is formed by joining a square of side 6 cm to a right-angled triangle, as shown ($AB = 9$ cm, $DC = 6$ cm, $AD = 6$ cm and angle $DAB = 90^\circ$). The lamina is freely suspended from D and hangs in equilibrium. Find the angle that DC makes with the vertical.



[7]

- 16** A uniform lamina is formed by removing a semicircle of radius $2a$ from a larger semicircle of radius $3a$. Both semicircles have the same centre O and the same straight edge, as shown. The centre of mass of a uniform semicircular lamina of radius r is at a distance $\frac{4r}{3\pi}$ from the centre of its straight edge. Find the distance of the centre of mass of the lamina from O.

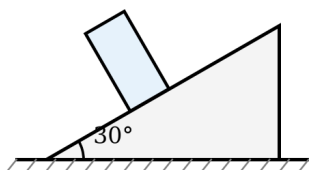


[6]

- 17** A uniform rod AB of mass 6 kg and length 4 m has particles of mass 2 kg and 4 kg attached at A and B respectively. The loaded rod is suspended from a light vertical string attached at the point P and rests horizontally. Find AP.

[4]

- 18** A uniform solid cylinder of radius 0.3 m and height 1.0 m stands with its flat base on a rough plane. The plane is slowly tilted. Assume that the cylinder does not slip.

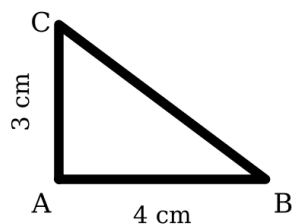


- (a) Show that the cylinder topples when the plane is inclined at about 31° to the horizontal.
- (b) The coefficient of friction between the cylinder and the plane is 0.5. Determine whether the cylinder topples or slips first.

[4]

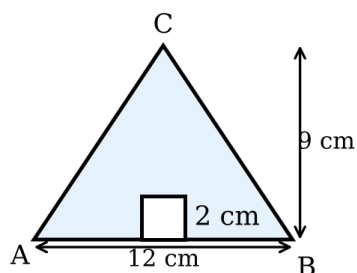
[2]

- 19** A uniform wire is bent into the shape of a right-angled triangle ABC with $AB = 4$ cm, $AC = 3$ cm and angle $BAC = 90^\circ$, as shown.



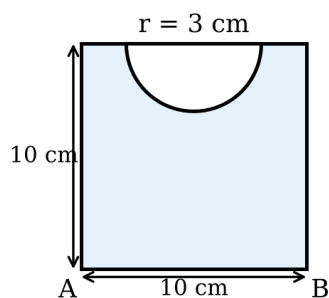
- (a) Find the distances of the centre of mass of the wire from AB and from AC. [5]
 (b) The wire is freely suspended from B and hangs in equilibrium. Find the angle that BA makes with the vertical. [3]

- 20** A uniform lamina is formed by removing a square of side 2 cm from an isosceles triangle ABC, in which $AB = 12$ cm, the perpendicular height of C above AB is 9 cm and $CA = CB$. The square has one side on AB and is placed symmetrically about the axis of symmetry of the triangle, as shown.



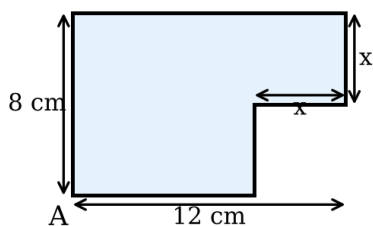
- (a) Find the distance of the centre of mass of the lamina from AB. [5]
 (b) The lamina is freely suspended from A and hangs in equilibrium. Find the angle that AB makes with the vertical. [3]

- 21** A uniform square lamina of side 10 cm has a semicircular hole of radius 3 cm cut from the middle of one edge, as shown. The centre of mass of a uniform semicircular lamina of radius r is at a distance $\frac{4r}{3\pi}$ from the centre of its straight edge.



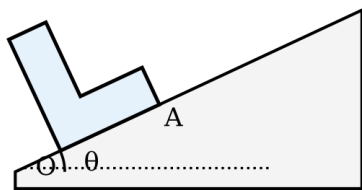
- (a) Find the distance of the centre of mass of the remaining lamina from the edge opposite the hole. [5]
 (b) The lamina is freely suspended from the corner A and hangs in equilibrium. Find the angle that AB makes with the vertical. [3]

- 22** A uniform rectangular lamina measures 12 cm by 8 cm. A square of side x cm is removed from one corner, as shown (the diagram shows $x = 4$). The distance of the centre of mass of the remaining lamina from the left-hand edge AD is d cm.



