

Null and Alternative Hypotheses

1 (2 marks)

Null hypothesis: the coin is fair. $H_0: p = 0.5$

B1

The coin is biased in either direction. $H_1: p \neq 0.5$

B1

Answer: $H_0: p = 0.5$, $H_1: p \neq 0.5$

2 marks

2 (2 marks)

Let p = the proportion of red sweets in the population of sweets

B1

$H_0: p = 0.2$ and $H_1: p > 0.2$ (the student's belief gives a one-tailed alternative)

B1

Answer: $H_0: p = 0.2$, $H_1: p > 0.2$

2 marks

3 (2 marks)

p = the probability that she wins a first serve

B1

$H_0: p = 0.6$, $H_1: p < 0.6$ (one-tailed: the coach suspects a decrease)

B1

Answer: $H_0: p = 0.6$, $H_1: p < 0.6$

2 marks

4 (2 marks)

μ = the mean daily rainfall in May (mm)

B1

$H_0: \mu = 2.1$, $H_1: \mu \neq 2.1$ (two-tailed: a change in either direction)

B1

Answer: $H_0: \mu = 2.1$, $H_1: \mu \neq 2.1$

2 marks

5 (2 marks)

λ = the mean number of customers per minute

B1

$H_0: \lambda = 3$, $H_1: \lambda > 3$ (one-tailed: the owner suspects an increase)

B1

Answer: $H_0: \lambda = 3$, $H_1: \lambda > 3$

2 marks

6 (2 marks)

Let ρ be the population product moment correlation coefficient between age and cholesterol level.

B1

$H_0: \rho = 0$, $H_1: \rho > 0$

B1

Answer: $H_0: \rho = 0$, $H_1: \rho > 0$

2 marks

7 (4 marks)

(a) $H_0: p = 0.7$, $H_1: p > 0.7$

B1

The alternative only looks for an increase, so the test is one-tailed (upper tail)

B1

Answer: $H_0: p = 0.7$, $H_1: p > 0.7$; one-tailed

2 marks

(b) $H_0: \mu = 500$, $H_1: \mu \neq 500$

B1

The inspector suspects a change in either direction, so the test is two-tailed

B1

Answer: $H_0: \mu = 500$, $H_1: \mu \neq 500$; two-tailed

2 marks

8 (4 marks)

(a) The null hypothesis must contain equality (it is the assumption we test); Ben has swapped H_0 and H_1

E1

$H_0: p = 0.25$, $H_1: p \neq 0.25$

B1

Answer: H_0 must say $p = 0.25$; $H_1: p \neq 0.25$

2 marks

(b) Hypotheses are statements about the population parameter p , not about a statistic from the sample

E1

The value $7/20 = 0.35$ comes from the sample, so it must not be used in the hypotheses, which must be written before the data are used. H_1 should be $p \neq 0.25$

E1

Answer: Hypotheses must be about the population parameter p and not use the sample result

2 marks

9 (2 marks)

(a) The null hypothesis is the statement about the population parameter that is assumed to be true during the test (it always contains equality) **B1**

(b) The alternative hypothesis is the statement about the parameter that is accepted if there is sufficient evidence to reject the null hypothesis **B1**

Answer: H_0 : assumed true, includes equality; H_1 : what we conclude if H_0 is rejected **2 marks**

10 (4 marks)

(i) is valid **B1**

(ii) is invalid: H_0 must contain the equality sign **B1**

(iii) is invalid: H_1 must be about the population parameter μ , not a sample statistic **B1**

(iv) is invalid: H_0 and H_1 must refer to the same value (0.4) **B1**

Answer: Only (i) is valid **4 marks**

11 (2 marks)

p = the proportion of bruised apples in the orchard **B1**

$H_0: p = 0.12$, $H_1: p \neq 0.12$ (two-tailed: the inspector has no direction in mind) **B1**

Answer: $H_0: p = 0.12$, $H_1: p \neq 0.12$ **2 marks**

12 (3 marks)

(a) One-tailed, since H_1 only considers values of μ above 35 (an increase) **B1 B1**

(b) $H_0: \mu = 35$, $H_1: \mu \neq 35$ **B1**

Answer: (a) one-tailed (upper tail); (b) $H_1: \mu \neq 35$ **3 marks**

13 (3 marks)

The test statistic is the quantity calculated from the sample that is used to decide whether to reject H_0 **B1**

Here it is X , the number of items in the sample with the characteristic **B1**

Under H_0 , $X \sim B(40, 0.35)$ **B1**

Answer: X = number with the characteristic, $X \sim B(40, 0.35)$ under H_0 **3 marks**

14 (5 marks)

(a) $H_0: p = 0.35$ **B1**

$H_1: p \neq 0.35$ **B1**

Answer: $H_0: p = 0.35$, $H_1: p \neq 0.35$ **2 marks**

(b) $X \sim B(40, 0.35)$ **B1**

Answer: $X \sim B(40, 0.35)$ **1 mark**

(c) $P(X \geq 18) = 1 - P(X \leq 17)$ **M1**

$= 0.124$ (calculator) **A1**

Answer: 0.124 **2 marks**

15 (6 marks)

(a) $H_0: \mu = 85$ **B1**

$H_1: \mu > 85$ **B1**

Answer: $H_0: \mu = 85$, $H_1: \mu > 85$ **2 marks**

(b) $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ with $\mu = 85$ and $\sigma^2/n = \frac{12^2}{25} = 5.76$ **M1**

$\bar{X} \sim N(85, 5.76)$, standard deviation 2.4 **A1**

Answer: $\bar{X} \sim N(85, 5.76)$ **2 marks**

(c) $z = \frac{90 - 85}{2.4} = 2.083$ **M1**

$P(Z \geq 2.083) = 0.0186$ **A1**

Answer: 0.0186 **2 marks**

16 (6 marks)

(a) Let p = proportion of trains on time. $H_0: p = 0.9$, $H_1: p < 0.9$

B1

One-tailed (lower tail)

B1

Answer: $H_0: p = 0.9$, $H_1: p < 0.9$; one-tailed

2 marks

(b) Let μ = mean mass of a loaf. $H_0: \mu = 800$, $H_1: \mu \neq 800$

B1

Two-tailed: error could be in either direction

B1

Answer: $H_0: \mu = 800$, $H_1: \mu \neq 800$; two-tailed

2 marks

(c) Let λ = mean number of emails per hour. $H_0: \lambda = 12$, $H_1: \lambda > 12$

B1

One-tailed (upper tail)

B1

Answer: $H_0: \lambda = 12$, $H_1: \lambda > 12$; one-tailed

2 marks

17 (4 marks)

(a) $0.032 < 0.05$ so reject H_0

M1

There is sufficient evidence at the 5% level that the proportion of customers using mobile banking is greater than 0.4

A1

(b) $0.032 > 0.01$ so do not reject H_0

M1

Insufficient evidence at the 1% level that the proportion of customers using mobile banking is greater than 0.4

A1

Answer: (a) reject H_0 ; (b) do not reject H_0

4 marks

18 (3 marks)

In a two-tailed test the 5% is split between the two tails, 2.5% in each

E1

$0.034 > 0.025$

M1

So do not reject H_0 : insufficient evidence that p differs from 0.5

A1

Answer: Compare with 0.025: do not reject H_0

3 marks

19 (2 marks)

Not rejecting H_0 only means there is insufficient evidence against it at this significance level

E1

It does not prove H_0 is true: a different or larger sample might give a different conclusion

E1

Answer: Failing to reject H_0 is not proof that H_0 is true

2 marks

20 (2 marks)

The sample is not random: it only includes students who arrive by bike, so every student sampled cycles and it is biased towards cyclists

E1

January weather is likely to be unrepresentative of the whole year

E1

Answer: Biased (not random) sample; unrepresentative time of year

2 marks

21 (7 marks)

(a) The hypotheses must be decided before looking at the data; choosing the direction from the sample result biases the test in favour of rejecting H_0

E1

Answer: Hypotheses must be chosen before the data are seen

1 mark

(b) No direction was suggested in advance, so $H_0: p = 0.5$, $H_1: p \neq 0.5$

B1

Answer: $H_0: p = 0.5$, $H_1: p \neq 0.5$

1 mark

(c) $X \sim B(20, 0.5)$, $P(X \geq 14) = 1 - P(X \leq 13)$
 $= 0.0577$

M1

A1

Answer: 0.0577

2 marks

(d) By symmetry $P(X \leq 6) = P(X \geq 14)$

M1

Total = $2 \times 0.0577 = 0.1153$

A1

Answer: 0.1153

2 marks

(e) His one-tailed probability 0.0577 is half the two-tailed value 0.1153, so he would overstate the evidence against a fair coin

E1

Answer: He would make the evidence look twice as strong

1 mark

22 (7 marks)

(a) A hypothesis test never proves anything; it measures how likely a result is if H_0 is true E1

A p-value of 0.032 means such a result would still happen about 3% of the time when $p = 0.4$, so the claim could be true E1

Answer: A significant result is evidence, not proof 2 marks

(b) The p-value is $P(\text{result at least this extreme} \mid H_0 \text{ is true})$, not $P(H_0 \text{ is true})$ E1

H_0 is a statement about a fixed parameter value; it is not a random event with a probability E1

Answer: p-value is $P(\text{data} \mid H_0)$, not $P(H_0)$ 2 marks

(c) $0.032 < 0.05$, so reject H_0 M1

There is sufficient evidence at the 5% level to suggest that the proportion of customers who use a loyalty card is less than 40% A1

Answer: Reject H_0 : evidence that proportion is below 40% 2 marks

(d) $0.032 > 0.01$, so do not reject H_0 : insufficient evidence at the 1% level that the proportion is below 40% B1

Answer: Do not reject H_0 1 mark

23 (7 marks)

(a) $P(X \geq 10) = 1 - P(X \leq 9) = 0.0480$ B1

Answer: 0.0480 1 mark

(b) $H_0: p = 0.3$, $H_1: p > 0.3$ B1

$0.0480 < 0.05$ M1

Reject H_0 : sufficient evidence that Amira's probability of winning is more than 0.3 A1

Answer: Reject H_0 3 marks

(c) $H_0: p = 0.3$, $H_1: p \neq 0.3$ B1

Two-tailed: compare 0.0480 with 0.025: $0.0480 > 0.025$ M1

Conclusion: do not reject H_0 , insufficient evidence at the 5% level that her win rate differs from 0.3 A1

Answer: Not significant in the two-tailed test 3 marks

24 (6 marks)

Under H_0 , $X \sim B(25, 0.2)$ and $P(X \geq 9) = 1 - P(X \leq 8) = 0.0468$ M1 A1

A (one-tailed): $0.0468 < 0.05$, so reject H_0 : evidence that the fault rate is greater than 0.2 M1 A1

B (two-tailed): compare with $0.05/2 = 0.025$: $0.0468 > 0.025$, so do not reject H_0 M1

Researcher A's hypotheses suit a claimed increase, as the direction is stated before the data are seen E1

Answer: A rejects H_0 ; B does not; A's hypotheses suit an increase 6 marks

25 (7 marks)

(a) p = probability that a patient taking the tablet suffers drowsiness B1

$H_0: p = 0.15$, $H_1: p > 0.15$ B1

Answer: $H_0: p = 0.15$, $H_1: p > 0.15$ 2 marks

(b) $X \sim B(30, 0.15)$ B1

Answer: $X \sim B(30, 0.15)$ 1 mark

(c) $P(X \geq 8) = 1 - P(X \leq 7) = 0.0698$ M1 A1

$0.0698 > 0.05$ so do not reject H_0 : insufficient evidence that more than 15% of patients suffer drowsiness A1

Answer: Do not reject H_0 3 marks

(d) The alternative hypothesis must be set before the data are collected, otherwise the test is biased towards rejecting H_0 E1

Answer: H_1 must be fixed before the data are seen 1 mark

26 (5 marks)

Under H_0 , $X \sim B(24, \frac{1}{6})$. Find the smallest c with $P(X \geq c) \leq 0.10$

M1

$$P(X \geq 6) = 0.1995 > 0.10$$

A1

$$P(X \geq 7) = 0.0912 \leq 0.10$$

A1

So reject H_0 if $X \geq 7$

A1

$$\text{Probability of wrongly rejecting } H_0 = P(X \geq 7 \mid p = 1/6) = 0.0912$$

B1

Answer: $X \geq 7$, probability 0.0912

5 marks

27 (9 marks)

(a) $P(X \leq 3) = 0.1719$ when $X \sim B(10, 0.5)$

M1

$0.1719 > 0.05$, so do not reject H_0 : no significant evidence that the treatment is worse

A1

Answer: $P(X \leq 3) = 0.1719 > 0.05$, do not reject H_0

2 marks

(b) H_0 is rejected when $P(X \leq 3) \leq 0.05$ where $X \sim B(10, p_0)$

M1

$$P(X \leq 3) = (1 - p)^{10} + 10p(1 - p)^9 + 45p^2(1 - p)^8 + 120p^3(1 - p)^7$$

M1

$P(X \leq 3)$ decreases as p increases, so solve $P(X \leq 3) = 0.05$

M1

Using a calculator's equation solver, $p = 0.607$, so H_0 is rejected for $p_0 \geq 0.607$

A1

Answer: $p_0 = 0.607$

4 marks

(c) $P(X \leq 3) = 0.6496$ when $X \sim B(10, 0.3)$

M1

This is quite likely, so the data are also consistent with $p = 0.3$ (and with every p up to 0.607)

A1

Not rejecting H_0 only means the data do not contradict it; many other values of p fit equally well, so H_0 is not proved

E1

Answer: Data are consistent with many values of p , so H_0 is not shown to be true

3 marks

28 (10 marks)

(a) Each test wrongly rejects with probability 0.05, independently, so $Y \sim B(20, 0.05)$

B1

$$P(Y \geq 1) = 1 - 0.95^{20} = 0.642$$

M1

Answer: 0.642

2 marks

(b) $E(Y) = 20 \times 0.05 = 1$

B1

Answer: 1

1 mark

(c) Even when nothing is going on, she is more likely than not (0.642) to get at least one "significant" result

E1

On average one of her 20 tests would give a false positive

E1

Improvement: use a smaller significance level for each test, or repeat/confirm the result with new data

E1

Answer: Many tests make false positives likely; use a smaller level for each test

3 marks

(d) $P(\text{no wrong rejection}) = (1 - \alpha)^{20}$

M1

$$1 - (1 - \alpha)^{20} = 0.05, \text{ so } (1 - \alpha)^{20} = 0.95$$

M1

$$1 - \alpha = 0.95^{1/20} = 0.99744$$

M1

$$\alpha = 0.00256 \text{ (about 0.26\%)}$$

A1

Answer: $\alpha = 0.00256$

4 marks

29 (8 marks)

(a) Each test is independent with $P(\text{not significant}) = 0.95$, so $P(\text{none significant}) = 0.95^k$

M1

$$P(\text{at least one}) = 1 - 0.95^k$$

A1

(b) $1 - 0.95^k > 0.5$ gives $0.95^k < 0.5$

M1

$$k > \frac{\ln 0.5}{\ln 0.95} = 13.5, \text{ so } k = 14$$

M1 A1

(c) $(1 - \alpha)^8 = 0.95$

M1

$$\alpha = 1 - 0.95^{1/8} = 0.00639$$

M1 A1

Answer: (b) $k = 14$ (c) $\alpha = 0.00639$

8 marks

30 (8 marks)

