

Displacement, Velocity and Acceleration

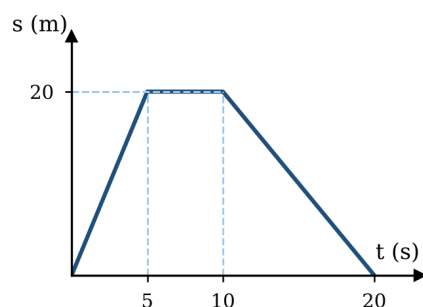
- 1** A car travels 150 km in 2.5 hours. Find its average speed in km h^{-1} . [2]
- 2** A runner runs 100 m east along a straight track and then turns round and runs 40 m west. Find (a) the distance she has run, (b) her displacement from her starting point, giving the direction. [3]
- 3** The velocity of a ball increases from 4 m s^{-1} to 16 m s^{-1} in 3 seconds. Find its average acceleration. [2]
- 4** Convert 108 km h^{-1} to m s^{-1} , and find the time taken to travel 450 m at this speed. [3]
- 5** A particle moves along a straight line. At time $t = 0$ its position is $x = -3 \text{ m}$ and at $t = 5 \text{ s}$ its position is $x = 7 \text{ m}$. Find its displacement and its average velocity. [3]
- 6** A train slows down from 30 m s^{-1} to 12 m s^{-1} in 6 seconds, with constant acceleration. Find the acceleration and explain the meaning of its sign. [3]
- 7** A cyclist rides 12 km at 24 km h^{-1} and then 8 km at 16 km h^{-1} . Find her average speed for the whole journey. [4]
- 8** Mia walks 3 km east in 40 minutes and then 1.5 km west in 20 minutes along a straight road. Find (a) her average speed, (b) her average velocity, for the whole walk, in km h^{-1} . [5]
- 9** A lift starts from rest and accelerates uniformly at 1.5 m s^{-2} . Find its speed after 4 seconds, and the time it takes to reach a speed of 7.5 m s^{-1} . [4]
- 10** A particle moves on the x-axis. Its position x metres at time t seconds is recorded in the table. The motion between readings is at constant velocity. Find the average velocity of the particle in each of the intervals 0 to 2 s, 2 to 4 s, 4 to 6 s and 6 to 8 s. [4]
- | | | | | | |
|--------------|----------|----------|----------|----------|----------|
| t (s) | 0 | 2 | 4 | 6 | 8 |
| x (m) | 3 | 11 | 11 | 5 | -7 |
- 11** Two cars start 300 m apart on a straight road and drive towards each other at constant speeds of 12 m s^{-1} and 18 m s^{-1} . Find the time until they meet and the distance the faster car has travelled by then. [4]
- 12** A train of length 200 m travels at a constant 20 m s^{-1} through a tunnel of length 1000 m. Find the time between the front of the train entering the tunnel and the rear of the train leaving it. [3]
- 13** An aircraft landing at 60 m s^{-1} decelerates uniformly at 2.4 m s^{-2} until it stops. Find the time it takes to stop. [3]
- 14** A cyclist rides to a village at an average speed of 15 km h^{-1} and returns along the same road at 10 km h^{-1} . Jo says: "The average speed for the round trip is $\frac{1}{2}(15 + 10) = 12.5 \text{ km h}^{-1}$." Show that Jo is wrong and find the correct average speed. [5]

- 15** A runner A leaves a point O at $t = 0$ and runs along a straight road at 5 m s^{-1} . A runner B leaves O 8 seconds later at 7 m s^{-1} along the same road in the same direction. Find the time after A starts when B catches A, and the distance from O at which this happens. [5]
- 16** A tennis ball travelling horizontally at 24 m s^{-1} hits a wall and rebounds along the same line at 16 m s^{-1} . The contact lasts 0.025 s. Taking the initial direction of motion as positive, find the average acceleration of the ball during contact, with its direction. [4]
- 17** A passenger train of length 150 m travels at 30 m s^{-1} . It overtakes a goods train of length 250 m travelling at 20 m s^{-1} on a parallel track in the same direction. Find the time from the moment the front of the passenger train is level with the rear of the goods train to the moment the rear of the passenger train is level with the front of the goods train. [4]
- 18** A car travels for 40 minutes at 72 km h^{-1} , then stops for 10 minutes, then travels a further 60 km at 90 km h^{-1} . Find the average speed for the whole journey including the stop. [5]
- 19** A particle moves along the x-axis. At $t = 0$ it is at $x = 2 \text{ m}$. It moves to $x = 14 \text{ m}$ at $t = 4 \text{ s}$ and then to $x = 6 \text{ m}$ at $t = 10 \text{ s}$. Find (a) the total distance travelled, (b) the average speed, (c) the displacement and (d) the average velocity over the 10 seconds. [5]
- 20** Ali and Ben each run 400 m. Ben runs 2 m s^{-1} faster than Ali and finishes 10 seconds before him. Find their speeds. [5]
- 21** Two cyclists, A and B, start at the same time from opposite ends of a straight road of length 90 km and ride towards each other, A at 15 km h^{-1} and B at 30 km h^{-1} . A bird starts with A and flies back and forth between them at 40 km h^{-1} , turning instantly each time it reaches a cyclist, until they meet.
(a) Find the time until the cyclists meet and the distance from A's start at which they meet. [3]
(b) Find the total distance flown by the bird, and explain why you do not need to know where it turns. [2]
- 22** A ball is thrown vertically upwards. Pat says: "The acceleration is negative, so the ball is always slowing down." Take upwards as positive and the acceleration as -9.8 m s^{-2} throughout the flight.
(a) Explain why Pat is wrong, referring to the sign of the velocity. [2]
(b) The ball leaves the hand at 14.7 m s^{-1} . Find its velocity after 1 s and after 2.5 s, and state whether it is rising or falling at each time. [3]
- 23** A sports car accelerates uniformly from rest to 60 miles per hour in 8.5 seconds. Take 1 mile = 1609 m.
(a) Convert 60 mph to m s^{-1} . [2]
(b) Find the acceleration of the car. [2]
(c) Express this as a multiple of $g = 9.8 \text{ m s}^{-2}$. [1]
- 24** A lift moves between two floors. It accelerates uniformly from rest at 0.8 m s^{-2} for 5 s, then moves at constant speed for 12 s, then decelerates uniformly to rest at 1.0 m s^{-2} .
(a) Find the greatest speed of the lift and the time it takes to decelerate. [3]
(b) Find the total time of the journey and the total distance travelled. [4]
- 25** A motorist drives 180 km at a constant speed. If she had driven 15 km h^{-1} faster, her journey would have taken 36 minutes less. Find her actual speed. [6]

- 26** A particle moves along a straight line through a point O. Its velocity (positive in the direction OA) is 3 m s^{-1} for $0 \leq t \leq 4$, then -2 m s^{-1} for $4 < t \leq 10$, then 5 m s^{-1} for $10 < t \leq 12$, where t is in seconds. It starts at O.
- Find the position of the particle at $t = 4, 10$ and 12 . [3]
 - Find the total distance travelled and the average speed for the 12 seconds. [3]
 - Find the average velocity for the 12 seconds. [1]
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- 27** A journey is made in two equal-distance stages at speeds u and v (both in km h^{-1}), and a second journey is made in two equal-time stages at speeds u and v .
- Show that the average speed for the equal-distance journey is $\frac{2uv}{u+v}$. [3]
 - Show that the average speed for the equal-time journey is $\frac{1}{2}(u+v)$. [2]
 - Show that the average speed in (a) is never greater than that in (b), and state when they are equal. Evaluate both for $u = 60, v = 40$. [4]
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- 28** Two runners start together from the same point on a circular track of circumference 400 m and run at constant speeds of 5 m s^{-1} and 7 m s^{-1} .
- If they run in the same direction, find the time until the faster runner first catches the slower one. [3]
 - If they run in opposite directions, find the time until they first meet, and the number of times they meet in 12 minutes (not counting the start). [4]
 - In the same-direction case, find how many times the faster runner has passed the slower in 12 minutes. [2]
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- 29** Two ferries operate between ports A and B, 24 km apart. At 12:00 ferry 1 leaves A for B at 12 km h^{-1} and ferry 2 leaves B for A at 18 km h^{-1} . Each ferry turns round immediately on reaching its destination and continues.
- Find the time and the distance from A at which the ferries first meet. [3]
 - Show that the ferries next meet 2.4 hours after leaving, and find where this is. [5]
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- 30** Car P leaves town A at 09:00 for town B, 120 km away, travelling at a constant 60 km h^{-1} . Car Q leaves A at 09:20 on the same road.
- Find the constant speed at which Q must travel to arrive at B at the same moment as P. [3]
 - Instead Q travels at 80 km h^{-1} . Find how far from A, and at what time, Q overtakes P, and how long before P it reaches B. [5]

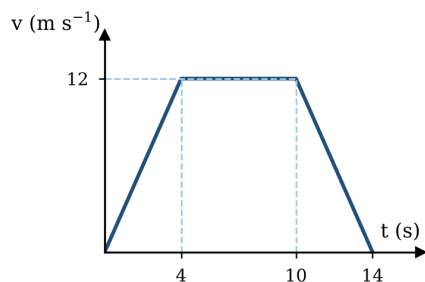
Motion Graphs

- 1** The displacement-time graph shows the journey of a cyclist along a straight road, with displacement s metres from her home at time t seconds.
- Find her speed during the first 5 seconds, state how long she is stationary and find her speed on the return.



