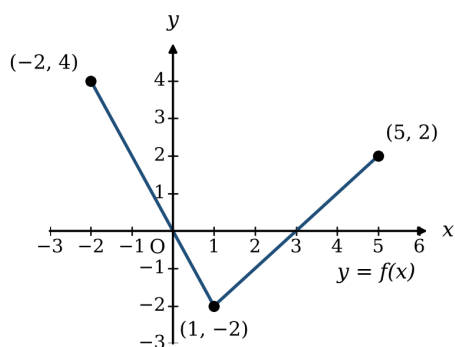


Function Notation, Domain and Range

- 1 Given that $f(x) = 2x^2 - 5x + 3$, find the value of
(a) $f(3)$, (b) $f(-1)$. [2]
- 2 The function g is defined by $g(x) = \sqrt{x - 4}$, for $x \in \mathbb{R}$, $x \geq 4$.
(a) Find $g(13)$. [1]
(b) State the range of g . [1]
(c) Explain why $x = 2$ is not in the domain of g . [1]
- 3 State the largest possible real domain of $f(x) = \frac{1}{x + 4}$. [1]
- 4 The function g is defined by $g(x) = x^2 + 3$, for $x \in \mathbb{R}$. State the range of g . [1]
- 5 The function h is defined by $h(x) = 2^x$, for $x \in \mathbb{R}$. Find $h(3)$ and $h(-2)$. [2]
- 6 The function f is defined by $f(x) = 3x - 1$, with domain $0, 1, 2, 3, 4$.
(a) List the values in the range of f . [2]
(b) Explain why f is a one-to-one function. [1]
- 7 The function f is defined by $f(x) = \frac{3x + 1}{x - 2}$, for $x \in \mathbb{R}$, $x \neq 2$.
(a) Find $f(-1)$. [1]
(b) Solve $f(x) = 4$. [1]
- 8 The function f is defined by $f(x) = x^2 - 6x + 11$, for $x \in \mathbb{R}$.
(a) Write $f(x)$ in the form $(x + a)^2 + b$. [2]
(b) Hence state the range of f . [1]
(c) Explain why f is a many-to-one function. [1]
- 9 The function f is defined by $f(x) = 2x + 1$ for $x < 3$, and $f(x) = 10 - x$ for $x \geq 3$.
(a) Find $f(-2)$, $f(3)$ and $f(8)$.
(b) Solve $f(x) = 5$. [4]
- 10 Find the largest real domain of each function.
(a) $f(x) = \sqrt{2x - 6}$ [2]
(b) $g(x) = \ln(5 - x)$ [2]
(c) $h(x) = \frac{1}{(x - 1)(x + 3)}$ [2]
- 11 Find the range of each function.
(a) $g(x) = (x - 2)^2 + 1$, for $0 \leq x \leq 5$ [3]
(b) $f(x) = 3 - 2x$, for $-1 < x \leq 4$ [2]
- 12 The function f is defined by $f(x) = x^2 - 4$, for $x \in \mathbb{R}$.
(a) Explain why f is a many-to-one function. [1]
(b) The domain is changed to $x \geq k$. State the smallest value of k for which f is one-to-one, and the range of f for this domain. [2]
- 13 The function f is defined by $f(x) = ax + b$, where a and b are constants. Given that $f(2) = 7$ and $f(-1) = -2$, find a and b . [3]

- 14** The diagram shows the graph of $y = f(x)$. It consists of two straight line segments and is defined for $-2 \leq x \leq 5$.

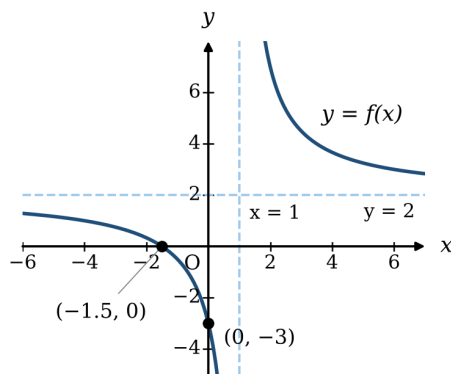
The end points of the graph are $(-2, 4)$ and $(5, 2)$ and the graph has a vertex at $(1, -2)$.



- (a) State the domain and the range of f . [2]
 (b) Solve $f(x) = 0$. [2]
 (c) Find the value of $f(f(-2))$. [2]
-
- 15** The function f is defined by $f(x) = 2x^2 - 8x + 5$, for $x \geq k$, where k is a constant.
- (a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants to be found. [3]
 (b) Given that f is a one-to-one function, state the least possible value of k . [1]
 (c) For this value of k , state the range of f . [1]
 (d) Using this value of k , solve $f(x) = 5$. [2]
-
- 16** The function g is defined by $g(x) = \frac{3x - 1}{x + 2}$, for $x \in \mathbb{R}$, $x \neq -2$.
- (a) Find $g(0)$ and solve $g(x) = 2$. [3]
 (b) Show that $g(x) = 3 - \frac{7}{x + 2}$. [2]
 (c) Hence explain why $g(x)$ can never equal 3. [1]
-
- 17** The function h is defined by $h(x) = e^{2x} - 3$, for $x \in \mathbb{R}$.
- (a) State the range of h . [1]
 (b) Solve $h(x) = 5$, giving your answer in the form $p \ln 2$. [3]
-
- 18** The function f is defined by $f(x) = 3 \sin x + 1$, for $0 \leq x \leq 2\pi$.
- (a) Find the range of f . [2]
 (b) Solve $f(x) = 2.5$, giving exact answers. [3]
-
- 19** The function f is defined by $f(x) = x^2 - 3x$, for $x \in \mathbb{R}$.
- (a) Show that $f(x + h) - f(x) = h(2x + h - 3)$. [2]
 (b) Find $f(a + 1) - f(a - 1)$ in terms of a , simplifying your answer. [2]
-
- 20** Find the range of each function.
- (a) $f(x) = \frac{6}{x + 1}$, for $x \geq 2$ [2]
 (b) $g(x) = \ln(x - 1) + 3$, for $x \geq 4$ [2]
 (c) $h(x) = e^{-x} + 2$, for $x \geq 0$ [2]
-
- 21** A farmer uses 40 m of fencing to make a rectangular enclosure against a straight wall. The side against the wall needs no fencing. The two sides perpendicular to the wall each have length x metres. The area of the enclosure is $A(x)$ square metres.
- (a) Show that $A(x) = 40x - 2x^2$. [2]
 (b) State the domain of A in this context. [2]
 (c) Find the range of A . [2]
 (d) Find the values of x for which the area is 150 m^2 . [2]

- 22 The function f is defined by $f(x) = \frac{2x+3}{x-1}$, for $x \in \mathbb{R}$, $x \neq 1$.

The diagram shows the graph of $y = f(x)$, with its asymptotes shown dashed.



- Show that $f(x) = 2 + \frac{5}{x-1}$. [2]
- Hence find the range of f , explaining your answer. [3]
- Find the exact values of x for which $f(x) = x$. [3]

- 23 A ball is thrown upwards from a point 1.8 m above the ground. Its height H metres above the ground, t seconds after being thrown, is modelled by $H(t) = 1.8 + 12t - 5t^2$.

- Express $H(t)$ in the form $p - q(t - r)^2$, where p , q and r are constants, and hence state the maximum height of the ball. [3]
- Find the time at which the ball hits the ground, giving your answer to 3 significant figures. [2]
- State the domain and range of H in this context. [2]
- Explain why the model gives values of H that are not sensible for $t > 2.54$. [1]

- 24 The function f is defined by $f(x) = \ln(x^2 - x - 6)$.

- Find the largest real domain of f . [2]
- Explain why $f(0)$ is not defined. [1]
- Solve $f(x) = 0$. [3]

- 25 The function f is defined by $f(x) = 2x + a$ for $x < 3$, and $f(x) = x^2 - b$ for $x \geq 3$, where a and b are constants.

Given that $f(1) = 0$ and that f is continuous at $x = 3$:

- Find a and b . [3]
- Find the range of f . [3]
- Solve $f(x) = 12$. [2]

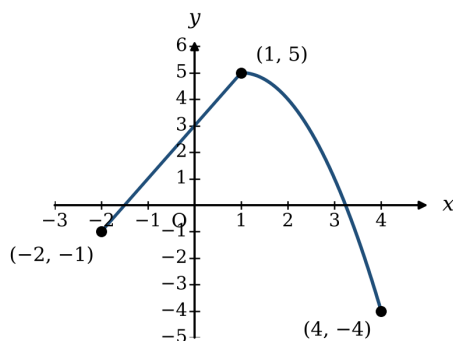
- 26 The function f is odd and has period 4. For $0 \leq x \leq 2$, $f(x) = x(2 - x)$.

- Find $f(5)$, $f(-3)$ and $f(10)$. [3]
- Sketch the graph of $y = f(x)$ for $-4 \leq x \leq 4$, marking the coordinates of the turning points and of the points where it meets the x -axis. [4]
- Solve $f(x) = \frac{3}{4}$ for $0 \leq x \leq 4$. [4]

- 27 The function f is defined by $f(x) = \frac{3}{x^2 + 2x + 4}$, for $x \in \mathbb{R}$.

