

## Solving Quadratic Equations

### 1 (2 marks)

$$(x - 3)(x - 4) = 0$$

M1

$$x = 3 \text{ or } x = 4$$

A1

**Answer:  $x = 3, x = 4$**

2 marks

### 2 (1 mark)

$$x^2 = 9, \text{ so } x = \pm 3$$

A1

**Answer:  $x = \pm 3$**

1 mark

### 3 (2 marks)

$$3x(x - 4) = 0$$

M1

$$x = 0 \text{ or } x = 4$$

A1

**Answer:  $x = 0, x = 4$**

2 marks

### 4 (2 marks)

$$2x - 5 = 0 \text{ or } x + 3 = 0$$

M1

$$x = \frac{5}{2} \text{ or } x = -3$$

A1

**Answer:  $x = \frac{5}{2}, x = -3$**

2 marks

### 5 (2 marks)

$$(x + 5)(x - 3) = 0$$

M1

$$x = -5 \text{ or } x = 3$$

A1

**Answer:  $x = -5, x = 3$**

2 marks

### 6 (2 marks)

$$(x - 5)^2 = 0$$

M1

$$x = 5 \text{ (repeated root)}$$

A1

**Answer:  $x = 5$**

2 marks

### 7 (3 marks)

$$(2x - 1)(x + 3) = 0$$

M1 A1

$$x = \frac{1}{2} \text{ or } x = -3$$

A1

**Answer:  $x = \frac{1}{2}, x = -3$**

3 marks

### 8 (3 marks)

$$(3x + 1)(2x - 3) = 0$$

M1 A1

$$x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$$

A1

**Answer:  $x = -\frac{1}{3}, x = \frac{3}{2}$**

3 marks

### 9 (3 marks)

$$(x - 3)^2 - 9 + 2 = 0$$

M1 A1

$$(x - 3)^2 = 7, \text{ so } x = 3 \pm \sqrt{7}$$

A1

**Answer:  $x = 3 \pm \sqrt{7}$**

3 marks

### 10 (3 marks)

$$x = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4}$$

M1 A1

$$x = 2.58 \text{ or } x = -0.581$$

A1

**Answer:  $x = 2.58, x = -0.581$**

3 marks

**11 (4 marks)**

$$(2x - 3)^2 - (x + 4)^2 = 4x^2 - 12x + 9 - (x^2 + 8x + 16)$$

M1

$$= 3x^2 - 20x - 7, \text{ so } 3x^2 - 20x - 32 = 0$$

A1

$$(3x + 4)(x - 8) = 0$$

M1

$$x = 8 \text{ or } x = -\frac{4}{3}$$

A1

$$\text{Answer: } x = 8, x = -\frac{4}{3}$$

4 marks

**12 (4 marks)**

$$\text{Let the integers be } n \text{ and } n + 1: n(n + 1) = 156$$

M1

$$n^2 + n - 156 = 0$$

A1

$$(n - 12)(n + 13) = 0, \text{ so } n = 12 \text{ (n must be positive)}$$

M1

The integers are 12 and 13

A1

**Answer: 12 and 13**

4 marks

**13 (4 marks)**

$$f(x + 2) = (x + 2)^2 - 6(x + 2) + 4$$

M1

$$= x^2 - 2x - 4$$

A1

$$x^2 - 6x + 4 = x^2 - 2x - 4$$

M1

$$-4x = -8, \text{ so } x = 2$$

A1

**Answer: x = 2**

4 marks

**14 (5 marks)**

$$\text{Let } y = x^2: y^2 - 10y + 9 = 0$$

M1 A1

$$(y - 1)(y - 9) = 0, \text{ so } y = 1 \text{ or } y = 9$$

M1

$$x^2 = 1 \text{ or } x^2 = 9$$

A1

$$x = \pm 1 \text{ or } x = \pm 3$$

A1

**Answer: x = ±1, x = ±3**

5 marks

**15 (5 marks)**

$$\text{Let } y = \sqrt{x}, \text{ so } x = y^2$$

M1

$$y^2 - 7y + 12 = 0$$

A1

$$(y - 3)(y - 4) = 0, \text{ so } y = 3 \text{ or } y = 4$$

M1 A1

$$x = 9 \text{ or } x = 16$$

A1

**Answer: x = 9, x = 16**

5 marks

**16 (6 marks)**

$$\text{Multiply by } (x - 1)(x + 2): 3(x + 2) + 2(x - 1) = (x - 1)(x + 2)$$

M1 A1

$$5x + 4 = x^2 + x - 2$$

A1

$$x^2 - 4x - 6 = 0$$

A1

$$x = \frac{4 \pm \sqrt{16 + 24}}{2}$$

M1

$$x = 2 \pm \sqrt{10}$$

A1

**Answer: x = 2 ± √10**

6 marks

**17 (6 marks)**

$$\text{(a) } (2x + 3)(x - 1) = 33$$

M1

$$2x^2 + x - 3 = 33, \text{ so } 2x^2 + x - 36 = 0$$

A1

**Answer: Shown**

2 marks

$$\text{(b) } (2x + 9)(x - 4) = 0$$

M1 A1

$$x = 4 \text{ (} x = -\frac{9}{2} \text{ is impossible for a length)}$$

A1

Sides 11 cm and 3 cm, perimeter 28 cm

A1

**Answer: x = 4; 28 cm**

4 marks

**18 (4 marks)**Substitute  $x = 3$ :  $9 + 3k - 15 = 0$ 

M1

 $k = 2$ 

A1

 $x^2 + 2x - 15 = (x - 3)(x + 5) = 0$ 

M1

The other root is  $x = -5$ 

A1

**Answer:  $k = 2$ ;  $x = -5$** 

4 marks

**19 (5 marks)**Pythagoras:  $x^2 + (x + 7)^2 = 169$ 

M1

 $2x^2 + 14x + 49 = 169$ 

A1

 $x^2 + 7x - 60 = 0$ 

A1

 $(x + 12)(x - 5) = 0$ 

M1

 $x = 5$  (reject  $x = -12$ )

A1

**Answer:  $x = 5$** 

5 marks

**20 (5 marks)** $2 + 12t - 5t^2 = 9$ 

M1

 $5t^2 - 12t + 7 = 0$ 

A1

 $(5t - 7)(t - 1) = 0$ 

M1 A1

 $t = 1$  s or  $t = 1.4$  s

A1

**Answer:  $t = 1$  or  $t = 1.4$** 

5 marks

**21 (5 marks)**Square both sides:  $x + 6 = x^2$ 

M1

 $x^2 - x - 6 = 0$ 

A1

 $(x - 3)(x + 2) = 0$ , so  $x = 3$  or  $x = -2$ 

M1 A1

Check:  $x = -2$  gives  $\sqrt{4} = 2 \neq -2$ , so reject.  $x = 3$ 

E1

**Answer:  $x = 3$** 

5 marks

**22 (5 marks)**(a) Sum of the roots:  $\alpha + 2\alpha = 9$ 

M1 A1

 $\alpha = 3$ 

A1

**Answer:  $\alpha = 3$** 

3 marks

(b) Product of the roots:  $\alpha \times 2\alpha = 2\alpha^2 = 18$ 

M1

 $k = 18$ 

A1

**Answer:  $k = 18$** 

2 marks

**23 (7 marks)**Let  $u = x^2 - 3x$ :  $u^2 - 2u - 8 = 0$ 

M1 A1

 $(u - 4)(u + 2) = 0$ , so  $u = 4$  or  $u = -2$ 

M1 A1

 $x^2 - 3x - 4 = 0$  gives  $x = 4$  or  $x = -1$ 

M1 A1

 $x^2 - 3x + 2 = 0$  gives  $x = 1$  or  $x = 2$ 

A1

**Answer:  $x = -1, 1, 2, 4$** 

7 marks

**24 (5 marks)**Let the integers be  $n - 1$ ,  $n$ ,  $n + 1$ :  $(n - 1)^2 + n^2 + (n + 1)^2 = 365$ 

M1

 $3n^2 + 2 = 365$ 

A1

 $n^2 = 121$ , so  $n = 11$  or  $n = -11$ 

M1 A1

The integers are 10, 11, 12 or  $-12$ ,  $-11$ ,  $-10$ 

A1

**Answer: 10, 11, 12 or  $-12$ ,  $-11$ ,  $-10$** 

5 marks

**25 (4 marks)**

$$x = \frac{2\sqrt{3} \pm \sqrt{12 + 36}}{2}$$

M1 A1

$$\sqrt{48} = 4\sqrt{3}, \text{ so } x = \frac{2\sqrt{3} \pm 4\sqrt{3}}{2}$$

M1

$$x = 3\sqrt{3} \text{ or } x = -\sqrt{3}$$

A1

$$\text{Answer: } x = 3\sqrt{3}, x = -\sqrt{3}$$

4 marks

**26 (7 marks)**

$$(a) x^2 + (x + 3.5)^2 = 6.5^2$$

M1

$$x^2 + x^2 + 7x + 12.25 = 42.25$$

A1

$$2x^2 + 7x - 30 = 0$$

A1

**Answer: Shown**

3 marks

$$(b) (2x - 5)(x + 6) = 0$$

M1 A1

$$x = 2.5 \text{ (} x = -6 \text{ is rejected)}$$

A1

$$\text{Height} = 2.5 + 3.5 = 6 \text{ m}$$

A1

**Answer: 6 m**

4 marks

**27 (8 marks)**

$$(a) (x - k)^2 - k^2 + k^2 - 1 = 0, \text{ i.e. } (x - k)^2 = 1$$

M1 A1

$$x - k = \pm 1, \text{ so } x = k + 1 \text{ or } x = k - 1$$

A1

**Answer: Shown**

3 marks

$$(b) (k + 1) - (k - 1) = 2$$

E1

**Answer: Difference is 2**

1 mark

$$(c) \text{Product } (k + 1)(k - 1) = k^2 - 1 = 8$$

M1

$$k^2 = 9, \text{ so } k = 3 \text{ or } k = -3$$

A1

$$k = 3: \text{ roots } 4 \text{ and } 2$$

A1

$$k = -3: \text{ roots } -2 \text{ and } -4$$

A1

**Answer: k = 3 (roots 2, 4) or k = -3 (roots -4, -2)**

4 marks

**28 (8 marks)**

$$(a) (kx - 2)(x - k) = kx^2 - k^2x - 2x + 2k$$

M1 A1

$$x = \frac{2}{k} \text{ or } x = k$$

A1

$$\text{Answer: } x = k, x = \frac{2}{k}$$

3 marks

$$(b) \text{Repeated root when } k = \frac{2}{k}$$

M1

$$k^2 = 2, \text{ so } k = \pm\sqrt{2}$$

A1

$$\text{Answer: } k = \pm\sqrt{2}$$

2 marks

$$(c) \alpha + \beta = k + \frac{2}{k}$$

B1

$$(k - \sqrt{2})^2 \geq 0 \text{ gives } k^2 + 2 \geq 2\sqrt{2}k$$

M1

$$\text{Dividing by } k > 0: k + \frac{2}{k} \geq 2\sqrt{2}$$

A1

**Answer: Shown**

3 marks

**29 (6 marks)**

$$\text{Let the roots be } p \text{ and } q: pq = 24 \text{ and } p + q = k$$

M1 A1

$$\text{Integer pairs with product 24: (1, 24), (2, 12), (3, 8), (4, 6) and the same pairs of negative numbers}$$

