

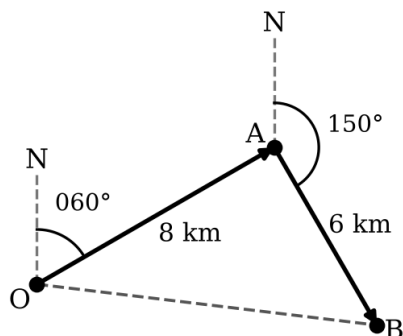
Scalars and Vectors

- 1 Mass, velocity, time, weight, distance and displacement are six quantities used in mechanics. Write down the three that are vector quantities. [3]

- 2 The vectors $\mathbf{p}$ and $\mathbf{q}$ are given by $\mathbf{p} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 5\mathbf{j}$. Find $\mathbf{p} + \mathbf{q}$ and hence find $|\mathbf{p} + \mathbf{q}|$. [3]

- 3 Find the unit vector in the direction of $12\mathbf{i} + 5\mathbf{j}$. [2]

- 4 A hiker walks 8 km on a bearing of 060° . She then walks 6 km on a bearing of 150° . Find the distance of the hiker from her starting point, and the bearing of her final position from her starting point.



[4]

- 5 A particle moves with constant velocity $(-5\mathbf{i} + 12\mathbf{j}) \text{ m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north respectively. Find the speed of the particle, and the bearing on which it is moving. [4]

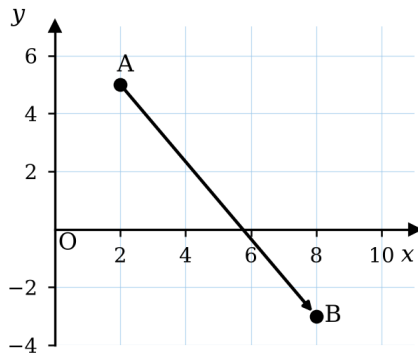
- 6 A ball is kicked with speed 20 m s^{-1} at 30° above the horizontal. Taking $\mathbf{i}$ as horizontal and $\mathbf{j}$ as vertically upwards, express the velocity of the ball in the form $p\mathbf{i} + q\mathbf{j}$, giving p and q to 3 significant figures. [3]

- 7 A force of magnitude 10 N acts on a bearing of 120° . Taking $\mathbf{i}$ as due east and $\mathbf{j}$ as due north, write the force in the form $p\mathbf{i} + q\mathbf{j}$, giving p and q correct to 3 significant figures. [3]

- 8 An aircraft points due north with an airspeed of 120 km h^{-1} . A wind blows from the west at 30 km h^{-1} . Find the speed of the aircraft relative to the ground and the bearing on which it actually travels. [4]

- 9 The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j}$. The vector $\mathbf{a} + \lambda\mathbf{b}$ is parallel to the vector $\mathbf{i} + 3\mathbf{j}$.
 (a) Find the value of λ . [3]
 (b) Find $|\mathbf{a} + \lambda\mathbf{b}|$ for this value of λ , giving your answer in the form $k\sqrt{10}$. [2]

- 10** Relative to a fixed origin O, the points A and B have position vectors $(2\mathbf{i} + 5\mathbf{j})$ m and $(8\mathbf{i} - 3\mathbf{j})$ m respectively.

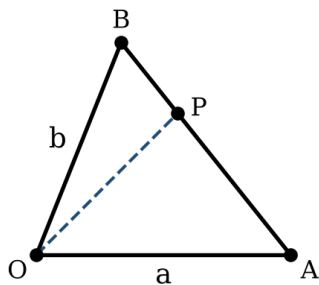


- (a) Find the vector AB and the distance AB. [3]
 (b) A particle moves in a straight line from A to B at constant velocity, taking 4 seconds. Find the velocity of the particle and its speed. [3]

- 11** At time $t = 0$ a particle is at the point with position vector $(-6\mathbf{i} + 4\mathbf{j})$ m. It moves with constant velocity $(3\mathbf{i} + 5\mathbf{j})$ m s⁻¹.
 (a) Find the position vector of the particle at time t seconds.
 (b) Find the time at which the particle is due north of the origin, and its distance from the origin then. [5]

- 12** A beetle moves across a flat table with constant velocity $(2\mathbf{i} - \mathbf{j})$ cm s⁻¹, starting at the point with position vector $(\mathbf{i} + 7\mathbf{j})$ cm. Find the time at which the beetle crosses the line $y = x$, and the distance it has travelled by then. [5]

- 13** In triangle OAB, $OA = a$ and $OB = b$. The point P lies on AB with $AP : PB = 2 : 1$.



