

A LEVEL MATHS • EXTREME PAPERS

MATHS



MODEL ANSWERS

EXTREME PAPER: DECISION MATHEMATICS

SCORE

/80

PERCENTAGE

%

STEMUP

Paper: Decision Mathematics (80 marks)

1 (8 marks) Edexcel | OCR A | OCR MEI

(a) First fit, in order: 7, 8 go in box 1 (15); 6 does not fit (21) so box 2; 7 goes in box 2 (13); 9 needs box 3

5 goes in box 1 (20); 4 goes in box 2 (17); 10 goes in box 3 (19); the last 4 fits nowhere (box 2 would be 21, box 3 would be 23). Boxes: [7 8 5], [6 7 4], [9 10], [4]: 4 boxes

First-fit decreasing: 10, 9, 8, 7, 7, 6, 5, 4, 4 packs as [10 9], [8 7 5], [7 6 4], [4]: 4 boxes

Answer: 4 boxes by each method (FF: [7 8 5], [6 7 4], [9 10], [4]) 3 marks

(b) Key idea: $60 = 3 \times 20$, so with 3 boxes every box must be exactly full.

The total mass is $7 + 8 + 6 + 7 + 9 + 5 + 4 + 10 + 4 = 60$ kg, so 3 boxes hold all the parcels only if each holds exactly 20 kg

The 10 kg parcel needs 10 kg more from 4, 4, 5, 6, 7, 7, 8, 9: three parcels would total at least $4 + 4 + 5 = 13 > 10$, so exactly two, and the only pair summing to 10 is 4 and 6 (there is one 5). The 9 kg parcel needs 11 kg from 4, 5, 7, 7, 8: three would total at least $4 + 5 + 7 = 16 > 11$, so a pair, and the only pair is 4 and 7

The remaining parcels 5, 7, 8 total 20, so the packing [10 6 4], [9 7 4], [8 7 5] exists and is forced

Answer: Unique: [10 6 4], [9 7 4], [8 7 5] 3 marks

(c) Suppose boxes $i < j$ both end with loads at most $C/2$. Let p be the first parcel placed in box j ; then p is at most the final load of box j , so p is at most $C/2$

When p was placed, box i held at most its final load, at most $C/2$, so box i could take p (the total is at most $C/2 + C/2 = C$). First fit would then have put p in box i or an earlier box, not box j : contradiction

Answer: Proof by contradiction 2 marks

2 (10 marks) Edexcel | OCR A | OCR MEI

(a) Permanent labels in order: S 0; C 7; A 9; B 13 (via C: $7 + 6$; via A would be 14); E 19 (via C: $7 + 12$)

D 22 (via E: $19 + 3$; via B would be 23); F 29 (via E: $19 + 10$); G 31 (via E: $19 + 12$; via F would be 35)

T: from F $29 + 10 = 39$, from G $31 + 9 = 40$, so $T = 39$. Trace back T from F, F from E, E from C, C from S: route S C E F T

Answer: S C E F T, length 39 km 4 marks

(b) True route S A C E F T has length $9 - 3 + 12 + 10 + 10 = 38$, which is less than 39

Dijkstra permanently labels C (7) before A (9). When A is labelled, the new arc offers C a label of

6, but C is already permanent and is not updated, so every later label stays 1 too large and T is 39

Answer: True distance 38 (S A C E F T); Dijkstra gives 39 3 marks

(c) Key idea: a route with k arcs changes length by exactly $3k$, so routes with more arcs are penalised more.

New length of S A C E F T (5 arcs) = $38 + 15 = 53$. Every S to T route has at least 4 arcs; routes with exactly 4 arcs have original length at least 39 (S C E F T, 39, is the shortest), so new length at least 51; routes with at least 5 arcs have original length at least 38, so new length at least 53

So Dijkstra on the new network (all lengths non-negative, so valid) returns S C E F T with new length 51; subtracting 3×4 gives 39, but the true distance is 38. The shift favours routes with fewer arcs, so it changes which route is shortest

Answer: Her method returns 39 (route S C E F T), not 38 3 marks

3 (10 marks) Edexcel | OCR A | OCR MEI

(a) Sorted: AB 3, FH 3, BC 4, CE 4, DG 4, GH 4 are all accepted (no cycles); BE 5 rejected (cycle B C E)