[3]

- 2 The velocity-time graph shows the motion of a car. Find the acceleration during the first 4 seconds and the deceleration during the last 4 seconds.

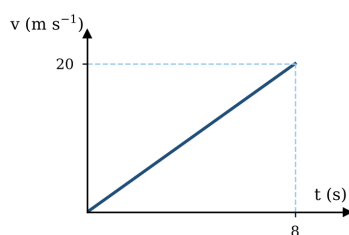


[3]

- 3 State what is represented by (a) the gradient of a displacement-time graph, (b) the gradient of a velocity-time graph, (c) the area under a velocity-time graph.

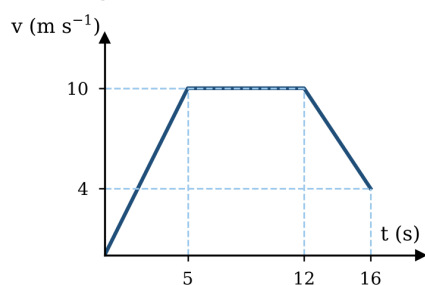
[3]

- 4 A particle accelerates uniformly from rest to 20 m s^{-1} in 8 seconds, as shown. Find the distance it travels in this time.



[2]

- 5 Describe the motion of the object shown in the velocity-time graph, giving the acceleration in each stage.

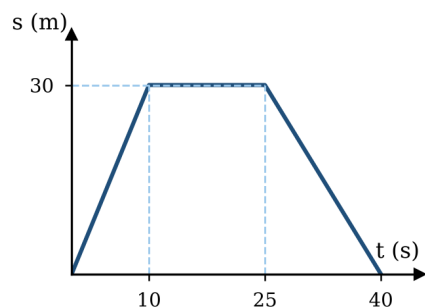


[3]

- 6 A train accelerates uniformly from rest at 0.5 m s^{-2} to a speed of 20 m s^{-1} , travels at this speed for T seconds, then decelerates uniformly at 1 m s^{-2} to rest. The total distance travelled is 1200 m. Sketch the velocity-time graph and find T .

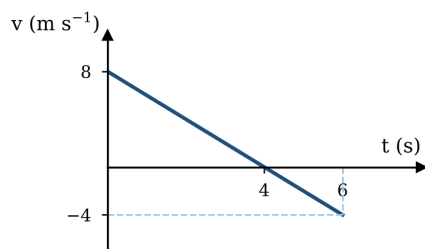
[5]

- 7 The graph shows the displacement from a starting point of a runner on a straight path. Find (a) the total distance run, (b) the average speed for the whole 40 seconds, (c) the average velocity for the whole 40 seconds.



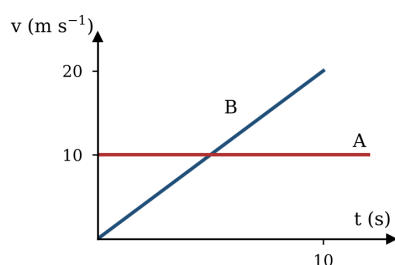
[4]

- 8** A particle moves on a straight line. Its velocity $v \text{ m s}^{-1}$ (positive to the right) varies as shown. Find the displacement and the total distance travelled in the first 6 seconds.



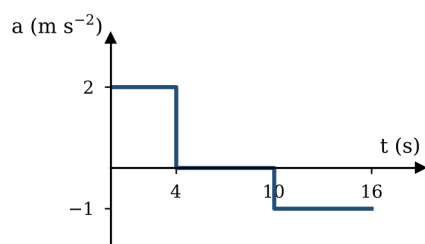
[4]

- 9** Car A travels at a constant 10 m s^{-1} . Car B starts from the same point at the same time from rest and accelerates uniformly to 20 m s^{-1} in 10 seconds. Show that, after 10 seconds, the cars are level.



[4]

- 10** A particle starts from rest. Its acceleration $a \text{ m s}^{-2}$ is 2 for $0 \leq t \leq 4$, zero for $4 \leq t \leq 10$ and -1 for $10 \leq t \leq 16$, as shown. Find its velocity at $t = 4, 10$ and 16 .



[4]

- 11** A walker leaves home and walks in a straight line at a constant 6 m s^{-1} to a shop. After 40 s she arrives, stays there for 20 s, and then returns home at a constant 4 m s^{-1} .

(a) Sketch the displacement-time graph for the whole trip, marking the key values on both axes.

[3]

(b) Find the total time for the trip and the average speed for the whole trip.

[3]

- 12** A train accelerates uniformly from rest at 1.5 m s^{-2} for 8 s, then travels at constant speed for 10 s, and then decelerates uniformly at 2 m s^{-2} until it stops.

(a) Sketch the velocity-time graph for the journey, marking the key values on both axes.

[3]

(b) Find the total distance travelled.

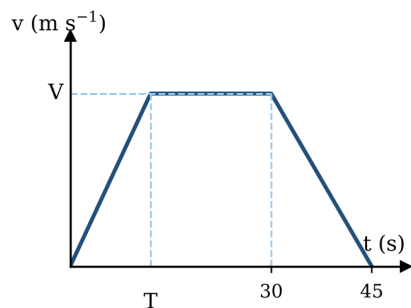
[2]

- 13** A train travels between two stations 2.1 km apart. It accelerates uniformly from rest for 20 s, then travels at constant speed $V \text{ m s}^{-1}$, and finally decelerates uniformly to rest in 30 s. The total time is 100 s. Sketch the velocity-time graph and find V .

[4]

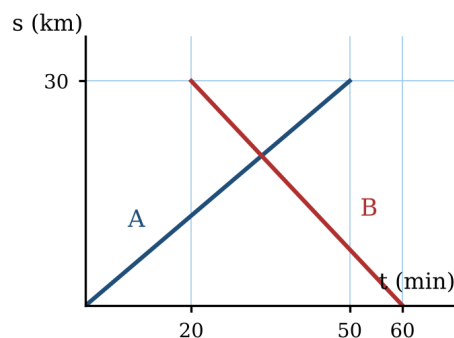
- 14** A runner accelerates uniformly from rest at 0.5 m s^{-2} for T seconds, then runs at constant speed until $t = 30$, and finally decelerates uniformly to rest at $t = 45$. The total distance run is 189 m.

Show that $T^2 - 75T + 756 = 0$ and hence find T .



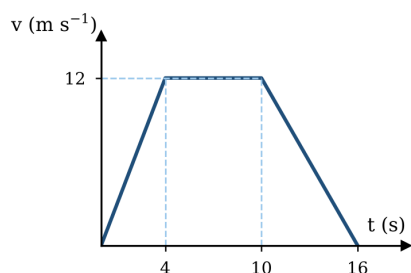
[6]

- 15** Two cyclists ride on the same straight road between towns P and Q, 30 km apart. The graph shows the distance s km from P of cyclist A, who leaves P at time 0 and arrives at Q at $t = 50$ minutes, and of cyclist B, who leaves Q at $t = 20$ minutes and arrives at P at $t = 60$ minutes. Find (a) the speed of each cyclist in km h^{-1} , (b) the time at which they pass, and (c) how far from P they are then.



[6]

- 16** The velocity of a particle for $0 \leq t \leq 16$ is shown in the diagram. Write down an expression for v in terms of t for each stage.



[4]

- 17** A driver travelling at 25 m s^{-1} sees an obstacle and brakes. She takes 0.6 s to react, during which the speed stays 25 m s^{-1} , and then decelerates uniformly at 5 m s^{-2} until the car stops. Sketch the velocity-time graph and find the total distance travelled from the moment she sees the obstacle.

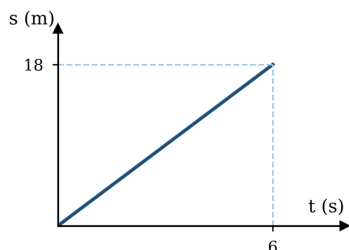
[5]

- 18** Two trains, each 130 m long, travel in the same direction on parallel tracks. At time $t = 0$ the front of each train is level with a signal. Train A travels at a constant 12 m s^{-1} . Train B is moving at 4 m s^{-1} at $t = 0$, accelerates uniformly to 10 m s^{-1} in 10 s, and then continues at 10 m s^{-1} .

- Find the acceleration of B and the distance it travels in the first 10 s. [2]
- Show that, for $t \geq 10$, the front of A is $(2t + 30) \text{ m}$ ahead of the front of B. [2]
- Find the time at which the front of B is level with the rear of A. [2]

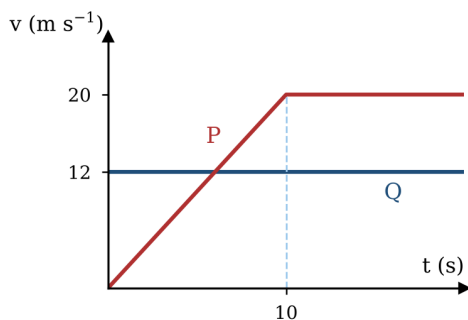
- 19** A lift in a tall building takes 20 s to go between two floors 112 m apart. It accelerates uniformly from rest at 1 m s^{-2} , then travels at a constant speed $V \text{ m s}^{-1}$, then decelerates uniformly at 2 m s^{-2} to rest.
- (a) Show that the lift spends $(20 - 1.5V)$ seconds at constant speed. [2]
- (b) Show that $3V^2 - 80V + 448 = 0$ and find V . [2]

- 20** The displacement-time graph of a toy car is a straight line from (0, 0) to (6, 18). A student writes: "The line is straight, so the acceleration is constant. The gradient is $\frac{18}{6} = 3$, so the acceleration is 3 m s^{-2} , and the area under the graph is the distance travelled, $\frac{1}{2} \times 6 \times 18 = 54 \text{ m}$."