- Show that the denominator of $f(x)$ is positive for all real x , so that the domain of f is $\mathbb{R}$. [2]
- Find the range of f . [3]
- Find the exact values of x for which $f(x) = \frac{1}{2}$. [3]
- The domain of f is now restricted to $x \geq -1$. State the new range of f and, giving a reason, say whether f is now one-to-one. [2]

- 28** The function f is defined by
 $f(x) = 2x + 3$ for $-2 \leq x < 1$,
 $f(x) = 5 - (x - 1)^2$ for $1 \leq x \leq 4$.
 The diagram shows a sketch of $y = f(x)$.



- (a) Find $f(1)$ and explain why the graph has no break at $x = 1$. [2]
 (b) Find the range of f . [3]
 (c) Find the set of values of k for which the equation $f(x) = k$ has exactly two solutions. [4]

- 29** The function f is defined by $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for $x \in \mathbb{R}$.

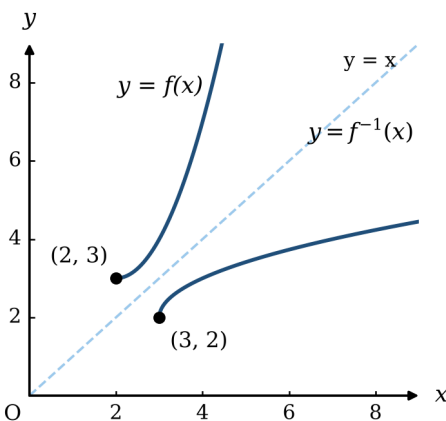
- (a) Show that $f(x) = 1 - \frac{2}{x^2 + 1}$. [2]
 (b) Hence find the range of f . [3]
 (c) Show that f is an even function, and state the geometrical meaning of this. [2]
 (d) Solve $f(x) = \frac{1}{2}$. [2]
 (e) Explain why the equation $f(x) = 1$ has no solutions. [1]

- 30** The function g is defined by $g(x) = \frac{x}{x^2 + 1}$, for $x \in \mathbb{R}$.

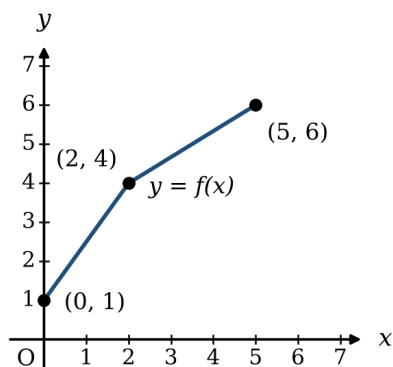
- (a) Show that g is an odd function. [2]
 (b) Show that g is many-to-one by finding two values of x for which $g(x) = \frac{2}{5}$. [3]
 (c) Let $y = g(x)$. Show that $yx^2 - x + y = 0$, and hence use the discriminant to show that $-\frac{1}{2} \leq g(x) \leq \frac{1}{2}$. [3]
 (d) Find the values of x at which g takes its greatest and least values. [2]

Composite and Inverse Functions

- 1** The functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = x^2$, for $x \in \mathbb{R}$.
 Find (a) $fg(2)$, (b) $gf(2)$. [2]
- 2** The function f is defined by $f(x) = x^3 - 1$, for $x \in \mathbb{R}$. Find $f^{-1}(x)$. [2]
- 3** The function f is defined by $f(x) = 3x - 5$, for $x \in \mathbb{R}$.
 Find $f^{-1}(x)$ and hence evaluate $f^{-1}(7)$. [3]
- 4** The function f is defined by $f(x) = \frac{x}{2} + 3$, for $x \in \mathbb{R}$. Find $f^{-1}(x)$. [2]
- 5** The function f is defined by $f(x) = 2x + 3$, for $x \in \mathbb{R}$. Find $ff(x)$. [2]

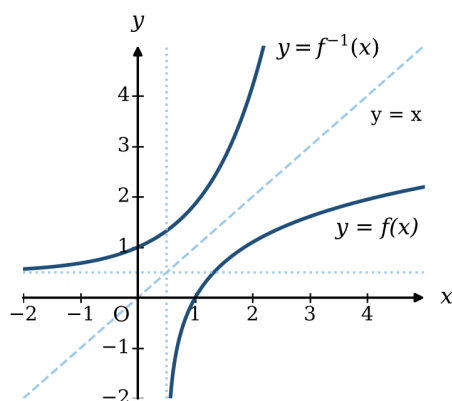
- 6** The functions f and g are defined by $f(x) = x^2 + 1$, for $x \in \mathbb{R}$, and $g(x) = \frac{1}{x-2}$, for $x \in \mathbb{R}, x \neq 2$.
- (a) Find $gf(x)$. [2]
- (b) Solve $gf(x) = \frac{1}{3}$. [2]
-
- 7** The function f is defined by $f(x) = \frac{2x+1}{x-3}$, for $x \in \mathbb{R}, x \neq 3$.
- (a) Find $f^{-1}(x)$. [3]
- (b) State the domain and the range of f^{-1} . [2]
-
- 8** The functions f and g are defined by $f(x) = 3x + 2$ and $g(x) = x - 4$, for $x \in \mathbb{R}$. Show that the equation $fg(x) = gf(x)$ has no solutions. [3]
-
- 9** The function f is defined by $f(x) = \frac{5}{x-2}$, for $x \in \mathbb{R}, x \neq 2$. Find $f^{-1}(x)$ and state its domain. [3]
-
- 10** The function f is defined by $f(x) = e^{2x} - 1$, for $x \in \mathbb{R}$. Find $f^{-1}(x)$, and state its domain. [3]
-
- 11** The function f is defined by $f(x) = (x-3)^2 + 1$, for $x \geq 3$. Find $f^{-1}(x)$ and state its domain. [3]
-
- 12** The function f is defined by $f(x) = x^2 - 4x + 7$, for $x \geq 2$.
The diagram shows the graphs of $y = f(x)$ and $y = f^{-1}(x)$, together with the line $y = x$.
- 
- (a) Find the range of f . [2]
- (b) Find $f^{-1}(x)$. [3]
- (c) Write down the domain of f^{-1} , and explain how the diagram shows that the two graphs are related. [1]
-
- 13** The functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = x^2 + 3$, for $x \in \mathbb{R}$.
- (a) Find $fg(x)$ and $gf(x)$, simplifying each. [2]
- (b) Solve the equation $fg(x) = gf(x)$, giving your answers as exact values. [3]
- (c) Find the range of gf . [2]
-
- 14** The functions f and g are defined by $f(x) = \sqrt{x+3}$, for $x \geq -3$, and $g(x) = x^2 - 5$, for $x \in \mathbb{R}$.
- (a) Find $gf(x)$, stating its domain. [2]
- (b) Find $fg(x)$ and the largest real domain of fg . [2]
- (c) Solve $fg(x) = 5$. [2]
-
- 15** The function f is defined by $f(x) = x^2 + 6x + 2$, for $x \geq -3$.
- (a) Write $f(x)$ in the form $(x+a)^2 + b$ and state the range of f . [2]
- (b) Find $f^{-1}(x)$ and state its domain. [3]
- (c) Find $f^{-1}(2)$. [1]

- 16** The diagram shows the graph of $y = f(x)$ for $0 \leq x \leq 5$. It consists of two straight line segments joining $(0, 1)$, $(2, 4)$ and $(5, 6)$.



- (a) Write down $f^{-1}(4)$ and $f^{-1}(6)$. [2]
 (b) Find the value of $f(f(0))$. [2]
 (c) Sketch the graph of $y = f^{-1}(x)$, giving the coordinates of the end points and of the point where the gradient changes. [2]
-
- 17** The temperature F , in degrees Fahrenheit, is given in terms of the temperature C in degrees Celsius by $F = \frac{9}{5}C + 32$. The Kelvin temperature K is given by $K = C + 273.15$.
- (a) Find C in terms of F . [2]
 (b) Hence express K in terms of F , and find the temperature in degrees Fahrenheit at which $K = 300$. Give your answer to 1 decimal place. [3]
-
- 18** The functions f , g and h are defined by $f(x) = x + 2$, $g(x) = 3x$ and $h(x) = x^2$, for $x \in \mathbb{R}$.
- (a) Find $fgh(x)$ and $hgf(x)$, simplifying each. [4]
 (b) Solve the equation $fgh(x) = hgf(x)$, giving your answers as exact values. [5]
 (c) State the range of hgf . [2]
-
- 19** The function f is defined by $f(x) = 2 - \ln(x + 1)$, for $x > -1$.
- (a) Find $f^{-1}(x)$, and state its domain and range. [4]
 (b) Evaluate $f^{-1}(2)$, and solve $f(x) = 1$, giving an exact answer. [3]
-
- 20** The functions f and g are defined by $f(x) = x^2 - 4x + k$, for $x \in \mathbb{R}$, and $g(x) = 2x - 1$, for $x \in \mathbb{R}$, where k is a constant.
- (a) Find the range of f in terms of k . [2]
 (b) Given that $fg(2) = 13$, find the value of k . [3]
 (c) Using this value of k , find the smallest value of a for which f is one-to-one on the domain $x \geq a$, and find $f^{-1}(x)$ for this domain. [4]
-
- 21** The function f is defined by $f(x) = 3x - 2$, for $x \in \mathbb{R}$, and $gf(x) = 9x^2 - 12x + 5$.
- (a) Find $g(x)$. [3]
 (b) Show that the equation $g(x) = f^{-1}(x)$ has no real solutions. [3]