M1 A1

$$\text{The sums are 25, 14, 11, 10 and their negatives}$$

M1

$$k = \pm 10, \pm 11, \pm 14, \pm 25$$

A1

**Answer: k = ±10, ±11, ±14, ±25**

6 marks

**30 (8 marks)**

A power equals 1 when the base is 1, the index is 0, or the base is  $-1$  with an even index

M1

Base 1:  $x^2 - 5x + 4 = 0$ , so  $x = 1$  or  $x = 4$

M1 A1

Index 0:  $x^2 - 9x + 20 = 0$ , so  $x = 4$  or  $x = 5$  (base  $5 \neq 0$ )

M1 A1

Base  $-1$ :  $x^2 - 5x + 6 = 0$ , so  $x = 2$  or  $x = 3$

M1

Indices at  $x = 2$  and  $x = 3$ : 6 and 2, both even, so both valid

A1

$x = 1, 2, 3, 4, 5$

A1

**Answer:  $x = 1, 2, 3, 4, 5$**

8 marks

**The Discriminant****1 (2 marks)**

$$b^2 - 4ac = (-3)^2 - 4(2)(5) = -31$$

M1

Negative, so there are no real roots

A1

**Answer:  $-31$ ; no real roots**

2 marks

**2 (2 marks)**

$$b^2 - 4ac = 36 - 36 = 0$$

M1

The discriminant is zero, so there is one repeated root

A1

**Answer: Shown**

2 marks

**3 (3 marks)**

$$b^2 - 4ac = 100 - 4k$$

M1

No real roots:  $100 - 4k < 0$

M1

$$k > 25$$

A1

**Answer:  $k > 25$**

3 marks

**4 (2 marks)**

$$16 - 4k = 0$$

M1

$$k = 4$$

A1

**Answer:  $k = 4$**

2 marks

**5 (2 marks)**

$$b^2 - 4ac = 1 - 4 = -3$$

M1

$-3 < 0$ , so no real roots

A1

**Answer: Shown**

2 marks

**6 (2 marks)**

Crossing the x-axis needs  $x^2 - 2x + 5 = 0$ ;  $b^2 - 4ac = 4 - 20 = -16$

M1

No real roots, so the curve does not meet the x-axis

A1

**Answer: Shown**

2 marks

**7 (3 marks)**

$$b^2 - 4ac = 36 - 12k$$

M1

$$36 - 12k = 0$$

A1

$$k = 3$$

A1

**Answer:  $k = 3$**

3 marks

**8 (4 marks)**

Real roots:  $k^2 - 36 \geq 0$

M1 A1

$$(k - 6)(k + 6) \geq 0$$

M1

$$k \leq -6 \text{ or } k \geq 6$$

A1

**Answer:  $k \leq -6$  or  $k \geq 6$**

4 marks

**9 (3 marks)**

$$b^2 - 4ac = 16 - 8k$$

M1

$$\text{No real roots: } 16 - 8k < 0$$

A1

$$k > 2$$

A1

**Answer:  $k > 2$** 

3 marks

**10 (3 marks)**

$$b^2 - 4ac = k^2 + 12$$

M1 A1

$$k^2 \geq 0, \text{ so } k^2 + 12 > 0 \text{ for all } k, \text{ giving two distinct real roots}$$

E1

**Answer: Shown**

3 marks

**11 (4 marks)**

$$x^2 + 3 = 2x + k, \text{ so } x^2 - 2x + (3 - k) = 0$$

M1 A1

$$\text{Tangent: discriminant} = 0, 4 - 4(3 - k) = 0$$

M1

$$k = 2$$

A1

**Answer:  $k = 2$** 

4 marks

**12 (3 marks)**

$$\text{Curve A does not meet the x-axis: } b^2 - 4ac < 0$$

B1

$$\text{Curve B touches the x-axis: } b^2 - 4ac = 0$$

B1

$$\text{Curve C crosses the x-axis twice: } b^2 - 4ac > 0$$

B1

**Answer: A: negative, B: zero, C: positive**

3 marks

**13 (3 marks)**

The graph must lie above the x-axis, so there are no real roots

M1

$$16 - 4k < 0$$

A1

$$k > 4$$

A1

**Answer:  $k > 4$** 

3 marks

**14 (5 marks)**

$$b^2 - 4ac = 64 - 4k^2 = 0$$

M1 A1

$$k^2 = 16, \text{ so } k = 4 \text{ or } k = -4$$

A1

$$k = 4: 4(x + 1)^2 = 0, \text{ so } x = -1$$

M1

$$k = -4: -4(x - 1)^2 = 0, \text{ so } x = 1$$

A1

**Answer:  $k = 4$  ( $x = -1$ ) or  $k = -4$  ( $x = 1$ )**

5 marks

**15 (5 marks)**

$$b^2 - 4ac = k^2 - 8(k + 6)$$

M1

$$= k^2 - 8k - 48 = (k - 12)(k + 4)$$

A1

$$\text{No real roots: } (k - 12)(k + 4) < 0$$

M1

Sketch of a positive parabola, negative between the roots

M1

$$-4 < k < 12$$

A1

**Answer:  $-4 < k < 12$** 

5 marks

**16 (4 marks)**The equation is a quadratic only when  $k \neq 2$ 

B1

$$b^2 - 4ac = [2(k + 1)]^2 - 4(k - 2)(k + 4)$$

M1

$$= 4k^2 + 8k + 4 - 4k^2 - 8k + 32 = 36$$

A1

 $36 > 0$  for every  $k$ , so there are two distinct real roots for all  $k \neq 2$  (when  $k = 2$  the equation is linear)

E1

**Answer: Shown**

4 marks

**17 (5 marks)**

$$\text{(a) } x^2 - 3x + 5 = x + 1, \text{ so } x^2 - 4x + 4 = 0$$

M1 A1

$$b^2 - 4ac = 16 - 16 = 0, \text{ so L touches C}$$

A1

**Answer: Shown**

3 marks

|                                                                                                                          |         |
|--------------------------------------------------------------------------------------------------------------------------|---------|
| (b) $(x - 2)^2 = 0$ , so $x = 2$                                                                                         | M1      |
| $y = 2 + 1 = 3$ , so P is (2, 3)                                                                                         | A1      |
| <b>Answer: P(2, 3)</b>                                                                                                   | 2 marks |
| <b>18 (5 marks)</b>                                                                                                      |         |
| $x^2 - 3x + 5 = kx + 1$ , so $x^2 - (3 + k)x + 4 = 0$                                                                    | M1 A1   |
| Tangent: $(3 + k)^2 - 16 = 0$                                                                                            | M1      |
| $3 + k = \pm 4$                                                                                                          | A1      |
| $k = 1$ or $k = -7$                                                                                                      | A1      |
| <b>Answer: <math>k = 1</math> or <math>k = -7</math></b>                                                                 | 5 marks |
| <b>19 (4 marks)</b>                                                                                                      |         |
| $b^2 - 4ac = (k + 2)^2 - 4k$                                                                                             | M1      |
| $= k^2 + 4k + 4 - 4k = k^2 + 4$                                                                                          | A1      |
| $k^2 + 4 \geq 4 > 0$ for all $k$                                                                                         | M1      |
| So the discriminant is positive and the roots are real                                                                   | E1      |
| <b>Answer: Shown</b>                                                                                                     | 4 marks |
| <b>20 (4 marks)</b>                                                                                                      |         |
| Touching the x-axis means a repeated root: $36 - 4k = 0$                                                                 | M1 A1   |
| $k = 9$                                                                                                                  | A1      |
| $x^2 - 6x + 9 = (x - 3)^2$ , so the point is (3, 0)                                                                      | A1      |
| <b>Answer: <math>k = 9</math>; (3, 0)</b>                                                                                | 4 marks |
| <b>21 (5 marks)</b>                                                                                                      |         |
| $x^2 - 4x + 3 - k = 0$                                                                                                   | M1      |
| $b^2 - 4ac = 16 - 4(3 - k) = 4 + 4k$                                                                                     | A1      |
| Real roots: $4 + 4k \geq 0$ , so $k \geq -1$                                                                             | M1 A1   |
| The minimum value of $y = (x - 1)(x - 3)$ is $-1$ , so the horizontal line $y = k$ meets the graph only when $k \geq -1$ | E1      |
| <b>Answer: <math>k \geq -1</math></b>                                                                                    | 5 marks |
| <b>22 (6 marks)</b>                                                                                                      |         |
| $x^2 + x + 3 = kx + 2$ , so $x^2 + (1 - k)x + 1 = 0$                                                                     | M1 A1   |
| No intersection: $(1 - k)^2 - 4 < 0$                                                                                     | M1      |
| $(1 - k - 2)(1 - k + 2) < 0$ , i.e. $(-1 - k)(3 - k) < 0$                                                                | A1      |
| $(k + 1)(k - 3) < 0$                                                                                                     | M1      |
| $-1 < k < 3$                                                                                                             | A1      |
| <b>Answer: <math>-1 &lt; k &lt; 3</math></b>                                                                             | 6 marks |
| <b>23 (7 marks)</b>                                                                                                      |         |
| (a) Other side = $10 - x$ , so area $A = x(10 - x)$                                                                      | M1      |
| $x^2 - 10x + A = 0$                                                                                                      | A1      |
| <b>Answer: Shown</b>                                                                                                     | 2 marks |
| (b) $x$ is real, so $b^2 - 4ac \geq 0$ : $100 - 4A \geq 0$                                                               | M1 A1   |
| $A \leq 25$                                                                                                              | A1      |
| <b>Answer: Shown</b>                                                                                                     | 3 marks |
| (c) $A = 25$ gives a repeated root $x = 5$                                                                               | M1      |
| It is a square of side 5 m                                                                                               | A1      |
| <b>Answer: A square of side 5 m</b>                                                                                      | 2 marks |
| <b>24 (7 marks)</b>                                                                                                      |         |
| (a) (1): $k^2 - 16 \geq 0$ , so $k \leq -4$ or $k \geq 4$                                                                | M1 A1   |
| (2): $16 - 4k \geq 0$ , so $k \leq 4$                                                                                    | M1 A1   |
| <b>Answer: (1): <math>k \leq -4</math> or <math>k \geq 4</math>; (2): <math>k \leq 4</math></b>                          | 4 marks |
| (b) For $-4 < k < 4$ only (2) has real roots                                                                             | M1 A1   |

