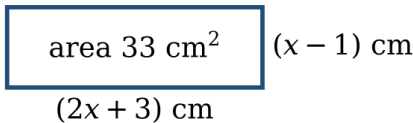


Solving Quadratic Equations

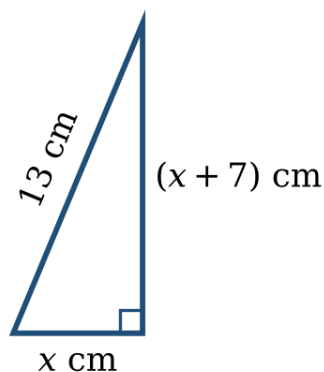
- 1 Solve $x^2 - 7x + 12 = 0$. [2]
- 2 Solve $x^2 - 9 = 0$. [1]
- 3 Solve $3x^2 - 12x = 0$. [2]
- 4 Solve $(2x - 5)(x + 3) = 0$. [2]
- 5 Solve $x^2 + 2x - 15 = 0$. [2]
- 6 Solve $x^2 - 10x + 25 = 0$. [2]
- 7 Solve $2x^2 + 5x - 3 = 0$. [3]
- 8 Solve $6x^2 - 7x - 3 = 0$. [3]
- 9 Solve $x^2 - 6x + 2 = 0$ by completing the square, giving your answers as surds. [3]
- 10 Use the quadratic formula to solve $2x^2 - 4x - 3 = 0$, giving your answers to 3 significant figures. [3]
- 11 Solve $(2x - 3)^2 - (x + 4)^2 = 25$. [4]
- 12 The product of two consecutive positive integers is 156. Find the integers. [4]
- 13 The function f is defined by $f(x) = x^2 - 6x + 4$. Solve the equation $f(x) = f(x + 2)$. [4]
- 14 Solve $x^4 - 10x^2 + 9 = 0$. [5]
- 15 Solve $x - 7\sqrt{x} + 12 = 0$. [5]
- 16 Solve $\frac{3}{x-1} + \frac{2}{x+2} = 1$, giving your answers as exact surds. [6]
- 17 The diagram shows a rectangle of length $(2x + 3)$ cm and width $(x - 1)$ cm. Its area is 33 cm^2 .



area 33 cm^2 $(x - 1) \text{ cm}$
 $(2x + 3) \text{ cm}$

 - (a) Show that $2x^2 + x - 36 = 0$. [2]
 - (b) Hence find the value of x and the perimeter of the rectangle. [4]
- 18 Given that $x = 3$ is a root of the equation $x^2 + kx - 15 = 0$, find the value of the constant k and the other root. [4]

- 19** The diagram shows a right-angled triangle. The two shorter sides have lengths x cm and $(x + 7)$ cm and the hypotenuse is 13 cm. Find the value of x .



[5]

- 20** A ball is thrown upwards. Its height h metres above the ground after t seconds is modelled by $h = 2 + 12t - 5t^2$. Find the times at which the height of the ball is 9 m.

[5]

- 21** Solve $\sqrt{x + 6} = x$.

[5]

- 22** The roots of the equation $x^2 - 9x + k = 0$ are α and 2α .

(a) Find the value of α .

[3]

(b) Hence find the value of k .

[2]

- 23** Solve $(x^2 - 3x)^2 - 2(x^2 - 3x) - 8 = 0$.

[7]

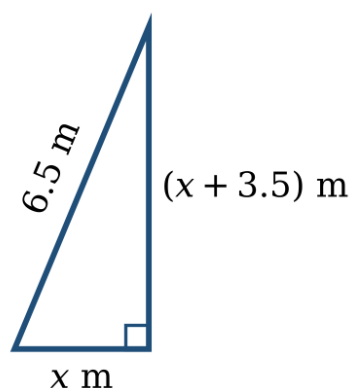
- 24** The sum of the squares of three consecutive integers is 365. Find the integers. (There are two sets.)

[5]

- 25** Solve $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$.