- (a) When $p = 0.5$ the distribution of X is symmetrical about $\frac{n}{2}$, so $P(X \geq n - x) = P(X \leq x)$ M1
- The probability of a result at least as extreme in either tail is therefore $2P(X \leq x)$ E1
- (b) $2P(X \leq 10) = 2 \times 0.0494 = 0.0987$ M1 A1
- $0.0987 > 0.05$ so do not reject H_0 A1
- (c) Need $P(X \leq x) \leq 0.025$: $P(X \leq 9) = 0.0214$, $P(X \leq 10) = 0.0494$ M1
- Largest $x = 9$, so $x = 10$ from part (b) is not in the rejection region A1 A1
- Answer: (b) 0.0987, do not reject H_0 (c) $x = 9$** 8 marks

Significance Levels and Critical Regions**1 (2 marks)**

- Need the largest c with $P(X \leq c) \leq 0.05$; $0.0282 < 0.05$ but $0.1493 > 0.05$ M1
- Critical region is $X = 0$ (that is, $X \leq 0$) A1
- Answer: $X = 0$** 2 marks

2 (3 marks)

- Lower tail test: need the largest c with $P(X \leq c) \leq 0.10$ M1
- $P(X \leq 3) = 0.0905 \leq 0.10$ but $P(X \leq 4) = 0.2173 > 0.10$ M1
- Critical region $X \leq 3$; actual significance level 0.0905 (9.05%) A1
- Answer: $X \leq 3$; actual level 0.0905** 3 marks

3 (2 marks)

- (a) The set of values of the test statistic for which H_0 is rejected B1
- (b) The probability of rejecting H_0 when H_0 is true (probability of incorrectly rejecting H_0) B1
- Answer: (a) values leading to rejection of H_0 ; (b) $P(\text{reject } H_0 \mid H_0 \text{ true})$** 2 marks

4 (4 marks)

- Lower tail: need $P(X \leq c) \leq 0.05$. $P(X \leq 1) = 0.0107 \leq 0.05$ but $P(X \leq 2) = 0.0547 > 0.05$ M1 A1
- Critical region: $X \leq 1$ A1
- Actual significance level = 0.0107 B1
- Answer: $X \leq 1$, actual significance level 0.0107** 4 marks

5 (2 marks)

- (a) $P(Z > z) = 0.01$ gives $z = 2.326$ B1
- (b) $P(Z > z) = 0.025$ gives $z = \pm 1.960$ B1
- Answer: (a) 2.326 (b) ± 1.960** 2 marks

6 (2 marks)

- (a) 9 is not in the critical region, so H_0 is not rejected B1
- (b) 12 is in the critical region, so H_0 is rejected B1
- Answer: (a) not rejected (b) rejected** 2 marks

7 (4 marks)

- Each tail has probability at most 0.025 B1
- Lower tail: $P(X \leq 3) = 0.0160 \leq 0.025$ but $P(X \leq 4) = 0.0510 > 0.025$ M1 A1
- Upper tail: $P(X \geq 13) = 0.0210 \leq 0.025$ but $P(X \geq 12) = 0.0565 > 0.025$, so critical region is $X \leq 3$ or $X \geq 13$ A1
- Answer: $X \leq 3$ or $X \geq 13$** 4 marks

8 (4 marks)

- (a) $P(X \leq 1) = 0.0070$ M1
- $P(X \geq 12) = 0.0107$ M1
- Actual significance level = $0.0070 + 0.0107 = 0.0178$ A1
- Answer: 0.0178** 3 marks

(b) X is discrete, so the tail probabilities jump in steps and the largest critical regions with probability ≤ 0.025 in each tail have a total below 0.05	E1
Answer: Discrete distribution: cannot reach exactly 2.5% in each tail	1 mark
9 (4 marks)	
Upper tail: need $P(X \geq c) \leq 0.05$. $P(X \geq 8) = 0.0321 \leq 0.05$ but $P(X \geq 7) = 0.0867 > 0.05$	M1 A1
Critical region: $X \geq 8$	A1
Actual significance level = 0.0321	B1
Answer: $X \geq 8$, actual significance level 0.0321	4 marks
10 (6 marks)	
Lower tail: $P(X \leq 1) = 0.0121$ and $P(X \leq 2) = 0.0486$. Since 0.0121 is closer to 0.025 than 0.0486, use $X \leq 1$	M1 A1
Upper tail: $P(X \geq 10) = 0.0672$, $P(X \geq 11) = 0.0299$ and $P(X \geq 12) = 0.0120$. The closest to 0.025 is 0.0299, so use $X \geq 11$	M1 A1
Critical region: $X \leq 1$ or $X \geq 11$	A1
Actual significance level = $0.0121 + 0.0299 = 0.0420$	B1
Answer: $X \leq 1$ or $X \geq 11$; actual significance level 0.0420	6 marks
11 (4 marks)	
Upper tail: need $P(X \geq c) \leq 0.05$. $P(X \geq 7) = 0.0335 \leq 0.05$ but $P(X \geq 6) = 0.0839 > 0.05$	M1 A1
Critical region: $X \geq 7$	A1
Actual significance level = 0.0335	B1
Answer: $X \geq 7$, actual significance level 0.0335	4 marks
12 (4 marks)	
Under H_0 , $\bar{X} \sim N(500, \frac{8^2}{16})$, standard deviation 2.00	B1
Lower critical value: $z = -1.645$, so $\bar{x} = 500 - 1.645 \times 2.00 = 497$	M1 A1
Critical region: $\bar{X} \leq 497$	A1
Answer: $\bar{X} \leq 497$	4 marks
13 (4 marks)	
Significance level = $P(X \geq 9 p = 0.25) = 1 - P(X \leq 8)$	M1
= 0.0409	A1
$P(X \geq 9 p = 0.4) = 1 - P(X \leq 8)$	M1
= 0.4044	A1
Answer: 0.0409 and 0.4044	4 marks
14 (5 marks)	
(a) $\bar{X} \sim N(50, \frac{6^2}{36})$	M1
$\bar{X} \sim N(50, 1)$	A1
Answer: $\bar{X} \sim N(50, 1)$	2 marks
(b) Two-tailed: each tail 0.025, so $z = \pm 1.96$	B1
$a = 50 - 1.96 \times 1 = 48.04$, $b = 50 + 1.96 \times 1 = 51.96$	M1 A1
Answer: $\bar{X} < 48.04$ or $\bar{X} > 51.96$	3 marks
15 (6 marks)	
(a) $H_0: p = 0.2$, $H_1: p > 0.2$	B1
$X \sim B(25, 0.2)$	B1
Answer: $H_0: p = 0.2$, $H_1: p > 0.2$; $X \sim B(25, 0.2)$	2 marks
(b) Upper tail: need smallest c with $P(X \geq c) \leq 0.05$	M1
$P(X \geq 9) = 0.0468 \leq 0.05$ but $P(X \geq 8) = 0.1091 > 0.05$	M1 A1
Answer: $X \geq 9$	3 marks
(c) Actual level = 0.0468: the probability of concluding that the proportion of red-flowering seeds exceeds 0.2 when it is in fact 0.2	B1

Answer: 0.0468**1 mark****16 (6 marks)**Lower tail: need $P(X \leq c) \leq 0.05$. $P(X \leq 6) = 0.0238 \leq 0.05$ but $P(X \leq 7) = 0.0553 > 0.05$ **M1 A1**Upper tail: need $P(X \geq c) \leq 0.05$. $P(X \geq 18) = 0.0320 \leq 0.05$ but $P(X \geq 17) = 0.0633 > 0.05$ **M1 A1**Critical region: $X \leq 6$ or $X \geq 18$ **A1**Actual significance level = $0.0238 + 0.0320 = 0.0557$ **B1****Answer: $X \leq 6$; $X \geq 18$, actual significance level 0.0557****6 marks****17 (6 marks)**Lower tail: $P(X \leq 2) = 0.0138$, $P(X \leq 3) = 0.0424$ and $P(X \leq 4) = 0.0996$. The closest to 0.03 is 0.0424, so use $X \leq 3$ **M1 A1**Upper tail: $P(X \geq 13) = 0.0638$, $P(X \geq 14) = 0.0342$ and $P(X \geq 15) = 0.0173$. The closest to 0.03 is 0.0342, so use $X \geq 14$ **M1 A1**Critical region: $X \leq 3$ or $X \geq 14$ **A1**Probability of a Type I error = $0.0424 + 0.0342 = 0.0766$ **B1****Answer: $X \leq 3$ or $X \geq 14$; Type I error probability 0.0766****6 marks****18 (6 marks)**(a) The critical region is symmetrical about μ_0 , so $\mu_0 = \frac{55.73 + 64.27}{2} = 60$ **B1****Answer: $\mu_0 = 60$** **1 mark**(b) Under H_0 , $\bar{X} \sim N(60, \frac{9^2}{12})$, standard deviation 2.60 **B1**

$$z = \frac{64.27 - 60}{2.60} = 1.644$$
 M1

 $P(\bar{X} > 64.27) = 0.0501$, so the significance level is $2 \times 0.0501 = 0.1003$ (about 10%) **A1****Answer: about 10% (0.1003)****3 marks**(c) The population is stated to be normally distributed, so $\bar{X}$ is exactly normal for any sample size. The central limit theorem was not needed; it is only required when the population is not normal and n is large **E1****Answer: Not needed: the population is normal****2 marks****19 (5 marks)**(a) Under H_0 , $X \sim B(25, 0.35)$; need $P(X \leq c) \leq 0.05$. $P(X \leq 4) = 0.0320$, $P(X \leq 5) = 0.0826$ **M1 A1**Critical region $X \leq 4$, actual significance level 0.0320 **A1****Answer: $X \leq 4$, 0.0320****3 marks**(b) 5 is not in the critical region **M1**Do not reject H_0 : insufficient evidence at the 5% level that the new batch is less effective **A1****Answer: do not reject H_0** **2 marks****20 (5 marks)**(a) Type I error: reject H_0 when true. $P(X \geq 7 | p = 0.3) = 1 - P(X \leq 6) = 0.0386$ **M1**(b) Type II error: fail to reject H_0 when it is false **B1**

$$P(X \leq 6 | p = 0.6) = 0.3348$$
 M1

Answer: (a) 0.0386 (b) 0.3348**5 marks****21 (8 marks)**(a) n is large and p is small **E1**

$$X \approx \text{Po}(250 \times 0.02) = \text{Po}(5)$$
 B1

Answer: $X \approx \text{Po}(5)$ **2 marks**(b) Lower tail: $P(X \leq 0) = 0.0067 \leq 0.025$ and $P(X \leq 1) = 0.0404 > 0.025$ **M1 A1**Upper tail: $P(X \geq 11) = 0.0137 \leq 0.025$ and $P(X \geq 10) = 0.0318 > 0.025$, so the critical region is $X = 0$ or $X \geq 11$ **M1 A1****Answer: $X = 0$ or $X \geq 11$** **4 marks**

(c) Poisson: $0.0067 + 0.0137 = 0.0204$

M1

Binomial $B(250, 0.02)$: $0.0064 + 0.0128 = 0.0192$, very close to the approximation

A1

Answer: 0.0204 (Poisson), 0.0192 (binomial)

2 marks

22 (8 marks)

(a) Under H_0 , $\bar{X} \sim N(50, \frac{8^2}{n})$; critical value for $\bar{X}$ is $50 + 1.645 \times \frac{8}{\sqrt{n}}$

M1 B1

Need $52 \geq 50 + 1.645 \times \frac{8}{\sqrt{n}}$, so $\sqrt{n} \geq \frac{1.645 \times 8}{2} = 6.579$

M1

$n \geq 43.29$, so smallest $n = 44$

A1

Answer: $n = 44$

4 marks

(b) Critical value for the upper tail uses $z = 1.96$

B1

Need $2 \geq 1.96 \times \frac{8}{\sqrt{n}}$, so $\sqrt{n} \geq \frac{1.96 \times 8}{2} = 7.84$

M1 A1

$n \geq 61.46$, so smallest $n = 62$

A1

Answer: $n = 62$

4 marks

23 (8 marks)

(a) Under H_0 , $X \approx N(36, 25.2)$, standard deviation 5.02

B1 M1

Need $P(X \geq c) \leq 0.05$ with continuity correction: $P(X > c - 0.5) \leq 0.05$

M1

$\frac{c - 0.5 - 36}{5.02} \geq 1.645$ so $c \geq 44.8$, i.e. $c = 45$

M1 A1

(b) $P(X \geq 45) = 0.0473 \leq 0.05$ and $P(X \geq 44) = 0.0694 > 0.05$

M1 A1

Critical region $X \geq 45$, actual significance level 0.0473

A1

Answer: (a) $X \geq 45$ (b) $X \geq 45$, 0.0473

8 marks

24 (6 marks)

(a) Under H_0 , $\bar{X} \sim N(100, \frac{225}{n})$; upper 2.5% point $z = 1.96$

M1

$b = 100 + 1.96 \times \frac{15}{\sqrt{n}}$ (shown), and by symmetry $a = 100 - \frac{1.96 \times 15}{\sqrt{n}}$

A1 B1

(b) $1.96 \times \frac{15}{\sqrt{n}} \leq 5$

M1

$\sqrt{n} \geq \frac{1.96 \times 15}{5} = 5.88$ so $n \geq 34.6$, $n = 35$

M1 A1

Answer: $n = 35$

6 marks

25 (9 marks)

(a) Lower tail: need $P(X \leq c) \leq 0.025$. $P(X \leq 1) = 0.0174 \leq 0.025$ but $P(X \leq 2) = 0.0620 > 0.025$

M1 A1

Upper tail: need $P(X \geq c) \leq 0.025$. $P(X \geq 12) = 0.0201 \leq 0.025$ but $P(X \geq 11) = 0.0426 > 0.025$

M1 A1

Critical region $X \leq 1$ or $X \geq 12$

A1

Actual significance level = $0.0174 + 0.0201 = 0.0374$

B1

Answer: $X \leq 1$ or $X \geq 12$, 0.0374

6 marks

(b) $P(\text{reject}) = P(X \leq 1) + P(X \geq 12)$ with $X \sim \text{Po}(9)$

M1 M1

= $0.0012 + 0.1970 = 0.1982$

A1

Answer: 0.1982

3 marks

26 (5 marks)

Need $P(X \geq c) \leq 0.01$ under $B(18, 0.5)$

M1

$P(X \geq 14) = 0.0154 > 0.01$

A1

$P(X \geq 15) = 0.0038 \leq 0.01$, so $c = 15$

A1 B1

X is discrete, so the probabilities jump and the largest allowed region may have probability strictly below 1%

E1

Answer: $c = 15$, 0.0038

5 marks

27 (10 marks)

(a) Each tail has probability at most 0.025

B1

$$P(X \leq 5) = 0.0207 \leq 0.025 \text{ but } P(X \leq 6) = 0.0577 > 0.025$$

M1

By symmetry the critical region is $X \leq 5$ or $X \geq 15$

A1

Answer: $X \leq 5$ or $X \geq 15$

3 marks

(b) $2 \times 0.0207 = 0.0414$

B1

Answer: 0.0414

1 mark

(c) $P(X \leq 5) + P(X \geq 15)$ with $X \sim B(20, 0.3)$

M1

$$= 0.4164 + 0.0000 = 0.4164$$

A1

Answer: 0.4164

2 marks

(d) Power = $P(X \leq 5) + P(X \geq 15)$ with $X \sim B(20, 0.8) = 0.8042$

B1

$p = 0.8$ is further from 0.5 than $p = 0.3$, so X is much more likely to fall in the critical region

E1

Answer: 0.8042

2 marks

(e) At $p = 0.5$ the power equals the significance level, 0.0414

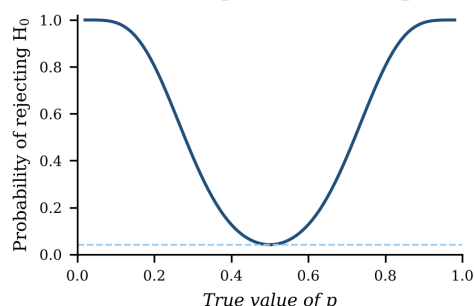
B1

The power rises towards 1 as p moves further from 0.5 in either direction (see the graph)