For  $k > 4$  only (1) has real roots. So  $-4 < k < 4$  or  $k > 4$

**Answer:**  $-4 < k < 4$  or  $k > 4$

A1

3 marks

## 25 (6 marks)

A line through  $(0, -1)$  is  $y = mx - 1$

$$x^2 = mx - 1, \text{ so } x^2 - mx + 1 = 0$$

$$\text{Tangent: } m^2 - 4 = 0$$

$$m = \pm 2$$

$$y = 2x - 1 \text{ and } y = -2x - 1$$

**Answer:**  $y = 2x - 1$  and  $y = -2x - 1$

B1

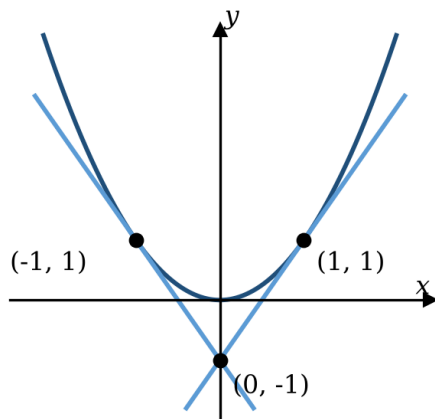
M1 A1

M1

A1

A1

6 marks



## 26 (6 marks)

$$x^2 - 6x + 11 = m + 4x - x^2, \text{ so } 2x^2 - 10x + 11 - m = 0$$

The curves touch, so the discriminant is zero:  $100 - 8(11 - m) = 0$

$$8m = -12, \text{ so } m = -\frac{3}{2}$$

$$2x^2 - 10x + \frac{25}{2} = 0 \text{ gives } 2(x - \frac{5}{2})^2 = 0, \text{ so } x = \frac{5}{2}$$

$$y = (\frac{5}{2})^2 - 6(\frac{5}{2}) + 11 = \frac{9}{4}, \text{ so } P \text{ is } (\frac{5}{2}, \frac{9}{4})$$

**Answer:**  $m = -\frac{3}{2}$ ;  $P(\frac{5}{2}, \frac{9}{4})$

M1 A1

M1

A1

M1

A1

6 marks

## 27 (8 marks)

(a)  $y^2 + ky + 4 = 0$

Four distinct real  $x$  need two distinct positive real values of  $y$

$$\text{Distinct real: } k^2 - 16 > 0, \text{ so } k < -4 \text{ or } k > 4$$

Product of roots  $= 4 > 0$ , so the roots have the same sign; sum  $= -k > 0$  needs  $k < 0$

$$\text{Hence } k < -4$$

**Answer: Shown**

(b) Repeated positive  $y$ :  $k = -4$

$$y = 2, \text{ so } x^2 = 2$$

$$x = \pm\sqrt{2}$$

**Answer:**  $k = -4$ ;  $x = \pm\sqrt{2}$

B1

M1

M1

M1

A1

5 marks

M1

A1

A1

3 marks

## 28 (6 marks)

$$\text{Touching the } x\text{-axis: } p^2 - 4q = 0$$

$$\text{Through } (1, 4): 1 + p + q = 4, \text{ so } q = 3 - p$$

$$p^2 - 4(3 - p) = 0, \text{ so } p^2 + 4p - 12 = 0$$

$$(p + 6)(p - 2) = 0$$

$$p = 2, q = 1 \text{ or } p = -6, q = 9$$

**Answer:**  $p = 2, q = 1$  or  $p = -6, q = 9$

M1 A1

M1

A1

M1

A1

6 marks

**29 (7 marks)**

$$y(x^2 + 2) = 2x - 1, \text{ so } yx^2 - 2x + (2y + 1) = 0$$

M1 A1

For real  $x$  the discriminant is non-negative:  $4 - 4y(2y + 1) \geq 0$

M1

$$2y^2 + y - 1 \leq 0$$

A1

$$(2y - 1)(y + 1) \leq 0$$

M1

$$-1 \leq y \leq \frac{1}{2}$$

A1

When  $y = 0$  the equation is linear with solution  $x = \frac{1}{2}$ , so the working holds for every  $y$  in the interval

E1

**Answer:**  $-1 \leq y \leq \frac{1}{2}$

7 marks

**30 (8 marks)**

**(a)** Expand:  $2x^2 - 2(a + b)x + a^2 + b^2 - 2c = 0$

M1 A1

$$b^2 - 4ac = 4(a + b)^2 - 8(a^2 + b^2 - 2c)$$

M1 A1

$$= -4(a - b)^2 + 16c$$

A1

Real roots:  $-4(a - b)^2 + 16c \geq 0$ , so  $c \geq \frac{(a - b)^2}{4}$

A1

**Answer: Shown**

6 marks

**(b)** The sum equals  $2c$  for some real  $x$  only if  $2c \geq \frac{(a - b)^2}{2}$

M1

Least value  $\frac{(a - b)^2}{2}$

A1

**Answer:**  $\frac{(a - b)^2}{2}$

2 marks

## Simultaneous Equations

**1 (2 marks)**

Add the equations:  $2x = 10$ , so  $x = 5$

M1

$$y = 7 - 5 = 2$$

A1

**Answer:**  $x = 5, y = 2$

2 marks

**2 (2 marks)**

Add:  $3x = 12$ , so  $x = 4$

M1

$$y = 11 - 8 = 3$$

A1

**Answer:**  $x = 4, y = 3$

2 marks

**3 (2 marks)**

Substitute:  $3x + 2x + 1 = 11$ , so  $x = 2$

M1

$$y = 5$$

A1

**Answer:**  $x = 2, y = 5$

2 marks

**4 (2 marks)**

From the second equation  $y = 5 - x$ ;  $3x + 10 - 2x = 12$

M1

$$x = 2, y = 3$$

A1

**Answer:**  $x = 2, y = 3$

2 marks

**5 (3 marks)**

Multiply the second equation by 2:  $6x + 2y = 18$

M1

Add to the first:  $11x = 22$ , so  $x = 2$

A1

$$y = 9 - 6 = 3$$

A1

**Answer:**  $x = 2, y = 3$

3 marks

**6 (3 marks)**From the second equation  $y = 2x - 1$ 

B1

 $4x + 6x - 3 = 17$ , so  $x = 2$ 

M1

 $y = 3$ 

A1

**Answer:  $x = 2, y = 3$** 

3 marks

**7 (4 marks)** $x^2 = x + 2$ 

M1

 $x^2 - x - 2 = 0$ , so  $(x - 2)(x + 1) = 0$ 

A1

 $x = 2$  or  $x = -1$ 

A1

 $(2, 4)$  and  $(-1, 1)$ 

A1

**Answer:  $(2, 4)$  and  $(-1, 1)$** 

4 marks

**8 (5 marks)** $y = 5 - x$ :  $x^2 + (5 - x)^2 = 17$ 

M1

 $2x^2 - 10x + 8 = 0$ 

A1

 $x^2 - 5x + 4 = 0$ , so  $x = 1$  or  $x = 4$ 

M1 A1

 $(1, 4)$  and  $(4, 1)$ 

A1

**Answer:  $x = 1, y = 4$  or  $x = 4, y = 1$** 

5 marks

**9 (5 marks)** $y = 7 - 2x$ :  $x(7 - 2x) = 6$ 

M1

 $2x^2 - 7x + 6 = 0$ 

A1

 $(2x - 3)(x - 2) = 0$ 

M1 A1

 $x = \frac{3}{2}, y = 4$  or  $x = 2, y = 3$ 

A1

**Answer:  $x = \frac{3}{2}, y = 4$  or  $x = 2, y = 3$** 

5 marks

**10 (4 marks)** $x^2 - 3x + 1 = x - 1$ 

M1

 $x^2 - 4x + 2 = 0$ 

A1

 $x = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$ 

A1

 $y = x - 1$  gives the points  $(2 + \sqrt{2}, 1 + \sqrt{2})$  and  $(2 - \sqrt{2}, 1 - \sqrt{2})$ 

A1

**Answer:  $(2 + \sqrt{2}, 1 + \sqrt{2})$  and  $(2 - \sqrt{2}, 1 - \sqrt{2})$** 

4 marks

**11 (5 marks)**Let  $a$  and  $c$  be the costs:  $3a + 2c = 28$  and  $2a + 5c = 37$ 

B1

Eliminate  $c$ :  $15a + 10c = 140$  and  $4a + 10c = 74$ 

M1

 $11a = 66$ , so  $a = 6$ 

A1

 $c = (28 - 18) \div 2 = 5$ 

M1

Adult £6, child £5

A1

**Answer: Adult £6, child £5**

5 marks

**12 (5 marks)** $x^2 + (x + 1)^2 = 25$ 

M1

 $2x^2 + 2x - 24 = 0$ , so  $x^2 + x - 12 = 0$ 

A1

 $(x + 4)(x - 3) = 0$ 

M1 A1

 $(3, 4)$  and  $(-4, -3)$ 

A1

**Answer:  $(3, 4)$  and  $(-4, -3)$** 

5 marks

**13 (4 marks)**Multiply the first by 6:  $3x + 2y = 24$ 

M1

Multiply this by 3 and the second by 2:  $9x + 6y = 72$  and  $10x - 6y = 4$ 

M1

Add:  $19x = 76$ , so  $x = 4$ 

A1

 $y = (24 - 12) \div 2 = 6$ 

A1

**Answer:  $x = 4, y = 6$** 

4 marks

**14 (6 marks)** $y = 1 - x: x^2 - x(1 - x) + (1 - x)^2 = 7$ 

M1

 $x^2 - x + x^2 + 1 - 2x + x^2 = 7$ 

A1

 $3x^2 - 3x - 6 = 0$ , so  $x^2 - x - 2 = 0$ 

A1

 $(x - 2)(x + 1) = 0$ 

M1

 $x = 2, y = -1$ 

A1

 $x = -1, y = 2$ 

A1

**Answer: (2, -1) and (-1, 2)**

6 marks

**15 (7 marks)**(a)  $x^2 - x - 6 = 2x - 2$ , so  $x^2 - 3x - 4 = 0$ 

M1 A1

 $(x - 4)(x + 1) = 0$ , so  $x = -1$  or  $x = 4$ 

A1

A(-1, -4) and B(4, 6)