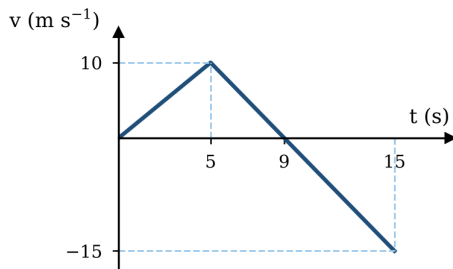
- (a) Identify two errors in the student's statements. [2]
- (b) Write down the correct velocity, acceleration and distance travelled in 6 s. [2]
- (c) Sketch the velocity-time graph for this motion and show where the distance appears on it. [2]

- 21** Particle Q moves with a constant speed 12 m s^{-1} along a straight line. Particle P is at the same point at $t = 0$ and accelerates uniformly from rest to 20 m s^{-1} in 10 s, and then moves at constant speed 20 m s^{-1} in the same direction, as shown.



- (a) Show that the greatest distance of P behind Q occurs at $t = 6$ and find this distance. [4]
- (b) Find the time at which P catches Q. [4]

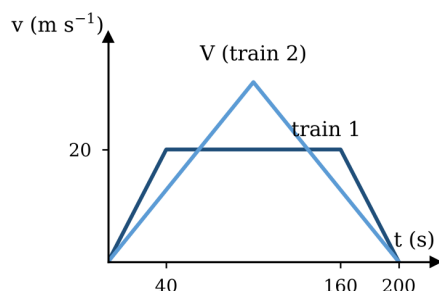
- 22** The velocity $v \text{ m s}^{-1}$ of a particle moving on a straight line, measured positive to the right of a point O, is shown for $0 \leq t \leq 15$. The particle starts at O.



- (a) Find the acceleration in each of the three stages. [3]
- (b) Show that the particle is back at O when $t = 15$, and find the greatest distance of the particle from O and the total distance travelled. [5]

- 23** A car accelerates uniformly from rest for 10 s to a speed $V \text{ m s}^{-1}$, travels at V for 30 s, and then decelerates uniformly to rest with a deceleration twice the size of its initial acceleration. The total distance travelled is 540 m. Find V and the acceleration in the first stage. [5]

- 24** Two trains start together from station A at time $t = 0$ and stop at station B. The speed-time graphs show their journeys. Train 1 accelerates uniformly for 40 s to 20 m s^{-1} , travels at this speed until $t = 160$ s, and then decelerates uniformly to rest at $t = 200$ s. Train 2 accelerates uniformly to a greatest speed V and then decelerates uniformly to rest, both stages taking 100 s.



- Find the distance AB. [2]
- Find V and the acceleration of train 2. [2]
- Find the times, other than 0 and 200 s, at which the two trains have the same speed. [3]
- Find the distance between the two trains when $t = 60$. [3]

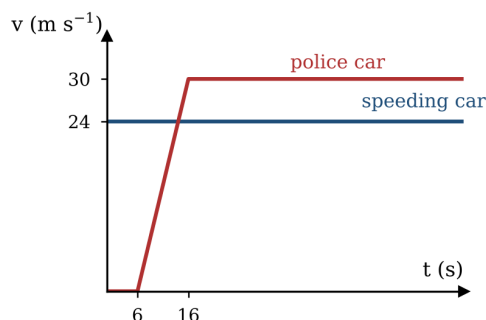
- 25** A car starts from rest at a point O and accelerates uniformly at 1.5 m s^{-2} for 12 s, after which it travels at constant speed. A motorcycle starts from rest at O 10 s after the car, accelerates uniformly at 3 m s^{-2} until it reaches 24 m s^{-1} , and then continues at this constant speed.

- Sketch the speed-time graphs of both vehicles on the same axes, using $t = 0$ as the time at which the car starts. [3]
- Find the time at which the motorcycle overtakes the car, and the distance from O at which this happens. [5]

- 26** A train travels between two stations 3 km apart in 150 s. It accelerates uniformly from rest at 0.5 m s^{-2} to a speed $V \text{ m s}^{-1}$, travels at V for some time and then decelerates uniformly at 0.75 m s^{-2} to rest.

- Show that the time spent at constant speed is $(150 - \frac{10V}{3})$ seconds. [2]
- Show that $V^2 - 90V + 1800 = 0$ and find V , giving a reason for rejecting any root. [6]

- 27** A speeding car passes a stationary police car at $t = 0$ and continues at a constant 24 m s^{-1} . The police car sets off 6 s later, accelerates uniformly at 3 m s^{-2} until it reaches 30 m s^{-1} , and then travels at this speed.

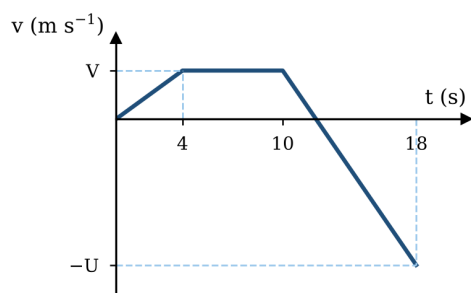


- Show that the police car reaches its top speed at $t = 16$ and that it has not yet caught the speeding car. [4]
- Find the time and distance from the police car's starting point at which it catches the speeding car. [4]

- 28** A train travels a distance D between stations. It accelerates uniformly from rest at a to a speed V , travels at V , then decelerates uniformly at b to rest, taking a total time T .

- Show that $T = \frac{D}{V} + \frac{V}{2}(\frac{1}{a} + \frac{1}{b})$. [5]
- With $a = 0.5 \text{ m s}^{-2}$, $b = 1 \text{ m s}^{-2}$, $D = 1200 \text{ m}$ and $T = 90 \text{ s}$, find the cruising speed V . [4]
- Find the least time in which the train could travel the 1200 m (with a and b as in (b)), by considering the case with no constant-speed stage. [3]

- 29** A particle moves on a straight line through O, starting at O at $t = 0$. Its velocity $v \text{ m s}^{-1}$ increases uniformly from 0 to V at $t = 4$, stays at V until $t = 10$ and then decreases uniformly to $-U$ at $t = 18$, passing through zero on the way, as shown. The particle is again at O when $t = 18$, and the magnitude of its acceleration during the last stage is 1.5 m s^{-2} .



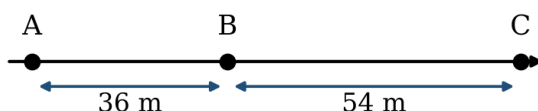
- Show that $U = 3V$ and hence find V and U . [5]
 - Find the time at which the particle is at its greatest distance from O and find this distance. [3]
 - Find the total distance the particle travels in the 18 seconds. [1]
- 30** Anna and Ben set off together at 09:00 from A to walk and cycle to B, sharing one bicycle. Anna cycles at 15 km h^{-1} for 40 minutes, leaves the bicycle at the side of the road, and walks on at 5 km h^{-1} . Ben walks at 5 km h^{-1} until he reaches the bicycle and then cycles at 15 km h^{-1} . They arrive at B at the same time.
- Find the distance from A to the bicycle. [2]
 - Let $AB = D \text{ km}$. Find D , and the time at which they arrive at B. [4]
 - On the same axes sketch the distance-time graphs for Anna and Ben, marking the key values. [2]
 - Find how far apart they are at 10:00. [2]

SUVAT Equations

- A car starts from rest and accelerates uniformly at 3 m s^{-2} for 8 seconds. Find its final speed and the distance it travels. [4]
- A cyclist travelling at 12 m s^{-1} brakes with a constant deceleration of 2 m s^{-2} for 5 seconds. Find her speed at the end of this time. [2]
- A particle moving in a straight line with constant acceleration increases its speed from 5 m s^{-1} to 17 m s^{-1} in 4 seconds. Find its acceleration and the distance it travels in this time. [4]
- A sledge starts from rest and accelerates uniformly down a slope. After travelling 80 m its speed is 20 m s^{-1} . Find its acceleration. [2]
- A train travelling at 20 m s^{-1} brakes uniformly and comes to rest after 50 m. Find the deceleration and the time taken to stop. [4]
- A ball rolls along a smooth horizontal floor with an initial speed of 6 m s^{-1} and a constant acceleration of 1.5 m s^{-2} . Find the distance it travels in the first 4 seconds. [2]
- A car travelling at 30 m s^{-1} brakes uniformly and stops in a distance of 90 m. Find the deceleration and the time taken to stop. [5]
- A cyclist is travelling at 4 m s^{-1} when she begins to accelerate at 2 m s^{-2} . She travels 21 m while accelerating. Find the time taken and her final speed. [5]

