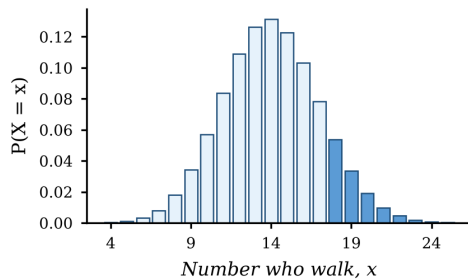


Null and Alternative Hypotheses

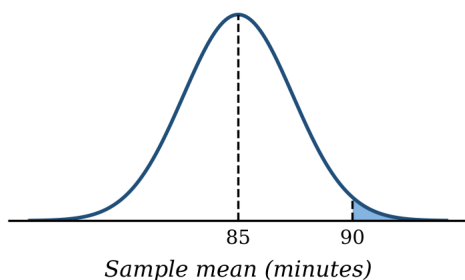
- 1 A coin is tossed to test whether it is biased. Let p be the probability that the coin lands heads. State suitable null and alternative hypotheses for the test. [2]
- 2 A sweet manufacturer claims that 20% of its sweets are red. A student thinks that the proportion of red sweets is actually higher than this. Define a parameter p and state the hypotheses the student should use. [2]
- 3 A tennis player says that she wins 60% of her first serves. Her coach thinks the true proportion is lower. Define the parameter p and state suitable hypotheses. [2]
- 4 The mean daily rainfall in a town in May is recorded as 2.1 mm. A meteorologist wants to test whether it has changed. Define the parameter μ and state suitable hypotheses. [2]
- 5 The number of customers entering a shop in one minute is modelled by a Poisson distribution with mean 3. The owner thinks that a new sign will increase the mean. Define the parameter λ and state suitable hypotheses. [2]
- 6 A scientist collects data on the age and the cholesterol level of 20 adults. She wants to test whether there is positive correlation between the two quantities in the population. State suitable hypotheses, defining any symbol you use. [2]
- 7 For each situation, write down suitable hypotheses and state whether the test is one-tailed or two-tailed.
 - (a) Previously 70% of patients were cured by a standard drug. A trial is carried out to see whether a new drug gives a higher cure rate. Let p be the cure rate for the new drug. [2]
 - (b) A machine is set to fill bags with a mean mass of 500 g. An inspector suspects that the mean mass of the bags is no longer 500 g. Let μ g be the population mean mass. [2]
- 8 A greenhouse owner claims that 25% of her tomatoes split before ripening. A sample of 20 tomatoes is taken and 7 of them have split. The owner wishes to test whether the proportion of split tomatoes has changed. Let p be the proportion of tomatoes that split. Two students write down hypotheses:
 - (a) Ben writes $H_0: p \neq 0.25$, $H_1: p = 0.25$. Explain what is wrong and write down the correct hypotheses. [2]
 - (b) Cara writes $H_0: p = 0.25$, $H_1: \bar{p} = \frac{7}{20}$. Explain what is wrong. [2]
- 9 Explain what is meant by (a) a null hypothesis, (b) an alternative hypothesis. [2]
- 10 A student writes four pairs of hypotheses for tests about a population proportion p . State which are valid and, for each invalid pair, give a reason.
 - (i) $H_0: p = 0.4$, $H_1: p > 0.4$ (ii) $H_0: p > 0.4$, $H_1: p = 0.4$ (iii) $H_0: p = 0.4$, $H_1: \bar{p} = 0.5$ (iv) $H_0: p = 0.4$, $H_1: p \neq 0.45$ [4]
- 11 A sample of 25 apples from an orchard is checked. The grower has always said that 12% of his apples are bruised. An inspector suspects that the proportion is different from 12%. Define the parameter p and state suitable hypotheses. [2]
- 12 A test is carried out on a population mean using the hypotheses $H_0: \mu = 35$ and $H_1: \mu > 35$.
 - (a) State, with a reason, whether this is a one-tailed or a two-tailed test.
 - (b) Write down the hypotheses for the corresponding two-tailed test. [3]

- 13** Define the term "test statistic" and write down the test statistic and its distribution under H_0 for a binomial test with $H_0: p = 0.35$ based on a sample of 40 items. [3]

- 14** A teacher claims that 35% of the students in her school walk to school. A random sample of 40 students is taken and 18 of them walk to school. Let p be the proportion of all students who walk to school. The diagram shows the probability distribution of the number of walkers in a sample of 40 if the teacher is correct. The shaded bars show 18 or more walkers.



- (a) The head of year thinks the proportion is different from 35%. State the hypotheses for a test of this belief. [2]
 (b) Write down the distribution of the number of walkers X in the sample if H_0 is true. [1]
 (c) Calculate $P(X \geq 18)$ under H_0 . Give your answer to 3 significant figures. [2]
- 15** The time, X minutes, that a patient waits in a clinic is normally distributed with standard deviation 12. The clinic manager believes that the mean waiting time has increased from its previous value of 85 minutes. A random sample of 25 patients has a mean waiting time of 90 minutes. The diagram shows the distribution of the sample mean when the mean waiting time is 85 minutes.



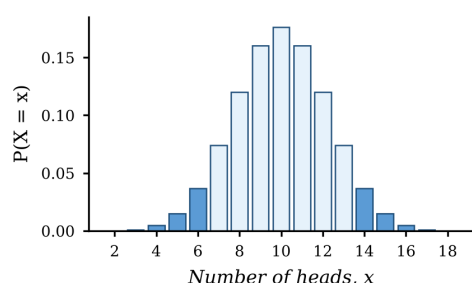
- (a) State suitable hypotheses for a test of the manager's belief. [2]
 (b) Find the distribution of the sample mean $\bar{X}$ under H_0 . [2]
 (c) Find $P(\bar{X} \geq 90)$ under H_0 , giving your answer to 3 significant figures. [2]
- 16** For each claim, write down suitable hypotheses (defining the parameter) and state whether the test should be one-tailed or two-tailed.
- (a) "At least 90% of train services arrive on time." A passenger group thinks that the proportion is lower. [2]
 (b) A bakery says its loaves have mean mass 800 g. A trading standards officer wants to check whether the mean mass is correct. [2]
 (c) Emails arrive at a server at a mean rate of 12 per hour. After a change in software the manager thinks the rate has risen. [2]
- 17** A hypothesis test with $H_0: p = 0.4$ and $H_1: p > 0.4$ gives a p-value of 0.032. Here p is the proportion of customers of a bank who use mobile banking. Write down your conclusion in context if the test was carried out at
- (a) the 5% level
 (b) the 1% level. [4]

- 18** In a two-tailed test of $H_0: p = 0.5$ against $H_1: p \neq 0.5$ at the 5% significance level, a student finds $P(X \leq 4) = 0.034$ for the observed value. The student concludes: "0.034 < 0.05, so reject H_0 ."
Explain the error and write down the correct conclusion. [3]

- 19** A test of $H_0: p = 0.25$ is carried out and H_0 is not rejected. Khalid says: "This proves that $p = 0.25$."
Comment on Khalid's statement. [2]

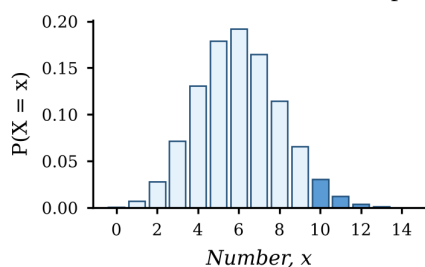
- 20** A headteacher wants to test whether more than 30% of students in her school cycle to school. She stands at the school gate one morning in January and asks the first 20 students who arrive by bike whether they cycle every day.
Give two reasons why this sample is not suitable for the test. [2]

- 21** Ravi tosses a coin 20 times to test whether it is fair, where p is the probability of heads. He gets 14 heads. Ravi looks at his results and says: "There are more heads than tails, so I will use the alternative hypothesis $p > 0.5$."
The diagram shows the distribution of the number of heads X when $p = 0.5$. The shaded bars show $X \leq 6$ and $X \geq 14$.



- (a) Explain why Ravi's choice of alternative hypothesis is not appropriate. [1]
 (b) Write down the hypotheses Ravi should use if he wants to test whether the coin is fair. [1]
 (c) Find $P(X \geq 14)$ when $p = 0.5$. [2]
 (d) Find the probability of a result at least as extreme as 14 heads in either direction, that is $P(X \leq 6) + P(X \geq 14)$. [2]
 (e) Use your answers to explain how Ravi's choice affects the strength of evidence he appears to have. [1]
- 22** A café owner claims that 40% of customers use a loyalty card. A manager believes that the proportion is lower than this. She carries out a hypothesis test at the 5% significance level using a random sample of customers, and the test statistic gives a p-value of 0.032.
Two colleagues give conclusions.
- (a) Anil says: "The p-value shows that the café owner's claim is definitely false." Explain why this is not correct. [2]
 (b) Bea says: "There is a 0.032 probability that H_0 is true." Explain why this is not correct. [2]
 (c) Write down the correct conclusion of the test, in context. [2]
 (d) State what the conclusion would be if the test were carried out at the 1% level. [1]

- 23** Amira plays an online game. The game says that she should win 30% of her matches. In a random sample of 20 matches she wins 10. Let p be the probability that she wins a match and $X \sim B(20, p)$. The diagram shows the distribution of X when $p = 0.3$ and the shaded bars are $X \geq 10$.



- (a) Find $P(X \geq 10)$ when $p = 0.3$. [1]
- (b) Amira believes the game is easier than claimed. Test this at the 5% level. [3]
- (c) Her friend Zane wants to know if her win rate is different from 0.3, with the same data. State the hypotheses, and conclusion at the 5% level. [3]
-
- 24** Two researchers use the same data to test a claim that 20% of components are faulty: in a random sample of 25 components, 9 are faulty. Researcher A uses $H_1: p > 0.2$. Researcher B uses $H_1: p \neq 0.2$. Both use the 5% significance level. Show that the two researchers reach different conclusions, and state which researcher's hypotheses suit a claim that the fault rate has increased. [6]
-
- 25** A pharmacist says that 15% of patients who take a certain tablet suffer from drowsiness. A doctor believes that the figure is too low. She asks 30 randomly chosen patients taking the tablet and 8 report drowsiness.
- (a) Define a suitable parameter and state the hypotheses. [2]
- (b) State the distribution of the number of patients with drowsiness, X , under H_0 . [1]
- (c) Find $P(X \geq 8)$ and state your conclusion at the 5% level. [3]
- (d) Explain why the doctor must not use the 8 patients to decide the form of H_1 after seeing the results. [1]
-
- 26** A fair six-sided die is rolled 24 times and the number of sixes X is recorded. A test of $H_0: p = \frac{1}{6}$ against $H_1: p > \frac{1}{6}$ is to be carried out, where p is the probability of a six. Find the smallest number of sixes that would lead to rejection of H_0 at the 10% level, and find the actual probability of rejecting H_0 wrongly using that rule. [5]
-
- 27** In a trial, a new treatment is given to 10 patients and 3 of them recover. Past records show that the probability of recovery without treatment is 0.5. The researcher wants to know whether the treatment is worse than no treatment. Let X be the number of recoveries among 10 patients and p the recovery probability with the treatment.
- (a) Test, at the 5% significance level, $H_0: p = 0.5$ against $H_1: p < 0.5$, stating your conclusion. [2]
- (b) The researcher now tests $H_0: p = p_0$ against $H_1: p < p_0$ for different values of p_0 , using the same data. Find, to 3 significant figures, the least value of p_0 for which H_0 is rejected at the 5% level. [4]
- (c) Find $P(X \leq 3)$ when $p = 0.3$ and use it, together with your answer to (b), to explain why failing to reject $H_0: p = 0.5$ in part (a) does not show that the recovery probability is 0.5. [3]

