

Scalars and Vectors

1 (3 marks)

A vector has both magnitude and direction.

Velocity is a vector

B1

Weight is a vector (it is a force acting downwards)

B1

Displacement is a vector

B1

Answer: velocity, weight and displacement

3 marks

2 (3 marks)

$$\mathbf{p} + \mathbf{q} = (3 + 1)\mathbf{i} + (2 - 5)\mathbf{j} = 4\mathbf{i} - 3\mathbf{j}$$

B1

$$|\mathbf{p} + \mathbf{q}| = \sqrt{4^2 + (-3)^2} = \sqrt{25}$$

M1

$$|\mathbf{p} + \mathbf{q}| = 5$$

A1

Answer: $\mathbf{p} + \mathbf{q} = 4\mathbf{i} - 3\mathbf{j}$, $|\mathbf{p} + \mathbf{q}| = 5$

3 marks

3 (2 marks)

$$|12\mathbf{i} + 5\mathbf{j}| = \sqrt{144 + 25} = 13$$

M1

$$\text{Unit vector} = \frac{1}{13}(12\mathbf{i} + 5\mathbf{j})$$

A1

Answer: $\frac{1}{13}(12\mathbf{i} + 5\mathbf{j})$

2 marks

4 (4 marks)

The bearings differ by $150^\circ - 60^\circ = 90^\circ$, so the two legs are perpendicular. $OB^2 = 8^2 + 6^2 = 100$

M1

$OB = 10 \text{ km}$

A1

$\tan \theta = \frac{6}{8}$, so $\theta = 36.9^\circ$ (angle between OA and OB)

M1

Bearing of B from O = $60^\circ + 36.9^\circ = 096.9^\circ$

A1

Answer: 10 km, on a bearing of 096.9°

4 marks

5 (4 marks)

$$\text{Speed} = \sqrt{(-5)^2 + 12^2} = \sqrt{169}$$

M1

$$\text{Speed} = 13 \text{ m s}^{-1}$$

A1

The particle moves north-west: the angle west of north is $\tan^{-1}(\frac{5}{12}) = 22.6^\circ$

M1

$$\text{Bearing} = 360^\circ - 22.6^\circ = 337.4^\circ$$

A1

Answer: 13 m s^{-1} , bearing 337.4°

4 marks

6 (3 marks)

$$\text{Horizontal component} = 20 \cos 30^\circ = 17.3$$

M1 A1

$$\text{Vertical component} = 20 \sin 30^\circ = 10$$

A1

Answer: $(17.3\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$

3 marks

7 (3 marks)

A bearing of 120° is 30° south of east.

$$\text{East component} = 10 \sin 120^\circ = 8.66$$

M1 A1

$$\text{North component} = 10 \cos 120^\circ = -5.00$$

A1

Answer: $(8.66\mathbf{i} - 5\mathbf{j}) \text{ N}$

3 marks

8 (4 marks)

The wind adds a velocity of 30 km h^{-1} due east to the 120 km h^{-1} due north. Ground speed =

M1 A1

$$\sqrt{120^2 + 30^2}$$

$$= 124 \text{ km h}^{-1}$$

$$\tan \theta = \frac{30}{120}, \text{ so } \theta = 14.0^\circ \text{ east of north}$$

M1

$$\text{Bearing} = 014.0^\circ$$

A1

Answer: 124 km h^{-1} on a bearing of 014.0°

4 marks

9 (5 marks)

(a) $a + \lambda b = (2 + \lambda)\mathbf{i} + (3 + 4\lambda)\mathbf{j}$

M1

Parallel to $\mathbf{i} + 3\mathbf{j}$, so the $\mathbf{j}$ component is 3 times the $\mathbf{i}$ component: $3 + 4\lambda = 3(2 + \lambda)$

M1

$$\lambda = 3$$

A1

Answer: $\lambda = 3$

3 marks

(b) $a + 3b = 5\mathbf{i} + 15\mathbf{j}$

M1

$$|a + 3b| = \sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$$

A1

Answer: $5\sqrt{10}$

2 marks

10 (6 marks)

(a) $\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (8 - 2)\mathbf{i} + (-3 - 5)\mathbf{j} = 6\mathbf{i} - 8\mathbf{j}$

B1

$$AB = \sqrt{6^2 + (-8)^2} = \sqrt{100}$$

M1

$$AB = 10 \text{ m}$$

A1

Answer: $\mathbf{AB} = 6\mathbf{i} - 8\mathbf{j}$, distance 10 m

3 marks

(b) $\text{Velocity} = \frac{\mathbf{AB}}{4}$

M1

$$= (1.5\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$$

A1

$$\text{Speed} = \sqrt{1.5^2 + 2^2} = 2.5 \text{ m s}^{-1}$$

A1

Answer: $(1.5\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$, speed 2.5 m s^{-1}

3 marks

11 (5 marks)

$$\mathbf{r} = (-6 + 3t)\mathbf{i} + (4 + 5t)\mathbf{j}$$

B1

Due north of O means the $\mathbf{i}$ component is zero: $-6 + 3t = 0$, so $t = 2$

M1 A1

$\mathbf{r} = 14\mathbf{j}$, so the distance is 14 m

M1 A1

Answer: (a) $(-6 + 3t)\mathbf{i} + (4 + 5t)\mathbf{j}$, (b) $t = 2 \text{ s}$, distance 14 m

5 marks

12 (5 marks)

Position at time t : $(1 + 2t)\mathbf{i} + (7 - t)\mathbf{j}$

B1

On $y = x$: $1 + 2t = 7 - t$, so $3t = 6$ and $t = 2$

M1 A1

$$\text{Speed} = \sqrt{2^2 + (-1)^2} = \sqrt{5} \text{ cm s}^{-1}$$

M1

$$\text{Distance} = 2\sqrt{5} = 4.47 \text{ cm}$$

A1

Answer: $t = 2 \text{ s}$, distance $2\sqrt{5} = 4.47 \text{ cm}$

5 marks

13 (6 marks)

(a) $\mathbf{AB} = \mathbf{b} - \mathbf{a}$

B1

$$\mathbf{OP} = \mathbf{OA} + \mathbf{AP} = \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a})$$

M1

$$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$

A1

Answer: shown

3 marks

(b) $\mathbf{OP} = \frac{1}{3}(6\mathbf{i}) + \frac{2}{3}(2\mathbf{i} + 5\mathbf{j}) = (2 + \frac{4}{3})\mathbf{i} + \frac{10}{3}\mathbf{j} = \frac{10}{3}\mathbf{i} + \frac{10}{3}\mathbf{j}$

M1 A1

$$|\mathbf{OP}| = \frac{10}{3}\sqrt{2} = \sqrt{200/9}$$

A1

Answer: $\frac{10}{3}\sqrt{2}$

3 marks

14 (5 marks)

$$\text{Total displacement} = (3 - 8 + 2)\mathbf{i} + (4 + 2 - 10)\mathbf{j} = -3\mathbf{i} - 4\mathbf{j} \text{ km}$$

M1 A1

$$\text{Return flight is the negative: } 3\mathbf{i} + 4\mathbf{j}, \text{ length } \sqrt{3^2 + 4^2} = 5.00 \text{ km}$$

M1 A1

$$\tan \alpha = \frac{3}{4} \text{ with the vector in the north-east quadrant: } \alpha = 36.9^\circ; \text{ bearing} = 036.9^\circ$$

A1

Answer: 5 km on a bearing of 036.9°

5 marks

15 (5 marks)

$$p^2 + (p + 7)^2 = 169$$

M1

$$2p^2 + 14p - 120 = 0, \text{ so } p^2 + 7p - 60 = 0$$

M1

$$(p + 12)(p - 5) = 0, \text{ and } p > 0 \text{ so } p = 5$$

M1 A1

$$\text{Velocity} = 5\mathbf{i} + 12\mathbf{j}; \text{ bearing} = \tan^{-1}\left(\frac{5}{12}\right) = 022.6^\circ$$