- 9** A sprinter accelerates uniformly from rest to 9 m s^{-1} in 2.5 s and then runs at a constant 9 m s^{-1} until she has covered 100 m . Find the total time for the race. [5]
- 10** A car moves along a straight road with constant acceleration. It passes a point A with speed 6 m s^{-1} and, 5 s later, passes a point B with speed 14 m s^{-1} .
 (a) Find the acceleration. [2]
 (b) Find the distance AB. [2]
 (c) The car continues and passes a point C, 20 m beyond B. Find the speed at C. [3]
- 11** A driver is travelling at 24 m s^{-1} when she sees a hazard. During her reaction time of 0.5 s the car continues at 24 m s^{-1} ; she then brakes with constant deceleration and stops. The total distance from seeing the hazard to stopping is 60 m . Find the deceleration. [4]
- 12** A car A passes a point O travelling at a constant 20 m s^{-1} . At the same instant a car B starts from rest at O and accelerates uniformly at 2 m s^{-2} in the same direction. Find the time at which B overtakes A, and B's speed then. [5]
- 13** A car is travelling at 30 m s^{-1} and is 60 m behind a lorry travelling at a constant 20 m s^{-1} on the same road. The driver of the car brakes, producing a constant deceleration of 2.5 m s^{-2} until its speed matches that of the lorry. Show that the car does not collide with the lorry and find the closest distance between them. [6]
- 14** A particle moves along a straight line through a point O with constant acceleration -2 m s^{-2} . It passes O with velocity 10 m s^{-1} . Find the times at which the particle is 24 m from O on the same side as its initial motion, and explain why there are two such times. [5]
- 15** A particle moves in a straight line with constant acceleration. Its speed at the start of a section of the motion is 2 m s^{-1} and at the end it is 14 m s^{-1} . Show that its speed at the midpoint of the section is 10 m s^{-1} . [5]
- 16** A ball is released at the top of a slope with an initial speed of 2 m s^{-1} and accelerates uniformly at 0.8 m s^{-2} down the slope, taking 5 s to reach the bottom. It then moves on horizontal ground at the speed it had at the bottom, until it hits a wall 30 m from the bottom of the slope. Find the length of the slope, and the total time from release until the ball hits the wall. [6]
- 17** A particle moves in a straight line with constant acceleration a . Its initial velocity is u , its velocity at time t is v and it has displacement s from its starting point.
 (a) By eliminating t from $v = u + at$ and $s = ut + \frac{1}{2}at^2$, show that $v^2 = u^2 + 2as$. [4]
 (b) A cyclist travelling at 15 m s^{-1} brakes with deceleration 3 m s^{-2} . Find the distance she travels before her speed has halved. [3]
- 18** A particle moves in a straight line with constant acceleration $a \text{ m s}^{-2}$. It travels 14 m in the third second of its motion and 20 m in the fifth second. Find the initial speed and the acceleration. [6]
- 19** A student solves: "A car travelling at 15 m s^{-1} brakes with a deceleration of 3 m s^{-2} . Find the distance it travels in 8 s ." Her solution is $s = 15 \times 8 - \frac{1}{2} \times 3 \times 64 = 24 \text{ m}$.
 (a) Explain why her answer is wrong, finding the time the car takes to stop. [3]
 (b) Find the correct distance travelled in 8 s . [2]

- 20** A ball is struck along a straight horizontal groove, passing a mark O with speed $u \text{ m s}^{-1}$ and moving with constant deceleration 1.2 m s^{-2} . It passes a mark P, on the same side of O, when $t = 2$ and again when $t = 7$ (where t is the time in seconds after passing O).
- (a) Show that $u = 5.4$. [3]
- (b) Find the distance OP, and the greatest distance of the ball from O. [3]
-
- 21** A particle moves in a straight line with constant acceleration and passes through points A, B and C in that order. It takes 3 s to travel from A to B and 2 s to travel from B to C. The distances AB and BC are 39 m and 36 m. Find the acceleration of the particle and its speed at A. [6]
-
- 22** A car moves along a straight road with constant acceleration from a point A to a point B. Its speed at A is 6 m s^{-1} and its speed at B is 14 m s^{-1} , and it takes 10 s to travel from A to B. The point M is the midpoint of AB.
- (a) Find the acceleration of the car and the distance AB. [4]
- (b) Find the speed of the car as it passes M. [2]
- (c) Find the time taken to reach M. [2]
-
- 23** The points A, O and B lie in that order on a straight line, with $AO = 4 \text{ m}$ and $OB = 9 \text{ m}$. A particle moving along the line passes O with speed $u \text{ m s}^{-1}$ in the direction OA. It has constant acceleration of $a \text{ m s}^{-2}$ in the direction OB, and reaches B 3 seconds later with speed 8 m s^{-1} .
- (a) Find u and a . [5]
- (b) Show that the particle does not reach A. [3]
-
- 24** A cyclist passes a point O at a constant speed of 8 m s^{-1} . Five seconds later a car passes O, travelling in the same direction at 22 m s^{-1} and decelerating uniformly at 2 m s^{-2} . Let τ seconds be the time after the car passes O.
- (a) Show that the cyclist and the car are level when $\tau^2 - 14\tau + 40 = 0$. [4]
- (b) Find the times after the cyclist passes O at which they are level, and the distance from O each time. [3]
-
- 25** Ana is running along a straight track at a constant 8 m s^{-1} and is 160 m from the finishing line. Bea is 16 m behind Ana, running at 7 m s^{-1} . At this instant Bea starts to accelerate uniformly, while Ana continues at constant speed.
- (a) Find the acceleration of Bea if the two women cross the line together. [4]
- (b) Find the acceleration Bea needs so that she is 4 m beyond the finishing line at the instant Ana reaches it. [2]
- (c) In the case in (a), find Bea's speed as she crosses the line. [2]
-
- 26** A car moves along a straight road with constant acceleration $a \text{ m s}^{-2}$. It passes points A, B and C in that order. The distances AB and BC are 36 m and 54 m respectively, and the car takes 3 seconds to travel from A to B and 3 seconds from B to C, as shown.



- (a) Show that the speed at B is 15 m s^{-1} . [3]
- (b) Find the acceleration and the speed at A. [4]
- (c) Prove that, for any motion with constant acceleration a , the distances covered in two successive equal time intervals T differ by aT^2 . [3]

- 27** A car is travelling at 25 m s^{-1} on a dry road when the driver sees a wet patch ahead and brakes with constant deceleration 5 m s^{-2} . On reaching the wet patch the car continues to brake with a smaller constant deceleration of 2.5 m s^{-2} until it stops. The car travels a total of 85 m from the start of braking until it stops.
- (a) Let the speed of the car on reaching the wet patch be $V \text{ m s}^{-1}$. Show that $\frac{625 - V^2}{10} + \frac{V^2}{5} = 85$. [3]
- (b) Find V and the total time for which the car is braking. [5]
-
- 28** A particle moves in a straight line with constant acceleration a , initial speed u , and travels distances s_n in the n -th second of its motion.
- (a) Show that the distance travelled in the n -th second is $u + a(n - \frac{1}{2})$. [3]
- (b) The particle covers 13 m in the third second and 25 m in the seventh second. Find u and a . [3]
- (c) Find the time taken for the particle to travel 140 m from the start, and the distance it covers in the 8th second. [3]
-
- 29** A car A passes a point O at $t = 0$ with speed 30 m s^{-1} and decelerates uniformly at 2 m s^{-2} . At the same instant a car B starts from rest at O and accelerates uniformly at 3 m s^{-2} in the same direction. Both cars continue to move in a straight line.
- (a) Show that A would come to rest at $t = 15$ and find its distance from O then. [2]
- (b) Show that B overtakes A at $t = 12$ and find the distance from O at which this happens, justifying that A is still moving. [5]
- (c) Find the speeds of the cars when B overtakes A and, if B's acceleration were 2 m s^{-2} instead, find when B would reach A and comment. [3]
-
- 30** Two cars travel along a straight road in the same direction, both at 20 m s^{-1} , with the rear car 12 m behind the front car. The driver of the front car brakes and decelerates uniformly at 5 m s^{-2} . The driver of the rear car reacts 0.4 s later and then brakes, decelerating uniformly at 4 m s^{-2} .
- (a) Show that the cars collide, and find the time, after the front car starts to brake, at which this happens. [7]
- (b) Find the speed of each car at the instant of collision. [2]

Vertical Motion

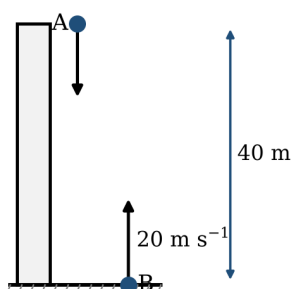
- 1** A stone is dropped from rest from the top of a cliff and takes 4 s to reach the sea. Find the speed with which it hits the sea and the height of the cliff. (Take $g = 9.8 \text{ m s}^{-2}$.) [4]
-
- 2** A ball is thrown vertically upwards with a speed of 14.7 m s^{-1} . Find the time taken to reach its highest point and the greatest height reached above the point of projection. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 3** A ball is dropped from rest from a height of 19.6 m above the ground. Find the time it takes to reach the ground and its speed on landing. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 4** A ball is thrown vertically downwards at 5 m s^{-1} from a high window. Find its speed after 2 seconds and the distance it has fallen in this time. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 5** A ball is thrown vertically upwards with speed 24.5 m s^{-1} . Find its velocity (with direction) after (a) 1 second, (b) 4 seconds. Take $g = 9.8 \text{ m s}^{-2}$. [4]

- 6** A stone is thrown vertically upwards at 12 m s^{-1} from a bridge 30 m above a river. Find the speed with which the stone hits the river and the time it takes to reach the river (the stone misses the bridge on the way down). Take $g = 9.8 \text{ m s}^{-2}$. [6]

- 7** A ball is projected vertically upwards from ground level. When it is 10 m above the ground it is moving upwards with speed 7 m s^{-1} . Find the speed of projection. Take $g = 9.8 \text{ m s}^{-2}$. [3]

