

Displacement, Velocity and Acceleration

1 (2 marks)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{150}{2.5}$$

M1

$$= 60 \text{ km h}^{-1}$$

A1

Answer: 60 km h⁻¹

2 marks

2 (3 marks)

$$\text{Distance} = 100 + 40 = 140 \text{ m}$$

B1

$$\text{Displacement} = 100 - 40 = 60 \text{ m}$$

M1

in the direction east (the original direction)

A1

Answer: (a) 140 m, (b) 60 m east

3 marks

3 (2 marks)

$$a = \frac{v - u}{t} = \frac{16 - 4}{3}$$

M1

$$= 4 \text{ m s}^{-2}$$

A1

Answer: 4 m s⁻²

2 marks

4 (3 marks)

$$108 \text{ km h}^{-1} = \frac{108 \times 1000}{3600} = 30 \text{ m s}^{-1}$$

M1 A1

$$\text{Time} = \frac{450}{30} = 15 \text{ s}$$

B1

Answer: 30 m s⁻¹, 15 s

3 marks

5 (3 marks)

$$\text{Displacement} = 7 - (-3) = 10 \text{ m}$$

M1 A1

$$\text{Average velocity} = \frac{10}{5} = 2 \text{ m s}^{-1}$$

A1

Answer: 10 m, 2 m s⁻¹

3 marks

6 (3 marks)

$$a = \frac{12 - 30}{6}$$

M1

$$= -3 \text{ m s}^{-2}$$

A1

The negative sign shows the acceleration is opposite to the direction of motion, so the train slows down (a deceleration of 3 m s⁻²)

E1

Answer: -3 m s⁻² (deceleration)

3 marks

7 (4 marks)

$$\text{Time for the first part} = \frac{12}{24} = 0.5 \text{ h}$$

M1

$$\text{Time for the second part} = \frac{8}{16} = 0.5 \text{ h}$$

A1

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{20}{1}$$

M1

$$= 20 \text{ km h}^{-1}$$

A1

Answer: 20 km h⁻¹

4 marks

8 (5 marks)

$$\text{Total time} = 60 \text{ minutes} = 1 \text{ h}$$

B1

$$(a) \text{ Total distance} = 3 + 1.5 = 4.5 \text{ km, so average speed} = 4.5 \text{ km h}^{-1}$$

M1 A1

$$(b) \text{ Displacement} = 3 - 1.5 = 1.5 \text{ km east}$$

M1

$$\text{Average velocity} = 1.5 \text{ km h}^{-1} \text{ east}$$

A1

Answer: (a) 4.5 km h⁻¹, (b) 1.5 km h⁻¹ east

5 marks

9 (4 marks)

$$v = u + at = 0 + 1.5 \times 4$$

M1

$$= 6 \text{ m s}^{-1}$$

A1

$$7.5 = 1.5t$$

M1

$$t = 5 \text{ s}$$

A1

Answer: 6 m s⁻¹, 5 s

4 marks

10 (4 marks)

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time}}.$$

$$0 \text{ to } 2: \frac{11 - 3}{2} = 4 \text{ m s}^{-1}$$

M1 A1

$$2 \text{ to } 4: \frac{11 - 11}{2} = 0 \text{ m s}^{-1}$$

A1

$$4 \text{ to } 6: \frac{5 - 11}{2} = -3 \text{ m s}^{-1}; 6 \text{ to } 8: \frac{-7 - 5}{2} = -6 \text{ m s}^{-1}$$

A1

Answer: 4, 0, -3, -6 m s⁻¹

4 marks

11 (4 marks)

$$\text{Closing speed} = 12 + 18 = 30 \text{ m s}^{-1}$$

M1

$$\text{Time} = \frac{300}{30} = 10 \text{ s}$$

A1

$$\text{Distance of the faster car} = 18 \times 10$$

M1

$$= 180 \text{ m}$$

A1

Answer: 10 s, 180 m

4 marks

12 (3 marks)

$$\text{Distance travelled} = 1000 + 200 = 1200 \text{ m}$$

M1 A1

$$\text{Time} = \frac{1200}{20} = 60 \text{ s}$$

A1

Answer: 60 s

3 marks

13 (3 marks)

$$v = u + at: 0 = 60 - 2.4t$$

M1 A1

$$t = 25 \text{ s}$$

A1

Answer: 25 s

3 marks

14 (5 marks)

$$\text{Let the one-way distance be } d \text{ km. Time out} = \frac{d}{15}, \text{ time back} = \frac{d}{10}$$

M1

$$\text{Total time} = \frac{d}{15} + \frac{d}{10} = \frac{d}{6} \text{ hours}$$

M1 A1

$$\text{Average speed} = \frac{2d}{\frac{d}{6}} = 12 \text{ km h}^{-1}$$

M1 A1

12 differs from 12.5 because more time is spent travelling at the slower speed

Answer: 12 km h⁻¹ (not 12.5)

5 marks

15 (5 marks)

$$\text{Position of A: } 5t. \text{ Position of B: } 7(t - 8)$$

B1 B1

$$5t = 7(t - 8)$$

M1

$$56 = 2t, \text{ so } t = 28 \text{ s}$$

A1

$$\text{Distance} = 5 \times 28 = 140 \text{ m}$$

A1

Answer: 28 s, 140 m

5 marks

16 (4 marks)

Initial velocity = 24, final velocity = -16

B1

$$a = \frac{-16 - 24}{0.025}$$

M1 A1

= -1600 m s⁻², i.e. 1600 m s⁻² directed away from the wall

A1

Answer: 1600 m s⁻² away from the wall

4 marks

17 (4 marks)Relative speed = 30 - 20 = 10 m s⁻¹

M1 A1

Relative distance = 150 + 250 = 400 m

M1

$$\text{Time} = \frac{400}{10} = 40 \text{ s}$$

A1

Answer: 40 s

4 marks

18 (5 marks)

$$\text{First distance} = 72 \times \frac{40}{60} = 48 \text{ km}$$

M1 A1

$$\text{Last time} = \frac{60}{90} \text{ h} = 40 \text{ minutes}$$

M1

Total distance = 108 km, total time = 40 + 10 + 40 = 90 min = 1.5 h

A1

$$\text{Average speed} = \frac{108}{1.5} = 72 \text{ km h}^{-1}$$

A1

Answer: 72 km h⁻¹

5 marks

19 (5 marks)

$$(a) (14 - 2) + (14 - 6) = 12 + 8 = 20 \text{ m}$$

M1 A1

$$(b) \frac{20}{10} = 2 \text{ m s}^{-1}$$

A1

$$(c) 6 - 2 = 4 \text{ m}$$

B1

$$(d) \frac{4}{10} = 0.4 \text{ m s}^{-1}$$

B1

Answer: (a) 20 m, (b) 2 m s⁻¹, (c) 4 m, (d) 0.4 m s⁻¹

5 marks

20 (5 marks)Let Ali's speed be v. Times: $\frac{400}{v}$ and $\frac{400}{(v+2)}$

B1

$$\frac{400}{v} - \frac{400}{(v+2)} = 10$$

M1

$$400(v+2) - 400v = 10v(v+2), \text{ so } v^2 + 2v - 80 = 0$$

M1 A1

$$(v+10)(v-8) = 0, v = 8$$

A1

Answer: Ali 8 m s⁻¹, Ben 10 m s⁻¹

5 marks

21 (5 marks)

$$(a) \text{ Closing speed } 45 \text{ km h}^{-1}, \text{ so } t = \frac{90}{45} = 2 \text{ hours}$$

M1 A1

$$\text{Distance} = 15 \times 2 = 30 \text{ km}$$

A1

Answer: 2 h, 30 km

3 marks

$$(b) \text{ The bird flies for the whole 2 hours at } 40 \text{ km h}^{-1}: 80 \text{ km}$$

M1 A1

Answer: 80 km

2 marks

22 (5 marks)(a) Slowing down needs velocity and acceleration in opposite directions: true on the way up ($v > 0, a < 0$)

E1

On the way down $v < 0$ and $a < 0$ have the same sign, so the ball speeds up

E1

Answer: speeds up on the way down

2 marks

$$(b) v = u + at: \text{ after } 1 \text{ s}, v = 14.7 - 9.8 = 4.9 \text{ m s}^{-1} \text{ (rising)}$$

M1 A1

$$\text{after } 2.5 \text{ s}, v = 14.7 - 24.5 = -9.8 \text{ m s}^{-1} \text{ (falling)}$$

A1

Answer: 4.9 m s⁻¹ rising; -9.8 m s⁻¹ falling

3 marks

23 (5 marks)

(a) $60 \times 1609 = 96\,540$ m per hour

M1

$\div 3600 = 26.8 \text{ m s}^{-1}$

A1

Answer: 26.8 m s^{-1}

2 marks

(b) $a = \frac{26.8}{8.5}$

M1

$= 3.15 \text{ m s}^{-2}$

A1

Answer: 3.15 m s^{-2}

2 marks

(c) $\frac{3.15}{9.8} = 0.32 \text{ g}$

B1

Answer: about 0.32 g

1 mark

24 (7 marks)

(a) $v = 0.8 \times 5 = 4 \text{ m s}^{-1}$

M1 A1

Deceleration time = $\frac{4}{1.0} = 4 \text{ s}$

A1

Answer: 4 m s^{-1} , 4 s

3 marks

(b) Total time = $5 + 12 + 4 = 21 \text{ s}$

B1

Acceleration stage: $\frac{1}{2}(0 + 4) \times 5 = 10 \text{ m}$; constant: $4 \times 12 = 48 \text{ m}$; deceleration: $\frac{1}{2}(4 + 0) \times 4 = 8 \text{ m}$

M1 A1

Total = 66 m

A1

Answer: 21 s, 66 m

4 marks

25 (6 marks)

Let the speed be $x \text{ km h}^{-1}$: times $\frac{180}{x}$ and $\frac{180}{(x + 15)}$

B1

$\frac{180}{x} - \frac{180}{(x + 15)} = 0.6$

M1 A1

$180 \times 15 = 0.6x(x + 15)$, so $x^2 + 15x - 4500 = 0$

M1

$(x + 75)(x - 60) = 0$

M1

$x = 60$ (speed positive)

A1

Answer: 60 km h^{-1}

6 marks

26 (7 marks)

(a) $t = 4$: $3 \times 4 = 12 \text{ m}$

B1

$t = 10$: $12 + (-2)(6) = 0 \text{ m}$, so it is back at O

M1 A1

$t = 12$: $0 + 5 \times 2 = 10 \text{ m}$

Answer: 12 m, 0 m, 10 m

3 marks

(b) Distance = $12 + 12 + 10 = 34 \text{ m}$

M1 A1

Average speed = $\frac{34}{12} = \frac{17}{6} = 2.83 \text{ m s}^{-1}$

A1

Answer: 34 m, 2.83 m s^{-1}

3 marks

(c) $\frac{10}{12} = \frac{5}{6} \text{ m s}^{-1}$ in the direction OA

B1

Answer: $\frac{5}{6} \text{ m s}^{-1}$

1 mark

27 (9 marks)

(a) Let each stage be d : total time = $\frac{d}{u} + \frac{d}{v} = \frac{d(u + v)}{uv}$

M1 A1

Average speed = $\frac{2d}{\frac{d(u + v)}{uv}} = \frac{2uv}{(u + v)}$

A1

Answer: shown

3 marks

(b) Let each stage last T : distance = $uT + vT$, time = $2T$

M1

Average speed = $\frac{(u + v)T}{2T} = \frac{1}{2}(u + v)$

A1

Answer: shown

2 marks

$$(c) \frac{1}{2}(u+v) - \frac{2uv}{(u+v)} = \frac{(u+v)^2 - 4uv}{2(u+v)} = \frac{(u-v)^2}{2(u+v)}$$

M1 A1

This is ≥ 0 , with equality only when $u = v$

E1

For 60 and 40: equal-distance 48 km h^{-1} , equal-time 50 km h^{-1}

B1

Answer: 48 and 50 km h^{-1}

4 marks

28 (9 marks)

(a) The faster runner gains $7 - 5 = 2 \text{ m s}^{-1}$ and must gain one lap, 400 m

M1 A1

$$t = \frac{400}{2} = 200 \text{ s}$$

A1

Answer: 200 s

3 marks

(b) Closing speed $5 + 7 = 12 \text{ m s}^{-1}$; they meet when together they have covered 400 m

M1

$$t = \frac{400}{12} = \frac{100}{3} \text{ s} = 33.3 \text{ s}$$

A1

Meetings at $t = \frac{100}{3}n$, with $n \geq 1$ and $\frac{100n}{3} \leq 720$

M1

$n \leq 21.6$, so 21 meetings

A1

Answer: 33.3 s, 21 meetings

4 marks

(c) Passes at $t = 200k \leq 720$, $k = 1, 2, 3$

M1

so 3 times

A1

Answer: 3 times

2 marks

29 (8 marks)

(a) Closing speed 30 km h^{-1} ; $t = \frac{24}{30} = 0.8 \text{ h}$

M1 A1

Distance from A = $12 \times 0.8 = 9.6 \text{ km}$

A1

Answer: after 0.8 h (12:48), 9.6 km from A

3 marks

(b) Ferry 1 reaches B at $t = 2 \text{ h}$; ferry 2 reaches A at $t = \frac{4}{3} \text{ h}$

B1 B1

At the next meeting the ferries have together covered $3 \times 24 = 72 \text{ km}$ (they are again approaching each other)

M1

$$t = \frac{72}{30} = 2.4 \text{ h}$$

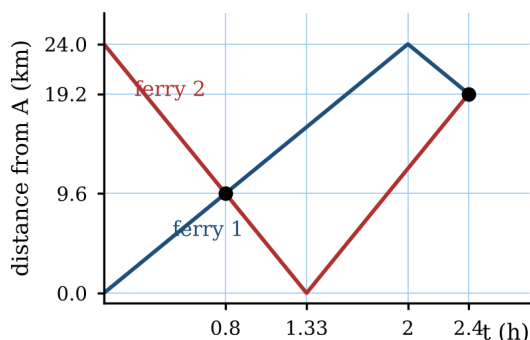
A1

Ferry 1: $24 - 12(2.4 - 2) = 19.2 \text{ km}$ from A (check ferry 2: $18 \times 2.4 - 24 = 19.2 \text{ km}$)

A1

Answer: 2.4 h, 19.2 km from A

5 marks



30 (8 marks)

(a) P arrives after $\frac{120}{60} = 2 \text{ h}$, i.e. at 11:00

B1

Q has $2 - \frac{1}{3} = \frac{5}{3} \text{ h}$

M1

$$\text{Speed} = \frac{120}{(5/3)} = 72 \text{ km h}^{-1}$$

A1

Answer: 72 km h^{-1}

3 marks

(b) Distances equal: $60t = 80(t - \frac{1}{3})$ with t hours after 09:00

M1 A1

$$20t = \frac{80}{3}, \text{ so } t = \frac{4}{3} \text{ h (10:20)}$$

A1

$$\text{Distance} = 60 \times \frac{4}{3} = 80 \text{ km}$$

A1

$$Q \text{ reaches B at } t = \frac{1}{3} + 1.5 = \frac{11}{6} \text{ h, which is } \frac{1}{6} \text{ h} = 10 \text{ minutes before P}$$

A1

Answer: 80 km from A at 10:20; 10 minutes before P

5 marks

Motion Graphs

1 (3 marks)

$$\text{Speed} = \text{gradient} = \frac{20}{5} = 4 \text{ m s}^{-1}$$

B1

Stationary from $t = 5$ to $t = 10$, i.e. for 5 s

B1

$$\text{Return speed} = \frac{20}{10} = 2 \text{ m s}^{-1}$$

B1

Answer: 4 m s⁻¹, 5 s, 2 m s⁻¹

3 marks

2 (3 marks)

$$\text{Acceleration} = \text{gradient} = \frac{12}{4} = 3 \text{ m s}^{-2}$$

M1 A1

$$\text{Deceleration} = \frac{12}{4} = 3 \text{ m s}^{-2}$$

A1

Answer: 3 m s⁻², 3 m s⁻²

3 marks

3 (3 marks)

(a) velocity

B1

(b) acceleration

B1

(c) displacement (change in position)

B1

Answer: velocity; acceleration; displacement

3 marks

4 (2 marks)

$$\text{Area of the triangle} = \frac{1}{2} \times 8 \times 20$$

M1

$$= 80 \text{ m}$$

A1

Answer: 80 m

2 marks

5 (3 marks)

$$0 \text{ to } 5 \text{ s: uniform acceleration } \frac{10}{5} = 2 \text{ m s}^{-2}$$

B1

5 to 12 s: constant velocity 10 m s⁻¹ (acceleration 0)

B1

$$12 \text{ to } 16 \text{ s: uniform deceleration } \frac{10 - 4}{4} = 1.5 \text{ m s}^{-2}$$

B1

Answer: 2 m s⁻², 0, deceleration 1.5 m s⁻²

3 marks

6 (5 marks)

$$\text{Accelerating time} = \frac{20}{0.5} = 40 \text{ s, decelerating time} = \frac{20}{1} = 20 \text{ s}$$