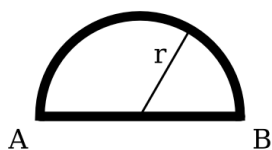
- (a) Show that $d = \frac{576 - 12x^2 + 0.5x^3}{96 - x^2}$. [4]
- (b) Given that $d = 5.2$, show that $x^3 - 13.6x^2 + 153.6 = 0$, and verify that $x = 4$ is a solution. [3]
- (c) Show that there is no other solution with $0 < x \leq 8$. [3]

- 23** A uniform L-shaped lamina is made from rectangles 8 cm by 3 cm and 3 cm by 6 cm. It rests in a vertical plane with the face OA in contact with a rough plane that is inclined at an angle θ to the horizontal, with O the lower end of the contact edge, as shown. The angle θ is slowly increased. The centre of mass of the lamina is 2.929 cm from the 9 cm edge through O and 3.429 cm from OA (the edge OA is 8 cm long).



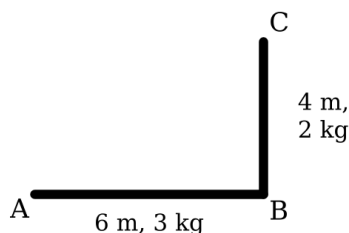
- (a) Find the value of θ at which the lamina is about to topple, assuming it does not slip. [4]
- (b) The coefficient of friction between the lamina and the plane is 0.7. Show that the lamina slips before it topples, and find the angle at which it slips. [3]

- 24** A uniform wire is bent into the shape of a semicircle of radius r with its diameter AB, the diameter being formed from a piece of the same wire, as shown. The centre of mass of a uniform semicircular arc of radius r is at a distance $\frac{2r}{\pi}$ from the centre of the diameter.



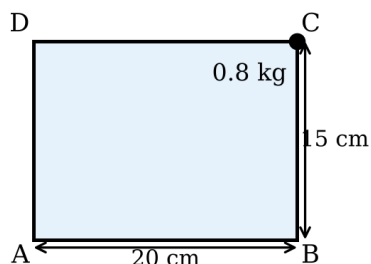
- (a) Show that the distance of the centre of mass of the whole wire from AB is $\frac{2r}{\pi + 2}$. [4]
- (b) The wire is freely suspended from A and hangs in equilibrium. Given that $r = 0.5$ m, find the angle that AB makes with the vertical. [3]

- 25** A uniform rod AB of mass 3 kg and length 6 m is rigidly joined at B to one end of a uniform rod BC of mass 2 kg and length 4 m, with BC perpendicular to AB, as shown.



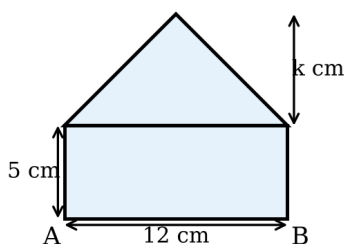
- (a) Find the distances of the centre of mass of the combined body from AB and from BC. [5]
 (b) The body is freely suspended from C and hangs in equilibrium. Find the angle that CB makes with the vertical. [3]

- 26** A uniform rectangular lamina ABCD has AB = 20 cm, BC = 15 cm and mass 2.4 kg. A particle of mass 0.8 kg is attached to the lamina at the corner C, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



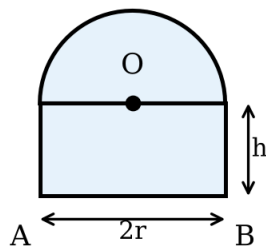
- (a) Find the distances of the centre of mass of the loaded lamina from AD and from AB. [5]
 (b) The loaded lamina hangs in a vertical plane with AB horizontal, supported by two vertical strings attached at A and B. Find the tension in each string. [5]
 (c) The strings are removed and the loaded lamina is freely suspended from A. Find the angle that AB makes with the vertical. [2]

- 27** A uniform lamina is formed by joining an isosceles triangle of base 12 cm and height k cm to the top of a rectangle measuring 12 cm by 5 cm, as shown. The centre of mass of the lamina is at height y cm above the base AB.