- 28** A researcher carries out 20 independent hypothesis tests, each at the 5% significance level. Unknown to her, the null hypothesis is true in every one of the tests. Let Y be the number of tests in which H_0 is (wrongly) rejected.
- State the distribution of Y and find the probability that at least one null hypothesis is rejected. Give your answer to 3 significant figures. [2]
 - Find the expected number of tests in which H_0 is rejected. [1]
 - The researcher reports every significant result as a genuine discovery. Use your answers to explain why this is unsafe, and suggest one way of improving her method. [3]
 - Find the significance level α that each test should use so that the probability of at least one wrong rejection among the 20 tests is 0.05. Give your answer to 3 significant figures. [4]
-
- 29** A researcher carries out k independent hypothesis tests, each at the 5% significance level, and all the null hypotheses are true.
- Show that the probability that at least one test is (wrongly) significant is $1 - 0.95^k$.
 - Find the least value of k for which this probability exceeds 0.5.
 - With $k = 8$ tests, find the significance level α that should be used for each test so that the probability of at least one wrong rejection is exactly 0.05, giving your answer to 3 significant figures. [8]
-
- 30** A coin is tossed n times, giving X heads. A two-tailed test of $H_0: p = 0.5$ is carried out and the observed value is $x < \frac{n}{2}$.
- Explain why the two-sided p -value is $2P(X \leq x)$ when $p = 0.5$.
 - For $n = 30$ and $x = 10$, find the two-sided p -value to 3 significant figures and state the conclusion at the 5% level.
 - Find the largest value of x (for $n = 30$) for which H_0 is rejected at the 5% level. [8]

Significance Levels and Critical Regions

- 1** A test has $H_0: p = 0.3$ and $H_1: p < 0.3$, using $X \sim B(10, 0.3)$ at the 5% significance level. You are given that $P(X \leq 0) = 0.0282$ and $P(X \leq 1) = 0.1493$. Find the critical region for the test. [2]
-
- 2** $X \sim B(15, 0.4)$. The cumulative probabilities are given in the table. A test of $H_0: p = 0.4$ against $H_1: p < 0.4$ is carried out at the 10% significance level. Find the critical region, and state the actual probability of rejecting H_0 when it is true.
- | x | 0 | 1 | 2 | 3 | 4 | 5 |
|---------------|--------|--------|--------|--------|--------|--------|
| $P(X \leq x)$ | 0.0005 | 0.0052 | 0.0271 | 0.0905 | 0.2173 | 0.4032 |
- [3]
-
- 3** Explain what is meant by (a) the critical region, (b) the significance level of a hypothesis test. [2]
-
- 4** A coin is tossed 10 times, with X the number of heads. A test of $H_0: p = 0.5$ against $H_1: p < 0.5$ is carried out at the 5% significance level, using $X \sim B(10, 0.5)$ under H_0 . Find the critical region for the test and state the actual significance level.

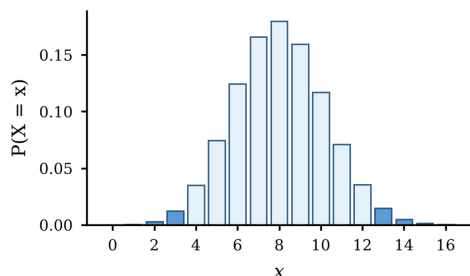
x	0	1	2	3
$P(X \leq x)$	0.0010	0.0107	0.0547	0.1719

[4]

5 $Z \sim N(0, 1)$. Write down the critical value(s) of z , to 3 decimal places, for (a) a one-tailed upper test at the 1% level, (b) a two-tailed test at the 5% level. [2]

6 A test of $H_0: p = 0.4$ against $H_1: p > 0.4$ using $X \sim B(15, p)$ has critical region $X \geq 10$. For each observed value of X , state whether H_0 is rejected: (a) $X = 9$, (b) $X = 12$. [2]

7 The random variable $X \sim B(20, p)$. A two-tailed test of $H_0: p = 0.4$ against $H_1: p \neq 0.4$ is carried out at the 5% significance level.
Find the critical region for the test. The diagram shows the distribution of X when $p = 0.4$.



[4]

8 A test of $H_0: p = 0.25$ against $H_1: p \neq 0.25$ uses the statistic $X \sim B(25, p)$. The test rejects H_0 if $X \leq 1$ or $X \geq 12$.

(a) Find the probability of rejecting H_0 when $p = 0.25$. This is the actual significance level of the test. [3]

(b) Explain why the actual significance level is less than the nominal 5%. [1]

9 A manufacturer believes that the proportion of faulty items has increased from 0.2.
A test of $H_0: p = 0.2$ against $H_1: p > 0.2$ is carried out at the 5% significance level, using $X \sim B(20, 0.2)$ under H_0 .
Find the critical region for the test and state the actual significance level.

x	5	6	7	8
$P(X \leq x)$	0.8042	0.9133	0.9679	0.9900

[4]

10 A test of $H_0: p = 0.15$ against $H_1: p \neq 0.15$ is based on $X \sim B(40, p)$ under H_0 . The critical region is to be chosen so that the probability of rejecting H_0 in each tail is as close as possible to 2.5%.

Find the critical region and state the actual significance level of the test. [6]

11 The number of faults per metre of wire is modelled by a Poisson distribution.
A test of $H_0: \lambda = 3$ against $H_1: \lambda > 3$ is carried out at the 5% significance level, using $X \sim \text{Po}(3)$ under H_0 .

Find the critical region and state the actual significance level.

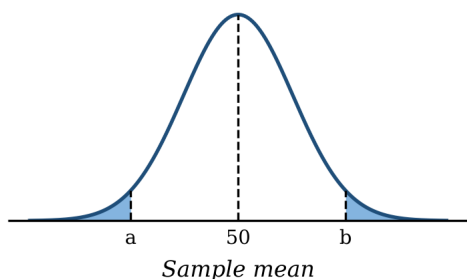
x	4	5	6	7
$P(X \leq x)$	0.8153	0.9161	0.9665	0.9881

[4]

12 A machine fills tubs with ice cream. The mass of a tub is normally distributed.
 $H_0: \mu = 500$ is tested against $H_1: \mu < 500$ at the 5% significance level using the mean $\bar{X}$ of a random sample of size 16, where the population standard deviation is 8 g.
Find the critical region for $\bar{X}$, giving values to 3 significant figures. [4]

- 13** A test of $H_0: p = 0.25$ against $H_1: p > 0.25$ uses $X \sim B(20, p)$ and the critical region $X \geq 9$. Find the actual significance level of the test, and the probability of rejecting H_0 when in fact $p = 0.4$. [4]

- 14** The random variable X is normally distributed with mean μ and standard deviation 6. A random sample of 36 observations is taken. The sample mean $\bar{X}$ is used to test $H_0: \mu = 50$ against $H_1: \mu \neq 50$ at the 5% significance level. The critical region is shown shaded, with boundaries a and b .



- (a) Write down the distribution of $\bar{X}$ when H_0 is true. [2]
 (b) Find the values of a and b , and hence the critical region. Give your answers to 4 significant figures. [3]

- 15** A seed company says that 20% of its seeds produce red flowers. A gardener suspects the proportion is higher than this. She plants 25 seeds, and X of them produce red flowers. She will test the claim at the 5% significance level, where p is the proportion of seeds producing red flowers.
 (a) State the hypotheses and the distribution of X under H_0 . [2]
 (b) Find the critical region for X . [3]
 (c) Find the actual significance level of the test, and state what this probability means in context. [1]

- 16** A pollster claims that 30% of people in a town recycle all their glass. An environmental group thinks the proportion is different. A test of $H_0: p = 0.3$ against $H_1: p \neq 0.3$ is carried out at the 10% significance level, using $X \sim B(40, 0.3)$ under H_0 .

Find the critical region for the test and state the actual significance level.

x	5	6	7	8	15	16	17	18
$P(X \leq x)$	0.0086	0.0238	0.0553	0.1110	0.8849	0.9367	0.9680	0.9852

[6]

- 17** The number of enquiries received by a helpline in one day is modelled by a Poisson distribution with mean λ . A two-tailed test of $H_0: \lambda = 8$ against $H_1: \lambda \neq 8$ is carried out using the number of enquiries X on a single day. The critical region is chosen so that the probability of rejecting H_0 in each tail is as close as possible to 3%. Find the critical region and the probability of a Type I error. [6]

- 18** A random variable X is normally distributed with standard deviation 9. A two-tailed test of $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ is based on the mean $\bar{X}$ of a random sample of size 12. The critical region is $\bar{X} < 55.73$ or $\bar{X} > 64.27$.
 (a) Write down the value of μ_0 . [1]
 (b) Find the significance level of the test. [3]
 (c) Explain whether the central limit theorem was needed in this question. [2]

- 19** A drug is known to cure 35% of patients with an infection. A doctor wants to test whether a new batch of the drug is less effective. She gives it to 25 patients and records the number X cured.
- (a) Find the critical region for a test at the 5% level, and the actual significance level. [3]
- (b) Only 5 patients are cured. State the conclusion of the test. [2]
-
- 20** A test uses $X \sim B(12, p)$. $H_0: p = 0.3$ is rejected in favour of $H_1: p > 0.3$ if $X \geq 7$.
- (a) Find the probability of a Type I error.
- (b) Find the probability of a Type II error when the true value is $p = 0.6$. Explain what a Type II error is. [5]
-
- 21** A factory states that 2% of its fuses are faulty. A quality inspector tests the claim by taking a random sample of 250 fuses and counting the number X that are faulty. She will test $H_0: p = 0.02$ against $H_1: p \neq 0.02$ at the 5% significance level, where p is the proportion of faulty fuses.
- (a) Explain why X can be approximated by a Poisson distribution and write down the approximating distribution under H_0 . [2]
- (b) Find the critical region for a two-tailed test at the 5% significance level using the Poisson approximation, with at most 2.5% in each tail. [4]
- (c) Find the actual significance level using the Poisson approximation, and compare it with the exact value found using the binomial distribution. [2]
-
- 22** The scores on a skills test are normally distributed with standard deviation 8. The mean score is believed to be 50. A trainer thinks a new method of teaching will change the mean. A random sample of n students taught by the new method has a mean score of 52.
- (a) The trainer tests $H_0: \mu = 50$ against $H_1: \mu > 50$ at the 5% level. Find the smallest value of n for which the sample mean of 52 is in the critical region. [4]
- (b) The trainer instead tests $H_0: \mu = 50$ against $H_1: \mu \neq 50$ at the 5% level. Find the smallest value of n for which the sample mean of 52 is significant. [4]
-
- 23** A test of $H_0: p = 0.3$ against $H_1: p > 0.3$ at the 5% level is based on $X \sim B(120, p)$.
- (a) Use a normal approximation to find the critical region for X .
- (b) Using your calculator with the binomial distribution, find the exact critical region and the actual significance level. [8]
-
- 24** Scores on a test are normally distributed with standard deviation 15. $H_0: \mu = 100$ is tested against $H_1: \mu \neq 100$ at the 5% level using the mean of a random sample of size n . The critical region is $\bar{X} \leq a$ or $\bar{X} \geq b$.
- (a) Show that $b = 100 + \frac{1.96 \times 15}{\sqrt{n}}$ and write down a .
- (b) Find the smallest n for which the upper critical value is at most 105. [6]
-
- 25** The number of flaws X in a sheet of glass is modelled by a Poisson distribution. The manufacturer says the mean number of flaws per sheet is 6. A quality inspector carries out a two-tailed test at the 5% level using the number of flaws in one sheet.
- (a) Find the critical region and actual significance level. [6]
- (b) The process changes and the true mean becomes 9. Find the probability that the test correctly rejects H_0 . [3]
-
- 26** A test of $H_0: p = 0.5$ against $H_1: p > 0.5$ uses $X \sim B(18, p)$. The critical region is $X \geq c$. Find the least value of c such that the significance level of the test does not exceed 1%, and the actual significance level for that c . Explain why the actual significance level is lower than 1%. [5]