[4]

- 26** A ladder of length 6.5 m leans against a vertical wall on horizontal ground. The foot of the ladder is x m from the wall and the top is $(x + 3.5)$ m above the ground, as shown.



(a) Show that $2x^2 + 7x - 30 = 0$.

[3]

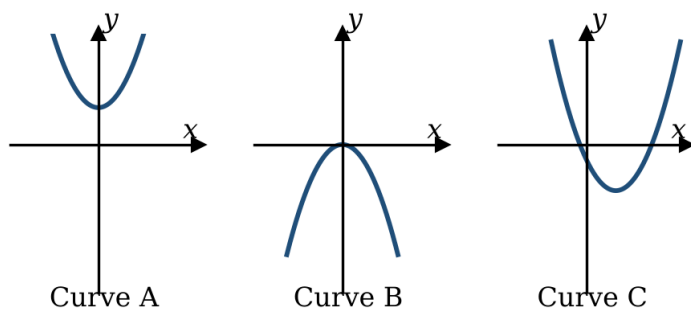
(b) Hence find how high up the wall the ladder reaches.

[4]

- 27** k is a real constant.
- (a) By completing the square, or otherwise, show that the roots of $x^2 - 2kx + k^2 - 1 = 0$ are $x = k + 1$ and $x = k - 1$. [3]
- (b) Explain why the two roots always differ by 2, whatever the value of k . [1]
- (c) Given that the product of the roots is 8, find the possible values of k and the roots in each case. [4]
-
- 28** k is a non-zero constant. The equation $kx^2 - (k^2 + 2)x + 2k = 0$ has roots α and β .
- (a) Solve the equation for x in terms of k . [3]
- (b) Find the values of k for which the equation has a repeated root. [2]
- (c) Given that $k > 0$, show that $\alpha + \beta \geq 2\sqrt{2}$. [3]
-
- 29** Find all the integer values of k for which the equation $x^2 - kx + 24 = 0$ has two integer roots. [6]
-
- 30** Find all the real values of x for which $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$. [8]
-

The Discriminant

- 1** Find the discriminant of $2x^2 - 3x + 5$ and state the number of real roots of $2x^2 - 3x + 5 = 0$. [2]
-
- 2** Show that $x^2 - 6x + 9 = 0$ has exactly one distinct real root. [2]
-
- 3** Find the range of values of k for which the equation $x^2 + 10x + k = 0$ has no real roots. [3]
-
- 4** Find the value of k for which $x^2 + 4x + k = 0$ has equal roots. [2]
-
- 5** Use the discriminant to show that $x^2 + x + 1 = 0$ has no real roots. [2]
-
- 6** Show that the curve $y = x^2 - 2x + 5$ does not cross the x -axis. [2]
-
- 7** The equation $kx^2 - 6x + 3 = 0$ has equal roots. Find the value of k . [3]
-
- 8** Find the set of values of k for which $x^2 + kx + 9 = 0$ has real roots. [4]
-
- 9** Find the set of values of k for which $2x^2 - 4x + k = 0$ has no real roots. [3]
-
- 10** Show that $x^2 - kx - 3 = 0$ has two distinct real roots for every real value of k . [3]
-
- 11** The line $y = 2x + k$ is a tangent to the curve $y = x^2 + 3$. Find the value of k . [4]
-
- 12** The diagram shows the graphs of three quadratic functions of the form $y = ax^2 + bx + c$, labelled A, B and C. For each graph, state whether $b^2 - 4ac$ is positive, zero or negative.