A1

**Answer: A(-1, -4), B(4, 6)**

4 marks

(b)  $AB^2 = (4 - (-1))^2 + (6 - (-4))^2 = 25 + 100$ 

M1 A1

 $AB = \sqrt{125} = 5\sqrt{5}$ 

A1

**Answer:  $5\sqrt{5}$** 

3 marks

**16 (6 marks)**From the first equation  $z = 9 - x - 2y$ 

B1

Substituting into the second:  $2x - y + 27 - 3x - 6y = 4$ , so  $x + 7y = 23$ 

M1 A1

Substituting into the third:  $3x + y - 18 + 2x + 4y = 7$ , so  $x + y = 5$ 

M1 A1

Subtracting:  $6y = 18$ , so  $y = 3$ ,  $x = 2$  and  $z = 1$ 

A1

**Answer:  $x = 2, y = 3, z = 1$** 

6 marks

**17 (6 marks)**Perimeter:  $2x + 2y = 34$ , so  $y = 17 - x$ 

B1

Pythagoras:  $x^2 + y^2 = 169$ 

B1

 $x^2 + (17 - x)^2 = 169$ , so  $x^2 - 17x + 60 = 0$ 

M1 A1

 $(x - 5)(x - 12) = 0$ 

M1

The rectangle is 5 cm by 12 cm

A1

**Answer: 5 cm by 12 cm**

6 marks

**18 (4 marks)** $x^2 - y^2 = (x - y)(x + y) = 3(x + y) = 15$ 

M1 A1

 $x + y = 5$ , and  $x - y = 3$ 

A1

 $x = 4, y = 1$ 

A1

**Answer:  $x = 4, y = 1$** 

4 marks

**19 (5 marks)** $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{5}{xy}$ 

M1

 $\frac{5}{xy} = \frac{5}{6}$ , so  $xy = 6$ 

A1

 $x(5 - x) = 6$ , so  $x^2 - 5x + 6 = 0$ 

M1

 $x = 2$  or  $x = 3$ 

A1

(2, 3) and (3, 2)

A1

**Answer: (2, 3) and (3, 2)**

5 marks

**20 (5 marks)****(a)** When  $x = 1$  the line gives  $y = 5$ 

B1

$1 + k + 7 = 5$ , so  $k = -3$

A1

**Answer:  $k = -3$** 

2 marks

**(b)**  $x^2 - 3x + 7 = 2x + 3$ , so  $x^2 - 5x + 4 = 0$ 

M1

$(x - 1)(x - 4) = 0$ , so  $x = 4$

A1

$y = 2(4) + 3 = 11$ : the point is  $(4, 11)$

A1

**Answer:  $(4, 11)$** 

3 marks

**21 (8 marks)****(a)**  $x^2 - 2x - 2 = x + 3$ , so  $x^2 - 3x - 5 = 0$ 

M1 A1

The sum of the  $x$ -coordinates is 3, so the midpoint has  $x = \frac{3}{2}$ 

M1

$y = \frac{3}{2} + 3 = \frac{9}{2}$ : the midpoint is  $(\frac{3}{2}, \frac{9}{2})$

A1

**Answer:  $(\frac{3}{2}, \frac{9}{2})$** 

4 marks

**(b)**  $x_A - x_B = \sqrt{b^2 - 4ac} = \sqrt{9 + 20} = \sqrt{29}$

M1 A1

The line has gradient 1, so the  $y$ -difference is also  $\sqrt{29}$ 

M1

$AB = \sqrt{29 + 29} = \sqrt{58}$

A1

**Answer:  $\sqrt{58}$** 

4 marks

**22 (5 marks)**

$x^2 + (3x - 10)^2 = 10$

M1

$10x^2 - 60x + 90 = 0$ , so  $x^2 - 6x + 9 = 0$

A1

 $(x - 3)^2 = 0$ , a repeated root, so the line is a tangent

M1 E1

$x = 3$ ,  $y = 3(3) - 10 = -1$ : the point is  $(3, -1)$

A1

**Answer: Shown;  $(3, -1)$** 

5 marks

**23 (8 marks)****(a)**  $x^2 - 4x + 3 = -x^2 + 2x + 3$ 

M1

$2x^2 - 6x = 0$ , so  $2x(x - 3) = 0$

A1

$x = 0$  or  $x = 3$

A1

 $P(0, 3)$  and  $Q(3, 0)$ 

A1

**Answer:  $P(0, 3), Q(3, 0)$** 

4 marks

**(b)** Midpoint  $(\frac{3}{2}, \frac{3}{2})$ 

B1

Gradient of  $PQ = \frac{0 - 3}{3 - 0} = -1$

M1

Perpendicular gradient = 1

A1

$y - \frac{3}{2} = x - \frac{3}{2}$ , so  $y = x$

A1

**Answer:  $y = x$** 

4 marks

**24 (6 marks)**

$(x + y)^2 = x^2 + y^2 + 2xy = 34 + 30 = 64$

M1 A1

$(x - y)^2 = 34 - 30 = 4$

M1

$x + y = \pm 8$  and  $x - y = \pm 2$

A1

$x + y = 8$ :  $(5, 3)$  or  $(3, 5)$

A1

$x + y = -8$ :  $(-3, -5)$  or  $(-5, -3)$

A1

**Answer:  $(5, 3), (3, 5), (-3, -5), (-5, -3)$** 

6 marks

**25 (6 marks)**

$$x^2 - 4x + 9 = kx, \text{ so } x^2 - (4 + k)x + 9 = 0$$

M1 A1

$$\text{One point of contact: } (4 + k)^2 - 36 = 0$$

M1

$$k = 2 \text{ or } k = -10$$

A1

$$k = 2: x^2 - 6x + 9 = 0, x = 3, \text{ point } (3, 6)$$

A1

$$k = -10: x^2 + 6x + 9 = 0, x = -3, \text{ point } (-3, 30)$$

A1

**Answer:  $k = 2$  at  $(3, 6)$ ;  $k = -10$  at  $(-3, 30)$**

6 marks

**26 (6 marks)**

(a) Multiply the first by 3:  $3x + 6y = 15$

M1

Subtracting:  $(k - 6)y = -8$ , which has no solution if  $k = 6$

M1

$$k = 6$$

A1

**Answer:  $k = 6$**

3 marks

(b) For  $k \neq 6$ ,  $y = \frac{-8}{k-6}$  and then  $x$  is found uniquely

E1

The lines have gradients  $-\frac{1}{2}$  and  $-\frac{3}{k}$

M1

At  $k = 6$  the gradients are equal and the lines are distinct, so they are parallel and never meet

E1

**Answer: Parallel lines when  $k = 6$**

3 marks

**27 (8 marks)**

Factorise the first equation:  $(x - 2y)(x + y) = 0$

M1 A1

Case 1:  $x = 2y$ . Then  $4y^2 + y^2 = 20$ , so  $y^2 = 4$ ,  $y = \pm 2$

M1 A1

$(4, 2)$  and  $(-4, -2)$

A1

Case 2:  $x = -y$ . Then  $2y^2 = 20$ , so  $y = \pm\sqrt{10}$

M1 A1

$(-\sqrt{10}, \sqrt{10})$  and  $(\sqrt{10}, -\sqrt{10})$

A1

**Answer:  $(4, 2)$ ,  $(-4, -2)$ ,  $(\sqrt{10}, -\sqrt{10})$ ,  $(-\sqrt{10}, \sqrt{10})$**

8 marks

**28 (7 marks)**

(a)  $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$

M1 A1

$$72 = 216 - 18ab$$

M1

$$ab = 8$$

A1

**Answer:  $ab = 8$**

4 marks

(b)  $a$  and  $b$  are roots of  $t^2 - 6t + 8 = 0$

M1

$$(t - 2)(t - 4) = 0$$

A1

$$(a, b) = (2, 4) \text{ or } (4, 2)$$

A1

**Answer:  $(2, 4)$  or  $(4, 2)$**

3 marks

**29 (9 marks)**

(a)  $x^2 + (x + k)^2 = 26$

M1

$$2x^2 + 2kx + k^2 - 26 = 0$$

A1

**Answer: Shown**

2 marks

(b)  $(x_A - x_B)^2 = (\text{sum})^2 - 4(\text{product}) = k^2 - 2(k^2 - 26) = 52 - k^2$

M1 A1

The line has gradient 1, so  $AB^2 = 2(x_A - x_B)^2 = 104 - 2k^2$

M1 A1

$$AB^2 = 32, \text{ so } 2k^2 = 72 \text{ and } k = 6$$

A1

$$2x^2 + 12x + 10 = 0, \text{ so } (x + 1)(x + 5) = 0$$

M1

$$A(-1, 5) \text{ and } B(-5, 1)$$

A1

**Answer:  $k = 6$ ;  $(-1, 5)$  and  $(-5, 1)$**

7 marks

**30 (6 marks)**

$$x - y = (\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})$$

M1

$$\text{So } 5(\sqrt{x} - \sqrt{y}) = 5, \text{ i.e. } \sqrt{x} - \sqrt{y} = 1$$

A1

$$\text{Adding to } \sqrt{x} + \sqrt{y} = 5: 2\sqrt{x} = 6, \text{ so } \sqrt{x} = 3$$

M1 A1

$$\sqrt{y} = 2$$

A1

$$x = 9 \text{ and } y = 4$$

A1

**Answer:  $x = 9, y = 4$** 

6 marks

**Polynomial Equations****1 (2 marks)**

$$x = 0, x - 2 = 0 \text{ or } x + 5 = 0$$

M1

$$x = 0, 2 \text{ or } -5$$

A1

**Answer:  $x = 0, 2, -5$** 

2 marks

**2 (3 marks)**

$$x(x^2 - 4) = 0$$

M1

$$x(x - 2)(x + 2) = 0$$

A1

$$x = 0, 2 \text{ or } -2$$

A1

**Answer:  $x = 0, 2, -2$** 

3 marks

**3 (3 marks)**

$$x(x^2 - 5x + 6) = 0$$

M1

$$x(x - 2)(x - 3) = 0$$

A1

$$x = 0, 2 \text{ or } 3$$

A1

**Answer:  $x = 0, 2, 3$** 

3 marks

**4 (2 marks)**

$$\text{Substitute } x = 2: 2^3 - 3(2) - 2$$

M1

$$= 8 - 6 - 2 = 0, \text{ so } x = 2 \text{ is a root}$$

A1

**Answer: Shown**

2 marks

**5 (2 marks)**

$$x^3 = -8$$

M1

$$x = -2$$

A1

**Answer:  $x = -2$** 

2 marks

**6 (2 marks)**

$$x^2 = 4 \text{ or } x^2 = 9$$

M1

$$x = \pm 2 \text{ or } x = \pm 3$$

A1

**Answer:  $x = \pm 2, \pm 3$** 

2 marks

**7 (4 marks)**

$$f(1) = 1 - 7 + 6 = 0, \text{ so } (x - 1) \text{ is a factor}$$

M1 A1

$$x^3 - 7x + 6 = (x - 1)(x^2 + x - 6) = (x - 1)(x + 3)(x - 2)$$

M1

$$x = 1, 2 \text{ or } -3$$

A1

**Answer:  $x = 1, 2, -3$** 

4 marks

**8 (4 marks)**

$$f(-2) = -16 - 4 + 26 - 6 = 0, \text{ so } (x + 2) \text{ is a factor}$$

M1 A1

$$2x^3 - x^2 - 13x - 6 = (x + 2)(2x^2 - 5x - 3) = (x + 2)(2x + 1)(x - 3)$$

M1

$$x = -2, -\frac{1}{2} \text{ or } 3$$

A1

**Answer:  $x = -2, -\frac{1}{2}, 3$** 

4 marks

**9 (4 marks)**

$$\text{Let } y = x^2: y^2 - 5y + 4 = 0$$

M1

$$(y - 1)(y - 4) = 0, \text{ so } y = 1 \text{ or } y = 4$$

A1

$$x^2 = 1 \text{ or } x^2 = 4$$

M1

$$x = \pm 1 \text{ or } x = \pm 2$$

A1

$$\text{Answer: } x = \pm 1, \pm 2$$

4 marks

**10 (5 marks)**

$$\text{Substitute } x = 1: 1 - 7 + k - 8 = 0$$

M1

$$k = 14$$

A1

$$x^3 - 7x^2 + 14x - 8 = (x - 1)(x^2 - 6x + 8)$$

M1

$$(x - 1)(x - 2)(x - 4)$$

A1

$$\text{The other roots are } x = 2 \text{ and } x = 4$$

A1

$$\text{Answer: } k = 14; x = 2, 4$$

5 marks

**11 (4 marks)**

$$y = 0: x(x^2 - x - 6) = 0$$

M1

$$x(x - 3)(x + 2) = 0$$

A1

$$x = 0, 3 \text{ or } -2$$

A1

$$(-2, 0), (0, 0) \text{ and } (3, 0)$$

A1

$$\text{Answer: } (-2, 0), (0, 0), (3, 0)$$

4 marks

**12 (4 marks)**

$$\text{Do not divide by } (x - 2). \text{ Rearrange: } (x - 2)(x^2 + 4x - 3) - 2(x - 2) = 0$$