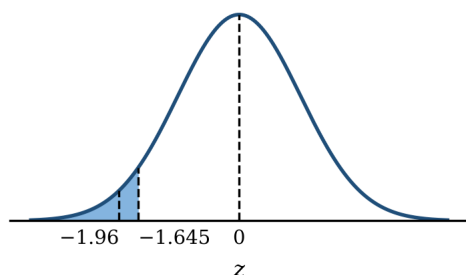
- 27** A two-tailed test of $H_0: p = 0.5$ against $H_1: p \neq 0.5$ is carried out at the 5% significance level using $X \sim B(20, p)$. The power of a test at a particular value of p is the probability that H_0 is rejected when p has that value.
- Find the critical region for the test. [3]
 - Find the actual significance level of the test. [1]
 - Find the power of the test when $p = 0.3$. [2]
 - Find the power of the test when $p = 0.8$, and explain why it is larger than the power in (c). [2]
 - Describe how the power of the test changes as p moves away from 0.5, and say what its value is at $p = 0.5$. [2]
-
- 28** The random variable $X \sim B(15, p)$. A test of $H_0: p = 0.4$ against $H_1: p > 0.4$ is based on X . The test has the form "reject H_0 if $X \geq c$ " and is carried out at significance level α , where the critical region is chosen as large as possible subject to $P(\text{Type I error}) \leq \alpha$.
- Show that the critical region is $X \geq 10$ for all values of α satisfying $0.0338 \leq \alpha < 0.0950$. [4]
 - The true value of p is 0.6. Find the probability of a Type II error when the critical region is $X \geq 10$. [2]
 - The researcher decides to use $\alpha = 0.10$ instead. State the critical region and find the new probability of a Type II error when $p = 0.6$. [2]
 - Use your answers to (b) and (c) to comment on the relationship between the significance level and the probability of a Type II error. [2]
-
- 29** A test of $H_0: p = 0.3$ against $H_1: p > 0.3$ is carried out at the 5% level using $X \sim B(20, p)$.
- Find the critical region.
 - Find the probability of rejecting H_0 when $p = 0.4, 0.5$ and 0.6 , and the probability of a Type II error in each case.
 - Comment on how the probability of a Type II error changes as the true value of p moves further from 0.3, and suggest how it could be reduced. [10]
-
- 30** A coin is suspected to favour heads. To test $H_0: p = 0.5$ against $H_1: p > 0.5$ at the 5% level a sample of n tosses is used, with n at least 10. A test is said to be adequate if it rejects H_0 with probability at least 0.8 when $p = 0.65$.
- For $n = 20$, find the critical region and the probability of rejecting H_0 when $p = 0.65$. Hence state whether $n = 20$ is adequate.
 - Find the smallest n (with a corresponding critical region) for which the test is adequate, giving the critical region and the probability of rejection. [11]

One-Tailed and Two-Tailed Tests

- 1** State whether each test is one-tailed or two-tailed, and if one-tailed say which tail.
- A company wants to know if the mean delivery time has changed. [1]
 - A farmer thinks that a new fertiliser has increased the proportion of seeds that germinate. [1]
 - A school claims that fewer than 10% of its students arrive late. An inspector checks whether the true proportion is lower than 10%. [1]
-
- 2** Z is a standard normal variable. Using your calculator, write down, to 3 decimal places, the critical z -values for a test at the 5% significance level when the test is
- one-tailed (upper tail); [1]
 - two-tailed. [1]

- 3** State whether each test is one-tailed or two-tailed.
(a) $H_0: p = 0.2$, $H_1: p < 0.2$ (b) $H_0: \mu = 12$, $H_1: \mu \neq 12$ (c) $H_0: \rho = 0$, $H_1: \rho \neq 0$ (d) $H_0: \lambda = 4$, $H_1: \lambda > 4$ [2]
- 4** $Z \sim N(0, 1)$. Find the critical value(s) of z , to 3 decimal places, for a test at the 10% significance level when the test is (a) one-tailed (upper), (b) two-tailed. [2]
- 5** In a two-tailed test of $H_0: p = 0.3$ at the 5% level, $X \sim B(10, 0.3)$ and the observed value is 0. You are given that $P(X \leq 0) = 0.0282$. State, with a reason, whether H_0 is rejected. [2]
- 6** A test has $H_0: p = 0.5$, $H_1: p > 0.5$ and is carried out at the 5% level. The p-value of the observed result is 0.0577. State the conclusion in the context where p is the probability that a newborn baby in a hospital is a girl. [2]
- 7** A coin is tossed 10 times and lands heads 9 times. Test, at the 5% significance level, whether the coin is biased, stating your hypotheses and conclusion. [4]
- 8** A spinner is supposed to land on blue with probability 0.3. In 20 spins it lands on blue 10 times. Let p be the probability that the spinner lands on blue. Let X be the number of blues in 20 spins.
(a) Test $H_0: p = 0.3$ against $H_1: p > 0.3$ at the 5% significance level. [2]
(b) Test $H_0: p = 0.3$ against $H_1: p \neq 0.3$ at the 5% significance level, and comment on your answer to (a). [2]
- 9** A pitch-and-putt player normally holes 40% of her putts from 3 metres. After a coaching session she holes 3 putts out of 15 and thinks the coaching has made her worse. Test, at the 5% significance level, whether her probability of holing a putt has decreased. [5]
- 10** A newsagent says that 10% of his customers buy a lottery ticket. A competitor thinks the proportion is higher. In a random sample of 20 customers, 5 buy a lottery ticket. Test, at the 5% significance level, whether the proportion of customers who buy a lottery ticket is greater than 0.1. [5]
- 11** A sports centre has averaged 2 equipment failures per week. After a refit, 6 failures occur in a randomly chosen week. Test, at the 5% significance level, whether the failure rate has increased. [5]
- 12** A sample of 12 houses has product moment correlation coefficient $r = 0.61$ between floor area and selling price. The one-tailed critical value of the product moment correlation coefficient for $n = 12$ at the 5% significance level is 0.4973. Test, at the 5% significance level, whether there is positive correlation between floor area and selling price. [4]
- 13** The mass of a pack of butter is normally distributed with standard deviation 4 g. The packs are labelled 250 g. A customer weighs a random sample of 16 packs and finds that the sample mean is 247.8 g. Test, at the 5% significance level, whether the mean mass of a pack is less than 250 g. [5]
- 14** A call centre claims that 30% of calls are answered within 10 seconds. A customer group thinks that the proportion is higher. In a random sample of 20 calls, only 1 was answered within 10 seconds. A student argues: " $P(X \leq 1) = 0.0076$ which is less than 0.05, so the result is significant and the claim is rejected in favour of the customer group's belief."
(a) Write down the hypotheses and calculate the probability that is appropriate for this test. [3]
(b) Explain the student's error and state the correct conclusion. [2]

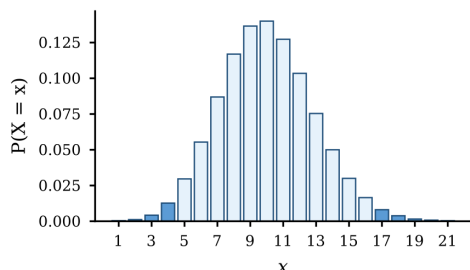
- 15** A machine fills bags of flour. The mass of a bag is normally distributed with standard deviation 4 g. The mean mass is meant to be 250 g. A random sample of 25 bags has a mean mass of 248.6 g. The diagram shows the standard normal curve with the 5% one-tailed critical value (shaded region) and the two-tailed critical value marked.



- (a) Test at the 5% significance level whether the mean mass is less than 250 g. [3]
 (b) Test at the 5% significance level whether the mean mass is different from 250 g. [2]
 (c) Explain why the two conclusions differ. [1]
-
- 16** A call-centre manager says that 60% of calls are resolved at the first attempt. An analyst thinks the proportion has changed. In a random sample of 25 calls, 10 are resolved at the first attempt. Test, at the 5% significance level, whether the proportion of calls resolved at the first attempt has changed from 0.6. [5]
-
- 17** The number of potholes on a 1 km stretch of road has a Poisson distribution with mean 3.5. After resurfacing, a surveyor inspects 2 km and finds 2 potholes. Test, at the 5% significance level, whether the mean number of potholes per kilometre has decreased. [5]
-
- 18** A school bus takes a mean of 35 minutes with standard deviation 6 minutes, which is normally distributed. After a new route is introduced, a random sample of 20 journeys has mean 37.9 minutes. Test, at the 5% significance level, whether the mean journey time has changed. [5]
-
- 19** A researcher records the heights and arm spans of 15 adults and obtains $r = 0.51$. She claims that there is a correlation, positive or negative, between them. (Notice she has not stated the direction.) The two-tailed critical value of the product moment correlation coefficient for $n = 15$ at the 5% significance level is 0.5140. Test, at the 5% significance level, whether there is correlation between height and arm span. [4]
-
- 20** A company says that 40% of its employees cycle to work. In a random sample of 20 employees, 3 cycle. A union representative says the percentage is lower.
 (a) Test, at the 5% level, the union representative's claim.
 (b) A second representative says the company's figure is simply wrong and has no direction in mind. Carry out a two-tailed test at the 5% level using the same data and comment on the difference. [7]
-
- 21** Historically, 22% of patients taking a certain medicine experience a side effect. A new batch is made, and 4 of a random sample of 40 patients taking it experience the side effect. A doctor wants to test, at the 10% significance level, whether the proportion of patients with the side effect has changed. Let p be the probability that a patient has the side effect.
 (a) State the hypotheses and the distribution of the number of patients X with the side effect under H_0 . [2]
 (b) Find the critical region for the test, and the probability of rejecting H_0 when it is true. [4]
 (c) State the conclusion of the test in context. [1]