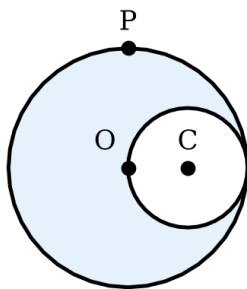
- (a) Show that $y = \frac{150 + 30k + 2k^2}{60 + 6k}$. [5]
 (b) Find the value of k for which the centre of mass lies on the join of the triangle and the rectangle. [3]
 (c) When $k = 6$, the lamina is freely suspended from A. Find the angle that AB makes with the vertical. [3]

- 28** A uniform lamina consists of a rectangle ABCD of width $2r$ and height h , with a semicircle of radius r attached along the edge CD, as shown (O is the midpoint of CD). The centre of mass of a uniform semicircular lamina of radius r is at a distance $\frac{4r}{3\pi}$ from its straight edge.



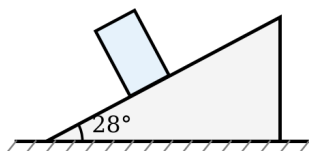
- (a) Show that the centre of mass of the lamina lies on CD if $h = r\sqrt{2/3}$. [5]
 (b) Now suppose $h = r$. The lamina is freely suspended from A and hangs in equilibrium. Find the angle that AB makes with the vertical. [7]

- 29** A uniform circular lamina has centre O and radius $2a$. A circular hole of radius a is cut out, with its centre C at a distance a from O, so that the hole touches the edge of the lamina, as shown. P is the point of the rim at the top of the lamina, with OP perpendicular to OC.



- (a) Show that the centre of mass of the remaining lamina is at a distance $\frac{a}{3}$ from O, on the opposite side to the hole. [4]
 (b) The lamina is freely suspended from P and hangs in equilibrium. Find the angle that OP makes with the vertical. [4]

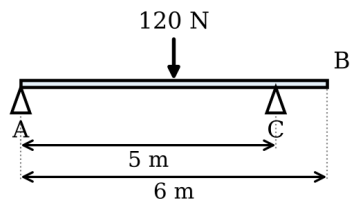
- 30** A uniform solid cylinder of mass 10 kg, radius 0.3 m and height 0.8 m has a particle of mass 2 kg fixed to the centre of its top face. The cylinder stands with its base on a rough plane which is slowly tilted, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the height of the centre of mass of the loaded cylinder above its base. [3]
 (b) Assuming the cylinder does not slip, find the angle of the plane when the cylinder is on the point of toppling. [4]
 (c) Find the least value of the coefficient of friction for which the cylinder topples before it slips. [3]
 (d) In fact $\mu = 0.55$. Find the angle of the plane when the cylinder begins to slip, and state what happens at this angle. [3]

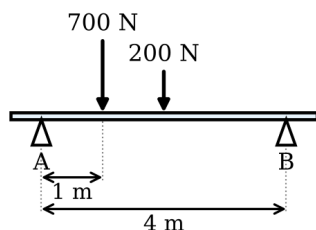
Rods, Beams and Ladders

- 1 A uniform beam AB of length 6 m and weight 120 N rests horizontally on two supports, one at A and one at C, where $AC = 5$ m, as shown. Find the reaction at each support.



[3]

- 2 A uniform plank of weight 200 N and length 5 m rests horizontally on supports at A and B, which are 4 m apart and each 0.5 m from an end of the plank. A man of weight 700 N stands on the plank 1 m from A, as shown. Find the reactions at A and at B.

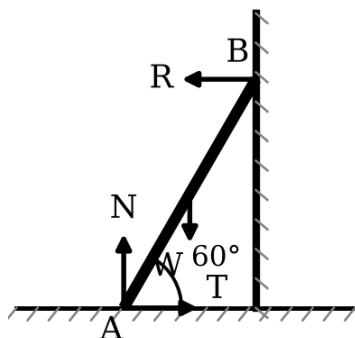


[5]

- 3 A uniform ladder rests with its upper end against a smooth vertical wall and its lower end on rough horizontal ground, as shown. Name each of the four forces R , N , F and W acting on the ladder and state the direction of each.

[4]

- 4 A uniform ladder of weight 200 N rests with its top against a smooth vertical wall and its foot on smooth horizontal ground. It is held at 60° to the horizontal by a horizontal rope attached to the foot of the ladder and to the base of the wall, as shown. Find the tension T in the rope.