E1

Answer: Power equals 0.0414 at $p = 0.5$ and increases to 1 as p moves away

2 marks

**28 (10 marks)**

(a) $P(X \geq 10) = 0.0338$

B1

$$P(X \geq 9) = 0.0950$$

B1

$X \geq 10$ is allowed when $P(X \geq 10) \leq \alpha$, that is $\alpha \geq 0.0338$

M1

$X \geq 9$ is not allowed when $P(X \geq 9) > \alpha$, that is $\alpha < 0.0950$; so the critical region is exactly $X \geq 10$ on this range

A1

Answer: shown

4 marks

(b) Type II: $X \leq 9$ when $X \sim B(15, 0.6)$

M1

$$P(X \leq 9) = 0.5968$$

A1

Answer: 0.5968

2 marks

(c) 0.10 lies between $P(X \geq 9) = 0.0950$ and $P(X \geq 8) = 0.2131$, so critical region is $X \geq 9$

M1

Type II: $P(X \leq 8) = 0.3902$ when $p = 0.6$

A1

Answer: $X \geq 9$; 0.3902

2 marks

(d) Larger α enlarges the critical region, so the Type II error probability falls (0.5968 to 0.3902)

E1

Reducing the chance of a Type I error increases the chance of a Type II error, so there is a trade-off

E1

Answer: Raising α lowers the Type II error probability: a trade-off

2 marks

29 (10 marks)

(a) $P(X \geq 10) = 0.0480 \leq 0.05$, $P(X \geq 9) = 0.1133 > 0.05$

M1 A1

Critical region $X \geq 10$

A1

(b) $P(X \geq 10)$ with $p = 0.4$: 0.2447

M1 A1

$p = 0.5$: 0.5881; $p = 0.6$: 0.8725

A1

Type II error = $1 - P(\text{reject})$: 0.7553, 0.4119, 0.1275

M1 A1

(c) The probability of a Type II error falls as the true p moves further from 0.3

E1

Increase the sample size (or raise the significance level)

E1

Answer: (a) $X \geq 10$ (b) Type II: 0.7553, 0.4119, 0.1275

10 marks

30 (11 marks)

(a) Under H_0 , $X \sim B(20, 0.5)$: $P(X \geq 15) = 0.0207 \leq 0.05$ and $P(X \geq 14) = 0.0577 > 0.05$ M1 A1

Critical region $X \geq 15$ A1

$P(X \geq 15 | p = 0.65) = 0.2454$ M1 A1

$0.2454 < 0.8$ so $n = 20$ is not adequate E1

(b) Search $n = 21, 22, \dots$ using the calculator: for each n find the least c with $P(X \geq c) \leq 0.05$ M1 M1

under $p = 0.5$, then $P(X \geq c | 0.65)$

$n = 69$: $c = 42$, $P(X \geq 42 | p = 0.65) = 0.8021 \geq 0.8$ A1 A1

$n = 69$ is the least value that works (all smaller n fail) B1

Answer: (a) $X \geq 15$, 0.2454, not adequate (b) $n = 69$, $X \geq 42$, 0.8021 11 marks

One-Tailed and Two-Tailed Tests

1 (3 marks)

(a) Two-tailed (a change in either direction) B1

Answer: Two-tailed 1 mark

(b) One-tailed, upper tail B1

Answer: One-tailed, upper tail 1 mark

(c) One-tailed, lower tail B1

Answer: One-tailed, lower tail 1 mark

2 (2 marks)

(a) $P(Z > z) = 0.05$, so $z = 1.645$ B1

Answer: 1.645 1 mark

(b) Each tail has 0.025, so $z = \pm 1.960$ B1

Answer: ± 1.960 1 mark

3 (2 marks)

(a) one-tailed (b) two-tailed (c) two-tailed (d) one-tailed B2

Answer: (a) one (b) two (c) two (d) one 2 marks

4 (2 marks)

(a) $P(Z > z) = 0.10$ gives $z = 1.282$ B1

(b) $P(Z > z) = 0.05$ gives $z = \pm 1.645$ B1

Answer: (a) 1.282 (b) ± 1.645 2 marks

5 (2 marks)

Compare with 0.025: $0.0282 > 0.025$ M1

Do not reject H_0 A1

Answer: do not reject H_0 2 marks

6 (2 marks)

$0.0577 > 0.05$, so do not reject H_0 M1

Insufficient evidence at the 5% level that the probability that a baby is a girl is greater than 0.5 A1

Answer: do not reject H_0 2 marks

7 (4 marks)

$H_0: p = 0.5$, $H_1: p \neq 0.5$, where p is the probability of heads (two-tailed) B1

Under H_0 , $X \sim B(10, 0.5)$; $P(X \geq 9) = 0.0107$ M1

Two-tailed test at 5%: compare with 0.025; $0.0107 < 0.025$ M1

Reject H_0 : there is evidence at the 5% level that the coin is biased A1

Answer: Reject H_0 : the coin is biased 4 marks

8 (4 marks)

(a) $P(X \geq 10) = 0.0480$ when $X \sim B(20, 0.3)$ M1

$0.0480 < 0.05$, so reject H_0 : evidence that blue is more likely than 0.3 A1

Answer: Reject H_0 2 marks

(b) Two-tailed: compare $P(X \geq 10) = 0.0480$ with 0.025; $0.0480 > 0.025$, so do not reject H_0 M1

The same data are significant for a one-tailed test but not for a two-tailed test, because the 5% is shared between the two tails E1

Answer: Do not reject H_0 2 marks

9 (5 marks)

Let p = the probability that she holes a putt. $H_0: p = 0.4$, $H_1: p < 0.4$ B1

Under H_0 , $X \sim B(15, 0.4)$ where X is the number of successes in the sample M1

$P(X \leq 3) = 0.0905$ (calculator) A1

Compare with 5%: $0.0905 > 5\%$ M1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that her probability of holing a putt has decreased. A1

Answer: $P(X \leq 3) = 0.0905$, do not reject H_0 5 marks

10 (5 marks)

Let p = the probability that a customer buys a lottery ticket. $H_0: p = 0.1$, $H_1: p > 0.1$ B1

Under H_0 , $X \sim B(20, 0.1)$ where X is the number of successes in the sample M1

$P(X \geq 5) = 0.0432$ (calculator) A1

Compare with 5%: $0.0432 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the proportion of customers who buy a lottery ticket is greater than 0.1. A1

Answer: $P(X \geq 5) = 0.0432$, reject H_0 5 marks

11 (5 marks)

Let λ = the mean number of failures per week. $H_0: \lambda = 2$, $H_1: \lambda > 2$ B1

Under H_0 , $X \sim \text{Po}(2)$ M1

$P(X \geq 6) = 0.0166$ (calculator) A1

Compare with 5%: $0.0166 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the failure rate has increased. A1

Answer: $P(X \geq 6) = 0.0166$, reject H_0 5 marks

12 (4 marks)

Let ρ = the population product moment correlation coefficient between floor area and selling price. $H_0: \rho = 0$, $H_1: \rho > 0$ B1

Critical value 0.4973 for $n = 12$ M1

$r = 0.610$, and $r > 0.4973$ A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that there is positive correlation between floor area and selling price. E1

Answer: critical value 0.4973, reject H_0 4 marks

13 (5 marks)

Let μ = the mean mass of a pack (g). $H_0: \mu = 250$, $H_1: \mu < 250$ B1

Under H_0 : $\bar{X} \sim N(250, \frac{4^2}{16})$ so the standard deviation of $\bar{X}$ is $\frac{4}{\sqrt{16}} = 1.00$ M1

$$z = \frac{247.8 - 250}{1.00} = -2.200$$
M1

$P(Z \leq -2.200) = 0.0139$, compared with 5% A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean mass of a pack is less than 250 g. A1

Answer: $z = -2.200$, p-value 0.0139, reject H_0 5 marks

14 (5 marks)**(a)** $H_0: p = 0.3, H_1: p > 0.3$ **B1**Upper tail test, so need $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.7^{20}$
= 0.9992**M1****A1****Answer: 0.9992****3 marks****(b)** The student used the lower tail, but H_1 is $p > 0.3$, so a result in the lower tail gives no support for H_1 **E1**0.9992 > 0.05, so do not reject H_0 : no evidence that the proportion is higher than 0.3**E1****Answer: Do not reject H_0** **2 marks****15 (6 marks)****(a)** $H_0: \mu = 250, H_1: \mu < 250$. Under $H_0, \bar{X} \sim N(250, \frac{4^2}{25})$, standard deviation 0.8**B1**

$$z = \frac{248.6 - 250}{0.8} = -1.75$$

M1-1.75 < -1.645, so reject H_0 : evidence that the mean mass is below 250 g**A1****Answer: Reject H_0** **3 marks****(b)** $H_1: \mu \neq 250$; the critical values are ± 1.96 **M1**-1.75 > -1.96, so do not reject H_0 : no evidence at 5% that the mean differs from 250 g**A1****Answer: Do not reject H_0** **2 marks****(c)** In the two-tailed test only 2.5% is in each tail, so the critical value (1.96) is further out than for the one-tailed test (1.645)**E1****Answer: Critical value is further out in a two-tailed test****1 mark****16 (5 marks)**Let p = the probability a call is resolved at the first attempt. $H_0: p = 0.6, H_1: p \neq 0.6$ **B1**Under $H_0, X \sim B(25, 0.6)$ where X is the number of successes in the sample**M1** $P(X \leq 10) = 0.0344$ (calculator)**A1**Two-tailed test, so compare with $5\%/2 = 2.5\%$: $0.0344 > 2.5\%$ **M1**Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that the proportion of calls resolved at the first attempt has changed from 0.6.**A1****Answer: $P(X \leq 10) = 0.0344$, do not reject H_0** **5 marks****17 (5 marks)**Let λ = the mean number of potholes per kilometre. $H_0: \lambda = 3.5, H_1: \lambda < 3.5$ **B1**Under H_0 , the mean for 2 km is $3.5 \times 2 = 7$, so $X \sim \text{Po}(7)$ **M1** $P(X \leq 2) = 0.0296$ (calculator)**A1**Compare with 5%: $0.0296 < 5\%$ **M1**Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean number of potholes per kilometre has decreased.**A1****Answer: $P(X \leq 2) = 0.0296$, reject H_0** **5 marks****18 (5 marks)**Let μ = the mean journey time (minutes). $H_0: \mu = 35, H_1: \mu \neq 35$ **B1**Under $H_0: \bar{X} \sim N(35, \frac{6^2}{20})$ so the standard deviation of $\bar{X}$ is $\frac{6}{\sqrt{20}} = 1.34$ **M1**

$$z = \frac{37.9 - 35}{1.34} = 2.162$$

M1 $P(Z \geq 2.162) = 0.0153$, compared with 2.5% (two-tailed, so half of the level)**A1**Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean journey time has changed.**A1****Answer: $z = 2.162$, p-value 0.0153, reject H_0** **5 marks**

19 (4 marks)

Let ρ = the population product moment correlation coefficient between height and arm span. $H_0: \rho = 0$, $H_1: \rho \neq 0$ B1

Critical value ± 0.5140 for $n = 15$ M1

$r = 0.510$, and $|r|$ is not beyond 0.5140 A1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that there is correlation between height and arm span. E1

Answer: critical value 0.5140, do not reject H_0 4 marks

20 (7 marks)

(a) Let p = proportion who cycle. $H_0: p = 0.4$, $H_1: p < 0.4$ B1

$X \sim B(20, 0.4)$: $P(X \leq 3) = 0.0160$ M1 A1

$0.0160 < 0.05$ so reject H_0 A1

(b) $H_1: p \neq 0.4$; compare 0.0160 with 0.025 M1

$0.0160 < 0.025$, so reject H_0 A1

The two-tailed test needs more extreme evidence (the tail probability is compared with 0.025 , not 0.05), so a result that is just significant one-tailed may fail it; here the evidence is strong enough for both E1

Answer: (a) reject H_0 ; (b) reject H_0 7 marks

21 (7 marks)

(a) $H_0: p = 0.22$, $H_1: p \neq 0.22$ B1

$X \sim B(40, 0.22)$ B1

Answer: $X \sim B(40, 0.22)$ 2 marks

(b) Two-tailed at 10%: 5% in each tail B1

Lower: $P(X \leq 4) = 0.0422 \leq 0.05$, $P(X \leq 5) = 0.0989 > 0.05$ M1

Upper: $P(X \geq 14) = 0.0415 \leq 0.05$, $P(X \geq 13) = 0.0830 > 0.05$ M1

Critical region $X \leq 4$ or $X \geq 14$; actual level 0.0837 A1

Answer: $X \leq 4$ or $X \geq 14$; 0.0837 4 marks

(c) 4 is in the critical region ($4 \leq 4$), so reject H_0 : evidence at the 10% level that the proportion experiencing the side effect has changed A1

Answer: Reject H_0 1 mark

22 (8 marks)

(a) The test is two-tailed, so the significance level must be shared: each tail is 2.5%, not 5% E1

He should compare 0.0481 with 0.025 , and $0.0481 > 0.025$, so H_0 should not have been rejected E1

Answer: He compared with 0.05 instead of 0.025 2 marks

(b) Lower: $P(X \leq 6) = 0.0172 \leq 0.025$, $P(X \leq 7) = 0.0435 > 0.025$ M1

Upper: $P(X \geq 18) = 0.0212 \leq 0.025$, $P(X \geq 17) = 0.0481 > 0.025$ M1

Critical region $X \leq 6$ or $X \geq 18$ A1

17 is not in the critical region, so do not reject H_0 : no evidence at the 5% level that the proportion of purchasers is not 40% A1

Answer: $X \leq 6$ or $X \geq 18$; do not reject H_0 4 marks

(c) $P(X \leq 6) + P(X \geq 18)$ M1

$= 0.0172 + 0.0212 = 0.0384$ A1

Answer: 0.0384 2 marks

23 (8 marks)

(a) $P(X \leq 10) = 0.0494 \leq 0.05$, $P(X \leq 11) = 0.1002 > 0.05$ M1 A1

Reject for $X \leq 10$ A1

(b) Each tail 0.025: $P(X \leq 9) = 0.0214 \leq 0.025$, $P(X \leq 10) = 0.0494 > 0.025$ M1 A1

By symmetry reject for $X \leq 9$ or $X \geq 21$ A1 A1

(c) The same data are being compared with a more demanding (2.5%) tail in the two-tailed test, since the two-tailed test also guards against the opposite direction E1

Answer: (a) $X \leq 10$ (b) $X \leq 9$ or $X \geq 21$ 8 marks

24 (7 marks)

Let p = proportion supporting. $H_0: p = 0.4$, $H_1: p > 0.4$

B1

Under H_0 , $X \sim B(200, 0.4) \approx N(80, 48)$, standard deviation 6.93

M1 A1

$$P(X \geq 92) \approx P(Z \geq \frac{91.5 - 80}{6.93})$$

M1

$$= P(Z \geq 1.660) = 0.0485$$

A1

$0.0485 < 0.05$ so reject H_0

M1

Sufficient evidence at the 5% level that support for the tram line is higher than 40%

A1

Answer: p-value 0.0485, reject H_0

7 marks

25 (7 marks)

(a) $H_0: \rho = 0$, $H_1: \rho > 0$. $0.41 > 0.4000$, so reject H_0

B1 M1 A1

There is sufficient evidence at the 5% level of positive correlation between age and reaction time