- 22** A company claims that 40% of visitors to its website make a purchase. A rival thinks the true proportion is different. In a random sample of 30 visitors, 17 make a purchase. Dan tests at the 5% significance level and writes: " $H_0: p = 0.4$, $H_1: p \neq 0.4$. $P(X \geq 17) = 0.0481$. This is less than 0.05, so reject H_0 ."
- (a) Explain the mistake in Dan's working. [2]
- (b) Find the critical region for the test and use it to give a correct conclusion. [4]
- (c) State the probability that the critical region you found in (b) leads to a Type I error. [2]
-
- 23** A fair-coin claim $p = 0.5$ is tested with 30 tosses, X being the number of heads.
- (a) Find the set of values of X for which H_0 is rejected in a one-tailed test ($H_1: p < 0.5$) at the 5% level.
- (b) Find the set of values of X for which H_0 is rejected in a two-tailed test at the 5% level.
- (c) Explain why a value of X in the set from
- (a) but not in
- (b) does not contradict the idea that the two tests use the same data. [8]
-
- 24** A pollster claims that 40% of adults in a city support a new tram line. A councillor thinks that the support is higher. In a random sample of 200 adults, 92 support the tram line. Using a suitable normal approximation, test the councillor's claim at the 5% level. Show your approximation and continuity correction. [7]
-
- 25** A student reads that the correlation between age and reaction time is expected to be positive. He collects data for 18 people and finds $r = 0.41$. The one-tailed 5% critical value for $n = 18$ is 0.4000 and the two-tailed 5% critical value is 0.4683.
- (a) Carry out a one-tailed test and state your conclusion in context.
- (b) Carry out the two-tailed test and explain why the conclusions differ (or agree).
- (c) State which test is appropriate and why. [7]
-
- 26** The number of typing errors on a page of a report is modelled by a Poisson distribution with mean 0.6 per page. A new proofreader claims that errors have become less frequent. She checks 5 pages chosen at random and finds that there are none.
- (a) State suitable hypotheses and the distribution of the number of errors in 5 pages under H_0 . [2]
- (b) Test the proofreader's claim at the 5% level. [3]
- (c) Find the least number of consecutive error-free pages that would allow H_0 to be rejected at the 5% level. [3]
-
- 27** $X \sim B(n, 0.5)$. A two-tailed test of $H_0: p = 0.5$ is carried out using X .
- (a) Show that $P(X \leq c) = P(X \geq n - c)$ for every integer c , when $p = 0.5$. [3]
- (b) Hence explain why, for $p = 0.5$, the critical region of a two-tailed test has the form $X \leq c$ or $X \geq n - c$. [2]
- (c) For $n = 24$, find the critical region for a test at the 5% significance level, and the actual significance level. [3]
- (d) Explain why the critical region is not symmetrical when $p \neq 0.5$. [1]

- 28** The proportion of customers who return an online order is 0.2. A company introduces a new policy and takes a random sample of 50 orders to test for a change in the proportion p that are returned. Let X be the number returned. The diagram shows the distribution of X when $p = 0.2$. The test is at the 5% significance level, and the power of a test at a given value of p is the probability of rejecting H_0 when p has that value.

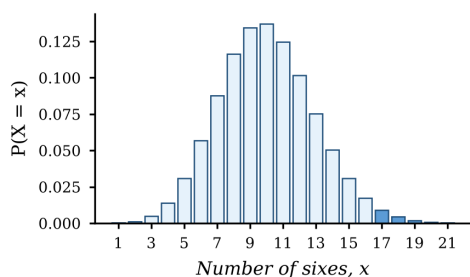


- (a) The company tests $H_0: p = 0.2$ against $H_1: p \neq 0.2$. Find the critical region and the actual significance level. [4]
- (b) Find the power of this test when the true value of p is 0.35. [2]
- (c) The company's director is sure that the return rate cannot have gone down. Using $H_1: p > 0.2$ at the 5% significance level, find the critical region and the power when $p = 0.35$. [3]
- (d) Compare the two powers and explain what this shows about using a one-tailed test. [1]
-
- 29** A sample of 13 independent trials gives $X \sim B(13, p)$. A test of $H_0: p = 0.5$ is carried out using the observed value $x = 3$.
- (a) Show that $P(X \leq 3) = \frac{189}{4096}$, and hence find the probability of a result at least as extreme as $x = 3$ in either direction. [4]
- (b) Explain why a one-tailed test at the 5% level ($H_1: p < 0.5$) rejects H_0 but a two-tailed test at the 5% level does not. [2]
- (c) A statistician says that choosing the one-tailed test after seeing $x = 3$ is "cheating". Explain, with reference to the probability of a Type I error, what is wrong. [1]
-
- 30** A drinks company claims that 25% of its bottles contain a prize. A consumer group tests the claim with a random sample of 40 bottles, with X the number containing a prize.
- (a) The group suspects that fewer than 25% of bottles contain a prize. Find the critical region at the 5% level and the actual significance level. [4]
- (b) Find the two-tailed critical region at the 5% level and the actual significance level. [5]
- (c) In the sample, 5 bottles have a prize. State the conclusion in each case, and explain why the conclusions agree or disagree. [3]

Binomial Hypothesis Testing

- 1** A coin is tossed 10 times and lands heads 9 times. Test at the 5% significance level whether the coin is biased towards heads. [3]
-
- 2** A shooter usually hits the target with probability 0.25. After new training she hits the target 2 times in 20 attempts. She thinks the training has made things worse. Test, at the 5% significance level, whether the probability of a hit has fallen. [3]
-
- 3** $X \sim B(10, 0.5)$. Find $P(X \geq 8)$ and state whether this is significant at the 5% level in a test of $H_0: p = 0.5$ against $H_1: p > 0.5$. [2]
-
- 4** A new drug is thought to cure a disease in 70% of cases. In a trial on 12 patients the number cured is X . Write down the distribution of X if the drug cures 70% of cases, and find $P(X \leq 5)$. [2]

- 5 In a test of $H_0: p = 0.2$ against $H_1: p > 0.2$ using a sample of size 20, the observed number of successes is 7. Given that $P(X \geq 7) = 0.0867$ for $X \sim B(20, 0.2)$, state the conclusion at the 5% level. [2]
- 6 State what the number X represents and its distribution under H_0 in the following test. "A bakery says that 5% of its muffins are underbaked. A tester thinks that it is more. He examines 30 muffins." [2]
- 7 A manufacturer states that 8% of its light bulbs are defective. An inspector believes the true proportion is higher. She tests a random sample of 30 bulbs and finds that 6 are defective. Carry out a hypothesis test at the 5% significance level. [5]
- 8 In a large population 40% of voters support a policy. After a campaign, a researcher asks a random sample of 15 voters and only 1 supports the policy. Test at the 10% significance level whether the proportion supporting the policy has changed. Use critical regions. [5]
- 9 A garden centre says that 80% of its bulbs flower in the first year. A customer thinks it is less. She plants 12 bulbs and 7 flower. Test, at the 5% significance level, whether the probability of flowering is less than 0.8. [5]
- 10 Historically 15% of students in a college pass a driving test first time. After a new course, 6 of a random sample of 20 students pass first time. Test, at the 5% significance level, whether the new course has increased the first-time pass rate. [5]
- 11 A multiple-choice test has 10 questions, each with 4 options, one correct. A student gets 6 correct and claims she is not guessing. Test, at the 5% significance level, whether the student is not simply guessing. [5]
- 12 A theatre says that 90% of booked seats are taken up. The manager checks 15 random bookings and only 11 are taken up. Test, at the 1% significance level, whether the proportion of booked seats taken up is lower than 0.9. [5]
- 13 A die is thought to be biased. It is rolled 18 times and shows an even number 13 times. Test, at the 10% significance level, whether there is evidence that the probability of an even number is not 0.5. [5]
- 14 A die is rolled 60 times and a six occurs 17 times. A player thinks the die is biased so that sixes are more likely. Let p be the probability of rolling a six. The diagram shows the distribution of the number of sixes X if the die is fair, with $X \geq 17$ shaded. [2]



- (a) State suitable hypotheses and the distribution of X under H_0 . [2]
- (b) Test at the 1% significance level whether the die is biased towards sixes. [3]
- (c) State, with a reason, what your conclusion would be at the 5% significance level. [1]

- 15** A television company claims that 40% of viewers watch a programme to the end. A rival company suspects that the proportion is different. In a random sample of 50 viewers, 12 watched to the end. Let p be the proportion of all viewers who watch to the end.
- (a) Test the company's claim at the 5% significance level, using a p-value. [4]
- (b) Find the critical region for the test, and verify that your conclusion in (a) agrees with it. [3]

- 16** Light bulbs are sold in boxes of 25. A supplier states that 6% of its bulbs are faulty. The number of faulty bulbs in a box is modelled by $X \sim B(25, 0.06)$.
- (a) Find the probability that a box contains exactly 2 faulty bulbs. [2]
- (b) Find the probability that a box contains at least 2 faulty bulbs. [2]
- (c) Three boxes are chosen at random. Find the probability that at least one of them contains at least 2 faulty bulbs. [2]
- (d) A new box is found to contain 5 faulty bulbs. Test, at the 5% significance level, whether this is evidence that the proportion of faulty bulbs is greater than 6%. [4]

- 17** A bakery claims that 1% of its rolls are underbaked. An inspector thinks the proportion is higher. She examines a random sample of 400 rolls and finds that 9 are underbaked. Using a Poisson approximation to the binomial distribution, test the inspector's belief at the 5% significance level. [5]

- 18** A fruit grower says that 25% of her oranges weigh more than 200 g. A buyer takes a random sample of 60 oranges and finds that 22 weigh more than 200 g. Test, at the 1% significance level, whether the proportion of oranges over 200 g is more than 0.25. [5]

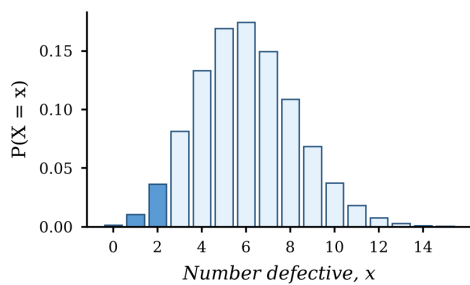
- 19** A factory claims that 4% of its light bulbs are faulty. A buyer inspects 10 randomly chosen boxes, each of 5 bulbs; the number of faulty bulbs in each box is shown in the table. Treat the 50 bulbs as a random sample. Test, at the 5% significance level, whether there is evidence that the proportion of faulty bulbs is more than 4%.

Box	1	2	3	4	5	6	7	8	9	10
Number of faulty bulbs	0	1	0	0	2	0	0	1	0	0

[5]

- 20** A swimming coach says that 60% of her squad swim 100 m in under 70 seconds. A rival coach thinks the proportion is lower. In a random sample of 25 swimmers from the squad, X swim under 70 seconds.
- (a) Find the critical region for the test at the 5% level. [3]
- (b) In the sample, 11 swim under 70 seconds. State the conclusion. [2]

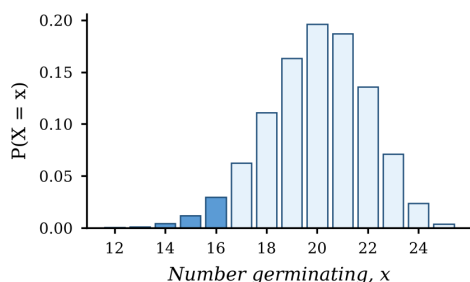
- 21** A factory has a defect rate of 15%. After a new process is introduced, the manager takes a random sample of 40 items and counts the number X that are defective. She will test at the 5% significance level whether the defect rate has fallen. Let p be the defect rate with the new process. The diagram shows the distribution of X when $p = 0.15$.