- 8** A stone is dropped from rest. Find the distance it falls during the third second of its motion. Take $g = 9.8 \text{ m s}^{-2}$. [4]

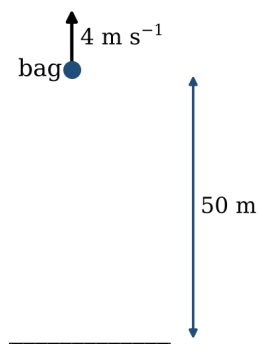
- 9** A ball A is dropped from rest from the top of a tower of height 40 m. At the same instant a ball B is thrown vertically upwards from the foot of the tower at 20 m s^{-1} . Show that the balls meet 2 seconds after release and find the height at which this occurs, and the velocity of each ball when they meet. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

- 10** A ball is thrown vertically upwards at 15 m s^{-1} from a point 10 m above horizontal ground. Find the time taken to reach the ground and the speed on hitting it. Take $g = 9.8 \text{ m s}^{-2}$. [6]

- 11** A hot-air balloon is rising at a constant 4 m s^{-1} . When it is 50 m above the ground, a sandbag is released from it. Find the time taken for the sandbag to reach the ground, and its speed on landing. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

- 12** A ball is thrown vertically upwards from ground level. It passes a point P twice, 2 s and 5 s after being thrown. Find the speed of projection and the height of P above the ground. Take $g = 9.8 \text{ m s}^{-2}$. [5]

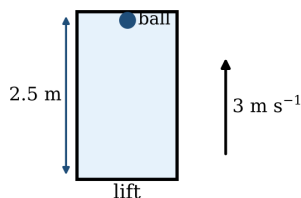
- 13** A particle is projected vertically upwards from ground level with speed $u \text{ m s}^{-1}$. Show that (a) the greatest height is $\frac{u^2}{2g}$ and (b) the total time of flight is $\frac{2u}{g}$. Hence find the speed needed for a flight time of 6 s. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 14** A ball is dropped from a height of 20 m and takes 2.1 s to reach the ground when the experiment is carried out. Using the model of free fall with $g = 9.8 \text{ m s}^{-2}$, find the predicted time. Find the average acceleration the observed time implies, and suggest a reason for the difference. [5]
- 15** A ball is thrown vertically upwards with a speed of 18 m s^{-1} from a balcony above horizontal ground. The ball lands on the ground 5 s later. Find the height of the balcony and the speed with which the ball hits the ground. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 16** A ball is projected vertically upwards from horizontal ground with a speed of 24.5 m s^{-1} . Find the length of time for which the ball is at least 20 m above the ground. [5]
- 17** A stone is dropped from rest from the top of a tower and hits the ground with a speed of 34.3 m s^{-1} . Find the height of the tower, and the time the stone takes to fall the last 20 m. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 18** A ball is projected vertically upwards from horizontal ground with a speed of 19.6 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the velocity and the height of the ball 3 s after projection. [3]
 (b) Find the total distance the ball travels in the first 3 s. [2]
- 19** A model rocket is launched vertically from the ground. Its engine gives it a constant upward acceleration of 15 m s^{-2} for 8 seconds and then cuts out. After that the rocket moves freely under gravity. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the speed and height of the rocket when the engine cuts out. [3]
 (b) Find the greatest height reached by the rocket. [3]
 (c) Find the total time from launch until the rocket returns to the ground. [3]
- 20** A stone A is dropped from rest from the top of a vertical cliff. One second later a stone B is thrown vertically downwards from the same point with speed 15 m s^{-1} . B overtakes A before reaching the sea. Find the time after the release of A at which B overtakes A, and the distance fallen then. Take $g = 9.8 \text{ m s}^{-2}$. [7]
- 21** A ball is thrown vertically upwards at 20 m s^{-1} from ground level. A window has its lower edge 10 m and its upper edge 14 m above the ground. Find the total time for which the ball is level with the window (between the two edges), counting both its upward and its downward passage. Take $g = 9.8 \text{ m s}^{-2}$. [7]
- 22** A ball is thrown vertically upwards at 14.7 m s^{-1} . Aisha says: "At the highest point the ball is stationary, so the acceleration there is zero." Ben says: "The acceleration is always 9.8 downwards, so at the top, after a moment, the ball must be going back up."
 (a) Explain what is wrong with Aisha's statement and with Ben's statement. [3]
 (b) Show that the velocity at $t = 1.5$ is zero and find the velocity at $t = 1.6$, and comment. [3]
- 23** A ball is thrown vertically upwards from a point O and passes a point P, h metres above O, at times t_1 and t_2 seconds after projection. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Show that $h = \frac{1}{2}g t_1 t_2$. [4]
 (b) Find h when the ball passes P at 0.5 s and 3.5 s, and the speed of projection. [3]
- 24** A stone is dropped down a well. The sound of the splash is heard 3.0 seconds after the stone is released. The speed of sound is 340 m s^{-1} . Find the depth of the well, giving your answer to the nearest metre. Take $g = 9.8 \text{ m s}^{-2}$. [7]

- 25** Particle A is projected vertically upwards from horizontal ground with a speed of 14.7 m s^{-1} . At the same instant particle B is projected vertically upwards with a speed of 4.9 m s^{-1} from a point 10 m vertically above A's starting point. Neither particle has yet landed. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the time at which the particles are at the same height, and that height. [5]
- (b) Find the velocity of each particle at this instant. [3]

- 26** Two balls A and B are thrown vertically upwards from the same point on the ground with the same speed 19.6 m s^{-1} . A is thrown at $t = 0$ and B is thrown T seconds later, where $0 < T < 4$. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the balls are at the same height when $t = 2 + \frac{T}{2}$. [5]
- (b) For $T = 2$, find the height at which they meet and the velocity of each ball at that moment. [4]
- (c) Explain why no meeting is possible in the air if $T \geq 4$. [2]

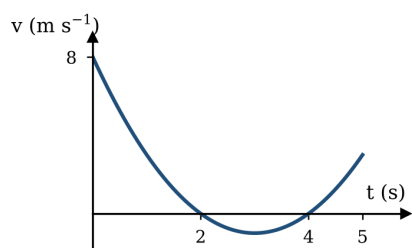
- 27** A lift is moving upwards at a constant speed of 3 m s^{-1} . A ball is released from rest relative to the lift at the ceiling, which is 2.5 m above the floor of the lift. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the time the ball takes to hit the floor of the lift. [3]
- (b) Find the velocity of the ball, relative to the ground, at the instant it hits the floor. [3]
- (c) Find the greatest height the ball reaches above the point of release, relative to the ground. [3]
- 28** A stone is dropped from rest from the top of a tower and, in the last second of its fall, it travels half of the total height of the tower. Let the total time of fall be T seconds.
- (a) Show that $T^2 - 4T + 2 = 0$. [4]
- (b) Find T and the height of the tower, and explain why one root is rejected. [4]
- 29** A ball is dropped from rest from a height of 19.6 m onto a hard floor. Each time it hits the floor it rebounds vertically with half of the speed with which it hit the floor. Air resistance is neglected. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the ball first hits the floor after 2 s with speed 19.6 m s^{-1} , and find the greatest height after the first bounce. [4]
- (b) Find the time between the first and second impact, and show that the times between successive impacts form a geometric progression. [4]
- (c) Find the total time from release until the ball stops bouncing, and comment on whether this is realistic. [3]
- 30** A ball is projected vertically upwards with speed U from a point which is $3H$ above horizontal ground, where H is the greatest height the ball reaches above the point of projection. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that $H = \frac{U^2}{2g}$. [2]
- (b) Show that the speed with which the ball hits the ground is $2U$. [3]
- (c) Show that the time taken for the ball to reach the ground is $\frac{3U}{g}$. [3]

Variable Acceleration

- 1 The displacement of a particle from a fixed point O at time t seconds is $s = 3t^2 + 2t$ metres. Find its velocity when $t = 2$ and its acceleration. [4]
- 2 A particle moves in a straight line with velocity $v = 6t - 2 \text{ m s}^{-1}$ at time t seconds. When $t = 0$ the particle is at the origin. Find its acceleration and its displacement from the origin when $t = 3$. [4]
- 3 A particle moves on a straight line so that its displacement from a point O after t seconds is $s = t^3 - 6t$ metres. Find the velocity when $t = 2$ and the acceleration when $t = 3$. [4]
- 4 A particle starts from rest with acceleration $a = 6t \text{ m s}^{-2}$. Find its velocity at time $t = 2$. [3]
- 5 The velocity of a particle is $v = 3t^2 - 12 \text{ m s}^{-1}$. Find the times at which the particle is instantaneously at rest. [3]
- 6 A particle moves on a straight line so that its displacement from a point O after t seconds is $s = t^3 - 6t^2 + 9t$ metres. Find the times at which the particle is at rest and its displacement from O at each of these times. [5]
- 7 A particle moves in a straight line with velocity $v = -3t^2 + 12t \text{ m s}^{-1}$ for $t \geq 0$. Find the greatest velocity of the particle, the time at which it comes to rest again, and its displacement from the starting point at that time. [6]
- 8 A particle moves in a straight line with acceleration $(6t - 4) \text{ m s}^{-2}$. When $t = 0$ its velocity is 3 m s^{-1} and its displacement from a point O is 2 m. Find its velocity and displacement when $t = 2$. [5]
- 9 A particle moves on a line with velocity $v = t^2 - 4t + 3 \text{ m s}^{-1}$. Find the times when the particle is at rest, and its acceleration at each of these times. State the minimum velocity. [5]
- 10 A particle moves along a straight line so that its displacement from a point O is $s = -t^3 + 5t^2$ metres at time t seconds. Find the time at which its acceleration is zero and its velocity at this time. [4]
- 11 A particle moves along a straight line with velocity $v = t^2 - 6t + 8 \text{ m s}^{-1}$ for $0 \leq t \leq 5$, as shown. Find the displacement of the particle from its starting point at $t = 5$ and the total distance it travels in this time. [7]