- 22** The function f is defined by $f(x) = \ln(2x - 1)$, for $x > \frac{1}{2}$. The function g is defined by $g(x) = e^{2x}$, for $x \in \mathbb{R}$. The diagram shows the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

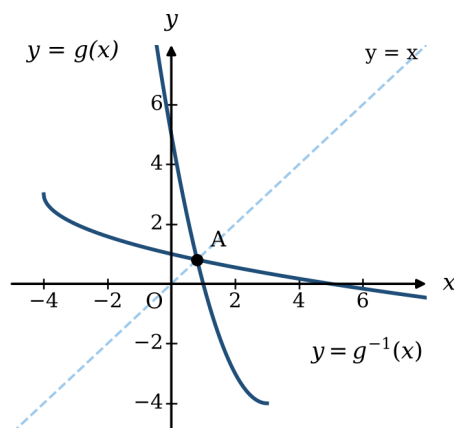


- Find $f^{-1}(x)$. [2]
 - State the domain and the range of f^{-1} . [2]
 - Find the exact solution of the equation $fg(x) = \ln 7$. [3]
 - Find the largest possible domain of fg . [2]
-
- 23** The functions f and g are defined by $f(x) = 2e^x - 1$, for $x \in \mathbb{R}$, and $g(x) = \ln(x + 4)$, for $x > -4$.
- State the range of f . [1]
 - Find $f^{-1}(x)$ and state its domain. [3]
 - Show that $fg(x) = 2x + 7$, and state the domain and range of fg . [4]
 - Solve $f(x) = 7$, giving an exact answer. [2]
-
- 24** The function f is defined by $f(x) = 2x + 1$ for $x \leq 1$, and $f(x) = x^2 + 2$ for $x > 1$.
- Show that f is continuous at $x = 1$, and state the range of f . [3]
 - Find $f^{-1}(x)$, stating clearly its form for $x \leq 3$ and for $x > 3$. [4]
 - Solve $f^{-1}(x) = 2$. [2]
-
- 25** The functions f and g are defined by $f(x) = x^2 - 2x - 3$, for $x \geq 1$, and $g(x) = 2x - 1$, for $x \in \mathbb{R}$.
- Find the range of f . [2]
 - Show that $fg(x) = 4x^2 - 8x$, and state the domain of fg . [3]
 - Solve $fg(x) = 0$. [2]
-
- 26** The functions f and g are defined by $f(x) = x^2 - 6$, for $x \in \mathbb{R}$, and $g(x) = \ln x$, for $x > 0$.
- State the range of f and the range of g . [2]
 - Explain why $gf(1)$ cannot be evaluated. [2]
 - Find the largest possible domain of gf , giving your answer in exact form. [2]
 - Solve the equation $gf(x) = \ln 10$. [2]
-
- 27** The function f is defined by $f(x) = x^2 - 6x + 10$, for $x \geq 3$. The function g is defined by $g(x) = \sqrt{x - 1}$, for $x \geq 1$.
- Find the range of f . [2]
 - Find $f^{-1}(x)$ and state its domain. [3]
 - Show that $gf(x) = x - 3$, and state its domain. [2]
 - Explain why the domain of fg is not $x \geq 1$. Find the domain and the range of fg . [3]

28 The function f is defined by $f(x) = x^2 - 6x + 5$, for $x \in \mathbb{R}$.

A student claims that f has the inverse $f^{-1}(x) = 3 + \sqrt{x + 4}$, for $x \geq -4$.

The function g is defined by $g(x) = x^2 - 6x + 5$, for $x \leq 3$. The diagram shows the graphs of $y = g(x)$ and $y = g^{-1}(x)$, and the line $y = x$. The graphs of g and g^{-1} meet at the point A.



- Explain what is wrong with the student's claim, and state for which restricted domain of f the expression $3 + \sqrt{x + 4}$ is the inverse. [2]
- Find $g^{-1}(x)$ and state its domain and range. [4]
- Find the exact coordinates of A. [3]

29 The function f is defined by $f(x) = \frac{2x + 5}{kx - 2}$, for $x \in \mathbb{R}$, $x \neq \frac{2}{k}$, where k is a positive constant.

- Show that $ff(x) = x$ for all x in the domain, provided $5k + 4 \neq 0$. [3]
- Write down $f^{-1}(x)$, giving a reason. [2]
- For $k = 1$, solve $f(x) = x$. [2]
- For $k = 1$, state the range of f , and explain why the graph of $y = f(x)$ is symmetrical in the line $y = x$. [2]

30 The function f is defined by $f(x) = x^2 - 2x + 2$, for $x \geq 1$.

- Find $f^{-1}(x)$, stating its domain. [3]
- Solve $f(x) = x$. [2]
- Suppose that the graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at (a, b) where $a \neq b$. Explain why (b, a) also lies on both graphs, and use the fact that f is increasing to show that this is impossible. [4]
- Hence state the coordinates of all points of intersection of $y = f(x)$ and $y = f^{-1}(x)$, and verify that $ff(x) = x$ has the same solutions. [3]

Graph Transformations

1 Describe the single transformation that maps the graph of $y = x^2$ onto the graph of

- $y = (x - 3)^2$, [1]
- $y = x^2 + 5$, [1]
- $y = -x^2$. [1]

2 The curve $y = f(x)$ has a minimum point at $(3, -2)$. Write down the coordinates of the minimum point of the curve with equation

- $y = f(x) + 4$, (b) $y = f(x - 1)$. [2]

3 Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of

- $y = f(x - 5)$, (b) $y = f(x) - 3$, (c) $y = 2f(x)$. [3]

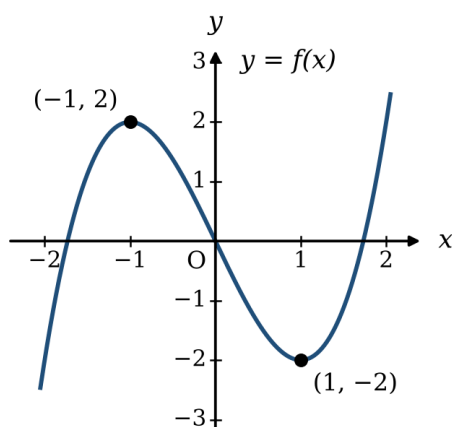
- 4** The curve $y = f(x)$ has a maximum point at $(2, 5)$. Write down the coordinates of the maximum point of
(a) $y = f(x + 1)$, (b) $y = f(x) + 2$, (c) $y = 3f(x)$. [3]

- 5** The curve $y = \sqrt{x}$ is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$. Write down the equation of the new curve. [2]

- 6** The curve $y = 2^x$ is reflected (a) in the x -axis, (b) in the y -axis. Write down the equation of each image. [2]

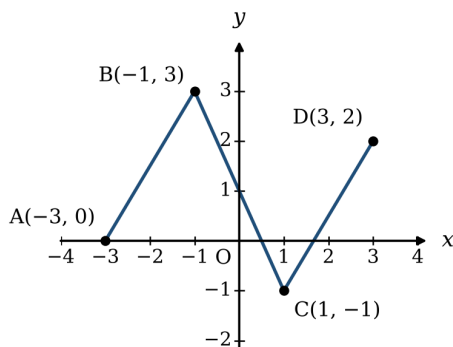
- 7** The point $P(4, 6)$ lies on the curve $y = f(x)$. Write down the coordinates of the image of P on the curve with equation
(a) $y = 3f(x)$, [1]
(b) $y = f(2x)$, [1]
(c) $y = -f(x)$, [1]
(d) $y = f(-x)$. [1]

- 8** The diagram shows the curve $y = f(x)$, where $f(x) = x^3 - 3x$. It has a maximum point at $(-1, 2)$ and a minimum point at $(1, -2)$.



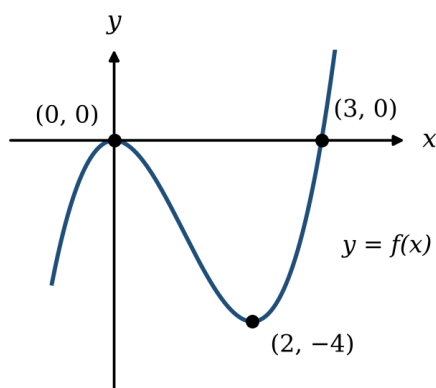
- (a) Write down the coordinates of the turning points of the curve $y = f(x - 2)$. [2]
(b) Write down the coordinates of the turning points of the curve $y = 2f(x)$. [2]
(c) State the coordinates of the maximum point of $y = f(-x)$. [1]
- 9** By completing the square, describe the translation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 4x + 7$. [2]
- 10** Describe the single transformation that maps the graph of $y = \sin x$ onto the graph of
(a) $y = \sin 2x$, (b) $y = 3 \sin x$, (c) $y = \sin(x + \frac{\pi}{3})$. [3]
- 11** Show that translating the graph of $y = e^x$ by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ gives the same curve as a stretch of $y = e^x$ parallel to the y -axis, and state the scale factor. [3]
- 12** Answer both parts of this question.
(a) The curve $y = x^2$ is mapped onto the curve $y = x^2 + 6x + 5$ by a single translation. Find the translation vector. [3]
(b) Describe a sequence of two stretches and a translation that maps the curve $y = \sin x$ onto the curve $y = 2 \sin 3x + 1$. [3]

- 13** The diagram shows the graph of $y = f(x)$, which consists of three straight line segments joining the points $A(-3, 0)$, $B(-1, 3)$, $C(1, -1)$ and $D(3, 2)$.
In each of the following parts, sketch the new graph and mark the coordinates of the images of A , B , C and D . (You may sketch on separate axes.)