- (a) State the hypotheses and find the critical region. [4]
 (b) State the probability of a Type I error for this test. [1]
 (c) In the sample, 3 items are defective. State the conclusion of the test. [1]
 (d) Suppose that the defect rate has in fact fallen to 5%. Find the probability that the test fails to detect this. [2]
-
- 22** A bank states that 20% of its customers use online banking. A manager believes the proportion is higher. In a sample of 20 customers, 7 use online banking. A student writes the following test at the 5% level. " $H_0: p = 0.2$, $H_1: p > 0.2$. $X \sim B(20, 0.2)$. $P(X \geq 7) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321$. This is less than 0.05, so reject H_0 ."
- (a) Identify the error in the student's calculation. [2]
 (b) Carry out the test correctly and state the conclusion in context. [3]
 (c) Find the critical region for the test, and use it to confirm your conclusion in (b). [3]
-
- 23** A school claims that 70% of its Year 13 students go to university. A journalist believes that the figure is lower and asks 18 randomly chosen Year 13 students, of whom 9 go to university.
- (a) Carry out a test at the 5% level.
 (b) The journalist then says: "The result is not significant, which proves that the school's figure is right." Comment.
 (c) Find the smallest number of students out of 18 that would have to go to university for the journalist's test not to reject H_0 . [8]
-
- 24** A new vaccine is claimed to reduce the infection rate from 10%. In a trial of n volunteers none gets infected, and this is used to test $H_0: p = 0.1$ against $H_1: p < 0.1$ at the 5% level.
- (a) Show that the p-value is 0.9^n .
 (b) Find the smallest n for which zero infections gives a significant result.
 (c) Why might a trial with so few infections still give unreliable conclusions? [6]
-
- 25** A sweet maker claims that 20% of packets contain a free toy. A consumer group thinks fewer packets contain a toy. In a first sample of 20 packets, 1 contains a toy.
- (a) Test the claim at the 5% level.
 (b) A second independent sample of 30 packets contains 3 toys. Carry out the test again with the second sample.
 (c) Combine both samples and test again at the 5% level. Comment on the results. [9]
-
- 26** Zara tests $H_0: p = 0.35$ against $H_1: p > 0.35$ at the 5% level with $X \sim B(20, p)$. She works out $P(X \geq 10)$ as $1 - P(X \leq 10) = 0.0532$ and concludes that H_0 should be not rejected.
- (a) Explain the mistake in Zara's working.
 (b) Carry out the test correctly if 10 successes are observed. [6]

- 27** A supplier claims that at most 5% of the components it sells are faulty. A buyer tests a random sample of n components and finds that exactly 3 are faulty. Let p be the proportion of faulty components and X the number of faulty components in the sample. The buyer tests $H_0: p = 0.05$ against $H_1: p > 0.05$ at the 5% significance level.
- Show that, when $n = 20$, the result of 3 faulty components is not significant. [3]
 - Find the largest value of n for which a result of 3 faulty components is significant. You may assume that $P(X \geq 3)$ increases as n increases. [4]
 - Explain, in context, why a larger sample makes 3 faulty components less convincing evidence. [2]
 - For $n = 16$, state the critical region and the probability of a Type I error. [1]

- 28** A seed supplier claims that 80% of its sunflower seeds germinate. A gardener plants 25 seeds and 16 germinate. She believes that the true germination rate is lower. Let p be the probability that a seed germinates and X the number of seeds that germinate out of 25. The diagram shows the distribution of X when $p = 0.8$.

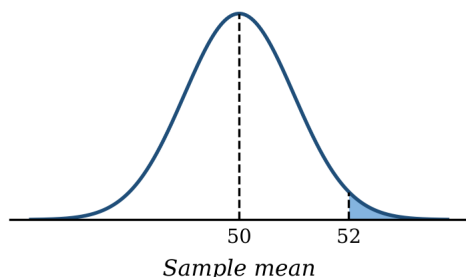


- State two assumptions needed for X to be modelled by a binomial distribution. [2]
 - Test the supplier's claim at the 5% significance level. [4]
 - Find the critical region for the test and the probability of a Type I error. [3]
 - The true germination rate is in fact 0.6. Find the probability that this test correctly rejects H_0 . [2]
- 29** $X \sim B(15, p)$ is observed to be 2. A test of $H_0: p = p_0$ against $H_1: p < p_0$ is carried out at the 5% level. Find, to 2 decimal places, the least value of p_0 for which H_0 would be rejected, and explain why every larger value of p_0 would also be rejected. Give a one-sentence interpretation for the value of p . [8]
- 30** A supplier ships components in batches. A batch is accepted if a random sample of 20 components contains at most 2 defective components. Let p be the proportion of defective components in the batch.
- Find the probability that a batch is accepted when $p = 0.05, 0.15$ and 0.30 . [3]
 - The rule is regarded as a test of $H_0: p = 0.05$ against $H_1: p > 0.05$, rejecting the batch if $X \geq 3$. Find the significance level of the test. [2]
 - The supplier wants the significance level to be below 1%, so the sample is increased to 40 with a new rejection rule $X \geq c$. Find the least c , and the probability of correctly rejecting a batch with $p = 0.15$ using this rule. [4]

Normal Hypothesis Testing

- 1 The random variable X is normally distributed with standard deviation 5. A random sample of 25 observations has mean 52. Test, at the 5% significance level, $H_0: \mu = 50$ against $H_1: \mu > 50$.

The diagram shows the distribution of the sample mean under H_0 .



[3]

- 2 The lifetime of a type of battery is normally distributed with standard deviation 6 hours. The manufacturer claims the mean lifetime is 100 hours. A random sample of 36 batteries has mean lifetime 98.5 hours. Test, at the 5% level, whether the mean lifetime is less than claimed.

[3]

- 3 $X \sim N(80, 6^2)$. A random sample of 36 observations is taken. Write down the distribution of the sample mean $\bar{X}$.

[2]

- 4 $X \sim N(\mu, 8^2)$. Under $H_0: \mu = 50$ a sample of size 25 has mean 53.4. Calculate the value of the test statistic $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$.

[2]

- 5 $Z \sim N(0, 1)$. Use your calculator to find $P(Z \geq 1.83)$, and state whether a result with $z = 1.83$ is significant in a one-tailed upper test at the 5% level.

[2]

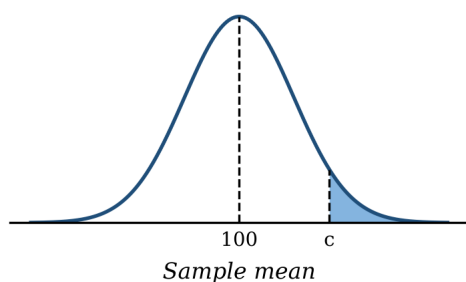
- 6 In a two-tailed test at the 5% level the critical values of z are ± 1.960 . State whether H_0 is rejected when (a) $z = 1.87$, (b) $z = -2.31$.

[2]

- 7 A machine produces rods whose lengths are normally distributed with standard deviation 3 mm. The mean length should be 20 mm. A random sample of 9 rods has mean length 18.2 mm. Test, at the 5% significance level, whether the mean length has changed.

[4]

- 8 The random variable $X \sim N(\mu, 4^2)$. A test of $H_0: \mu = 100$ against $H_1: \mu > 100$ is carried out at the 5% significance level, using the mean $\bar{X}$ of a random sample of size 16. The critical region is shown shaded in the diagram.



- (a) Find the critical value c of the critical region $\bar{X} > c$, giving your answer to 2 decimal places.
- (b) The sample mean is 102.1. State the conclusion of the test.

[3]

[2]

- 9** The lifetime of a type of bulb is normally distributed with standard deviation 90 hours. The makers claim a mean lifetime of 1200 hours. A random sample of 30 bulbs has mean lifetime 1168 hours.
Test, at the 5% significance level, whether the mean lifetime is less than 1200 hours. [5]

- 10** Adult heights in a region are normally distributed with standard deviation 7 cm. A study of 40 adults from one village has a mean height of 172.6 cm. The regional mean is 170 cm.
Test, at the 1% significance level, whether the mean height in the village is greater than 170 cm. [5]

- 11** The time taken by a student to solve a puzzle is normally distributed with standard deviation 20 seconds and mean 150 seconds. After a hint is given to 16 students, their mean time is 139 seconds.
Test, at the 5% significance level, whether the mean time has changed. [5]

- 12** The masses, in grams, of a random sample of 8 jars of jam are shown in the table. The mass of a jar is normally distributed with standard deviation 2.5 g and the label states a mean of 50 g.
Calculate the sample mean and test, at the 5% level, whether the mean mass is greater than 50 g.

48.2	51.5	49.0	52.3	50.8	53.1	49.7	51.9
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[6]

- 13** The variance of the daily output of a machine is 64 units^2 , with a normal distribution. The mean daily output is claimed to be 400 units. Over 9 randomly chosen days the mean output was 394 units.
Test, at the 10% significance level, whether the mean daily output is not 400 units. [5]

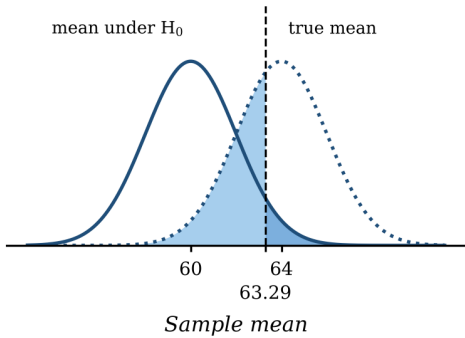
- 14** A company claims that its chocolate bars have a mean mass of 100 g. A consumer group weighs a random sample of 10 bars. The masses, in grams, are shown in the table. The mass of a bar is normally distributed with standard deviation 1.5 g.
Test, at the 5% significance level, whether the mean mass is less than 100 g.

98.2	101.4	97.6	99.3	96.8
100.1	98.9	97.4	99.7	98.1

[6]

- 15** The time, in seconds, taken by students to complete a puzzle is normally distributed with standard deviation 9. The mean time is believed to be 82 seconds. After a change to the instructions, a random sample of 36 students has mean time 85.5 seconds. Test, using a p-value, whether the mean time has changed, at
(a) the 5% level and
(b) the 1% level. [6]

- 16** A nutritionist claims that the mean daily sugar intake of teenagers is 90 g. A random sample of 60 teenagers gives $\Sigma x = 5700$ and $\Sigma x^2 = 560\,000$, where x grams is the daily sugar intake.
(a) Calculate the sample mean and an unbiased estimate of the population variance. [3]
(b) Using s^2 as an estimate of σ^2 , test, at the 5% significance level, whether the mean daily sugar intake of teenagers is greater than 90 g. [4]
(c) Give a reason why it is reasonable to use s^2 in place of σ^2 here. [1]
(d) Find the greatest value of the sample standard deviation s , for a sample of the same size and mean, for which H_0 would still be rejected at the 5% level. [2]