B1 B1

$$\text{Distances: } \frac{1}{2} \times 40 \times 20 = 400 \text{ m; } 20T; \frac{1}{2} \times 20 \times 20 = 200 \text{ m}$$

M1

$$400 + 20T + 200 = 1200$$

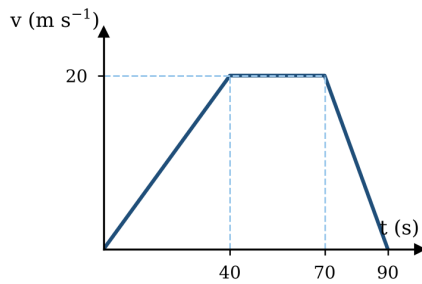
M1

$$T = 30.0 \text{ s}$$

A1

Answer: T = 30 s

5 marks



7 (4 marks)

(a) Out 30 m, then 30 m back: distance = 60 m

M1 A1

(b) Average speed = $\frac{60}{40} = 1.5 \text{ m s}^{-1}$

A1

(c) The displacement is zero, so the average velocity is 0

B1

Answer: (a) 60 m, (b) 1.5 m s^{-1} , (c) 0

4 marks

8 (4 marks)

Positive part 0 to 4 s: area = $\frac{1}{2} \times 4 \times 8 = 16 \text{ m}$

M1 A1

Negative part 4 to 6 s: area = $\frac{1}{2} \times 2 \times 4 = 4 \text{ m}$ (to the left)

A1

Displacement = $16 - 4 = 12 \text{ m}$; distance = $16 + 4 = 20 \text{ m}$

A1

Answer: displacement 12 m, distance 20 m

4 marks

9 (4 marks)

Distance of A after 10 s = $10 \times 10 = 100 \text{ m}$

B1

B: area of triangle = $\frac{1}{2} \times 10 \times 20$

M1 A1

= 100 m, the same, so the cars are level

E1

Answer: both 100 m

4 marks

10 (4 marks)

The change in velocity is the area under the acceleration-time graph.

t = 4: $2 \times 4 = 8 \text{ m s}^{-1}$

M1 A1

t = 10: unchanged, 8 m s^{-1}

A1

t = 16: $8 - 1 \times 6 = 2 \text{ m s}^{-1}$

A1

Answer: 8, 8, 2 m s^{-1}

4 marks

11 (6 marks)

(a) Straight line from (0, 0) to (40, 240) with gradient 6

B1

Horizontal line from t = 40 to t = 60 at s = 240

B1

Straight line down to (120, 0), with the values marked

B1

Answer: sketch

3 marks

(b) Return time = $\frac{240}{4} = 60 \text{ s}$, so the trip takes $60 + 60 = 120 \text{ s}$

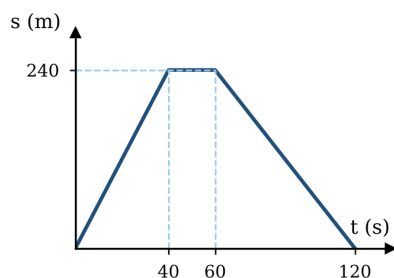
M1 A1

Average speed = $\frac{480}{120} = 4 \text{ m s}^{-1}$

A1

Answer: 120 s, 4 m s^{-1}

3 marks



12 (5 marks)

(a) Line from the origin to (8, 12)

B1

Horizontal line to $t = 18$

B1

Line down to the t -axis at $t = 24$

B1

Answer: sketch

3 marks

(b) $\text{Area} = \frac{1}{2}(10 + 24) \times 12$

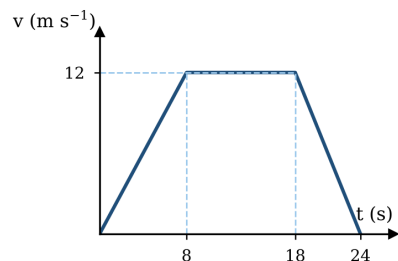
M1

$= 204 \text{ m}$

A1

Answer: 204 m

2 marks



13 (4 marks)

The graph is a trapezium: the constant-speed stage lasts $100 - 20 - 30 = 50 \text{ s}$

B1

$\text{Distance} = \frac{1}{2}(50 + 100)V = 2100$

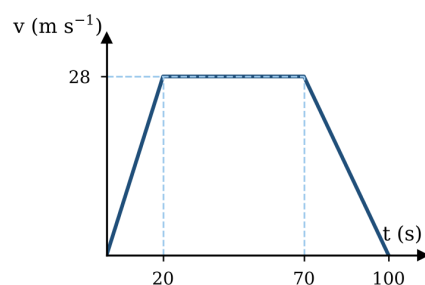
M1 A1

$V = 28 \text{ m s}^{-1}$

A1

Answer: 28 m s⁻¹

4 marks



14 (6 marks)

Greatest speed $V = 0.5T$

B1

$\text{Distance} = \frac{1}{2}(30 - T + 45)V = \frac{1}{2}(75 - T) \times 0.5T$

M1 A1

$\frac{1}{4}T(75 - T) = 189$, so $T^2 - 75T + 756 = 0$

A1

$(T - 12)(T - 63) = 0$

M1

$T \leq 30$, so $T = 12$

A1

Answer: $T = 12$

6 marks

15 (6 marks)

(a) A: $\frac{30}{50} \text{ km per minute} = 0.6 \times 60 = 36 \text{ km h}^{-1}$

M1 A1

B: $\frac{30}{40} \text{ km per minute} = 45 \text{ km h}^{-1}$

A1

(b) $s_A = 0.6t$, $s_B = 30 - 0.75(t - 20)$

M1

$0.6t = 45 - 0.75t$, so $t = \frac{100}{3} \text{ minutes}$

A1

(c) $s = 0.6 \times \frac{100}{3} = 20 \text{ km}$

A1

Answer: 36 and 45 km h⁻¹; $t = \frac{100}{3} \text{ min}$; 20 km from P

6 marks

16 (4 marks)

$$0 \leq t \leq 4: \text{gradient } \frac{12}{4} = 3, \text{ so } v = 3t$$

M1 A1

$$4 \leq t \leq 10: v = 12$$

B1

$$10 \leq t \leq 16: \text{gradient } -2, v = 12 - 2(t - 10) = 32 - 2t$$

A1

Answer: 3t; 12; 32 - 2t

4 marks

17 (5 marks)

$$\text{Reaction distance} = 25 \times 0.6 = 15 \text{ m}$$

B1

$$\text{Time to stop} = \frac{25}{5} = 5 \text{ s, so the graph is a trapezium}$$

M1

$$\text{Braking distance} = \frac{1}{2} \times 5 \times 25 = 62.5 \text{ m}$$

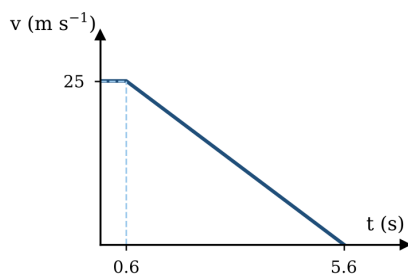
M1 A1

$$\text{Total} = 77.5 \text{ m}$$

A1

Answer: 77.5 m

5 marks

**18 (6 marks)**

$$(a) a = \frac{10 - 4}{10} = 0.6 \text{ m s}^{-2}$$

B1

$$s = \frac{1}{2}(4 + 10) \times 10 = 70 \text{ m}$$

B1

Answer: 0.6 m s⁻², 70 m

2 marks

$$(b) A: 12t. B: 70 + 10(t - 10) = 10t - 30$$

M1

$$\text{Difference} = 12t - (10t - 30) = 2t + 30$$

A1

Answer: shown

2 marks

$$(c) \text{ The front of A is 130 m ahead of its rear, so } 2t + 30 = 130$$

M1

$$t = 50 \text{ s}$$

A1

Answer: 50 s

2 marks

19 (7 marks)

$$(a) \text{ Time accelerating} = \frac{V}{1} = V, \text{ time decelerating} = \frac{V}{2}$$

M1

$$\text{Constant stage} = 20 - V - \frac{V}{2} = 20 - 1.5V$$

A1

Answer: shown

2 marks

$$(b) \text{ Distance} = \frac{1}{2}V^2 + \frac{1}{2}V(V/2) + V(20 - 1.5V) = 112$$

M1 A1

$$\frac{3}{4}V^2 + 20V - 1.5V^2 = 112 \text{ gives } 20V - \frac{3}{4}V^2 = 112$$

M1

$$3V^2 - 80V + 448 = 0$$

A1

$$(3V - 56)(V - 8) = 0, V = 8 \text{ (} V = \frac{56}{3} \text{ gives a negative time at constant speed)}$$

A1

Answer: V = 8 m s⁻¹

5 marks

20 (6 marks)

(a) A straight displacement-time graph means constant velocity, and its gradient is velocity (3 m s⁻¹), so the acceleration is zero

E1

The area under a displacement-time graph has no physical meaning

E1

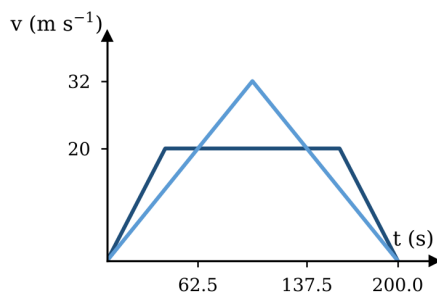
Answer: gradient is velocity; area meaningless

2 marks

(b) Velocity 3 m s⁻¹, acceleration 0

B1

Distance = 18 m (the final displacement)	B1
Answer: 3 m s^{-1}, 0, 18 m	2 marks
(c) A horizontal line at $v = 3$ from $t = 0$ to 6	B1
Area of the rectangle $6 \times 3 = 18 \text{ m}$	B1
Answer: horizontal line $v = 3$	2 marks
21 (8 marks)	
(a) P's speed is $2t$ for $t \leq 10$; P is slower than Q until $2t = 12$, so $t = 6$	M1 A1
Distance of Q = $12 \times 6 = 72 \text{ m}$, of P = $\frac{1}{2} \times 6 \times 12 = 36 \text{ m}$	M1
Gap = 36 m	A1
Answer: 36 m	4 marks
(b) At $t = 10$: P has gone 100 m, Q 120 m, so P has not yet caught Q	B1
For $t \geq 10$: $P = 100 + 20(t - 10) = 20t - 100$, $Q = 12t$	M1 A1
$20t - 100 = 12t$, so $t = 12.5 \text{ s}$	A1
Answer: 12.5 s	4 marks
22 (8 marks)	
(a) 0 to 5 s: $\frac{10}{5} = 2 \text{ m s}^{-2}$	B1
5 to 9 s: $\frac{0 - 10}{4} = -2.5 \text{ m s}^{-2}$	M1
9 to 15 s: $\frac{-15 - 0}{6} = -2.5 \text{ m s}^{-2}$	A1
Answer: 2, -2.5, -2.5 m s⁻²	3 marks
(b) Area 0 to 9: $\frac{1}{2} \times 9 \times 10 = 45 \text{ m}$ to the right	M1 A1
Area 9 to 15: $\frac{1}{2} \times 6 \times 15 = 45 \text{ m}$ to the left	M1
Displacement = $45 - 45 = 0$, so it is at O	A1
Greatest distance 45 m (at $t = 9$); total distance 90 m	B1
Answer: 45 m, 90 m	5 marks
23 (5 marks)	
Initial acceleration = $\frac{V}{10}$, so deceleration $\frac{2V}{10}$ and the braking time is 5 s	B1 M1
Distance = $\frac{1}{2}(10)V + 30V + \frac{1}{2}(5)V = 37.5V$	M1 A1
$37.5V = 540$, so $V = 14.4 \text{ m s}^{-1}$; acceleration = 1.44 m s^{-2}	A1
Answer: $V = 14.4 \text{ m s}^{-1}$, $a = 1.44 \text{ m s}^{-2}$	5 marks
24 (10 marks)	
(a) Area of trapezium = $\frac{1}{2}(120 + 200) \times 20$	M1
= 3200 m	A1
Answer: 3200 m	2 marks
(b) $\frac{1}{2} \times 200 \times V = 3200$, so $V = 32 \text{ m s}^{-1}$	M1 A1
Answer: $V = 32 \text{ m s}^{-1}$, acceleration 0.32 m s^{-2}	2 marks
(c) Train 2 reaches 20 m s^{-1} at $t = \frac{20}{0.32} = 62.5 \text{ s}$	M1 A1
By symmetry of train 2, the other time is $200 - 62.5 = 137.5 \text{ s}$	A1
Answer: 62.5 s and 137.5 s	3 marks
(d) Train 1: $\frac{1}{2} \times 40 \times 20 + 20 \times 20 = 800 \text{ m}$	M1 A1
Train 2: $\frac{1}{2} \times 0.32 \times 60^2 = 576 \text{ m}$, so the distance apart is 224 m	A1
Answer: 224 m	3 marks

**25 (8 marks)**

(a) Car: line from the origin to (12, 18), then horizontal

B1

Motorcycle: starts at $t = 10$, reaches 24 m s^{-1} at $t = 18$, then horizontal

B1

Key values marked on the axes

B1

Answer: sketch

3 marks

(b) Car for $t \geq 12$: $\frac{1}{2} \times 12 \times 18 + 18(t - 12) = 18t - 108$

B1

Motorcycle at $t = 18$: $\frac{1}{2} \times 8 \times 24 = 96 \text{ m}$

M1 A1

Motorcycle for $t \geq 18$: $96 + 24(t - 18) = 24t - 336$

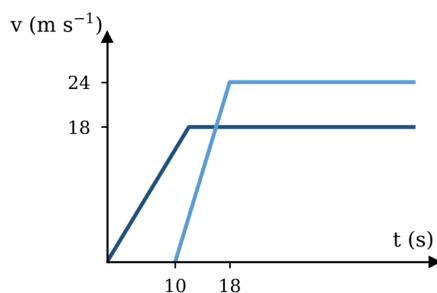
M1

$18t - 108 = 24t - 336$ gives $t = 38 \text{ s}$ and distance 576 m

A1

Answer: $t = 38 \text{ s}$, 576 m

5 marks

**26 (8 marks)**

(a) Accelerating time $= 2V$, decelerating time $= \frac{4V}{3}$

M1

Constant stage $= 150 - 2V - \frac{4V}{3} = 150 - \frac{10V}{3}$

A1

Answer: shown

2 marks

(b) Distance $= \frac{1}{2}V(2V) + \frac{1}{2}V(\frac{4V}{3}) + V(150 - \frac{10V}{3}) = 3000$

M1 A1

$$V^2 + \frac{2}{3}V^2 + 150V - \frac{10}{3}V^2 = 3000$$

M1

$$150V - \frac{5}{3}V^2 = 3000, \text{ so } V^2 - 90V + 1800 = 0$$

A1

$$(V - 30)(V - 60) = 0$$

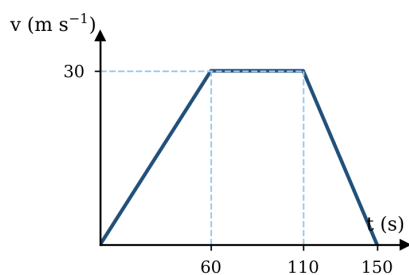
M1

$V = 60$ gives a negative constant-speed time ($150 - 200$), so $V = 30$

A1

Answer: $V = 30 \text{ m s}^{-1}$

6 marks



27 (8 marks)

(a) Time to reach 30: $\frac{30}{3} = 10$ s after starting, so $t = 16$

M1 A1

Police distance = $\frac{1}{2} \times 10 \times 30 = 150$ m; speeding car = $24 \times 16 = 384$ m

M1

$150 < 384$, so not yet caught

A1

Answer: shown

4 marks

(b) For $t > 16$: police = $150 + 30(t - 16)$

M1

$150 + 30t - 480 = 24t$

M1 A1

$6t = 330$, so $t = 55$ s; distance = $24 \times 55 = 1320$ m

A1

Answer: $t = 55$ s, 1320 m

4 marks

28 (12 marks)

(a) Accelerating time $\frac{V}{a}$, decelerating time $\frac{V}{b}$, constant stage $T - \frac{V}{a} - \frac{V}{b}$

M1 A1

Distance = $\frac{V^2}{2a} + \frac{V^2}{2b} + V(T - \frac{V}{a} - \frac{V}{b}) = D$

M1

$D = VT - \frac{V^2}{2}(\frac{1}{a} + \frac{1}{b})$

A1

Divide by V and rearrange: $T = \frac{D}{V} + \frac{V}{2}(\frac{1}{a} + \frac{1}{b})$

A1

Answer: shown

5 marks

(b) $\frac{V}{2}(\frac{1}{a} + \frac{1}{b}) = 1.5V$; so $\frac{1200}{V} + 1.5V = 90$

M1

$1.5V^2 - 90V + 1200 = 0$, $V^2 - 60V + 800 = 0$

M1 A1

$(V - 20)(V - 40) = 0$; $V = 40$ needs $3V = 120 > 90$ s, impossible; so $V = 20$