- (a) $y = f(x - 2)$. [2]
 (b) $y = -f(x)$. [2]
 (c) $y = f(2x)$. [2]

- 14** The diagram shows the curve $y = f(x)$, where $f(x) = x^3 - 3x^2$. The curve has a maximum at $(0, 0)$, a minimum at $(2, -4)$, and crosses the x -axis at $(3, 0)$.



- (a) Sketch the curve $y = f(x + 1)$, giving the coordinates of the turning points and of the points where it meets the axes. [4]
 (b) Write down the coordinates of the turning points of $y = -f(x)$. [2]

- 15** The graph of $y = \cos x$ is transformed onto the graph of $y = 2 \cos 3x - 1$.
 (a) Describe a sequence of two stretches and a translation that achieves this. [3]
 (b) State the maximum value, the minimum value and the period of $y = 2 \cos 3x - 1$. [2]

- 16** The curve C has equation $y = x^2$. It is transformed by a translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ followed by a stretch, scale factor $\frac{1}{2}$, parallel to the x -axis.

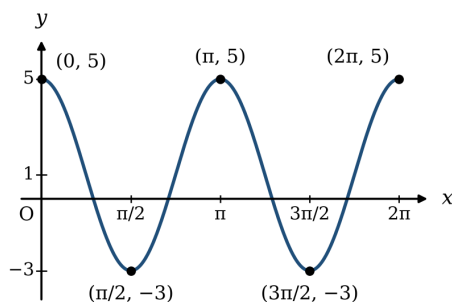
- (a) Show that the equation of the final curve is $y = (2x + 4)^2$. [3]
 (b) Write down the coordinates of the vertex of the final curve. [1]

- 17** The graph of $y = x^2$ is reflected in the x -axis and then translated by $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
 (a) Find the equation of the resulting curve. [2]
 (b) Find the coordinates of the maximum point, and the exact x -coordinates of the points where the curve meets the x -axis. [3]

- 18** The points $(-2, 1)$, $(0, 4)$ and $(3, -2)$ lie on the curve $y = f(x)$. Find the coordinates of the corresponding points on the curve $y = 3 - f(x + 2)$. [4]

- 19** The curve C has equation $y = 2x^2 - 12x + 13$.
- Write the equation of C in the form $y = a(x + b)^2 + c$. [3]
 - Describe a sequence of three transformations that maps the graph of $y = x^2$ onto C, and write down the coordinates of the minimum point of C. [4]
 - Find the exact x-coordinates of the points where C meets the x-axis. [3]

- 20** The diagram shows the graph of $y = a + b \cos(cx)$, for $0 \leq x \leq 2\pi$, where a, b and c are positive constants. The curve has maximum points at $(0, 5)$, $(\pi, 5)$ and $(2\pi, 5)$, and minimum points at $(\frac{\pi}{2}, -3)$ and $(\frac{3\pi}{2}, -3)$.



- Find the values of a, b and c. [3]
- Describe a sequence of transformations that maps $y = \cos x$ onto this curve. [3]
- Find the number of solutions of the equation $1 + 4 \cos 2x = 3$ in the interval $0 \leq x \leq 2\pi$, explaining how you know. [2]

- 21** The curve C has equation $y = x^2 - 4x + 1$. It is reflected in the x-axis and then translated by $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ to give the curve C_1 .

- Show that the equation of C_1 is $y = -x^2 + 8x - 10$. [3]
- Find the coordinates of the maximum point of C_1 and the exact x-coordinates of its intersections with the x-axis. [3]
- Find the equation of the curve obtained if the translation is carried out first, and then the reflection. Comment on the result. [3]

- 22** The curve C has equation $y = 2^x$.

- Show that the translation of C by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ is the same curve as the stretch of C parallel to the y-axis with scale factor 8. [2]
- C is reflected in the y-axis and then translated by $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$. Write down the equation of the new curve and sketch it, stating its asymptote and the coordinates of the points where it meets the axes. [4]

- 23** The curve C has equation $y = f(x)$, where $f(x) = x^2 + bx + c$. The curve $y = f(x)$ is translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the curve with equation $y = x^2 - 10x + 27$.

- Show that the translated curve has equation $y = x^2 + (b - 6)x + c - 3b + 7$. [3]
- Find the values of b and c. [3]

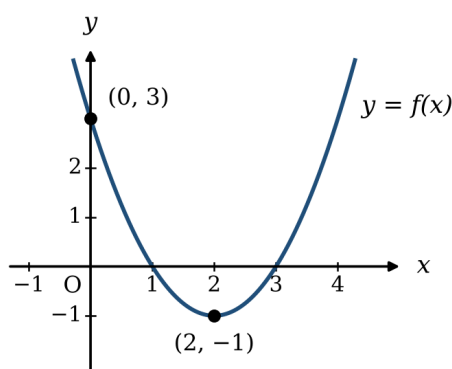
- 24** The function f is defined by $f(x) = x^2 - 4x + 3$.

- Show that $f(4 - x) = f(x)$ for all x. [2]
- The graph of $y = f(4 - x)$ is the image of $y = f(x)$ under a reflection. Name the line of reflection, and deduce a line of symmetry of $y = f(x)$. [3]
- Write down a line of symmetry of $y = f(x + 1)$. [1]

- 25** The curve with equation $y = f(x)$ meets the x -axis at the points $(1, 0)$ and $(5, 0)$, and has a minimum point at $(3, -4)$.
- Find the two values of α for which the graph of $y = f(x + \alpha)$ passes through the origin. [2]
 - Find the two values of k , $k > 0$, for which the graph of $y = f(kx)$ passes through the point $(10, 0)$. [3]
 - Write down the coordinates of the minimum point of the curve $y = 2f(x - 1) + 3$. [3]

- 26** The diagram shows the graph of $y = f(x)$, where $f(x) = x^2 - 4x + 3$. The curve has its minimum at $(2, -1)$ and crosses the y -axis at $(0, 3)$.

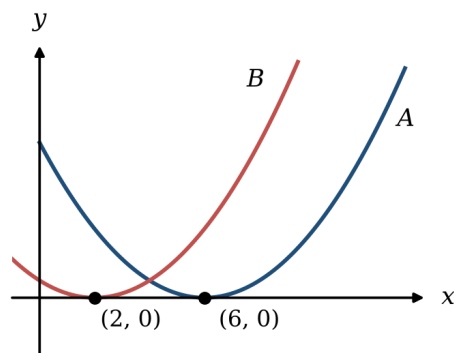
The curve C_1 is obtained from $y = f(x)$ by a stretch parallel to the x -axis with scale factor $\frac{1}{2}$, followed by a translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$. The curve C_2 is obtained from $y = f(x)$ by the same two transformations applied in the opposite order.



- Find an equation for C_1 in the form $y = f(\dots)$. [2]
 - Find an equation for C_2 in the form $y = f(\dots)$. [2]
 - Find the coordinates of the minimum point of each of C_1 and C_2 , and hence explain why the order of the transformations matters. [3]
 - Find the x -coordinates of the points where C_1 meets the line $y = 3$. [3]
- 27** Two students make statements about transformations.
- Student A: "The graph of $y = (x - 4)^2$ is the graph of $y = x^2$ translated 4 units to the left."
- Student B: "The graph of $y = 2x^2$ is the graph of $y = x^2$ stretched parallel to the x -axis with scale factor 2."
- Explain the mistake in each statement. [4]
 - Show that $y = 2x^2$ can also be obtained from $y = x^2$ by a stretch parallel to the x -axis, and state the scale factor. [3]
 - Show that the graph of $y = 3e^x$ can be obtained from $y = e^x$ by a translation, and state the translation vector. [2]
- 28** The cubic equation $x^3 - 4x^2 + x + 6 = 0$ has roots $-1, 2$ and 3 . Use transformations of the curve $y = x^3 - 4x^2 + x + 6$ to find a cubic equation, with integer coefficients and leading coefficient 1, whose roots are
- each 2 more than the original roots, [4]
 - twice the original roots, [3]
 - the negatives of the original roots. [3]

29

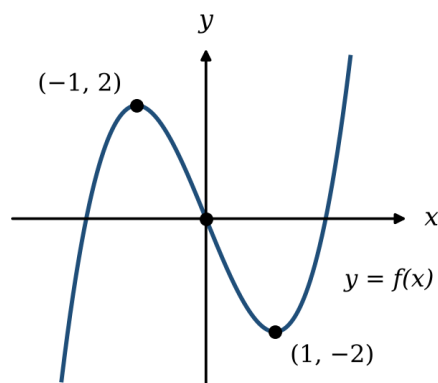
The graph of $y = x^2$ is transformed in two ways. In transformation A it is first translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and then stretched, scale factor 3, parallel to the x-axis. In transformation B it is first stretched, scale factor 3, parallel to the x-axis and then translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$. The diagram shows the two images.



- Find the equation of the image under A, and the equation of the image under B. [4]
- Find the coordinates of the vertex of each image, and hence identify which curve in the diagram is which. [3]
- Explain in words why the two vertices differ, and state the translation which, applied after the stretch, would give the same curve as A. [2]
- Find the exact x-coordinates of the points where the two curves intersect. [3]

30

The diagram shows the curve $y = f(x)$, where $f(x) = x^3 - 3x$. It has a maximum at $(-1, 2)$ and a minimum at $(1, -2)$.