- 14** A particle starts from rest at a point O and moves in a straight line with acceleration $(4 - 2t) \text{ m s}^{-2}$. Find (a) the time at which the particle is next at rest, (b) the time at which it returns to O and its speed then. [6]
-
- 15** A car is travelling at 20 m s^{-1} when the driver applies the brakes at $t = 0$. The deceleration at time t seconds after braking begins is $kt \text{ m s}^{-2}$, where k is a constant. The car comes to rest after 5 seconds. Find k and the distance travelled while braking. [6]
-
- 16** A particle moves in a straight line with velocity $v = 2t^2 - 10t + 12 \text{ m s}^{-1}$. Find the time interval during which the particle moves in the negative direction, and its displacement during this interval. [5]
-
- 17** A particle moves on a straight line. Its displacement from the fixed point O at time t seconds is $s = t^3 - 3t^2 + 4t$ metres, $t \geq 0$.
 (a) Show that the particle is always moving in the positive direction. [3]
 (b) Hence explain why the particle never returns to O for $t > 0$, and find its acceleration when its velocity is least. [4]
-
- 18** A train leaves a station at $t = 0$ and moves along a straight track. A model for its speed is $v = \frac{t(60 - t)}{50} \text{ m s}^{-1}$ for $0 \leq t \leq 60$, after which it stops.
 (a) Find the greatest speed of the train according to the model. [3]
 (b) Find the distance between the stations and the average speed of the train. [4]
-
- 19** A student is told that a particle moves with acceleration $a = 2t \text{ m s}^{-2}$ from an initial speed of 3 m s^{-1} , and writes: " $v = u + at = 3 + 2t \times t = 3 + 2t^2$."
 (a) Explain why the student's method is not valid. [2]
 (b) Find the correct expressions for v and for the displacement s from the starting point. [3]
 (c) Compare the student's value of v at $t = 3$ with the correct value. [2]
-
- 20** A particle moves in a straight line with velocity $v = (pt^2 + qt) \text{ m s}^{-1}$, where p and q are constants. At $t = 1$ the acceleration of the particle is 2 m s^{-2} and at $t = 3$ its velocity is 18 m s^{-1} . Find p and q , and the displacement of the particle between $t = 0$ and $t = 3$. [7]
-
- 21** A particle moves in a straight line with acceleration $(2t - 8) \text{ m s}^{-2}$. At $t = 0$ the particle is at O with velocity 12 m s^{-1} .
 (a) Show that $v = (t - 2)(t - 6)$ and find the times at which the particle is at rest. [3]
 (b) Find the displacement from O at $t = 2$ and $t = 6$, and the total distance travelled in the first 6 seconds. [4]
-
- 22** A car is travelling at 32 m s^{-1} when the driver begins to brake. At time t seconds after braking begins, the deceleration of the car is $(2 + \frac{t}{2}) \text{ m s}^{-2}$.
 (a) Show that the car stops after 8 seconds. [4]
 (b) An obstacle is 150 m ahead of the car at the instant the driver begins to brake. Find the distance between the car and the obstacle when the car has stopped. [4]
-
- 23** A particle moves along a straight line. At time t seconds its velocity is $v = t^2 + kt + 4 \text{ m s}^{-1}$, where k is a constant. The minimum value of the velocity occurs when $t = 2.5$.
 (a) Find the value of k . [2]
 (b) Find the times at which the particle is instantaneously at rest. [2]
 (c) Find the total distance travelled by the particle in the interval $0 \leq t \leq 5$. [4]
-

- 24** A particle starts from rest at a point O and moves along a straight line. For $0 \leq t \leq 6$ its acceleration at time t seconds is $(6 - t) \text{ m s}^{-2}$. After $t = 6$ the particle moves with constant velocity.
- Find the velocity of the particle when $t = 6$. [3]
 - Show that the particle is 72 m from O when $t = 6$. [3]
 - Find the time at which the particle is 1000 m from O. [3]
-
- 25** A train starts from rest at O and moves along a straight track. For $0 \leq t \leq 6$ its acceleration is $(12t - 2t^2) \text{ m s}^{-2}$. From $t = 6$ it decelerates uniformly at 4 m s^{-2} until it stops at a signal.
- Find the speed of the train and its distance from O when $t = 6$. [4]
 - Find the total distance from O to the signal and the total time taken. [4]
 - Show that the train's acceleration is greatest at $t = 3$ in the first stage, and find its value. [2]
-
- 26** Two particles P and Q move along the same straight line, both starting from a point O at $t = 0$. Particle P has velocity $3t^2 \text{ m s}^{-1}$ and particle Q moves with constant velocity 12 m s^{-1} , both in the same direction.
- Find the displacement of P from O at time t . [2]
 - Find the greatest distance by which Q is ahead of P, and the time at which this occurs. [4]
 - Find the time at which P overtakes Q and their common distance from O, giving answers to 3 significant figures. [4]
-
- 27** A particle moves in a straight line with displacement $s = at^2 + bt + c$ from a point O, where a , b and c are constants.
- Find the velocity at time t , and show that the average velocity between times t_1 and t_2 equals the instantaneous velocity at the midpoint of the interval. [5]
 - Given $s = 3t^2 - 2t + 1$, find the average velocity over $2 \leq t \leq 6$ and the time at which the instantaneous velocity equals it. [3]
 - Explain why this result does not hold in general when the acceleration is not constant, giving an example. [2]
-
- 28** A particle P moves along a straight line starting from O at $t = 0$, with velocity $v = kt(6 - t)^2 \text{ m s}^{-1}$ for $0 \leq t \leq 6$, where k is a positive constant. The greatest speed of P is 16 m s^{-1} .
- Show that the greatest speed occurs at $t = 2$ and hence find k . [5]
 - Find the distance travelled by P in the 6 seconds and its average speed. [4]
 - Show that P is never moving in the negative direction, and find its acceleration at $t = 6$. [2]
-
- 29** A particle P moves along a straight line through a point O. At time t seconds, $t \geq 0$, its velocity is $(t^2 - 7t + 10) \text{ m s}^{-1}$, and P is at O when $t = 0$.
- Find the times at which P is instantaneously at rest. [2]
 - Find the distance travelled by P before it first comes to rest. [2]
 - Find the displacement of P from O at the instant when the total distance it has travelled is 12 m. [3]
-
- 30** A particle moves in a straight line. Its velocity $v \text{ m s}^{-1}$ at time t seconds is given by $v = 4t - t^2$ for $0 \leq t \leq 3$, and $v = 9 - 2t$ for $t > 3$.
- Find the greatest speed of the particle for $0 \leq t \leq 3$. [2]
 - Find the displacement of the particle from its starting point when $t = 3$. [2]
 - Find the time after $t = 3$ at which the particle is instantaneously at rest. [2]
 - Find the total distance travelled by the particle in the first 6 seconds. [4]

Kinematics Using Calculus

- 1** A particle P has position vector $\mathbf{r} = (t^2)\mathbf{i} + (3t)\mathbf{j}$ metres at time t seconds. Find its velocity at $t = 2$ and its speed at that instant. [4]

- 2 The velocity of a particle at time t seconds is $\mathbf{v} = ((3t^2 - 2)\mathbf{i} + 4t\mathbf{j}) \text{ m s}^{-1}$. Find the acceleration at $t = 2$. [3]

- 3 A particle moves on the x -axis with displacement $x = 3 \sin 2t$ metres from the origin at time t seconds. Find its velocity when $t = \frac{\pi}{6}$. [3]

- 4 A particle moves along a straight line with velocity $\mathbf{v} = 4e^{-t} \text{ m s}^{-1}$, where t is the time in seconds. Find its acceleration when $t = 0$ and when $t = 2$, giving exact answers. [3]

- 5 A particle has position vector $\mathbf{r} = ((2t + 1)\mathbf{i} + (t^2 - 3)\mathbf{j}) \text{ m}$ at time t seconds. Find its distance from the origin when $t = 3$. [3]

- 6 A particle starts at the origin with velocity $(3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ and moves with constant acceleration $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-2}$. Find its velocity and position vector when $t = 3$. [4]

- 7 A particle moves so that its position vector at time t seconds is $\mathbf{r} = ((t^2 - 4t)\mathbf{i} + (3t - t^2)\mathbf{j}) \text{ m}$. Find the time at which the particle is moving parallel to $\mathbf{j}$ and its speed then. [5]

- 8 A particle has acceleration $(6\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$. At $t = 0$ its velocity is $(3\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$ and it is at the origin. Find its velocity and position at $t = 2$. [6]

- 9 A particle moves on a straight line with displacement $s = 2te^{-t}$ metres from O at time t seconds. Show that the particle is at rest at $t = 1$ and find its displacement from O then, giving an exact answer. [5]

- 10 A particle moves with position vector $\mathbf{r} = (3t)\mathbf{i} + (4t - 2t^2)\mathbf{j}$ metres. Find the least speed of the particle and the time at which it occurs. [5]