[3]

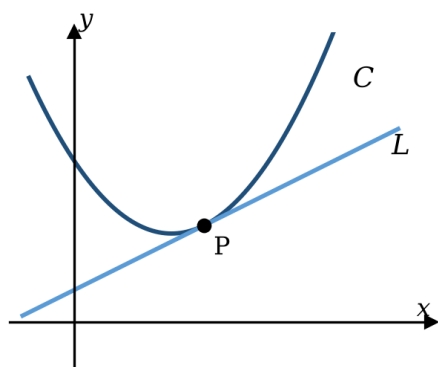
13 Find the set of values of k for which $x^2 - 4x + k > 0$ for all real values of x . [3]

14 Find the values of k for which $kx^2 + 8x + k = 0$ has equal roots, and solve the equation in each case. [5]

15 Find the set of values of k for which the equation $2x^2 + kx + (k + 6) = 0$ has no real roots. [5]

16 Show that the equation $(k - 2)x^2 + 2(k + 1)x + (k + 4) = 0$ has two distinct real roots for every real value of k except $k = 2$. [4]

17 The diagram shows the curve C with equation $y = x^2 - 3x + 5$ and the line L with equation $y = x + 1$.



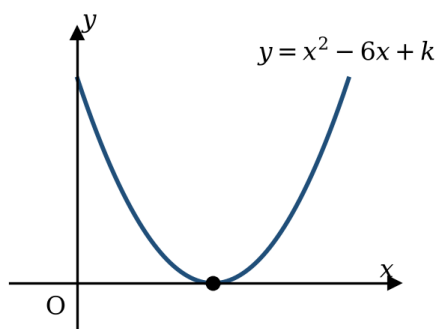
(a) Show that L is a tangent to C . [3]

(b) Find the coordinates of the point P where L touches C . [2]

18 The line $y = kx + 1$ is a tangent to the curve $y = x^2 - 3x + 5$. Find the possible values of k . [5]

19 Show that the equation $kx^2 + (k + 2)x + 1 = 0$ has real roots for every real value of k . [4]

20 The diagram shows the curve $y = x^2 - 6x + k$, which touches the x -axis. Find the value of k and the coordinates of the point where the curve touches the axis.



[4]

21 Find the set of values of k for which the equation $(x - 1)(x - 3) = k$ has real roots. Interpret your answer by describing the graph of $y = (x - 1)(x - 3)$. [5]

22 Find the set of values of k for which the line $y = kx + 2$ does not meet the curve $y = x^2 + x + 3$. [6]

23 A rectangle has perimeter 20 m and area $A \text{ m}^2$. One side has length $x \text{ m}$.

(a) Show that $x^2 - 10x + A = 0$. [2]

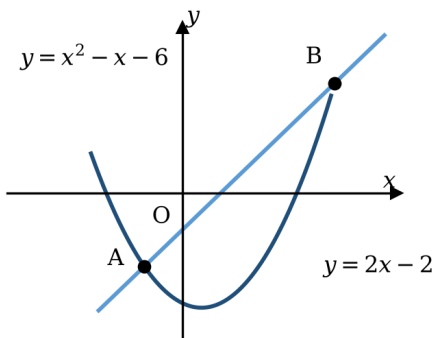
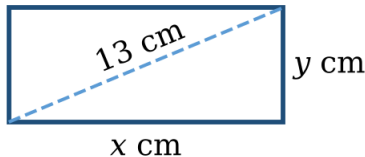
(b) Use the discriminant to show that $A \leq 25$. [3]

(c) Describe the rectangle with the largest area. [2]