A1

Answer: $V = 20$ m s⁻¹

4 marks

(c) No cruising: $D = \frac{1}{2}V^2(\frac{1}{0.5} + \frac{1}{1}) = 1.5V^2$, so $V^2 = 800$, $V = 20\sqrt{2}$

M1 A1

$T = \frac{D}{V} + 1.5V = \frac{1200}{28.28} + 42.43 = 84.9$ s

A1

Answer: 84.9 s

3 marks

29 (9 marks)

(a) Area: $\frac{1}{2}(4)V + 6V + \frac{1}{2}(8)(V - U) = 0$

M1 A1

$2V + 6V + 4V - 4U = 0$, so $U = 3V$

A1

Gradient: $\frac{V+U}{8} = 1.5$, so $\frac{4V}{8} = 1.5$

M1

$V = 3$, $U = 9$

A1

Answer: $V = 3$, $U = 9$

5 marks

(b) $v = 0$ when $10 + \frac{3}{1.5} = 12$ s

M1 A1

Distance = $\frac{1}{2}(4)(3) + 6(3) + \frac{1}{2}(2)(3) = 6 + 18 + 3 = 27$ m

A1

Answer: $t = 12$ s, 27 m

3 marks

(c) $27 + 27 = 54$ m (the return is also 27 m)

B1

Answer: 54 m

1 mark

30 (10 marks)

(a) Distance = $15 \times \frac{40}{60}$

M1

= 10 km

A1

Answer: 10 km

2 marks

(b) Anna's total time = $\frac{2}{3} + \frac{(D-10)}{5}$ hours

M1

Ben's total time = $\frac{10}{5} + \frac{(D-10)}{15}$ hours

M1

$\frac{2}{3} + \frac{(D-10)}{5} = 2 + \frac{(D-10)}{15}$ gives $\frac{2(D-10)}{15} = \frac{4}{3}$, so $D = 20$ km

A1

Total time = $\frac{2}{3} + 2 = 2$ h 40 min, so they arrive at 11:40

A1

Answer: D = 20 km, 11:40

4 marks

(c) Anna: (0, 0) to (40, 10) to (160, 20)

B1

Ben: (0, 0) to (120, 10) to (160, 20)

B1

Answer: sketch

2 marks

(d) Anna: $10 + 5 \times \frac{20}{60} = \frac{35}{3}$ km; Ben: 5 km

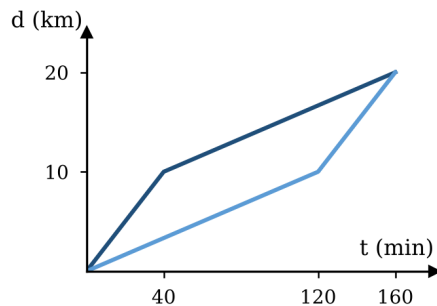
M1

Difference = $\frac{20}{3} = 6.67$ km

A1

Answer: 6.67 km

2 marks



SUVAT Equations

1 (4 marks)

Using $v = u + at$: $v = 0 + 3 \times 8$

M1

$v = 24$

A1

Using $s = ut + \frac{1}{2}at^2$: $s = 0 \times 8 + \frac{1}{2} \times 3 \times 8^2$

M1

$s = 96$

A1

Answer: 24 m s^{-1} , 96 m

4 marks

2 (2 marks)

Using $v = u + at$: $v = 12 + (-2) \times 5$

M1

$v = 2$

A1

Answer: 2 m s^{-1}

2 marks

3 (4 marks)

Using $v = u + at$: $17 = 5 + a \times 4$

M1

$a = 3$

A1

Using $s = \frac{1}{2}(u + v)t$: $s = \frac{1}{2}(5 + 17) \times 4$

M1

$s = 44$

A1

Answer: 3 m s^{-2} , 44 m

4 marks

4 (2 marks)

Using $v^2 = u^2 + 2as$: $20^2 = 0^2 + 2 \times a \times 80$

M1

$a = 2.5$

A1

Answer: 2.5 m s^{-2}

2 marks

5 (4 marks)

Using $v^2 = u^2 + 2as$: $0^2 = 20^2 + 2 \times a \times 50$

M1

$a = -4$

A1

Using $s = \frac{1}{2}(u + v)t$: $50 = \frac{1}{2}(20 + 0) \times t$

M1

$t = 5$

A1

Answer: 4 m s^{-2} , 5 s

4 marks

6 (2 marks)

$$\text{Using } s = ut + \frac{1}{2}at^2: s = 6 \times 4 + \frac{1}{2} \times 1.5 \times 4^2$$

M1

$$s = 36$$

A1

Answer: 36 m

2 marks

7 (5 marks)Known: $s = 90$, $u = 30$, $v = 0$; find a , t

B1

$$\text{Using } v^2 = u^2 + 2as: 0^2 = 30^2 + 2 \times a \times 90$$

M1

$$a = -5$$

A1

$$\text{Using } s = \frac{1}{2}(u + v)t: 90 = \frac{1}{2}(30 + 0) \times t$$

M1

$$t = 6$$

A1

Answer: 5 m s⁻², 6 s

5 marks

8 (5 marks)Known: $s = 21$, $u = 4$, $a = 2$; find t , v

B1

$$\text{Using } s = ut + \frac{1}{2}at^2: 21 = 4 \times t + \frac{1}{2} \times 2 \times t^2$$

M1

$$t^2 + 4t - 21 = 0, \text{ so } (t + 7)(t - 3) = 0 \text{ and } t > 0: t = 3$$

A1

$$\text{Using } v^2 = u^2 + 2as: v^2 = 4^2 + 2 \times 2 \times 21$$

M1

$$v = 10$$

A1

Answer: 3 s, 10 m s⁻¹

5 marks

9 (5 marks)

$$\text{Acceleration stage: } s = \frac{1}{2}(0 + 9) \times 2.5 = 11.25 \text{ m}$$

M1 A1

$$\text{Remaining distance} = 100 - 11.25 = 88.75 \text{ m}$$

M1

$$\text{Time at constant speed} = \frac{88.75}{9} = 9.861 \text{ s}$$

A1

$$\text{Total time} = 2.5 + 9.861 = 12.4 \text{ s}$$

A1

Answer: 12.4 s

5 marks

10 (7 marks)

$$\text{(a) } a = \frac{v - u}{t} = \frac{14 - 6}{5}$$

M1

$$= 1.6 \text{ m s}^{-2}$$

A1

Answer: 1.6 m s⁻²

2 marks

$$\text{(b) } s = \frac{1}{2}(u + v)t = \frac{1}{2}(6 + 14) \times 5$$

M1

$$= 50 \text{ m}$$

A1

Answer: 50 m

2 marks

$$\text{(c) } v^2 = 14^2 + 2 \times 1.6 \times 20$$

M1 A1

$$= 260, \text{ so } v = \sqrt{260} = 16.1 \text{ m s}^{-1}$$

A1

Answer: 16.1 m s⁻¹

3 marks

11 (4 marks)

$$\text{Thinking distance} = 24 \times 0.5 = 12 \text{ m}$$

B1

$$\text{Braking distance} = 60 - 12 = 48 \text{ m}$$

M1

$$0 = 24^2 + 2a(48)$$

M1

$$a = -6 \text{ m s}^{-2}, \text{ so the deceleration is } 6 \text{ m s}^{-2}$$

A1

Answer: 6 m s⁻²

4 marks

12 (5 marks)

$$A: s = 20t. \quad B: s = \frac{1}{2}(2)t^2 = t^2$$

B1 M1

$$20t = t^2, \text{ so } t(t - 20) = 0$$

M1

$$t = 20 \text{ s}$$

A1

$$B\text{'s speed} = 0 + 2 \times 20 = 40 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 20 \text{ s, } 40 \text{ m s}^{-1}$$

5 marks

13 (6 marks)

$$\text{Time for the car to slow to } 20 \text{ m s}^{-1}: 20 = 30 - 2.5t, \text{ so } t = 4 \text{ s}$$

M1 A1

$$\text{Car distance} = \frac{1}{2}(30 + 20) \times 4 = 100 \text{ m}$$

M1

$$\text{Lorry distance} = 20 \times 4 = 80 \text{ m}$$

B1

$$\text{The car gains } 100 - 80 = 20 \text{ m, so the gap is } 60 - 20 = 40 \text{ m}$$

M1

$$\text{The speeds are then equal, so the gap stays } 40 \text{ m: no collision, closest distance } 40 \text{ m}$$

A1

$$\text{Answer: } 40 \text{ m, no collision}$$

6 marks

14 (5 marks)

$$s = ut + \frac{1}{2}at^2: 24 = 10t - t^2$$

M1 A1

$$t^2 - 10t + 24 = 0, \text{ so } (t - 4)(t - 6) = 0$$

M1 A1

$t = 4 \text{ s}$ and $t = 6 \text{ s}$: the particle passes the point on the way out, comes to rest, and passes it again on the way back

E1

$$\text{Answer: } t = 4 \text{ s and } t = 6 \text{ s}$$

5 marks

15 (5 marks)

$$\text{Let the length of the section be } 2d \text{ and the acceleration } a. \text{ For the whole section } 14^2 = 2^2 + 2a(2d), \text{ so } 4ad = 192$$

M1 A1

$$\text{For the first half: } v^2 = 2^2 + 2a(d)$$

M1

$$v^2 = 4 + 2ad = 4 + 96$$

M1

$$v^2 = 100, \text{ so } v = 10 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 10 \text{ m s}^{-1}$$

5 marks

16 (6 marks)

$$\text{Slope: } s = ut + \frac{1}{2}at^2 = 2 \times 5 + \frac{1}{2}(0.8)(25)$$

M1 A1

$$= 20 \text{ m}$$

A1

$$\text{Speed at the bottom: } v = 2 + 0.8 \times 5 = 6 \text{ m s}^{-1}$$

M1

$$\text{Time on the ground} = \frac{30}{6} = 5 \text{ s}$$

M1

$$\text{Total time} = 5 + 5 = 10 \text{ s}$$

A1

$$\text{Answer: } 20 \text{ m, } 10 \text{ s}$$

6 marks

17 (7 marks)

$$(a) \text{ From the first, } t = \frac{v - u}{a}$$

B1

$$s = u\left(\frac{v - u}{a}\right) + \frac{1}{2}a\left(\frac{v - u}{a}\right)^2$$

M1

$$2as = 2u(v - u) + (v - u)^2 = (v - u)(2u + v - u) = (v - u)(v + u)$$

M1

$$= v^2 - u^2, \text{ so } v^2 = u^2 + 2as$$

A1

$$\text{Answer: shown}$$

4 marks

$$(b) v = 7.5: 7.5^2 = 15^2 + 2(-3)s$$

M1 A1

$$6s = 225 - 56.25 = 168.75, \text{ so } s = 28.125 \text{ m} \approx 28.1 \text{ m}$$

A1

$$\text{Answer: } 28.1 \text{ m}$$

3 marks

18 (6 marks)

Distance in the n -th second = $s(n) - s(n - 1)$, with $s(t) = ut + \frac{1}{2}at^2$

M1

Third second: $(3u + 4.5a) - (2u + 2a) = u + 2.5a = 14$

M1 A1

Fifth second: $(5u + 12.5a) - (4u + 8a) = u + 4.5a = 20$

A1

Subtracting: $2a = 6$, so $a = 3 \text{ m s}^{-2}$

M1

$u = 14 - 7.5 = 6.5 \text{ m s}^{-1}$

A1

Answer: $u = 6.5 \text{ m s}^{-1}$, $a = 3 \text{ m s}^{-2}$

6 marks

19 (5 marks)

(a) Time to stop: $0 = 15 - 3t$, so $t = 5 \text{ s}$

M1 A1

The car is stationary after 5 s, so the formula (which assumes constant acceleration) cannot be used up to 8 s

E1

Answer: the car stops after 5 s

3 marks

(b) Distance in 5 s: $\frac{1}{2}(15 + 0) \times 5$

M1

= 37.5 m (and no more movement after)

A1

Answer: 37.5 m

2 marks

20 (6 marks)

(a) Displacement at $t = 2$: $2u - 0.6 \times 4 = 2u - 2.4$; at $t = 7$: $7u - 0.6 \times 49 = 7u - 29.4$

M1

Equal: $2u - 2.4 = 7u - 29.4$

M1

$5u = 27$, so $u = 5.4$

A1

Answer: shown

3 marks

(b) $OP = 2(5.4) - 2.4 = 8.4 \text{ m}$

M1 A1

Greatest distance when $v = 0$: $t = \frac{5.4}{1.2} = 4.5 \text{ s}$, $s = \frac{5.4^2}{2 \times 1.2} = 12.15 \text{ m}$

A1

Answer: 8.4 m, 12.15 m

3 marks

21 (6 marks)

A to B: $39 = 3u + \frac{1}{2}a \times 3^2$

M1 A1

A to C (5 s, 75 m): $75 = 5u + \frac{1}{2}a \times 5^2$

M1 A1

Solving: $a = 2 \text{ m s}^{-2}$ and $u = 10 \text{ m s}^{-1}$

M1 A1

Answer: $a = 2 \text{ m s}^{-2}$, speed at A = 10 m s^{-1}

6 marks

22 (8 marks)

(a) $a = \frac{14 - 6}{10} = 0.8 \text{ m s}^{-2}$

M1 A1

$AB = \frac{1}{2}(6 + 14) \times 10 = 100 \text{ m}$

M1 A1

Answer: 0.8 m s^{-2} , 100 m

4 marks

(b) $v^2 = 6^2 + 2 \times 0.8 \times 50 = 116$

M1

$v = \sqrt{116} = 10.8 \text{ m s}^{-1}$

A1

Answer: 10.8 m s^{-1}

2 marks

(c) $t = \frac{\sqrt{116} - 6}{0.8}$

M1

= 5.96 s

A1

Answer: 5.96 s

2 marks

23 (8 marks)

(a) Take the direction OB as positive. Then u becomes $-u$: $9 = -3u + \frac{1}{2}a \times 9$

M1 A1

$$8 = -u + 3a$$

M1 A1

Solving: $u = 2$ and $a = \frac{10}{3}$

A1

Answer: $u = 2$, $a = \frac{10}{3}$

5 marks

(b) Particle stops momentarily when $0 = (-2)^2 + 2 \times \frac{10}{3} \times s$, so $s = -0.6$

M1 A1

This is 0.6 m from O towards A, which is less than 4 m, so A is not reached

E1

Answer: greatest distance 0.6 m < 4 m

3 marks

24 (7 marks)

(a) Cyclist: $s = 8(\tau + 5)$

B1

Car: $s = 22\tau - \tau^2$

M1 A1

$$8\tau + 40 = 22\tau - \tau^2 \text{ gives } \tau^2 - 14\tau + 40 = 0$$

A1

Answer: shown

4 marks

(b) $(\tau - 4)(\tau - 10) = 0$, so $\tau = 4$ or 10

M1

Times after the cyclist passes O: 9 s and 15 s

A1

Distances $8 \times 9 = 72$ m and $8 \times 15 = 120$ m

A1

Answer: 9 s (72 m) and 15 s (120 m)

3 marks

25 (8 marks)

(a) Ana's time $= \frac{160}{8} = 20$ s

M1 A1

Bea: $176 = 7 \times 20 + \frac{1}{2}a \times 20^2$

M1

$$a = 0.18 \text{ m s}^{-2}$$

A1

Answer: 0.18 m s^{-2}

4 marks

(b) Bea runs $176 + 4 = 180$ m in 20 s: $180 = 7 \times 20 + \frac{1}{2}a \times 400$

M1

$$a = 0.2 \text{ m s}^{-2}$$

A1

Answer: 0.2 m s^{-2}

2 marks

(c) $v = 7 + 0.18 \times 20$

M1

$$= 10.6 \text{ m s}^{-1}$$

A1

Answer: 10.6 m s^{-1}

2 marks

26 (10 marks)

(a) The speed at B is the average velocity over AC, as the times are equal: use $s = vt - \frac{1}{2}at^2$ for

M1

AB and $s = vt + \frac{1}{2}at^2$ for BC

Adding: $36 + 54 = 6v_B$

M1

$$v_B = 15 \text{ m s}^{-1}$$

A1

Answer: shown

3 marks

(b) $BC - AB = 54 - 36 = 18 = a \times 3^2$ (each stage lasts 3 s)

M1 A1

$$a = 2 \text{ m s}^{-2}$$

A1

Speed at A $= 15 - 2 \times 3 = 9 \text{ m s}^{-1}$

A1

Answer: $a = 2 \text{ m s}^{-2}$, $u = 9 \text{ m s}^{-1}$

4 marks

(c) First interval: $s_1 = uT + \frac{1}{2}aT^2$

B1

First two intervals: $s_1 + s_2 = 2uT + 2aT^2$, so $s_2 = uT + \frac{3}{2}aT^2$

M1

$$s_2 - s_1 = aT^2$$

A1

Answer: proved

3 marks

27 (8 marks)