A1

Answer: p = 5, bearing 022.6°

5 marks

16 (7 marks)

$$\text{(a) Displacement in 2 hours} = (2\mathbf{i} + 12\mathbf{j}) - (10\mathbf{i} + 4\mathbf{j}) = -8\mathbf{i} + 8\mathbf{j}$$

M1

$$\text{Velocity} = -4\mathbf{i} + 4\mathbf{j} \text{ km h}^{-1}$$

A1

$$\text{Answer: } -4\mathbf{i} + 4\mathbf{j} \text{ km h}^{-1}$$

2 marks

$$\text{(b) } \mathbf{r}_A = (10 - 4t)\mathbf{i} + (4 + 4t)\mathbf{j}$$

B1

$$\text{Equate the } \mathbf{j} \text{ components: } 4 + 4t = 22 - 2t$$

M1

$$t = 3, \text{ so they meet at 15:00}$$

A1

$$\text{Equate the } \mathbf{i} \text{ components: } 10 - 12 = 6 + k$$

M1

$$k = -8$$

A1

Answer: 15:00, k = -8

5 marks

17 (5 marks)

$$\mathbf{r}_P = (2 + 4t)\mathbf{i} + (10 - 3t)\mathbf{j} \text{ and } \mathbf{r}_Q = (22 - 2t)\mathbf{i} + 4\mathbf{j}$$

B1 B1

$$P \text{ is due west of } Q \text{ when their } \mathbf{j} \text{ components are equal: } 10 - 3t = 4$$

M1

$$t = 2 \text{ hours}$$

A1

$$\mathbf{r}_P = 10\mathbf{i} + 4\mathbf{j} \text{ and } \mathbf{r}_Q = 18\mathbf{i} + 4\mathbf{j}, \text{ so the distance is } 18 - 10 = 8 \text{ km}$$

A1

Answer: t = 2 h, distance 8 km

5 marks

18 (8 marks)

$$\text{(a) The velocity components are 5 (across) and 3 (along the bank): speed} = \sqrt{5^2 + 3^2}$$

M1

$$= \sqrt{34} = 5.83 \text{ m s}^{-1}$$

A1

Answer: 5.83 m s⁻¹

2 marks

$$\text{(b) Across the river the boat moves at } 5 \text{ m s}^{-1}; \text{ time} = \frac{60}{5}$$

M1

$$= 12 \text{ s}$$

A1

Answer: 12 s

2 marks

$$\text{(c) Downstream speed is } 3 \text{ m s}^{-1} \text{ for } 12 \text{ s: distance} = 3 \times 12$$

M1

$$= 36 \text{ m}$$

A1

Answer: 36 m

2 marks

$$\text{(d) } \tan \theta = \frac{5}{3} \text{ (across : along)}$$

M1

$$\theta = 59.0^\circ$$

A1

Answer: 59.0°

2 marks

19 (6 marks)

(a) Speed is the magnitude of the velocity vector, so the components cannot simply be added; the $\mathbf{i}$ and $\mathbf{j}$ directions are perpendicular.

E1

$$\text{Speed} = \sqrt{(-2)^2 + 6^2} = \sqrt{40}$$

M1

$$= 2\sqrt{10} \text{ m s}^{-1}$$

A1

Answer: $2\sqrt{10} \text{ m s}^{-1}$

3 marks

(b) The cyclist finishes at the starting point, so the displacement (final position minus initial position) is the zero vector, not 12 km.

E1

Displacement = 0

B1

The distance travelled is $3 + 4 + 5 = 12$ km (the return leg is $\sqrt{3^2 + 4^2} = 5$ km)

B1

Answer: displacement 0, distance 12 km

3 marks

20 (7 marks)

(a) At time t hours after 12:00: A is at $(4 + 2t)\mathbf{i} + (3 + t)\mathbf{j}$, B is at $(-2 + 4t)\mathbf{i} + (9 - t)\mathbf{j}$

B1 B1

$\mathbf{i}$ components equal: $4 + 2t = -2 + 4t$, so $t = 3$

M1

$\mathbf{j}$ components at $t = 3$: $3 + 3 = 6$ and $9 - 3 = 6$, equal, so the boats collide

M1

Position vector = $(10\mathbf{i} + 6\mathbf{j})$ km at 15:00

A1

Answer: $t = 3$ (15:00), position $(10\mathbf{i} + 6\mathbf{j})$ km

5 marks

(b) At $t = 1$: A = $6\mathbf{i} + 4\mathbf{j}$, B = $2\mathbf{i} + 8\mathbf{j}$, so AB = $4\mathbf{i} - 4\mathbf{j}$

M1

Distance = $\sqrt{32} = 4\sqrt{2} = 5.66$ km

A1

Answer: $4\sqrt{2} = 5.66$ km

2 marks

21 (4 marks)

(a) AB = $(5 - 2)\mathbf{i} + (7 - 1)\mathbf{j} = 3\mathbf{i} + 6\mathbf{j}$

M1

BC = $(11 - 5)\mathbf{i} + (19 - 7)\mathbf{j} = 6\mathbf{i} + 12\mathbf{j}$

M1

BC = 2AB, so AB and BC are parallel and share B, hence A, B, C are collinear

E1

Answer: shown

3 marks

(b) BC = 2AB, so AB : BC = 1 : 2

B1

Answer: 1 : 2

1 mark

22 (7 marks)

(a) The magnitude of a sum depends on the angle between the vectors.

E1

Magnitude 5 occurs only when they are perpendicular (Pythagoras), e.g. the sum of $3\mathbf{i}$ and $4\mathbf{j}$

E1

Answer: depends on the angle

2 marks

(b) Largest $3 + 4 = 7$ when the vectors point in the same direction

B1

Smallest $4 - 3 = 1$ when they point in opposite directions

B1

Answer: 7 (parallel), 1 (opposite)

2 marks

(c) $|a + b|^2 = (3 + 4\cos \theta)^2 + (4\sin \theta)^2 = 9 + 24\cos \theta + 16 = 25 + 24\cos \theta$

M1 A1

$25 + 24\cos \theta = 37$, so $\cos \theta = \frac{1}{2}$ and $\theta = 60^\circ$

A1

Answer: $\theta = 60^\circ$

3 marks

23 (7 marks)

(a) The northward component of the airspeed must cancel the wind: $200 \sin \varphi = 40$, where φ is the angle north of east

M1 A1

$\varphi = 11.5^\circ$, so bearing = $090^\circ - 11.5^\circ = 078.5^\circ$

A1

Answer: 078.5°

3 marks

(b) Ground speed = $200 \cos \varphi = \sqrt{200^2 - 40^2}$

M1

= 196 km h^{-1}

A1

Time = $\frac{300}{196} = 1.53$ hours = 91.9 minutes

A1

Answer: 196 km h^{-1} , 91.9 minutes

3 marks

(c) The wind is again perpendicular to the track so the ground speed and the time are the same:

B1

91.9 minutes

Answer: 91.9 minutes

1 mark

24 (8 marks)

(a) Change in position = $(2\mathbf{i} + 0.5\mathbf{j}) - (4\mathbf{i} - \mathbf{j}) = -2\mathbf{i} + \frac{3}{2}\mathbf{j}$

M1

Velocity = $\frac{1}{0.5} \times \text{this} = -4\mathbf{i} + 3\mathbf{j} \text{ km h}^{-1}$

A1

Answer: $-4\mathbf{i} + 3\mathbf{j} \text{ km h}^{-1}$

2 marks

(b) $\mathbf{r} = (4\mathbf{i} - \mathbf{j}) + t(-4\mathbf{i} + 3\mathbf{j})$

M1

$$\mathbf{r} = (4 - 4t)\mathbf{i} + (-1 + 3t)\mathbf{j} \text{ km}$$

$$\text{Answer: } (4 - 4t)\mathbf{i} + (-1 + 3t)\mathbf{j}$$

A1

2 marks

$$\text{(c) At 09:30, } t = 1.5 \text{ and the buoy is at } (4 - 6)\mathbf{i} + (-1 + 4.5)\mathbf{j} = -2\mathbf{i} + \frac{7}{2}\mathbf{j}$$

