

A LEVEL MATHS • EXTREME PAPERS

MATHS



EXAM PAPER

EXTREME PAPER: MECHANICS

SCORE

/90

PERCENTAGE

%

STEMUP

Paper: Mechanics (90 marks)

1 Edexcel | AQA | OCR A | OCR MEI | CIE

Two particles A and B move along the same straight line. At time t seconds ($t \geq 0$) their displacements from a fixed point O of the line are x_A metres and x_B metres.

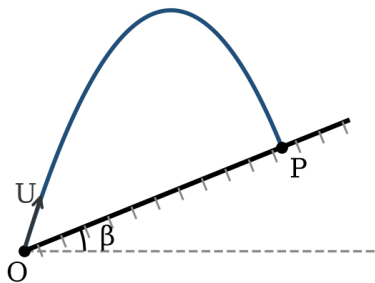
Particle A leaves O at $t = 0$ with velocity $(8t - t^2) \text{ m s}^{-1}$. Particle B moves with constant velocity 12 m s^{-1} and at $t = 0$ is at the point with displacement $-c$ metres, where c is a positive constant.

The particles are said to meet whenever $x_A = x_B$ for some $t > 0$.

- Show that A and B meet when $t(t - 6)^2 = 3c$, and hence find the set of values of c for which they meet exactly three times. [5]
- Given that $c = 9$, find the three times at which the particles meet. Give exact answers. [2]
- For $c = 9$, find the total distance travelled by A between the first and the third meetings. [3]

2 Edexcel | AQA | OCR A | OCR MEI | CIE

A plane is inclined at an angle β to the horizontal, where $0^\circ < \beta < 90^\circ$. A particle is projected from a point O on the plane with speed U at an angle α above the plane, in the vertical plane containing a line of greatest slope, and with its initial velocity directed up the slope. The particle moves freely under gravity (g is the acceleration due to gravity) and strikes the plane at the point P. The particle strikes the plane at right angles.



- Show that $\tan \alpha \tan \beta = \frac{1}{2}$. [5]
- Hence show that $OP = \frac{2U^2 \sin \beta}{g(1 + 3 \sin^2 \beta)}$. [3]
- The speed U is fixed and β may be varied (with α adjusted so that the particle still strikes the plane at right angles). Find the value of β for which OP is greatest, and find this greatest value of OP in terms of U and g . [4]

3 Edexcel | AQA | OCR A | OCR MEI | CIE

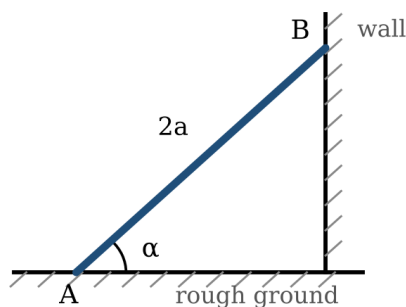
A particle P of mass m lies on a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. The coefficient of friction between P and the plane is $\frac{1}{4}$. A fixed smooth barrier is attached to the plane at its lowest point O, perpendicular to the plane. The particle is released from rest at a point on the line of greatest slope through O, at a distance d up the plane from O.

P moves down the plane, and every collision between P and the barrier is perfectly elastic (P rebounds with the speed it had just before the collision). P is modelled as a particle and g is the acceleration due to gravity.

- Show that P first reaches the barrier after a time $\sqrt{5d/g}$, and that after its first rebound P travels a distance $\frac{1}{2}d$ up the plane before coming instantaneously to rest. [4]
- Show that P does not remain at rest at the top of any rise, and find the total distance travelled by P until it finally comes to rest. [3]
- Find, in terms of d and g , the total time from release until P finally comes to rest. Give your answer in an exact simplified form. [4]

4 Edexcel | AQA | OCR A | OCR MEI | CIE

A uniform ladder AB, of weight W and length $2a$, has its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the ground. The coefficient of friction between the ladder and the ground is μ_1 and between the ladder and the wall is μ_2 , where $\mu_1\mu_2 < 1$.



- (a) The ladder is on the point of slipping, with A moving away from the wall. Show that $\tan \alpha = \frac{1 - \mu_1\mu_2}{2\mu_1}$. [5]
- (b) Now $\mu_1 = \frac{1}{2}$, $\mu_2 = \frac{1}{4}$ and $\tan \alpha = 1$. A person of weight W stands on the ladder at the point P with $AP = x$. Find the greatest value of x for which the ladder does not slip. Give x as a multiple of a . [4]
- (c) Now suppose that $\mu_1\mu_2 \geq 1$ and that the ladder carries no person. Show that the ladder can rest in equilibrium whatever the angle α , where $0^\circ < \alpha < 90^\circ$. [4]

5 CIE

A particle of mass m is projected vertically upwards from a point O with speed u . In addition to its weight, the particle experiences air resistance of magnitude mkv^2 , where v is its speed and k is a positive constant. The acceleration due to gravity is g .