- 17** Fish in a lake have lengths that are normally distributed with standard deviation 4.2 cm. After a pollution incident a biologist measures 45 fish and finds a mean of 21.5 cm, compared with 22.6 cm before. She thinks the pollution has reduced the mean length.
Test, at the 1% significance level, whether the mean length of fish has decreased. [5]
- 18** The scores on a standard aptitude test are normally distributed with mean 100 and standard deviation 15. A teacher thinks her class of 36 students has a different mean from the national mean. Their total score is 3744.
Test the teacher's belief at the 5% significance level. [5]
- 19** The thickness of a plastic sheet is normally distributed with standard deviation 0.8 mm. The target mean is 12.5 mm. A technician checks 50 sheets and thinks the mean thickness is too high.
(a) Find the critical region for the sample mean at the 5% level. [3]
(b) The sample mean is 12.71 mm. State the conclusion. [2]
- 20** A manufacturer claims that its batteries last for a mean of 40 hours. The lifetimes are normally distributed with standard deviation 5 hours. A consumer tests n batteries and obtains a mean of 38.8 hours.
Find the least value of n for which the consumer would reject $H_0: \mu = 40$ against $H_1: \mu < 40$ at the 5% significance level. [5]
- 21** A test scores variable X is normally distributed with standard deviation 12. A teacher tests $H_0: \mu = 60$ against $H_1: \mu > 60$ at the 5% level using the mean of a random sample of 36 scores.
The diagram shows the distribution of the sample mean under H_0 (solid curve) and when the true mean is 64 (dotted curve). The dark region is the probability of a Type I error, and the light region is the probability of a Type II error.
- 
- (a) Find the critical region for $\bar{X}$, giving the boundary to 2 decimal places. [3]
(b) A sample has mean 63.8. State the conclusion of the test. [1]
(c) The true mean is 64. Find the probability of a Type II error. [3]
(d) Without further calculation, say how the probability of a Type II error would change if the true mean were 66, and explain why. [1]
- 22** The mass of a bag of cement is normally distributed with standard deviation 10 kg. The mean is claimed to be 200 kg. A random sample of 25 bags has a mean mass of 204 kg. A student tests, at the 5% level, $H_0: \mu = 200$ against $H_1: \mu \neq 200$ and writes:
" $z = \frac{204 - 200}{10} = 0.4$, which is less than 1.96, so do not reject H_0 ."
- (a) Explain the error in the student's working. [2]
(b) Carry out the test correctly. [4]
(c) Explain why taking a larger sample makes a given difference between $\bar{x}$ and μ more likely to be significant. [2]

- 23** A manufacturer claims that 35% of its customers buy the premium model. A rival believes that the proportion is higher. In a random sample of 150 customers, 64 bought the premium model.
Use a normal approximation to the binomial distribution to test the rival's belief at the 5% level. [7]
- 24** The number of customers arriving at a bank per day is modelled by a Poisson distribution with mean 40. After a promotion, 52 customers arrive on a randomly chosen day. Test, at the 5% level, whether the mean number of arrivals has increased, using a normal approximation. Compare your p-value with the exact Poisson value. [7]
- 25** The scores of a population are normally distributed with standard deviation 10. A test of $H_0: \mu = 50$ against $H_1: \mu > 50$ is carried out at the 5% level using the mean of a sample of size 25.
(a) Find the critical region.
(b) Find the probability of a Type II error if in fact $\mu = 56$.
(c) State the effect on this probability of increasing the sample size. [8]
- 26** The time, T minutes, taken by trainees to assemble a unit is normally distributed with mean μ and standard deviation σ . It is found that 10% of trainees take less than 12 minutes and 5% take more than 27 minutes.
(a) Find μ and σ , giving each to 1 decimal place. [5]
(b) A trainer believes that a new method has reduced the mean assembly time. Using $\sigma = 5.1$, test this belief at the 5% significance level, given that a random sample of 9 trainees using the new method has mean time 15.2 minutes. [4]
- 27** A mobile phone company claims that 30% of its customers upgrade their phone each year. An analyst believes the proportion is higher. In a random sample of 100 customers, 38 upgraded. Let p be the proportion who upgrade and X the number in the sample who upgrade.
(a) Write down the hypotheses and show that X can be approximated by a normal distribution, stating the distribution. [2]
(b) Use the normal approximation, with a continuity correction, to test H_0 at the 5% significance level. [4]
(c) Calculate the exact probability $P(X \geq 38)$ using the binomial distribution and state the conclusion it gives. [2]
(d) A student omits the continuity correction and uses $z = \frac{38 - 30}{\sqrt{21}}$. Find the conclusion they reach and explain why it is unreliable. [2]
- 28** A population is normally distributed with standard deviation 8. A test of $H_0: \mu = 50$ against $H_1: \mu > 50$ is to be carried out at the 5% significance level, using the mean $\bar{X}$ of a random sample of size n . The power of the test at a given value of μ is the probability that H_0 is rejected when μ has that value.
(a) Show that the critical region is $\bar{X} > 50 + \frac{1.645 \times 8}{\sqrt{n}}$. [2]
(b) Show that the power of the test when $\mu = 53$ is $P(Z > 1.645 - \frac{3\sqrt{n}}{8})$, where $Z \sim N(0, 1)$. [3]
(c) Find the smallest sample size n for which the power at $\mu = 53$ is at least 0.9. [4]
(d) Verify your answer to (c) by finding the power when $n = 61$ and when $n = 60$. [2]

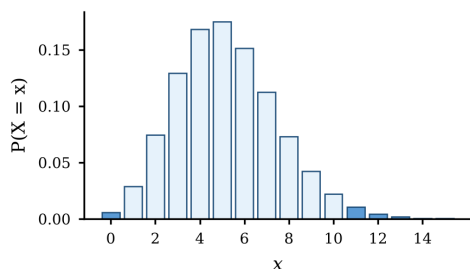
- 29** Two machines, A and B, fill packets. The masses, in grams, of the packets are normally distributed with standard deviations 6 for machine A and 8 for machine B. A random sample of 12 packets from A has mean 52.3 and an independent random sample of 15 packets from B has mean 48.1. An engineer believes that the mean mass of packets from A is greater than that from B.
- State suitable hypotheses and write down the distribution of $\bar{X}_A - \bar{X}_B$ under H_0 . [3]
 - Carry out the test at the 5% significance level. [3]
 - State two assumptions made in carrying out this test. [2]
 - Samples of equal size n are taken from each machine, with the same sample means. Find the smallest n for which the difference in sample means would be significant at the 5% level. [2]
-
- 30** The masses of eggs from a farm are normally distributed with standard deviation 5 g. The farmer says the mean mass is 62 g. Boxes hold 6 eggs. A shop owner thinks the mean is lower. She weighs the eggs in 4 boxes, a random sample of 24 eggs, and finds the total mass is 1452 g.
- Test the shop owner's belief at the 5% level. [5]
 - The farmer says that the sample was taken from four boxes and is not random because eggs in a box are often from the same hen. Comment on how this affects the validity of the test. [2]
 - Find, to 1 decimal place, the greatest sample mean (for a random sample of 24 eggs) that would lead to H_0 being rejected at the 5% level. [3]

Poisson Hypothesis Testing

- The number of accidents at a junction is modelled by a Poisson distribution with mean 4 per month. After a new road layout is introduced, 9 accidents occur in one month. Test, at the 5% significance level, whether the mean number of accidents per month has increased. [3]
- A call centre receives calls at a mean rate of 6 per hour. After advertising stops, 2 calls are received in one hour. Test, at the 5% significance level, whether the mean rate of calls has decreased. [3]
- Give two conditions needed for the number of events in a fixed interval to be modelled by a Poisson distribution, and give one reason why the number of cars passing a school gate in a minute might not be Poisson. [3]
- Customers arrive at a till at a mean rate of 1.5 per minute. Write down the distribution of the number of customers arriving in 4 minutes, and the mean. [2]
- $X \sim \text{Po}(4.5)$. Use your calculator to find $P(X \geq 9)$ and state whether it is significant at the 5% level in a one-tailed test. [2]
- The number of emails Ben receives in an hour is modelled by $\text{Po}(\lambda)$. He normally gets 5 per hour, but believes it has decreased. Write down hypotheses for a test using the number of emails in one hour. [2]
- A website receives error reports at a mean rate of 2.5 per day. After an update, 13 reports are received over a period of 3 days. Test, at the 5% significance level, whether the rate of error reports has increased. State any assumption you make. [5]

- 8** $X \sim \text{Po}(\lambda)$. A two-tailed test of $H_0: \lambda = 5.2$ against $H_1: \lambda \neq 5.2$ is carried out at the 5% significance level, using one observation of X . The diagram shows the distribution of X under H_0 .

Find the critical region and the actual significance level.



[4]

- 9** A busy road has averaged 3 accidents per month. In the month after a new speed limit, there is 1 accident.

Test, at the 10% significance level, whether the mean number of accidents per month has decreased.

[5]

- 10** A bookshop sells an average of 2 copies of a novel per day. After a review, 9 copies are sold in 3 days.

Test, at the 5% significance level, whether the rate of sales has increased.

[5]

- 11** Faults occur in a cable at a rate of 0.4 per metre. In a 10 m length of a new cable, 1 fault is found.

Test, at the 5% significance level, whether the new cable has fewer faults per metre.

[5]

- 12** A telephone helpline receives 8 calls per hour on average. In a randomly chosen hour after a news item, 14 calls are received.

Test, at the 1% significance level, whether the mean number of calls per hour has increased.

[5]

- 13** A lake has an average of 4 swans per 100 m of shoreline. A survey of 100 m of a nearby lake finds 9 swans and a naturalist claims the density is different.

Test, at the 5% significance level, whether the mean number of swans per 100 m is not 4.

[5]

- 14** Customers arrive at a service desk at random at a mean rate of 5 per hour.

(a) Find the probability that more than 7 customers arrive in a given hour.

[2]

(b) Find the probability that exactly 18 customers arrive in a given 3 hour period.

[2]

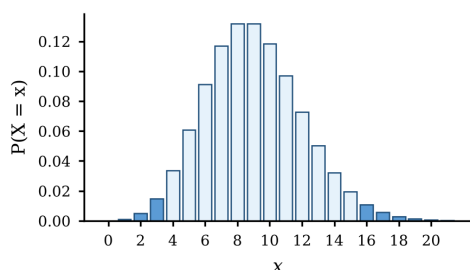
(c) After a training programme the manager records just 1 arrival in a randomly chosen hour and believes that the arrival rate has fallen. Test this belief at the 5% significance level.

[4]

(d) Give one reason why the conclusion in (c) may not be reliable.

[1]

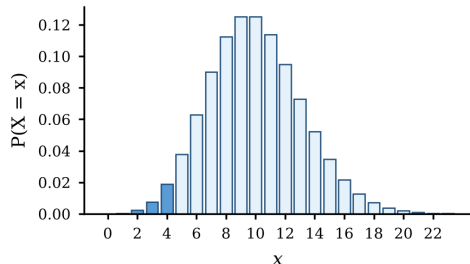
- 15** Patients arrive at a hospital emergency department at a mean rate of 9 per hour during the evening. The manager believes that a new nearby clinic has changed the arrival rate. In a randomly chosen hour, 3 patients arrive. Let λ be the mean number of arrivals per hour. The diagram shows the distribution of the number of arrivals X in an hour under H_0 .