M1 A1

$$\text{Patrol boat displacement in 0.5 h} = -2\mathbf{i} + \frac{7}{2}\mathbf{j} - (8\mathbf{i} + 2\mathbf{j}) = 6\mathbf{i} + \frac{3}{2}\mathbf{j}, \text{ so velocity} = 12\mathbf{i} + 3\mathbf{j}$$

M1

$$\text{Speed} = \sqrt{12^2 + 3^2} = \sqrt{153} = 12.4 \text{ km h}^{-1}$$

A1

$$\text{Answer: } 12\mathbf{i} + 3\mathbf{j} \text{ km h}^{-1}, \text{ speed } 12.4 \text{ km h}^{-1}$$

4 marks

25 (6 marks)

$$\text{(a) Angle south of east} = \arctan\left(\frac{3}{4}\right) = 36.87^\circ$$

M1

$$\text{Bearing} = 90^\circ + 36.9^\circ = 126.9^\circ$$

A1

$$\text{Answer: } 126.9^\circ$$

2 marks

$$\text{(b) After 2 hours the boat is at } 6\mathbf{i} + 8\mathbf{j} \text{ km}$$

B1

$$\text{Second stage for } s \text{ hours: } \mathbf{j}\text{-component } 8 - 3s = 0$$

M1

$$s = \frac{8}{3}, \text{ so the time is } 2 + \frac{8}{3} = 4 \text{ h } 40 \text{ min after noon, that is } 16:40$$

A1

$$\text{Distance} = 6 + 4 \times \frac{8}{3} = \frac{50}{3} = 16.7 \text{ km}$$

A1

$$\text{Answer: } 16:40, 16.7 \text{ km}$$

4 marks

26 (11 marks)

$$\text{(a) P: } (5 + 6t)\mathbf{i} + (18 + 2t)\mathbf{j}$$

B1

$$\text{Q: } (25 + 2t)\mathbf{i} + (13 + 5t)\mathbf{j}$$

B1

$$\text{Answer: P: } (5 + 6t)\mathbf{i} + (18 + 2t)\mathbf{j}, \text{ Q: } (25 + 2t)\mathbf{i} + (13 + 5t)\mathbf{j}$$

2 marks

$$\text{(b) QP} = \text{position of P} - \text{position of Q} = (-20 + 4t)\mathbf{i} + (5 - 3t)\mathbf{j}$$

M1

$$d^2 = (-20 + 4t)^2 + (5 - 3t)^2$$

M1

$$= 400 - 160t + 16t^2 + 25 - 30t + 9t^2 = 25t^2 - 190t + 425$$

A1

$$\text{Answer: shown}$$

3 marks

$$\text{(c) } d^2 = 25\left(t - \frac{19}{5}\right)^2 + 425 - 25 \times \frac{361}{25}$$

M1

$$= 25\left(t - 3.8\right)^2 + 64, \text{ least when } t = 3.8 \text{ (3 hours 48 minutes after noon)}$$

A1

$$\text{Least } d = \sqrt{64} = 8 \text{ km}$$

A1

$$\text{Answer: } 8 \text{ km, at } t = 3.8 \text{ hours}$$

3 marks

$$\text{(d) } d^2 < 100: 25t^2 - 190t + 325 < 0, \text{ i.e. } t^2 - 7.6t + 13 < 0$$

M1

$$\text{Roots: } t = \frac{7.6 \pm \sqrt{57.76 - 52}}{2} = \frac{7.6 \pm 2.4}{2}, \text{ so } t = 2.6 \text{ and } t = 5$$

A1

$$\text{Duration} = 5 - 2.6 = 2.4 \text{ hours}$$

A1

$$\text{Answer: } 2.4 \text{ hours}$$

3 marks

27 (9 marks)

$$\text{(a) } \mathbf{OM} = \mathbf{OA} + \mathbf{AM} = \mathbf{a} + \frac{1}{2}\mathbf{AB}$$

M1

$$= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

A1

$$\text{Answer: shown}$$

2 marks

$$\text{(b) } \mathbf{OG} = \frac{2}{3}\mathbf{OM} = \frac{1}{3}(\mathbf{a} + \mathbf{b})$$

B1

$$\mathbf{AG} = \mathbf{OG} - \mathbf{OA} = \frac{1}{3}(\mathbf{a} + \mathbf{b}) - \mathbf{a} = \frac{1}{3}(\mathbf{b} - 2\mathbf{a})$$

M1

$$\mathbf{ON} = \frac{1}{2}\mathbf{b}, \text{ so } \mathbf{AN} = \frac{1}{2}\mathbf{b} - \mathbf{a} = \frac{1}{2}(\mathbf{b} - 2\mathbf{a})$$

M1

$$\text{So } \mathbf{AG} = \frac{2}{3}\mathbf{AN}: \mathbf{AG} \text{ is parallel to } \mathbf{AN} \text{ and } \mathbf{A} \text{ is common, so } \mathbf{A}, \mathbf{G} \text{ and } \mathbf{N} \text{ are collinear}$$

E1

$$\mathbf{AG} = \frac{2}{3}\mathbf{AN} \text{ means } \mathbf{AG} : \mathbf{GN} = 2 : 1$$

A1

$$\text{Answer: } \mathbf{AG} : \mathbf{GN} = 2 : 1$$

5 marks

$$(c) \text{ OG} = \frac{1}{3}(a + b) = \frac{1}{3}(8\mathbf{i} + 6\mathbf{j}), \text{ so } G = \left(\frac{8}{3}, 2\right)$$

M1

$$N = (1, 3) \text{ and } A + \frac{2}{3}AN = (6, 0) + \frac{2}{3}(-5, 3) = \left(\frac{8}{3}, 2\right) \text{ as required}$$

A1

$$\text{Answer: } G = \left(\frac{8}{3}, 2\right)$$

2 marks

28 (9 marks)

$$(a) p = \frac{1}{2}(a + b), q = \frac{1}{2}(b + c)$$

B1

$$r = \frac{1}{2}(c + d), s = \frac{1}{2}(d + a)$$

B1

Answer: midpoint formulae

2 marks

$$(b) PQ = q - p = \frac{1}{2}(b + c) - \frac{1}{2}(a + b) = \frac{1}{2}(c - a)$$

M1 A1

$$SR = r - s = \frac{1}{2}(c + d) - \frac{1}{2}(d + a) = \frac{1}{2}(c - a)$$

M1

PQ = SR, so PQ and SR are equal and parallel, hence PQRS is a parallelogram

E1

Answer: proved

4 marks

$$(c) P(3, 0.5), Q(6.75, 3.25), R(4.25, 5.25), S(0.5, 2.5)$$

B1 B1

$$PQ = (3.75, 2.75) \text{ and } SR = (4.25 - 0.5, 5.25 - 2.5) = (3.75, 2.75)$$

B1

$$\text{Answer: } PQ = SR = (3.75, 2.75)$$

3 marks

29 (9 marks)

$$(a) \text{ Position of S: } (-16 + 6t)\mathbf{i} + (8 - 2t)\mathbf{j}$$

B1

$$d^2 = (-16 + 6t)^2 + (8 - 2t)^2$$

M1

$$= 256 - 192t + 36t^2 + 64 - 32t + 4t^2 = 40t^2 - 224t + 320$$

A1

Answer: shown

3 marks

$$(b) d^2 = 40(t - 2.8)^2 + 320 - 40 \times 7.84$$

M1

Least when $t = 2.8$, i.e. at 11:48

A1

$$d_{\min}^2 = 6.4, \text{ so } d_{\min} = 2.53 \text{ km}$$

A1

Answer: 2.53 km at 11:48

3 marks

$$(c) d = 10: 40t^2 - 224t + 220 = 0, \text{ i.e. } 10t^2 - 56t + 55 = 0$$

M1

$$t = \frac{56 \pm \sqrt{3136 - 2200}}{20} = \frac{56 \pm \sqrt{936}}{20}$$

M1

$$\text{Difference of roots} = \frac{\sqrt{936}}{10} = 3.06 \text{ hours (about 184 minutes)}$$