You may use the fact that, for motion in a straight line, acceleration $= v \frac{dv}{dx}$.

- (a) Show that the greatest height H reached above O is $H = \frac{1}{2k} \ln(1 + \frac{ku^2}{g})$. [4]
- (b) The particle then falls from rest at the highest point. Let w be its speed when it returns to O. Show that $\frac{1}{w^2} = \frac{1}{u^2} + \frac{k}{g}$. [3]
- (c) Given that $u^2 = \frac{3g}{k}$, find the time taken for the downward journey from the highest point back to O, in terms of g and k . [3]
- (d) Still with $u^2 = \frac{3g}{k}$, find the time of ascent, given that $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan(\frac{x}{a}) + \text{constant}$. Hence find, in terms of g and k and to 3 significant figures, by how much the descent takes longer than the ascent. [2]

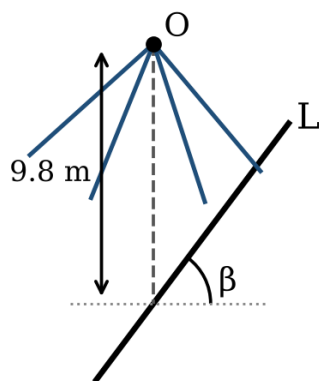
6 Edexcel (IAL) | CIE

A smooth horizontal straight line is perpendicular to a fixed smooth vertical wall. Particle A of mass $3m$ and particle B of mass m lie on the line on the same side of the wall, with B at rest 9 m from the wall and A 27 m from the wall (so B lies between A and the wall). A is set moving towards B with speed 6 m s^{-1} . All collisions, between A and B and between B and the wall, are perfectly elastic: after a collision the speed of separation equals the speed of approach. Momentum is conserved in every collision between A and B. The particles are small enough to be modelled as points.

- Show that exactly five collisions occur in total (three between A and B and two between B and the wall), and find the final velocities of A and B. [7]
- Show that the last collision between A and B takes place at the same point as the first, and find the time between the first and the last collisions. [2]
- Find the total magnitude of the impulse exerted by the wall on B, and confirm your answer by considering the momentum of the system. [2]

7 Edexcel | AQA | OCR A | OCR MEI | CIE

Take $g = 9.8\text{ m s}^{-2}$. Smooth straight rigid wires are fixed in a vertical plane, all passing through a point O, each making an angle α with the horizontal, where $0^\circ < \alpha < 90^\circ$ and α may differ from wire to wire. A small bead is released from rest at O and slides down a wire. Several beads are released from O at the same instant, one on each wire.

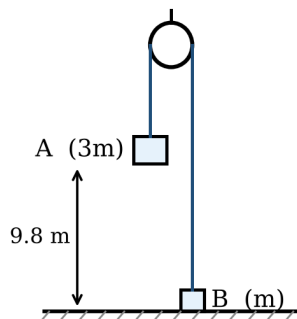


- Show that the distance of a bead from O after time t seconds is $\frac{1}{2} g t^2 \sin \alpha$. [2]
- Show that, at a given time t , all the beads lie on a circle, and find its radius in terms of g and t . [3]
- A straight line L lies in the same vertical plane. It passes through the point 9.8 m vertically below O and is inclined at an angle β to the horizontal, where $\cos \beta = \frac{3}{5}$. A bead slides from O down a wire that ends on L. Find the least possible time for the bead to reach L, and the angle that the corresponding wire makes with the downward vertical. [5]

8 Edexcel (IAL) | CIE

Take $g = 9.8 \text{ m s}^{-2}$. Particles A and B, of masses $3m$ and m , are attached to the ends of a light inextensible string that passes over a smooth light fixed pulley. B rests on horizontal ground and A is held at rest 9.8 m above the ground with the string taut and both straight parts vertical. The system is released from rest. The pulley is so high that B never reaches it.

When A hits the ground it comes to rest and does not rebound. Whenever the string becomes taut after being slack there is a sudden jerk: the tension is impulsive, the string stays taut afterwards, and B and A move with a common speed (A upwards, B downwards, or the reverse).



- (a) Show that A reaches the ground after 2 s , with speed 9.8 m s^{-1} . [2]
- (b) After A lands the string goes slack. Show that it becomes taut again 2 s later, and find the common speed of the particles immediately afterwards. (The impulse of the tension on each particle equals that particle's change of momentum.) [4]
- (c) Show that A hits the ground again after a further 1 s , and that the whole process then repeats with all speeds and times scaled by a constant factor. Hence find the total time from release until the motion of A finally ceases. [4]
- (d) The model predicts infinitely many jerks within a finite time. State one modelling assumption that makes this prediction unrealistic. [1]