- Show that the equation $f(x) + k = 0$ has three distinct real roots if and only if $-2 < k < 2$. [4]
- Find the values of a for which the curve $y = f(x - a)$ passes through the origin. [3]
- The curve $y = f(x - 1) + k$ touches the x-axis. Find the two possible values of k , and the x-coordinate of the point of contact in each case. [5]

Polynomial Graphs

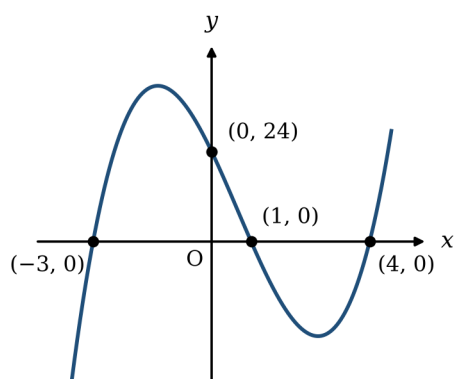
- The curve C has equation $y = (x - 1)(x + 2)(x - 4)$.
 - Write down the coordinates of the points where C crosses the x-axis. [1]
 - Find the coordinates of the point where C crosses the y-axis. [1]
- Sketch the curve with equation $y = x(x - 3)(x + 2)$, showing the coordinates of the points where it crosses the axes. [3]
- Write down the roots of $f(x) = (x + 1)(2x - 3)(x - 4)$. [2]

4 Sketch the curve with equation $y = x^2(x - 4)$, showing the coordinates of the points where it meets the axes. [3]

5 Sketch the curve with equation $y = (1 - x)(x + 2)(x + 4)$, showing the coordinates of the points where it meets the axes. [3]

6 Sketch the curve with equation $y = (x - 2)^2(x + 1)$, showing the coordinates of the points where it meets the axes. [4]

7 The diagram shows the curve $y = f(x)$, where f is a cubic. The curve crosses the x -axis at $(-3, 0)$, $(1, 0)$ and $(4, 0)$ and the y -axis at $(0, 24)$.



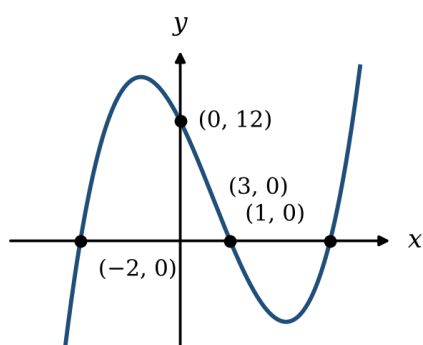
(a) Find $f(x)$ in factorised form. [3]

(b) Hence write $f(x)$ in the form $ax^3 + bx^2 + cx + d$. [2]

8 Sketch the curve with equation $y = x^2(x - 2)(x + 2)$, showing the coordinates of the points where it meets the axes. [4]

9 Sketch the curve with equation $y = (x - 1)^3$, showing the coordinates of the points where it meets the axes. [3]

10 The diagram shows the curve $y = f(x)$, where f is a cubic. The curve crosses the x -axis at $(-2, 0)$, $(1, 0)$ and $(3, 0)$ and the y -axis at $(0, 12)$.



(a) Write $f(x)$ in the form $k(x + 2)(x - 1)(x - 3)$ and find the value of k . [3]

(b) Find $f(4)$. [1]

11 By sketching the curve $y = (x + 1)(x - 2)(x - 4)$, or otherwise, solve the inequality $(x + 1)(x - 2)(x - 4) > 0$. [3]

12 Describe the behaviour of y as $x \rightarrow \infty$ and as $x \rightarrow -\infty$ for each curve.

(a) $y = x^3 - 4x$ [2]

(b) $y = (2 - x)(x + 1)(x + 3)$ [2]

(c) $y = x^2(x - 1)(x + 4)$ [2]

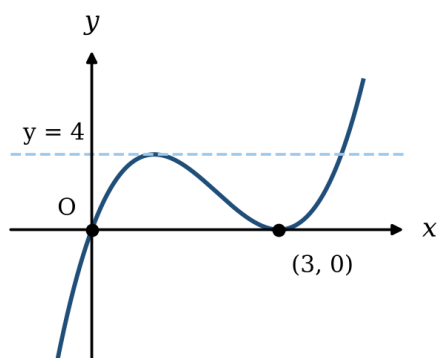
- 13** The curve C has equation $y = x^3 - 3x^2 - 4x + 12$.
- (a) Show that $(x - 3)$ is a factor of $x^3 - 3x^2 - 4x + 12$ and hence factorise it completely. [3]
- (b) Sketch C, showing the coordinates of the points where it crosses the axes. [3]

- 14** Answer all three parts.
- (a) Sketch, on the same axes, the curve $y = x^3 - 4x$ and the line $y = x$, marking the coordinates of the points where the curve meets the axes. [3]
- (b) Find the exact x-coordinates of the points where the line meets the curve. [3]
- (c) Explain how your sketch confirms that there are exactly three solutions. [1]

- 15** The curve C has equation $y = x^2(x + 2)(3 - x)$.
- (a) Sketch C, showing the coordinates of the points where it meets the axes. [4]
- (b) Hence find the values of x for which $y > 0$. [2]

- 16** $f(x) = x^3 + ax^2 + bx - 6$, where a and b are constants. $(x - 2)$ is a factor of $f(x)$, and when $f(x)$ is divided by $(x + 1)$ the remainder is -12 .
- (a) Find the values of a and b . [4]
- (b) Show that $f(x) = 0$ has only one real root. [2]

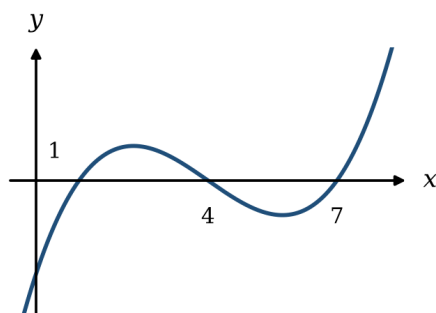
- 17** The diagram shows the curve C with equation $y = x^3 - 6x^2 + 9x$ and the line $y = 4$.



- (a) Factorise $x^3 - 6x^2 + 9x$ completely. [2]
- (b) Show that $x = 1$ is a root of $x^3 - 6x^2 + 9x - 4 = 0$ and find the other roots. [3]
- (c) Explain what the repeated root tells you about the line and the curve. [1]
- 18** The function f is defined by $f(x) = x^3 - x$.
- (a) Factorise $f(x)$ completely. [2]
- (b) Write down the equation of the curve $y = f(x + 2)$ in factorised form, and sketch it, showing where it meets the axes. [3]
- 19** The curves C_1 and C_2 have equations $y = x^2(x - 3)$ and $y = x(3 - x)$ respectively.
- (a) Show that the x-coordinates of the points where C_1 meets C_2 satisfy $x(x - 3)(x + 1) = 0$, and find the coordinates of these points. [4]
- (b) Sketch C_1 and C_2 on the same axes, showing the coordinates of the points where they meet the axes and the coordinates of their points of intersection. [3]

20

The height y kilometres of a section of a road above sea level is modelled by $y = \frac{1}{10}(x - 1)(x - 4)(x - 7)$ for $0 \leq x \leq 8$, where x is the horizontal distance in km from the start of the section. The diagram shows the graph of the model.



- Find the height of the road at the start of the section. [2]
- Find the values of x for which the road is below sea level. [2]
- Give one reason why this model may not be realistic. [1]

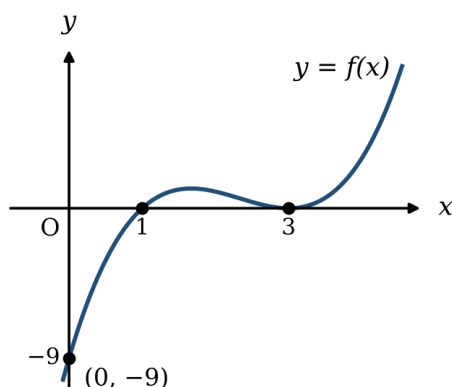
21

The curve C has equation $y = x^4 - 5x^2 + 4$.

- Factorise $x^4 - 5x^2 + 4$ completely. [2]
- Sketch C , showing the coordinates of the points where it meets the axes. [3]
- Find the set of values of k for which the equation $x^4 - 5x^2 + 4 = k$ has four distinct real solutions. [3]

22

The diagram shows the curve $y = f(x)$. The curve touches the x -axis at $(3, 0)$, crosses the x -axis at $(1, 0)$ and crosses the y -axis at $(0, -9)$.



- Show that $f(x) = (x - 1)(x - 3)^2$. [3]
- State the equation of the curve $y = f(x + 2)$, and the coordinates of the points where it meets the axes. [3]
- The curve $y = f(x + a)$ passes through the origin. Find the possible values of a . [2]

23

A cubic curve touches the x -axis at $(-2, 0)$, crosses the x -axis at $(3, 0)$, and crosses the y -axis at $(0, -12)$.