M1

$$(x - 2)(x^2 + 4x - 5) = 0$$

A1

$$(x - 2)(x + 5)(x - 1) = 0$$

M1

$$x = 2, x = -5 \text{ or } x = 1$$

A1

$$\text{Answer: } x = 2, -5, 1$$

4 marks

**13 (4 marks)**

$$f(-1) = -1 + 2 + 5 - 6 = 0$$

M1

$$f(x) = (x + 1)(x^2 + x - 6)$$

M1 A1

$$(x + 1)(x + 3)(x - 2) = 0, \text{ so } x = -1, -3 \text{ or } 2$$

A1

$$\text{Answer: } x = -1, -3, 2$$

4 marks

**14 (6 marks)**

$$f(1) = 1 + 1 - 7 - 1 + 6 = 0 \text{ and } f(-1) = 1 - 1 - 7 + 1 + 6 = 0$$

M1 A1

$$(x - 1)(x + 1) = x^2 - 1 \text{ is a factor}$$

A1

$$f(x) = (x^2 - 1)(x^2 + x - 6)$$

M1 A1

$$= (x - 1)(x + 1)(x + 3)(x - 2), \text{ so } x = 1, -1, -3 \text{ or } 2$$

A1

$$\text{Answer: } x = 1, -1, -3, 2$$

6 marks

**15 (5 marks)**

$$f(-1) = -1 - 3 + 4 = 0, \text{ so } (x + 1) \text{ is a factor}$$

M1 A1

$$x^3 - 3x^2 + 4 = (x + 1)(x^2 - 4x + 4)$$

M1

$$= (x + 1)(x - 2)^2$$

A1

$$x = -1 \text{ or } x = 2 \text{ (repeated)}$$

A1

$$\text{Answer: } x = -1, x = 2 \text{ (repeated)}$$

5 marks

**16 (5 marks)**

$$y = a(x + 2)(x - 1)(x - 3)$$

B1

$$x = 0, y = -12: a(2)(-1)(-3) = 6a = -12, \text{ so } a = -2$$

M1 A1

$$(x + 2)(x - 1)(x - 3) = x^3 - 2x^2 - 5x + 6$$

M1

$$y = -2x^3 + 4x^2 + 10x - 12$$

A1

$$\text{Answer: } y = -2x^3 + 4x^2 + 10x - 12$$

5 marks

**17 (5 marks)**

$$(x + 1)(x - 2)(x - 3) = (x + 1)(x^2 - 5x + 6) = x^3 - 4x^2 + x + 6$$

M1 A1

The cubic terms cancel:  $x^3 - 2x^2 - x^3 + 4x^2 - x - 6 = 2x^2 - x - 6$

M1

$$2x^2 - x - 15 = 0, \text{ so } (2x + 5)(x - 3) = 0$$

A1

$$x = 3 \text{ or } x = -\frac{5}{2}$$

A1

**Answer:  $x = 3, x = -\frac{5}{2}$**

5 marks

**18 (4 marks)**

$$x^2(x - 3) - 4(x - 3) = 0$$

M1 A1

$$(x - 3)(x^2 - 4) = 0$$

A1

$$x = 3, 2 \text{ or } -2$$

A1

**Answer:  $x = 3, 2, -2$**

4 marks

**19 (6 marks)**

$$f(1) = 1 - 5 + 5 - 1 = 0, \text{ so } (x - 1) \text{ is a factor}$$

M1 A1

$$x^3 - 5x^2 + 5x - 1 = (x - 1)(x^2 - 4x + 1)$$

M1 A1

$$x^2 - 4x + 1 = 0: x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

M1

$$x = 1, x = 2 + \sqrt{3} \text{ or } x = 2 - \sqrt{3}$$

A1

**Answer:  $x = 1, 2 \pm \sqrt{3}$**

6 marks

**20 (5 marks)**

$$\text{Let } y = x^3: y^2 - 7y - 8 = 0$$

M1 A1

$$(y - 8)(y + 1) = 0, \text{ so } y = 8 \text{ or } y = -1$$

M1 A1

$$x = 2 \text{ or } x = -1$$

A1

**Answer:  $x = 2 \text{ or } x = -1$**

5 marks

**21 (10 marks)**

(a) Base is a square of side  $(24 - 2x)$ , height  $x$

B1

$$V = x(24 - 2x)^2 = x(576 - 96x + 4x^2)$$

M1

$$= 4x^3 - 96x^2 + 576x$$

A1

**Answer: Shown**

3 marks

(b)  $V(4) = 256 - 1536 + 2304 = 1024$

B1

$$4x^3 - 96x^2 + 576x - 1024 = 0, \text{ so } x^3 - 24x^2 + 144x - 256 = 0$$

M1

$$\text{Divide by } (x - 4): (x - 4)(x^2 - 20x + 64)$$

M1 A1

$$(x - 4)^2(x - 16) = 0, \text{ so } x = 4 \text{ or } x = 16$$

A1

**Answer:  $x = 4 \text{ or } x = 16$**

5 marks

(c)  $x$  must be less than 12 so that the base has positive side

E1

The repeated root means the cubic touches the line  $V = 1024$  at  $x = 4$ , so  $1024 \text{ cm}^3$  is the maximum volume

E1

**Answer:  $x < 12$ ; 1024 is the maximum**

2 marks

**22 (6 marks)**

$$x^3 - 4x = 3x - 6$$

M1

$$x^3 - 7x + 6 = 0$$

A1

$$f(1) = 0, \text{ so } (x - 1)(x^2 + x - 6) = (x - 1)(x + 3)(x - 2)$$

M1 A1

$$x = 1, 2 \text{ or } -3$$

A1

$$y = 3x - 6: (1, -3), (2, 0) \text{ and } (-3, -15)$$

A1

**Answer:  $(1, -3), (2, 0), (-3, -15)$**

6 marks

**23 (6 marks)**

(a)  $x^3 - 1 = (x - 1)(x^2 + x + 1)$  M1

Discriminant of  $x^2 + x + 1$  is  $1 - 4 = -3 < 0$  M1So  $x = 1$  is the only real root E1**Answer: Shown** 3 marks

(b)  $x^6 - 1 = (x^3 - 1)(x^3 + 1)$  M1

 $x^3 = 1$  or  $x^3 = -1$ , each with exactly one real root M1 $x = 1$  or  $x = -1$  A1**Answer:  $x = 1, x = -1$**  3 marks**24 (7 marks)**

(a)  $(x^2 - 4)^2 = x^4 - 8x^2 + 16$  M1

$f(x) = (x - 2)^2(x + 2)^2$  A1

 $x = 2$  and  $x = -2$ , both repeated A1**Answer:  $x = \pm 2$ , both repeated** 3 marks

(b)  $(x^2 - 4)^2 = 9$ , so  $x^2 - 4 = \pm 3$  M1 A1

$x^2 = 7$  or  $x^2 = 1$  A1

$x = \pm\sqrt{7}$  or  $x = \pm 1$  A1

**Answer:  $x = \pm 1, \pm\sqrt{7}$**  4 marks**25 (7 marks)**(a) Let the roots be  $m - d, m, m + d$ : the sum of the roots is  $3m$  M1For  $x^3 - 6x^2 + \dots$ , the sum of the roots is 6 M1So  $3m = 6$  and  $m = 2$  A1**Answer: Shown** 3 marks

(b)  $f(2) = 8 - 24 + 2q - 6 = 0$ , so  $q = 11$  M1 A1

$x^3 - 6x^2 + 11x - 6 = (x - 2)(x^2 - 4x + 3)$  M1

The other roots are  $x = 1$  and  $x = 3$  A1**Answer:  $q = 11$ ; roots 1 and 3** 4 marks**26 (7 marks)**

(a)  $(x - 1)(x + 5) = x^2 + 4x - 5 = m - 5$  M1 A1

$(x + 1)(x + 3) = x^2 + 4x + 3 = m + 3$ , so the equation is  $(m - 5)(m + 3) = 9$  A1

**Answer: Shown** 3 marks

(b)  $m^2 - 2m - 15 = 9$ , so  $m^2 - 2m - 24 = 0$  and  $(m - 6)(m + 4) = 0$  M1

$m = 6$ :  $x^2 + 4x - 6 = 0$ , so  $x = -2 \pm \sqrt{10}$  A1 A1

$m = -4$ :  $x^2 + 4x + 4 = 0$ , so  $x = -2$  A1

**Answer:  $x = -2, x = -2 \pm \sqrt{10}$**  4 marks**27 (8 marks)**

(a)  $f'(x) = 3x^2 - 3 = 0$ , so  $x = 1$  or  $x = -1$  M1 A1

$f(1) = k - 2$  A1

$f(-1) = k + 2$  A1

**Answer:  $x = \pm 1; y = k - 2$  and  $k + 2$**  4 marks(b) The curve is a positive cubic: local maximum at  $x = -1$  and local minimum at  $x = 1$  B1