- 11 A particle has velocity $(-3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north. Find its speed and the bearing of its direction of motion. [4]

- 12 A particle P has position vector $\mathbf{r} = ((t^2 - 4t + 5)\mathbf{i} + (t^3 - 4t)\mathbf{j}) \text{ m}$ at time t seconds, $t \geq 0$. Find the time at which P is due east of the origin O (other than $t = 0$) and the velocity of P at that time. [5]

- 13 A particle of mass 2 kg moves on a smooth horizontal plane under a single horizontal force $\mathbf{F} = ((12t)\mathbf{i} + 8\mathbf{j}) \text{ N}$, where t is the time in seconds. The particle is at rest at the origin when $t = 0$. Find the position vector of the particle when $t = 2$ and its distance from the origin. [6]

- 14 A particle moves along a straight line with velocity $\mathbf{v} = 10e^{-0.5t} \text{ m s}^{-1}$, where t is in seconds, and it starts at the origin. Find (a) the acceleration at $t = 2$, (b) the time at which the speed is 5 m s^{-1} , (c) the displacement as t increases without limit. [6]

- 15 The displacement of a particle from O is $s = 8 \ln(1 + t)$ metres at time t seconds, $t \geq 0$. Find (a) its velocity at $t = 3$, (b) its acceleration at $t = 3$, (c) the time at which its speed is 1 m s^{-1} . [6]

- 16 A particle moves along a straight line with velocity $\mathbf{v} = 4 \cos(\frac{\pi}{2}t) \text{ m s}^{-1}$, for $0 \leq t \leq 4$, where t is in seconds, starting from the origin. Find the times at which the particle is at rest, its greatest displacement from the origin and the total distance travelled in the 4 seconds. [6]

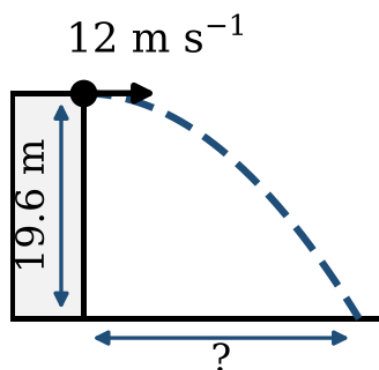
- 17 A particle P of mass 0.8 kg moves under the action of a single force, so that its position vector relative to a fixed origin O at time t seconds is $\mathbf{r} = (3t^2 - 2t)\mathbf{i} + (t^3 - 4t)\mathbf{j}$ metres.
 (a) Find the magnitude of the force acting on P when $t = 2$. [3]
 (b) Find the value of t when the velocity of P is parallel to the vector $\mathbf{i} + \mathbf{j}$. [3]

- 18** The position vector of a particle at time t seconds is $\mathbf{r} = 2t\mathbf{i} + (t^2 - 3t)\mathbf{j}$ metres, relative to a fixed origin O .
- (a) Find the cartesian equation of the path of the particle, in the form $y = f(x)$. [2]
 (b) Find the position of the particle at the instant when its velocity is parallel to $\mathbf{i}$. [4]
-
- 19** A particle moves so that its position vector at time t seconds is $\mathbf{r} = ((t - 3)\mathbf{i} + (2t - 4)\mathbf{j})$ m relative to the origin O .
- (a) Show that the square of the distance of the particle from O is $5t^2 - 22t + 25$. [2]
 (b) Find the least distance of the particle from O . [4]
-
- 20** Particle A has position vector $\mathbf{r}_A = (2t)\mathbf{i} + (t^2)\mathbf{j}$ and particle B has position vector $\mathbf{r}_B = (t + 3)\mathbf{i} + (18 - 3t)\mathbf{j}$, in metres at time t seconds.
- (a) Show that the particles collide and find the time and position. [4]
 (b) Find the velocity of each particle at the collision and the angle between their directions of motion. [4]
-
- 21** A cyclist accelerates from rest so that her speed v m s⁻¹ after t seconds is $v = 12(1 - e^{-t/2})$, as modelled.
- (a) Find her initial acceleration and the limiting value of her speed. [3]
 (b) Find the time at which her speed is 9 m s⁻¹, and the distance she has travelled by then. [5]
-
- 22** The height h metres of a float above the sea bed at time t hours is modelled by $h = 5 + 2 \sin(\frac{\pi}{6}t)$.
- (a) Find the vertical velocity of the float at $t = 0$ and show that its greatest speed is $\frac{\pi}{3}$ m h⁻¹. [4]
 (b) Find the times in the first 12 hours when the float is momentarily at rest and its height then. [3]
-
- 23** A particle has position vector $\mathbf{r} = (t^2)\mathbf{i} + (t^3)\mathbf{j}$ metres. Jo says: "The speed at $t = 2$ is the rate of change of the distance from the origin, $d|\mathbf{r}|/dt$."
- (a) Find the speed of the particle at $t = 2$. [3]
 (b) Find $d|\mathbf{r}|/dt$ at $t = 2$ and hence show that Jo's statement is wrong. [4]
-
- 24** A particle moves with velocity $\mathbf{v} = ((at^2 + 3)\mathbf{i} + bt\mathbf{j})$ m s⁻¹, where a and b are constants. At $t = 2$ its acceleration is $(8\mathbf{i} - 3\mathbf{j})$ m s⁻². When $t = 0$ its position vector is $(\mathbf{i} + 2\mathbf{j})$ m. Find a and b , and the speed and position vector of the particle at $t = 1$. [7]
-
- 25** A particle P of mass 0.5 kg moves under a variable force so that its acceleration at time t seconds is $\mathbf{a} = (2t - 6)\mathbf{i} + 4\mathbf{j}$ m s⁻². At $t = 0$ the particle is at the point with position vector $(-9\mathbf{i} + 20\mathbf{j})$ m and has velocity $(9\mathbf{i} - 12\mathbf{j})$ m s⁻¹.
- (a) Show that P is instantaneously at rest when $t = 3$. [3]
 (b) Show that when $t = 3$ the particle is on the y -axis, and find its distance from its initial position. [4]
 (c) Find the magnitude of the force acting on P when $t = 4$. [2]
-
- 26** A particle moves along a straight line starting at O when $t = 0$, with velocity $\mathbf{v} = 4te^{-t/2}$ m s⁻¹ for $t \geq 0$.
- (a) Show that the greatest speed of the particle is $\frac{8}{e}$ m s⁻¹ and state when it occurs. [4]
 (b) Show, using integration by parts, that the displacement at time t is $s = 16 - 8(t + 2)e^{-t/2}$. [5]
 (c) Find the displacement as $t \rightarrow \infty$ and the time at which the particle is 10 m from O , to 3 s.f. (use a calculator to solve). [3]

- 27** A particle moves in the plane with position vector $\mathbf{r} = ((t^2 - 4t)\mathbf{i} + (t^2 - 6t)\mathbf{j})$ metres at time t seconds.
- Find $\mathbf{v}$ and $\mathbf{a}$, and show that the speed v is given by $v^2 = 8t^2 - 40t + 52$. [4]
 - Find the least speed of the particle and verify that at this instant the velocity and acceleration are perpendicular. [5]
 - Prove that, for any motion with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$, $d(|\mathbf{v}|^2)/dt = 2 \mathbf{a} \cdot \mathbf{v}$, and explain the result in (b). [3]
-
- 28** A particle of mass 2 kg moves in a straight line on a smooth horizontal surface under a horizontal force of $(18 - 6t)$ N in the direction of motion, where t is the time in seconds. The particle starts from rest at O.
- Find the velocity and displacement of the particle at time t . [4]
 - Find the greatest speed during the first 6 seconds, and the time at which the particle is next at rest. [3]
 - Find the time at which the particle returns to O, and its velocity then, assuming the same force acts throughout. [3]
-
- 29** A particle moves in a straight line. Its deceleration is proportional to its velocity: $a = -0.5v$, where $v \text{ m s}^{-1}$ is the velocity and t is the time in seconds. At $t = 0$ the particle is at O with velocity 10 m s^{-1} .
- By separating variables, show that $v = 10e^{-0.5t}$. [4]
 - Find the displacement s as a function of t , and show that the particle never travels further than 20 m from O. [3]
 - Find the time at which the speed has dropped to 1 m s^{-1} and the distance travelled by then. [3]
-
- 30** A particle moves with constant acceleration $4\mathbf{i} \text{ m s}^{-2}$. At time $t = 0$ it is at the point with position vector $\mathbf{j}$ m, relative to a fixed origin O, and it has velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$.
- Find the position vector of the particle at time t seconds. [4]
 - Find the distance of the particle from O when $t = 3$. [2]
 - Show that the path of the particle has cartesian equation $x = \frac{(y - 1)(y + 2)}{2}$. [3]

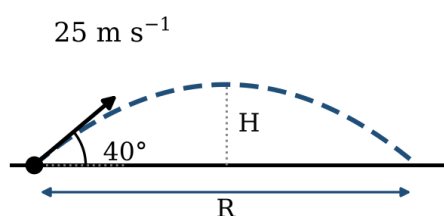
Projectiles

- 1** A ball is thrown horizontally at 12 m s^{-1} from the top of a vertical cliff 19.6 m above horizontal ground. Find the time taken to reach the ground and the horizontal distance from the foot of the cliff at which it lands. Take $g = 9.8 \text{ m s}^{-2}$.