- Find an equation for the curve in the form $y = k(x + 2)^2(x - 3)$, and show that $k = 1$. [3]
- Sketch the curve. [2]
- Solve $(x + 2)^2(x - 3) = -12$, giving exact answers. [3]
- Find the set of values of x for which $(x + 2)^2(x - 3) \geq 0$. [2]

24

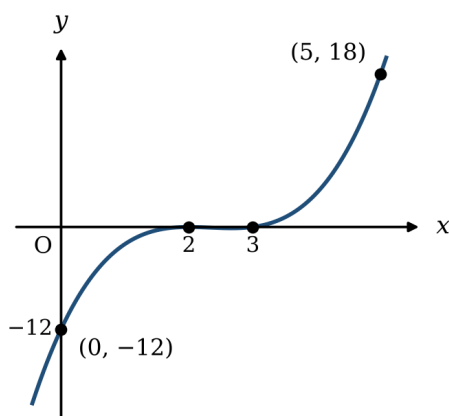
$f(x) = 2x^3 - 3x^2 - 11x + 6$.

- Show that $(x - 3)$ is a factor of $f(x)$, and factorise $f(x)$ completely. [4]
- Sketch the curve $y = f(x)$, showing the coordinates of the points where it meets the axes. [3]
- Hence solve $f(x) < 0$. [2]

- 25** $f(x) = x^3 + px^2 + qx + 12$, where p and q are constants. $(x - 2)$ and $(x + 3)$ are both factors of $f(x)$.
- (a) Find the values of p and q . [4]
- (b) Show that $f(x) = (x - 2)^2(x + 3)$. [2]
- (c) Sketch $y = f(x)$ and hence solve $f(x) > 0$. [4]

- 26** The curve C has equation $y = (x + p)(x - q)^2$, where p and q are positive constants.
- (a) Sketch C , showing the coordinates of the points where it meets the axes in terms of p and q . [4]
- (b) The curve C is translated by $\begin{pmatrix} -q \\ 0 \end{pmatrix}$. Find an equation of the image, and state what happens at the origin. [3]
- (c) Find the set of values of x for which $(x + p)(x - q)^2 \geq 0$. [2]

- 27** The diagram shows the curve $y = f(x)$, where f is a cubic. The curve touches the x -axis at $(2, 0)$, crosses the x -axis at one other point $(b, 0)$, and passes through $(0, -12)$ and $(5, 18)$.



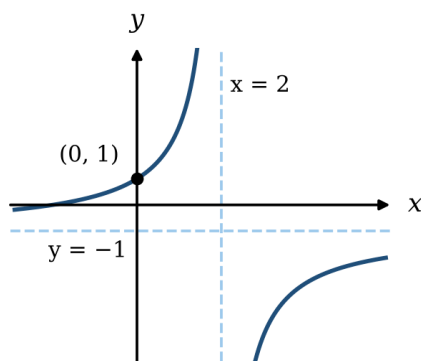
- (a) Find $f(x)$ in the form $k(x - 2)^2(x - b)$, stating the values of k and b . [4]
- (b) Show that the line $y = 4$ meets the curve at exactly one point, and find its coordinates. [3]
- (c) Find the exact coordinates of the minimum point of the curve, and hence find the range of values of k for which $f(x) = k$ has three distinct real solutions. [4]
- 28** A box with no lid is made from a rectangular sheet of card 20 cm by 30 cm. Squares of side x cm are cut from each corner and the sides are folded up. The volume of the box is $V \text{ cm}^3$.
- (a) Show that $V = 4x(10 - x)(15 - x)$. [2]
- (b) State the range of values of x for which the model is valid, and sketch the graph of $y = V$ for all real x , marking the points where $V = 1000$. [3]
- (c) Find, in exact form, the values of x in the valid range for which the volume is 1000 cm^3 . [4]
- 29** The curve C has equation $y = x^3 - 3a^2x + 2a^3$, where a is a positive constant.
- (a) Show that C can be written as $y = (x - a)^2(x + 2a)$. [3]
- (b) Hence describe the points where C meets the x -axis, and state the y -intercept in terms of a . [3]
- (c) Given that C passes through the point $(0, 16)$, find a and sketch C . [3]
- (d) For this value of a , use differentiation to find the coordinates of the turning points of C , and hence find the values of k for which $x^3 - 12x + 16 = k$ has three distinct real solutions. [5]

- 30** $f(x) = x^3 - 9x^2 + kx - 24$, where k is a constant, and $(x - 3)$ is a factor of $f(x)$.
- (a) Show that $k = 26$. [2]
- (b) Factorise $f(x)$ completely. [3]
- (c) Sketch the curve $y = f(x)$, showing where it meets the axes. [3]
- (d) The roots of $f(x) = 0$ form an arithmetic sequence. Write down the common difference, and find the equation of the curve $y = f(x + 1)$ in factorised form, stating whether its roots still form an arithmetic sequence. [4]
- (e) Solve $f(x) < 0$. [2]

Reciprocal and Rational Graphs

- 1** Write down the equations of the asymptotes of the curve with equation
- (a) $y = \frac{1}{x - 3}$, [1]
- (b) $y = \frac{1}{x} + 2$. [1]
- 2** Sketch the graph of $y = \frac{2}{x}$, stating the equations of the asymptotes and the coordinates of two points on the curve. [3]
- 3** Sketch the graph of $y = -\frac{1}{x}$, showing the equations of any asymptotes and the coordinates of two points on the curve. [3]
- 4** Sketch the graph of $y = \frac{1}{x^2}$, showing the equations of any asymptotes and the coordinates of two points on the curve. [3]
- 5** Solve the equation $\frac{1}{x - 2} = 3$. [2]
- 6** Sketch the graph of $y = \frac{1}{x + 2}$, showing the equations of the asymptotes and the coordinates of any points where the curve crosses the axes. [4]
- 7** Sketch the graph of $y = \frac{3}{x - 1} + 2$, showing clearly the equations of the asymptotes and the coordinates of the points where the curve crosses the axes. [5]
- 8** Sketch the graph of $y = \frac{1}{(x - 2)^2}$, showing the equations of the asymptotes and the coordinates of the point where the curve crosses the y -axis. [3]
- 9** The curve C has equation $y = \frac{x + 1}{x - 2}$, $x \neq 2$.
- (a) Show that $y = 1 + \frac{3}{x - 2}$. [2]
- (b) Sketch C , showing the asymptotes and the coordinates of the points where it meets the axes. [3]
- 10** Solve the equation $\frac{1}{x + 1} = x - 1$, giving your answers as exact values. [3]
- 11** The curve $y = \frac{4}{x}$ meets the line $y = x - 3$ at two points. Find the coordinates of these points. [4]

- 12** The diagram shows the curve with equation $y = \frac{a}{x-2} + b$, where a and b are constants. The asymptotes are $x = 2$ and $y = -1$, and the curve passes through $(0, 1)$.

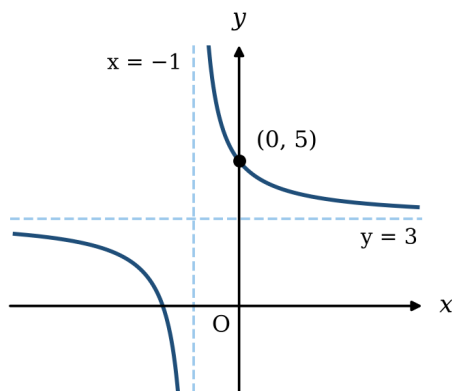


- (a) Write down the value of b . [1]
 (b) Find the value of a . [2]

- 13** Describe the single transformation that maps the graph of $y = \frac{1}{x}$ onto the graph of

- (a) $y = \frac{1}{x+3}$, [1]
 (b) $y = \frac{5}{x}$, [1]
 (c) $y = \frac{1}{x} - 4$, [1]
 (d) $y = -\frac{1}{x}$. [1]

- 14** The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{a}{x+b} + c$ and a , b and c are constants. The asymptotes of the curve are $x = -1$ and $y = 3$, and the curve passes through $(0, 5)$.



- (a) Find the values of a , b and c . [3]
 (b) Find the coordinates of the point where the curve crosses the x -axis. [2]
 (c) Solve the inequality $f(x) > 3$. [1]

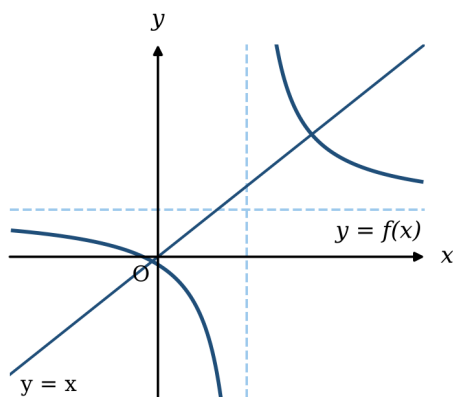
- 15** The curve C has equation $y = \frac{3-x}{x+1}$, $x \neq -1$.

- (a) Show that $y = -1 + \frac{4}{x+1}$. [2]
 (b) Sketch C , showing the equations of the asymptotes and the coordinates of the points where C meets the axes. [4]

- 16** The graph of $y = \frac{1}{x}$ is stretched parallel to the y -axis with scale factor 3 and then translated by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

- (a) Write down the equation of the resulting curve and the equations of its asymptotes. [3]
 (b) Find the coordinates of the points where the curve meets the axes. [3]

- 17** The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{2x+1}{x-3}$, $x \neq 3$, and the line $y = x$.