- (a) State suitable hypotheses and the distribution of X under H_0 . [2]
 (b) Test at the 5% significance level whether the arrival rate has changed. [3]
 (c) Find the critical region for the test and the actual significance level. [2]
-
- 16** An airline loses an average of 1.2 bags per flight. After a new baggage system, 10 flights are monitored and 5 bags are lost in total. Test, at the 5% significance level, whether the mean number of bags lost per flight has decreased. [5]
-
- 17** A web server records 0.5 crashes per week on average. Over a period of 12 weeks with new software, there are 11 crashes. Test, at the 5% significance level, whether the crash rate has increased. [5]
-
- 18** Meteorites strike a research site at a mean rate of 3.5 per year. In a randomly chosen year, 8 strikes are observed and a researcher thinks the rate has changed. Test, at the 10% significance level, whether the mean rate of strikes is not 3.5 per year. [5]
-
- 19** The number of flaws X on a 1 m^2 sheet of metal is $\text{Po}(\lambda)$. A test of $H_0: \lambda = 4.5$ against $H_1: \lambda \neq 4.5$ is carried out at the 10% significance level using one sheet. Cumulative probabilities for $\text{Po}(4.5)$ are shown. Find the critical region and the actual significance level, and state the conclusion if 11 flaws are found.
- | x | 0 | 1 | 2 | 3 | 4 | 8 | 9 | 10 | 11 | 12 |
|---------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $P(X \leq x)$ | 0.0111 | 0.0611 | 0.1736 | 0.3423 | 0.5321 | 0.9597 | 0.9829 | 0.9933 | 0.9976 | 0.9992 |
- [7]
-
- 20** The number of vehicles passing a checkpoint in an hour is Poisson with mean 36. After a diversion, 28 vehicles pass in an hour. Use a normal approximation to test at the 5% level whether the mean has decreased. [6]
-
- 21** A bus company records a mean of 40 breakdowns per day across its fleet. After servicing changes, 52 breakdowns are recorded on a randomly chosen day. Let λ be the mean number of breakdowns per day. The company tests $H_0: \lambda = 40$ against $H_1: \lambda > 40$ at the 5% significance level, assuming that breakdowns occur as a Poisson process.
- (a) Use the exact Poisson distribution to carry out the test. [3]
 (b) Show that $\text{Po}(40)$ can be approximated by $N(40, 40)$, and use this approximation, with a continuity correction, to test H_0 . [4]
 (c) Comment on the accuracy of the approximation. [1]

- 22** A fabric supplier says that flaws occur at a mean rate of 2.5 per metre of cloth. A weaver believes that a new supplier's cloth has fewer flaws. She will test this at the 5% significance level using the total number of flaws X in a randomly chosen 4 metres of the new cloth, modelling X as Poisson. Let λ be the mean number of flaws in 4 metres.

The diagram shows the distribution of X when the new cloth is the same as the old.



- (a) State the hypotheses and the distribution of X under H_0 . [2]
- (b) Find the critical region and the probability of a Type I error. [2]
- (c) The new cloth really has 1.25 flaws per metre. Find the probability that the test fails to detect the improvement. [3]
-
- 23** Flaws occur in a roll of fabric at random at a mean rate of 0.25 per metre. A buyer inspects a 20 m length of fabric from a new supplier and counts X flaws, and wants to test whether the supplier's fabric has more flaws.
- (a) State suitable hypotheses and the distribution of X under H_0 . [2]
- (b) Find the critical region at the 5% level and the actual significance level. [4]
- (c) The true mean for the new supplier is 0.5 flaws per metre. Find the probability that the buyer's test correctly rejects H_0 . [2]
- (d) Find the smallest length of fabric (to the nearest metre) that the buyer must inspect so that finding no flaws at all would be significant in a one-tailed test of $H_0: \lambda = 0.25$ against $H_1: \lambda < 0.25$ at the 5% level. [2]

- 24** The table shows the number of complaints x received by a hotel on each of 60 randomly chosen days. In a previous year the mean was 1.4 complaints per day. Calculate the mean number of complaints per day for these 60 days, and use the total number of complaints with a normal approximation to test at the 5% level whether the mean is now lower.

x	0	1	2	3	4	5
f	12	21	15	8	3	1

- [8]
- 25** A student tests $H_0: \lambda = 6$ against $H_1: \lambda \neq 6$ using the number of events X in one unit of time, $X \sim \text{Po}(\lambda)$. Her rule is to reject H_0 when $X \leq 1$ or $X \geq 12$.
- (a) Find the significance level of the test.
- (b) Find the probability that the test correctly rejects H_0 when $\lambda = 9$.
- (c) The student then repeats the test using two units of time with total $T \sim \text{Po}(2\lambda)$, rejecting H_0 when $T \leq 5$ or $T \geq 20$. Find the new significance level and comment. [8]
-
- 26** Calls to a help desk arrive at a mean rate of 3.8 per minute. After a system change, the manager records the number of calls in each of 8 randomly chosen one-minute intervals: 3, 5, 2, 6, 4, 7, 3, 5. She believes the mean rate has increased.
- (a) State hypotheses and the distribution of the total number of calls in the 8 intervals under H_0 . [2]
- (b) Test her belief at the 5% level. [4]
- (c) Explain why it is reasonable to add the counts, and state one assumption of the test. [2]

- 27** An office manager records the number of emails X received by a department in each of 60 randomly chosen 10-minute intervals. The results are shown in the table (7+ means 7, and you may treat these intervals as having exactly 7 emails).

The manager has believed that the mean rate is 2.5 emails per 10-minute interval, and wants to test, at the 5% significance level, whether the rate has increased.

Emails, x	0	1	2	3	4	5	6	7+
Frequency	3	9	14	15	10	6	2	1

- (a) Calculate the mean number of emails per interval. [2]
- (b) Calculate the sample variance (using divisor $n - 1$) and explain why your answers support a Poisson model. [3]
- (c) Explain why the total number of emails in the 60 intervals can be modelled by $Po(150)$ if H_0 is true. [1]
- (d) Carry out the test, using a normal approximation with a continuity correction. [4]
-
- 28** The number of defects X in a roll of plastic sheet is modelled by a Poisson distribution with mean λ . Historically $\lambda = 4$. The firm adopts the rule that, using one roll, it will reject $H_0: \lambda = 4$ in favour of $H_1: \lambda > 4$ if X is in a critical region chosen at the 1% significance level. The power of the test at a value of λ is the probability of rejecting H_0 when λ has that value.
- (a) Find the critical region and the actual probability of a Type I error. [3]
- (b) Find the power of the test when $\lambda = 8$ and when $\lambda = 12$. [2]
- (c) Find the smallest integer value of λ for which the power of the test is at least 0.8. [3]
- (d) The factory's engineer wants to detect an increase in λ from 4 to 8 with high probability. Use your answers to comment on whether this test is suitable, and suggest one change. [2]
-
- 29** Cars arrive at a toll booth at random at a mean rate of λ per minute. Historically $\lambda = 2$. The manager observes the number of cars arriving in a randomly chosen period of t minutes and rejects $H_0: \lambda = 2$ in favour of $H_1: \lambda > 2$ if the number of cars is at least $3t + 1$.
- (a) For $t = 3$, show that the rule is to reject H_0 if at least 10 cars arrive, and find the significance level.
- (b) For $t = 5$, find the corresponding significance level (rejecting H_0 if at least 16 cars arrive).
- (c) Find the probability of rejecting H_0 when $\lambda = 3.5$ for $t = 3$ and $t = 5$, and comment on which test is better at detecting the increase. [10]
-
- 30** The number of defects X in a 5 m roll of cable is modelled by $Po(\lambda)$. A test of $H_0: \lambda = 3$ against $H_1: \lambda > 3$ is carried out at the 5% level using one roll.
- (a) Find the critical region and the actual significance level.
- (b) The true mean is 6. Find the power of the test, i.e. the probability that H_0 is rejected.
- (c) Show that for large λ the power is close to 1, and find the smallest true value of λ (to 1 decimal place) for which the power exceeds 0.9. [10]

Correlation Hypothesis Testing

- 1** A sample of 10 pairs of data has product moment correlation coefficient $r = 0.72$. The one-tailed 5% critical value for $n = 10$ is 0.5494. Test, at the 5% significance level, whether there is evidence of positive correlation. State your hypotheses, where ρ is the population correlation coefficient. [2]
-
- 2** For a random sample of 15 students, the correlation coefficient between hours of screen time and sleep time is $r = -0.42$. The one-tailed 5% critical value for $n = 15$ is 0.4409. Test at the 5% significance level whether there is evidence of negative correlation. [3]

3 A researcher collects pairs of data on 14 students: hours of exercise per week and resting heart rate. She believes that more exercise gives a lower resting heart rate. State suitable hypotheses for a test of this belief, defining ρ . [2]

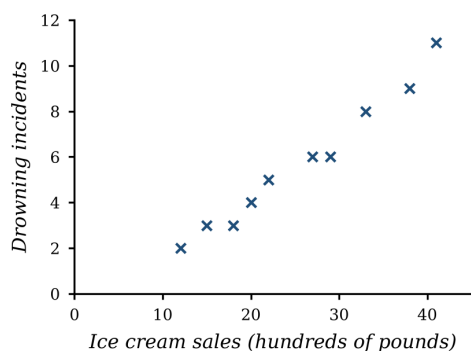
4 Describe the strength and type of correlation indicated by (a) $r = -0.91$, (b) $r = 0.08$. [2]

5 The one-tailed 5% critical value for a sample of size 9 is 0.5822. A sample of size 9 gives $r = 0.62$. Test for positive correlation at the 5% level. [2]

6 A student finds that the value of r for a sample of data is 0.02, and says: "So there is no relationship at all between the two variables." Give one reason why this conclusion may be wrong. [2]

7 A sample of 12 pairs of observations has $r = 0.58$. Using the two-tailed 5% critical value for $n = 12$, which is 0.5760, test whether there is any correlation between the two variables. State your conclusion in terms of ρ . [4]

8 In each of 10 months a seaside town records its ice cream sales and the number of drowning incidents on its beaches. The scatter diagram shows the data. The product moment correlation coefficient is $r = 0.988$. The two-tailed 5% critical value for $n = 10$ is 0.6319.



(a) Test, at the 5% significance level, whether there is correlation between ice cream sales and drowning incidents. [3]

(b) A newspaper concludes that ice cream causes drowning. Explain why this conclusion is not justified and suggest a more likely reason for the correlation. [1]

9 Eight students sit a maths and a physics test. The product moment correlation coefficient between their scores is $r = 0.79$.
The one-tailed critical value of the product moment correlation coefficient for $n = 8$ at the 5% significance level is 0.6215.
Test, at the 5% significance level, whether there is positive correlation between maths and physics scores. [4]

10 For 20 cities, the correlation between the mean temperature and the sales of woollen hats is $r = -0.46$.
The one-tailed critical value of the product moment correlation coefficient for $n = 20$ at the 5% significance level is 0.3783.
Test, at the 5% significance level, whether there is negative correlation between temperature and sales of woollen hats. [4]

11 A sample of 15 pairs of observations of rainfall and the number of cars on a road gives $r = 0.43$. The researcher has no idea of the direction of any correlation.
The two-tailed critical value of the product moment correlation coefficient for $n = 15$ at the 5% significance level is 0.5140.
Test, at the 5% significance level, whether there is correlation between rainfall and traffic volume. [4]

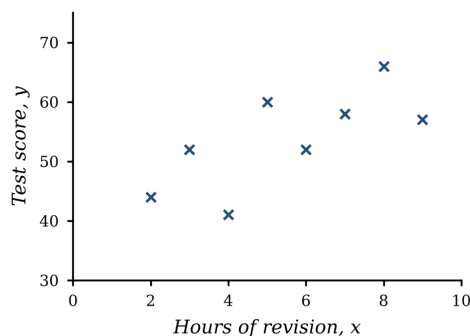
- 12** A sample of 25 people gives a product moment correlation coefficient of 0.35 between height and shoe size, where the sample is random.
The one-tailed critical value of the product moment correlation coefficient for $n = 25$ at the 1% significance level is 0.4622.
Test, at the 1% significance level, whether there is positive correlation between height and shoe size. [4]

- 13** The table shows the marks of eight students in two tests. Test 1 is marked out of 50 and Test 2 is marked out of 30.