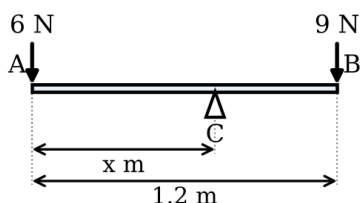


[4]

- 5 A uniform plank of length 4 m and mass 20 kg is balanced on a pivot at its midpoint. A child of mass 30 kg sits 1.5 m from the pivot on one side. Find the distance from the pivot at which a child of mass 24 kg must sit on the other side to keep the plank horizontal.

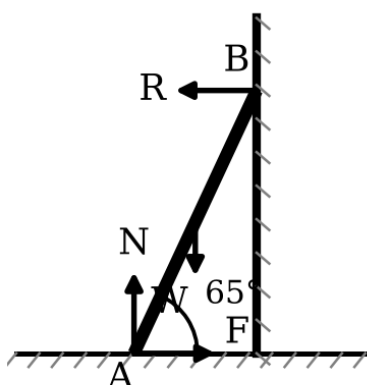
[4]

- 6 Particles of weight 6 N and 9 N are attached to the ends A and B of a light rod of length 1.2 m. The rod rests horizontally on a pivot at the point C, as shown. Find the distance AC and the reaction of the pivot on the rod.



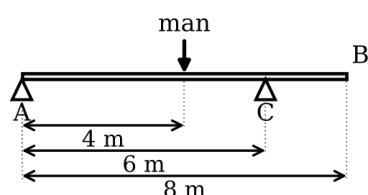
[4]

- 7 A uniform ladder AB of mass 15 kg and length 4 m rests with its upper end B against a smooth vertical wall and its lower end A on rough horizontal ground. The ladder makes an angle of 65° with the ground, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the normal reaction at the wall, the normal reaction and the friction force at the ground, and the least possible value of the coefficient of friction.



[7]

- 8 A uniform beam AB of mass 50 kg and length 8 m rests horizontally on supports at A and at C, where $AC = 6 \text{ m}$. A man of mass 80 kg stands on the beam 4 m from A, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the reactions at A and C.

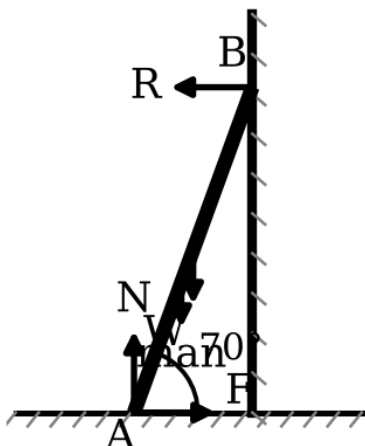


[6]

- 9 A uniform ladder rests with its top against a smooth vertical wall and its foot on rough horizontal ground, where the coefficient of friction is 0.4. Find the smallest angle that the ladder can make with the ground if it is to remain in equilibrium.

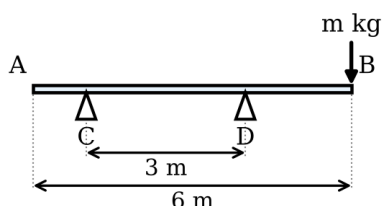
[6]

- 10** A uniform ladder of mass 20 kg and length 5 m rests with its upper end against a smooth vertical wall and its lower end on rough horizontal ground. The ladder makes an angle of 70° with the horizontal. A man of mass 70 kg stands on the ladder at a point 2 m from its lower end, as shown. The ladder remains in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$. Find the magnitude of the friction force at the foot of the ladder, and the least possible value of the coefficient of friction.



[7]

- 11** A uniform beam AB of mass 40 kg and length 6 m rests horizontally on supports at C and D, where $AC = 1 \text{ m}$ and $AD = 4 \text{ m}$. A block of mass $m \text{ kg}$ is placed at B, as shown. The beam is on the point of tipping about D. Take $g = 9.8 \text{ m s}^{-2}$. Find m .



[4]

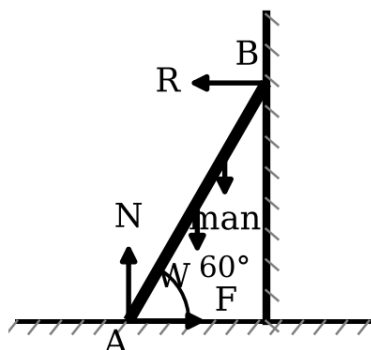
- 12** A uniform horizontal beam AB of mass 40 kg and length 3 m is smoothly hinged at A to a vertical wall. It is held in equilibrium by a light cable BC attached to the wall above A, making an angle of 50° with the beam, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the tension in the cable and the magnitude of the force exerted on the beam by the hinge.