Next BD 6 joins A, B, C, E to D, G, H, F: total $3 + 3 + 4 + 4 + 4 + 4 + 6 = 28$. EF 6 would do equally well, so the trees differ in BD or EF

Answer: 28; two trees, one using BD and one using EF 2 marks

(b) Key idea: AG is in a minimum spanning tree exactly when w is not more than the largest edge on the best alternative route from A to G (the minimax route)

The six edges of weight at most 4 form two components, A, B, C, E and D, G, H, F, and any other

route from A to G must use an edge of weight at least 6 (BD or EF): the best alternative has largest edge 6

(i) $w \leq 6$. (ii) $w < 6$, since at $w = 6$ there is a minimum spanning tree that avoids AG (it uses BD or EF)

A1

Answer: (i) $w \leq 6$ (ii) $w < 6$

3 marks

(c) The six edges AB, FH, BC, CE, DG, GH are in every minimum spanning tree and leave two components

M1

Exactly one more edge is needed, of weight 6, joining the components: the choices are BD, EF and AG, so 3 trees (weight 28)

A1

Answer: 3

2 marks

(d) Key idea: Kruskal's algorithm only meets e after every lighter edge, and decides on e by whether u and v are already connected.

(Only if) Suppose such a path P exists and T is any spanning tree containing e. Removing e splits T into two parts, with u and v in different parts, so some edge f of P joins the two parts. Then T with e replaced by f is a spanning tree of smaller weight, so T is not minimal. So e is in no minimum spanning tree

B1 M1

(If) Suppose no such path exists. Run Kruskal's algorithm, taking e before the other edges of weight w. Every edge accepted before e has weight less than w, and these cannot connect u to v, so e is accepted. Kruskal's algorithm gives a minimum spanning tree, and it contains e

A1

Answer: Proof (exchange argument and Kruskal)

3 marks

4 (12 marks) Edexcel | OCR MEI

(a) Degrees: A 2, B 3, C 4, D 3, E 5, F 3, G 3, H 3. The odd vertices are B, D, E, F, G, H

B1

Pair them using shortest paths: 15 pairings. Least distances: BD 4, EG 5, FH 2, so $BD + EG + FH = 11$. Next best is $BE + DG + FH = 5 + 5 + 2 = 12$; every other pairing is at least 14

M1

Repeat B D, E G and F H. Length = $58 + 11 = 69$ km

A1

Answer: 69 km (repeat BD, EG, FH)

3 marks

(b) A route from A to H needs A and H to be the only odd vertices once repeated roads are added. H is already odd, but A is even and must become odd, while B, D, E, F, G must become even.

M1

Repeating a path between two vertices changes the parity of both, so pair up A, B, D, E, F, G

Best pairing: AB 3, DE 4, FG 6 (F H G) = 13. (Next: $AB + DG + EF = 3 + 5 + 6 = 14$.) Length = $58 + 13 = 71$ km

A1

$71 > 69$: A is even, so starting at A forces extra repeats to make A odd; an open route is not automatically shorter

E1

Answer: 71 km (repeat AB, DE, FH, HG)

3 marks

(c) Total length of roads is $53 + x$. Pairing BD, EG, FH costs $4 + x + 2 = 6 + x$; pairing BE, DG, FH costs $5 + 5 + 2 = 12$, independent of x. Every other pairing costs more than the cheaper of these for all $0 < x \leq 12$

B1

Pairing BD, EG, FH is best when $6 + x \leq 12$, that is $x \leq 6$; at $x = 6$ the two pairings tie

M1

Length = $53 + x + 6 + x = 59 + 2x$ for $0 < x \leq 6$; and $53 + x + 12 = 65 + x$ for $6 \leq x \leq 12$ (both give 71 at $x = 6$)

A1

Answer: $59 + 2x$ for $0 < x \leq 6$; $65 + x$ for $6 \leq x \leq 12$

3 marks

(d) Key idea: the two ends of the route stay odd, so they are 'free' and the other odd junctions are paired.

If both ends are odd junctions, the other four odd junctions must be paired by repeated roads. If an end is A or C (even) then at least five odd junctions and that junction need a repeated road at them; each repeated road serves at most two of these, and the cheapest road at each of B, D, E, F, G, H, A, C is 3, 4, 2, 2, 4, 2, 3, 2, so the repeats cost at least half of $(13 + 2) = 7.5$

M1

For two odd ends, the repeats cost the least total of two disjoint pairings of the remaining four.