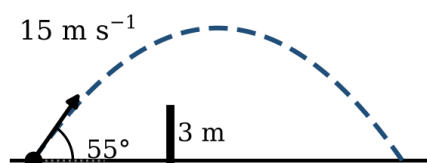
- 2** A ball is projected with speed 20 m s^{-1} at an angle of 30° above the horizontal. Find the horizontal and vertical components of its initial velocity. [2]

- 3 A projectile is launched with horizontal component of velocity 8 m s^{-1} and vertical component 15 m s^{-1} . Find its initial speed and the angle of projection above the horizontal. [3]
- 4 A ball is projected from level ground with an initial vertical component of velocity 14.7 m s^{-1} . Find the time taken to reach its greatest height, and the greatest height above the ground. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 5 A ball is thrown horizontally at 8 m s^{-1} from a high window. Find its speed 2 seconds after it is thrown. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 6 A ball is projected from the origin O so that its position vector at time t seconds is $\mathbf{r} = (12t)\mathbf{i} + (15t - 4.9t^2)\mathbf{j}$ metres, where $\mathbf{i}$ is horizontal and $\mathbf{j}$ vertically upwards. Find its position when $t = 2$. [3]
- 7 A ball is kicked from level ground with speed 25 m s^{-1} at 40° above the horizontal. Find (a) the time of flight, (b) the range, (c) the greatest height reached. Take $g = 9.8 \text{ m s}^{-2}$.



- [7]
- 8 A stone is thrown horizontally at 20 m s^{-1} from the top of a tower of height 45 m . Find the time to reach the ground, the speed on impact and the angle the velocity makes with the horizontal at that instant. Take $g = 9.8 \text{ m s}^{-2}$. [7]
- 9 A ball is thrown horizontally from the top of a building 40 m high and lands 60 m from the foot of the building. Find the speed with which it was thrown. [4]
- 10 A particle is projected from level ground with speed u at an angle θ above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the range on level ground is $R = \frac{u^2 \sin 2\theta}{g}$. [4]
- (b) Hence state the angle of projection for the maximum range at a given speed and find the maximum range when $u = 30 \text{ m s}^{-1}$. [2]
- (c) Find the two possible angles of projection if the range is 60 m when $u = 30 \text{ m s}^{-1}$. [3]
- 11 A tennis ball is served horizontally at 25 m s^{-1} from a point 2.5 m above the ground. The net is 12 m horizontally from the server and is 0.9 m high. Show that the ball clears the net and find the distance from the server at which the ball first hits the ground. Take $g = 9.8 \text{ m s}^{-2}$. [7]
- 12 A ball is projected from level ground with speed 20 m s^{-1} at 60° above the horizontal. Find the least speed of the ball during its flight, and the height above the ground at which this occurs. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 13 A particle is projected from the origin with speed u at an angle θ above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the path satisfies $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$. [3]
- (b) A ball is projected at 18 m s^{-1} at 40° . Use the equation to find its height when it has travelled 20 m horizontally. [3]

- 14** A ball is kicked from level ground with speed 15 m s^{-1} at 55° above the horizontal towards a vertical wall of height 3 m whose base is 8 m from the point of projection. Determine whether the ball passes over the wall and, if so, by how much.



[5]

- 15** Stone A is thrown horizontally at 20 m s^{-1} from the top of a cliff 44.1 m high. At the same instant stone B is dropped from rest from the same point. Show that they reach the sea at the same time and find the distance between the points where they land.

[5]

- 16** A ball is projected from horizontal ground with speed 35 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the greatest height reached by the ball. [2]
 (b) Find the horizontal range of the ball. [2]
 (c) Find the length of time for which the ball is at least 15 m above the ground. [3]

- 17** A particle is projected from horizontal ground and moves freely under gravity. Three seconds after projection it is moving with speed 12 m s^{-1} at 30° below the horizontal. Find the speed and the angle of projection. Take $g = 9.8 \text{ m s}^{-2}$.

[6]

- 18** Two projectiles are launched from level ground with the same speed u , one at angle θ above the horizontal and the other at $90^\circ - \theta$. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Show that they have the same range. [3]
 (b) Show that the ratio of their greatest heights is $\tan^2 \theta : 1$. [3]
 (c) For $u = 28 \text{ m s}^{-1}$ and range 60 m , find the two angles of projection and the greatest height in each case. [4]

- 19** A ball is projected from a point O on level ground with speed $u \text{ m s}^{-1}$ at 45° above the horizontal. It passes through the point P, which is 30 m horizontally from O and 10 m above the ground. Find u , and the speed of the ball at P. Take $g = 9.8 \text{ m s}^{-2}$.

[7]

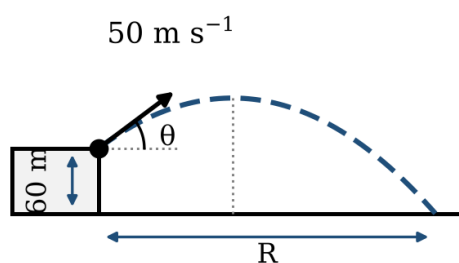
- 20** A stunt motorcyclist leaves the end of a ramp, which is 4 m above the ground, with speed 25 m s^{-1} at 20° above the horizontal and lands on level ground. Model the motorcyclist and bike as a particle. Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Find the time of flight and the horizontal distance from the end of the ramp to the landing point. [5]
 (b) Find the speed and the angle to the horizontal of the motorcyclist's velocity on landing. [3]

- 21** A ball is projected from the ground at 14.7 m s^{-1} at angle θ above the horizontal. Kim says: "At the highest point the ball is momentarily at rest, so I can use the time to the top to find the range from $R = \text{speed} \times \text{time}$." Take $g = 9.8 \text{ m s}^{-2}$.

- (a) Explain the error in Kim's reasoning. [2]
 (b) For $\theta = 30^\circ$, find the range correctly and show the value that Kim's method gives (time to the top $\times u \cos \theta$), and state the ratio. [4]

- 22** A ball is projected from a point O on level ground with initial velocity $(12\mathbf{i} + 16\mathbf{j}) \text{ m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors horizontally and vertically upwards. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the velocity of the ball at time t and the time at which the ball is moving at 45° below the horizontal. [4]
- (b) Find the time at which the velocity of the ball is perpendicular to its initial velocity. [3]
-
- 23** A golf ball is hit from a tee on level ground with speed 35 m s^{-1} at 25° above the horizontal. It lands on a green which is 4 m above the level of the tee. Find the time of flight and the horizontal distance from the tee to the landing point, and explain why the quadratic equation for the time has two positive roots. Take $g = 9.8 \text{ m s}^{-2}$. [7]
-
- 24** A ball is projected from horizontal ground. It reaches its greatest height of 30 m when it has travelled a horizontal distance of 40 m. Find the angle and the speed of projection. Take $g = 9.8 \text{ m s}^{-2}$. [6]
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- 25** A ball is hit from a point 1.2 m above horizontal ground, with speed $U \text{ m s}^{-1}$ at an angle of 45° above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Given that $U = 11$, find the greatest height above the ground reached by the ball. [3]
- (b) The ball hits the top of a vertical pole of height 2.0 m, which stands on the ground 9.6 m horizontally from the point of projection. Find U . [4]
-
- 26** A golfer hits a ball from a tee on a cliff, 15 m above level ground, with speed $V \text{ m s}^{-1}$ at an angle of 35° above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) The ball must land at least 120 m horizontally from the base of the cliff. Find the least possible value of V . [4]
- (b) When $V = 40$, find the speed of the ball, and the angle its direction of motion makes with the horizontal, 4 seconds after it is hit. [3]
-
- 27** A particle is projected from level ground with speed u at an angle θ above the horizontal. Its greatest height is H and its range is R . Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that $H = \frac{u^2 \sin^2 \theta}{2g}$ and $R = \frac{u^2 \sin 2\theta}{g}$. [4]
- (b) Show that $\tan \theta = \frac{4H}{R}$. [3]
- (c) A ball has a range of 60 m and a greatest height of 15 m. Find the angle of projection and the speed of projection. [4]
-
- 28** A gun on the edge of a cliff 60 m above the sea fires a shell with speed 50 m s^{-1} at an angle θ above the horizontal, where $\sin \theta = 0.6$. Model the shell as a particle. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the greatest height of the shell above the sea. [3]
- (b) Find the time taken for the shell to hit the sea and the horizontal distance travelled. [5]
- (c) Find the speed and direction of the shell as it hits the sea. [3]

- 29** A ball is projected from a point O on level ground with speed $u \text{ m s}^{-1}$. It is to pass through a point P which is 40 m horizontally from O and 10 m above the ground. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that, with $T = \tan \theta$, the angle of projection θ satisfies $7840T^2 - 40u^2T + (7840 + 10u^2) = 0$. [4]
- (b) Show that the ball can reach P only if $u^4 - 196u^2 - 153664 \geq 0$, and hence find the least possible value of u to 3 significant figures. [5]
- (c) Find the angle of projection when u has this least value. [2]
-
- 30** A target is released from rest at a point T which is 20 m above level ground and 30 m horizontally from a point O on the ground. At the instant the target is released, a projectile is fired from O with speed 25 m s^{-1} directly towards T. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that $\tan \theta = \frac{2}{3}$ for the angle of projection and find the time at which the projectile reaches the vertical line through T. [3]
- (b) Show that at this time the projectile and the target are at the same height, and find that height. [5]
- (c) Find the least speed of projection for the collision to occur before the target reaches the ground. [3]
-