Three distinct roots need the maximum above the axis and the minimum below it M1

$k + 2 > 0$  and  $k - 2 < 0$  A1

$-2 < k < 2$  A1

**Answer: Shown** 4 marks**28 (8 marks)**

(a)  $(x^2 + 2x - 1)^2 = x^4 + 4x^3 + 4x^2 - 2x^2 - 4x + 1$  M1 A1

$= x^4 + 4x^3 + 2x^2 - 4x + 1$  A1

**Answer: Shown** 3 marks

(b)  $(x^2 + 2x - 1)^2 = 0$ , so  $x^2 + 2x - 1 = 0$

M1

$$x = \frac{-2 \pm \sqrt{4 + 4}}{2}$$

A1

$$x = -1 \pm \sqrt{2}$$

A1

**Answer:  $x = -1 \pm \sqrt{2}$**

3 marks

(c) The quartic is a perfect square, so each root occurs twice

E1

The graph touches the x-axis at each root and never goes below it

E1

**Answer: Graph touches the axis twice, lies above it**

2 marks

### 29 (8 marks)

(a)  $x = 0$  is not a root, so divide by  $x^2$ :  $6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$

M1

$$6(x^2 + \frac{1}{x^2}) - 35(x + \frac{1}{x}) + 62 = 0$$

A1

$$x^2 + \frac{1}{x^2} = t^2 - 2$$

M1

$$6(t^2 - 2) - 35t + 62 = 0, \text{ so } 6t^2 - 35t + 50 = 0$$

A1

**Answer: Shown**

4 marks

(b)  $(2t - 5)(3t - 10) = 0$

M1

$$t = \frac{5}{2} \text{ or } t = \frac{10}{3}$$

A1

$$x + \frac{1}{x} = \frac{5}{2} \text{ gives } 2x^2 - 5x + 2 = 0, \text{ so } x = 2 \text{ or } x = \frac{1}{2}$$

A1

$$x + \frac{1}{x} = \frac{10}{3} \text{ gives } 3x^2 - 10x + 3 = 0, \text{ so } x = 3 \text{ or } x = \frac{1}{3}$$

A1

**Answer:  $x = 2, \frac{1}{2}, 3, \frac{1}{3}$**

4 marks

### 30 (9 marks)

(a) Touching means  $f(2) = 0$  and  $f'(2) = 0$

E1

$$f(2) = 8 + 4a + 2b - 4 = 0, \text{ so } 2a + b = -2$$

M1 A1

$$f'(x) = 3x^2 + 2ax + b, \text{ so } 12 + 4a + b = 0$$

M1

$$\text{Subtracting: } 2a = -10, \text{ so } a = -5 \text{ and } b = 8$$

A1

**Answer:  $a = -5, b = 8$**

5 marks

(b)  $x^3 - 5x^2 + 8x - 4 = (x - 2)(x^2 - 3x + 2)$

M1 A1

$$= (x - 2)^2(x - 1)$$

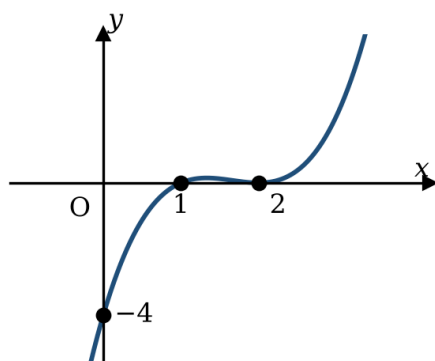
A1

Touches the axis at  $x = 2$ , crosses at  $x = 1$ , meets the y-axis at  $(0, -4)$

B1

**Answer:  $(x - 2)^2(x - 1)$ ; see sketch**

4 marks



## Linear and Quadratic Inequalities

### 1 (2 marks)

$$3x > 15$$

M1

$$x > 5$$

A1

**Answer:  $x > 5$**

2 marks

### 2 (2 marks)

$$-2x \leq 6$$

M1

Dividing by  $-2$  reverses the inequality:  $x \geq -3$

A1

**Answer:  $x \geq -3$**

2 marks

### 3 (2 marks)

$(x - 3)(x + 3) < 0$ , the graph is below the axis between the roots

M1

$$-3 < x < 3$$

A1

**Answer:  $-3 < x < 3$**

2 marks

### 4 (3 marks)

$(x - 2)(x - 3) \geq 0$ , roots 2 and 3

M1

Positive parabola: above the axis outside the roots

M1

$$x \leq 2 \text{ or } x \geq 3$$

A1

**Answer:  $x \leq 2 \text{ or } x \geq 3$**

3 marks

### 5 (2 marks)

$$2x + 6 < x + 10$$

M1

$$x < 4$$

A1

**Answer:  $x < 4$**

2 marks

### 6 (3 marks)

Subtract 1 from each part:  $-4 < 2x \leq 8$

M1

Divide by 2:  $-2 < x \leq 4$

A1 A1

**Answer:  $-2 < x \leq 4$**

3 marks

### 7 (3 marks)

$$(x + 5)(x - 3) < 0$$

M1

Roots  $-5$  and  $3$ , below the axis between them

A1

$$-5 < x < 3$$

A1

**Answer:  $-5 < x < 3$**

3 marks

### 8 (4 marks)

$$(2x + 3)(x - 2) > 0$$

M1 A1

Roots  $-\frac{3}{2}$  and  $2$ ; positive parabola

M1

$$x < -\frac{3}{2} \text{ or } x > 2$$

A1

**Answer:  $x < -\frac{3}{2} \text{ or } x > 2$**

4 marks

### 9 (3 marks)

$$2x + 1 > 9 \text{ gives } x > 4$$

B1

$$3x - 4 < 17 \text{ gives } x < 7$$

B1

Both:  $4 < x < 7$

B1

**Answer:  $4 < x < 7$**

3 marks

**10 (4 marks)**

$$x^2 - 4x \geq 0, \text{ so } x(x - 4) \geq 0$$

M1 A1

Roots 0 and 4; positive parabola

M1

$$x \leq 0 \text{ or } x \geq 4$$

A1

**Answer:  $x \leq 0$  or  $x \geq 4$** 

4 marks

**11 (4 marks)**

$$x^2 - 3x - 10 < 0$$

M1

$$(x - 5)(x + 2) < 0$$

A1

Roots 5 and  $-2$ , below the axis between them

M1

$$-2 < x < 5$$

A1

**Answer:  $-2 < x < 5$** 

4 marks

**12 (3 marks)**(a) The curve is below the x-axis between  $x = -1$  and  $x = 3$ 

B1

**Answer:  $-1 < x < 3$** 

1 mark

(b) The curve is on or above the x-axis outside the roots

M1

$$x \leq -1 \text{ or } x \geq 3$$

A1

**Answer:  $x \leq -1$  or  $x \geq 3$** 

2 marks

**13 (4 marks)**

$$n^2 - 10n + 13 = 0 \text{ gives } n = \frac{10 \pm \sqrt{48}}{2} = 5 \pm 2\sqrt{3}$$

M1 A1

$$5 - 2\sqrt{3} < n < 5 + 2\sqrt{3}, \text{ that is } 1.54 < n < 8.46 \text{ (to 3 s.f.)}$$

A1

 $n = 2, 3, 4, 5, 6, 7, 8$ , so there are 7 integers

A1

**Answer: 7**

4 marks

**14 (5 marks)**

$$x^2 - x - 6 < 0: (x - 3)(x + 2) < 0, \text{ so } -2 < x < 3$$

M1 A1

$$2x + 1 > 3: x > 1$$

B1

Both inequalities hold when

M1

$$1 < x < 3$$

A1

**Answer:  $1 < x < 3$** 

5 marks

**15 (5 marks)**The graph must not touch or cross the axis, so  $b^2 - 4ac < 0$ 

M1

$$4k^2 - 4(k + 6) < 0$$

A1

$$k^2 - k - 6 < 0, \text{ so } (k - 3)(k + 2) < 0$$

M1 A1

$$-2 < k < 3$$

A1

**Answer:  $-2 < k < 3$** 

5 marks

**16 (5 marks)**The cubic has roots  $x = -3, 0$  and  $2$ 

B1

Sketch: positive cubic, below the axis for  $x < -3$  and for  $0 < x < 2$ 

M1 A1

It is above the axis between  $-3$  and  $0$ , and for  $x > 2$ 

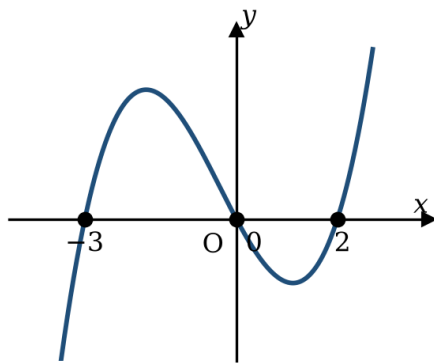
M1

$$-3 < x < 0 \text{ or } x > 2$$

A1

**Answer:  $-3 < x < 0$  or  $x > 2$** 

5 marks



### 17 (5 marks)

$$x + 3 > x^2 - 3x$$

M1

$$x^2 - 4x - 3 < 0$$

A1

$$\text{Roots: } x = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{7}$$

M1 A1

$$2 - \sqrt{7} < x < 2 + \sqrt{7}$$

A1

$$\text{Answer: } 2 - \sqrt{7} < x < 2 + \sqrt{7}$$

5 marks

### 18 (6 marks)

$$\text{Area: } x(x + 5) < 84, \text{ so } x^2 + 5x - 84 < 0$$

M1 A1

$$(x + 12)(x - 7) < 0, \text{ so } -12 < x < 7; \text{ with } x > 0 \text{ this gives } 0 < x < 7$$

M1 A1

$$\text{Perimeter: } 2x + 2(x + 5) \geq 26, \text{ so } 4x \geq 16 \text{ and } x \geq 4$$

M1

$$4 \leq x < 7$$

A1

$$\text{Answer: } 4 \leq x < 7$$

6 marks

### 19 (5 marks)

$$\frac{3x}{x+2} - 2 \leq 0$$

M1

$$\frac{3x - 2(x+2)}{x+2} = \frac{x-4}{x+2} \leq 0$$

A1

The critical values are  $x = -2$  (excluded, since the fraction is undefined) and  $x = 4$

M1

The fraction is negative for  $-2 < x < 4$

A1

and zero at  $x = 4$ , so  $-2 < x \leq 4$

A1

$$\text{Answer: } -2 < x \leq 4$$

5 marks

### 20 (4 marks)

$$x^2 - 4x + 1 = 0 \text{ gives } x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

M1 A1

The graph is on or below the axis between the roots

M1

$$2 - \sqrt{3} \leq x \leq 2 + \sqrt{3}$$

A1

$$\text{Answer: } 2 - \sqrt{3} \leq x \leq 2 + \sqrt{3}$$

4 marks

### 21 (6 marks)

$$2x^2 - 7x + 3 = (2x - 1)(x - 3) \leq 0, \text{ so } \frac{1}{2} \leq x \leq 3$$