- (a) Show that $OP = \frac{1}{3}a + \frac{2}{3}b$. [3]
 (b) Given that $a = 6\mathbf{i}$ and $b = 2\mathbf{i} + 5\mathbf{j}$, find $|OP|$, giving your answer in the form $k\sqrt{2}$. [3]

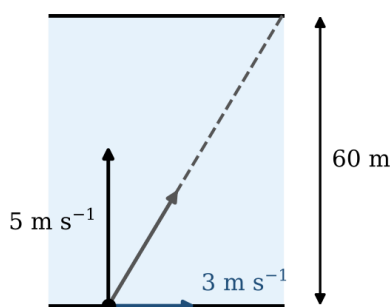
- 14** A drone flies in three straight legs, with displacements $(3\mathbf{i} + 4\mathbf{j})$ km, $(-8\mathbf{i} + 2\mathbf{j})$ km and $(2\mathbf{i} - 10\mathbf{j})$ km, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. It then flies straight back to its starting point. Find the total displacement after the three legs, and the distance and bearing of the final return flight. [5]

- 15** A particle moves with constant velocity $(p\mathbf{i} + (p + 7)\mathbf{j})$ m s⁻¹, where $p > 0$, and its speed is 13 m s⁻¹. Find p , and the bearing on which the particle moves. [5]

- 16** At noon ship A is at the point with position vector $(10\mathbf{i} + 4\mathbf{j})$ km relative to a fixed origin O, and at 14:00 it is at $(2\mathbf{i} + 12\mathbf{j})$ km, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Ship A moves with constant velocity. Ship B moves with constant velocity $(2\mathbf{i} - 2\mathbf{j})$ km h^{-1} and, t hours after noon, has position vector $[(2t + k)\mathbf{i} + (22 - 2t)\mathbf{j}]$ km, where k is a constant.
- (a) Find the velocity of ship A. [2]
- (b) The ships meet. Find the time they meet and the value of k . [5]

- 17** At time $t = 0$ boat P is at the point with position vector $(2\mathbf{i} + 10\mathbf{j})$ km relative to a fixed origin, moving with velocity $(4\mathbf{i} - 3\mathbf{j})$ km h^{-1} . At the same time boat Q is at $(22\mathbf{i} + 4\mathbf{j})$ km, moving with velocity $-2\mathbf{i}$ km h^{-1} . The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are due east and due north. Find the time at which P is due west of Q, and the distance between the boats at that time. [5]

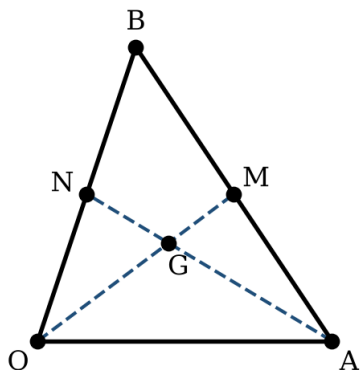
- 18** A river is 60 m wide with straight parallel banks. The water flows at a constant 3 m s^{-1} parallel to the banks.
- A boat has a speed of 5 m s^{-1} in still water. The boat leaves one bank and always points directly across the river, perpendicular to the banks, as shown.



- (a) Find the speed of the boat relative to the bank. [2]
- (b) Find the time taken to cross the river. [2]
- (c) Find the distance the boat is carried downstream by the time it reaches the opposite bank. [2]
- (d) Find the acute angle between the path of the boat and the bank. [2]
- 19** Ayesha has made two statements about the vectors and scalars in a mechanics problem.
- (a) Statement 1: "A ball has velocity $(-2\mathbf{i} + 6\mathbf{j})$ m s^{-1} , so its speed is $-2 + 6 = 4 \text{ m s}^{-1}$." Explain the error in her working and find the correct speed of the ball, giving your answer as an exact value. [3]
- (b) Statement 2: "A cyclist rides 3 km east, then 4 km north, then rides straight back to the start. The displacement for the whole trip is 12 km." Explain the error in her statement and state the displacement and the distance travelled. [3]
- 20** Relative to a harbour at the origin O, two boats A and B are at the points with position vectors $(4\mathbf{i} + 3\mathbf{j})$ km and $(-2\mathbf{i} + 9\mathbf{j})$ km at 12:00. Boat A moves with constant velocity $(2\mathbf{i} + \mathbf{j})$ km h^{-1} and boat B with constant velocity $(4\mathbf{i} - \mathbf{j})$ km h^{-1} .
- (a) Show that the boats collide, and find the time of the collision and the position vector of the point of collision. [5]
- (b) Find the distance between the boats at 13:00. [2]
- 21** Relative to an origin O, the points A, B and C have position vectors $(2\mathbf{i} + \mathbf{j})$ m, $(5\mathbf{i} + 7\mathbf{j})$ m and $(11\mathbf{i} + 19\mathbf{j})$ m respectively.
- (a) Show that A, B and C are collinear. [3]
- (b) Find the ratio AB : BC. [1]