E1

(b) $H_1: \rho \neq 0$. $0.41 < 0.4683$, so do not reject H_0

M1

The two-tailed critical value is larger because the 5% is split between two tails

E1

(c) One-tailed: the student expected a positive correlation before collecting the data

E1

Answer: (a) reject (b) do not reject

7 marks

26 (8 marks)

(a) Let λ = mean number of errors per page: $H_0: \lambda = 0.6$, $H_1: \lambda < 0.6$

B1

Under H_0 , $X \sim \text{Po}(3)$

B1

Answer: $H_0: \lambda = 0.6$, $H_1: \lambda < 0.6$; $X \sim \text{Po}(3)$

2 marks

(b) $P(X = 0) = e^{-3} = 0.0498$

M1 A1

$0.0498 < 0.05$, so reject H_0 : sufficient evidence that the error rate has fallen

A1

Answer: 0.0498: reject

3 marks

(c) $P(\text{no errors in } k \text{ pages}) = e^{-0.6k} < 0.05$

M1

$$0.6k > \ln 20 = 3.00 \text{ so } k > 4.99$$

M1

$$k = 5$$

A1

Answer: $k = 5$

3 marks

27 (9 marks)

(a) $P(X = r) = \frac{n!}{r!(n-r)!} (0.5)^n = P(X = n - r)$

M1

So the distribution is symmetrical about $n/2$

A1

$$P(X \geq n - c) = P(X = n - c) + \dots + P(X = n) = P(X = c) + \dots + P(X = 0) = P(X \leq c)$$

M1

Answer: shown

3 marks

(b) The two tails have equal probability, so the cut-offs are symmetrical: c on the left corresponds to $n - c$ on the right

E1

Each tail must have probability at most half the significance level, and the same value of c satisfies this in both tails

E1

Answer: Symmetry gives equal tails

2 marks

(c) $P(X \leq 6) = 0.0113 \leq 0.025$ but $P(X \leq 7) = 0.0320 > 0.025$

M1

Critical region $X \leq 6$ or $X \geq 18$ (since $n - c = 18$)

A1

$$\text{Actual level} = 2 \times 0.0113 = 0.0227$$

A1

Answer: $X \leq 6$ or $X \geq 18$; 0.0227

3 marks

(d) The distribution of X is skewed when $p \neq 0.5$, so the points with 2.5% in each tail are not equally far from the mean

E1

Answer: Distribution is skewed

1 mark

28 (10 marks)

(a) Lower: $P(X \leq 4) = 0.0185 \leq 0.025$, $P(X \leq 5) = 0.0480 > 0.025$

M1

Upper: $P(X \geq 17) = 0.0144 \leq 0.025$, $P(X \geq 16) = 0.0308 > 0.025$

M1

Critical region $X \leq 4$ or $X \geq 17$

A1

$$\text{Actual level} = 0.0185 + 0.0144 = 0.0329$$

A1

Answer: $X \leq 4$ or $X \geq 17$; 0.0329

4 marks

(b) $P(X \leq 4) + P(X \geq 17)$ with $X \sim B(50, 0.35)$

M1

$$= 0.6112$$

Answer: 0.6112

A1

2 marks

(c) $P(X \geq 16) = 0.0308 \leq 0.05$ but $P(X \geq 15) = 0.0607 > 0.05$, so critical region $X \geq 16$

M1 A1

Power = $P(X \geq 16)$ with $X \sim B(50, 0.35) = 0.7199$

A1

Answer: $X \geq 16$; power 0.7199

3 marks

(d) The one-tailed test is more powerful (0.7199 against 0.6112) for a change in the predicted direction, but it has no power to detect a decrease

E1

Answer: One-tailed is more powerful in its direction only

1 mark

29 (9 marks)

$$(a) P(X \leq 3) = (1 + 13 + 78 + 286) \times (1/2)^{13} = \frac{378}{8192} = \frac{189}{4096}$$

M1 A1

$$\text{By symmetry } P(X \geq 10) = \frac{189}{4096}, \text{ so } P(X \leq 3 \text{ or } X \geq 10) = \frac{189}{2048} = 0.0923$$

M1 A1

(b) One-tailed: $0.0461 < 0.05$, so reject H_0

M1

Two-tailed: $0.0461 > 0.025$ (equivalently $0.0923 > 0.05$), so do not reject H_0

A1

(c) If the direction is chosen after seeing which tail the data fall in, the test really rejects for extreme results in either tail

E1

Then the true probability of rejecting a true H_0 is about double the stated level (up to 10% instead of 5%), so the stated significance level is wrong

E1

The hypotheses must be fixed before the data are collected

E1

Answer: (a) 0.0923 (b) one-tailed rejects, two-tailed does not

9 marks

30 (12 marks)

(a) Under H_0 , $X \sim B(40, 0.25)$; $P(X \leq 5) = 0.0433 \leq 0.05$, $P(X \leq 6) = 0.0962 > 0.05$

M1 A1

Critical region $X \leq 5$, actual level 0.0433

A1 B1

Answer: $X \leq 5$, 0.0433

4 marks

(b) Each tail 0.025. Lower: $P(X \leq 4) = 0.0160$, $P(X \leq 5) = 0.0433$

M1 A1

Upper: $P(X \geq 17) = 0.0116$, $P(X \geq 16) = 0.0262$

M1 A1

Critical region $X \leq 4$ or $X \geq 17$, actual level 0.0276

A1

Answer: $X \leq 4$ or $X \geq 17$, actual level 0.0276

5 marks

(c) $5 \leq 5$, so reject H_0 in (a); 5 is not in the critical region of (b), so do not reject H_0 in (b)

M1 A1

A smaller critical region in (b) because each tail uses only 2.5%

E1

Answer: (a) reject H_0 ; (b) do not reject H_0

3 marks

Binomial Hypothesis Testing

1 (3 marks)

$H_0: p = 0.5$, $H_1: p > 0.5$, where $p = P(\text{heads})$. Under H_0 , $X \sim B(10, 0.5)$

B1

$$P(X \geq 9) = 0.0107$$

M1

$0.0107 < 0.05$, so reject H_0 : there is evidence that the coin is biased towards heads

A1

Answer: Reject H_0

3 marks

2 (3 marks)

$H_0: p = 0.25$, $H_1: p < 0.25$. Under H_0 , $X \sim B(20, 0.25)$

B1

$$P(X \leq 2) = 0.0913$$

M1

$0.0913 > 0.05$, so do not reject H_0 : no evidence that the probability of a hit has fallen

A1

Answer: Do not reject H_0

3 marks

3 (2 marks)

$$P(X \geq 8) = 1 - P(X \leq 7) = 0.0547$$

M1

$0.0547 > 0.05$, so not significant

A1

Answer: 0.0547

2 marks

4 (2 marks)

$X \sim B(12, 0.7)$

B1

$P(X \leq 5) = 0.0386$

B1

Answer: 0.0386

2 marks

5 (2 marks)

$0.0867 > 0.05$

M1

Do not reject H_0 : no evidence that $p > 0.2$

A1

Answer: do not reject

2 marks

6 (2 marks) X = the number of underbaked muffins in the sample of 30

B1

Under H_0 , $X \sim B(30, 0.05)$

B1

Answer: $X \sim B(30, 0.05)$

2 marks

7 (5 marks)Let p = proportion of defective bulbs. $H_0: p = 0.08$, $H_1: p > 0.08$

B1

Under H_0 , $X \sim B(30, 0.08)$

B1

$P(X \geq 6) = 1 - P(X \leq 5)$

M1

$= 0.0293$

A1

 $0.0293 < 0.05$, so reject H_0 : there is sufficient evidence that the proportion of defective bulbs is greater than 8%

A1

Answer: Reject H_0

5 marks

8 (5 marks) $H_0: p = 0.4$, $H_1: p \neq 0.4$; under H_0 , $X \sim B(15, 0.4)$; 5% in each tail

B1

Lower: $P(X \leq 2) = 0.0271 \leq 0.05$, $P(X \leq 3) = 0.0905 > 0.05$

M1 A1

Upper: $P(X \geq 10) = 0.0338 \leq 0.05$, $P(X \geq 9) = 0.0950 > 0.05$

M1

Critical region $X \leq 2$ or $X \geq 10$; 1 lies in it, so reject H_0 : evidence that the proportion has changed

A1

Answer: Reject H_0

5 marks

9 (5 marks)Let p = the probability that a bulb flowers in the first year. $H_0: p = 0.8$, $H_1: p < 0.8$

B1

Under H_0 , $X \sim B(12, 0.8)$ where X is the number of successes in the sample

M1

$P(X \leq 7) = 0.0726$ (calculator)

A1

Compare with 5%: $0.0726 > 5\%$

M1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that the probability of flowering is less than 0.8.

A1

Answer: $P(X \leq 7) = 0.0726$, do not reject H_0

5 marks

10 (5 marks)Let p = the probability that a student passes first time. $H_0: p = 0.15$, $H_1: p > 0.15$

B1

Under H_0 , $X \sim B(20, 0.15)$ where X is the number of successes in the sample

M1

$P(X \geq 6) = 0.0673$ (calculator)

A1

Compare with 5%: $0.0673 > 5\%$

M1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that the new course has increased the first-time pass rate.

A1

Answer: $P(X \geq 6) = 0.0673$, do not reject H_0

5 marks

11 (5 marks)

Let p = the probability that she answers a question correctly. $H_0: p = 0.25$, $H_1: p > 0.25$ B1

Under H_0 , $X \sim B(10, 0.25)$ where X is the number of successes in the sample M1

$P(X \geq 6) = 0.0197$ (calculator) A1

Compare with 5%: $0.0197 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the student is not simply guessing. A1

Answer: $P(X \geq 6) = 0.0197$, reject H_0 5 marks

12 (5 marks)

Let p = the probability that a booked seat is taken up. $H_0: p = 0.9$, $H_1: p < 0.9$ B1

Under H_0 , $X \sim B(15, 0.9)$ where X is the number of successes in the sample M1

$P(X \leq 11) = 0.0556$ (calculator) A1

Compare with 1%: $0.0556 > 1\%$ M1

Do not reject H_0 : there is insufficient evidence at the 1% level to suggest that the proportion of booked seats taken up is lower than 0.9. A1

Answer: $P(X \leq 11) = 0.0556$, do not reject H_0 5 marks

13 (5 marks)

Let p = the probability of an even number. $H_0: p = 0.5$, $H_1: p \neq 0.5$ B1

Under H_0 , $X \sim B(18, 0.5)$ where X is the number of successes in the sample M1

$P(X \geq 13) = 0.0481$ (calculator) A1

Two-tailed test, so compare with $10\%/2 = 5\%$: $0.0481 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 10% level to suggest that the die is biased towards even or odd numbers. A1

Answer: $P(X \geq 13) = 0.0481$, reject H_0 5 marks

14 (6 marks)

(a) $H_0: p = \frac{1}{6}$, $H_1: p > \frac{1}{6}$ B1

$X \sim B(60, \frac{1}{6})$ B1

Answer: $H_0: p = \frac{1}{6}$, $H_1: p > \frac{1}{6}$; $X \sim B(60, \frac{1}{6})$ 2 marks

(b) $P(X \geq 17) = 1 - P(X \leq 16) = 0.0164$ M1

$0.0164 > 0.01$ M1

Do not reject H_0 : no evidence at the 1% level that sixes are more likely A1

Answer: Do not reject H_0 3 marks

(c) $0.0164 < 0.05$, so reject H_0 at the 5% level: significant evidence that the die is biased B1

Answer: Reject H_0 at 5% 1 mark

15 (7 marks)

(a) $H_0: p = 0.4$, $H_1: p \neq 0.4$; under H_0 , $X \sim B(50, 0.4)$ B1

Lower tail: $P(X \leq 12) = 0.0133$ M1

Two-tailed, so compare with 0.025: $0.0133 < 0.025$ M1

Reject H_0 : evidence that the proportion who watch to the end is not 40% A1

Answer: Reject H_0 4 marks

(b) $P(X \leq 12) = 0.0133 \leq 0.025$, $P(X \leq 13) = 0.0280 > 0.025$ M1

$P(X \geq 28) = 0.0160 \leq 0.025$, $P(X \geq 27) = 0.0314 > 0.025$, so critical region $X \leq 12$ or $X \geq 28$ A1

$12 \leq 12$, so 12 lies in the critical region, which agrees with part (a) B1

Answer: $X \leq 12$ or $X \geq 28$ 3 marks

16 (10 marks)

(a) $X \sim B(25, 0.06)$ M1

$P(X = 2) = 0.2602$ A1

Answer: 0.2602 2 marks

(b) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 0.4473$ Answer: 0.4473	M1 A1 2 marks
(c) $P(\text{none of the 3 boxes has at least 2 faulty}) = (1 - 0.4473)^3 = 0.1688$ $P(\text{at least one}) = 1 - 0.1688 = 0.8312$ Answer: 0.8312	M1 A1 2 marks
(d) $H_0: p = 0.06, H_1: p > 0.06$ $P(X \geq 5) = 1 - P(X \leq 4) = 0.0150$ $0.0150 < 0.05$, so reject H_0 : there is sufficient evidence at the 5% level that the proportion of faulty bulbs is greater than 6% Answer: $P(X \geq 5) = 0.0150$: reject H_0	B1 M1 A1 A1 4 marks
17 (5 marks) Let p = the proportion of underbaked rolls. $H_0: p = 0.01, H_1: p > 0.01$ n is large and p is small, so $X \sim B(400, 0.01)$ is approximated by $Po(400 \times 0.01) = Po(4)$ $P(X \geq 9) = 1 - P(X \leq 8) = 0.0214$ $0.0214 < 0.05$ Reject H_0 : there is sufficient evidence at the 5% level that the proportion of underbaked rolls is greater than 1% (the exact binomial probability is 0.0208, giving the same conclusion) Answer: $P(X \geq 9) = 0.0214$: reject H_0	B1 M1 A1 M1 A1 5 marks
18 (5 marks) Let p = the probability that an orange weighs more than 200 g. $H_0: p = 0.25, H_1: p > 0.25$ Under $H_0, X \sim B(60, 0.25)$ where X is the number of successes in the sample $P(X \geq 22) = 0.0298$ (calculator) Compare with 1%: $0.0298 > 1\%$ Do not reject H_0 : there is insufficient evidence at the 1% level to suggest that the proportion of oranges over 200 g is more than 0.25. Answer: $P(X \geq 22) = 0.0298$, do not reject H_0	B1 M1 A1 M1 A1 5 marks
19 (5 marks) Total faulty = 5. Let p = $P(\text{a bulb is faulty})$: $H_0: p = 0.04, H_1: p > 0.04$ $X \sim B(50, 0.04)$ under H_0 $P(X \geq 5) = 0.0490$ $0.0490 < 0.05$ Reject H_0 : sufficient evidence that the proportion of faulty bulbs is more than 4% Answer: $P(X \geq 5) = 0.0490$: reject H_0	B1 M1 A1 M1 A1 5 marks
20 (5 marks) (a) $H_0: p = 0.6, H_1: p < 0.6; X \sim B(25, 0.6)$ $P(X \leq 10) = 0.0344 \leq 0.05, P(X \leq 11) = 0.0778 > 0.05$ Answer: $X \leq 10$ (b) 11 is not in the critical region Do not reject H_0 : insufficient evidence at the 5% level that the proportion is less than 0.6 Answer: do not reject	B1 M1 A1 3 marks M1 A1 2 marks
21 (8 marks) (a) $H_0: p = 0.15, H_1: p < 0.15; X \sim B(40, 0.15)$ under H_0 $P(X \leq 2) = 0.0486 \leq 0.05$ $P(X \leq 3) = 0.1302 > 0.05$, so critical region is $X \leq 2$ Answer: $X \leq 2$ (b) $P(X \leq 2 p = 0.15) = 0.0486$ Answer: 0.0486 (c) 3 is not in the critical region ($P(X \leq 3) = 0.1302 > 0.05$), so do not reject H_0 : insufficient evidence that the defect rate has fallen	B1 M1 A1 A1 4 marks B1 1 mark B1