(a) Dry section: $V^2 = 25^2 - 2(5)s_1$, so $s_1 = \frac{625 - V^2}{10}$

M1 A1

Wet section: $0 = V^2 - 2(2.5)s_2$, so $s_2 = \frac{V^2}{5}$

A1

Answer: shown

3 marks

(b) $62.5 - \frac{V^2}{10} + \frac{V^2}{5} = 85$, so $\frac{V^2}{10} = 22.5$

M1 A1

$V^2 = 225$, $V = 15 \text{ m s}^{-1}$

A1

Time on the dry section: $\frac{25 - 15}{5} = 2 \text{ s}$; on the wet section: $\frac{15}{2.5} = 6 \text{ s}$

M1

Total time = 8 s

A1

Answer: $V = 15 \text{ m s}^{-1}$, 8 s

5 marks

28 (9 marks)

(a) $s_n = [un + \frac{1}{2}an^2] - [u(n-1) + \frac{1}{2}a(n-1)^2]$

M1

$= u + \frac{1}{2}a(2n-1)$

M1 A1

Answer: shown

3 marks

(b) $u + 2.5a = 13$ and $u + 6.5a = 25$

M1

$4a = 12$, so $a = 3 \text{ m s}^{-2}$

A1

$u = 13 - 7.5 = 5.5 \text{ m s}^{-1}$

A1

Answer: $u = 5.5 \text{ m s}^{-1}$, $a = 3 \text{ m s}^{-2}$

3 marks

(c) $140 = 5.5t + 1.5t^2$, so $3t^2 + 11t - 280 = 0$

M1

$(3t + 35)(t - 8) = 0$, so $t = 8 \text{ s}$

A1

8th second: $5.5 + 3 \times 7.5 = 28 \text{ m}$

A1

Answer: $t = 8 \text{ s}$, 28 m

3 marks

29 (10 marks)

(a) $0 = 30 - 2t$, so $t = 15 \text{ s}$; $s = \frac{1}{2}(30)(15) = 225 \text{ m}$

M1 A1

Answer: $t = 15 \text{ s}$, 225 m

2 marks

(b) A: $s = 30t - t^2$; B: $s = 1.5t^2$

B1 B1

$30t - t^2 = 1.5t^2$, so $2.5t^2 = 30t$, $t = 12$

M1 A1

$12 < 15$ so A is still moving; distance = $1.5 \times 144 = 216 \text{ m}$

A1

Answer: $t = 12 \text{ s}$, 216 m

5 marks

(c) At $t = 12$: A has $30 - 24 = 6 \text{ m s}^{-1}$, B has 36 m s^{-1}

B1

With $a = 2$: $t^2 = 30t - t^2$, so $t = 15 \text{ s}$, exactly when A stops

M1

B reaches A just as A stops at 225 m

A1

Answer: 6 and 36 m s^{-1} ; $t = 15 \text{ s}$

3 marks

30 (9 marks)

(a) Measure from the front car's initial position. Front car: $x_F = 20t - 2.5t^2$, and it stops at $t = 4$

M1 A1

Rear car travels 8 m in 0.4 s. With $\tau = t - 0.4$: $x_R = -12 + 8 + 20\tau - 2\tau^2$

M1 A1

$x_F = 20(\tau + 0.4) - 2.5(\tau + 0.4)^2 = 18\tau + 7.6 - 2.5\tau^2$

M1

Equal positions: $\tau^2 + 4\tau - 23.2 = 0$, so $\tau = 3.215$

A1

$t = 3.62 \text{ s}$, which is less than 4 s, so the front car is still moving and the cars collide

A1

Answer: collision at $t = 3.62 \text{ s}$

7 marks

(b) Front: $20 - 5 \times 3.615 = 1.92 \text{ m s}^{-1}$

M1

Rear: $20 - 4 \times 3.215 = 7.14 \text{ m s}^{-1}$

A1

Answer: front 1.92 m s^{-1} , rear 7.14 m s^{-1}

2 marks

Vertical Motion

1 (4 marks)

Take downwards as positive, so $a = 9.8$.

$$v = u + at = 0 + 9.8 \times 4 = 39.2 \text{ m s}^{-1}$$

M1 A1

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2}(9.8)(16) = 78.4 \text{ m}$$

M1 A1

Answer: 39.2 m s^{-1} , 78.4 m

4 marks

2 (4 marks)

At the top $v = 0$: $0 = 14.7 - 9.8t$, so $t = 1.5 \text{ s}$

M1 A1

$$0 = 14.7^2 - 2(9.8)s, \text{ so } s = \frac{216.09}{19.6} = 11.025 \text{ m}$$

M1 A1

Answer: 1.5 s , 11.0 m

4 marks

3 (4 marks)

Downwards positive: $19.6 = \frac{1}{2}(9.8)t^2$

M1

$$t^2 = 4, \text{ so } t = 2 \text{ s}$$

A1

$$v = 0 + 9.8 \times 2$$

M1

$$= 19.6 \text{ m s}^{-1}$$

A1

Answer: 2 s , 19.6 m s^{-1}

4 marks

4 (4 marks)

Downwards positive: $v = 5 + 9.8 \times 2$

M1

$$= 24.6 \text{ m s}^{-1}$$

A1

$$s = 5 \times 2 + \frac{1}{2}(9.8)(4)$$

M1

$$= 29.6 \text{ m}$$

A1

Answer: 24.6 m s^{-1} , 29.6 m

4 marks

5 (4 marks)

Upwards positive: $v = 24.5 - 9.8t$

M1

(a) $v = 14.7 \text{ m s}^{-1}$ upwards

A1

(b) $v = 24.5 - 39.2 = -14.7$

M1

so 14.7 m s^{-1} downwards

A1

Answer: 14.7 m s^{-1} up; 14.7 m s^{-1} down

4 marks

6 (6 marks)

Upwards positive: $u = 12$, $a = -9.8$, $s = -30$

B1

$$v^2 = 12^2 + 2(-9.8)(-30) = 144 + 588 = 732$$

M1 A1

$$v = -27.1, \text{ so the speed is } 27.1 \text{ m s}^{-1}$$

A1

$$-27.06 = 12 - 9.8t$$

M1

$$t = 3.99 \text{ s}$$

A1

Answer: 27.1 m s^{-1} , 3.99 s

6 marks

7 (3 marks)

Upwards positive: $7^2 = u^2 - 2(9.8)(10)$

M1 A1

$$u^2 = 49 + 196 = 245, \text{ so } u = 15.7 \text{ m s}^{-1}$$

A1

Answer: 15.7 m s^{-1}

3 marks

8 (4 marks)

$$\text{Distance fallen in } 3 \text{ s} = \frac{1}{2}(9.8)(9) = 44.1 \text{ m}$$

M1 A1

$$\text{Distance fallen in } 2 \text{ s} = \frac{1}{2}(9.8)(4) = 19.6 \text{ m}$$

A1

$$\text{Third second: } 44.1 - 19.6 = 24.5 \text{ m}$$

A1

Answer: 24.5 m

4 marks

9 (6 marks)

$$\text{Height of A above the ground: } 40 - 4.9t^2; \text{ height of B: } 20t - 4.9t^2$$

B1 B1

$$\text{Equal: } 40 - 4.9t^2 = 20t - 4.9t^2, \text{ so } t = 2 \text{ s}$$

M1 A1

$$\text{Height} = 40 - 4.9 \times 4 = 20.4 \text{ m}$$

A1

$$\text{Velocities (upwards positive): A: } -9.8 \times 2 = -19.6 \text{ m s}^{-1}; \text{ B: } 20 - 19.6 = 0.4 \text{ m s}^{-1}$$

B1

Answer: t = 2 s, 20.4 m; A 19.6 m s⁻¹ down, B 0.4 m s⁻¹ up

6 marks

10 (6 marks)

$$\text{Upwards positive, taking the ground as } s = -10: -10 = 15t - 4.9t^2$$

M1 A1

$$4.9t^2 - 15t - 10 = 0, t = \frac{15 \pm \sqrt{225 + 196}}{9.8}$$

M1

$$t = 3.62 \text{ s (positive root)}$$

A1

$$v = 15 - 9.8(3.624) = -20.52$$

M1

$$\text{speed} = 20.5 \text{ m s}^{-1}$$

A1

Answer: 3.62 s, 20.5 m s⁻¹

6 marks

11 (6 marks)

$$\text{The bag initially moves upward at } 4 \text{ m s}^{-1}: u = 4, s = -50, a = -9.8$$

B1

$$-50 = 4t - 4.9t^2, \text{ so } 4.9t^2 - 4t - 50 = 0$$

M1 A1

$$t = \frac{4 + \sqrt{16 + 980}}{9.8} = 3.63 \text{ s}$$

A1

$$v = 4 - 9.8(3.629) = -31.56$$

M1

$$\text{speed} = 31.6 \text{ m s}^{-1}$$

A1

Answer: 3.63 s, 31.6 m s⁻¹

6 marks

12 (5 marks)

$$\text{The motion is symmetrical, so the highest point is reached at } t = \frac{2 + 5}{2} = 3.5 \text{ s}$$

M1 A1

$$u = 9.8 \times 3.5 = 34.3 \text{ m s}^{-1}$$

A1

$$\text{Height of P: } s = 34.3 \times 2 - 4.9 \times 4$$

M1

$$= 49 \text{ m}$$

A1

Answer: 34.3 m s⁻¹, 49 m

5 marks

13 (5 marks)

$$(a) \text{ At the top } v = 0: 0 = u^2 - 2gs, \text{ so } s = \frac{u^2}{2g}$$

M1 A1

$$(b) \text{ Back at the ground } s = 0: 0 = ut - \frac{1}{2}gt^2, \text{ so } t(u - \frac{1}{2}gt) = 0$$

M1

$$t = \frac{2u}{g}$$

A1

$$6 = \frac{2u}{9.8}, \text{ so } u = 29.4 \text{ m s}^{-1}$$

A1

Answer: 29.4 m s⁻¹

5 marks

14 (5 marks)

$$\text{Predicted: } 20 = \frac{1}{2}(9.8)t^2, t^2 = 4.082, t = 2.02 \text{ s}$$

M1 A1

$$\text{Observed: } 20 = \frac{1}{2}a(2.1)^2, \text{ so } a = \frac{40}{4.41} = 9.07 \text{ m s}^{-2}$$

M1 A1

The observed acceleration is less than 9.8 because air resistance (ignored by the model) opposes the motion

E1

Answer: 2.02 s; 9.07 m s⁻²; air resistance

5 marks

15 (5 marks)

$$\text{Taking upwards as positive: } -h = 18 \times 5 - \frac{1}{2} \times 9.8 \times 5^2$$

M1 A1

$$h = 122.5 - 90 = 32.5 \text{ m}$$

A1

$$v = 18 - 9.8 \times 5 = -31$$

M1

$$\text{Speed on impact} = 31 \text{ m s}^{-1}$$

A1

Answer: 32.5 m, 31 m s⁻¹

5 marks

16 (5 marks)

$$20 = 24.5t - 4.9t^2$$

M1 A1

$$4.9t^2 - 24.5t + 20 = 0$$

M1

$$t = 1.027 \text{ and } t = 3.973$$

A1

$$\text{Time} = 3.973 - 1.027 = 2.95 \text{ s}$$

A1

Answer: 2.95 s

5 marks

17 (5 marks)

$$v^2 = 2gh: 34.3^2 = 2 \times 9.8 \times h$$

M1

$$h = 60.02 \text{ m}$$

A1

$$\text{Fall of the first 40.02 m: } 40.02 = \frac{1}{2} \times 9.8 \times t^2, \text{ so } t = 2.858 \text{ s}$$

M1 A1

$$\text{Total time} = \frac{34.3}{9.8} = 3.5 \text{ s, so the last 20 m takes 0.642 s}$$

A1

Answer: tower 60.0 m, 0.642 s

5 marks

18 (5 marks)

$$\text{(a) } v = 19.6 - 9.8 \times 3 = -9.8, \text{ so } 9.8 \text{ m s}^{-1} \text{ downwards}$$

M1

$$s = 19.6 \times 3 - 4.9 \times 9$$

M1

$$= 14.7 \text{ m}$$

A1

Answer: 9.8 m s⁻¹ downwards, 14.7 m

3 marks

$$\text{(b) Greatest height} = \frac{19.6^2}{2 \times 9.8} = 19.6 \text{ m}$$

M1

$$\text{Distance} = 19.6 + (19.6 - 14.7) = 24.5 \text{ m}$$

A1

Answer: 24.5 m

2 marks

19 (9 marks)

$$\text{(a) } v = 15 \times 8 = 120 \text{ m s}^{-1}$$

M1 A1

$$s = \frac{1}{2}(15)(64) = 480 \text{ m}$$

A1

Answer: 120 m s⁻¹, 480 m

3 marks

$$\text{(b) Extra height: } 0 = 120^2 - 2(9.8)s$$

M1 A1

$$s = 734.7 \text{ m, so the greatest height} = 1215 \text{ m}$$

A1

Answer: 1215 m

3 marks

$$\text{(c) Time to rise further: } \frac{120}{9.8} = 12.24 \text{ s}$$

B1

$$\text{Fall from the top: } 1215 = 4.9t^2, t = 15.74 \text{ s}$$

M1

$$\text{Total} = 8 + 12.24 + 15.74 = 36.0 \text{ s}$$

A1

Answer: 36.0 s

3 marks

20 (7 marks)

Let t be the time since A was released. Downwards positive.

$$A: s = 4.9t^2$$

$$B \text{ (released 1 s later): } s = 15(t - 1) + 4.9(t - 1)^2$$

$$\text{Equate with } \tau = t - 1: 4.9(\tau + 1)^2 = 15\tau + 4.9\tau^2$$

$$9.8\tau + 4.9 = 15\tau, \text{ so } \tau = \frac{4.9}{5.2} = 0.9423$$

$$t = 1.94 \text{ s}$$

$$\text{Distance} = 4.9(1.942)^2 = 18.5 \text{ m}$$

Answer: $t = 1.94 \text{ s}$, 18.5 m

B1
B1 M1
M1
A1
A1
A1
7 marks

21 (7 marks)

$$\text{Height } h = 20t - 4.9t^2$$

$$h = 10: 4.9t^2 - 20t + 10 = 0, t = \frac{20 \pm \sqrt{400 - 196}}{9.8}$$

$$\text{Going up: } t_1 = \frac{20 - \sqrt{204}}{9.8} = 0.5834 \text{ s}$$

$$h = 14: 4.9t^2 - 20t + 14 = 0, \text{ going up: } t_2 = \frac{20 - \sqrt{125.6}}{9.8} = 0.8972 \text{ s}$$

$$\text{Time between edges going up} = 0.3138 \text{ s, the same coming down: total} = 2 \times 0.3138 = 0.628 \text{ s}$$

Answer: 0.628 s

B1
M1 A1
A1
M1 A1
A1
7 marks

22 (6 marks)

(a) Aisha: velocity is zero only for an instant; the acceleration is still 9.8 m s^{-2} downwards (the velocity is changing from positive to negative)

Ben: the acceleration is downwards, so the velocity becomes more negative: the ball moves back down

Answer: $a = 9.8$ downwards throughout; ball then moves down

$$(b) v = 14.7 - 9.8(1.5) = 0$$

$$\text{At } t = 1.6: v = 14.7 - 15.68 = -0.98 \text{ m s}^{-1}$$

Answer: 0 ; -0.98 m s^{-1} (moving down)

E1 E1
E1
3 marks
M1
M1 A1
3 marks

23 (7 marks)

$$(a) h = ut - \frac{1}{2}gt^2, \text{ so } \frac{1}{2}gt^2 - ut + h = 0 \text{ has roots } t_1 \text{ and } t_2$$

$$\text{The product of the roots is } \frac{h}{\frac{1}{2}g} = \frac{2h}{g}$$

$$t_1 t_2 = \frac{2h}{g}, \text{ so } h = \frac{1}{2}g t_1 t_2$$

Answer: shown

$$(b) h = \frac{1}{2}(9.8)(0.5)(3.5) = 8.575 \text{ m}$$

$$t_1 + t_2 = \frac{2u}{g} = 4, \text{ so } u = 19.6 \text{ m s}^{-1}$$

Answer: 8.575 m , 19.6 m s^{-1}

M1 A1
M1
A1
4 marks
M1 A1
A1
3 marks

24 (7 marks)

$$\text{Let the fall take } T \text{ seconds: } d = 4.9T^2$$

$$\text{The sound takes } \frac{d}{340} = \frac{4.9T^2}{340} \text{ s}$$

$$T + \frac{4.9T^2}{340} = 3, \text{ so } 4.9T^2 + 340T - 1020 = 0$$

$$T = \frac{-340 + \sqrt{340^2 + 4(4.9)(1020)}}{9.8} = 2.880 \text{ s}$$

$$d = 4.9 \times 2.880^2 = 40.65 \text{ m} \approx 41 \text{ m}$$

Answer: 41 m

B1
B1
M1 A1
M1 A1
A1
7 marks

25 (8 marks)**(a)** Height of A: $14.7t - 4.9t^2$

B1

Height of B: $10 + 4.9t - 4.9t^2$

B1

Equating: $9.8t = 10$

M1

$$t = \frac{50}{49} = 1.02 \text{ s}$$

A1

Height = 9.90 m

A1

Answer: 1.02 s, 9.90 m

5 marks

(b) A: $14.7 - 9.8 \times \frac{50}{49} = 4.7 \text{ m s}^{-1}$ upwards

M1 A1

B: $4.9 - 10 = -5.1$, so 5.1 m s^{-1} downwards

A1

Answer: A: 4.7 m s⁻¹ up; B: 5.1 m s⁻¹ down

3 marks

26 (11 marks)**(a)** $h_A = 19.6t - 4.9t^2$; $h_B = 19.6(t - T) - 4.9(t - T)^2$

B1 B1

Equate: $19.6T = 4.9(t^2 - (t - T)^2) = 4.9(2tT - T^2)$

M1 A1

Dividing by $4.9T$: $4 = 2t - T$, so $t = 2 + \frac{T}{2}$

A1

Answer: shown

5 marks

(b) $t = 3 \text{ s}$: $h = 19.6 \times 3 - 4.9 \times 9 = 14.7 \text{ m}$

M1 A1

A: $v = 19.6 - 29.4 = -9.8 \text{ m s}^{-1}$ (falling)