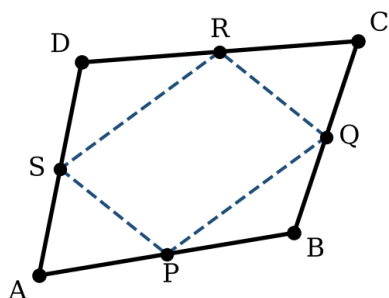
- 22** Tom says: "If two vectors have magnitudes 3 and 4, their sum always has magnitude 5."
 (a) Explain why Tom is wrong in general. [2]
 (b) Write down the largest and smallest possible magnitudes of the sum, and describe the vectors in each case. [2]
 (c) The two vectors are $\mathbf{a} = 3\mathbf{i}$ and $\mathbf{b} = 4 \cos \theta \mathbf{i} + 4 \sin \theta \mathbf{j}$. Find the value of θ , between 0° and 180° , for which $|\mathbf{a} + \mathbf{b}| = \sqrt{37}$. [3]
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- 23** An aircraft has an airspeed of 200 km h^{-1} . A steady wind blows towards the south at 40 km h^{-1} . The pilot wishes to fly directly east from A to a town B, 300 km away.
 (a) Find the bearing on which the pilot must point the aircraft. [2]
 (b) Find the speed of the aircraft relative to the ground and the time taken to reach B, giving the time in minutes. [3]
 (c) Find the time taken for the return flight due west from B to A at the same airspeed, and comment. [1]
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- 24** A drifting buoy is at the point with position vector $(4\mathbf{i} - \mathbf{j}) \text{ km}$ at 08:00 and $(2\mathbf{i} + 0.5\mathbf{j}) \text{ km}$ at 08:30, relative to a fixed origin O, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. The buoy moves with constant velocity.
 At 09:00 a patrol boat leaves the point with position vector $(-8\mathbf{i} + 2\mathbf{j}) \text{ km}$ and moves with constant velocity so that it intercepts the buoy at 09:30.
 (a) Find the velocity of the buoy, in km h^{-1} . [2]
 (b) Find the position vector of the buoy t hours after 08:00. [2]
 (c) Find the velocity of the patrol boat and its speed. [4]
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- 25** A boat leaves a point O at noon and travels with constant velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ km h}^{-1}$ for 2 hours, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. It then changes course and travels with constant velocity $(4\mathbf{i} - 3\mathbf{j}) \text{ km h}^{-1}$.
 (a) Find the bearing of the boat's course during the second stage. [2]
 (b) Find the time at which the boat is due east of O, and its distance from O at that time. [4]
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- 26** At noon, ship P is at the point with position vector $(5\mathbf{i} + 18\mathbf{j}) \text{ km}$ and ship Q is at the point with position vector $(25\mathbf{i} + 13\mathbf{j}) \text{ km}$, relative to a lighthouse at the origin. Ship P moves with constant velocity $(6\mathbf{i} + 2\mathbf{j}) \text{ km h}^{-1}$ and ship Q moves with constant velocity $(2\mathbf{i} + 5\mathbf{j}) \text{ km h}^{-1}$.
 At time t hours after noon the distance between the ships is $d \text{ km}$.
 (a) Write down the position vectors of P and Q at time t . [2]
 (b) Show that $d^2 = 25t^2 - 190t + 425$. [3]
 (c) Find the least distance between the ships, and the time at which this occurs. [3]
 (d) The ships are in danger if they are less than 10 km apart. Find, in hours, how long the danger lasts. [3]
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- 27** The diagram shows a thin triangular plate OAB. Relative to the corner O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$. The point M is the midpoint of AB, the point N is the midpoint of OB, and G is the point on OM such that $OG = \frac{2}{3}OM$.



- (a) Show that $OM = \frac{1}{2}(\mathbf{a} + \mathbf{b})$. [2]
 (b) Show that G lies on AN and find the ratio AG : GN. [5]
 (c) Given that O is the origin, A is the point (6, 0) and B is the point (2, 6), find the coordinates of G, and verify that your answer is consistent with part (b). [2]

- 28** ABCD is any quadrilateral with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ relative to an origin. The points P, Q, R and S are the midpoints of AB, BC, CD and DA respectively, as shown. Prove that PQRS is a parallelogram.

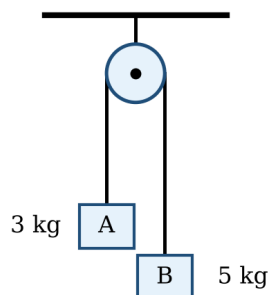


- (a) Write down the position vectors of P, Q, R and S in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$. [2]
 (b) Find PQ and SR, and hence prove that PQRS is a parallelogram. [4]
 (c) The quadrilateral ABCD has A(0, 0), B(6, 1), C(7.5, 5.5), D(1, 5) as in the figure. Find the coordinates of P, Q, R, S and verify that $PQ = SR$ numerically. [3]
- 29** A lighthouse L is at the origin O of a coordinate system with $\mathbf{i}$ and $\mathbf{j}$ unit vectors east and north and distances in km. A ship S is at the point with position vector $(-16\mathbf{i} + 8\mathbf{j})$ km at 09:00 and moves with constant velocity $(6\mathbf{i} - 2\mathbf{j})$ km h^{-1} . The ship can be seen from the lighthouse only when it is within 10 km of it.
- (a) Show that, t hours after 09:00, the distance d km of the ship from the lighthouse satisfies $d^2 = 40t^2 - 224t + 320$. [3]
 (b) Find the least distance of the ship from the lighthouse and the time at which this occurs. [3]
 (c) For how long is the ship visible from the lighthouse? [3]
- 30** At 10:00 a ship S is at the point with position vector $(-20\mathbf{i} - 10\mathbf{j})$ km relative to a fixed origin O, and at 10:30 it is at $(-16\mathbf{i} - 7\mathbf{j})$ km, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. S moves with constant velocity. A lighthouse L is at a fixed point. At 11:00 L is due north of S and at 12:00 L is on a bearing of 315° from S.
- (a) Find the velocity of S, and its speed. [3]
 (b) Find the position vector of S t hours after 10:00. [2]
 (c) Find the position vector of L. [4]