Answer: Do not reject H_0 **1 mark****(d)** Fail to detect: $X \geq 3$ when $X \sim B(40, 0.05)$ **M1** $P(X \geq 3) = 0.3233$ **A1****Answer: 0.3233****2 marks****22 (8 marks)****(a)** The complement of $X \geq 7$ is $X \leq 6$, not $X \leq 7$ **E1**So $P(X \geq 7) = 1 - P(X \leq 6)$, and the student has found $P(X \geq 8)$ instead**E1****Answer: $P(X \geq 7) = 1 - P(X \leq 6)$** **2 marks****(b)** $P(X \geq 7) = 1 - P(X \leq 6) = 0.0867$ **M1 A1** $0.0867 > 0.05$, so do not reject H_0 : insufficient evidence that more than 20% of customers use online banking**A1****Answer: Do not reject H_0** **3 marks****(c)** $P(X \geq 8) = 0.0321 \leq 0.05$ **M1** $P(X \geq 7) = 0.0867 > 0.05$, so critical region $X \geq 8$ **A1**7 is not in the critical region, so H_0 is not rejected**B1****Answer: $X \geq 8$** **3 marks****23 (8 marks)****(a)** $H_0: p = 0.7$, $H_1: p < 0.7$; $X \sim B(18, 0.7)$ **B1** $P(X \leq 9) = 0.0596$ **M1 A1** $0.0596 > 0.05$ so do not reject H_0 **A1****(b)** Failing to reject H_0 does not prove it; it only says there is insufficient evidence against it**E1****(c)** CR is $X \leq 8$, as $P(X \leq 8) = 0.0210 \leq 0.05$ and $P(X \leq 9) = 0.0596 > 0.05$ **M1 A1**So H_0 is not rejected for $X \geq 9$ **A1****Answer: (a) do not reject (c) 9****8 marks****24 (6 marks)****(a)** $X \sim B(n, 0.1)$, $P(X \leq 0) = P(X = 0) = 0.9^n$ **M1 A1****(b)** $0.9^n < 0.05$: $n \ln 0.9 < \ln 0.05$ **M1** $n > \frac{\ln 0.05}{\ln 0.9} = 28.4$, so $n = 29$ **A1 A1****(c)** Possible answers: the volunteers may not be a random sample, or may differ from the general population; there may be other reasons for no infections**E1****Answer: $n = 29$** **6 marks****25 (9 marks)****(a)** $H_0: p = 0.2$, $H_1: p < 0.2$. $X \sim B(20, 0.2)$: $P(X \leq 1) = 0.0692$ **B1 M1 A1** $0.0692 > 0.05$ so do not reject H_0 **A1****(b)** $X \sim B(30, 0.2)$: $P(X \leq 3) = 0.1227 > 0.05$, so do not reject H_0 **M1 A1****(c)** Combined: 4 toys in 50 packets, $X \sim B(50, 0.2)$: $P(X \leq 4) = 0.0185$ **M1 A1** $0.0185 < 0.05$, so reject H_0 ; the larger sample gives stronger evidence even though each part alone was not significant**E1****Answer: (a) do not reject (b) do not reject (c) reject****9 marks****26 (6 marks)****(a)** $P(X \geq 10) = 1 - P(X \leq 9)$, not $1 - P(X \leq 10)$: she has found $P(X \geq 11)$ **E1 E1****(b)** $1 - P(X \leq 9) = 0.1218$ **M1 A1** $0.1218 > 0.05$, so do not reject H_0 **M1**Insufficient evidence at the 5% level that $p > 0.35$ **A1****Answer: (b) 0.1218: do not reject H_0** **6 marks**

27 (10 marks)**(a)** $X \sim B(20, 0.05)$

B1

$$P(X \geq 3) = 1 - P(X \leq 2)$$

M1

$$= 0.0755 > 0.05, \text{ so not significant}$$

A1

Answer: shown

3 marks

(b) Significant when $P(X \geq 3) \leq 0.05$ with $X \sim B(n, 0.05)$

M1

$$n = 16: P(X \geq 3) = 0.0429 \leq 0.05$$

M1 A1

$$n = 17: P(X \geq 3) = 0.0503 > 0.05, \text{ so the largest value is } n = 16$$

A1

Answer: $n = 16$

4 marks

(c) The expected number of faulty components under H_0 is $0.05n$, which grows with n

E1

A fixed count of 3 is then a smaller proportion of the sample, closer to what H_0 predicts

E1

Answer: Expected number under H_0 increases with n

2 marks

(d) Critical region $X \geq 3$, Type I error probability 0.0429

B1

Answer: $X \geq 3$; 0.0429

1 mark

28 (11 marks)**(a)** Each seed germinates independently of the others

E1

The probability of germination is the same (0.8) for every seed, and the number of seeds is fixed at 25

E1

Answer: Independence and constant probability

2 marks

(b) $H_0: p = 0.8, H_1: p < 0.8$

B1

Under $H_0, X \sim B(25, 0.8)$

B1

$$P(X \leq 16) = 0.0468$$

M1

$$0.0468 < 0.05, \text{ so reject } H_0: \text{evidence that the germination rate is below 80\%}$$

A1

Answer: Reject H_0

4 marks

(c) $P(X \leq 16) = 0.0468 \leq 0.05$

M1

$$P(X \leq 17) = 0.1091 > 0.05, \text{ so critical region } X \leq 16$$

A1

$$\text{Probability of Type I error} = 0.0468$$

B1

Answer: $X \leq 16$; 0.0468

3 marks

(d) Reject when $X \leq 16$ with $X \sim B(25, 0.6)$

M1

$$P(X \leq 16) = 0.7265$$

A1

Answer: 0.7265

2 marks

29 (8 marks)Reject H_0 if $P(X \leq 2 | p_0) \leq 0.05$

M1

$$g(p) = P(X \leq 2) = (1 - p)^{15} + 15p(1 - p)^{14} + 105p^2(1 - p)^{13}$$

M1 A1

$$g(0.36) = 0.0528 > 0.05 \text{ and } g(0.37) = 0.0450 < 0.05$$

M1 A1

$$\text{So the least value is } p_0 = 0.37$$

A1

 $g(p)$ is a decreasing function of p (more successes are expected as p grows), so for larger p_0 the probability is even smaller

E1

Values of p of about 0.37 or more can be rejected at the 5% level, so 0.37 acts as an upper bound for p

E1

Answer: $p_0 = 0.37$

8 marks

30 (9 marks)**(a)** $X \sim B(20, p), P(\text{accept}) = P(X \leq 2)$

M1

$$p = 0.05: 0.9245; p = 0.15: 0.4049$$

A1

$$p = 0.30: 0.0355$$

A1

Answer: 0.9245, 0.4049, 0.0355

3 marks

(b) $P(\text{reject} | p = 0.05) = P(X \geq 3) = 1 - P(X \leq 2)$

M1

$$= 0.0755$$

A1

Answer: 0.0755

2 marks

(c) $P(X \geq c | p = 0.05) \leq 0.01$ with $X \sim B(40, 0.05)$: $P(X \geq 7) = 0.0034, P(X \geq 6) = 0.0139$

M1 A1

$$c = 7$$

A1

$$P(X \geq 7 | p = 0.15) = 0.3933$$

B1

Answer: $c = 7, 0.3933$ **4 marks**

Normal Hypothesis Testing

1 (3 marks)

Under H_0 , $\bar{X} \sim N(50, \frac{5^2}{25})$, so standard deviation of $\bar{X}$ is 1

B1

$$z = \frac{52 - 50}{1} = 2.00$$

M1

$2.00 > 1.645$ (critical value), so reject H_0 : evidence that the mean is greater than 50

A1**Answer: Reject H_0** **3 marks**

2 (3 marks)

$H_0: \mu = 100$, $H_1: \mu < 100$. Under H_0 , $\bar{X} \sim N(100, \frac{6^2}{36})$, standard deviation 1

B1

$$z = \frac{98.5 - 100}{1} = -1.50$$

M1

$-1.50 > -1.645$, so do not reject H_0 : no evidence that the mean lifetime is below 100 hours

A1**Answer: Do not reject H_0** **3 marks**

3 (2 marks)

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ with $\mu = 80$

B1

Variance $\frac{36}{36} = 1$, so $\bar{X} \sim N(80, 1)$

B1**Answer: $\bar{X} \sim N(80, 1)$** **2 marks**

4 (2 marks)

$$z = \frac{53.4 - 50}{8/\sqrt{25}} = \frac{3.4}{1.6}$$

M1

$$z = 2.125$$

A1**Answer: 2.125****2 marks**

5 (2 marks)

$$P(Z \geq 1.83) = 0.0336$$

B1

$0.0336 < 0.05$, so significant

B1**Answer: 0.0336****2 marks**

6 (2 marks)

(a) $|1.87| < 1.96$, so do not reject H_0

B1

(b) $|-2.31| > 1.96$, so reject H_0

B1**Answer: (a) do not reject (b) reject****2 marks**

7 (4 marks)

$H_0: \mu = 20$, $H_1: \mu \neq 20$. Under H_0 , $\bar{X} \sim N(20, \frac{3^2}{9})$, standard deviation 1

B1

$$z = \frac{18.2 - 20}{1} = -1.80$$

M1

Two-tailed, critical values ± 1.96 , and $1.80 < 1.96$

M1

Do not reject H_0 : no evidence that the mean length has changed

A1**Answer: Do not reject H_0** **4 marks**

8 (5 marks)

(a) Under H_0 , $\bar{X} \sim N(100, \frac{4^2}{16})$, standard deviation 1

B1

$$c = 100 + 1.645 \times 1$$

M1

$$c = 101.64$$

A1

Answer: 101.64

3 marks

(b) $102.1 > 101.64$, so it lies in the critical region

M1

Reject H_0 : evidence that the mean has increased above 100

A1

Answer: Reject H_0

2 marks

9 (5 marks)

Let μ = the mean lifetime (hours). $H_0: \mu = 1200$, $H_1: \mu < 1200$

B1

Under H_0 : $\bar{X} \sim N(1200, \frac{90^2}{30})$ so the standard deviation of $\bar{X}$ is $\frac{90}{\sqrt{30}} = 16.4$

M1

$$z = \frac{1168 - 1200}{16.4} = -1.947$$

M1

$P(Z \leq -1.947) = 0.0257$, compared with 5%

A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean lifetime is less than 1200 hours.

A1

Answer: $z = -1.947$, p-value 0.0257, reject H_0

5 marks

10 (5 marks)

Let μ = the mean height in the village (cm). $H_0: \mu = 170$, $H_1: \mu > 170$

B1

Under H_0 : $\bar{X} \sim N(170, \frac{7^2}{40})$ so the standard deviation of $\bar{X}$ is $\frac{7}{\sqrt{40}} = 1.11$

M1

$$z = \frac{172.6 - 170}{1.11} = 2.349$$

M1

$P(Z \geq 2.349) = 0.0094$, compared with 1%

A1

Reject H_0 : there is sufficient evidence at the 1% level to suggest that the mean height in the village is greater than 170 cm.

A1

Answer: $z = 2.349$, p-value 0.0094, reject H_0

5 marks

11 (5 marks)

Let μ = the mean time (seconds). $H_0: \mu = 150$, $H_1: \mu \neq 150$

B1

Under H_0 : $\bar{X} \sim N(150, \frac{20^2}{16})$ so the standard deviation of $\bar{X}$ is $\frac{20}{\sqrt{16}} = 5.00$

M1

$$z = \frac{139 - 150}{5.00} = -2.200$$

M1

$P(Z \leq -2.200) = 0.0139$, compared with 2.5% (two-tailed, so half of the level)

A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean time has changed.

A1

Answer: $z = -2.200$, p-value 0.0139, reject H_0

5 marks

12 (6 marks)

$$\bar{x} = \frac{406.5}{8} = 50.812$$

M1 A1

$H_0: \mu = 50$, $H_1: \mu > 50$; $\bar{X} \sim N(50, \frac{2.5^2}{8})$, standard deviation 0.884

B1 M1

$$z = 0.919, P(Z \geq z) = 0.1790$$

M1

$0.1790 > 0.05$, so do not reject H_0 : insufficient evidence that the mean mass is greater than 50 g

A1

Answer: $\bar{x} = 50.812$; $z = 0.919$

6 marks

13 (5 marks)

Let μ = the mean daily output. $H_0: \mu = 400$, $H_1: \mu \neq 400$

B1

Under H_0 : $\bar{X} \sim N(400, \frac{8^2}{9})$ so the standard deviation of $\bar{X}$ is $\frac{8}{\sqrt{9}} = 2.67$

M1

$$z = \frac{394 - 400}{2.67} = -2.250$$

M1

$P(Z \leq -2.250) = 0.0122$, compared with 5% (two-tailed, so half of the level)

A1

Reject H_0 : there is sufficient evidence at the 10% level to suggest that the mean daily output is not 400 units.

A1

Answer: $z = -2.250$, p-value 0.0122, reject H_0

5 marks

14 (6 marks)

Sum = 987.5, so $\bar{x} = \frac{987.5}{10} = 98.75$

B1

$H_0: \mu = 100$, $H_1: \mu < 100$

B1

Under H_0 , $\bar{X} \sim N(100, \frac{1.5^2}{10})$, standard deviation 0.4743

M1

$$z = \frac{98.75 - 100}{0.4743} = -2.635$$

M1

$-2.635 < -1.645$, so reject H_0

M1

There is sufficient evidence at the 5% level that the mean mass of a bar is less than 100 g

A1

Answer: Reject H_0

6 marks

15 (6 marks)

$H_0: \mu = 82$, $H_1: \mu \neq 82$. Under H_0 , $\bar{X} \sim N(82, \frac{9^2}{36})$, standard deviation 1.5

B1

$$z = \frac{85.5 - 82}{1.5} = 2.333$$

M1

p-value = $2 \times P(Z > 2.333) = 2 \times 0.0098 = 0.0196$

M1 A1

5% level: $0.0196 < 0.05$, so reject H_0 : evidence that the mean time has changed

A1

1% level: $0.0196 > 0.01$, so do not reject H_0 : insufficient evidence at this level

A1

Answer: Reject at 5%, do not reject at 1% ($p = 0.0196$)

6 marks

16 (10 marks)

(a) $\bar{x} = \frac{5700}{60} = 95.0$

B1

$$S_{xx} = 560\,000 - 60 \times 95.0^2 = 18500$$

M1

$$s^2 = \frac{18500}{59} = 314$$

A1

Answer: mean 95.0, $s^2 = 314$

3 marks

(b) $H_0: \mu = 90$, $H_1: \mu > 90$

B1

Under H_0 , $\bar{X} \approx N(90, \frac{314}{60})$, standard deviation 2.29

M1

$$z = \frac{95.0 - 90}{2.29} = 2.187, \text{ so } P(Z > 2.187) = 0.0144$$

A1

$0.0144 < 0.05$, so reject H_0 : sufficient evidence that the mean intake is greater than 90 g

A1

Answer: $z = 2.187$, reject H_0

4 marks

(c) The sample is large ($n = 60$), so s^2 is a good estimate of σ^2 and, by the central limit theorem, $\bar{X}$ is approximately normal

E1

Answer: Large sample

1 mark

(d) Reject if $\frac{95 - 90}{s/\sqrt{60}} > 1.645$, that is $s < \frac{5\sqrt{60}}{1.645}$

M1

$$s < 23.5$$

A1

Answer: $s < 23.5$

2 marks

17 (5 marks)

Let μ = the mean length of fish (cm). $H_0: \mu = 22.6$, $H_1: \mu < 22.6$

B1

Under H_0 : $\bar{X} \sim N(22.6, \frac{4.2^2}{45})$ so the standard deviation of $\bar{X}$ is $\frac{4.2}{\sqrt{45}} = 0.626$

M1

$$z = \frac{21.5 - 22.6}{0.626} = -1.757$$

M1

$P(Z \leq -1.757) = 0.0395$, compared with 1%

A1

Do not reject H_0 : there is insufficient evidence at the 1% level to suggest that the mean length of fish has decreased.