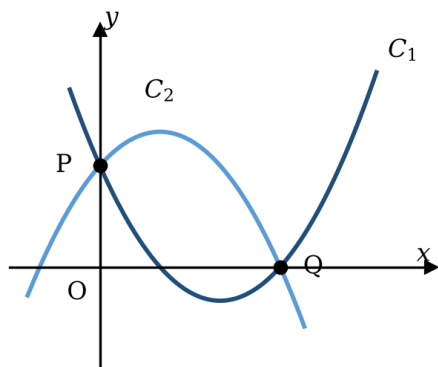
- 24** Consider the two equations $x^2 + kx + 4 = 0$ (1) and $x^2 + 4x + k = 0$ (2).
 (a) Find the set of values of k for which equation (1) has real roots, and the set for which equation (2) has real roots. [4]
 (b) Hence find the values of k for which exactly one of the equations has real roots. [3]
-
- 25** Two tangents to the curve $y = x^2$ pass through the point $(0, -1)$. Find the equations of the tangents. [6]
-
- 26** The curves with equations $y = x^2 - 6x + 11$ and $y = m + 4x - x^2$, where m is a constant, touch at a point P . Find the value of m and the coordinates of P . [6]
-
- 27** Let $f(x) = x^4 + kx^2 + 4$, where k is a real constant. Let $y = x^2$.
 (a) Show that $f(x) = 0$ has four distinct real roots only when $k < -4$. [5]
 (b) Find the value of k for which $f(x) = 0$ has exactly two distinct real roots, and find them. [3]
-
- 28** The curve $y = x^2 + px + q$ touches the x -axis and passes through the point $(1, 4)$, where p and q are constants. Find the possible values of p and q . [6]
-
- 29** Find the set of values taken by $y = \frac{2x - 1}{x^2 + 2}$ as x varies over the real numbers, by considering the discriminant of a suitable quadratic equation in x . [7]
-
- 30** a , b and c are real constants, and the equation $(x - a)^2 + (x - b)^2 = 2c$ has real roots.
 (a) Show that $c \geq \frac{(a - b)^2}{4}$. [6]
 (b) Hence find, in terms of a and b , the least possible value of $(x - a)^2 + (x - b)^2$ for real x . [2]

Simultaneous Equations

- 1** Solve the simultaneous equations $x + y = 7$ and $x - y = 3$. [2]
-
- 2** Solve $2x + y = 11$ and $x - y = 1$. [2]
-
- 3** Solve $y = 2x + 1$ and $3x + y = 11$. [2]
-
- 4** Solve $3x + 2y = 12$ and $x + y = 5$. [2]
-
- 5** Solve $5x - 2y = 4$ and $3x + y = 9$. [3]
-
- 6** Solve $4x + 3y = 17$ and $2x - y = 1$. [3]
-
- 7** Solve the simultaneous equations $y = x + 2$ and $y = x^2$. [4]
-
- 8** Solve $x + y = 5$ and $x^2 + y^2 = 17$. [5]
-
- 9** Solve $2x + y = 7$ and $xy = 6$. [5]
-
- 10** Find, as exact surds, the coordinates of the points where the line $y = x - 1$ meets the curve $y = x^2 - 3x + 1$. [4]
-
- 11** Three adult tickets and two child tickets cost £28. Two adult tickets and five child tickets cost £37. Find the cost of an adult ticket and of a child ticket. [5]
-
- 12** Solve $x^2 + y^2 = 25$ and $y = x + 1$. [5]

- 13** Solve $\frac{x}{2} + \frac{y}{3} = 4$ and $5x - 3y = 2$. [4]
-
- 14** Solve the simultaneous equations $x + y = 1$ and $x^2 - xy + y^2 = 7$. [6]
-
- 15** The diagram shows the curve $y = x^2 - x - 6$ and the line $y = 2x - 2$, which meet at the points A and B.
- 
- (a) Find the coordinates of A and B. [4]
 (b) Find the exact length of AB. [3]
-
- 16** Solve the simultaneous equations
 $x + 2y + z = 9$
 $2x - y + 3z = 4$
 $3x + y - 2z = 7$. [6]
-
- 17** A rectangle has sides x cm and y cm, perimeter 34 cm and diagonal 13 cm. Find the dimensions of the rectangle.
- 
- [6]
-
- 18** Solve $x - y = 3$ and $x^2 - y^2 = 15$. [4]
-
- 19** Solve $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$ and $x + y = 5$. [5]
-
- 20** The line $y = 2x + 3$ meets the curve $y = x^2 + kx + 7$ at the point where $x = 1$.
 (a) Find the value of the constant k . [2]
 (b) Find the coordinates of the other point of intersection. [3]
-
- 21** The line $y = x + 3$ meets the curve $y = x^2 - 2x - 2$ at the points A and B.
 (a) Find the coordinates of the midpoint of AB. [4]
 (b) Find the exact length of AB. [4]
-
- 22** Show that the line $y = 3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$ and find the coordinates of the point of contact. [5]
-

- 23** The diagram shows the curves $C_1: y = x^2 - 4x + 3$ and $C_2: y = -x^2 + 2x + 3$, which intersect at the points P and Q.