A1

B: $t - T = 1$, $v = 19.6 - 9.8 = 9.8 \text{ m s}^{-1}$ (rising)

A1

Answer: 14.7 m; A 9.8 m s⁻¹ down, B 9.8 m s⁻¹ up

4 marks

(c) The time of flight of A is $\frac{2 \times 19.6}{9.8} = 4 \text{ s}$

M1

If $T \geq 4$, A has already landed (or is just landing) when B is thrown

E1

Answer: A has landed before B is thrown

2 marks

27 (9 marks)**(a)** At release the ball has velocity 3 m s^{-1} upwards, the same as the lift; relative to the lift its initial velocity is 0 and the relative acceleration is 9.8 downwards

M1 A1

$$2.5 = \frac{1}{2}(9.8)\tau^2, \tau = 0.714 \text{ s}$$

A1

Answer: 0.714 s

3 marks

(b) $v = 3 - 9.8(0.7143)$

M1 A1

 $= -4.00 \text{ m s}^{-1}$, i.e. 4.00 m s^{-1} downwards

A1

Answer: 4.00 m s⁻¹ downwards

3 marks

(c) Upward speed at release is 3 m s^{-1} : $0 = 9 - 2(9.8)s$

M1 A1

$$s = \frac{9}{19.6} = 0.459 \text{ m}$$

A1

Answer: 0.459 m

3 marks

28 (8 marks)**(a)** Height $h = \frac{1}{2}gT^2$; the height fallen in the first $(T - 1)$ seconds is $\frac{1}{2}g(T - 1)^2$

M1 A1

$$\text{Last second: } \frac{1}{2}gT^2 - \frac{1}{2}g(T - 1)^2 = \frac{1}{2}(1/2 gT^2)$$

M1

$$T^2 - (T - 1)^2 = \frac{1}{2}T^2, \text{ so } 2T - 1 = \frac{1}{2}T^2, \text{ i.e. } T^2 - 4T + 2 = 0$$

A1

Answer: shown

4 marks

$$\text{(b) } T = \frac{4 \pm \sqrt{16 - 8}}{2} = 2 \pm \sqrt{2}$$

M1 A1

T must exceed 1 s (there is a last second): $T = 2 - \sqrt{2} = 0.586 < 1$ is rejected, so $T = 2 + \sqrt{2} = 3.414 \text{ s}$

E1

$$\text{Height} = 4.9 \times 3.414^2 = 57.1 \text{ m}$$

A1

Answer: T = 3.41 s, h = 57.1 m

4 marks

29 (11 marks)

(a) $19.6 = 4.9t^2$, $t = 2$ s; $v = 9.8 \times 2 = 19.6 \text{ m s}^{-1}$

M1 A1

Rebound speed 9.8 m s^{-1} : height $= \frac{9.8^2}{2 \times 9.8} = 4.9 \text{ m}$

M1 A1

Answer: 4.9 m

4 marks

(b) Rebound speeds 9.8, 4.9, 2.45, ...: time between impacts $= \frac{2v}{g}$: $2 \times 9.8/9.8 = 2$ s, then 1 s, 0.5 s, ...

M1 A1

Each is half the previous: geometric with first term 2 and common ratio $\frac{1}{2}$

M1 A1

Answer: 2 s, 1 s, 0.5 s, ...

4 marks

(c) Sum to infinity $= \frac{2}{1 - \frac{1}{2}} = 4$ s of bouncing

M1 A1

Total $= 2 + 4 = 6$ s: the infinitely many bounces take only a finite time, and a real ball also comes to rest at about this time

E1

Answer: 6 s

3 marks

30 (8 marks)

(a) At the greatest height $v = 0$: $0 = U^2 - 2gH$

M1

so $H = \frac{U^2}{2g}$

A1

Answer: shown

2 marks

(b) Displacement from projection to ground $= -3H$

B1

$v^2 = U^2 + 2g \times 3H = U^2 + 3U^2$

M1

$= 4U^2$, so the speed is $2U$

A1

Answer: shown

3 marks

(c) On reaching the ground the velocity is $-2U$: $-2U = U - gt$

M1 A1

$t = \frac{3U}{g}$

A1

Answer: shown

3 marks

Variable Acceleration**1 (4 marks)**

$v = \frac{ds}{dt} = 6t + 2$

M1 A1

When $t = 2$, $v = 14 \text{ m s}^{-1}$

A1

$a = \frac{dv}{dt} = 6 \text{ m s}^{-2}$

A1

Answer: 14 m s^{-1} , 6 m s^{-2}

4 marks

2 (4 marks)

$a = \frac{dv}{dt} = 6 \text{ m s}^{-2}$

M1 A1

$s = \int v \, dt = 3t^2 - 2t + c$, and $s = 0$ when $t = 0$ so $c = 0$

M1

When $t = 3$, $s = 21 \text{ m}$

A1

Answer: 6 m s^{-2} , 21 m

4 marks

3 (4 marks)

$v = 3t^2 - 6$

M1 A1

$t = 2$: $v = 6 \text{ m s}^{-1}$

A1

$a = 6t$, so at $t = 3$, $a = 18 \text{ m s}^{-2}$

A1

Answer: 6 m s^{-1} , 18 m s^{-2}

4 marks

4 (3 marks)

$$v = \int 6t \, dt = 3t^2 + c$$

M1 A1

$$v = 0 \text{ when } t = 0, \text{ so } c = 0 \text{ and } v = 3t^2$$

$$\text{At } t = 2, v = 12 \text{ m s}^{-1}$$

A1

Answer: 12 m s⁻¹

3 marks

5 (3 marks)

$$\text{At rest when } v = 0: 3t^2 - 12 = 0$$

M1

$$t^2 = 4$$

A1

$$t = 2 \text{ s } (t \geq 0)$$

A1

Answer: t = 2 s

3 marks

6 (5 marks)

$$v = \frac{ds}{dt} = 3t^2 - 12t + 9$$

M1 A1

$$\text{At rest: } 3(t^2 - 4t + 3) = 0, \text{ so } 3(t - 1)(t - 3) = 0$$

M1 A1

$$t = 1: s = 4 \text{ m}; t = 3: s = 0 \text{ m}$$

A1

Answer: t = 1 (s = 4 m), t = 3 (s = 0 m)

5 marks

7 (6 marks)

$$a = 12 - 6t = 0 \text{ for a stationary value of } v, \text{ so } t = 2$$

M1 A1

$$v_{\max} = 24 - 12 = 12 \text{ m s}^{-1}$$

A1

$$v = 0: 3t(4 - t) = 0, \text{ so } t = 4 \text{ s}$$

M1 A1

$$s = 6t^2 - t^3; \text{ at } t = 4, s = 96 - 64 = 32 \text{ m}$$

A1

Answer: 12 m s⁻¹; t = 4 s; 32 m

6 marks

8 (5 marks)

$$v = \int (6t - 4) \, dt = 3t^2 - 4t + c, \text{ with } v = 3 \text{ at } t = 0 \text{ so } c = 3$$

M1 A1

$$s = \int v \, dt = t^3 - 2t^2 + 3t + c', \text{ with } s = 2 \text{ at } t = 0 \text{ so } c' = 2$$

M1 A1

$$\text{At } t = 2: v = 12 - 8 + 3 = 7 \text{ m s}^{-1}, s = 8 - 8 + 6 + 2 = 8 \text{ m}$$

A1

Answer: 7 m s⁻¹, 8 m

5 marks

9 (5 marks)

$$v = (t - 1)(t - 3), \text{ so } v = 0 \text{ at } t = 1 \text{ and } t = 3$$

M1 A1

$$a = 2t - 4: \text{ at } t = 1, a = -2; \text{ at } t = 3, a = 2 \text{ (m s}^{-2}\text{)}$$

M1 A1

$$a = 0 \text{ at } t = 2, v = -1 \text{ m s}^{-1} \text{ (minimum)}$$

A1

Answer: t = 1, 3; a = -2, 2; minimum -1 m s⁻¹

5 marks

10 (4 marks)

$$v = 10t - 3t^2, a = 10 - 6t$$

M1 A1

$$a = 0 \text{ when } t = \frac{5}{3}$$

A1

$$v = 10\left(\frac{5}{3}\right) - 3\left(\frac{25}{9}\right) = \frac{50}{3} - \frac{25}{3} = \frac{25}{3} = \frac{25}{3} \text{ m s}^{-1}$$

A1

Answer: t = $\frac{5}{3}$ s, v = $\frac{25}{3}$ m s⁻¹

4 marks

11 (7 marks)

$$v = (t - 2)(t - 4): v > 0 \text{ for } t < 2 \text{ and } t > 4, v < 0 \text{ for } 2 < t < 4$$

M1 A1

$$s = \int v \, dt = \frac{1}{3}t^3 - 3t^2 + 8t$$

M1 A1

$$s(2) = \frac{20}{3}, s(4) = \frac{16}{3}, s(5) = \frac{20}{3}$$

M1

$$\text{Displacement at } t = 5: \frac{20}{3} \text{ m}$$

A1

$$\text{Distance} = \frac{20}{3} + \frac{4}{3} + \frac{4}{3} = \frac{28}{3} \text{ m}$$

A1

$$\text{Answer: displacement } \frac{20}{3} \text{ m, distance } \frac{28}{3} \text{ m}$$

7 marks

12 (6 marks)

$$s = 0: t(t^2 - 6t + 8) = 0, \text{ so } t(t - 2)(t - 4) = 0$$

M1 A1

$$t = 0, 2, 4 \text{ s}$$

A1

$$v = 3t^2 - 12t + 8$$

M1

$$t = 0: v = 8; t = 2: v = -4; t = 4: v = 8 \text{ (m s}^{-1}\text{)}$$

A1 A1

$$\text{Answer: } t = 0, 2, 4 \text{ s; } v = 8, -4, 8 \text{ m s}^{-1}$$

6 marks

13 (5 marks)

$$a = 2t - 8 = 0 \text{ gives } t = 4, \text{ where } v = 16 - 32 = -16$$

M1 A1 A1

$$\text{Check the ends: } v(0) = 0, v(6) = -12$$

M1

$$\text{Greatest speed} = 16 \text{ m s}^{-1} \text{ (at } t = 4\text{)}$$

A1

$$\text{Answer: } 16 \text{ m s}^{-1} \text{ at } t = 4$$

5 marks

14 (6 marks)

$$v = 4t - t^2 \text{ (using } v = 0 \text{ at } t = 0\text{)}$$

M1 A1

$$(a) v = t(4 - t) = 0 \text{ at } t = 4 \text{ s}$$

A1

$$s = 2t^2 - \frac{1}{3}t^3$$

B1

$$(b) s = 0: t^2(2 - \frac{t}{3}) = 0, \text{ so } t = 6 \text{ s}$$

M1

$$v(6) = 24 - 36 = -12, \text{ speed } 12 \text{ m s}^{-1}$$

A1

$$\text{Answer: (a) } 4 \text{ s, (b) } 6 \text{ s, } 12 \text{ m s}^{-1}$$

6 marks

15 (6 marks)

$$a = -kt, \text{ so } v = 20 - \frac{1}{2}kt^2$$

M1 A1

$$v = 0 \text{ at } t = 5: 20 = \frac{25}{2}k, \text{ so } k = 1.6$$

M1 A1

$$s = 20t - \frac{k}{6}t^3 = 20t - \frac{4}{15}t^3$$

M1

$$\text{At } t = 5: s = 100 - \frac{100}{3} = \frac{200}{3} \text{ m}$$

A1

$$\text{Answer: } k = 1.6, \frac{200}{3} \text{ m}$$

6 marks

16 (5 marks)

$$v = 2(t - 2)(t - 3) < 0 \text{ for } 2 < t < 3$$

M1 A1

$$\text{Displacement} = \int_2^3 v \, dt = [\frac{2}{3}t^3 - 5t^2 + 12t]$$

M1 A1

$$= (18 - 45 + 36) - (\frac{16}{3} - 20 + 24) = 9 - \frac{28}{3} = -\frac{1}{3}$$

A1

$$\text{Answer: } 2 < t < 3, \text{ displacement } -\frac{1}{3} \text{ m}$$

5 marks

17 (7 marks)

$$(a) v = 3t^2 - 6t + 4$$

M1

$$= 3(t - 1)^2 + 1$$

M1

This is at least 1 > 0, so the velocity is always positive

E1

Answer: shown

3 marks

(b) s is increasing and s(0) = 0, so s > 0 for t > 0

E1

$$a = 6t - 6 = 0 \text{ when } v \text{ is least: } t = 1$$

M1 A1

a = 0 at that time (the least velocity is 1 m s⁻¹)

A1

Answer: a = 0 at t = 1

4 marks

18 (7 marks)

$$(a) v = \frac{60t - t^2}{50}, \frac{dv}{dt} = \frac{60 - 2t}{50} = 0 \text{ when } t = 30$$

M1 A1

$$v_{\max} = \frac{30 \times 30}{50} = 18 \text{ m s}^{-1}$$

A1

Answer: 18 m s⁻¹

3 marks

$$(b) s = \int_0^{60} \frac{60t - t^2}{50} dt = \frac{1}{50} \left[30t^2 - \frac{1}{3}t^3 \right]_0^{60}$$

M1 A1

$$= \frac{1}{50} (108000 - 72000) = 720 \text{ m}$$

A1

$$\text{Average speed} = \frac{720}{60} = 12 \text{ m s}^{-1}$$

A1

Answer: 720 m, 12 m s⁻¹

4 marks

19 (7 marks)

(a) The equation v = u + at only applies when the acceleration is constant

E1

Here a changes with time, so v must be found by integrating the acceleration

E1

Answer: a is not constant

2 marks

$$(b) v = \int 2t dt = t^2 + c, \text{ with } v = 3 \text{ at } t = 0: v = t^2 + 3$$

M1 A1

$$s = \int v dt = \frac{1}{3}t^3 + 3t \text{ (s = 0 at t = 0)}$$

A1

$$\text{Answer: } v = t^2 + 3, s = \frac{1}{3}t^3 + 3t$$

3 marks

$$(c) \text{ Student: } 3 + 18 = 21 \text{ m s}^{-1}$$

B1

Correct: 9 + 3 = 12 m s⁻¹, so the student's value is too large

B1

Answer: 21 compared with 12

2 marks

20 (7 marks)

$$a = 2pt + q$$

M1

$$t = 1: 2p + q = 2$$

A1

$$t = 3: 9p + 3q = 18, \text{ so } 3p + q = 6$$

M1 A1

$$\text{Subtracting: } p = 4, q = -6$$

A1

$$v = 4t^2 - 6t, s = \int_0^3 v dt = \left[\frac{4}{3}t^3 - 3t^2 \right]_0^3$$

M1

$$= 36 - 27 = 9 \text{ m (v changes sign at } t = 1.5, \text{ so this is the displacement)}$$

A1

Answer: p = 4, q = -6, displacement 9 m

7 marks

21 (7 marks)

$$(a) v = \int (2t - 8) dt = t^2 - 8t + c, \text{ and } v = 12 \text{ at } t = 0 \text{ so } c = 12$$

M1 A1

$$v = t^2 - 8t + 12 = (t - 2)(t - 6), \text{ at rest at } t = 2 \text{ and } t = 6$$

A1

Answer: t = 2 s and t = 6 s

3 marks

$$(b) s = \frac{1}{3}t^3 - 4t^2 + 12t$$

M1

$$s(2) = \frac{32}{3} \text{ m}, s(6) = 72 - 144 + 72 = 0$$

A1 A1

$$\text{The particle goes out } \frac{32}{3} \text{ m and returns to O: distance} = \frac{64}{3} = 21.3 \text{ m}$$