Units and Modelling Assumptions

- 1 Write down the SI unit of (a) mass, (b) acceleration, (c) force.
 - (a) mass [1]
 - (b) acceleration [1]
 - (c) force [1]
- 2 Convert each of the following to the units shown.
 - (a) 54 km h^{-1} to m s^{-1} [2]
 - (b) $36\,000 \text{ cm}^2$ to m^2 [1]
- 3 Express (a) the newton and (b) the joule in terms of the base units kg, m and s. [2]
- 4 Convert 90 km h^{-1} to m s^{-1} , and 25 m s^{-1} to km h^{-1} . [2]
- 5 State what is meant by each of these modelling assumptions: (a) a string is light, (b) a surface is smooth. [2]
- 6 In a mechanics problem a ball is modelled as a particle on a smooth surface, attached to a light inextensible string.
Explain what is meant by each of the following modelling assumptions.
 - (a) particle [1]
 - (b) light inextensible string [1]
 - (c) smooth surface [1]
- 7 Use dimensions to check that each of the following is consistent. Here T is time, l is a length, g is the acceleration due to gravity, m is mass, v is speed and r is a radius.
 - (a) The period of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. [2]
 - (b) The formula $F = \frac{mv^2}{r}$ gives a force. [2]
- 8 Aluminium has density 2.7 g cm^{-3} . Convert this to kg m^{-3} . [2]
- 9 Using the dimensions M, L and T for mass, length and time, show that kinetic energy $\frac{1}{2}mv^2$ and work done Fs have the same dimensions. [3]
- 10 Two particles are connected by a light inextensible string. State what each of the words "light" and "inextensible" allows you to assume about (a) the tension in the string and (b) the accelerations of the two particles. [2]
- 11 A car travelling along a straight road is modelled as a particle.
 - (a) Give one reason why this is reasonable. (b) Describe a situation in which this model would not be suitable. [3]
- 12 The letters s, u, v, a and t represent displacement, initial speed, final speed, acceleration and time. Using dimensions, state which of the following are dimensionally consistent:
 - (i) $s = ut + \frac{1}{2}at^2$ (ii) $v = u + at^2$ (iii) $s = \frac{1}{2}(u + v)t$ [3]
- 13 A heater has a power rating of 2.4 kW, where the watt is defined by $1 \text{ W} = 1 \text{ J s}^{-1}$.
 - (a) Express the watt ($1 \text{ W} = 1 \text{ J s}^{-1}$) in terms of the base units kg, m and s. [2]
 - (b) The heater runs for one hour. Express the energy it uses, 2.4 kW h, in joules. [2]

- 14** Two particles A and B, of masses 3 kg and 5 kg, are connected by a light inextensible string that passes over a smooth fixed pulley, as shown. The particles move vertically. State four modelling assumptions that are made about this situation. For each, state one simplification it allows in the mathematical model.



[4]

- 15** The speed v of waves on shallow water is thought to depend only on the depth of the water h and the acceleration due to gravity g , so that $v = k g^a h^b$, where k is a dimensionless constant. By using dimensions, find the values of a and b , and write down the formula for v .

[5]

- 16** Newton's law of gravitation states that the force between two particles of masses m_1 and m_2 a distance r apart is $F = \frac{Gm_1m_2}{r^2}$. Find the dimensions of G , and hence its SI units.

[3]

- 17** A model for the air resistance on a car is $R = kv^2$, where R is in newtons and v is in m s^{-1} .

(a) Find the units of k .

[2]

(b) Given that $k = 0.4$ in these units, find the resistance at 30 m s^{-1} and the percentage increase in R when the speed rises by 10%.

[3]

- 18** A student times the swings of a simple pendulum of length l and obtains the period T shown in the table. She believes that T^2 is directly proportional to l .

Use the data to find $T^2 \div l$ for each length, and comment on whether her belief is supported.

Estimate the constant, and the value of g if $T^2 = \frac{4\pi^2 l}{g}$.

l (m)	0.250	0.500	0.750	1.00	1.50
T (s)	1.00	1.43	1.72	2.01	2.48

[5]

- 19** The period of oscillation of a mass m on a spring is $T = 2\pi\sqrt{m/k}$, where k is the spring constant. Find the dimensions of k and show that its SI units are kg s^{-2} , equivalent to N m^{-1} .

[3]

- 20** A tank is filled with 60 litres of petrol. The density of petrol is 0.74 g cm^{-3} . (1 litre = 1000 cm^3 .)

(a) Find the mass of the petrol in kg.

[2]

(b) Express the density of petrol in kg m^{-3} .

[1]

(c) The tank is a cuboid with a rectangular base measuring 0.5 m by 0.4 m. Find the depth of petrol in the tank, in cm.