M1

Least distances between odd junctions: FH 2; BD, DE, GH 4; BE, DG, EG 5; others at least 6.

Two disjoint pairs cost at least $2 + 4 = 6$ (FH with BD or DE; FH with GH shares H), and without FH at least $4 + 4 = 8$

So 6 is the least extra: FH + BD leaves E and G as the ends, and FH + DE leaves B and G.

A1

Shortest route $58 + 6 = 64$ km, starting at E and finishing at G, or starting at B and finishing at G (or the reverse of either)

Answer: 64 km: between E and G, or between B and G (either direction)

3 marks

5 (12 marks) Edexcel | OCR MEI

(a) Use shortest paths in the network: e.g. A to E = A C E = 9, A to F = A C E F = 14, B to E = B A C E = 12, C to D = C A B D = 14 (not C E D = 15), C to F = C E F = 12

M1

Complete table: see the table printed after this question's answers (symmetric)

A1

Answer: Table (see model answer)

2 marks

(b) From A: C (2); from C: B (5); from B: D (9); from D: F (4); from F: E (5); return E to A (9)

M1

Upper bound $2 + 5 + 9 + 4 + 5 + 9 = 34$ km, tour A C B D F E A

A1

Answer: 34 (A C B D F E A)

2 marks

(c) Minimum spanning tree of A, B, C, D, F using the table: AC 2, AB 3, DF 4, then BD 9 joins the pieces: weight 18

M1 A1

Add the two shortest distances from E: EF 5 and EC 7. Lower bound $18 + 5 + 7 = 30$

A1

Answer: 30

3 marks

(d) Tour A B D F E C A has length $3 + 9 + 4 + 5 + 7 + 2 = 30$, equal to the lower bound, so the shortest tour has length exactly 30 km

M1

Key idea: equality in the bound. Delete E from any tour of length 30: the rest is a path through A, B, C, D, F of weight at least 18, and the two edges at E cost at least $5 + 7 = 12$, so both must be equalities

M1

So E joins to F and C (the unique two shortest, since $ED = 8$ is next), and the path is a minimum spanning tree of weight 18. That tree is unique (AC 2, AB 3, DF 4, then BD 9 is the only cheapest link between A, B, C and D, F) and is the path C A B D F. The tour is C A B D F E C, the same as A B D F E C A

A1

Answer: 30 km: A B D F E C A, and its reverse only

3 marks

(e) Delete A: minimum spanning tree of B, C, D, E, F is BC 5, DF 4, EF 5, CE 7 = 21; the two shortest from A are 2 and 3. Bound = $21 + 5 = 26$

B1

A has two very cheap links (2 and 3), so deleting A adds back only 5 to a heavy tree (21);

E1

deleting E adds back 12 to a lighter tree (18). Deleting a vertex with expensive links removes more of the cost from the tree, which makes the bound tighter

Answer: 26; weaker because A's two shortest distances are small

2 marks

	A	B	C	D	E	F
A	-	3	2	12	9	14
B	3	-	5	9	12	13
C	2	5	-	14	7	12
D	12	9	14	-	8	4
E	9	12	7	8	-	5
F	14	13	12	4	5	-

6 (14 marks) Edexcel | AQA | OCR MEI

(a) Earliest start: A 0, B 0, C 5, D 5, E 5, F 5, G 8, H 5, I 5, J 5; earliest finish: A 5, B 5, C 8, D 9, E 9, F 9, G 15, H 9, I 7, J 10

M1 A1

Minimum completion time 15 hours (G finishes at 15). Backward pass: latest finish is 15 for D, E, F, G, H, I, J; C 8; B 5; A 10 (J must start by 10)

M1

Critical activities (earliest start = latest start): B, C, G

A1

Answer: 15 hours; critical activities B, C, G

4 marks

(b) A must precede D, F, I and J; the longest chain through A is A then J = $5 + 5 = 10$, so A can start as late as $15 - 10 = 5$ hours: $s = 5$

B1

If $s = 7$: A finishes at 12, then J takes 5 more hours: 17 hours, with critical path A, J