A1

Answer: $\frac{32}{3}$ m, 0; 21.3 m**4 marks****22 (8 marks)**

(a) $v = 32 - \int(2 + \frac{t}{2}) dt = 32 - 2t - \frac{t^2}{4}$

M1 A1

$v = 0: t^2 + 8t - 128 = 0$, that is $(t - 8)(t + 16) = 0$

M1

$t = 8$ s

A1**Answer: shown****4 marks**

(b) $s = \int(32 - 2t - \frac{t^2}{4}) dt = 32t - t^2 - \frac{t^3}{12}$

M1 A1

At $t = 8$: $s = 256 - 64 - \frac{512}{12} = \frac{448}{3}$ m

A1

Distance from obstacle $= 150 - \frac{448}{3} = \frac{2}{3}$ m

A1**Answer: $\frac{2}{3}$ m****4 marks****23 (8 marks)**

(a) $\frac{dv}{dt} = 2t + k = 0$ when $t = 2.5$

M1

$k = -5$

A1**Answer: $k = -5$** **2 marks**

(b) $t^2 - 5t + 4 = (t - 1)(t - 4) = 0$

M1

$t = 1$ and $t = 4$

A1**Answer: $t = 1, t = 4$** **2 marks**

(c) $s = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$

M1

0 to 1: $\frac{11}{6}$; 1 to 4: $-\frac{9}{2}$ so distance $\frac{9}{2}$; 4 to 5: $\frac{11}{6}$

A1 A1

Total distance $= \frac{11}{6} + \frac{9}{2} + \frac{11}{6} = \frac{49}{6}$ m

A1**Answer: $\frac{49}{6}$ m****4 marks****24 (9 marks)**

(a) $v = \int(6 - t) dt = 6t - \frac{t^2}{2}$ ($v = 0$ when $t = 0$)

M1 A1

$v(6) = 36 - 18 = 18$ m s⁻¹

A1**Answer: 18 m s⁻¹****3 marks**

(b) $x = \int(6t - \frac{t^2}{2}) dt = 3t^2 - \frac{t^3}{6}$

M1 A1

$x(6) = 108 - 36 = 72$ m

A1**Answer: shown****3 marks**

(c) For $t > 6$: $x = 72 + 18(t - 6)$

M1 A1

$72 + 18(t - 6) = 1000$, so $t = 57.56$ s

A1**Answer: 57.6 s****3 marks****25 (10 marks)**

(a) $v = \int(12t - 2t^2) dt = 6t^2 - \frac{2}{3}t^3$

M1 A1

$s = \int v dt = 2t^3 - \frac{1}{6}t^4$

A1

At $t = 6$: $v = 216 - 144 = 72$ m s⁻¹; $s = 432 - 216 = 216$ m

A1**Answer: 72 m s⁻¹, 216 m****4 marks**

(b) Deceleration time $= \frac{72}{4} = 18$ s

M1 A1

Braking distance $= \frac{72^2}{2 \times 4} = 648$ m

M1

Total distance = 864 m, total time = 6 + 18 = 24 s

Answer: 864 m, 24 s

A1

4 marks

(c) $a = 12t - 2t^2$, $a' = 12 - 4t = 0$ at $t = 3$

M1

$a(3) = 36 - 18 = 18 \text{ m s}^{-2}$

A1

Answer: 18 m s⁻²

2 marks

26 (10 marks)

(a) $s_p = \int 3t^2 dt = t^3$ ($s = 0$ at $t = 0$)

M1 A1

Answer: t^3

2 marks

(b) $\text{Gap} = 12t - t^3$; derivative $12 - 3t^2 = 0$, so $t = 2$

M1 A1

$t = 2$ gives the maximum (the gap increases then decreases)

E1

$\text{Gap} = 24 - 8 = 16 \text{ m}$

A1

Answer: 16 m at $t = 2$ s

4 marks

(c) $t^3 = 12t$, so $t^2 = 12$ ($t > 0$)

M1 A1

$t = 3.46 \text{ s}$

A1

Distance = $12 \times 3.464 = 41.6 \text{ m}$

A1

Answer: $t = 3.46 \text{ s}$, 41.6 m

4 marks

27 (10 marks)

(a) $v = 2at + b$

B1

Average velocity = $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$

M1

$= \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1} = a(t_1 + t_2) + b$

M1 A1

v at the midpoint $\frac{t_1 + t_2}{2} = 2a(\frac{t_1 + t_2}{2}) + b = a(t_1 + t_2) + b$, the same

A1

Answer: shown

5 marks

(b) $s(2) = 9$, $s(6) = 97$, average velocity = $\frac{88}{4} = 22 \text{ m s}^{-1}$

M1 A1

$v = 6t - 2 = 22$ at $t = 4$ (the midpoint)

A1

Answer: 22 m s⁻¹ at $t = 4$

3 marks

(c) For $s = t^3$, average velocity on $0 \leq t \leq 2$ is $\frac{8}{2} = 4$, but $v(1) = 3 \neq 4$

B1

The result relies on the velocity being linear in time (constant acceleration)

E1

Answer: e.g. $s = t^3$

2 marks

28 (11 marks)

(a) $v = k(t^3 - 12t^2 + 36t)$

B1

$\frac{dv}{dt} = k(3t^2 - 24t + 36) = 3k(t - 2)(t - 6) = 0$

M1 A1

$t = 2$ ($t = 6$ gives $v = 0$); $v(2) = k \times 2 \times 16 = 32k$

A1

$32k = 16$, so $k = 0.5$

A1

Answer: $t = 2$, $k = 0.5$

5 marks

(b) $s = \int_0^6 0.5(t^3 - 12t^2 + 36t) dt = 0.5[\frac{1}{4}t^4 - 4t^3 + 18t^2]_0^6$

M1 A1

$= 0.5(324 - 864 + 648) = 54 \text{ m}$

A1

Average speed = $\frac{54}{6} = 9 \text{ m s}^{-1}$

A1

Answer: 54 m, 9 m s⁻¹

4 marks

(c) $v = 0.5t(6 - t)^2 \geq 0$ for $t \geq 0$ (a square multiplied by positive t)

E1

$a = 1.5(t - 2)(t - 6) = 0$ at $t = 6$

A1

Answer: $a = 0$

2 marks

29 (7 marks)

(a) $(t - 2)(t - 5) = 0$

M1

$t = 2 \text{ and } t = 5$

A1

Answer: $t = 2, t = 5$

2 marks

(b) $\int_0^2 (t^2 - 7t + 10) dt = \left[\frac{t^3}{3} - \frac{7t^2}{2} + 10t \right]_0^2$

M1

$= \frac{26}{3} \text{ m}$

A1

Answer: $\frac{26}{3} \text{ m}$

2 marks

(c) Between $t = 2$ and $t = 5$ P returns $\frac{9}{2} \text{ m}$, so by $t = 5$ the total distance is $\frac{26}{3} + \frac{9}{2} = \frac{79}{6} \text{ m}$, which exceeds 12, so P is still returning

M1

It has returned $12 - \frac{26}{3} = \frac{10}{3} \text{ m}$

M1

Displacement $= \frac{26}{3} - \frac{10}{3} = \frac{16}{3} \text{ m}$

A1

Answer: $\frac{16}{3} \text{ m}$

3 marks

30 (10 marks)

(a) $\frac{dv}{dt} = 4 - 2t = 0$, so $t = 2$

M1

$v = 8 - 4 = 4 \text{ m s}^{-1}$

A1

Answer: 4 m s^{-1}

2 marks

(b) $\int_0^3 (4t - t^2) dt = \left[2t^2 - \frac{t^3}{3} \right]_0^3$

M1

$= 18 - 9 = 9 \text{ m}$

A1

Answer: 9 m

2 marks

(c) $9 - 2t = 0$

M1

$t = 4.5 \text{ s}$

A1

Answer: 4.5 s

2 marks

(d) From $t = 3$ to 4.5 : $[9t - t^2] = (40.5 - 20.25) - (27 - 9) = 2.25 \text{ m}$, so the particle is 11.25 m from the start at $t = 4.5$

M1 A1

From $t = 4.5$ to 6 the velocity is negative: $|(54 - 36) - 20.25| = 2.25 \text{ m}$

M1

Total distance $= 11.25 + 2.25 = 13.5 \text{ m}$

A1

Answer: 13.5 m

4 marks

Kinematics Using Calculus**1 (4 marks)**

$v = \frac{dr}{dt} = 2t\mathbf{i} + 3\mathbf{j}$

M1 A1

At $t = 2$: $v = 4\mathbf{i} + 3\mathbf{j}$

A1

Speed $= \sqrt{16 + 9} = 5 \text{ m s}^{-1}$

A1

Answer: $(4\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}, 5 \text{ m s}^{-1}$

4 marks

2 (3 marks)

$a = \frac{dv}{dt} = 6t\mathbf{i} + 4\mathbf{j}$

M1 A1

At $t = 2$: $a = (12\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-2}$

A1

Answer: $(12\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-2}$

3 marks

3 (3 marks)

$$v = \frac{dx}{dt} = 6 \cos 2t \quad \text{M1 A1}$$

$$t = \frac{\pi}{6}: v = 6 \cos\left(\frac{\pi}{3}\right) = 3 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 3 m s^{-1} 3 marks

4 (3 marks)

$$a = \frac{dv}{dt} = -4e^{-t} \quad \text{M1 A1}$$

$$t = 0: a = -4 \text{ m s}^{-2}; t = 2: a = -4e^{-2} \text{ m s}^{-2} \quad \text{A1}$$

Answer: $-4 \text{ m s}^{-2}; -4e^{-2} \text{ m s}^{-2}$ 3 marks

5 (3 marks)

$$r(3) = 7\mathbf{i} + 6\mathbf{j} \quad \text{M1 A1}$$

$$|r| = \sqrt{49 + 36} = \sqrt{85} = 9.22 \text{ m} \quad \text{A1}$$

Answer: $\sqrt{85} = 9.22 \text{ m}$ 3 marks

6 (4 marks)

$$v = u + at = (3 + 2 \times 3)\mathbf{i} + (-2 + 3)\mathbf{j} = 9\mathbf{i} + \mathbf{j} \quad \text{M1 A1}$$

$$r = ut + \frac{1}{2}at^2 = (9 + 9)\mathbf{i} + (-6 + 4.5)\mathbf{j} \quad \text{M1}$$

$$r = (18\mathbf{i} - 1.5\mathbf{j}) \text{ m} \quad \text{A1}$$

Answer: $v = (9\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$, $r = (18\mathbf{i} - 1.5\mathbf{j}) \text{ m}$ 4 marks

7 (5 marks)

$$v = (2t - 4)\mathbf{i} + (3 - 2t)\mathbf{j} \quad \text{M1 A1}$$

Parallel to $\mathbf{j}$: the $\mathbf{i}$ component is zero, $2t - 4 = 0$, so $t = 2$ M1 A1

$$v = (3 - 4)\mathbf{j} = -\mathbf{j}, \text{ speed} = 1 \text{ m s}^{-1} \quad \text{A1}$$

Answer: $t = 2 \text{ s}$, 1 m s^{-1} 5 marks

8 (6 marks)

$$v = \int a \, dt = (3t^2 + c_1)\mathbf{i} + (-2t + c_2)\mathbf{j}; \text{ with } v(0) = 3\mathbf{i} + \mathbf{j}: v = (3t^2 + 3)\mathbf{i} + (1 - 2t)\mathbf{j} \quad \text{M1 A1 A1}$$

$$r = \int v \, dt = (t^3 + 3t)\mathbf{i} + (t - t^2)\mathbf{j} \text{ (} r = 0 \text{ at } t = 0\text{)} \quad \text{M1 A1}$$

$$\text{At } t = 2: v = (15\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}, r = (14\mathbf{i} - 2\mathbf{j}) \text{ m} \quad \text{A1}$$

Answer: $v = (15\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$, $r = (14\mathbf{i} - 2\mathbf{j}) \text{ m}$ 6 marks

9 (5 marks)

$$v = \frac{ds}{dt} = 2e^{-t} + 2t(-e^{-t}) \text{ (product rule)} \quad \text{M1 A1}$$

$$= 2e^{-t}(1 - t) \quad \text{A1}$$

$$v = 0 \text{ only when } t = 1 \text{ (} e^{-t} > 0\text{)} \quad \text{A1}$$

$$s(1) = 2e^{-1} = \frac{2}{e} \text{ m} \quad \text{A1}$$

Answer: $\frac{2}{e} = 0.736 \text{ m}$ 5 marks

10 (5 marks)

$$v = 3\mathbf{i} + (4 - 4t)\mathbf{j} \quad \text{M1 A1}$$

$$\text{Speed}^2 = 9 + 16(1 - t)^2 \quad \text{M1}$$

$$\text{This is least when } t = 1 \quad \text{A1}$$

$$\text{Least speed} = 3 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 3 m s^{-1} at $t = 1$ 5 marks

11 (4 marks)

$$\text{Speed} = \sqrt{(-3)^2 + 4^2} = 5 \text{ m s}^{-1}$$

M1 A1

The direction is north-west: the angle west of north is $\tan^{-1}(\frac{3}{4}) = 36.9^\circ$

M1

$$\text{Bearing} = 360^\circ - 36.9^\circ = 323^\circ$$

A1

Answer: 5 m s⁻¹ on a bearing of 323°

4 marks

12 (5 marks)

Due east of O means the **j** component is zero and the **i** component is positive: $t^3 - 4t = 0$

M1

$$t(t - 2)(t + 2) = 0, \text{ so } t = 2 \text{ (} t > 0 \text{)}$$

A1

Check: $r(2) = \mathbf{i}$, due east

B1

$$\mathbf{v} = (2t - 4)\mathbf{i} + (3t^2 - 4)\mathbf{j}$$

M1

$$\text{At } t = 2: \mathbf{v} = 8\mathbf{j} \text{ m s}^{-1}$$

A1

Answer: t = 2 s, v = 8j m s⁻¹

5 marks

13 (6 marks)

$$\mathbf{F} = m\mathbf{a}; \mathbf{a} = \frac{\mathbf{F}}{2} = (6t\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-2}$$

M1 A1

$$\mathbf{v} = \int \mathbf{a} \, dt = 3t^2\mathbf{i} + 4t\mathbf{j} \text{ (} \mathbf{v} = 0 \text{ at } t = 0 \text{)}$$

M1 A1

$$\mathbf{r} = \int \mathbf{v} \, dt = t^3\mathbf{i} + 2t^2\mathbf{j}; \text{ at } t = 2: \mathbf{r} = (8\mathbf{i} + 8\mathbf{j}) \text{ m}$$

M1

$$\text{Distance} = \sqrt{64 + 64} = 8\sqrt{2} = 11.3 \text{ m}$$

A1

Answer: (8i + 8j) m, 8√2 = 11.3 m

6 marks

14 (6 marks)

$$(a) a = -5e^{-0.5t}; \text{ at } t = 2: a = -5e^{-1} = -1.84 \text{ m s}^{-2}$$

M1 A1

$$(b) 10e^{-0.5t} = 5, e^{-0.5t} = \frac{1}{2}, \text{ so } t = 2 \ln 2 = 1.39 \text{ s}$$

M1 A1

$$(c) s = \int v \, dt = -20e^{-0.5t} + c = 20(1 - e^{-0.5t})$$

M1

$$\text{As } t \rightarrow \infty, s \rightarrow 20 \text{ m}$$

A1

Answer: (a) -1.84 m s⁻²; (b) 1.39 s; (c) 20 m

6 marks

15 (6 marks)

$$(a) v = \frac{8}{1+t}; t = 3: v = 2 \text{ m s}^{-1}$$

M1 A1

$$(b) a = -\frac{8}{(1+t)^2}; t = 3: a = -0.5 \text{ m s}^{-2}$$

M1 A1

$$(c) \frac{8}{1+t} = 1, \text{ so } t = 7 \text{ s}$$

M1 A1

Answer: (a) 2 m s⁻¹, (b) -0.5 m s⁻², (c) 7 s

6 marks

16 (6 marks)

$$\text{At rest: } \cos(\frac{\pi}{2}t) = 0, \text{ so } \frac{\pi}{2}t = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}, t = 1 \text{ or } 3$$

M1 A1

$$s = \int 4 \cos(\frac{\pi}{2}t) \, dt = \frac{8}{\pi} \sin(\frac{\pi}{2}t) \text{ (} s = 0 \text{ at } t = 0 \text{)}$$

M1 A1

$$s(1) = \frac{8}{\pi} = 2.55 \text{ m}, s(3) = -\frac{8}{\pi}, s(4) = 0$$

A1

$$\text{Greatest displacement } 2.55 \text{ m; distance} = 4 \times \frac{8}{\pi} = 10.2 \text{ m}$$

A1

Answer: t = 1 and 3; 2.55 m; 10.2 m

6 marks

17 (6 marks)

$$(a) \mathbf{v} = (6t - 2)\mathbf{i} + (3t^2 - 4)\mathbf{j} \text{ and } \mathbf{a} = 6\mathbf{i} + 6t\mathbf{j}$$