[2]

- 21** A student drops a small steel ball from rest from different heights h metres and records the time of fall t seconds, as shown in the table. The student uses a model in which the ball is a particle falling freely under gravity, with $g = 9.8 \text{ m s}^{-2}$.

h (m)	1.0	2.0	4.0	6.0
t (s)	0.46	0.66	0.93	1.13

- (a) Show that, according to the model, $t = \sqrt{\frac{2h}{g}}$. [2]
- (b) For $h = 4.0$, find the time predicted by the model, and the percentage by which the measured time exceeds it. [3]
- (c) The measured times are all slightly greater than those predicted by the model. Give two reasons why. [2]

- 22** A student has made the following three errors in a mechanics problem set. In each part, identify the error and give the correct result.

- (a) "A trolley of mass 1200 g accelerates at 2.5 m s^{-2} , so the resultant force is $F = ma = 1200 \times 2.5 = 3000 \text{ N}$." [2]
Explain the error and find the correct resultant force.
- (b) "A man has a weight of 686 N, so his mass is 686 kg, because mass and weight are the same thing." [3]
Explain the error and find the correct mass, taking $g = 9.8 \text{ m s}^{-2}$.
- (c) " $72 \text{ km h}^{-1} = 72 \times 3600 \div 1000 = 259.2 \text{ m s}^{-1}$." [2]
Explain the error and find the correct speed in m s^{-1} .

- 23** The drag force F on a sphere moving through air is thought to depend on the air density ρ , the speed v and the cross-sectional area A , so that $F = k\rho^a v^b A^c$, where k is dimensionless.

- (a) Use dimensional analysis to find a , b and c . [4]
- (b) A sphere of cross-sectional area 0.05 m^2 moves at 12 m s^{-1} in air of density 1.2 kg m^{-3} and $k = 0.25$. Find F . [2]
- (c) Find the factor by which F changes when the speed is tripled. [1]

- 24** A golf ball is hit and its flight is modelled as that of a projectile moving under gravity only. The model predicts a range R .

- (a) State three modelling assumptions made. [3]
- (b) Explain how the actual range would compare with R if air resistance were included. [3]

- 25** Sam says: "The equation $E = mv$ is dimensionally consistent with kinetic energy because both contain m and v ." Chen says: "If an equation is dimensionally consistent, it must be correct."

- (a) Show that Sam is wrong. [3]
- (b) Show that Chen is wrong by giving a counterexample. [2]

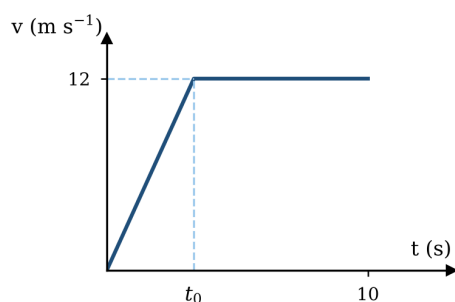
- 26** A box of mass 0.5 kg hangs in equilibrium from the lower end of a vertical rope of mass 0.5 kg . The model "a light rope" would ignore the mass of the rope. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the tension at the top of the rope using this model. [1]
- (b) Find the tension at the top of the rope if the mass of the rope is included. [2]
- (c) Find the percentage error made by the light-rope model. [2]

- 27** When a small sphere of radius r moves slowly with speed v through a fluid of viscosity η , the drag force F is thought to be proportional to $\eta^a v^b r^c$.
The dimensions of viscosity are $M L^{-1} T^{-1}$.
- (a) Use dimensional analysis to find a , b and c . [4]
- (b) Experiments show that $F = 6\pi\eta rv$. A steel ball of radius 2.0 mm and density 7800 kg m^{-3} falls at constant speed through oil with $\eta = 0.90 \text{ kg m}^{-1} \text{ s}^{-1}$. Taking $g = 9.8 \text{ m s}^{-2}$ and assuming the drag balances the weight, find the speed of the ball. [3]
- (c) State one force that was ignored in part (b) and say how including it would change the speed found. [2]

- 28** A model is needed for a 100 m sprint, which a runner completes in exactly 10 s, starting from rest. The runner is modelled as a particle moving in a straight line.
Model A: the acceleration is constant for the whole race.

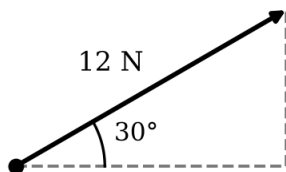
Model B: the runner accelerates uniformly for t_0 seconds to a top speed of 12 m s^{-1} , then runs at that constant speed for the rest of the race, as shown in the diagram.



- (a) Find the acceleration and the finishing speed predicted by Model A, and comment on whether Model A is realistic. [4]
- (b) For Model B, find the value of t_0 and the acceleration of the runner during the first stage. [4]
- (c) The runner is timed at 38 m after 5 s. Calculate the distance covered after 5 s according to each model, and state which model is better. [2]
- 29** The frequency f of small oscillations of a liquid drop is thought to depend on its surface tension γ , density ρ and radius r , so that $f = k\gamma^a \rho^b r^c$, where k is dimensionless. The surface tension γ has units N m^{-1} .
- (a) Show that the dimensions of γ are MT^{-2} . [2]
- (b) Use dimensional analysis to find a , b and c . [5]
- (c) Find the factor by which the frequency changes when the radius of the drop is doubled, with γ and ρ fixed. [2]
- 30** A cyclist rides on a level road at constant speed $v \text{ m s}^{-1}$. A model assumes that the only resistance is a force kv^2 newtons, so that the power P watts the cyclist develops is $P = kv^3$.
- (a) Find the units of k . [2]
- (b) Measurements show that $P = 120 \text{ W}$ when $v = 8 \text{ m s}^{-1}$. Find k . [2]
- (c) Use the model to find v when $P = 405 \text{ W}$. [2]
- (d) At $v = 12 \text{ m s}^{-1}$ the cyclist actually develops 330 W. Compare with the model and suggest one modelling assumption that could explain the difference. [4]

