

A LEVEL MATHS • EXTREME PAPERS

MATHS



EXAM PAPER

EXTREME PAPER: DECISION MATHEMATICS

SCORE

/80

PERCENTAGE

%

STEMUP

Paper: Decision Mathematics (80 marks)

1 Edexcel | OCR A | OCR MEI

A courier has nine parcels with masses, in kg, in the order they arrive:

7, 8, 6, 7, 9, 5, 4, 10, 4.

Each delivery box can hold a total mass of at most 20 kg, and a parcel cannot be split.

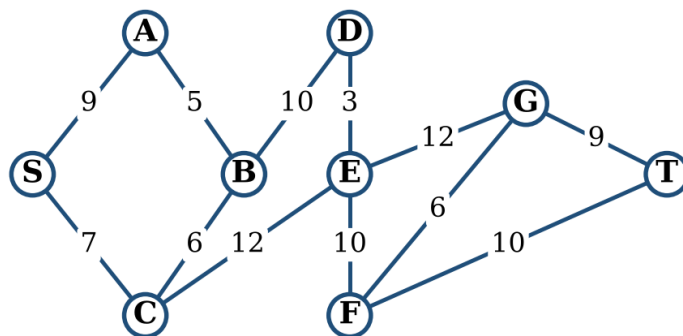
- Use the first-fit algorithm, with the parcels in the order given, and then the first-fit decreasing algorithm, to pack the parcels. State the number of boxes each uses. [3]
- Show that the parcels can be packed into 3 boxes, and prove that, apart from the order of the parcels within a box, this packing is the only way to use 3 boxes. (Both algorithms in (a) failed to find it.) [3]
- Capacity is now any whole number of kilograms C , and the parcels may arrive in any order. [2]
Prove that when first fit is used, at most one box ends with a load of at most $C/2$.

2 Edexcel | OCR A | OCR MEI

The network shows the lengths, in km, of roads joining a depot S to a customer T through seven junctions.

All roads can be used in both directions.

A courier wants the shortest route from S to T.

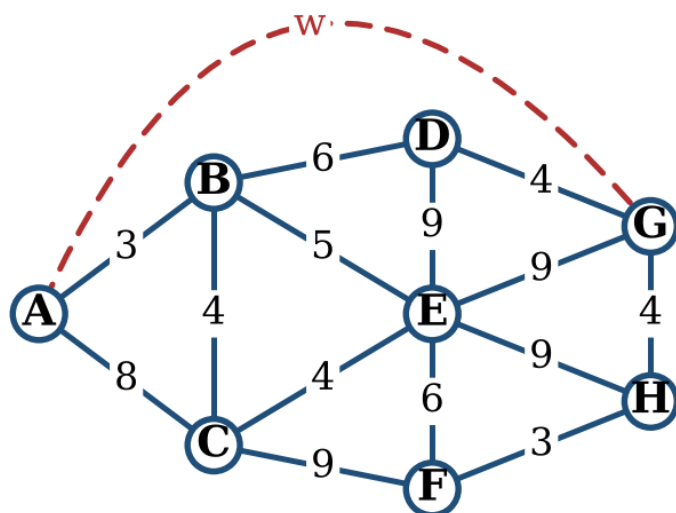


- Use Dijkstra's algorithm to find the shortest route from S to T and its length. Show the order in which vertices are given permanent labels, and all working values. [4]
- Return to the original network. The couriers' vehicles are electric and recover energy going downhill. A new one-way road from A to C has 'length' -3 (a gain). A student applies Dijkstra's algorithm exactly as in (a), using this one-way arc as well, and obtains the same answer as in (a).
Show that the student's answer is not the true shortest distance from S to T. Explain precisely why Dijkstra's algorithm breaks down here. [3]
- A second student tries to avoid the negative arc. She adds 3 to the length of every arc, so each road becomes two arcs (one in each direction) of length 3 more, and the one-way arc A to C becomes length 0. She runs Dijkstra's algorithm on the new network, and then subtracts 3 for each arc of the route found.
Show that her method also gives the wrong answer for the shortest distance from S to T, and explain why adding a constant to every arc length cannot work in general. [3]

3 Edexcel | OCR A | OCR MEI

A utility company plans a network of cables joining eight sites A to H. The solid lines show the possible connections and the cost of each, in thousands of pounds.

The company also considers a further connection AG (dashed) of cost w thousand pounds, where $w > 0$.

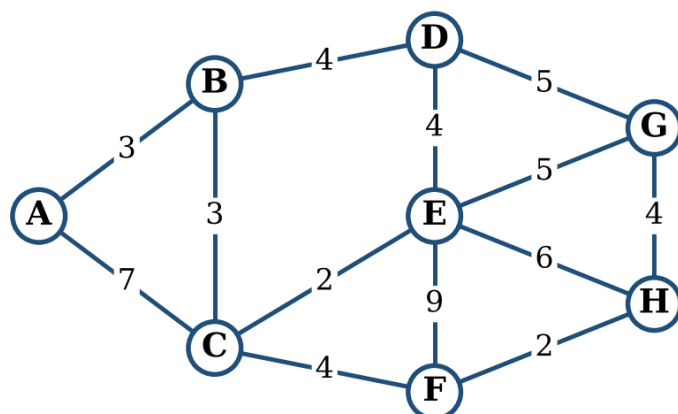


- Without the connection AG, use Kruskal's algorithm to find the weight of a minimum spanning tree. Explain why there is more than one minimum spanning tree and state the edges in which two such trees differ. [2]
- Find the set of values of w for which AG belongs to (i) at least one minimum spanning tree, (ii) every minimum spanning tree. [3]
- When $w = 6$, how many different minimum spanning trees are there? Justify your answer. [2]
- In a connected network, an edge e of weight w joins vertices u and v . Prove that e belongs to at least one minimum spanning tree if and only if there is no path from u to v made up entirely of edges of weight strictly less than w . [3]

4 Edexcel | OCR MEI

The network shows the roads, with lengths in km, of a small town. A gritting lorry must travel along every road at least once. Roads can be used in both directions.