A1

Answer: 3.06 hours

3 marks

30 (9 marks)

$$(a) \text{ Displacement in } 0.5 \text{ h} = 4\mathbf{i} + 3\mathbf{j}$$

M1

$$\text{Velocity} = 8\mathbf{i} + 6\mathbf{j} \text{ km h}^{-1}$$

A1

$$\text{Speed} = \sqrt{8^2 + 6^2} = 10 \text{ km h}^{-1}$$

A1

Answer: $8\mathbf{i} + 6\mathbf{j} \text{ km h}^{-1}$, 10 km h^{-1}

3 marks

$$(b) r_s = (-20\mathbf{i} - 10\mathbf{j}) + t(8\mathbf{i} + 6\mathbf{j})$$

M1

$$= (-20 + 8t)\mathbf{i} + (-10 + 6t)\mathbf{j} \text{ km}$$

A1

Answer: $(-20 + 8t)\mathbf{i} + (-10 + 6t)\mathbf{j}$

2 marks

$$(c) \text{ At } 11:00 (t = 1) \text{ S is at } -12\mathbf{i} - 4\mathbf{j}, \text{ and L is due north of S, so L has } \mathbf{i}\text{-coordinate } -12$$

B1

$$\text{At } 12:00 (t = 2) \text{ S is at } -4\mathbf{i} + 2\mathbf{j}$$

M1

$$\text{L is north-west of S, so L is } d \text{ km west and } d \text{ km north of S: } 12 = -4 - d, \text{ so } d = 8$$

M1

$$L = -12\mathbf{i} + 10\mathbf{j}$$

A1

Answer: $-12\mathbf{i} + 10\mathbf{j} \text{ km}$

4 marks

Units and Modelling Assumptions

1 (3 marks)

(a) The SI unit of mass is the kilogram

B1

Answer: kg

1 mark

(b) Acceleration is a change of speed in m s^{-1} per second

B1

Answer: m s^{-2}

1 mark

(c) Force = mass $\times$ acceleration, in kg m s^{-2} , called the newton

B1

Answer: N (= kg m s^{-2})

1 mark

2 (3 marks)

(a) $54 \text{ km h}^{-1} = \frac{54 \times 1000}{3600} \text{ m s}^{-1}$

M1

$= 15 \text{ m s}^{-1}$

A1

Answer: 15 m s^{-1}

2 marks

(b) $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, so $36\,000 \text{ cm}^2 = 3.6 \text{ m}^2$

B1

Answer: 3.6 m^2

1 mark

3 (2 marks)

N: force = mass $\times$ acceleration, so $1 \text{ N} = 1 \text{ kg m s}^{-2}$

B1

J: work = force $\times$ distance, so $1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}$

B1

Answer: $\text{N} = \text{kg m s}^{-2}$, $\text{J} = \text{kg m}^2 \text{ s}^{-2}$

2 marks

4 (2 marks)

$90 \text{ km h}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ m s}^{-1}$

B1

$25 \text{ m s}^{-1} = 25 \times \frac{3600}{1000} = 90 \text{ km h}^{-1}$

B1

Answer: 25 m s^{-1} ; 90 km h^{-1}

2 marks

5 (2 marks)

Light: the mass of the string is negligible, so the tension is the same throughout the string

B1

Smooth: there is no friction between the surface and the object

B1

Answer: light: no mass; smooth: no friction

2 marks

6 (3 marks)

(a) The object is treated as a point mass: its size, shape and any rotation are ignored, and all its mass acts at one point

B1

Answer: a point mass, size ignored

1 mark

(b) The string has negligible mass (so the tension is the same throughout) and does not stretch (so its length is fixed)

B1

Answer: negligible mass and fixed length

1 mark

(c) There is no friction between the surface and the object

B1

Answer: no friction

1 mark

7 (4 marks)

(a) Dimensions of $\frac{1}{g} = \frac{L}{L T^{-2}} = T^2$

M1

so $\sqrt{\frac{1}{g}}$ has dimension T, the same as the left-hand side, so the formula is consistent

A1

Answer: consistent

2 marks

(b) Dimensions of $\frac{m v^2}{r} = \frac{M (L T^{-1})^2}{L}$

M1

$= M L T^{-2}$, which is the dimension of force (mass $\times$ acceleration)

A1

Answer: dimension $M L T^{-2}$

2 marks

8 (2 marks)

$$1 \text{ g cm}^{-3} = \frac{10^{-3} \text{ kg}}{10^{-6} \text{ m}^3} = 1000 \text{ kg m}^{-3}$$

M1

$$2.7 \text{ g cm}^{-3} = 2700 \text{ kg m}^{-3}$$

A1

Answer: 2700 kg m⁻³

2 marks

9 (3 marks)

$$[v] = \text{LT}^{-1}, \text{ so } [\tfrac{1}{2}mv^2] = \text{M}(\text{LT}^{-1})^2 = \text{ML}^2\text{T}^{-2}$$

M1 A1

$$[Fs] = (\text{MLT}^{-2})(\text{L}) = \text{ML}^2\text{T}^{-2}, \text{ the same}$$

B1

Answer: both ML²T⁻²

3 marks

10 (2 marks)

Light: the tension is the same at every point of the string

B1

Inextensible: the string does not stretch, so both particles have the same acceleration (magnitude) along the string

B1

Answer: same tension; same acceleration

2 marks

11 (3 marks)

(a) The car is small compared with the distances involved and does not rotate, so its size can be ignored

B1

(b) Not suitable when the car is parking or turning in a small space, where its length and rotation matter

B1 B1

Answer: (a) size negligible compared with distance; (b) e.g. manoeuvring in a small space

3 marks

12 (3 marks)

$$[a] = \text{LT}^{-2}, [u] = [v] = \text{LT}^{-1}, [s] = \text{L}$$

B1

$$(i) [ut] = \text{L}, [at^2] = \text{L}: \text{consistent. (iii)} [(u + v)t] = \text{L}: \text{consistent}$$

B1

$$(ii) [at^2] = \text{L} \text{ but } [u] = \text{LT}^{-1}, \text{ so not consistent}$$

B1

Answer: (i) and (iii) only

3 marks

13 (4 marks)

$$(a) 1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$$

B1

$$1 \text{ W} = 1 \text{ J s}^{-1} = 1 \text{ kg m}^2 \text{ s}^{-3}$$

B1

Answer: kg m² s⁻³

2 marks

$$(b) 2.4 \text{ kW h} = 2400 \text{ W} \times 3600 \text{ s}$$

M1

$$= 8\,640\,000 \text{ J} = 8.64 \times 10^6 \text{ J}$$

A1

Answer: 8.64 × 10⁶ J

2 marks

14 (4 marks)

Light string: negligible mass, so the tension has the same magnitude on both sides of the pulley

B1

Inextensible string: A and B have the same speed and the same magnitude of acceleration

B1

Smooth pulley: no friction at the pulley, so there is no change in tension as the string passes over it

B1

Particles (and negligible air resistance): sizes ignored and no extra resistance forces, so only the weight and the tension act on each

B1

Answer: light, inextensible, smooth pulley, particles with no air resistance

4 marks

15 (5 marks)