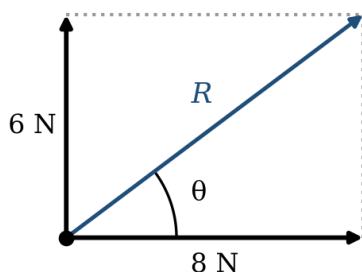
Resolving Forces and Vectors

- 1 A force of magnitude 12 N acts at 30° above the horizontal, as shown.
Find the horizontal and vertical components of the force.



[2]

- 2 Two forces of magnitudes 8 N and 6 N act on a particle at right angles to each other, as shown.
Find the magnitude of the resultant force R , and the angle θ it makes with the 8 N force.



[3]

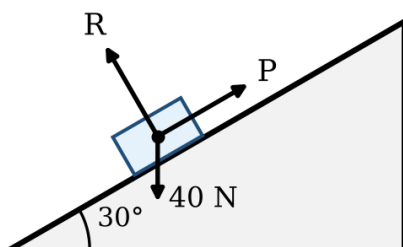
- 3 Three forces $(4\mathbf{i} - 2\mathbf{j})$ N, $(-\mathbf{i} + 5\mathbf{j})$ N and $(\mathbf{a}\mathbf{i} + \mathbf{b}\mathbf{j})$ N act on a particle which is in equilibrium.
Find a and b .

[3]

- 4 Three forces $F_1 = (2\mathbf{i} + 3\mathbf{j})$ N, $F_2 = (-5\mathbf{i} + \mathbf{j})$ N and $F_3 = (\mathbf{a}\mathbf{i} + \mathbf{b}\mathbf{j})$ N act on a particle. The particle is in equilibrium.
Find the values of a and b and the magnitude of F_3 .

[4]

- 5 A particle of weight 40 N rests on a smooth plane inclined at 30° to the horizontal. It is held in equilibrium by a force of magnitude P N acting up the line of greatest slope of the plane. The normal reaction of the plane on the particle is R N.

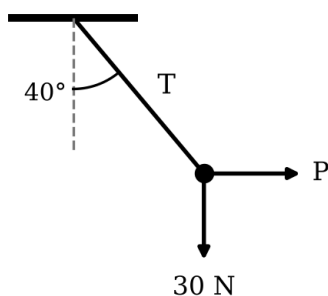


- (a) Find the components of the weight (i) parallel to the plane, (ii) perpendicular to the plane.
(b) Find P and R .

[2]

[2]

- 6 A particle of weight 30 N is attached to a light string, which makes an angle of 40° to the vertical. The particle is held in equilibrium by a horizontal force P, as shown. Find the tension T in the string and the magnitude of P.



[4]

- 7 A particle is in equilibrium under the action of three horizontal forces: 8 N due east, 6 N due north and F. Find the magnitude of F and the bearing of its line of action.

[4]

- 8 A block of mass 4 kg rests on horizontal ground. A force of 20 N pushes down on the block at 50° below the horizontal. Find the normal reaction between the block and the ground. Take $g = 9.8 \text{ m s}^{-2}$.

[3]

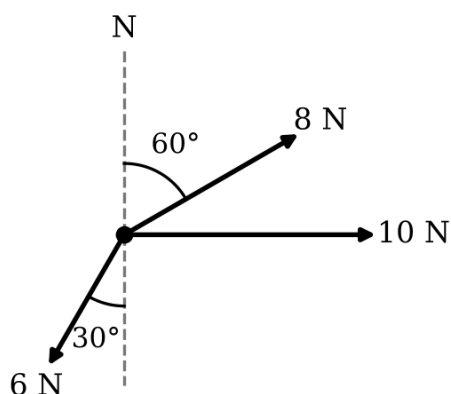
- 9 Two forces of magnitudes 20 N and 21 N act on a particle at right angles to each other.

- (a) Find the magnitude of their resultant, and the angle it makes with the 21 N force.
(b) A third force F is now applied so that the three forces are in equilibrium. State the magnitude of F and the angle it makes with the 21 N force.

[4]

[2]

- 10 Three horizontal forces act on a particle: 10 N due east, 8 N on a bearing of 060° and 6 N on a bearing of 210° , as shown.

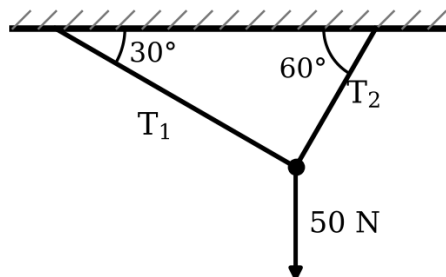


- (a) Find the magnitude of the resultant force.
(b) Find the bearing of the resultant.