A1

Answer: $z = -1.757$, p-value 0.0395, do not reject H_0

5 marks

18 (5 marks)

$$\frac{\bar{x}}{s} = \frac{3744}{36} = 104$$

B1

$H_0: \mu = 100$, $H_1: \mu \neq 100$; under H_0 , $\bar{X} \sim N(100, 2.5^2)$

M1

$$z = \frac{104 - 100}{2.5} = 1.6$$

M1

$1.6 < 1.96$, so (or $P(Z > 1.6) = 0.0548 > 0.025$) do not reject H_0

A1

Insufficient evidence that the class mean differs from 100

A1

Answer: $z = 1.6$, do not reject H_0

5 marks

19 (5 marks)

(a) $H_0: \mu = 12.5$, $H_1: \mu > 12.5$; $\bar{X} \sim N(12.5, \frac{0.8^2}{50})$

B1

$$\bar{x} > 12.5 + 1.645 \times \frac{0.8}{\sqrt{50}} = 12.686$$

M1 A1

Answer: $\bar{X} \geq 12.686$

3 marks

(b) $12.71 > 12.686$

M1

Reject H_0 : sufficient evidence that the mean thickness is above target

A1

Answer: reject

2 marks

20 (5 marks)

Reject if $\frac{38.8 - 40}{5/\sqrt{n}} < -1.645$

M1 B1

$$\frac{1.2\sqrt{n}}{5} > 1.645$$

M1

$$\sqrt{n} > 6.85 \text{ so } n > 46.98$$

M1

Least $n = 47$

A1

Answer: $n = 47$

5 marks

21 (8 marks)

(a) Under H_0 , $\bar{X} \sim N(60, \frac{12^2}{36})$, standard deviation 2

B1

$$\text{Boundary} = 60 + 1.645 \times 2$$

M1

Critical region $\bar{X} > 63.29$

A1

Answer: $\bar{X} > 63.29$

3 marks

(b) $63.8 > 63.29$, so reject H_0 : evidence that the mean score exceeds 60

A1

Answer: Reject H_0

1 mark

(c) Type II error: $\bar{X} \leq 63.29$ when $\bar{X} \sim N(64, 2^2)$

M1

$$z = \frac{63.29 - 64}{2} = -0.355$$

M1

$$P(Z \leq -0.355) = 0.3612$$

A1

Answer: 0.3612

3 marks

(d) It would be smaller (0.0877 in fact), because the true distribution is further from the critical value, so fewer sample means fall below it

E1

Answer: Smaller

1 mark

22 (8 marks)

(a) The test uses the sample mean $\bar{X}$, whose standard deviation is $\frac{\sigma}{\sqrt{n}}$, not σ E1

The student divided by 10 instead of $\frac{10}{\sqrt{25}} = 2$ E1

Answer: Should divide by $\sigma/\sqrt{n} = 2$ 2 marks

(b) Under H_0 , $\bar{X} \sim N(200, \frac{10^2}{25})$, standard deviation 2 B1

$$z = \frac{204 - 200}{2} = 2.00$$

M1

$$2.00 > 1.96$$

M1

Reject H_0 : evidence that the mean mass is not 200 kg A1

Answer: Reject H_0 4 marks

(c) The standard deviation of $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$, which decreases as n increases E1

So the same difference gives a larger z -value E1

Answer: sd of the sample mean falls as n rises 2 marks

23 (7 marks)

$H_0: p = 0.35$, $H_1: p > 0.35$ B1

$X \sim B(150, 0.35) \approx N(52.5, 34.125)$, standard deviation 5.84 M1 A1

$$P(X \geq 64) \approx P(Z \geq \frac{63.5 - 52.5}{5.84})$$

M1

$$= P(Z \geq 1.883) = 0.0298$$

A1

$$0.0298 < 0.05, \text{ so reject } H_0$$

M1

Sufficient evidence that more than 35% buy the premium model A1

Answer: 0.0298: reject H_0 7 marks

24 (7 marks)

$H_0: \lambda = 40$, $H_1: \lambda > 40$; $X \sim \text{Po}(40) \approx N(40, 40)$ B1 M1

$$P(X \geq 52) \approx P(Z \geq \frac{51.5 - 40}{\sqrt{40}}) = P(Z \geq 1.818)$$

M1 A1

$$= 0.0345$$

A1

$$0.0345 < 0.05 \text{ so reject } H_0$$

M1

Exact: $P(X \geq 52) = 0.0387$, very close to the approximation, same conclusion A1

Answer: 0.0345: reject H_0 7 marks

25 (8 marks)

(a) $\bar{X} \sim N(50, 2^2)$; critical value = $50 + 1.645 \times 2 = 53.29$ M1 A1

Critical region $\bar{X} \geq 53.29$ A1

(b) Type II error: H_0 not rejected although $\mu = 56$: $P(\bar{X} < 53.29)$, $\bar{X} \sim N(56, 4)$ M1 B1

$$= P(Z < \frac{53.29 - 56}{2}) = P(Z < -1.355)$$

M1

$$= 0.0877$$

A1

(c) Increasing n reduces the standard deviation of $\bar{X}$, so the probability of a Type II error decreases E1

Answer: (a) $\bar{X} \geq 53.29$ (b) 0.0877 8 marks

26 (9 marks)

(a) $P(T < 12) = 0.10$ gives $z = -1.2816$; $P(T > 27) = 0.05$ gives $z = 1.6449$ B1 B1

$$12 = \mu - 1.2816 \sigma \text{ and } 27 = \mu + 1.6449 \sigma$$

M1

$$\text{Subtracting: } 15 = 2.9264 \sigma, \text{ so } \sigma = 5.126$$

M1

$$\mu = 12 + 1.2816 \times 5.126 = 18.57, \text{ so } \mu = 18.6 \text{ and } \sigma = 5.1$$

A1

Answer: $\mu = 18.6$, $\sigma = 5.1$ 5 marks

(b) $H_0: \mu = 18.6$, $H_1: \mu < 18.6$ B1

Under H_0 , $\bar{T} \sim N(18.6, \frac{5.1^2}{9})$, standard deviation 1.7

M1

$$z = \frac{15.2 - 18.6}{1.7} = -2.000, \text{ so } P(Z < -2.000) = 0.0228$$

A1

$0.0228 < 0.05$, so reject H_0 : sufficient evidence that the mean assembly time has fallen

A1

Answer: $z = -2.000$, reject H_0

4 marks

27 (10 marks)

(a) $H_0: p = 0.3$, $H_1: p > 0.3$. $X \sim B(100, 0.3)$, $np = 30$ and $n(1 - p) = 70$ are both large

B1

X is approximately $N(30, 21)$, since $np = 30$ and $np(1 - p) = 21$

B1

Answer: $X \approx N(30, 21)$

2 marks

(b) $P(X \geq 38)$ becomes $P(Y > 37.5)$ where $Y \sim N(30, 21)$

M1

$$z = \frac{37.5 - 30}{\sqrt{21}} = 1.637$$

M1

$$P(Z > 1.637) = 0.0509$$

A1

$0.0509 > 0.05$, so do not reject H_0 : insufficient evidence that the proportion is higher than 0.3

A1

Answer: Do not reject H_0 ($p \approx 0.0509$)

4 marks

(c) $P(X \geq 38) = 1 - P(X \leq 37) = 0.0530$ (calculator)

M1

$0.0530 > 0.05$, so do not reject H_0 , agreeing with (b)

A1

Answer: 0.0530

2 marks

(d) $z = 1.746$, $P(Z > 1.746) = 0.0404 < 0.05$, so they would reject H_0

M1

X is discrete: the event $X \geq 38$ should start at 37.5, so omitting the correction understates the probability (0.0404 against the exact 0.0530) and gives a wrong conclusion

E1

Answer: Rejects H_0 wrongly

2 marks

28 (11 marks)

(a) Under H_0 , $\bar{X} \sim N(50, \frac{8^2}{n})$, so the standard deviation of $\bar{X}$ is $\frac{8}{\sqrt{n}}$

B1

Upper-tail 5%: $z = 1.645$, so critical value = $50 + 1.645 \times \frac{8}{\sqrt{n}}$

B1

Answer: shown

2 marks

(b) When $\mu = 53$, $\bar{X} \sim N(53, \frac{8^2}{n})$

B1

$$\text{Power} = P(\bar{X} > 50 + \frac{1.645 \times 8}{\sqrt{n}}) = P(Z > \frac{50 + 1.645 \times 8}{\sqrt{n}} - \frac{53}{8/\sqrt{n}})$$

M1

$$= P(Z > 1.645 - \frac{3\sqrt{n}}{8})$$

A1

Answer: shown

3 marks

(c) Need $P(Z > 1.645 - \frac{3\sqrt{n}}{8}) \geq 0.9$, so $1.645 - \frac{3\sqrt{n}}{8} \leq -1.2816$

M1 A1

$$\frac{3\sqrt{n}}{8} \geq 1.645 + 1.2816 = 2.9264, \text{ so } \sqrt{n} \geq 7.804$$

M1

$n \geq 60.90$, so the smallest sample size is $n = 61$

A1

Answer: $n = 61$

4 marks

(d) $n = 61$: power = $0.9004 \geq 0.9$

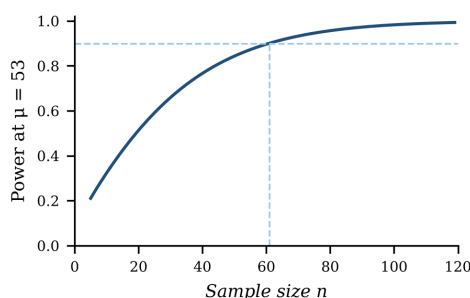
B1

$n = 60$: power = $0.8961 < 0.9$, so 61 is the smallest

B1

Answer: Powers 0.9004 and 0.8961

2 marks



29 (10 marks)

(a) $H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B$

B1

$\bar{X}_A \sim N(\mu_A, \frac{36}{12})$ and $\bar{X}_B \sim N(\mu_B, \frac{64}{15})$, independent

M1

Under $H_0: \bar{X}_A - \bar{X}_B \sim N(0, 3 + 4.27) = N(0, 7.27)$

A1

Answer: N(0, 7.27)

3 marks

(b) $z = \frac{52.3 - 48.1}{\sqrt{7.27}} = 1.558$

M1

$P(Z > 1.558) = 0.0596$

A1

0.0596 > 0.05, so do not reject H_0 : insufficient evidence that the mean of A is greater than the mean of B

A1

Answer: z = 1.558, do not reject H_0

3 marks

(c) The two samples are random and independent of each other

E1

Both populations are normally distributed (or the samples are large) with the stated standard deviations

E1

Answer: Independent random samples; normal populations

2 marks

(d) Need $\frac{4.2}{\sqrt{\frac{36+64}{n}}} > 1.645$, so $n > (1.645 \times 10/4.2)^2 = 15.34$

M1

$n = 16$

A1

Answer: n = 16

2 marks

30 (10 marks)

(a) $\bar{x} = \frac{1452}{24} = 60.5$

B1

$H_0: \mu = 62, H_1: \mu < 62; \bar{X} \sim N(62, \frac{25}{24})$

B1 M1

$z = \frac{60.5 - 62}{\sqrt{25/24}} = -1.470$

M1

$P(Z \leq -1.470) = 0.0708 > 0.05$, so do not reject H_0

A1

Answer: z = -1.470

5 marks

(b) The eggs within one box may be dependent (similar masses), so the observations are not independent

E1

The standard deviation of the mean would then be larger than $\frac{\sigma}{\sqrt{n}}$, so the test overstates the evidence against H_0

E1

Answer: Independence is violated; evidence may be overstated

2 marks

(c) Critical value: $62 - 1.645 \times \frac{5}{\sqrt{24}}$

M1 B1

$= 60.3$

A1

Answer: 60.3 g

3 marks

Poisson Hypothesis Testing**1 (3 marks)**

$H_0: \lambda = 4, H_1: \lambda > 4$; under $H_0, X \sim \text{Po}(4)$

B1

$P(X \geq 9) = 1 - P(X \leq 8) = 0.0214$

M1

0.0214 < 0.05, so reject H_0 : evidence that the mean number of accidents has increased

A1

Answer: Reject H_0

3 marks

2 (3 marks)

$H_0: \lambda = 6, H_1: \lambda < 6$; under $H_0, X \sim \text{Po}(6)$

B1

$P(X \leq 2) = 0.0620$

M1

0.0620 > 0.05, so do not reject H_0 : no evidence that the rate has decreased

A1

Answer: Do not reject H_0

3 marks

3 (3 marks)

Events occur independently of each other B1

Events occur at a constant average rate (singly and at random) B1

Traffic is not at a constant rate (e.g. rush hour) or cars travel in groups, so independence/constant rate fails E1

Answer: Independent events, constant rate; e.g. rush hour varies the rate 3 marks

4 (2 marks)

Mean for 4 minutes = $1.5 \times 4 = 6$ B1

$X \sim \text{Po}(6)$ B1

Answer: $X \sim \text{Po}(6)$ 2 marks

5 (2 marks)

$P(X \geq 9) = 1 - P(X \leq 8) = 0.0403$ M1

$0.0403 < 0.05$, so significant A1

Answer: 0.0403 2 marks

6 (2 marks)

$H_0: \lambda = 5$ B1

$H_1: \lambda < 5$ B1

Answer: $H_0: \lambda = 5, H_1: \lambda < 5$ 2 marks

7 (5 marks)

Over 3 days the mean under H_0 is $3 \times 2.5 = 7.5$, so $X \sim \text{Po}(7.5)$ M1

$H_0: \lambda = 2.5$ per day, $H_1: \lambda > 2.5$ per day B1

$P(X \geq 13) = 1 - P(X \leq 12) = 0.0427$ M1 A1

Reject H_0 at 5% since $0.0427 < 0.05$. (Assumption: reports occur independently and at a constant average rate) A1

Answer: Reject H_0 5 marks

8 (4 marks)

Each tail has probability at most 0.025 B1

Lower: $P(X \leq 0) = 0.0055 \leq 0.025$ but $P(X \leq 1) = 0.0342 > 0.025$ M1 A1

Upper: $P(X \geq 11) = 0.0177 \leq 0.025$ but $P(X \geq 10) = 0.0397 > 0.025$, so critical region $X = 0$ or $X \geq 11$; actual level 0.0232 A1

Answer: $X = 0$ or $X \geq 11$; 0.0232 4 marks

9 (5 marks)

Let λ = the mean number of accidents per month. $H_0: \lambda = 3, H_1: \lambda < 3$ B1

Under $H_0, X \sim \text{Po}(3)$ M1

$P(X \leq 1) = 0.1991$ (calculator) A1

Compare with 10%: $0.1991 > 10\%$ M1

Do not reject H_0 : there is insufficient evidence at the 10% level to suggest that the mean number of accidents per month has decreased. A1

Answer: $P(X \leq 1) = 0.1991$, do not reject H_0 5 marks

10 (5 marks)

Let λ = the mean number of copies sold per day. $H_0: \lambda = 2, H_1: \lambda > 2$ B1

Under H_0 , the mean for 3 days is $2 \times 3 = 6$, so $X \sim \text{Po}(6)$ M1

$P(X \geq 9) = 0.1528$ (calculator) A1

Compare with 5%: $0.1528 > 5\%$ M1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that the rate of sales has increased. A1

Answer: $P(X \geq 9) = 0.1528$, do not reject H_0 5 marks

11 (5 marks)

Let λ = the mean number of faults per metre. $H_0: \lambda = 0.4$, $H_1: \lambda < 0.4$

B1

Under H_0 , the mean for 10 m is $0.4 \times 10 = 4$, so $X \sim \text{Po}(4)$

M1

$P(X \leq 1) = 0.0916$ (calculator)

A1

Compare with 5%: $0.0916 > 5\%$

M1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that the new cable has fewer faults per metre.