B1

Answer: $s = 5$; 17 hours with A, J critical

2 marks

(c) Key idea: the total work is fixed, but in the first 5 hours only two activities can be going on. Total work = $5 + 5 + 3 + 4 + 4 + 4 + 7 + 4 + 2 + 5 = 43$ worker-hours; three workers over 15 hours provide 45

M1

Every activity except A and B needs A or B to be finished, and A and B each take 5 hours, so nothing else can start before time 5. In the first 5 hours at most 2 workers are busy, wasting at

M1

least 5 worker-hours

Then at most $45 - 5 = 40 < 43$ worker-hours are available, a contradiction

A1

Answer: Impossible: at most 40 worker-hours usable, 43 needed

3 marks

(d) A 0, B 0, C 5, D 5, E 5, F 5, G 8, H 9, I 9, J 9

M1

Check workers busy: 0 to 5: 2; 5 to 8: C D E F (4); 8 to 9: G D E F (4); 9 to 11: G H I J (4); 11 to 13: G H J; 13 to 14: G J; 14 to 15: G. Precedences hold (e.g. H after B)

A1

Answer: e.g. A0 B0 C5 D5 E5 F5 G8 H9 I9 J9

2 marks

(e) With completion time T , three workers give $3T$ worker-hours, of which at least 5 are wasted in the first 5 hours (as in (c)), so $3T \geq 43 + 5 = 48$

M1

Hence $T \geq 16$

A1

Schedule: A 0, B 0, C 5, D 5, E 5, F 8, G 9, H 12, I 9, J 11 finishes at 16 with at most 3 activities in progress at any time

B1

Answer: 16 hours

3 marks

7 (14 marks) Edexcel | OCR MEI

(a) Vertices of R: $(0, 0)$, $(0, \frac{23}{5})$, $(\frac{29}{8}, \frac{31}{8})$, $(4, \frac{7}{2})$, $(\frac{34}{5}, 0)$

M1

Profit $6x + 5y$ at these: 0, 23, $\frac{329}{8} = 41.125$, 41.5, 40.8 respectively

A1

Greatest is at $(4, 3.5)$ (the intersection of $5x + 4y = 34$ and $2x + 2y = 15$): profit 41.5

A1

Answer: $x = 4$, $y = 3.5$, profit 41.5

3 marks

(b) Key idea: write $6x + 5y = 5(x + y) + x$ and bound each piece using whole-number rounding.

$2x + 2y \leq 15$ gives $x + y \leq 7.5$, so $x + y \leq 7$ for whole numbers

B1

$5x + 4y \leq 34$ with $y \geq 0$ gives $x \leq 6.8$, so $x \leq 6$

B1

Hence $6x + 5y \leq 5 \times 7 + 6 = 41$

M1

$(6, 1)$ satisfies all constraints $(34, 28, 11)$ against $(34, 30, 23)$ and gives 41. Maximum 41 at $x = 6$, $y = 1$

A1

Answer: 41 at $(6, 1)$

4 marks

(c) From (b), every feasible whole-number point has $x + y \leq 7$ and $x \leq 6$

B1

Write $x + ky = k(x + y) + (1 - k)x$. For $0 < k \leq 1$ both coefficients are non-negative, so $x + ky \leq 7k + 6(1 - k) = 6 + k$

M1

$(6, 1)$ gives $6 + k$, so it is optimal for $0 < k \leq 1$

A1

For $k > 1$ the feasible point $(3, 4)$ ($5x + 4y = 31 \leq 34$, $2x + 2y = 14 \leq 15$, $x + 5y = 23 \leq 23$)

A1

gives $3 + 4k > 6 + k$, so $(6, 1)$ is not optimal. Answer $0 < k \leq 1$

Answer: $0 < k \leq 1$

4 marks

(d) Now $x \leq 7$ and still $x + y \leq 7$, so $6x + 5y = 5(x + y) + x \leq 35 + 7 = 42$

M1

$(7, 0)$ gives 42 and satisfies $35 \leq 35$, $28 \leq 30$, $7 \leq 23$. So greatest profit 42 at $(7, 0)$

A1

The real optimum moves from $(4, 3.5)$ to $(5, 2.5)$ (profit 42.5) but the whole-number optimum jumps from $(6, 1)$ to $(7, 0)$: rounding $x + y$ down to 7 makes all points on $x + y = 7$ competitive, and the largest x on that line wins

E1

Answer: 42 at $(7, 0)$

3 marks