M1 A1

$$x^2 - x - 2 = (x - 2)(x + 1) > 0, \text{ so } x < -1 \text{ or } x > 2$$

M1 A1

Intersection of the two sets

M1

$$2 < x \leq 3$$

A1

$$\text{Answer: } 2 < x \leq 3$$

6 marks

### 22 (7 marks)

$$\text{(a) } x^2 - 4 = x - 2, \text{ so } x^2 - x - 2 = 0$$

M1

$$(x - 2)(x + 1) = 0$$

A1

$$(2, 0) \text{ and } (-1, -3)$$

A1

$$\text{Answer: } (2, 0) \text{ and } (-1, -3)$$

3 marks

**(b)** Integer  $x$  from  $-1$  to  $2$ 

M1

$x = -1: y = -3. x = 0: y = -4, -3, -2$

A1

$x = 1: y = -3, -2, -1. x = 2: y = 0$

A1

8 points in total

A1

**Answer:**  $(-1, -3), (0, -4), (0, -3), (0, -2), (1, -3), (1, -2), (1, -1), (2, 0)$ 

4 marks

**23 (5 marks)****(a)**  $x$  may be negative or zero, and dividing by a negative number reverses the inequality

E1

Dividing loses the solutions with  $x < 0$ 

E1

**Answer:** Dividing by  $x$  can reverse the inequality and loses solutions

2 marks

**(b)**  $x(x - 4) > 0$ 

M1

$x < 0$  or  $x > 4$

A1

For example  $x = -1: 1 > -4$ , true, but  $-1$  is not greater than  $4$ 

B1

**Answer:**  $x < 0$  or  $x > 4$ 

3 marks

**24 (6 marks)**

**(a)**  $a^2 + b^2 - 2ab = (a - b)^2$

M1

A square is never negative, so  $a^2 + b^2 \geq 2ab$ 

E1

**Answer:** Shown

2 marks

**(b)** Take  $a = x$  and  $b = \frac{2}{x}$  in (a):  $x^2 + \frac{4}{x^2} \geq 2 \times x \times \frac{2}{x}$

M1 A1

$= 4$

A1

Equality when  $a = b: x = \frac{2}{x}$ , so  $x^2 = 2$ ,  $x = \pm\sqrt{2}$

A1

**Answer:** Shown;  $x = \pm\sqrt{2}$ 

4 marks

**25 (6 marks)**

$x^2 + 2x = kx - 5$ , so  $x^2 + (2 - k)x + 5 = 0$

M1 A1

Two distinct points:  $(2 - k)^2 - 20 > 0$ 

M1

$k^2 - 4k - 16 > 0$

A1

$k = \frac{4 \pm \sqrt{16 + 64}}{2} = 2 \pm 2\sqrt{5}$

M1

$k < 2 - 2\sqrt{5}$  or  $k > 2 + 2\sqrt{5}$

A1

**Answer:**  $k < 2 - 2\sqrt{5}$  or  $k > 2 + 2\sqrt{5}$ 

6 marks

**26 (6 marks)**

Let  $y = x^2: y^2 - 5y + 4 < 0$ , so  $(y - 1)(y - 4) < 0$

M1 A1

$1 < y < 4$ , so  $1 < x^2 < 4$

A1

$x^2 < 4$  gives  $-2 < x < 2$

M1

$x^2 > 1$  gives  $x < -1$  or  $x > 1$

M1

$-2 < x < -1$  or  $1 < x < 2$

A1

**Answer:**  $-2 < x < -1$  or  $1 < x < 2$ 

6 marks

**27 (7 marks)**

$f(3) = 27 - 27 - 12 + 12 = 0$ , so  $(x - 3)$  is a factor

M1 A1

$x^3 - 3x^2 - 4x + 12 = (x - 3)(x^2 - 4) = (x - 3)(x - 2)(x + 2)$

M1 A1

Roots  $-2, 2, 3$ ; positive cubic

B1

The graph is above the axis between  $-2$  and  $2$  and for  $x > 3$ 

M1

$-2 \leq x \leq 2$  or  $x \geq 3$

A1

**Answer:**  $-2 \leq x \leq 2$  or  $x \geq 3$ 

7 marks

**28 (8 marks)**

Multiply by  $(x - 2)^2$ , which is positive for  $x \neq 2$

M1

$$x(x - 2) > 3(x - 2)^2$$

A1

$$(x - 2)[x - 3(x - 2)] > 0$$

M1

$$(x - 2)(6 - 2x) > 0, \text{ so } (x - 2)(x - 3) < 0$$

M1 A1

Negative parabola reversed:  $2 < x < 3$

A1

$$\text{Check } x = 2.5: \frac{2.5}{0.5} = 5 > 3 \quad \square$$

B1

$$2 < x < 3$$

A1

**Answer:  $2 < x < 3$**

8 marks

**29 (9 marks)**

(a) The inequality is equivalent to  $1 + \sqrt{7} > \sqrt{10}$ ; both sides are positive, so square them

M1

$$(1 + \sqrt{7})^2 = 8 + 2\sqrt{7}, \text{ so we need } 8 + 2\sqrt{7} > 10, \text{ that is } \sqrt{7} > 1$$

M1

$$\text{This is true, so } 3 + \sqrt{7} > 2 + \sqrt{10}$$

A1

**Answer: Shown**

3 marks

(b)  $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

B1

$$(\sqrt{a} - \sqrt{b})^2 \geq 0 \text{ gives } a + b \geq 2\sqrt{ab}$$

M1

$$\text{So } (\sqrt{a} + \sqrt{b})^2 \leq 2(a + b)$$

A1

Both sides are positive, so taking square roots gives the result

A1

**Answer: Shown**

4 marks

(c) Take  $a = 5$  and  $b = 11$ :  $2(a + b) = 32$

M1

$$\sqrt{5} + \sqrt{11} \leq \sqrt{32} = 4\sqrt{2}$$

A1

**Answer: Shown**

2 marks

**30 (9 marks)**

(a)  $(a - b)^2 + (b - c)^2 + (c - a)^2 = 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$

M1 A1 A1

Halving gives the left-hand side

A1

**Answer: Shown**

4 marks

(b) Each square in (a) is  $\geq 0$ , so the right-hand side is  $\geq 0$

M1

Rearranging gives the result

E1

**Answer: Proved**

2 marks

(c) Equality when  $a = b = c$

B1

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 3(ab + bc + ca)$$

M1

$$36 \geq 3(ab + bc + ca), \text{ so } ab + bc + ca \leq 12$$

A1

**Answer: Equality at  $a = b = c$ ;  $ab + bc + ca \leq 12$**

3 marks

**Modulus Equations and Inequalities****1 (2 marks)**

$$|3 - 8| = |-5| = 5$$

M1

$$5 + 2 = 7$$

A1

**Answer: 7**

2 marks

**2 (2 marks)**

$$x - 3 = 5 \text{ or } x - 3 = -5$$

M1

$$x = 8 \text{ or } x = -2$$

A1

**Answer:  $x = 8, x = -2$**

2 marks

**3 (3 marks)**

$$2x + 1 = 7 \text{ or } 2x + 1 = -7$$

M1

$$x = 3$$

A1

$$x = -4$$

A1

**Answer:  $x = 3, x = -4$**

3 marks

**4 (2 marks)**The distance of  $x$  from 0 is less than 4

M1

$$-4 < x < 4$$

A1

**Answer:**  $-4 < x < 4$ 

2 marks

**5 (3 marks)**

$$-3 < x - 2 < 3$$

M1

Add 2:  $-1 < x < 5$

A1 A1

**Answer:**  $-1 < x < 5$ 

3 marks

**6 (3 marks)**

$$x + 1 \geq 2 \text{ or } x + 1 \leq -2$$

M1

$$x \geq 1$$

A1

$$x \leq -3$$

A1

**Answer:**  $x \leq -3 \text{ or } x \geq 1$ 

3 marks

**7 (3 marks)**The graph of  $y = x - 2$  with the part below the  $x$ -axis reflected in the axis

M1

V-shape with vertex at  $(2, 0)$ 

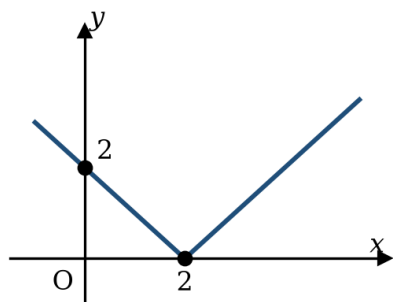
A1

 $y$ -intercept:  $x = 0$  gives  $y = 2$ , so  $(0, 2)$ 

A1

**Answer:** See sketch

3 marks

**8 (3 marks)**

$$-8 < 3x - 4 < 8$$

M1

$$-4 < 3x < 12$$

A1

$$-\frac{4}{3} < x < 4$$

A1

**Answer:**  $-\frac{4}{3} < x < 4$ 

3 marks

**9 (3 marks)**

$$2x - 5 > 3 \text{ or } 2x - 5 < -3$$

M1

$$x > 4$$

A1

$$x < 1$$

A1

**Answer:**  $x < 1 \text{ or } x > 4$ 

3 marks

**10 (4 marks)**

$$x + 3 = 2x - 1 \text{ or } x + 3 = -(2x - 1)$$

M1

First:  $x = 4$

A1

Second:  $x + 3 = -2x + 1$ , so  $3x = -2$

M1

$$x = -\frac{2}{3}$$

A1

**Answer:**  $x = 4, x = -\frac{2}{3}$ 

4 marks

**11 (4 marks)**

$$x^2 - 4 = 5 \text{ or } x^2 - 4 = -5$$

M1

$$x^2 = 9, \text{ so } x = \pm 3$$

A1

$$x^2 = -1 \text{ has no real solutions}$$

M1 A1

$$\text{Answer: } x = \pm 3$$

4 marks

**12 (4 marks)**

$$\text{Case } x \geq 0: x = 2x - 3, \text{ so } x = 3$$

M1 A1

$$\text{Case } x < 0: -x = 2x - 3, \text{ so } x = 1, \text{ which is not less than 0, so rejected}$$

M1

$$x = 3$$

A1

$$\text{Answer: } x = 3$$

4 marks

**13 (5 marks)**

$$\text{The critical values are } x = 0 \text{ and } x = \frac{3}{2}$$

B1

$$x \geq \frac{3}{2}: x = 2x - 3 - 3, \text{ so } x = 6, \text{ which is valid}$$

M1 A1

$$0 \leq x < \frac{3}{2}: x = (3 - 2x) - 3 = -2x, \text{ so } x = 0, \text{ which is valid}$$