[6]

- 13** A uniform plank AB of mass 30 kg and length 5 m rests horizontally on supports at its ends A and B. A man of mass 70 kg walks along the plank from A towards B. Find his distance from A when the reaction at B is twice the reaction at A. Take $g = 9.8 \text{ m s}^{-2}$.

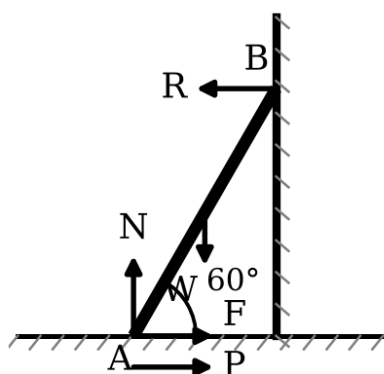
[5]

- 14** A uniform ladder of mass 25 kg and length 6 m rests with its top against a smooth vertical wall and its foot on rough horizontal ground, where the coefficient of friction is 0.4. The ladder makes an angle of 60° with the ground. A man of mass 75 kg climbs the ladder, as shown. Take $g = 9.8 \text{ m s}^{-2}$.
Find the greatest distance along the ladder from its foot that the man can climb before the ladder begins to slip.



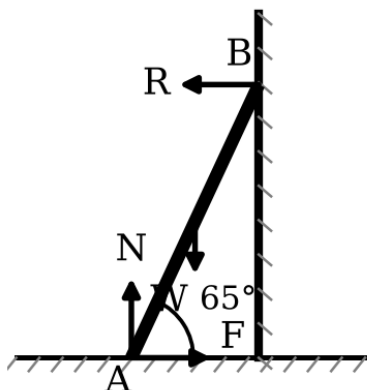
[8]

- 15** A uniform ladder of mass 20 kg rests with its upper end against a smooth vertical wall and its lower end on rough horizontal ground, where the coefficient of friction is 0.2. The ladder makes an angle of 60° with the ground. A horizontal force P , directed towards the wall, is applied to the foot of the ladder (as shown) to prevent it from slipping. Take $g = 9.8 \text{ m s}^{-2}$.
Find the least value of P .



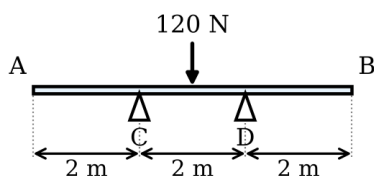
[7]

- 16** A non-uniform ladder AB of length 5 m and weight 150 N rests with its end B against a smooth vertical wall and its end A on rough horizontal ground. The centre of mass of the ladder is 2 m from A, and the ladder makes an angle of 65° with the ground. Find the least coefficient of friction needed to keep the ladder in equilibrium.



[7]

- 17** A uniform beam AB of length 6 m and weight 120 N rests horizontally on supports at C and D, where AC = 2 m and AD = 4 m, as shown. A load of weight 200 N is placed on the beam at a point P, where AP = x m. Find the range of values of x for which the beam remains in equilibrium resting on both supports.

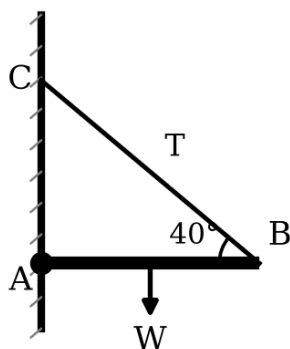


[7]

- 18** A uniform ladder of mass 20 kg and length 5 m rests with its top against a smooth vertical wall and its foot on rough horizontal ground, making an angle of 65° with the ground. A man of mass 60 kg can climb two thirds of the way up the ladder before it begins to slip. Find the coefficient of friction between the ladder and the ground. Take $g = 9.8 \text{ m s}^{-2}$.