Student	A	B	C	D	E	F	G	H
Test 1 mark (out of 50), x	30	41	22	35	44	38	27	19
Test 2 mark (out of 30), y	14	19	12	20	22	13	18	11

- (a) Use your calculator to find the product moment correlation coefficient between x and y . [1]
(b) Explain how the value of r would change if both sets of marks were converted to percentages. [1]
(c) The one-tailed 5% critical value of the product moment correlation coefficient for $n = 8$ is 0.6215. Test, at the 5% significance level, whether there is positive correlation between the two test marks. [3]

- 14** Eight students record the number of hours x spent revising for a test and their score y out of 80. The data are shown in the scatter diagram.
 $(x, y) = (2, 44), (3, 52), (4, 41), (5, 60), (6, 52), (7, 58), (8, 66), (9, 57)$.
The one-tailed 5% critical value of the product moment correlation coefficient for $n = 8$ is 0.6215. Calculate r , and test whether there is evidence of positive correlation between revision time and score.



- 15** A researcher records the age x (years) of 12 cars and their resale value y (hundreds of pounds). The summary statistics are $S_{xx} = 240.5$, $S_{yy} = 1860$ and $S_{xy} = -415$.
(a) Calculate the product moment correlation coefficient.
(b) The two-tailed 5% critical value for $n = 12$ is 0.5760. Test whether there is any correlation between age and resale value. [6]

- 16** A sports scientist measures the training time x (hours per week) and the 5 km running time y (minutes) of 12 athletes.
The summary statistics are $S_{xx} = 84.5$, $S_{yy} = 190.2$ and $S_{xy} = -71.3$. The one-tailed critical value for $n = 12$ at the 5% level is 0.4973.
Calculate r and test, at the 5% significance level, whether there is negative correlation between training time and running time. [5]

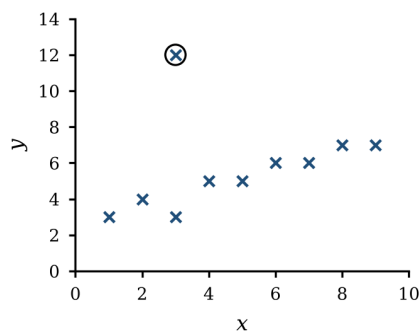
- 17** For a random sample of 16 used cars, x is the age in years and y is the price in thousands of pounds.
The summary statistics are $S_{xx} = 210.4$, $S_{yy} = 96.8$ and $S_{xy} = -88.5$. The one-tailed critical value for $n = 16$ at the 1% level is 0.5742.
Calculate r and test, at the 1% significance level, whether there is negative correlation between age and price. [5]

- 18** The table shows the rainfall x (cm) and the yield y (tonnes per hectare) of a crop on 8 farms. Use your calculator to find the product moment correlation coefficient, and test at the 5% level (the one-tailed critical value is 0.6215) whether there is positive correlation.

x	2.1	3.4	4.0	5.2	6.1	7.3	8.0	9.4
y	11.8	12.5	12.1	14.6	14.9	15.2	17.4	16.1

[5]

- 19** The scatter diagram shows 10 pairs of values. The circled point (3, 12) is thought to be an error. The one-tailed 5% critical values are 0.5822 for $n = 9$ and 0.5494 for $n = 10$. The correlation coefficients are $r = 0.313$ for all 10 points and $r = 0.951$ for the 9 points without the circled point. Test for positive correlation at the 5% level in each case and comment.

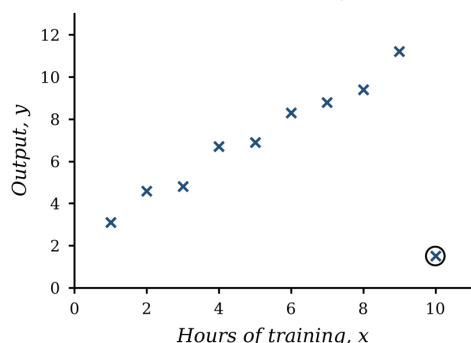


[5]

- 20** A teacher records the number of hours of sleep and the score in a memory test for 18 students and finds $r = 0.37$. She suggests a correlation exists but gives no direction. The two-tailed critical value of the product moment correlation coefficient for $n = 18$ at the 5% significance level is 0.4683.
Test, at the 5% significance level, whether there is correlation between hours of sleep and memory test score. [4]

- 21** A company records the training time x (hours) of 10 employees and their output y (units per hour). The data are shown in the scatter diagram, and the circled point is for an employee who was absent through illness on the day of measuring.

x : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; y : 3.1, 4.6, 4.8, 6.7, 6.9, 8.3, 8.8, 9.4, 11.2, 1.5



- Calculate r for all 10 employees and test, at the 5% level, for correlation. The two-tailed critical value for $n = 10$ is 0.6319. [4]
- Explain why it is reasonable to exclude the circled point, and recalculate r and repeat the test without it. The two-tailed critical value for $n = 9$ is 0.6664. [3]
- Comment on the effect of one extreme value on the correlation coefficient. [1]

- 22** The table shows the one-tailed 5% critical values of the product moment correlation coefficient for samples of size n .

Two scientists each find that their sample gives $r = 0.5$, one from a sample of size 8 and the other from a sample of size 20.

n	6	7	8	9	10	11	12	13
Critical value	0.7293	0.6694	0.6215	0.5822	0.5494	0.5214	0.4973	0.4762
n	14	15	16	17	18	19	20	
Critical value	0.4575	0.4409	0.4259	0.4124	0.4000	0.3887	0.3783	

- Test, at the 5% level, whether there is positive correlation for the sample of size 8. [3]
- Carry out the same test for the sample of size 20. [2]
- Find the smallest sample size for which $r = 0.5$ would be significant at the 5% level (one-tailed). [2]
- Explain why the same value of r gives different conclusions. [1]

- 23** A researcher finds $r = 0.35$ for a random sample of n pairs of values and tests $H_0: \rho = 0$ against $H_1: \rho > 0$ at the 5% level.

- Use the values below to explain why $r = 0.35$ is not significant for $n = 20$ but is for $n = 40$.
 - Find, using your calculator or the data shown, the smallest n for which it becomes significant, and comment on the practical meaning of a significant result with a small r .
- The one-tailed 5% critical values are: $n = 20$: 0.3783, $n = 30$: 0.3061, $n = 40$: 0.2638. [8]

- 24** The table shows the age x (years) and the systolic blood pressure y of ten patients.

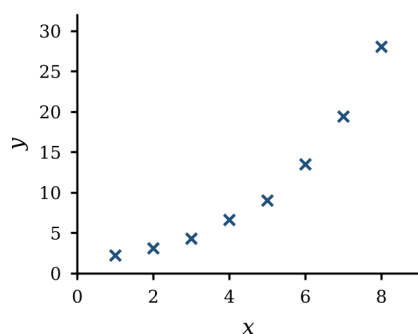
x	12	15	17	20	22	25	28	31	33	36
y	24	29	25	31	36	33	41	38	45	47

- Find the product moment correlation coefficient. [1]
- Test, at the 5% significance level, whether there is positive correlation, given the one-tailed critical value 0.5494. [3]
- A doctor says: "This proves that ageing causes high blood pressure." Comment. [2]
- A 70 year old patient has a blood pressure of 200. Explain why the data cannot be used to predict this value. [2]

- 25** Two variables have sample correlation coefficient $r = 0.53$ from 14 pairs. A two-tailed test at the 5% level has critical value 0.5324.
- Carry out the two-tailed test.
 - Another researcher says "the one-tailed critical value 0.4575 is smaller, so my result is more significant if I use a one-tailed test." Explain when a one-tailed test would be acceptable, and what is wrong with choosing it after seeing r .
 - State the change to the conclusion if the significance level is changed to 1% (two-tailed critical value 0.6614).

[7]

- 26** The scatter diagram shows the population y (thousands) of bacteria in a culture at times x hours. The one-tailed 5% critical value for $n = 8$ is 0.6215.



- The product moment correlation coefficient for x and y is 0.941. Test for positive correlation at the 5% level.
- Explain, referring to the diagram, why the test in (a) does not show that a linear model is suitable.
- The correlation coefficient between x and $\ln y$ is 0.9997. Test again at the 5% level and suggest a suitable model.

[2]

[2]

[3]

- 27** For 10 pairs of values (x, y) , $S_{xx} = 60$, $S_{yy} = 250$ and $S_{xy} = 96$. The one-tailed 5% critical value for $n = 10$ is 0.5494.

- Calculate r and test $H_0: \rho = 0$ against $H_1: \rho > 0$ at the 5% level.
- The data are recoded as $u = 2x + 3$ and $v = 20 - 5y$. Find S_{uu} , S_{vv} and S_{uv} , and hence r_{uv} .
- For $u = ax + b$ and $v = cy + d$ with $a, c \neq 0$, show that $r_{uv} = \frac{ac}{|ac|} r_{xy}$, where r_{xy} is the correlation coefficient of x and y .
- Explain why a test of the recoded data for positive correlation would be based on $H_1: \rho < 0$, and what this says about units of measurement.

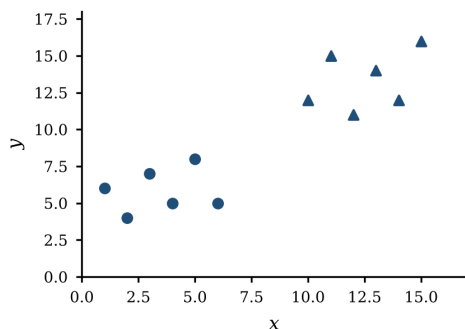
[3]

[3]

[3]

[1]

- 28** A scientist collects 12 pairs of readings (x, y) from two machines. Six readings come from Machine A (circles) and six from Machine B (triangles), as shown in the scatter diagram. The two-tailed 5% critical values are 0.8114 for $n = 6$ and 0.5760 for $n = 12$.
Machine A: (1, 6), (2, 4), (3, 7), (4, 5), (5, 8), (6, 5). Machine B: (10, 12), (11, 15), (12, 11), (13, 14), (14, 12), (15, 16).



- The product moment correlation coefficients are $r_A = 0.182$ for Machine A and $r_B = 0.381$ for Machine B. Test, at the 5% level, whether there is correlation for each machine separately. [3]
- Calculate r for all 12 readings together and carry out a test for correlation. [3]
- Explain, using the diagram, why the conclusions in (a) and (b) differ and why the pooled result may be misleading. [2]
- State what a sensible analysis of these data should do before testing for correlation. [2]

- 29** The ten data points (x, y) satisfy $y = (x - 5.5)^2$ exactly, where $x = 1, 2, \dots, 10$.
- Use your calculator to show that the product moment correlation coefficient is 0.
 - Hence explain carefully why the result of a correlation test at any significance level would be not significant, and why that does not mean x and y are unrelated.
 - Suggest a transformation of x so that the test would show strong correlation, and state the value of r obtained. [8]

- 30** A researcher tests for correlation between two variables using samples of size n , and uses the fact that for $H_0: \rho = 0$ the statistic $t = r\sqrt{\frac{n-2}{1-r^2}}$ follows a t -distribution with $n - 2$ degrees of freedom. A critical value r^* of the PMCC corresponds to a critical value t^* of t .

- Show that $r^* = \frac{t^*}{\sqrt{n-2+t^{*2}}}$.
- For $n = 10$ and a one-tailed 5% test, $t^* = 1.8595$ (8 degrees of freedom). Use the formula to verify the critical value 0.5494.
- Hence show that the critical value of r decreases as n increases for the same significance level (use the formula for large n). [9]