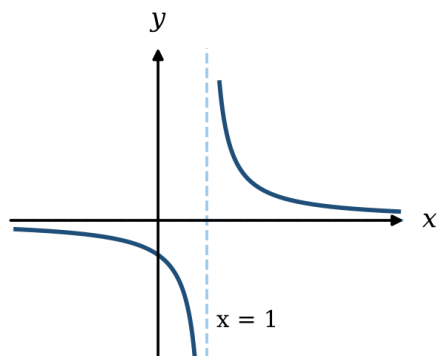


- (a) Show that $f(x) = 2 + \frac{7}{x-3}$. [2]
 (b) Hence write down the equations of the asymptotes, and find the coordinates of the points where the curve crosses the axes. [3]
 (c) Find the exact x -coordinates of the points where the line $y = x$ meets the curve. [2]

- 18** The curve C has equation $y = \frac{1}{x^2 - 4}$.

- (a) Write down the equations of the three asymptotes of C . [2]
 (b) Sketch C , showing the coordinates of the point where C meets the y -axis. [3]

- 19** The diagram shows the curve with equation $y = \frac{1}{x-1}$.



- (a) On the same axes sketch the line $y = 2$, and find the x -coordinate of the point where the line meets the curve. [3]
 (b) Hence solve the inequality $\frac{1}{x-1} < 2$. [2]

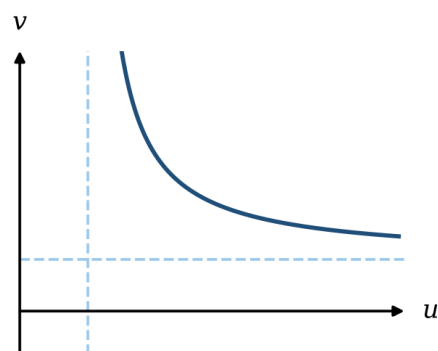
- 20** The curve C has equation $y = \frac{1}{(x+1)^2} - 4$.

- (a) Sketch C , showing the equations of the asymptotes and the coordinates of the points where C crosses the axes. [5]
 (b) Hence, or otherwise, solve the inequality $\frac{1}{(x+1)^2} > 4$. [3]

- 21** The function f is defined by $f(x) = \frac{x}{x^2 - 1}$, for $x \in \mathbb{R}$, $x \neq \pm 1$.

- (a) Show that f is an odd function, and state what this implies about the graph of $y = f(x)$. [3]
 (b) Write down the equations of the asymptotes of the graph. [2]
 (c) Sketch the graph of $y = f(x)$. [3]
 (d) Solve $f(x) > 0$. [2]

- 22** A thin lens has focal length 12 cm. An object is placed at a distance u cm from the lens ($u > 12$), and its image forms at a distance v cm from the lens, where $\frac{1}{u} + \frac{1}{v} = \frac{1}{12}$. The diagram shows the graph of v against u .

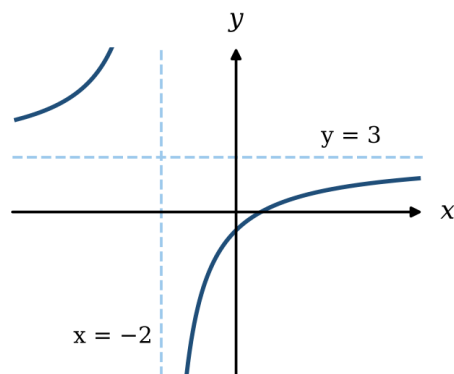


- (a) Show that $v = \frac{12u}{u - 12}$. [3]
- (b) Write down the equations of the asymptotes of the graph and say what each means in this context. [4]
- (c) Find the object distance for which the image distance is twice the object distance. [3]

- 23** The function f is defined by $f(x) = x + \frac{1}{x}$, for $x \in \mathbb{R}$, $x \neq 0$.

- (a) Show that for $x > 0$, $f(x) - 2 = \frac{(x - 1)^2}{x}$, and hence that $f(x) \geq 2$ for $x > 0$. [3]
- (b) Write down the equation of each asymptote of $y = f(x)$, and sketch the graph. [4]
- (c) Solve $f(x) = \frac{5}{2}$. [2]

- 24** The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3x - 2}{x + 2}$, $x \neq -2$.



- (a) Find the coordinates of the points where the graph meets the axes. [2]
- (b) Show that the line $y = x$ does not meet the graph. [3]
- (c) Show that $f(x) = 3 - \frac{8}{x + 2}$, and hence solve $f(x) < 0$. [3]

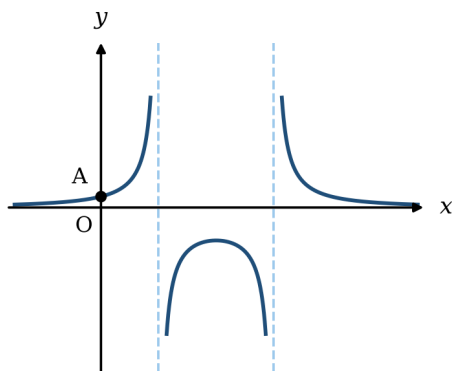
- 25** The curve C has equation $y = \frac{2x + 1}{x + 3}$, $x \neq -3$.

- (a) Show that $y = 2 - \frac{5}{x + 3}$. [2]
- (b) Describe a sequence of three transformations that maps the graph of $y = \frac{1}{x}$ onto C . [3]
- (c) Write down the equations of the asymptotes of C , and find the coordinates of the points where C meets the axes. [4]

26 The curve C has equation $y = \frac{3x+1}{x-2}$, $x \neq 2$, and the line L has equation $y = x + k$, where k is a constant.

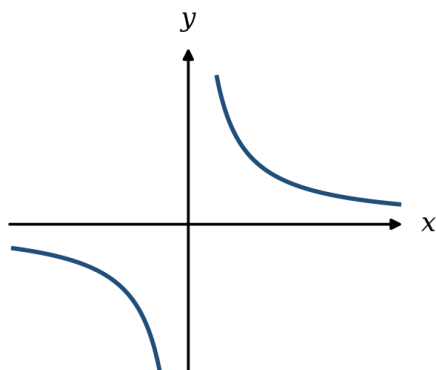
- Sketch C, showing the equations of the asymptotes and the coordinates of the points where C meets the axes. [3]
- Show that the x -coordinates of the points where L meets C satisfy $x^2 + (k-5)x - 2k - 1 = 0$, and prove that L meets C at two distinct points for every real value of k . [5]
- The x -coordinates of the two points are α and β . Given that $\alpha + \beta = 3$, find k and the value of $\alpha\beta$. [2]

27 The diagram shows the graph of $y = \frac{1}{(x-1)(x-3)}$. The curve crosses the y -axis at the point A.



- Write down the equations of the three asymptotes and find the coordinates of A. [3]
- By writing $(x-1)(x-3)$ in completed square form, find the range of the function. [3]
- Solve the inequality $\frac{1}{(x-1)(x-3)} > \frac{1}{3}$. [3]

28 The diagram shows the curve $y = \frac{4}{x}$. The line L has equation $y = k - x$, where k is a positive constant.



- Show that the x -coordinates of the points where L meets the curve satisfy $x^2 - kx + 4 = 0$. [2]
- Find the value of k for which L is a tangent to the curve, and the coordinates of the point of contact. [4]
- State, with reasons, the number of intersections when $k = 5$ and when $k = 3$. [2]
- Hence, or otherwise, solve the inequality $\frac{4}{x} > 5 - x$. [4]

29 The function f is defined by $f(x) = \frac{x^2 - 4}{x^2 - 1}$, for $x \in \mathbb{R}$, $x \neq \pm 1$.

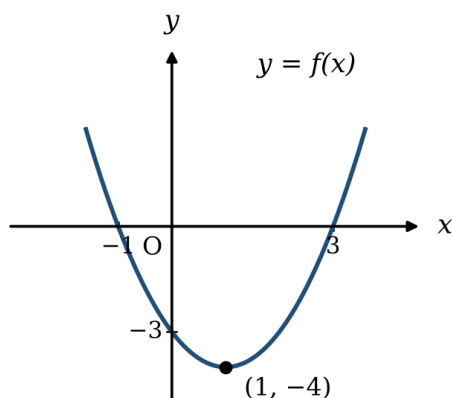
- Show that $f(x) = 1 - \frac{3}{x^2 - 1}$. [2]
- Write down the equations of the asymptotes, and state the symmetry of the graph. [3]
- Sketch $y = f(x)$, showing the coordinates of the points where it meets the axes. [4]
- Solve $f(x) \geq 0$. [3]
- Find the range of f . [5]

- 30** The curve C has equation $y = \frac{k}{x-a} + b$, where k, a and b are positive constants such that $ab > k$.
- (a) Write down the equations of the asymptotes of C. [2]
- (b) Show that C meets the y-axis at $(0, b - \frac{k}{a})$ and the x-axis at $(a - \frac{k}{b}, 0)$. [4]
- (c) Explain why both intercepts are positive. [3]
- (d) Sketch C, showing the asymptotes and both intercepts. [2]