[7]

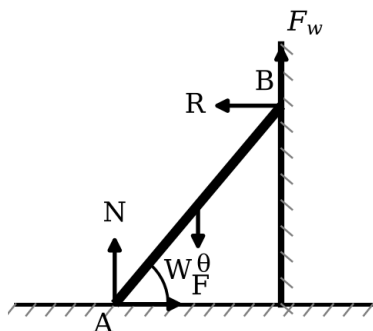
- 19** A uniform horizontal beam AB of mass 30 kg and length 4 m is smoothly hinged at A to a vertical wall and supported by a light cable BC, attached to the wall above A, which makes an angle of 40° with the beam, as shown. The cable will break if the tension in it exceeds 1200 N. A man of mass 80 kg walks along the beam from A. Take $g = 9.8 \text{ m s}^{-2}$. Find the greatest distance from A that he can safely walk.



[5]

- 20** A uniform ladder AB of mass 20 kg and length 5 m rests with its top B against a smooth vertical wall and its foot A on smooth horizontal ground. The ladder makes an angle of 60° with the ground and is held in equilibrium by a light horizontal rope. One end of the rope is attached to the ladder at the point C, where $AC = 3$ m, and the other end is attached to the wall at the same height as C. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the tension in the rope. [6]
- (b) The rope can withstand a tension of at most 200 N. Find the greatest distance AC at which the rope could be attached. [4]

- 21** A uniform ladder of weight W rests with its top B against a rough vertical wall and its foot A on rough horizontal ground, as shown. The coefficient of friction is 0.4 between the ladder and the ground and 0.25 between the ladder and the wall. The ladder is in limiting equilibrium, on the point of slipping, at an angle θ to the horizontal.

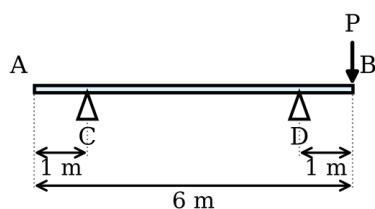


- (a) Show that the normal reaction at the wall is $R = \frac{4W}{11}$, and find the normal reaction at the ground in terms of W . [5]
- (b) Find θ . [6]

- 22** A uniform ladder of length $2a$ and weight W rests with its top against a smooth vertical wall and its foot on rough horizontal ground, where the coefficient of friction is μ . A man of weight w stands on the ladder at a distance s from its foot. The ladder makes an angle θ with the horizontal.

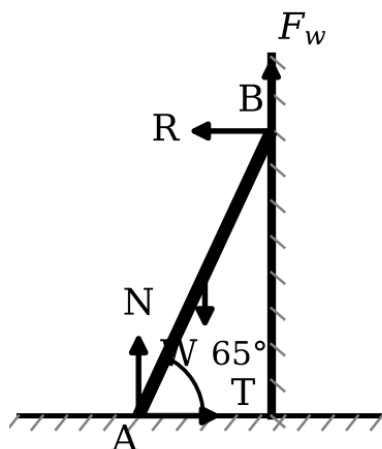
- (a) Show that the ladder is in equilibrium only if $\tan \theta \geq \frac{Wa + ws}{2a\mu(W + w)}$. [6]
- (b) Given that $W = 200 \text{ N}$, $w = 600 \text{ N}$, $\mu = 0.3$ and $a = 2.5 \text{ m}$, find the least angle θ when the man is 4 m from the foot of the ladder. [3]
- (c) Find the least angle needed if the man stands at the top of the ladder. [4]

- 23** A non-uniform beam AB of length 6 m rests horizontally on supports at C and D, where $AC = 1 \text{ m}$ and $DB = 1 \text{ m}$, as shown. The reaction at C is 90 N and the reaction at D is 150 N.



- (a) Find the weight of the beam and the distance of its centre of mass from A. [4]
- (b) A load of weight P is hung from B and the beam is on the point of tipping about D. Find P . [4]

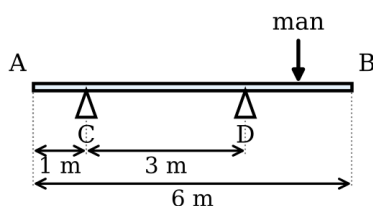
- 24** A uniform ladder of mass 20 kg and length 5 m rests with its top B against a rough vertical wall, where the coefficient of friction is 0.3, and its foot A on smooth horizontal ground. The ladder is held at 65° to the horizontal by a horizontal rope attached to A and to the base of the wall, as shown. The ladder is in limiting equilibrium, on the point of slipping down the wall. Take $g = 9.8 \text{ m s}^{-2}$. Find the tension in the rope and the normal reaction of the ground.