The total length of all the roads is 58 km.

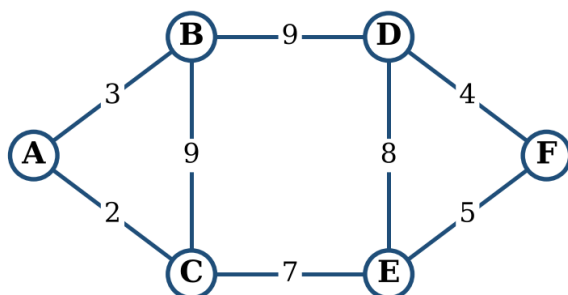


- The lorry starts and finishes at A. Find the length of a shortest route, justifying clearly which roads are repeated. [3]
- The lorry must now start at A and finish at H. Find the length of a shortest route, and explain why it is longer than the route in (a) even though it does not return to A. [3]
- Roadworks change the length of road EG to x km (instead of 5), where $0 < x \leq 12$. The lorry starts and finishes at A. Find the length of the shortest route as a function of x , showing the value of x at which the roads repeated change. [3]
- The lorry may now start at any junction and finish at any different junction. Find the length of the shortest possible route that uses every road, and state every pair of start and finish junctions that achieves it. [3]

5 Edexcel | OCR MEI

A surveyor must visit six villages A to F. The network shows the direct roads between them, with lengths in km. The surveyor starts at A, visits every village at least once and returns to A.

In this question, a 'tour' of the villages means a closed route visiting each village once, with the distance between two villages taken as the shortest distance in the network.



- Construct the table of least distances between the six villages. [2]
- Use the nearest neighbour algorithm, starting at A, on your table to find an upper bound for the length of the shortest tour. State the tour. [2]
- Delete vertex E and use the remaining villages to find a lower bound for the length of the shortest tour. [3]
- Hence determine the length of the shortest tour, exhibiting a tour of that length. Prove that, apart from its direction, this is the only tour of that length. [3]
- Calculate the lower bound obtained by deleting A instead of E, and explain why it is weaker. [2]

6 Edexcel | AQA | OCR MEI

A project consists of ten activities, A to J. Each activity needs exactly one worker for its whole duration and cannot be interrupted. The duration (in hours) of each activity and the activities that must be completed immediately before it can start are shown in the table.

Any worker can do any activity, and a worker can start a new activity as soon as they finish the previous one. Times are whole numbers of hours from the start.

Activity	Duration	Preceded by
A	5	-
B	5	-
C	3	B
D	4	A
E	4	B
F	4	A
G	7	C
H	4	B
I	2	A
J	5	A

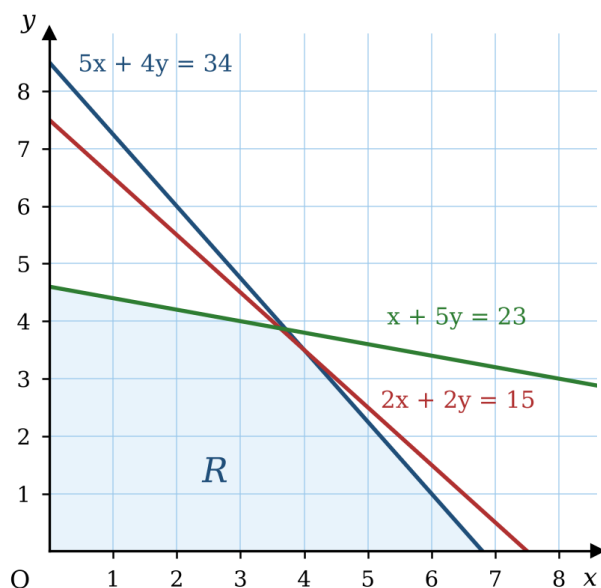
- (a) Carry out a forward pass and a backward pass. Hence find the minimum completion time of the project and list the critical activities. [4]
- (b) A delivery problem means that activity A cannot start before time s hours. Find the greatest value of s for which the minimum completion time stays 15 hours. When $s = 7$, find the new minimum completion time and the critical activities. [2]
- (c) Show that three workers cannot complete the project in the minimum time of 15 hours. [3]
- (d) Show that four workers can complete the project in 15 hours by giving a start time for each activity. [2]
- (e) Find, with proof, the least possible completion time using three workers. [3]

7 Edexcel | OCR MEI

A workshop makes two products, X and Y, in whole numbers. It makes x of X and y of Y in a week, and a profit of £ $(6x + 5y)$ hundred is made.

The constraints are $5x + 4y \leq 34$, $2x + 2y \leq 15$, $x + 5y \leq 23$, $x \geq 0$, $y \geq 0$.

The diagram shows the three boundary lines and the region R satisfying all the constraints when x and y can take any real values.



- If x and y could be any non-negative real numbers, find the maximum profit and the values of x and y that give it. Show your method. [3]
- Now x and y must be whole numbers. Prove that the maximum profit is 41, and state where it occurs. [4]
- The profit function is changed to $x + ky$, where $k > 0$ is a constant (x and y still whole numbers). Find the set of values of k for which the point $(6, 1)$ gives the greatest value. [4]
- The first constraint is changed to $5x + 4y \leq 35$, the others and the profit $6x + 5y$ unchanged. Find the new greatest whole-number profit, and the point where it occurs. Explain why the optimal whole-number point moves a long way along the boundary although the real-number optimum only moves a short distance. [3]