A1

Answer: $P(X \leq 1) = 0.0916$, do not reject H_0

5 marks

12 (5 marks)

Let λ = the mean number of calls per hour. $H_0: \lambda = 8$, $H_1: \lambda > 8$

B1

Under H_0 , $X \sim \text{Po}(8)$

M1

$P(X \geq 14) = 0.0342$ (calculator)

A1

Compare with 1%: $0.0342 > 1\%$

M1

Do not reject H_0 : there is insufficient evidence at the 1% level to suggest that the mean number of calls per hour has increased.

A1

Answer: $P(X \geq 14) = 0.0342$, do not reject H_0

5 marks

13 (5 marks)

Let λ = the mean number of swans per 100 m. $H_0: \lambda = 4$, $H_1: \lambda \neq 4$

B1

Under H_0 , $X \sim \text{Po}(4)$

M1

$P(X \geq 9) = 0.0214$ (calculator)

A1

Compare with $5\%/2 = 2.5\%$: $0.0214 < 2.5\%$

M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean number of swans per 100 m is not 4.

A1

Answer: $P(X \geq 9) = 0.0214$, reject H_0

5 marks

14 (9 marks)

(a) $X \sim \text{Po}(5)$; $P(X > 7) = 1 - P(X \leq 7)$

M1

$= 0.1334$

A1

Answer: 0.1334

2 marks

(b) Mean for 3 hours = 15, so $Y \sim \text{Po}(15)$

M1

$P(Y = 18) = 0.0706$

A1

Answer: 0.0706

2 marks

(c) $H_0: \lambda = 5$, $H_1: \lambda < 5$

B1

Under H_0 , $X \sim \text{Po}(5)$; $P(X \leq 1) = 0.0404$

M1 A1

$0.0404 < 0.05$, so reject H_0 : sufficient evidence that the arrival rate has fallen

A1

Answer: $P(X \leq 1) = 0.0404$: reject H_0

4 marks

(d) Only one hour was observed, and the rate may differ by time of day, so the hour may not be representative

E1

Answer: One hour is not representative

1 mark

15 (7 marks)

(a) $H_0: \lambda = 9$, $H_1: \lambda \neq 9$

B1

$X \sim \text{Po}(9)$

B1

Answer: $H_0: \lambda = 9$, $H_1: \lambda \neq 9$; $X \sim \text{Po}(9)$

2 marks

(b) Two-tailed, so compare with 0.025

B1

$P(X \leq 3) = 0.0212$

M1

$0.0212 < 0.025$, so reject H_0 : evidence that the arrival rate has changed

A1

Answer: Reject H_0

3 marks

(c) $P(X \leq 3) = 0.0212$, $P(X \leq 4) = 0.0550$; $P(X \geq 16) = 0.0220$, $P(X \geq 15) = 0.0415$

M1

Critical region $X \leq 3$ or $X \geq 16$; actual level 0.0433

A1

Answer: $X \leq 3$ or $X \geq 16$; 0.0433

2 marks

16 (5 marks)

Let λ = the mean number of bags lost per flight. $H_0: \lambda = 1.2$, $H_1: \lambda < 1.2$ B1

Under H_0 , the mean for 10 flights is $1.2 \times 10 = 12$, so $X \sim \text{Po}(12)$ M1

$P(X \leq 5) = 0.0203$ (calculator) A1

Compare with 5%: $0.0203 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the mean number of bags lost per flight has decreased. A1

Answer: $P(X \leq 5) = 0.0203$, reject H_0 5 marks

17 (5 marks)

Let λ = the mean number of crashes per week. $H_0: \lambda = 0.5$, $H_1: \lambda > 0.5$ B1

Under H_0 , the mean for 12 weeks is $0.5 \times 12 = 6$, so $X \sim \text{Po}(6)$ M1

$P(X \geq 11) = 0.0426$ (calculator) A1

Compare with 5%: $0.0426 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that the crash rate has increased. A1

Answer: $P(X \geq 11) = 0.0426$, reject H_0 5 marks

18 (5 marks)

Let λ = the mean number of strikes per year. $H_0: \lambda = 3.5$, $H_1: \lambda \neq 3.5$ B1

Under H_0 , $X \sim \text{Po}(3.5)$ M1

$P(X \geq 8) = 0.0267$ (calculator) A1

Compare with $10\%/2 = 5\%$: $0.0267 < 5\%$ M1

Reject H_0 : there is sufficient evidence at the 10% level to suggest that the mean rate of strikes is not 3.5 per year. A1

Answer: $P(X \geq 8) = 0.0267$, reject H_0 5 marks

19 (7 marks)

Lower tail: need $P(X \leq c) \leq 0.05$. $P(X \leq 0) = 0.0111 \leq 0.05$ but $P(X \leq 1) = 0.0611 > 0.05$ M1 A1

Upper tail: need $P(X \geq c) \leq 0.05$. $P(X \geq 9) = 0.0403 \leq 0.05$ but $P(X \geq 8) = 0.0866 > 0.05$ M1 A1

Critical region: $X \leq 0$ or $X \geq 9$ A1

Actual significance level = $0.0111 + 0.0403 = 0.0514$ B1

11 is in the critical region, so reject H_0 M1

Answer: $X \leq 0$ or $X \geq 9$, 0.0514; reject H_0 7 marks

20 (6 marks)

$H_0: \lambda = 36$, $H_1: \lambda < 36$; $X \approx N(36, 36)$ B1 M1

$P(X \leq 28) \approx P(Z \leq \frac{28.5 - 36}{6})$ M1

$= P(Z \leq -1.250) = 0.1056$ A1

$0.1056 > 0.05$, so do not reject H_0 M1

Insufficient evidence of a decrease in the mean A1

Answer: 0.1056: do not reject H_0 6 marks

21 (8 marks)

(a) Under H_0 , $X \sim \text{Po}(40)$ B1

$P(X \geq 52) = 1 - P(X \leq 51) = 0.0387$ (calculator) M1

$0.0387 < 0.05$, so reject H_0 A1

Answer: Reject H_0 3 marks

(b) For a Poisson distribution mean = variance = 40; 40 is large so $X \approx N(40, 40)$ B1

$P(X \geq 52) \approx P(Y > 51.5)$, $Y \sim N(40, 40)$ M1

$z = \frac{51.5 - 40}{\sqrt{40}} = 1.818$, probability 0.0345 M1

$0.0345 < 0.05$, so reject H_0 , agreeing with (a) A1

Answer: Reject H_0 4 marks

(c) The approximate probability 0.0345 is close to the exact value 0.0387 (difference 0.0042) E1

Answer: Close to the exact value**1 mark****22 (8 marks)****(a)** $H_0: \lambda = 10$, $H_1: \lambda < 10$ **B1** $X \sim \text{Po}(10)$ under H_0 **B1****Answer: $H_0: \lambda = 10$, $H_1: \lambda < 10$; $X \sim \text{Po}(10)$** **2 marks****(b)** $P(X \leq 4) = 0.0293 \leq 0.05$ **M1** $P(X \leq 5) = 0.0671 > 0.05$ **A1**Critical region $X \leq 4$; Type I error probability 0.0293**A1****Answer: $X \leq 4$; 0.0293****3 marks****(c)** True mean over 4 m is $4 \times 1.25 = 5$, so $X \sim \text{Po}(5)$ **B1**Fail to detect: $X \geq 5$ **M1** $P(X \geq 5) = 0.5595$ **A1****Answer: 0.5595****3 marks****23 (10 marks)****(a)** $H_0: \lambda = 0.25$, $H_1: \lambda > 0.25$ where λ = mean flaws per metre**B1** $X \sim \text{Po}(5)$ under H_0 **B1****Answer: $X \sim \text{Po}(5)$** **2 marks****(b)** Upper tail: need $P(X \geq c) \leq 0.05$. $P(X \geq 10) = 0.0318 \leq 0.05$ but $P(X \geq 9) = 0.0681 > 0.05$ **M1 A1**Critical region: $X \geq 10$ **A1**

Actual significance level = 0.0318

B1**Answer: $X \geq 10$, actual significance level 0.0318****4 marks****(c)** $P(X \geq 10)$ with $X \sim \text{Po}(10)$ **M1** $= 0.5421$ **A1****Answer: 0.5421****2 marks****(d)** $P(X = 0) = e^{-0.25L} < 0.05$ **M1** $L > 4 \ln 20 = 12.0$, so $L = 12$ m**A1****Answer: 12 m****2 marks****24 (8 marks)**Total = $0 \times 12 + 1 \times 21 + 2 \times 15 + 3 \times 8 + 4 \times 3 + 5 \times 1 = 92$, so mean = $\frac{92}{60} = 1.53$ **M1 A1** $H_0: \lambda = 1.4$, $H_1: \lambda < 1.4$; total T over 60 days, $T \sim \text{Po}(84) \approx N(84, 84)$ **B1 M1** $P(T \leq 92) \approx P(Z \leq \frac{92.5 - 84}{\sqrt{84}})$ **M1** $= P(Z \leq 0.927) = 0.8231$ **A1** $0.8231 > 0.05$, so do not reject H_0 **M1**

Insufficient evidence that the mean number of complaints has fallen

A1**Answer: mean 1.53; do not reject H_0** **8 marks****25 (8 marks)****(a)** $P(X \leq 1) + P(X \geq 12)$ under $\text{Po}(6)$ **M1** $= 0.0174 + 0.0201 = 0.0374$ **A1****(b)** $P(X \leq 1) + P(X \geq 12)$ under $\text{Po}(9) = 0.0012 + 0.1970$ **M1** $= 0.1982$ **A1****(c)** $T \sim \text{Po}(12)$ under H_0 : $P(T \leq 5) + P(T \geq 20) = 0.0203 + 0.0213$ **M1 A1** $= 0.0416$ **A1**

The significance level is larger than in (a); larger totals give more information and lets the test be tuned closer to the target level

E1**Answer: (a) 0.0374 (b) 0.1982 (c) 0.0416****8 marks****26 (8 marks)****(a)** $H_0: \lambda = 3.8$, $H_1: \lambda > 3.8$ where λ is the mean number of calls per minute**B1**Total T $\sim \text{Po}(30.4)$ under H_0 **B1****Answer: $T \sim \text{Po}(30.4)$** **2 marks**

(b) $T = 35$ **B1**

$$P(T \geq 35) = 1 - P(T \leq 34) = 0.2244$$

M1 A1

$0.2244 > 0.05$, so do not reject H_0

A1**Answer: 0.2244****4 marks****(c) The sum of independent Poisson variables with means 3.8 each is Poisson with mean 3.8×8** **E1**

Assumption: calls arrive independently and at a constant rate, and the intervals are independent

E1**Answer: Sum of independent Poisson is Poisson****2 marks****27 (10 marks)**

$$(a) \sum x f = 0 \times 3 + 1 \times 9 + 2 \times 14 + 3 \times 15 + 4 \times 10 + 5 \times 6 + 6 \times 2 + 7 \times 1 = 171$$

M1

$$\text{Mean} = \frac{171}{60} = 2.85$$

A1**Answer: 2.85****2 marks**

$$(b) \sum x^2 f = 0 + 9 + 56 + 135 + 160 + 150 + 72 + 49 = 631$$

M1

$$s^2 = \frac{1}{59}(631 - 60 \times 2.85^2) = 2.435$$

M1

For a Poisson distribution mean = variance, and 2.85 is close to 2.43, so the model is reasonable

E1**Answer: $s^2 = 2.435$** **3 marks**

(c) The sum of independent Poisson variables is Poisson with mean the sum of the means: $60 \times 2.5 = 150$

E1**Answer: Po(150)****1 mark**

(d) $H_0: \lambda = 2.5$, $H_1: \lambda > 2.5$ per interval; total $T \approx N(150, 150)$

B1

$$P(T \geq 171) \approx P(Y > 170.5)$$

M1

$$z = \frac{170.5 - 150}{\sqrt{150}} = 1.674, \text{ probability } 0.0471$$

A1

$0.0471 < 0.05$, so reject H_0 : evidence that the rate of emails has increased

A1**Answer: Reject H_0** **4 marks****28 (10 marks)**

$$(a) P(X \geq 10) = 0.0081 \leq 0.01$$

M1

$$P(X \geq 9) = 0.0214 > 0.01$$

A1

Critical region $X \geq 10$; Type I error probability 0.0081

A1**Answer: $X \geq 10$; 0.0081****3 marks**

$$(b) \lambda = 8: P(X \geq 10) = 0.2834$$

B1

$$\lambda = 12: P(X \geq 10) = 0.7576$$

B1**Answer: 0.2834 and 0.7576****2 marks**

(c) Need $P(X \geq 10) \geq 0.8$ for $X \sim \text{Po}(\lambda)$

M1

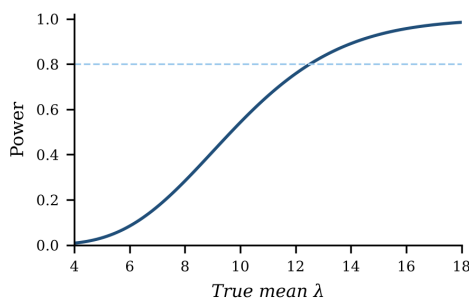
$$\lambda = 12: 0.7576 < 0.8; \lambda = 13: 0.8342 \geq 0.8$$

M1 A1**Answer: $\lambda = 13$** **3 marks**

(d) At $\lambda = 8$ the power is only 0.2834, so most such increases would go undetected

E1

Use a larger sample (several rolls, giving a larger mean) or a larger significance level to raise the power

E1**Answer: Not suitable: power only 0.2834****2 marks**

29 (10 marks)(a) $3t + 1 = 10$; under H_0 the number of cars in 3 minutes is $Po(6)$

M1

$$P(X \geq 10) = 1 - P(X \leq 9) = 0.0839$$

M1 A1

(b) $Po(10)$: $P(X \geq 16) = 0.0487$

M1 A1

(c) $t = 3$, $\lambda = 3.5$: $Po(10.5)$, $P(X \geq 10) = 0.6029$

M1 A1

 $t = 5$, $Po(17.5)$, $P(X \geq 16) = 0.6725$

M1 A1

The $t = 5$ test is much more likely to detect the increase (and has a similar significance level), so it is better; longer observation gives more information

E1

Answer: (a) 0.0839 (b) 0.0487 (c) 0.6029, 0.6725

10 marks

30 (10 marks)(a) Under H_0 , $X \sim Po(3)$: $P(X \geq 7) = 0.0335 \leq 0.05$, $P(X \geq 6) = 0.0839 > 0.05$

M1 A1

Critical region $X \geq 7$, actual significance level 0.0335

A1 B1

(b) Power = $P(X \geq 7 | \lambda = 6) = 0.3937$

M1 A1

(c) $P(X \geq 7 | \lambda) = 1 - e^{-\lambda}(1 + \lambda + \dots + \frac{\lambda^6}{6!}) \rightarrow 1$ as λ increases, because the exponential factor $e^{-\lambda}$ tends to zero