Modulus Graphs

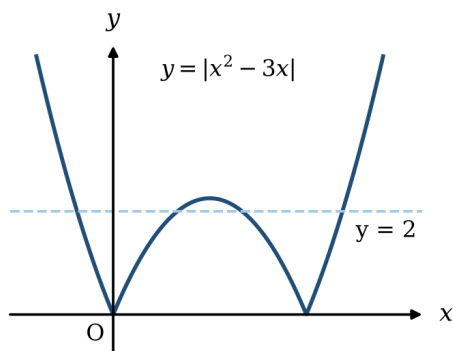
- 1** Evaluate (a) $|-4| + |3|$, (b) $|5 - 12|$. [2]
- 2** Sketch the graph of $y = |x - 3|$, showing the coordinates of the vertex and of the point where the graph crosses the y-axis. [3]
- 3** Solve $|x - 3| = 5$. [2]
- 4** Solve $|2x + 1| = 7$. [2]
- 5** Sketch the graph of $y = |x + 2|$, showing the coordinates of the vertex and of the point where the graph crosses the y-axis. [2]
- 6** The function f is defined by $f(x) = |3 - 2x|$. Find $f(-1)$ and $f(4)$. [2]
- 7** Answer both parts.
- (a) Sketch the graph of $y = |2x - 4|$, showing the coordinates of the points where it meets the axes. [2]
- (b) Solve the equation $|2x - 4| = 6$. [2]
- 8** Answer both parts.
- (a) Sketch the graph of $y = |x^2 - 4|$, showing the coordinates of the points where it meets the axes. [3]
- (b) Solve $|x^2 - 4| = 5$. [2]
- 9** Solve each inequality.
- (a) $|x - 4| < 3$ [2]
- (b) $|2x + 1| \geq 5$ [2]
- 10** Sketch the graph of $y = 4 - |x|$, showing the coordinates of the points where it meets the axes. [3]
- 11** Solve $|2x - 3| = x$. [4]
- 12** A machine fills bags of flour. A bag with mass m grams is accepted if $|m - 500| \leq 3$.
- (a) Write the condition without the modulus sign. [2]
- (b) Which of the masses 498.5 g, 503.2 g and 496.9 g would be rejected? [1]
- 13** Answer all three parts.
- (a) Solve the equation $|2x - 1| = |x + 4|$. [3]
- (b) Hence solve the inequality $|2x - 1| < |x + 4|$. [2]
- (c) Sketch the graphs of $y = |2x - 1|$ and $y = |x + 4|$ on the same axes, to illustrate your answers. [2]
- 14** Solve $|3x - 2| = |x|$. [4]

- 15** The diagram shows the graph of $y = f(x)$, where $f(x) = x^2 - 2x - 3$. The minimum point is $(1, -4)$.



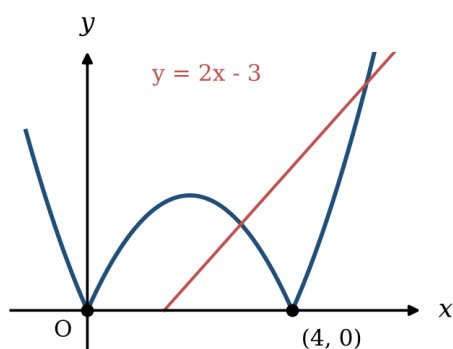
- (a) Sketch the graph of $y = |f(x)|$, showing the coordinates of the points where it meets the axes and of its local maximum point. [3]
- (b) Sketch the graph of $y = f(|x|)$, showing the coordinates of the points where it meets the axes. [3]
-
- 16** Consider the inequality $|x - 2| < |x + 1|$.
- (a) By squaring both sides, solve the inequality. [3]
- (b) Explain why squaring both sides is valid here. [1]
-
- 17** Let $f(x) = x - 2$.
- (a) Sketch $y = |f(x)|$ on a pair of axes, showing the coordinates of the points where it meets the axes. [2]
- (b) On the same axes sketch $y = f(|x|)$, showing where it meets the axes, and describe how the two graphs differ. [3]
-
- 18** Solve $||x| - 3| = 2$. [3]
-
- 19** Let $f(x) = \frac{1}{x - 2}$, for $x \in \mathbb{R}$, $x \neq 2$.
- (a) Sketch the graph of $y = |f(x)|$, showing the equation of each asymptote and the coordinates of the point where it meets the axes. [3]
- (b) Solve $|f(x)| > 1$. [3]
-
- 20** The function f is defined by $f(x) = |\ln x|$, for $x > 0$.
- (a) Sketch the graph of $y = f(x)$, showing the coordinates of the point where it meets the x-axis and the equation of its asymptote. [3]
- (b) Solve $f(x) = 2$. [2]
- (c) Solve $f(x) < 1$. [2]
-
- 21** Answer all three parts.
- (a) Sketch, on the same axes, the graphs of $y = |3x - 6|$ and $y = x + 2$, showing the coordinates of the points where each graph meets the axes. [3]
- (b) Solve $|3x - 6| = x + 2$. [3]
- (c) Hence solve $|3x - 6| > x + 2$. [2]
-
- 22** The functions f and g are defined by $f(x) = |2x - 5|$ and $g(x) = x^2 - 3$, for $x \in \mathbb{R}$.
- (a) Find $fg(x)$, and show that $gf(x) = 4x^2 - 20x + 22$. [3]
- (b) Solve $fg(x) = 3$. [3]
- (c) State the range of fg . [2]

- 23** The diagram shows the graph of $y = |x^2 - 3x|$ and the line $y = 2$.



- (a) Find the exact x -coordinates of the four points where the line meets the graph. [4]
 (b) Hence solve the inequality $|x^2 - 3x| < 2$. [3]
 (c) State, with a reason, the number of solutions of $|x^2 - 3x| = 3$. [1]

- 24** The diagram shows the graph of $y = |x^2 - 4x|$ and the line $y = 2x - 3$.



- (a) Show that the equation $|x^2 - 4x| = 2x - 3$ can only have solutions with $x \geq \frac{3}{2}$. [2]
 (b) Solve $|x^2 - 4x| = 2x - 3$, giving exact answers. [5]

- 25** The function f is defined by $f(x) = |x + 2| - |x - 1|$, for $x \in \mathbb{R}$.

- (a) Write $f(x)$ without modulus signs in each of the regions $x < -2$, $-2 \leq x \leq 1$ and $x > 1$. [3]
 (b) Sketch $y = f(x)$ and state the range of f . [3]
 (c) Solve $f(x) = 1$. [2]

- 26** The equation $|2x - k| = x + 3$, where k is a constant, has $x = 4$ as a solution.

- (a) Show that $k = 1$ or $k = 15$. [3]
 (b) For each of these values of k , find the other solution of the equation. [4]

- 27** A student solves $|2x - 1| = 3x + 2$ and writes:

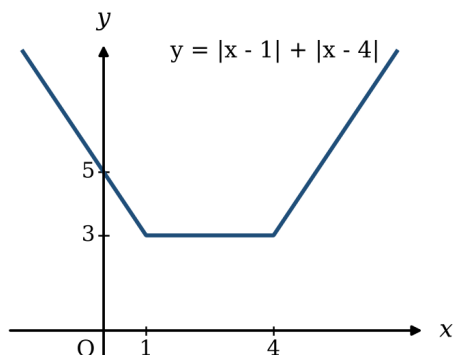
" $2x - 1 = 3x + 2$ gives $x = -3$. $1 - 2x = 3x + 2$ gives $x = -\frac{1}{5}$. So the solutions are $x = -3$ and $x = -\frac{1}{5}$."

- (a) Show that the student's answer contains an error, and explain how it arose. [2]
 (b) Solve the equation correctly. [3]
 (c) Hence solve $|2x - 1| > 3x + 2$. [2]
 (d) Sketch the graphs of $y = |2x - 1|$ and $y = 3x + 2$ to confirm your answer to part (c). [2]

- 28** In this question a , b and x are real numbers.

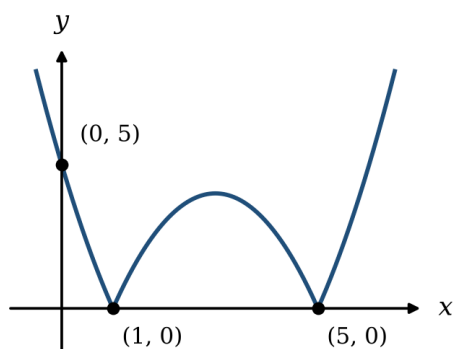
- (a) Prove that $|a + b| \leq |a| + |b|$. [4]
 (b) Deduce that $|x + 3| \leq |x| + 3$ for all x , stating when equality holds. [2]
 (c) Given that $|x - 2| < 1$, show that $|x^2 - 4| < 5$. [4]

- 29** The diagram shows the graph of $y = |x - 1| + |x - 4|$.



- Find the equation of the graph in each of the regions $x < 1$, $1 \leq x \leq 4$ and $x > 4$. [3]
- Solve $|x - 1| + |x - 4| = 7$. [2]
- Hence solve $|x - 1| + |x - 4| < 7$. [2]
- Find the set of values of k for which $|x - 1| + |x - 4| = k$ has exactly two solutions. [2]
- Prove that $|x - 1| + |x - 4| \geq 3$ for all real x . [1]

- 30** The diagram shows the graph of $y = |x^2 - 6x + 5|$.



- Show that the local maximum of the graph is at $(3, 4)$, explaining how it arises from the graph of $y = x^2 - 6x + 5$. [3]
- Find the number of solutions of $|x^2 - 6x + 5| = k$ when $k < 0$, $k = 0$, $0 < k < 4$, $k = 4$ and $k > 4$. [5]
- Solve $|x^2 - 6x + 5| = 3$, giving exact answers. [4]
- Hence solve $|x^2 - 6x + 5| < 3$. [3]