[4]

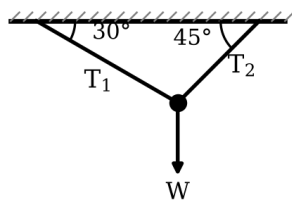
[2]

- 11** A particle of weight 50 N is suspended in equilibrium by two light inextensible strings. The strings are attached to a horizontal ceiling and make angles of 30° and 60° with the ceiling, as shown. The tensions in the strings are T_1 N and T_2 N. Find T_1 and T_2 .



[7]

- 12** A lamp of mass 12 kg is suspended in equilibrium by two light strings, one inclined at 30° and the other at 45° to the horizontal, as shown. Find the tensions T_1 and T_2 . Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 13** A particle of weight W rests on a rough plane inclined at 30° to the horizontal, and is on the point of sliding down the plane.
- (a) Show that the coefficient of friction is $\frac{1}{\sqrt{3}}$. [3]
- (b) Now suppose $\mu = \frac{1}{(2\sqrt{3})}$ and $W = 90 \text{ N}$. Find the least force P , acting up the plane and parallel to it, that keeps the particle at rest. [3]

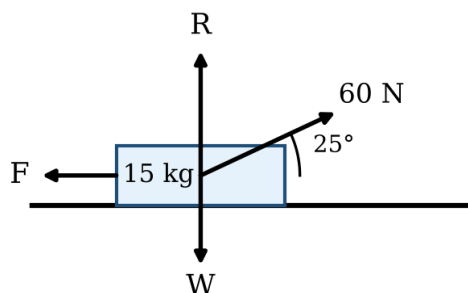
- 14** Three horizontal forces $(2\mathbf{i} + 5\mathbf{j}) \text{ N}$, $(-6\mathbf{i} + 3\mathbf{j}) \text{ N}$ and $(4\mathbf{i} - 2\mathbf{j}) \text{ N}$ act on a particle of mass 2 kg, which can move on a smooth horizontal table. Find the magnitude of the acceleration of the particle and the bearing of its direction of motion, taking $\mathbf{i}$ and $\mathbf{j}$ as east and north. [5]

- 15** Four horizontal forces act on a particle and keep it in equilibrium: 30 N on a bearing of 000° , 40 N on a bearing of 090° , X newtons on a bearing of 240° and Y newtons on a bearing of 180° . Find X and Y . [6]

- 16** Two points A and B are 2.5 m apart on a horizontal ceiling. A particle of weight 130 N hangs at a point C below the ceiling, supported by two light strings AC and BC , where $AC = 1.5 \text{ m}$ and $BC = 2 \text{ m}$.
- (a) Show that angle ACB is 90° . [2]
- (b) Find the tensions in the two strings. [4]

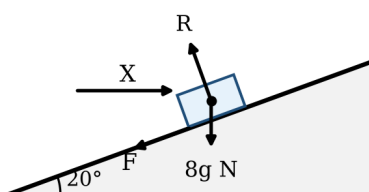
- 17** A particle P of weight 200 N is suspended in equilibrium by two light strings AP and BP , which are attached to fixed points A and B on a horizontal ceiling, on opposite sides of P . String AP makes an angle of 50° with the upward vertical and its tension is 100 N. String BP makes an angle θ with the horizontal. Find θ and the tension in BP . [6]

- 18** A box of mass 15 kg is pulled along rough horizontal ground by a light rope inclined at 25° above the horizontal, as shown. The tension in the rope is 60 N and the box moves in a straight line at constant speed. The friction force is F N and the normal reaction is R N. Take $g = 9.8 \text{ m s}^{-2}$.



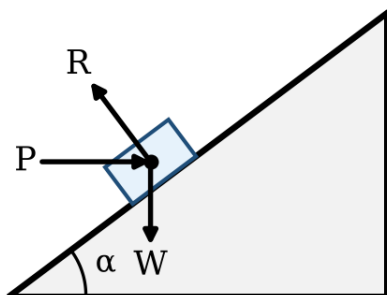
- Find F . [2]
 - Find R . [2]
 - The ground is modelled as rough, with coefficient of friction μ . Find the value of μ . [2]
 - State one modelling assumption you have made about the box. [1]
- 19** Daniel has made some statements about forces. For each, explain the error and give the correct information.
- "A force of 30 N acts at 40° above the horizontal, so its horizontal component is $30 \sin 40^\circ = 19.3$ N and its vertical component is $30 \cos 40^\circ = 23.0$ N." Explain the error and find the correct components. [3]
 - "Two forces of 6 N and 8 N act at a point, so the resultant is $6 + 8 = 14$ N." Explain the error. State the greatest and least possible magnitudes of the resultant and its magnitude when the forces are perpendicular. [3]
 - Find the magnitude of the resultant of the 6 N and 8 N forces when the angle between their lines of action is 60° . [2]

- 20** A block of mass 8 kg is pushed up a rough plane inclined at 20° to the horizontal by a horizontal force X newtons, as shown. The coefficient of friction between the block and the plane is 0.4. The block is on the point of sliding up the plane. Find the value of X . Take $g = 9.8 \text{ m s}^{-2}$.