M1 A1

$$\text{When } t = 2: \mathbf{a} = 6\mathbf{i} + 12\mathbf{j}, \text{ so } |\mathbf{F}| = 0.8 \times \sqrt{180} = 10.7 \text{ N}$$

A1

Answer: 10.7 N

3 marks

(b) Components equal: $6t - 2 = 3t^2 - 4$

M1

$$3t^2 - 6t - 2 = 0$$

A1

$$t = \frac{6 + \sqrt{60}}{6} = 2.29 \text{ s}$$

A1

Answer: 2.29 s

3 marks

18 (6 marks)

(a) $x = 2t$, so $t = \frac{x}{2}$

M1

$$y = \frac{x^2}{4} - \frac{3x}{2}$$

A1

Answer: $y = \frac{x^2}{4} - \frac{3x}{2}$

2 marks

(b) $v = 2i + (2t - 3)j$

M1 A1

Parallel to i when $2t - 3 = 0$, so $t = 1.5$

M1

$$r = 3i - 2.25j$$

A1

Answer: $3i - 2.25j$

4 marks

19 (6 marks)

(a) $|r|^2 = (t - 3)^2 + (2t - 4)^2$

M1

$$= t^2 - 6t + 9 + 4t^2 - 16t + 16 = 5t^2 - 22t + 25$$

A1

Answer: shown

2 marks

(b) $d(|r|^2)/dt = 10t - 22 = 0$, so $t = 2.2$

M1 A1

$$\text{Least } |r|^2 = 5(4.84) - 48.4 + 25 = 0.8$$

M1

$$\text{Least distance} = \sqrt{0.8} = 0.894 \text{ m}$$

A1

Answer: 0.894 m

4 marks

20 (8 marks)

(a) i components equal: $2t = t + 3$, so $t = 3$

M1 A1

j components at $t = 3$: A: 9; B: $18 - 9 = 9$, equal, so they collide

M1

Position: $6i + 9j$ m at $t = 3$ s

A1

Answer: $t = 3$ s, $(6i + 9j)$ m

4 marks

(b) $v_A = 2i + 2tj = 2i + 6j$; $v_B = i - 3j$

B1 B1

$$\cos \theta = \frac{2 - 18}{\sqrt{40}\sqrt{10}} = -0.8$$

M1

$$\theta = 143^\circ$$

A1

Answer: 143°

4 marks

21 (8 marks)

(a) $a = \frac{dv}{dt} = 6e^{-t/2}$

M1 A1

$t = 0$: $a = 6 \text{ m s}^{-2}$; as $t \rightarrow \infty$, $e^{-t/2} \rightarrow 0$, so $v \rightarrow 12 \text{ m s}^{-1}$

A1

Answer: 6 m s^{-2} , 12 m s^{-1}

3 marks

(b) $9 = 12(1 - e^{-t/2})$, so $e^{-t/2} = \frac{1}{4}$, $t = 2 \ln 4 = 2.77 \text{ s}$

M1 A1

$$s = \int v \, dt = 12t + 24e^{-t/2} + c, \text{ with } s = 0 \text{ at } t = 0: c = -24$$

M1 A1

$$s = 12(2.77) + 24(1/4) - 24 = 15.3 \text{ m}$$

A1

Answer: $t = 2 \ln 4 = 2.77 \text{ s}$, 15.3 m

5 marks

22 (7 marks)

$$(a) \frac{dh}{dt} = 2 \times \frac{\pi}{6} \cos\left(\frac{\pi}{6}t\right) = \frac{\pi}{3} \cos\left(\frac{\pi}{6}t\right)$$

M1 A1

$$t = 0: \frac{\pi}{3} = 1.05 \text{ m h}^{-1}$$

A1

The greatest speed is when $|\cos| = 1$, so $\frac{\pi}{3} \text{ m h}^{-1}$

A1

$$\text{Answer: } \frac{\pi}{3} \text{ m h}^{-1}$$

4 marks

$$(b) \frac{dh}{dt} = 0: \cos\left(\frac{\pi}{6}t\right) = 0, \frac{\pi}{6}t = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

M1

$$t = 3 \text{ h and } t = 9 \text{ h}$$

A1

Heights: 7 m at $t = 3$ and 3 m at $t = 9$

A1

Answer: $t = 3$ (7 m), $t = 9$ (3 m)

3 marks

23 (7 marks)

$$(a) \mathbf{v} = 2t\mathbf{i} + 3t^2\mathbf{j}, \text{ at } t = 2: \mathbf{v} = 4\mathbf{i} + 12\mathbf{j}$$

M1 A1

$$\text{Speed} = \sqrt{16 + 144} = 12.6 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 12.6 \text{ m s}^{-1}$$

3 marks

$$(b) |r| = \sqrt{t^4 + t^6}, \frac{d|r|}{dt} = \frac{4t^3 + 6t^5}{2\sqrt{t^4 + t^6}}$$

M1 A1

$$\text{At } t = 2: \frac{32 + 192}{2\sqrt{80}} = 12.5 \text{ m s}^{-1}$$

A1

This differs from the speed (12.6): the speed is the magnitude of the velocity, whereas $d|r|/dt$ only measures the component of velocity along the line from O

E1

Answer: 12.5, not equal

4 marks

24 (7 marks)

$$\mathbf{a} = 2a\mathbf{i} + b\mathbf{j}$$

M1

$$\text{At } t = 2: 4a = 8 \text{ and } b = -3, \text{ so } a = 2, b = -3$$

M1 A1 A1

$$\mathbf{v} = (2t^2 + 3)\mathbf{i} - 3t\mathbf{j}; \text{ at } t = 1: \mathbf{v} = 5\mathbf{i} - 3\mathbf{j}, \text{ speed} = \sqrt{34} = 5.83 \text{ m s}^{-1}$$

M1 A1

$$\mathbf{r} = \left(\frac{2}{3}t^3 + 3t + 1\right)\mathbf{i} + \left(-\frac{3}{2}t^2 + 2\right)\mathbf{j}; \text{ at } t = 1: \mathbf{r} = (4.667\mathbf{i} + 0.5\mathbf{j}) \text{ m}$$

A1

Answer: $a = 2$, $b = -3$; speed 5.83 m s^{-1} ; $\mathbf{r} = (4.667\mathbf{i} + 0.5\mathbf{j}) \text{ m}$

7 marks

25 (9 marks)

$$(a) \mathbf{v} = (t^2 - 6t)\mathbf{i} + 4t\mathbf{j} + \mathbf{c}, \text{ with } \mathbf{c} = 9\mathbf{i} - 12\mathbf{j}$$

M1 A1

$$\mathbf{v} = (t^2 - 6t + 9)\mathbf{i} + (4t - 12)\mathbf{j}, \text{ and } \mathbf{v}(3) = 0$$

A1

Answer: shown

3 marks

$$(b) \mathbf{r} = \left(\frac{t^3}{3} - 3t^2 + 9t - 9\right)\mathbf{i} + (2t^2 - 12t + 20)\mathbf{j}$$

M1 A1

$$\mathbf{r}(3) = 0\mathbf{i} + 2\mathbf{j}, \text{ so P is on the y-axis}$$

A1

$$\text{Distance from } (-9, 20) \text{ to } (0, 2) = \sqrt{81 + 324} = \sqrt{405} = 20.1 \text{ m}$$

A1

Answer: 20.1 m

4 marks

$$(c) \mathbf{a} = 2\mathbf{i} + 4\mathbf{j}$$

M1

$$|F| = 0.5 \times \sqrt{20} = 2.24 \text{ N}$$

A1

Answer: 2.24 N

2 marks

26 (12 marks)

$$(a) a = 4e^{-t/2} - 2te^{-t/2} = 4e^{-t/2}\left(1 - \frac{t}{2}\right)$$

M1 A1

$a = 0$ when $t = 2$; $a > 0$ before and $a < 0$ after, so v is greatest at $t = 2$

M1

$$v(2) = 8e^{-1} = \frac{8}{e} = 2.94 \text{ m s}^{-1}$$

A1

$$\text{Answer: } \frac{8}{e} = 2.94 \text{ m s}^{-1} \text{ at } t = 2$$

4 marks

$$(b) s = \int 4te^{-t/2} dt: \text{ take } u = t, dv = e^{-t/2} dt, \text{ so } du = dt, v = -2e^{-t/2}$$

M1

$$\int 4te^{-t/2} dt = 4[-2te^{-t/2} + 2 \int e^{-t/2} dt]$$

$$= -8te^{-t/2} - 16e^{-t/2} + c$$

$$s = 0 \text{ at } t = 0: c = 16, \text{ so } s = 16 - 8(t + 2)e^{-t/2}$$

Answer: shown

(c) As $t \rightarrow \infty$, $(t + 2)e^{-t/2} \rightarrow 0$, so $s \rightarrow 16 \text{ m}$

$$16 - 8(t + 2)e^{-t/2} = 10, \text{ i.e. } (t + 2)e^{-t/2} = 0.75$$

Solving numerically: $t = 4.24 \text{ s}$

Answer: 16 m; t = 4.24 s

M1 A1
A1
A1
5 marks
B1
M1
A1
3 marks

27 (12 marks)

(a) $\mathbf{v} = (2t - 4)\mathbf{i} + (2t - 6)\mathbf{j}$, $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}$

$$v^2 = (2t - 4)^2 + (2t - 6)^2$$

$$= 8t^2 - 40t + 52$$

Answer: shown

(b) $d(v^2)/dt = 16t - 40 = 0$, $t = 2.5$

$$v^2 = 8(6.25) - 100 + 52 = 2, \text{ least speed } \sqrt{2} = 1.41 \text{ m s}^{-1}$$

$$\mathbf{v}(2.5) = \mathbf{i} - \mathbf{j}; \mathbf{a} \cdot \mathbf{v} = 2(1) + 2(-1) = 0, \text{ so perpendicular}$$

Answer: $\sqrt{2} \text{ m s}^{-1}$, $\mathbf{a} \cdot \mathbf{v} = 0$

(c) $|\mathbf{v}|^2 = \mathbf{v} \cdot \mathbf{v}$, so $d(|\mathbf{v}|^2)/dt = \mathbf{a} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{a} = 2\mathbf{a} \cdot \mathbf{v}$

Speed is least when $d(|\mathbf{v}|^2)/dt = 0$, i.e. $\mathbf{a} \cdot \mathbf{v} = 0$ (a perpendicular to v)

Answer: proved

B1 B1
M1
A1
4 marks
M1 A1
A1
M1 A1
5 marks
M1 A1
E1
3 marks

28 (10 marks)

(a) $\mathbf{a} = \frac{\mathbf{F}}{m} = 9 - 3t$

$$\mathbf{v} = 9t - \frac{3}{2}t^2$$

$$s = \frac{9}{2}t^2 - \frac{1}{2}t^3$$

using $v = 0$ and $s = 0$ at $t = 0$

Answer: $\mathbf{v} = 9t - 1.5t^2$, $s = 4.5t^2 - 0.5t^3$

(b) $\mathbf{a} = 0$ at $t = 3$, $v = 27 - 13.5 = 13.5 \text{ m s}^{-1}$

$$v = 0: 1.5t(6 - t) = 0, t = 6 \text{ s}$$

Answer: 13.5 m s^{-1} , $t = 6 \text{ s}$

(c) $s = 0: 0.5t^2(9 - t) = 0$, so $t = 9 \text{ s}$

$$v(9) = 81 - 121.5 = -40.5 \text{ m s}^{-1}, \text{ i.e. } 40.5 \text{ m s}^{-1} \text{ back towards O}$$

Answer: $t = 9 \text{ s}$, 40.5 m s^{-1} towards O

M1
A1
A1
B1
4 marks
M1 A1
A1
3 marks
M1 A1
A1
3 marks

29 (10 marks)

(a) $\frac{dv}{dt} = -0.5v$, so $\int \frac{1}{v} dv = \int -0.5 dt$

$$\ln v = -0.5t + c$$

$$v = 10 \text{ at } t = 0: \ln 10 = c, \text{ so } v = 10e^{-0.5t}$$

Answer: shown

(b) $s = \int 10e^{-0.5t} dt = -20e^{-0.5t} + c$

$$s = 0 \text{ at } t = 0: s = 20(1 - e^{-0.5t}) < 20 \text{ for all } t$$

Answer: $s = 20(1 - e^{-0.5t}) < 20$

(c) $10e^{-0.5t} = 1$, so $t = 2 \ln 10 = 4.61 \text{ s}$

$$s = 20(1 - \frac{1}{10}) = 18 \text{ m}$$

Answer: 4.61 s, 18 m

M1 A1
A1
A1
4 marks
M1 A1
A1
3 marks
M1 A1
A1
3 marks

30 (9 marks)

$$(a) \mathbf{v} = 4t\mathbf{i} + (3\mathbf{i} + 2\mathbf{j}) = (4t + 3)\mathbf{i} + 2\mathbf{j}$$

M1 A1

$$\mathbf{r} = (2t^2 + 3t)\mathbf{i} + 2t\mathbf{j} + \mathbf{j}$$

M1

$$\mathbf{r} = (2t^2 + 3t)\mathbf{i} + (2t + 1)\mathbf{j}$$

A1

$$\text{Answer: } (2t^2 + 3t)\mathbf{i} + (2t + 1)\mathbf{j}$$

4 marks

$$(b) \mathbf{r}(3) = 27\mathbf{i} + 7\mathbf{j}$$

M1

$$\text{Distance} = \sqrt{27^2 + 7^2} = \sqrt{778} = 27.9 \text{ m}$$

A1

$$\text{Answer: } 27.9 \text{ m}$$

2 marks

$$(c) y = 2t + 1, \text{ so } t = \frac{y - 1}{2}$$

M1

$$x = 2 \times \frac{(y - 1)^2}{4} + 3 \times \frac{y - 1}{2} = \frac{(y - 1)(y - 1 + 3)}{2}$$

M1 A1

$$\text{Answer: shown}$$

3 marks

Projectiles**1 (4 marks)**

$$\text{Vertically: } 19.6 = \frac{1}{2}(9.8)t^2, \text{ so } t^2 = 4$$

M1 A1

$$t = 2 \text{ s}$$

A1

$$\text{Horizontal distance} = 12 \times 2 = 24 \text{ m}$$

A1

$$\text{Answer: } 2 \text{ s, } 24 \text{ m}$$

4 marks

2 (2 marks)

$$\text{Horizontal: } 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$$

B1

$$\text{Vertical: } 20 \sin 30^\circ = 10 \text{ m s}^{-1}$$

B1

$$\text{Answer: } 17.3 \text{ m s}^{-1} \text{ horizontally, } 10 \text{ m s}^{-1} \text{ vertically}$$

2 marks

3 (3 marks)

$$\text{Speed} = \sqrt{8^2 + 15^2} = 17 \text{ m s}^{-1}$$

M1 A1

$$\tan \theta = \frac{15}{8}, \theta = 61.9^\circ$$

A1

$$\text{Answer: } 17 \text{ m s}^{-1} \text{ at } 61.9^\circ$$

3 marks

4 (4 marks)

$$\text{At the top the vertical velocity is zero: } 0 = 14.7 - 9.8t, t = 1.5 \text{ s}$$

M1 A1

$$0 = 14.7^2 - 2(9.8)H$$

M1

$$H = \frac{216.09}{19.6} = 11.0 \text{ m}$$

A1

$$\text{Answer: } 1.5 \text{ s, } 11.0 \text{ m}$$

4 marks

5 (3 marks)

$$\text{Horizontal velocity stays } 8 \text{ m s}^{-1}; \text{ vertical velocity} = 9.8 \times 2 = 19.6 \text{ m s}^{-1} \text{ downwards}$$

M1 A1

$$\text{Speed} = \sqrt{8^2 + 19.6^2} = 21.2 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 21.2 \text{ m s}^{-1}$$

3 marks

6 (3 marks)

$$x = 12 \times 2 = 24$$

B1

$$y = 15 \times 2 - 4.9 \times 4 = 30 - 19.6 = 10.4$$

M1

$$\mathbf{r} = (24\mathbf{i} + 10.4\mathbf{j}) \text{ m}$$

A1

$$\text{Answer: } (24\mathbf{i} + 10.4\mathbf{j}) \text{ m}$$

3 marks

7 (7 marks)

Components: horizontal $25 \cos 40^\circ = 19.15$, vertical $25 \sin 40^\circ = 16.07$

B1

(a) Return to ground: $0 = 16.07t - 4.9t^2$, $t = \frac{2 \times 16.07}{9.8} = 3.28 \text{ s}$

M1 A1

(b) $R = 19.15 \times 3.280 = 62.8 \text{ m}$

M1 A1

(c) $H = \frac{16.07^2}{2 \times 9.8} = 13.2 \text{ m}$

M1 A1

Answer: 3.28 s, 62.8 m, 13.2 m

7 marks

8 (7 marks)