[8]

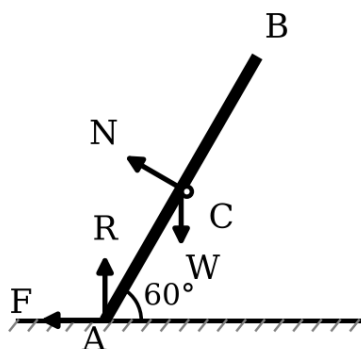
- 25** A uniform ladder of mass 25 kg and length 6 m rests with its top against a smooth vertical wall and its foot on rough horizontal ground, making an angle of 60° with the ground. A man of mass 75 kg stands on the ladder 4 m from its foot. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the normal reaction and the friction force at the ground. [4]
 - Hence find the magnitude and direction of the total force exerted by the ground on the ladder. [4]
 - Find the least coefficient of friction between the ladder and the ground. [2]

- 26** A uniform plank AB of mass 30 kg and length 6 m rests horizontally on supports at C and D, where $AC = 1 \text{ m}$ and $AD = 4 \text{ m}$, as shown. A man of mass 60 kg walks along the plank from A towards B. Take $g = 9.8 \text{ m s}^{-2}$.



- Find the reaction at D when the man is at the point C. [3]
- Find the distance of the man from A when the two reactions are equal. [3]
- Find the greatest distance from A that the man can walk without the plank tipping. [3]

- 27 A uniform ladder AB of length 8 m and weight 200 N rests with its foot A on rough horizontal ground and touches the smooth top edge C of a garden wall of height h m, as shown. The ladder makes an angle of 60° with the ground.

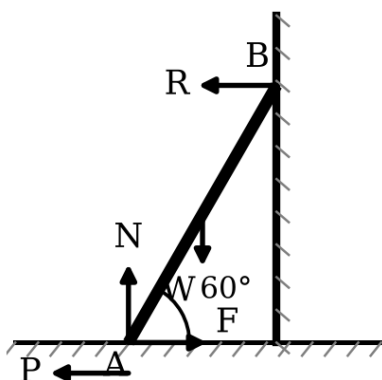


- (a) Given that $h = 2.4$, find the reaction at the wall edge, and the normal reaction and the friction force at A. Find the least coefficient of friction needed. [7]
- (b) For a different wall, the coefficient of friction is 0.4. Find the range of values of h for which the ladder can rest in equilibrium at this angle. [9]

- 28 A uniform ladder of weight W rests with its top against a rough vertical wall and its foot on rough horizontal ground. The coefficient of friction is μ_1 at the ground and μ_2 at the wall. The ladder is in limiting equilibrium, on the point of slipping, at an angle θ to the horizontal.

- (a) Show that the normal reaction at the wall is $R = \frac{\mu_1 W}{1 + \mu_1 \mu_2}$. [4]
- (b) Show that $\tan \theta = \frac{1 - \mu_1 \mu_2}{2\mu_1}$. [5]
- (c) Find θ when $\mu_1 = 0.5$ and $\mu_2 = 0.4$. [2]
- (d) Show that the formula reduces to the result for a smooth wall when $\mu_2 = 0$, and explain why the ladder can stand at a smaller angle when the wall is rough. [2]

- 29 A uniform ladder of mass 20 kg and length L rests with its top against a smooth vertical wall and its foot on rough horizontal ground, where the coefficient of friction is 0.5. The ladder makes an angle of 60° with the ground. A horizontal force P , directed away from the wall, is applied to the foot of the ladder, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that the friction force at the foot of the ladder is $F = R + P$, where R is the normal reaction at the wall, and find R . [4]
- (b) Find the greatest value of P for which the ladder remains in equilibrium. [4]
- (c) Show that if the angle of the ladder were smaller than a certain value, no such force P could be applied. Find that angle. [4]

- 30** A non-uniform ladder AB of length 5 m and weight 200 N has its centre of mass G at a distance 2 m from A. The end A rests on rough horizontal ground, with coefficient of friction 0.5, and the end B rests against a rough vertical wall, with coefficient of friction 0.4. The ladder is in limiting equilibrium, on the point of slipping at both A and B, and makes an angle θ with the horizontal.
- (a) Find the normal reaction of the ground and the normal reaction of the wall on the ladder. [6]
- (b) Find θ . [6]
- (c) The wall is now replaced by a smooth wall and the ladder is placed at the same angle θ . Find the least coefficient of friction at the ground needed for the ladder to remain in equilibrium, and comment on your answer. [5]
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