Dimensions: $[v] = L T^{-1}$, $[g] = L T^{-2}$, $[h] = L$

B1

$$L T^{-1} = (L T^{-2})^a L^b = L^{a+b} T^{-2a}$$

M1

Powers of T: $-1 = -2a$, so $a = \frac{1}{2}$

A1

Powers of L: $1 = a + b$, so $b = \frac{1}{2}$

A1

$$v = k\sqrt{g h}$$

B1

Answer: $a = b = \frac{1}{2}$, $v = k\sqrt{g h}$

5 marks

16 (3 marks)

$$G = \frac{Fr^2}{m_1 m_2}, \text{ so } [G] = \frac{MLT^{-2} \times L^2}{M^2}$$

M1

$$= M^{-1} L^3 T^{-2}$$

A1

SI units: $kg^{-1} m^3 s^{-2}$

A1

Answer: $M^{-1} L^3 T^{-2}$; $kg^{-1} m^3 s^{-2}$

3 marks

17 (5 marks)

(a) $k = \frac{R}{v^2}$: units $N/(m s^{-1})^2 = N s^2 m^{-2}$

M1

$$= kg m^{-1}$$

A1

Answer: $kg m^{-1}$

2 marks

(b) $R = 0.4 \times 30^2 = 360 \text{ N}$

B1

New $R = 0.4 \times (33)^2 = 435.6 \text{ N}$

M1

Increase = $\frac{75.6}{360} \times 100 = 21\%$

A1

Answer: 360 N, 21%

3 marks

18 (5 marks)

$T^2 \div l = 4.00, 4.09, 3.94, 4.04, 4.10$

M1 A1

These are all approximately equal (about 4.03), so the belief is supported

E1

Mean constant $k = 4.03 s^2 m^{-1}$

B1

$$g = \frac{4\pi^2}{k} = 9.78 m s^{-2}$$

A1

Answer: supported, k about 4.03, g about $9.8 m s^{-2}$

5 marks

19 (3 marks)

$$k = \frac{4\pi^2 m}{T^2}, \text{ so } [k] = M T^{-2}$$

M1 A1

$$N m^{-1} = kg m s^{-2} m^{-1} = kg s^{-2}$$

A1

Answer: MT^{-2}

3 marks

20 (5 marks)

(a) Volume = $60\,000 \text{ cm}^3$, so mass = $60\,000 \times 0.74 = 44\,400 \text{ g}$

M1

$$= 44.4 \text{ kg}$$

A1

Answer: 44.4 kg

2 marks

(b) $0.74 \text{ g cm}^{-3} = 0.74 \times \frac{10^{-3}}{10^{-6}} = 740 \text{ kg m}^{-3}$

B1

Answer: 740 kg m^{-3}

1 mark

(c) 60 litres = 0.06 m^3 , so depth = $\frac{0.06}{0.5 \times 0.4} \text{ m}$

M1

$$= 0.3 \text{ m} = 30 \text{ cm}$$

A1

Answer: 30 cm

2 marks

21 (7 marks)

(a) $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = g$, $s = h$: $h = \frac{1}{2}gt^2$

M1

$$t^2 = \frac{2h}{g}, \text{ so } t = \sqrt{\frac{2h}{g}} \quad (t > 0)$$

A1

Answer: shown

2 marks

(b) $t = \sqrt{\frac{2 \times 4.0}{9.8}} = 0.9035 \text{ s}$

M1

$$\text{Excess} = 0.93 - 0.904 = 0.026 \text{ s}$$

A1

$$\text{Percentage} = \frac{0.026}{0.904} \times 100 = 2.9\%$$

A1

Answer: 0.904 s, measured time is 2.9% greater

3 marks

(c) Air resistance acts on the ball, so its acceleration is less than g

E1

Reaction time: the student starts and stops the timer late, which adds a small error to every time

E1

Answer: air resistance; reaction time in timing

2 marks

22 (7 marks)(a) The mass was not converted to kilograms: $1200 \text{ g} = 1.2 \text{ kg}$

E1

$$F = 1.2 \times 2.5 = 3 \text{ N}$$

B1

Answer: 3 N

2 marks

(b) Mass is the amount of matter (kg); weight is the gravitational force on the mass (N), so they are different quantities

E1

$$W = mg, \text{ so } m = \frac{686}{9.8}$$

M1

$$m = 70 \text{ kg}$$

A1

Answer: 70 kg

3 marks

(c) To convert km h^{-1} to m s^{-1} multiply by $\frac{1000}{3600}$; the student inverted the factor

E1

$$72 \times \frac{1000}{3600} = 20 \text{ m s}^{-1}$$

B1

Answer: 20 m s⁻¹

2 marks

23 (7 marks)

(a) $[p] = \text{ML}^{-3}$, $[v] = \text{LT}^{-1}$, $[A] = \text{L}^2$, $[F] = \text{MLT}^{-2}$

B1

$$\text{M: } a = 1. \text{ T: } -b = -2, \text{ so } b = 2$$

M1 A1

$$\text{L: } -3a + b + 2c = 1, \text{ so } -3 + 2 + 2c = 1 \text{ and } c = 1$$

A1

Answer: a = 1, b = 2, c = 1

4 marks

(b) $F = 0.25 \times 1.2 \times 12^2 \times 0.05$

M1

$$= 2.16 \text{ N}$$

A1

Answer: 2.16 N

2 marks

(c) $F \propto v^2$, so the factor is $3^2 = 9$

B1

Answer: 9

1 mark

24 (6 marks)

(a) The ball is a particle (size and spin are ignored)

B1

Air resistance (and wind) is negligible

B1

The acceleration due to gravity is constant and the ground is horizontal

B1

Answer: particle; no air resistance; constant g

3 marks

(b) Air resistance opposes the motion of the ball

B1

so the horizontal speed decreases throughout the flight

B1

so the actual range is smaller than R

B1

Answer: smaller than R

3 marks

25 (5 marks)

(a) $[mv] = M \times LT^{-1} = MLT^{-1}$

M1

Energy has dimensions ML^2T^{-2}

B1

These differ, so $E = mv$ is not dimensionally consistent

E1

Answer: MLT^{-1} is not ML^2T^{-2}

3 marks

(b) $E = mv^2$ has dimensions ML^2T^{-2} , so is consistent

B1

but kinetic energy is $\frac{1}{2}mv^2$: dimensional analysis cannot detect numerical constants

E1

Answer: e.g. $E = mv^2$

2 marks

26 (5 marks)

(a) $T = 0.5g = 4.9 \text{ N}$

B1

Answer: 4.9 N

1 mark

(b) Total weight supported = $(0.5 + 0.5)g$

M1

$T = 9.8 \text{ N}$

A1

Answer: 9.8 N

2 marks

(c) Error = $9.8 - 4.9 = 4.9 \text{ N}$

M1

Percentage error = $\frac{4.9}{9.8} \times 100 = 50\%$

A1

Answer: 50%

2 marks

27 (9 marks)

(a) $[F] = M L T^{-2}$, and $(M L^{-1} T^{-1})^a (L T^{-1})^b L^c = M^a L^{-a+b+c} T^{-a-b}$

M1

Mass: $a = 1$

A1

Time: $-a - b = -2$, so $b = 1$

A1

Length: $-a + b + c = 1$, so $c = 1$

A1

Answer: $a = b = c = 1$

4 marks

(b) Mass = $7800 \times \frac{4}{3}\pi(0.002)^3 = 0.000261 \text{ kg}$, so $W = mg = 0.00256 \text{ N}$

M1

Constant speed: $6\pi\eta r v = W$, so $v = \frac{0.00256}{6\pi \times 0.90 \times 0.002}$

M1

$v = 0.0755 \text{ m s}^{-1}$

A1

Answer: 0.0755 m s⁻¹

3 marks

(c) The upthrust (buoyancy) from the oil also acts upwards on the ball

E1

The net downward force is smaller, so the constant speed would be smaller than the value found

E1

Answer: upthrust; speed would be smaller

2 marks

28 (10 marks)

(a) $s = \frac{1}{2}at^2$: $100 = \frac{1}{2}a(10)^2$

M1

$a = 2 \text{ m s}^{-2}$

A1

$v = at = 2 \times 10 = 20 \text{ m s}^{-1}$

A1

 $20 \text{ m s}^{-1} = 72 \text{ km h}^{-1}$ is faster than any human sprinter, so Model A is not realistic