Vertical motion: $45 = 4.9t^2$, $t = 3.03 \text{ s}$

M1 A1

Vertical speed at impact: $9.8 \times 3.030 = 29.7 \text{ m s}^{-1}$

M1 A1

Speed = $\sqrt{20^2 + 29.70^2} = 35.8 \text{ m s}^{-1}$

M1 A1

$\tan \theta = \frac{29.70}{20}$, $\theta = 56.0^\circ$ below the horizontal

A1

Answer: 3.03 s, 35.8 m s⁻¹, 56.0° below the horizontal

7 marks

9 (4 marks)

Vertical: $40 = 4.9t^2$, $t = 2.857 \text{ s}$

M1 A1

Horizontal: $60 = u \times 2.857$

M1

$u = 21.0 \text{ m s}^{-1}$

A1

Answer: 21.0 m s⁻¹

4 marks

10 (9 marks)

(a) Time of flight: $0 = u \sin \theta t - \frac{1}{2}gt^2$, so $t = \frac{2u \sin \theta}{g}$

M1 A1

$R = u \cos \theta \times \frac{2u \sin \theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g}$

M1

$= \frac{u^2 \sin 2\theta}{g}$

A1

Answer: shown

4 marks

(b) $\sin 2\theta = 1$ gives $\theta = 45^\circ$

B1

$R_{\max} = \frac{u^2}{g} = \frac{900}{9.8} = 91.8 \text{ m}$

B1

Answer: 45°, 91.8 m

2 marks

(c) $\sin 2\theta = \frac{60 \times 9.8}{900} = 0.6533$

M1

$2\theta = 40.8^\circ$ or $180^\circ - 40.8^\circ$

M1

$\theta = 20.4^\circ$ or 69.6°

A1

Answer: 20.4° or 69.6°

3 marks

11 (7 marks)

Time to reach the net: $t = \frac{12}{25} = 0.48 \text{ s}$

M1 A1

Height of ball at the net = $2.5 - 4.9(0.48)^2 = 1.37 \text{ m}$

M1 A1

$1.37 > 0.9$, so the ball clears the net

E1

Time to fall 2.5 m: $t = \sqrt{2 \times 2.5/9.8} = 0.7143 \text{ s}$

M1

Distance = $25 \times 0.7143 = 17.9 \text{ m}$

A1

Answer: clears the net; lands 17.9 m from the server

7 marks

12 (5 marks)

The horizontal velocity is constant: $20 \cos 60^\circ = 10 \text{ m s}^{-1}$

B1

The speed is least when the vertical velocity is zero, at the highest point

E1

Least speed = 10 m s^{-1}

A1

$$H = \frac{(20 \sin 60^\circ)^2}{2 \times 9.8} = \frac{300}{19.6}$$

M1

= 15.3 m

A1

Answer: 10 m s^{-1} at height 15.3 m

5 marks

13 (6 marks)

(a) $x = u \cos \theta t$, $y = u \sin \theta t - \frac{1}{2}gt^2$

B1

$$t = \frac{x}{u \cos \theta}$$

M1

$$y = u \sin \theta \frac{x}{u \cos \theta} - \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta} = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

A1

Answer: shown

3 marks

(b) $y = 20 \tan 40^\circ - \frac{9.8 \times 400}{2 \times 18^2 \times \cos^2 40^\circ}$

M1 A1

$$= 16.78 - 10.31 = 6.47 \text{ m}$$

A1

Answer: 6.47 m

3 marks

14 (5 marks)

Time to reach the wall: $t = \frac{8}{15 \cos 55^\circ} = 0.9298 \text{ s}$

M1 A1

$$\text{Height} = 15 \sin 55^\circ \times 0.9298 - 4.9(0.9298)^2$$

M1

$$= 7.19 \text{ m}$$

A1

$7.19 > 3$, so the ball clears the wall by 4.19 m

A1

Answer: clears by 4.19 m

5 marks

15 (5 marks)

Both have initial vertical velocity zero, so both satisfy $44.1 = 4.9t^2$

M1

$t = 3 \text{ s}$ for each stone

A1

A lands $20 \times 3 = 60 \text{ m}$ from the foot of the cliff

M1 A1

B lands at the foot of the cliff, so the distance apart is 60 m

A1

Answer: $t = 3 \text{ s}$ for both; 60 m

5 marks

16 (7 marks)

(a) $\sin \alpha = 0.6$, so the initial vertical speed is 21 m s^{-1} ; $0 = 21^2 - 2 \times 9.8 \times H$

M1

$$H = 22.5 \text{ m}$$

A1

Answer: 22.5 m

2 marks

(b) Time of flight = $\frac{2 \times 21}{9.8}$; horizontal speed 28

M1

$$R = 28 \times \frac{30}{7} = 120 \text{ m}$$

A1

Answer: 120 m

2 marks

(c) $15 = 21t - 4.9t^2$, so $4.9t^2 - 21t + 15 = 0$

M1 A1

$t = 0.9057$ and 3.380 , so the time is 2.47 s

A1

Answer: 2.47 s

3 marks

17 (6 marks)

Horizontal component of velocity: $12 \cos 30^\circ = 10.39 \text{ m s}^{-1}$

B1

Vertical component at $t = 3$: $-12 \sin 30^\circ = -6$; initially $-6 + 9.8 \times 3 = 23.4 \text{ m s}^{-1}$

M1 A1

$$u = \sqrt{10.39^2 + 23.4^2} = 25.6 \text{ m s}^{-1}$$

M1 A1

$\tan \theta = \frac{23.4}{10.39}$, so $\theta = 66.1^\circ$ above the horizontal

A1

Answer: 25.6 m s^{-1} at 66.1°

6 marks

18 (10 marks)

$$(a) R = \frac{u^2 \sin 2\theta}{g}$$

B1

For the angle $(90^\circ - \theta)$: $\sin(180^\circ - 2\theta) = \sin 2\theta$

M1

so the ranges are equal

A1

Answer: shown

3 marks

$$(b) H_1 = \frac{u^2 \sin^2 \theta}{2g}, H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

M1 A1

$$H_1 : H_2 = \sin^2 \theta : \cos^2 \theta = \tan^2 \theta : 1$$

A1

Answer: shown

3 marks

$$(c) \sin 2\theta = \frac{60 \times 9.8}{784} = 0.7500, 2\theta = 48.6^\circ \text{ or } 131^\circ$$

M1 A1

$$\theta = 24.3^\circ \text{ or } 65.7^\circ$$

A1

$$H = \frac{784 \sin^2 \theta}{19.6} = 6.77 \text{ m or } 33.2 \text{ m}$$

A1

Answer: 24.3° and 65.7°

4 marks

19 (7 marks)

$$\text{Using } y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} \text{ with } \theta = 45^\circ: 10 = 30 - \frac{9.8 \times 900}{2u^2 \times 0.5}$$

M1 A1

$$10 = 30 - \frac{8820}{u^2}, \text{ so } u^2 = 441$$

M1 A1

$$u = 21 \text{ m s}^{-1}$$

A1

$$\text{At P: } t = \frac{30}{21 \cos 45^\circ} = 2.020 \text{ s}, v_y = 21 \sin 45^\circ - 9.8(2.020) = -4.95$$

M1

$$\text{Speed} = \sqrt{(14.85)^2 + (-4.95)^2} = 15.7 \text{ m s}^{-1}$$

A1

Answer: u = 21, speed at P 15.7 m s⁻¹

7 marks

20 (8 marks)

$$(a) \text{ Vertical: } -4 = 25 \sin 20^\circ t - 4.9t^2, \text{ i.e. } 4.9t^2 - 8.551t - 4 = 0$$

M1 A1

$$t = \frac{8.551 + \sqrt{8.551^2 + 78.4}}{9.8} = 2.13 \text{ s}$$

M1 A1

$$\text{Horizontal distance} = 25 \cos 20^\circ \times 2.129 = 50.0 \text{ m}$$

A1

Answer: 2.13 s, 50.0 m

5 marks

$$(b) v_y = 25 \sin 20^\circ - 9.8(2.129) = -12.31, v_x = 23.49$$

M1

$$\text{Speed} = \sqrt{23.49^2 + 12.31^2} = 26.5 \text{ m s}^{-1}$$

A1

$$\text{Angle} = \tan^{-1}\left(\frac{12.3}{23.49}\right) = 27.7^\circ \text{ below the horizontal}$$

A1

Answer: 26.5 m s⁻¹ at 27.7° below horizontal

3 marks

21 (6 marks)

(a) Only the vertical component of the velocity is zero at the highest point; the horizontal component $u \cos \theta$ remains

E1

The speed there is $u \cos \theta$, not zero, and the range needs the whole flight time (twice the time to the top)

E1

Answer: horizontal velocity is not zero

2 marks

$$(b) \text{ Time to top} = \frac{14.7 \sin 30^\circ}{9.8} = 0.75 \text{ s; horizontal speed} = 14.7 \cos 30^\circ = 12.73$$

M1 A1

$$\text{Range} = 12.73 \times 2 \times 0.75 = 19.1 \text{ m}$$

M1

Kim's value is 9.55 m, half the range

A1

Answer: 19.1 m (Kim's value is half)

4 marks

22 (7 marks)

(a) $v = 12i + (16 - 9.8t)j$

M1 A1

45° below the horizontal: $v_y = -v_x$, so $16 - 9.8t = -12$

M1

$t = \frac{28}{9.8} = 2.86 \text{ s}$

A1

Answer: t = 2.86 s

4 marks

(b) Dot product zero: $12 \times 12 + 16(16 - 9.8t) = 0$

M1 A1

$400 = 156.8t$, $t = 2.55 \text{ s}$

A1

Answer: t = 2.55 s

3 marks

23 (7 marks)

Vertical: $4 = 35 \sin 25^\circ t - 4.9t^2$, $35 \sin 25^\circ = 14.79$

M1 A1

$4.9t^2 - 14.79t + 4 = 0$

A1

$t = \frac{14.79 \pm \sqrt{14.79^2 - 78.4}}{9.8}$

M1

$t = 0.300 \text{ s}$ or 2.72 s

A1

The ball is at height 4 m twice, once rising and once falling; it lands on the green on the way down, $t = 2.72 \text{ s}$

E1

Distance = $35 \cos 25^\circ \times 2.718 = 86.2 \text{ m}$

A1

Answer: 2.72 s, 86.2 m

7 marks

24 (6 marks)

Vertically: $0 = u_y^2 - 2 \times 9.8 \times 30$, so $u_y = 24.25 \text{ m s}^{-1}$

M1 A1

Time to greatest height = $\frac{24.25}{9.8} = 2.474 \text{ s}$

M1

$u_x = \frac{40}{2.474} = 16.17 \text{ m s}^{-1}$

A1

$\tan \theta = \frac{u_y}{u_x} = 1.5$, so $\theta = 56.3^\circ$

A1

$u = \sqrt{u_x^2 + u_y^2} = 29.1 \text{ m s}^{-1}$

A1

Answer: $\theta = 56.3^\circ$, $u = 29.1 \text{ m s}^{-1}$

6 marks

25 (7 marks)

(a) Initial vertical speed = $11 \sin 45^\circ = 7.778 \text{ m s}^{-1}$

M1

Rise = $\frac{(7.778)^2}{2 \times 9.8} = 3.087 \text{ m}$

M1

Greatest height = 4.29 m

A1

Answer: 4.29 m

3 marks

(b) $9.6 = U \cos 45^\circ \times t$ and $2.0 = 1.2 + U \sin 45^\circ \times t - 4.9t^2$

M1 A1

$t = \frac{9.6\sqrt{2}}{U}$, so $0.8 = 9.6 - 4.9 \times \frac{2 \times 9.6^2}{U^2}$

M1

$U^2 = 102.6$, $U = 10.1$

A1

Answer: U = 10.1

4 marks

26 (7 marks)

(a) $x = V \cos 35^\circ \times t$ and $-15 = V \sin 35^\circ \times t - 4.9t^2$

M1 A1

Eliminating t : $-15 = 120 \tan 35^\circ - \frac{4.9 \times 120^2}{V^2 \cos^2 35^\circ}$

M1

$V = 32.6 \text{ m s}^{-1}$

A1

Answer: V = 32.6

4 marks

(b) Horizontal: $40 \cos 35^\circ = 32.77$; vertical: $40 \sin 35^\circ - 9.8 \times 4 = -16.26$

M1 A1

Speed = 36.6 m s^{-1} at 26.4° below the horizontal

A1

Answer: 36.6 m s^{-1} , 26.4° below horizontal

3 marks

27 (11 marks)

(a) At the top $v_y = 0$: $0 = u^2 \sin^2 \theta - 2gH$, so $H = \frac{u^2 \sin^2 \theta}{2g}$

M1 A1

Flight time $\frac{2u \sin \theta}{g}$, so $R = u \cos \theta \times \frac{2u \sin \theta}{g} = \frac{u^2 \sin 2\theta}{g}$

M1 A1

Answer: shown

4 marks

(b) $\frac{H}{R} = \frac{u^2 \sin^2 \theta}{2g} \times \frac{g}{2u^2 \sin \theta \cos \theta}$

M1 A1

$= \frac{\sin \theta}{4 \cos \theta} = \frac{1}{4} \tan \theta$, so $\tan \theta = \frac{4H}{R}$

A1

Answer: shown

3 marks

(c) $\tan \theta = \frac{4 \times 15}{60} = 1$, so $\theta = 45^\circ$

M1 A1

$R = \frac{u^2 \sin 90^\circ}{g}$, so $u^2 = 60 \times 9.8 = 588$

M1

$u = 24.2 \text{ m s}^{-1}$

A1

Answer: 45° , 24.2 m s^{-1}

4 marks

28 (11 marks)

(a) Vertical component = $50 \times 0.6 = 30 \text{ m s}^{-1}$, horizontal = 40 m s^{-1}

B1

Extra height = $\frac{30^2}{2 \times 9.8} = 45.92 \text{ m}$

M1

Greatest height = $60 + 45.92 = 106 \text{ m}$

A1

Answer: 106 m

3 marks

(b) $-60 = 30t - 4.9t^2$, so $4.9t^2 - 30t - 60 = 0$

M1 A1

$t = \frac{30 + \sqrt{900 + 1176}}{9.8} = 7.71 \text{ s}$

M1 A1

Distance = $40 \times 7.711 = 308 \text{ m}$

A1

Answer: 7.71 s, 308 m

5 marks

(c) $v_y = 30 - 9.8 \times 7.711 = -45.56$

M1

Speed = $\sqrt{40^2 + 45.56^2} = 60.6 \text{ m s}^{-1}$

A1

Angle below horizontal = $\tan^{-1}\left(\frac{45.6}{40}\right) = 48.7^\circ$

A1

Answer: 60.6 m s^{-1} at 48.7° below the horizontal

3 marks

29 (11 marks)

(a) Path: $y = x \tan \theta - \frac{gx^2}{2u^2}(1 + \tan^2 \theta)$, using $\frac{1}{\cos^2 \theta} = 1 + \tan^2 \theta$

M1 A1

$10 = 40T - \frac{9.8 \times 1600}{2u^2}(1 + T^2)$

M1

$10u^2 = 40u^2T - 7840(1 + T^2)$, so $7840T^2 - 40u^2T + 7840 + 10u^2 = 0$

A1

Answer: shown

4 marks

(b) Real T requires discriminant ≥ 0 : $(40u^2)^2 - 4(7840)(7840 + 10u^2) \geq 0$

M1 A1

$1600u^4 - 313600u^2 - 245862400 \geq 0$, dividing by 1600: $u^4 - 196u^2 - 153664 \geq 0$

A1

Critical value: $u^2 = \frac{196 + \sqrt{196^2 + 4 \times 153664}}{2} = 502.1$

M1

$u \geq 22.4$, so the least speed is 22.4 m s^{-1}

A1

Answer: 22.4 m s^{-1}

5 marks

(c) $T = \frac{40u^2}{2 \times 7840} = \frac{u^2}{392} = 1.281$

M1

$\theta = 52.0^\circ$

A1

Answer: 52.0°

2 marks

30 (11 marks)

(a) Aimed at T: $\tan \theta = \frac{20}{30} = \frac{2}{3}$, $\cos \theta = \frac{3}{\sqrt{13}} = 0.8321$

M1 A1

Horizontal: $30 = 25 \cos \theta \times t$, $t = 1.44$ s

A1

Answer: $t = 1.44$ s

3 marks

(b) Projectile: $y = 25 \sin \theta t - 4.9t^2 = (25 \cos \theta t) \tan \theta - 4.9t^2 = 30 \times \frac{2}{3} - 4.9t^2 = 20 - 4.9t^2$

M1 A1 A1

Target: $20 - 4.9t^2$, the same: they collide

E1

$t = 1.442$ s: height = $20 - 4.9(1.442)^2 = 9.81$ m

A1

Answer: same height; 9.81 m

5 marks

(c) The target takes $\sqrt{2 \times 20/9.8} = 2.020$ s to fall

M1

Need $30 \leq u \cos \theta \times 2.020$, so $u \geq \frac{30}{2.020 \cos \theta}$

M1

$u \geq 17.8 \text{ m s}^{-1}$

A1

Answer: 17.8 m s^{-1}

3 marks