

A LEVEL MATHS · EXTREME PAPERS

MATHS



EXAM PAPER

EXTREME PAPER: STATISTICS

SCORE

/90

PERCENTAGE

%

STEMUP

Paper: Statistics (90 marks)

1 Edexcel | AQA | OCR A | OCR MEI | CIE

A fair coin is tossed repeatedly. Alia wins the game if the three consecutive results H, T, H appear before the three consecutive results H, H, T appear. Ben wins if H, H, T appears first. For example, the sequence T H H H T ends the game on the fifth toss, with a win for Ben. You may assume that the game ends with probability 1.

- Show that the probability that Alia wins the game is $\frac{1}{3}$. [5]
- The coin is now biased, with $P(H) = p$ and $P(T) = q = 1 - p$, where $0 < p < 1$. Show that the probability that Alia wins is $\frac{q}{1+q}$, and deduce that Ben is more likely than Alia to win, whatever the value of p . [3]
- For the fair coin, find the probability that Alia wins on the third or the fourth toss, given that Alia wins the game. [2]

2 Edexcel | AQA | OCR A | OCR MEI | CIE

Two scores X_1 and X_2 are independent and each has the distribution $N(\mu, \sigma^2)$. Let M be the larger and L the smaller of the two scores. Use the values $\Phi(0.8416) = 0.8$, $\Phi(1.2816) = 0.9$ and $\Phi(2) = 0.9772$, where Φ is the cumulative distribution function of $N(0, 1)$.

- Given that $P(M < 60) = 0.64$ and $P(L > 40) = 0.81$, find μ and σ , each to 3 significant figures. [5]
- Now n independent scores, each with the distribution $N(\mu, \sigma^2)$, are taken and the largest is M_n . Show that $P(M_n > \mu) = 1 - 2^{-n}$. [2]
- Let M_n and L_n be the largest and smallest of n independent scores, each with the distribution $N(\mu, \sigma^2)$. Show that $P(M_n > \mu + 2\sigma \text{ and } L_n < \mu - 2\sigma) = 1 - 2[\Phi(2)]^n + [2\Phi(2) - 1]^n$, and evaluate this for $n = 20$, to 3 significant figures. [4]

3 Edexcel | AQA | OCR A | OCR MEI | CIE

Measurements X taken from a process are normally distributed with standard deviation 12. The mean μ of the process is known to be either 80 or 84. To decide which, a random sample of n measurements is taken, with sample mean $\bar{x}$, and this decision rule is used:

conclude $\mu = 84$ if $\bar{x} > c$, otherwise conclude $\mu = 80$.

Error E1 is concluding $\mu = 84$ when $\mu = 80$. Error E2 is concluding $\mu = 80$ when $\mu = 84$.

- The constant c is chosen so that $P(E1) = P(E2)$. Show that $c = 82$ whatever the value of n . [3]
- Using $c = 82$, find the least n for which each error probability is at most 0.01. Use $\Phi(2.3263) = 0.99$. [3]
- Now $n = 36$ and $c = 82$. Before sampling, $P(\mu = 80) = 0.7$ and $P(\mu = 84) = 0.3$. Use $\Phi(1) = 0.8413$.
 - Find $P(E1)$ and $P(E2)$.
 - Given that the decision reached is " $\mu = 84$ ", find the probability that in fact $\mu = 80$. [5]

4 Edexcel | AQA | OCR A | OCR MEI | CIE

A manager interviews n candidates, one after another, in a random order, with all $n!$ orders equally likely. The candidates can be ranked from best to worst with no ties, but at each interview the manager sees only how that candidate compares with those already seen. Each candidate must be accepted or rejected immediately, and a rejected candidate cannot be recalled. The manager uses this rule, for a chosen integer k with $1 \leq k < n$:

reject the first k candidates; then accept the first later candidate who is better than every candidate seen so far; if there is no such candidate, accept the last candidate.

(a) For $n = 4$ and $k = 1$, show that the probability that the best candidate is accepted is $\frac{11}{24}$. [4]

(b) For general n and k , show that the probability that the best candidate is accepted is $\frac{k}{n} \sum_{j=k+1}^n \frac{1}{j-1}$. [4]

(c) For $n = 5$, find the value of k which gives the greatest probability of accepting the best candidate, and state this probability. [3]

5 CIE

The ten letters of the word STATISTICS (three S, three T, two I, one A, one C) are arranged in a row.