E1

Answer: $a = 2 \text{ m s}^{-2}$, $v = 20 \text{ m s}^{-1}$, not realistic

4 marks

(b) Distance = area under the graph: $\frac{1}{2} \times t_0 \times 12 + 12(10 - t_0) = 100$

M1

$6t_0 + 120 - 12t_0 = 100$, so $6t_0 = 20$

A1

$t_0 = \frac{10}{3} \text{ s (3.33 s)}$

M1

$a = \frac{12}{t_0} = 3.6 \text{ m s}^{-2}$

A1

Answer: $t_0 = \frac{10}{3} \text{ s}$, $a = 3.6 \text{ m s}^{-2}$

4 marks

(c) Model A: $\frac{1}{2} \times 2 \times 5^2 = 25 \text{ m}$. Model B: $\frac{1}{2} \times 3.6 \times \left(\frac{10}{3}\right)^2 + 12\left(5 - \frac{10}{3}\right) = 20 + 20 = 40 \text{ m}$

M1

Model B (error 2 m) is much closer to 38 m than Model A (error 13 m), so Model B is the better model

A1

Answer: A: 25 m, B: 40 m; Model B is better**2 marks****29 (9 marks)**

$$(a) [\gamma] = [\text{force}]/[\text{length}] = \text{MLT}^{-2}/\text{L}$$

$$= \text{MT}^{-2}$$

M1**A1****Answer: shown****2 marks**

$$(b) [f] = \text{T}^{-1}, [\gamma] = \text{MT}^{-2}, [\rho] = \text{ML}^{-3}, [r] = \text{L}$$

B1

$$\text{M: } a + b = 0. \text{ T: } -2a = -1, \text{ so } a = \frac{1}{2}$$

M1 A1

$$b = -\frac{1}{2}$$

A1

$$\text{L: } -3b + c = 0, \text{ so } c = 3b = -\frac{3}{2}$$

A1

$$\text{Answer: } a = \frac{1}{2}, b = -\frac{1}{2}, c = -\frac{3}{2}$$

5 marks

$$(c) f \propto r^{-3/2}$$

M1

$$\text{Factor} = 2^{-3/2} = 0.354$$

A1**Answer: 0.354****2 marks****30 (10 marks)**

$$(a) P = Fv \text{ gives } kv^3 = (kv^2)v, \text{ so } kv^2 \text{ is in newtons; } k = \text{N}/(\text{m s}^{-1})^2$$

$$= \text{kg m}^{-1}$$

M1**A1****Answer: kg m⁻¹****2 marks**

$$(b) k = \frac{P}{v^3} = \frac{120}{512}$$

M1

$$= 0.234 \text{ kg m}^{-1}$$

A1**Answer: 0.234 kg m⁻¹****2 marks**

$$(c) v^3 = \frac{405}{k} = \frac{405 \times 512}{120} = 1728$$

M1

$$v = 12 \text{ m s}^{-1}$$

A1**Answer: 12 m s⁻¹****2 marks**

$$(d) \text{ Model gives } P = 120/512 \times 1728 = 405 \text{ W}$$

M1 A1

The measured 330 W is lower than predicted by 75 W, about 19%

B1E.g. wind assistance, the road not level (downhill), or resistance not exactly proportional to v^2 **E1****Answer: model overestimates, e.g. tailwind****4 marks****Resolving Forces and Vectors****1 (2 marks)**

$$\text{Horizontal component} = 12 \cos 30^\circ = 10.4 \text{ N}$$

B1

$$\text{Vertical component} = 12 \sin 30^\circ = 6.00 \text{ N}$$

B1**Answer: horizontal 10.4 N, vertical 6.00 N****2 marks****2 (3 marks)**

$$R = \sqrt{8^2 + 6^2} = \sqrt{100}$$

M1

$$R = 10 \text{ N}$$

A1

$$\tan \theta = \frac{6}{8}, \text{ so } \theta = 36.9^\circ$$

A1**Answer: 10 N, $\theta = 36.9^\circ$** **3 marks****3 (3 marks)**

$$\text{Sum of forces} = 0: 4 - 1 + a = 0, \text{ so } a = -3$$

M1 A1

$$-2 + 5 + b = 0, \text{ so } b = -3$$

A1**Answer: a = -3, b = -3****3 marks**

4 (4 marks)

Equilibrium: $F_1 + F_2 + F_3 = 0$

M1

i: $2 - 5 + a = 0$, so $a = 3$

A1

j: $3 + 1 + b = 0$, so $b = -4$

A1

$|F_3| = \sqrt{3^2 + (-4)^2} = 5 \text{ N}$

A1

Answer: $a = 3$, $b = -4$, $|F_3| = 5 \text{ N}$

4 marks

5 (4 marks)

(a) Parallel to the plane: $40 \sin 30^\circ = 20.0 \text{ N}$

B1

Perpendicular to the plane: $40 \cos 30^\circ = 34.6 \text{ N}$

B1

Answer: 20.0 N and 34.6 N

2 marks

(b) Resolving parallel to the plane: $P = 40 \sin 30^\circ$; perpendicular to the plane: $R = 40 \cos 30^\circ$

M1

$P = 20.0$, $R = 34.6$

A1

Answer: $P = 20.0$, $R = 34.6$

2 marks

6 (4 marks)

Vertically: $T \cos 40^\circ = 30$

M1

$T = 39.2 \text{ N}$

A1

Horizontally: $P = T \sin 40^\circ$

M1

$P = 30 \tan 40^\circ = 25.2 \text{ N}$

A1

Answer: $T = 39.2 \text{ N}$, $P = 25.2 \text{ N}$

4 marks

7 (4 marks)

F must balance the resultant of the other two, which has magnitude $\sqrt{8^2 + 6^2} = 10 \text{ N}$

M1 A1

The resultant of the 8 N and 6 N has bearing $90^\circ - \tan^{-1}(\frac{6}{8}) = 53.1^\circ$

M1

F acts in the opposite direction: bearing = $53.1^\circ + 180^\circ = 233^\circ$

A1

Answer: 10 N on a bearing of 233°

4 marks

8 (3 marks)

Vertically: $R = 4g + 20 \sin 50^\circ$

M1 A1

$R = 39.2 + 15.3 = 54.5 \text{ N}$

A1

Answer: 54.5 N

3 marks

9 (6 marks)

(a) Magnitude = $\sqrt{20^2 + 21^2} = \sqrt{841} = 29 \text{ N}$

M1 A1

Angle = $\arctan(\frac{20}{21}) = 43.6^\circ$

M1 A1

Answer: 29 N at 43.6° to the 21 N force

4 marks

(b) F is equal and opposite to the resultant, so its magnitude is 29 N

B1

It makes an angle of $180^\circ - 43.6^\circ = 136^\circ$ with the 21 N force

B1

Answer: 29 N at 136° to the 21 N force

2 marks

10 (6 marks)

(a) Resolving east: $10 + 8 \sin 60^\circ + 6 \sin 210^\circ = 10 + 6.928 - 3 = 13.928 \text{ N}$

M1

Resolving north: $8 \cos 60^\circ + 6 \cos 210^\circ = 4 - 5.196 = -1.196 \text{ N}$

M1

$R = \sqrt{13.928^2 + (-1.196)^2}$

M1

$R = 14.0 \text{ N}$

A1

Answer: 14.0 N

4 marks

(b) The resultant points east and slightly south: angle below east = $\tan^{-1}(\frac{1.196}{13.928}) = 4.91^\circ$