E1

Trial with the calculator: power(10.5) = 0.8984, power(10.6) = 0.9034

M1 A1

Smallest $\lambda = 10.6$

A1

Answer: (a) $X \geq 7$, 0.0335 (b) 0.3937 (c) $\lambda = 10.6$

10 marks

Correlation Hypothesis Testing**1 (2 marks)** $H_0: \rho = 0$, $H_1: \rho > 0$

B1

0.72 > 0.5494, so reject H_0 : there is evidence of positive correlation

B1

Answer: Reject H_0

2 marks

2 (3 marks) $H_0: \rho = 0$, $H_1: \rho < 0$

B1

Critical region is $r < -0.4409$

M1

-0.42 > -0.4409, so do not reject H_0 : insufficient evidence of negative correlation

A1

Answer: Do not reject H_0

3 marks

3 (2 marks)

ρ is the population product moment correlation coefficient between exercise time and resting heart rate

B1

 $H_0: \rho = 0$, $H_1: \rho < 0$

B1

Answer: $H_0: \rho = 0$, $H_1: \rho < 0$

2 marks

4 (2 marks)

(a) strong negative correlation

B1

(b) very weak (almost no linear) correlation

B1

Answer: (a) strong negative (b) almost none

2 marks

5 (2 marks) $H_0: \rho = 0$, $H_1: \rho > 0$; 0.62 > 0.5822

M1

Reject H_0 : evidence of positive correlation

A1

Answer: reject

2 marks

6 (2 marks) r measures only linear correlation

B1

There could be a non-linear (e.g. curved) relationship, such as a quadratic pattern, with r close to zero

E1

Answer: r only measures linear correlation

2 marks

7 (4 marks)

$H_0: \rho = 0, H_1: \rho \neq 0$ (two-tailed)

B1

Critical values are ± 0.5760

M1

$0.58 > 0.5760$

M1

Reject H_0 : there is evidence at the 5% level of a linear correlation between the variables

A1

Answer: Reject H_0

4 marks

8 (4 marks)

(a) $H_0: \rho = 0, H_1: \rho \neq 0$

B1

$0.988 > 0.6319$

M1

Reject H_0 : there is evidence of correlation between ice cream sales and drowning incidents

A1

Answer: Reject H_0

3 marks

(b) Correlation does not imply causation; both are likely to increase in hot weather (a third variable), when more people eat ice cream and swim

E1

Answer: Hot weather affects both

1 mark

9 (4 marks)

Let ρ = the population product moment correlation coefficient between maths and physics scores.

B1

$H_0: \rho = 0, H_1: \rho > 0$

Critical value 0.6215 for $n = 8$

M1

$r = 0.790$, and $r > 0.6215$

A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that there is positive correlation between maths and physics scores.

E1

Answer: critical value 0.6215, reject H_0

4 marks

10 (4 marks)

Let ρ = the population product moment correlation coefficient between mean temperature and

B1

sales of woollen hats. $H_0: \rho = 0, H_1: \rho < 0$

Critical value -0.3783 for $n = 20$

M1

$r = -0.460$, and $r < -0.3783$

A1

Reject H_0 : there is sufficient evidence at the 5% level to suggest that there is negative correlation between temperature and sales of woollen hats.

E1

Answer: critical value 0.3783, reject H_0

4 marks

11 (4 marks)

Let ρ = the population product moment correlation coefficient between rainfall and number of cars. $H_0: \rho = 0, H_1: \rho \neq 0$

B1

Critical value ± 0.5140 for $n = 15$

M1

$r = 0.430$, and $|r|$ is not beyond 0.5140

A1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that there is correlation between rainfall and traffic volume.

E1

Answer: critical value 0.5140, do not reject H_0

4 marks

12 (4 marks)

Let ρ = the population product moment correlation coefficient between height and shoe size. $H_0:$

B1

$\rho = 0, H_1: \rho > 0$

Critical value 0.4622 for $n = 25$

M1

$r = 0.350$, and r is not beyond 0.4622

A1

Do not reject H_0 : there is insufficient evidence at the 1% level to suggest that there is positive correlation between height and shoe size.

E1

Answer: critical value 0.4622, do not reject H_0

4 marks

13 (5 marks)

(a) $r = 0.7389$ (calculator)

B1

Answer: $r = 0.739$

1 mark

(b) It would not change: converting to percentages is a linear scaling of each variable, and r is unaffected by linear coding with positive multipliers	E1
Answer: r is unchanged	1 mark
(c) $H_0: \rho = 0, H_1: \rho > 0$	B1
$0.7389 > 0.6215$	M1
Reject H_0 : there is sufficient evidence at the 5% level of positive correlation between the marks on the two tests	A1
Answer: $r = 0.739 > 0.6215$: reject H_0	3 marks

14 (6 marks)

Using a calculator's statistics mode: $r = 0.7173$	M1 A1
$H_0: \rho = 0, H_1: \rho > 0$	B1
Critical value 0.6215	B1
$0.7173 > 0.6215$	M1
Reject H_0 : there is evidence at the 5% level of positive correlation between revision time and test score	A1
Answer: $r = 0.717$, reject H_0	6 marks

15 (6 marks)

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-415}{\sqrt{240.5 \times 1860}}$	M1
$= -0.6205$	A1
$H_0: \rho = 0, H_1: \rho \neq 0$	B1
Critical values ± 0.5760	B1
$-0.6205 < -0.5760$, so r is in the critical region	M1
Reject H_0 : evidence of (negative) correlation between age and resale value	A1
Answer: $r = -0.620$; reject H_0	6 marks

16 (5 marks)

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-71.3}{\sqrt{84.5 \times 190.2}} = -0.5624$	M1 A1
Let ρ be the population correlation between training time and running time: $H_0: \rho = 0, H_1: \rho < 0$	B1
$r = -0.5624 < -0.4973$	M1
Reject H_0 : there is sufficient evidence at the 5% level to suggest that there is negative correlation between training time and running time.	A1
Answer: $r = -0.562$, reject H_0	5 marks

17 (5 marks)

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-88.5}{\sqrt{210.4 \times 96.8}} = -0.6201$	M1 A1
Let ρ be the population correlation between age and price: $H_0: \rho = 0, H_1: \rho < 0$	B1
$r = -0.6201 < -0.5742$	M1
Reject H_0 : there is sufficient evidence at the 1% level to suggest that there is negative correlation between age and price.	A1
Answer: $r = -0.620$, reject H_0	5 marks

18 (5 marks)

$r = 0.9200$ (calculator)	B1
$H_0: \rho = 0, H_1: \rho > 0$	B1
$0.9200 > 0.6215$	M1
Reject H_0	A1
There is sufficient evidence at the 5% level of positive correlation between rainfall and yield	E1
Answer: $r = 0.920$: reject H_0	5 marks

19 (5 marks) $H_0: \rho = 0, H_1: \rho > 0$

B1

All 10: $0.313 < 0.5494$, so do not reject H_0

M1 A1

Without the point: $0.951 > 0.5822$, so reject H_0

M1

One point can change the conclusion: r is sensitive to outliers

E1

Answer: all: do not reject; without: reject

5 marks

20 (4 marks)Let ρ = the population product moment correlation coefficient between sleep time and memory test score. $H_0: \rho = 0, H_1: \rho \neq 0$

B1

Critical value ± 0.4683 for $n = 18$

M1

 $r = 0.370$, and $|r|$ is not beyond 0.4683

A1

Do not reject H_0 : there is insufficient evidence at the 5% level to suggest that there is correlation between hours of sleep and memory test score.

E1

Answer: critical value 0.4683, do not reject H_0

4 marks

21 (8 marks)(a) $r = 0.379$ (calculator)

B1

 $H_0: \rho = 0, H_1: \rho \neq 0$

B1

 $0.379 < 0.6319$

M1

Do not reject H_0 : no significant correlation

A1

Answer: $r = 0.379$; not significant

4 marks

(b) The point is not representative: the output is low because of illness, not training

E1

Without it $r = 0.989 > 0.6664$

M1

Reject H_0 : strong evidence of positive correlation

A1

Answer: $r = 0.989$; reject H_0

3 marks

(c) The PMCC is not resistant to outliers: a single point can change r from 0.99 to 0.38 and reverse the conclusion

E1

Answer: One outlier can change the conclusion

1 mark

22 (8 marks)(a) $H_0: \rho = 0, H_1: \rho > 0$

B1

From the table the critical value for $n = 8$ is 0.6215

B1

 $0.5 < 0.6215$, so do not reject H_0

B1

Answer: Do not reject H_0

3 marks

(b) The critical value for $n = 20$ is 0.3783

M1

 $0.5 > 0.3783$, so reject H_0 : evidence of positive correlation

A1

Answer: Reject H_0

2 marks

(c) Need critical value < 0.5 ; $n = 11$ gives 0.5214 and $n = 12$ gives 0.4973

M1

Smallest n is 12

A1

Answer: $n = 12$

2 marks

(d) A larger sample gives stronger evidence: a given r is less likely to arise by chance in a larger sample, so the critical value falls as n increases

E1

Answer: Critical value falls as n increases

1 mark

23 (8 marks)(a) $n = 20$: $0.35 < 0.3783$ so not significant

M1 A1

 $n = 40$: $0.35 > 0.2638$ so significant

A1

Critical values fall as n grows, since a given r is less likely to occur by chance in a large sample

E1

(b) Critical values by n : $n = 23$ gives 0.3515 (> 0.35) and $n = 24$ gives 0.3438 (< 0.35)

M1 A1

Smallest $n = 24$

A1

A significant result means the correlation is unlikely to be zero, not that it is strong: with $r = 0.35$ only about 12% of the variation is explained linearly

E1

Answer: $n = 24$

8 marks

24 (8 marks)

(a) $r = 0.9498$ (calculator)

B1

Answer: 0.950

1 mark

(b) $H_0: \rho = 0, H_1: \rho > 0$

B1

$0.9498 > 0.5494$

M1

Reject H_0 : evidence of positive correlation between age and blood pressure

A1

Answer: reject H_0

3 marks

(c) Correlation does not imply causation

E1

Other factors (diet, weight, genetics) also affect blood pressure, and the sample of ten patients is small

E1

Answer: Correlation is not causation

2 marks

(d) 70 is far outside the range of the data (12 to 36): extrapolation is unreliable

E1

The relationship may not be linear outside the observed range

E1

Answer: extrapolation

2 marks

25 (7 marks)

(a) $H_0: \rho = 0, H_1: \rho \neq 0; 0.53 < 0.5324$, so do not reject H_0

B1 M1 A1

(b) One-tailed is acceptable only if the direction of correlation was predicted before the data were collected

E1

Choosing the tail after seeing the sign of r effectively doubles the true significance level

E1

(c) $0.53 < 0.6614$, so H_0 is no longer rejected at the 1% level

M1 A1

Answer: (a) do not reject (c) do not reject

7 marks

26 (7 marks)

(a) $H_0: \rho = 0, H_1: \rho > 0; 0.941 > 0.6215$

M1

Reject H_0 : evidence of positive correlation

A1

Answer: reject H_0

2 marks

(b) The scatter diagram shows a curved (exponential-type) pattern rather than a straight line

E1

A significant r only shows linear association is unlikely to be zero, not that the relationship is linear

E1

Answer: pattern is curved

2 marks

(c) $0.9997 > 0.6215$, so reject H_0

M1 A1

$\ln y$ is very close to linearly related to x , suggesting $y = ab^x$ (exponential growth)

E1

Answer: reject H_0 ; $y = ab^x$

3 marks

27 (10 marks)

(a) $r = \frac{96}{\sqrt{60 \times 250}} = 0.7838$

M1 A1

$0.7838 > 0.5494$, so reject H_0 : evidence of positive correlation

A1

Answer: $r = 0.784$; reject H_0

3 marks

(b) $S_{uu} = 2^2 S_{xx} = 240, S_{vv} = (-5)^2 S_{yy} = 6250$

M1 A1

$S_{uv} = 2 \times (-5) \times S_{xy} = -960, r_{uv} = \frac{-960}{\sqrt{240 \times 6250}} = -0.7838$

A1

Answer: $r_{uv} = -0.784$

3 marks

(c) Adding constants does not change deviations from the mean; so $S_{uu} = a^2 S_{xx}, S_{vv} = c^2 S_{yy}, S_{uv} = ac S_{xy}$

M1 A1

$$r_{uv} = \frac{ac S_{xy}}{\sqrt{a^2 c^2 S_{xx} S_{yy}}} = \frac{ac}{|ac|} \times \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{ac}{|ac|} r_{xy}$$

A1

Answer: shown

3 marks

(d) Here $ac < 0$, so the sign of r changes; a positive relation between x and y becomes a negative one between u and v . The size of r does not depend on the units, only the direction can reverse

E1

Answer: Sign reverses ($ac < 0$); size unchanged

1 mark

28 (10 marks)**(a)** $H_0: \rho = 0$, $H_1: \rho \neq 0$ for each machine **B1**Machine A: $0.182 < 0.8114$, do not reject H_0 **M1**Machine B: $0.381 < 0.8114$, do not reject H_0 ; no significant correlation within either machine **A1****Answer: Neither is significant** **3 marks****(b)** $r = 0.902$ (calculator) **M1** $0.902 > 0.5760$ **M1**Reject H_0 : significant correlation when the readings are pooled **A1****Answer: $r = 0.902$; reject H_0** **3 marks****(c)** The two machines form two separate clusters: Machine B has higher readings of both x and y , which produces an apparent linear trend across the clusters **E1**Within each machine there is no clear relationship, so the pooled correlation is caused by the machine (a hidden variable), not by x and y affecting each other **E1****Answer: Clusters create apparent correlation** **2 marks****(d)** Identify and separate the groups (stratify the data) and test within each group **E1**Collect more data within each group because six points give little power **E1****Answer: Separate the groups; use more data** **2 marks****29 (8 marks)****(a)** The data are symmetrical about $x = 5.5$, so $S_{xy} = \Sigma(x - 5.5)(y - \bar{y}) = 0$ by symmetry (pairs cancel) **M1 A1**So $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0$ (calculator gives 0.0000) **A1****(b)** $r = 0$ is never beyond a critical value (all critical values are positive), so $H_0: \rho = 0$ is not rejected **M1 A1**The relationship is perfect but non-linear (quadratic), and r measures only linear association **E1****(c)** Use $u = (x - 5.5)^2$ as the explanatory variable: then $y = u$ exactly, so $r = 1$ **M1 A1****Answer: (a) $r = 0$ (c) $u = (x - 5.5)^2$, $r = 1$** **8 marks****30 (9 marks)****(a)** $t^* = r^* \sqrt{\frac{n-2}{1-r^{*2}}}$, so $t^{*2}(1-r^{*2}) = r^{*2}(n-2)$ **M1 A1** $t^{*2} = r^{*2}(n-2+t^{*2})$, so $r^* = \frac{t^*}{\sqrt{n-2+t^{*2}}}$ **M1 A1****(b)** $r^* = \frac{1.8595}{\sqrt{8+1.8595^2}} = \frac{1.8595}{\sqrt{11.4577}} = 0.5493$ **M1 A1****(c)** For large n , t^* tends to the normal critical value z^* , a constant, so $r^* \approx \frac{z^*}{\sqrt{n}}$ **M1**This decreases as n increases, so a smaller correlation is needed to reject H_0 **A1**Hence the tabulated one-tailed 5% value for $n = 10$ is 0.5494 to 4 d.p. **B1****Answer: $r^* = t^*/\sqrt{n-2+t^{*2}}$; 0.5494** **9 marks**