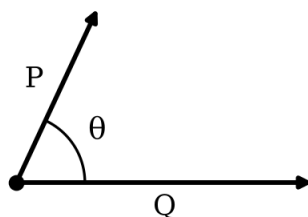
- 21** A broken-down van is towed by two light ropes attached to the front of the van. The ropes make angles of 20° and 35° on opposite sides of the direction of motion. The ropes pull the van with a total forward force of 2000 N and there is no sideways force overall.
- Write down two equations for the tensions T_1 (at 20°) and T_2 (at 35°). [2]
 - Find T_1 and T_2 . [3]
 - Find the sum of the two tensions and compare it with 2000 N, and explain the difference. [2]
- 22** A student resolves the weight of a 50 N block (mass 5.1 kg, take $g = 9.8$) on a smooth slope of angle 40° , writing: "Component down the slope = $50 \cos 40^\circ = 38.3$ N, so the normal reaction is $50 \sin 40^\circ = 32.1$ N."
- Explain the error and write down the correct values. [3]
 - Hence find the force F , acting up the slope parallel to it, required to hold the block at rest, and the acceleration of the block if F is removed. [3]

- 23** Three horizontal forces act on a particle which is in equilibrium: 12 N on a bearing of 000° , P newtons on a bearing of 090° and Q newtons on a bearing of 210° . Find P and Q. [6]
- 24** A box of mass 25 kg rests on rough horizontal ground. Two ropes are attached to the box and both are taut, in the same vertical plane. One rope pulls to the right with tension 80 N at 30° above the horizontal and the other pulls to the left with tension 120 N at 50° above the horizontal. The box is in equilibrium. Take $g = 9.8$.
- Find the normal reaction between the box and the ground. [3]
 - Find the magnitude of the friction force, and state its direction. [2]
 - The coefficient of friction is 0.1. Show that the box does stay in equilibrium. [2]
- 25** A particle of weight W newtons rests in equilibrium on a rough plane inclined at 25° to the horizontal. A force of magnitude 150 N acts on the particle in the vertical plane containing a line of greatest slope, at 30° above the plane and directed up the slope. The normal reaction of the plane on the particle is 120 N.
- Find the value of W. [3]
 - Find the magnitude of the friction force and state whether it acts up or down the plane. [3]
- 26** A box of mass 4 kg is on rough horizontal ground, with coefficient of friction 0.25. A force of magnitude P newtons acts on the box at an angle θ below the horizontal, pushing the box. The box is in limiting equilibrium and the friction force is 20 N. Take $g = 9.8$. Find θ and P. [6]
- 27** A block of mass 8 kg rests on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the block and the plane is 0.5. A horizontal force of magnitude P N, in the vertical plane containing a line of greatest slope, acts on the block as shown, and the block remains in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$.



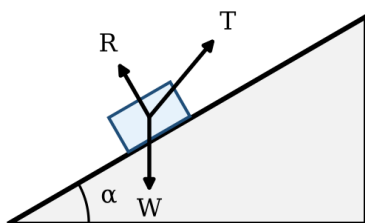
- Show that the normal reaction is $R = 0.8W + 0.6P$, where $W = 8g$. [2]
- Find the least value of P for which the block is in equilibrium. [4]
- Find the greatest value of P for which the block is in equilibrium. [3]

- 28** Two forces of magnitudes P and Q act at a point O . The angle between their lines of action is θ , where $0^\circ < \theta < 180^\circ$, as shown. The magnitude of the resultant is R .



- By resolving the forces parallel and perpendicular to the line of action of Q , show that $R^2 = P^2 + Q^2 + 2PQ \cos \theta$. [4]
- Given that $P = 5 \text{ N}$, $Q = 8 \text{ N}$ and $R = 7 \text{ N}$, find θ . [3]
- For these values, find the angle between the resultant and the 8 N force. [3]

- 29** A particle of weight W rests on a smooth plane inclined at an angle α to the horizontal. It is held in equilibrium by a light string attached to the particle and inclined at an angle β above the plane, in the vertical plane through a line of greatest slope, as shown. The tension is T and the normal reaction is R .



- Show that $T = \frac{W \sin \alpha}{\cos \beta}$ and $R = W \cos \alpha - W \sin \alpha \tan \beta$. [4]
- For $W = 50 \text{ N}$, $\alpha = 30^\circ$ and $\beta = 20^\circ$, find T and R . [2]
- Show that the particle is about to leave the plane when $\beta = 90^\circ - \alpha$, and find the value of β when $\alpha = 30^\circ$. [3]

- 30** A sledge of weight W is dragged at constant speed along rough horizontal ground by a rope inclined at an angle θ above the horizontal. The coefficient of friction is μ .

- Show that the tension in the rope is $T = \frac{\mu W}{(\cos \theta + \mu \sin \theta)}$. [4]
- Show that T is least when $\tan \theta = \mu$, and that the least value of T is $\frac{\mu W}{\sqrt{1 + \mu^2}}$. [5]
- For $W = 400 \text{ N}$ and $\mu = 0.5$, find the least tension and the angle at which the rope should be held. [2]