M1 A1

$$x < 0: -x = (3 - 2x) - 3 = -2x \text{ gives } x = 0, \text{ which is not less than 0, so there is no new solution}$$

$$\text{Check: } x = 0: 0 = 3 - 3 \text{ and } x = 6: 6 = 9 - 3. \text{ So } x = 0 \text{ or } x = 6$$

$$\text{Answer: } x = 0, x = 6$$

5 marks

**14 (5 marks)**

$$2x + 3 = x + 6 \text{ gives } x = 3$$

M1 A1

$$2x + 3 = -(x + 6) \text{ gives } 3x = -9, \text{ so } x = -3$$

M1 A1

$$\text{Check both: } |9| = 9 \square, |-3| = 3 = -3 + 6 \square$$

B1

$$\text{Answer: } x = 3, x = -3$$

5 marks

**15 (5 marks)**

$$x^2 - 5x = 6: (x - 6)(x + 1) = 0, \text{ so } x = 6 \text{ or } x = -1$$

M1 A1

$$x^2 - 5x = -6: (x - 2)(x - 3) = 0$$

M1 A1

$$x = -1, 2, 3 \text{ or } 6$$

A1

$$\text{Answer: } x = -1, 2, 3, 6$$

5 marks

**16 (5 marks)**

$$\text{Both sides are non-negative, so square: } (3x - 1)^2 < (x + 5)^2$$

M1

$$9x^2 - 6x + 1 < x^2 + 10x + 25$$

A1

$$8x^2 - 16x - 24 < 0, \text{ so } x^2 - 2x - 3 < 0$$

A1

$$(x - 3)(x + 1) < 0$$

M1

$$-1 < x < 3$$

A1

$$\text{Answer: } -1 < x < 3$$

5 marks

**17 (5 marks)**

$$\text{On the right-hand branch } y = 2x - 3: 2x - 3 = x + 1$$

M1

$$x = 4$$

A1

$$\text{On the left-hand branch } y = -(2x - 3): -2x + 3 = x + 1$$

M1

$$3x = 2, \text{ so } x = \frac{2}{3}$$

A1

$$\text{Both lie on the correct branches } (x > \frac{3}{2} \text{ and } x < \frac{3}{2})$$

E1

$$\text{Answer: } x = \frac{2}{3} \text{ and } x = 4$$

5 marks

**18 (6 marks)**

$$-5 < x^2 - 4x < 5$$

M1

$$x^2 - 4x < 5: (x - 5)(x + 1) < 0, \text{ so } -1 < x < 5$$

M1 A1

$$x^2 - 4x > -5: x^2 - 4x + 5 = (x - 2)^2 + 1 > 0 \text{ is true for all } x$$

M1 A1

$$-1 < x < 5$$

A1

$$\text{Answer: } -1 < x < 5$$

6 marks

**19 (4 marks)**

$$(a) -8 \leq m - 500 \leq 8$$

M1

$$492 \leq m \leq 508$$

A1

$$\text{Answer: } 492 \leq m \leq 508$$

2 marks

$$(b) \text{ The midpoint is } a = 16$$

B1

$$\text{The half-width is } b = 4, \text{ so } |x - 16| < 4$$

B1

$$\text{Answer: } |x - 16| < 4$$

2 marks

**20 (6 marks)**

Critical values  $x = -2$  and  $x = 1$  split the line into three regions

B1

$$x \geq 1: (x - 1) + (x + 2) = 2x + 1 = 5, \text{ so } x = 2 \quad \square$$

M1 A1

$$-2 \leq x < 1: (1 - x) + (x + 2) = 3 \neq 5, \text{ no solution}$$

M1 A1

$$x < -2: (1 - x) - (x + 2) = -2x - 1 = 5, \text{ so } x = -3 \quad \square$$

A1

$$\text{Answer: } x = 2 \text{ or } x = -3$$

6 marks

**21 (7 marks)**

$$x^2 - x - 6 = (x - 3)(x + 2): \text{ roots } -2 \text{ and } 3, \text{ y-intercept } -6$$

B1

Reflect the part below the axis: the graph meets the axes at  $(-2, 0)$ ,  $(3, 0)$ ,  $(0, 6)$

M1 A1

$$x^2 - x - 6 = 6: x^2 - x - 12 = 0, \text{ so } x = 4 \text{ or } x = -3$$

M1 A1

$$x^2 - x - 6 = -6: x(x - 1) = 0, \text{ so } x = 0 \text{ or } x = 1$$

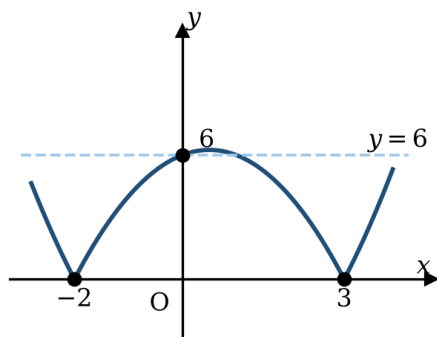
M1

$$x = -3, 0, 1, 4$$

A1

$$\text{Answer: } x = -3, 0, 1, 4$$

7 marks

**22 (6 marks)**

$$(a) x < -4: f(x) = -(x + 4) - (2 - x) = -6$$

M1 A1

$$-4 \leq x < 2: f(x) = (x + 4) - (2 - x) = 2x + 2; x \geq 2: f(x) = (x + 4) - (x - 2) = 6$$

A1

$$\text{Answer: } -6, 2x + 2, 6$$

3 marks

$$(b) x < -4: -6 > 2 \text{ is false}$$

B1

$$-4 \leq x < 2: 2x + 2 > 2, \text{ so } 0 < x < 2; x \geq 2: 6 > 2 \text{ is always true}$$

M1

$$\text{Combining: } x > 0$$

A1

$$\text{Answer: } x > 0$$

3 marks

**23 (8 marks)**

$$(a) |2x - 5| \text{ is least } (0) \text{ when } x = \frac{5}{2}$$

M1

$$\text{Vertex } \left(\frac{5}{2}, -3\right)$$

A1

$$\text{Range: } f(x) \geq -3$$

A1

$$\text{Answer: } \left(\frac{5}{2}, -3\right); f(x) \geq -3$$

3 marks

(b)  $|2x - 5| = 3$ , so  $2x - 5 = \pm 3$

$x = 4$  or  $x = 1$

**Answer:  $x = 1, x = 4$**

M1

A1

2 marks

(c)  $|2x - 5| < 5$

$-5 < 2x - 5 < 5$

$0 < x < 5$

**Answer:  $0 < x < 5$**

M1

A1

A1

3 marks

#### 24 (5 marks)

(a) The student ignored the lower bound: the inequality means  $-5 < 2x - 1 < 5$

$-4 < 2x < 6$

$-2 < x < 3$

**Answer:  $-2 < x < 3$**

E1

M1

A1

3 marks

(b)  $|x|$  is never negative, so  $|x| = -3$  is impossible

There are no solutions

**Answer: No solutions**

E1

B1

2 marks

#### 25 (5 marks)

$x^2 = |x|^2$ , so  $|x|^2 - 5|x| + 6 = 0$

Let  $u = |x|$ :  $(u - 2)(u - 3) = 0$

$|x| = 2$  or  $|x| = 3$

$x = \pm 2$  or  $x = \pm 3$

**Answer:  $x = \pm 2, \pm 3$**

M1

M1 A1

A1

A1

5 marks

#### 26 (7 marks)

(a) The graph never goes below  $y = 0$ : no solutions

(b) The line  $y = 0$  meets the graph at  $x = \pm 2$ : 2 solutions

(c) The line cuts both branches of the W: 4 solutions

(d) The line  $y = k$  passes through the local maximum  $(0, 4)$ : 3 solutions

(e) Only the two outer branches are above  $y = 4$ : 2 solutions

**Answer: (a) 0 (b) 2 (c) 4 (d) 3 (e) 2**

B1

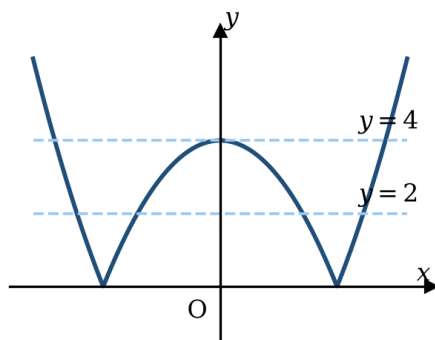
B1

B1 B1

B1

B1 B1

7 marks



#### 27 (8 marks)

Critical values  $x = -1$  and  $x = 3$

$x \geq 3$ :  $(x + 1) + (x - 3) = 2x - 2 < 8$ , so  $x < 5$

So  $3 \leq x < 5$

$-1 \leq x < 3$ :  $(x + 1) + (3 - x) = 4 < 8$ , always true

$x < -1$ :  $-(x + 1) + (3 - x) = 2 - 2x < 8$ , so  $x > -3$

$-3 < x < 5$

**Answer:  $-3 < x < 5$**

B1

M1 A1

A1

M1 A1

M1

A1

8 marks

### 28 (6 marks)

Since  $|x - 3| \geq 0$ , we need  $2x - 1 > 0$ , so  $x > \frac{1}{2}$

B1

$-(2x - 1) < x - 3 < 2x - 1$

M1

$x - 3 < 2x - 1$  gives  $x > -2$

A1

$x - 3 > -2x + 1$  gives  $3x > 4$ , so  $x > \frac{4}{3}$

M1 A1

Combining all conditions:  $x > \frac{4}{3}$

A1

**Answer:**  $x > \frac{4}{3}$

6 marks

### 29 (8 marks)

(a)  $|c|^2 = c^2$  for any real  $c$ , so  $|a + b|^2 = (a + b)^2$

B1

$ab$  is either  $|ab|$  or  $-|ab|$ , so  $ab \leq |ab| = |a||b|$

B1

**Answer: Shown**

2 marks

(b)  $|a + b|^2 = a^2 + 2ab + b^2 \leq a^2 + 2|a||b| + b^2$

M1 A1

$= (|a| + |b|)^2$

A1

Both sides are non-negative, so taking square roots gives the result

E1

**Answer: Proved**

4 marks

(c) Let  $a = x - 1$  and  $b = 4 - x$ , so  $a + b = 3$

M1

$3 = |a + b| \leq |a| + |b| = |x - 1| + |4 - x| = |x - 1| + |x - 4|$

A1

**Answer: Shown**

2 marks

### 30 (9 marks)

(a)  $D = |x| + |x - 4| + |x - 10|$

B1

**Answer:  $D = |x| + |x - 4| + |x - 10|$**

1 mark

(b) For  $4 \leq x < 10$ :  $D = x + (x - 4) + (10 - x)$

M1

$= x + 6$

A1

For  $0 \leq x < 4$ ,  $D = 14 - x$ , which decreases to 10 at  $x = 4$

M1

Least value 10, at  $x = 4$  (the house in the middle)

A1

**Answer:  $D = x + 6$ ; least  $D = 10$  at  $x = 4$**

4 marks

(c)  $x \geq 10$ :  $D = 3x - 14 \leq 16$ , so  $x \leq 10$

M1 A1

$0 \leq x < 10$ :  $D = 14 - x$  or  $x + 6$ , both  $\leq 16$  for  $0 \leq x \leq 10$

M1

$x < 0$ :  $D = 14 - 3x \leq 16$ , so  $x \geq -\frac{2}{3}$ . Overall  $-\frac{2}{3} \leq x \leq 10$

A1

**Answer:  $-\frac{2}{3} \leq x \leq 10$**

4 marks