M1

Bearing = $90^\circ + 4.91^\circ = 094.9^\circ$

A1

Answer: 094.9°

2 marks

11 (7 marks)

$$\text{Horizontally: } T_1 \cos 30^\circ = T_2 \cos 60^\circ$$

M1

$$\text{so } T_2 = T_1 \frac{\cos 30^\circ}{\cos 60^\circ} = \sqrt{3}T_1$$

A1

$$\text{Vertically: } T_1 \sin 30^\circ + T_2 \sin 60^\circ = 50$$

M1

$$\frac{1}{2}T_1 + \frac{\sqrt{3}}{2} \times \sqrt{3}T_1 = 50$$

A1

$$2T_1 = 50$$

M1

$$T_1 = 25 \text{ N}$$

A1

$$T_2 = 25\sqrt{3} = 43.3 \text{ N}$$

A1

$$\text{Answer: } T_1 = 25 \text{ N, } T_2 = 25\sqrt{3} = 43.3 \text{ N}$$

7 marks

12 (5 marks)

$$\text{Horizontally: } T_1 \cos 30^\circ = T_2 \cos 45^\circ$$

M1 A1

$$\text{Vertically: } T_1 \sin 30^\circ + T_2 \sin 45^\circ = 12g = 118$$

M1 A1

$$\text{Substituting } T_2 = T_1(\cos 30^\circ / \cos 45^\circ): T_1 = 86.1 \text{ N, } T_2 = 105 \text{ N}$$

A1

$$\text{Answer: } T_1 = 86.1 \text{ N, } T_2 = 105 \text{ N}$$

5 marks

13 (6 marks)

$$\text{(a) Parallel: } F = W \sin 30^\circ. \text{ Perpendicular: } R = W \cos 30^\circ$$

M1

$$\text{Limiting friction: } F = \mu R, \text{ so } \mu = \tan 30^\circ$$

M1

$$= \frac{1}{\sqrt{3}}$$

A1

Answer: shown

3 marks

$$\text{(b) } R = 90 \cos 30^\circ = 77.9 \text{ N, maximum friction} = \mu R = 22.5 \text{ N}$$

B1

$$\text{Friction acts up the plane; least } P \text{ when friction is at its maximum: } P + \mu R = 90 \sin 30^\circ = 45$$

M1

$$P = 45 - 22.5 = 22.5 \text{ N}$$

A1

Answer: 22.5 N

3 marks

14 (5 marks)

$$\text{Resultant force} = (2 - 6 + 4)\mathbf{i} + (5 + 3 - 2)\mathbf{j} = 0\mathbf{i} + 6\mathbf{j}$$

M1 A1

$$F = ma: a = \frac{6}{2} = 3 \text{ m s}^{-2}$$

M1 A1

$$\text{The resultant is due north, so the bearing is } 000^\circ$$

A1

$$\text{Answer: } 3 \text{ m s}^{-2} \text{ on a bearing of } 000^\circ$$

5 marks

15 (6 marks)

$$\text{Resolving due east: } 40 - X \sin 60^\circ = 0$$

M1 A1

$$X = \frac{80}{\sqrt{3}} = 46.2$$

A1

$$\text{Resolving due north: } 30 - X \cos 60^\circ - Y = 0$$

M1 A1

$$Y = 30 - \frac{X}{2} = 6.91$$

A1

$$\text{Answer: } X = 46.2, Y = 6.91$$

6 marks

16 (6 marks)

$$\text{(a) } 1.5^2 + 2^2 = 2.25 + 4 = 6.25 = 2.5^2$$

M1

$$\text{so by the converse of Pythagoras' theorem angle } ACB = 90^\circ$$

A1

Answer: shown

2 marks

$$\text{(b) AC makes angle } \alpha \text{ with the ceiling where } \cos \alpha = \frac{1.5}{2.5} = 0.6, \text{ and BC makes angle } \beta \text{ with cos}$$

B1

$$\beta = 0.8$$

$$\text{Horizontally: } 0.6T_A = 0.8T_B; \text{ vertically: } 0.8T_A + 0.6T_B = 130$$

M1 A1

$$T_A = 104 \text{ N and } T_B = 78 \text{ N}$$

A1

$$\text{Answer: } T_A = 104 \text{ N, } T_B = 78 \text{ N}$$

4 marks

17 (6 marks)

Let the tension in BP be T. Horizontally: $T \cos \theta = 100 \sin 50^\circ = 76.60$

M1 A1

Vertically: $T \sin \theta + 100 \cos 50^\circ = 200$, so $T \sin \theta = 135.7$

M1 A1

$$\tan \theta = \frac{135.7}{76.60}, \text{ so } \theta = 60.6^\circ$$

A1

$$T = \sqrt{76.60^2 + 135.7^2} = 156 \text{ N}$$

A1

Answer: $\theta = 60.6^\circ$, $T = 156 \text{ N}$

6 marks

18 (7 marks)

(a) Constant speed, so horizontally $F = 60 \cos 25^\circ$

M1

$$F = 54.4 \text{ N}$$

A1

Answer: 54.4 N

2 marks

(b) Vertically: $R + 60 \sin 25^\circ = 15 \times 9.8$

M1

$$R = 147 - 25.36 = 122 \text{ N}$$

A1

Answer: 122 N

2 marks

(c) Moving, so $F = \mu R$, giving $\mu = \frac{F}{R}$

M1

$$\mu = \frac{54.4}{122} = 0.447$$

A1

Answer: $\mu = 0.447$

2 marks

(d) The box is modelled as a particle (so the forces all act at one point and there is no turning effect)

B1

Answer: the box is a particle

1 mark

19 (8 marks)

(a) The angle is measured from the horizontal, so the horizontal component uses cos and the vertical component uses sin; Daniel has swapped them

E1

$$\text{Horizontal} = 30 \cos 40^\circ = 23.0 \text{ N}$$

B1

$$\text{Vertical} = 30 \sin 40^\circ = 19.3 \text{ N}$$

B1

Answer: horizontal 23.0 N, vertical 19.3 N

3 marks

(b) Forces are vectors, so they add by direction as well as magnitude: the sum of the magnitudes is only correct when the forces act in the same direction

E1

Greatest 14 N (same direction), least $8 - 6 = 2 \text{ N}$ (opposite directions)

B1

$$\text{Perpendicular: } \sqrt{6^2 + 8^2} = 10 \text{ N}$$

B1

Answer: greatest 14 N, least 2 N, perpendicular 10 N

3 marks

(c) Along the 8 N force: $8 + 6 \cos 60^\circ = 11$; perpendicular: $6 \sin 60^\circ = 3\sqrt{3}$

M1

$$R = \sqrt{121 + 27} = \sqrt{148} = 12.2 \text{ N}$$

A1

Answer: 12.2 N

2 marks

20 (6 marks)

$$\text{Perpendicular: } R = 8g \cos 20^\circ + X \sin 20^\circ$$

M1 A1

$$\text{Parallel: } X \cos 20^\circ = 8g \sin 20^\circ + F, \text{ with } F = 0.4R$$

M1 A1

$$X \cos 20^\circ = 8g \sin 20^\circ + 0.4(8g \cos 20^\circ + X \sin 20^\circ)$$

M1

$$X(\cos 20^\circ - 0.4 \sin 20^\circ) = 8g(\sin 20^\circ + 0.4 \cos 20^\circ), \text{ so } X = 70.1 \text{ N}$$

A1

Answer: $X = 70.1 \text{ N}$

6 marks

21 (7 marks)

(a) Sideways: $T_1 \sin 20^\circ = T_2 \sin 35^\circ$

B1

$$\text{Forwards: } T_1 \cos 20^\circ + T_2 \cos 35^\circ = 2000$$

B1

Answer: equations as above

2 marks

(b) From the first, $T_1 = T_2(\sin 35^\circ / \sin 20^\circ) = 1.677 T_2$

M1

$$T_2(1.677 \cos 20^\circ + \cos 35^\circ) = 2000$$

M1

$$T_1 = 1400 \text{ N}, T_2 = 835 \text{ N}$$

A1

Answer: $T_1 = 1400 \text{ N}$, $T_2 = 835 \text{ N}$

3 marks

(c) $T_1 + T_2 = 2240 \text{ N}$, greater than 2000 N

B1

Each rope has a sideways component that cancels the other's, so some tension is wasted