(a) Find the number of different arrangements. [2]

(b) Find the number of arrangements in which no two S are adjacent. [3]

(c) Show that the number of arrangements in which no two S are adjacent and no two T are adjacent is 12 120.

(Hint: arrange A, I, I, C first.)

[6]

6 Edexcel | AQA | OCR A | OCR MEI

For a set of bivariate data (x, y) of size n , $S_{xx} = \sum(x - \bar{x})^2$, $S_{xy} = \sum(x - \bar{x})(y - \bar{y})$ and $S_{yy} = \sum(y - \bar{y})^2$.

The regression line of y on x has gradient $\frac{S_{xy}}{S_{xx}}$ and passes through $(\bar{x}, \bar{y})$; the product moment correlation coefficient is $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.

Group P has $m = 8$ observations with mean $(2, 5)$, $S_{xx} = 12$, $S_{xy} = 18$ and $S_{yy} = 30$. Group Q has $n = 12$ observations with mean $(6, 3)$, $S_{xx} = 20$, $S_{xy} = 14$ and $S_{yy} = 16$.

(a) Two groups have sizes m and n , and x -means $\bar{x}_1$ and $\bar{x}_2$. Show that for the combined

data $S_{xx} = S_{xx}^{(1)} + S_{xx}^{(2)} + \frac{mn}{m+n}(\bar{x}_1 - \bar{x}_2)^2$, where $S_{xx}^{(1)}$ and $S_{xx}^{(2)}$ are the values for the separate groups.

The corresponding results $S_{xy} = S_{xy}^{(1)} + S_{xy}^{(2)} + \frac{mn}{m+n}(\bar{x}_1 - \bar{x}_2)(\bar{y}_1 - \bar{y}_2)$ and $S_{yy} =$

$S_{yy}^{(1)} + S_{yy}^{(2)} + \frac{mn}{m+n}(\bar{y}_1 - \bar{y}_2)^2$ may be used in the rest of the question. [4]

(b) Find the product moment correlation coefficient for group P, for group Q, and for the combined data, each to 3 significant figures. [3]

(c) Find the equation of the regression line of y on x for the combined data, in the form $y = (a - x)/b$ where a and b are integers. [3]

(d) The y -mean of group Q is changed to $5 + d$, with everything else unchanged. Find the value of d for which the combined correlation coefficient is zero. [2]

7 Edexcel | AQA | OCR A | OCR MEI | CIE

The random variable X has the binomial distribution $B(n, p)$, where n is a positive integer, $0 < p < 1$ and $q = 1 - p$. For a function g , $E(g(X)) = \sum g(r) P(X = r)$, summed over all $r = 0, 1, \dots, n$.

- (a) By considering expansions of $(q + p)^n$ and $(q - p)^n$, show that $P(X \text{ is even}) = \frac{1 + (q - p)^n}{2}$. [3]
- (b) For $n = 9$, find the value of p for which $P(X \text{ is even}) = 0.55$, to 3 significant figures. [2]
- (c) Show that $E\left(\frac{1}{X + 1}\right) = \frac{1 - q^{n+1}}{(n + 1)p}$. [4]
- (d) Hence, or otherwise, show that $E\left(\frac{1}{(X + 1)(X + 2)}\right) = \frac{1 - q^{n+2} - (n + 2)p q^{n+1}}{(n + 1)(n + 2)p^2}$. [3]

8 CIE | Edexcel IAL

For $r = 1, 2, 3, \dots$ and $\lambda > 0$ let $F_r(\lambda) = P(X \geq r)$, where X has the Poisson distribution $Po(\lambda)$.

- (a) Show that $\frac{dF_r}{d\lambda} = \frac{e^{-\lambda} \lambda^{r-1}}{(r - 1)!}$. [4]
- (b) Hence show that $\int_0^\lambda t^2 e^{-t} dt = 2 - e^{-\lambda}(\lambda^2 + 2\lambda + 2)$. [3]
- (c) The mean rate Λ of a random count N is itself a continuous random variable with probability density function $e^{-\lambda}$ for $\lambda > 0$. Given $\Lambda = \lambda$, N has the distribution $Po(\lambda)$, so that $P(N = n) = \int_0^\infty P(N = n | \Lambda = \lambda) e^{-\lambda} d\lambda$. Using $\int_0^\infty x^n e^{-kx} dx = \frac{n!}{k^{n+1}}$, show that $P(N = n) = \left(\frac{1}{2}\right)^{n+1}$. [3]
- (d) Given that $N \geq 1$, find the probability that N is even. [2]