E1

Answer: 2240 N: sideways components cancel**2 marks****22 (6 marks)**

(a) The angle between the weight and the normal to the slope is 40° , so the component perpendicular to the slope is $50 \cos 40^\circ$ and the component down the slope is $50 \sin 40^\circ$
 Correct values: down the slope 32.1 N, normal reaction 38.3 N

E1 M1**Answer: down slope 32.1 N, R = 38.3 N****3 marks**(b) $F = 50 \sin 40^\circ = 32.1 \text{ N}$ **B1**Without F: $5.1a = 50 \sin 40^\circ = 32.14$ **M1** $a = 6.30 \text{ m s}^{-2}$ (that is $g \sin 40^\circ = 6.30$)**A1****Answer: F = 32.1 N, a = 6.30 m s⁻²****3 marks****23 (6 marks)**Resolve north: $12 + Q \cos 210^\circ = 0$ **M1 A1** $\cos 210^\circ = -\frac{\sqrt{3}}{2}$, so $Q = \frac{24}{\sqrt{3}} = 8\sqrt{3} = 13.9 \text{ N}$ **M1 A1**Resolve east: $P + Q \sin 210^\circ = 0$, $\sin 210^\circ = -\frac{1}{2}$ **M1** $P = \frac{Q}{2} = 4\sqrt{3} = 6.93 \text{ N}$ **A1****Answer: P = 4√3 = 6.93 N, Q = 8√3 = 13.9 N****6 marks****24 (7 marks)**(a) $R + 80 \sin 30^\circ + 120 \sin 50^\circ = 25 \times 9.8$ **M1 A1** $R = 245 - 40 - 91.9 = 113 \text{ N}$ **A1****Answer: 113 N****3 marks**(b) $120 \cos 50^\circ = 77.1$ and $80 \cos 30^\circ = 69.28$, so friction = 7.85 N**M1**

Friction acts to the right

A1**Answer: 7.85 N to the right****2 marks**(c) Maximum friction = $0.1 \times 113 = 11.3 \text{ N}$ **M1**

11.3 is greater than the 7.85 N needed, so the box does not move

A1**Answer: maximum friction 11.3 N exceeds 7.85 N****2 marks****25 (6 marks)**(a) Perpendicular to the plane: $120 + 150 \sin 30^\circ = W \cos 25^\circ$ **M1 A1** $W = \frac{195}{\cos 25^\circ} = 215$ **A1****Answer: 215 N****3 marks**(b) Along the plane: $150 \cos 30^\circ = W \sin 25^\circ + F$ **M1 A1** $F = 129.9 - 90.93 = 39.0 \text{ N}$, acting down the plane**A1****Answer: 39.0 N down the plane****3 marks****26 (6 marks)** $F = 0.25R$, so $R = \frac{20}{0.25} = 80 \text{ N}$ **M1 A1**Vertically: $R = 4 \times 9.8 + P \sin \theta$, so $P \sin \theta = 80 - 39.2 = 40.8$ **M1 A1**Horizontally: $P \cos \theta = 20$ **M1** $\tan \theta = \frac{40.8}{20} = 2.04$, so $\theta = 63.9^\circ$ and $P = \sqrt{20^2 + 40.8^2} = 45.4 \text{ N}$ **A1****Answer: θ = 63.9°, P = 45.4 N****6 marks****27 (9 marks)**(a) $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$ Perpendicular to the plane: $R = W \cos \alpha + P \sin \alpha$ **M1** $R = 0.8W + 0.6P$ **A1****Answer: shown****2 marks**(b) Least P: the block is about to slide down, so friction $F = 0.5R$ acts up the plane**M1**

Parallel to the plane: $P \cos \alpha + F = W \sin \alpha$, so $0.8P + 0.5(0.8W + 0.6P) = 0.6W$

M1

$$1.1P = 0.2W, \text{ so } P = \frac{2W}{11}$$

A1

$$P = \frac{2 \times 78.4}{11} = 14.3 \text{ N}$$

A1

Answer: 14.3 N

4 marks

(c) Greatest P: the block is about to slide up, so friction acts down the plane: $P \cos \alpha = W \sin \alpha + 0.5R$

M1

$$0.8P = 0.6W + 0.5(0.8W + 0.6P), \text{ so } 0.5P = W$$

A1

$$P = 2W = 157 \text{ N}$$

A1

Answer: 157 N

3 marks

28 (10 marks)

(a) Parallel to Q: $Q + P \cos \theta$

M1

Perpendicular to Q: $P \sin \theta$

A1

$$R^2 = (Q + P \cos \theta)^2 + (P \sin \theta)^2$$

M1

$$= Q^2 + 2PQ \cos \theta + P^2(\cos^2 \theta + \sin^2 \theta) = P^2 + Q^2 + 2PQ \cos \theta, \text{ using } \cos^2 \theta + \sin^2 \theta = 1$$

A1

Answer: shown

4 marks

(b) $49 = 25 + 64 + 80 \cos \theta$

M1

$$\cos \theta = -\frac{1}{2}$$

A1

$$\theta = 120^\circ$$

A1

Answer: $\theta = 120^\circ$

3 marks

(c) Parallel to Q: $8 + 5 \cos 120^\circ = 5.5$; perpendicular: $5 \sin 120^\circ = \frac{5\sqrt{3}}{2}$

M1

$$\tan \varphi = \frac{5\sqrt{3}}{2 \div 5.5} = \frac{5\sqrt{3}}{11}$$

A1

$$\varphi = 38.2^\circ$$

A1

Answer: 38.2°

3 marks

29 (9 marks)

(a) Parallel to the plane: $T \cos \beta = W \sin \alpha$

M1 A1

$$\text{So } T = \frac{W \sin \alpha}{\cos \beta}$$

A1

Perpendicular: $R + T \sin \beta = W \cos \alpha$, so $R = W \cos \alpha - W \sin \alpha \tan \beta$

A1

Answer: shown

4 marks

(b) $T = \frac{50 \sin 30^\circ}{\cos 20^\circ} = 26.6 \text{ N}$

M1

$$R = 50 \cos 30^\circ - 26.60 \sin 20^\circ = 34.2 \text{ N}$$

A1

Answer: $T = 26.6 \text{ N}$, $R = 34.2 \text{ N}$

2 marks

(c) Leaving the plane means $R = 0$: $W \cos \alpha = W \sin \alpha \tan \beta$

M1

$$\tan \beta = \cot \alpha = \tan(90^\circ - \alpha), \text{ so } \beta = 90^\circ - \alpha$$

M1 E1

Answer: $\beta = 60^\circ$ for $\alpha = 30^\circ$

3 marks

30 (11 marks)

(a) Vertical: $R + T \sin \theta = W$, so $R = W - T \sin \theta$

M1 A1

Horizontal: $T \cos \theta = \mu R = \mu(W - T \sin \theta)$

M1

$T(\cos \theta + \mu \sin \theta) = \mu W$, giving the result

A1

Answer: shown

4 marks

(b) T is least when the denominator $f(\theta) = \cos \theta + \mu \sin \theta$ is greatest

M1

$$f'(\theta) = -\sin \theta + \mu \cos \theta = 0 \text{ gives } \tan \theta = \mu$$

M1 A1

$$\text{With } \tan \theta = \mu: \cos \theta = \frac{1}{\sqrt{1 + \mu^2}}, \sin \theta = \frac{\mu}{\sqrt{1 + \mu^2}}, \text{ so } f = \sqrt{1 + \mu^2}$$

M1

$$T_{\min} = \frac{\mu W}{\sqrt{1 + \mu^2}}$$

A1

Answer: shown

5 marks

(c) $\theta = \tan^{-1} 0.5 = 26.6^\circ$

B1

$$T_{\min} = \frac{0.5 \times 400}{\sqrt{1.25}} = 179 \text{ N}$$

B1**Answer: 179 N at 26.6° above the horizontal****2 marks**