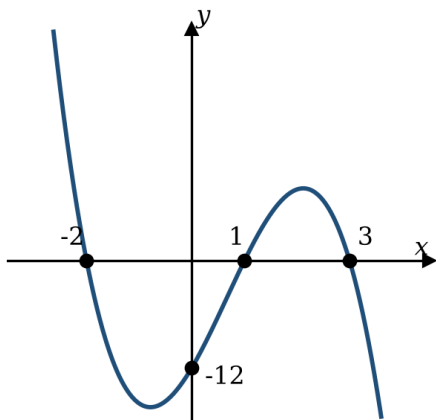


- (a) Find the coordinates of P and Q. [4]
 (b) Find the equation of the perpendicular bisector of PQ. [4]
- 24** Solve the simultaneous equations $x^2 + y^2 = 34$ and $xy = 15$. [6]
- 25** The line $y = kx$ meets the curve $y = x^2 - 4x + 9$ at exactly one point. Find the possible values of k and the coordinates of the point of contact in each case. [6]
- 26** Consider the simultaneous equations $x + 2y = 5$ and $3x + ky = 7$, where k is a constant.
 (a) Find the value of k for which the equations have no solution. [3]
 (b) Explain why the equations have a unique solution for all other values of k , and give a geometrical interpretation of the case $k = 6$. [3]
- 27** Solve the simultaneous equations $x^2 - xy - 2y^2 = 0$ and $x^2 + y^2 = 20$. [8]
- 28** a and b are real numbers such that $a + b = 6$ and $a^3 + b^3 = 72$.
 (a) Show that $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$, and hence find the value of ab . [4]
 (b) Hence find the possible values of a and b . [3]
- 29** The line $y = x + k$, where $k > 0$, meets the circle $x^2 + y^2 = 26$ at the points A and B.
 (a) Show that the x -coordinates of A and B satisfy $2x^2 + 2kx + k^2 - 26 = 0$. [2]
 (b) Given that the length of AB is $4\sqrt{2}$, find the value of k and the coordinates of A and B. [7]
- 30** Solve the simultaneous equations $\sqrt{x} + \sqrt{y} = 5$ and $x - y = 5$, where x and y are positive. [6]

Polynomial Equations

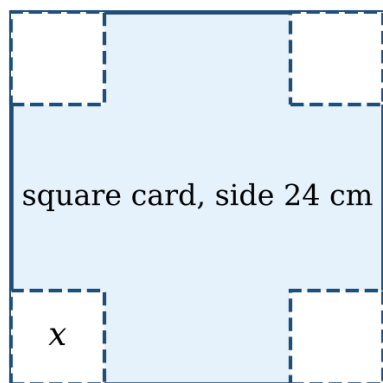
- 1** Solve $x(x - 2)(x + 5) = 0$. [2]
- 2** Solve $x^3 - 4x = 0$. [3]
- 3** Solve $x^3 - 5x^2 + 6x = 0$. [3]
- 4** Show that $x = 2$ is a root of $x^3 - 3x - 2 = 0$. [2]
- 5** Find the real solution of $x^3 + 8 = 0$. [2]
- 6** Solve $(x^2 - 4)(x^2 - 9) = 0$. [2]
- 7** Solve $x^3 - 7x + 6 = 0$. [4]

- 8** Solve $2x^3 - x^2 - 13x - 6 = 0$. [4]
- 9** Solve $x^4 - 5x^2 + 4 = 0$. [4]
- 10** Given that $x = 1$ is a root of $x^3 - 7x^2 + kx - 8 = 0$, find the value of k and the other roots. [5]
- 11** Find the coordinates of the points where the curve $y = x^3 - x^2 - 6x$ crosses the x -axis. [4]
- 12** Solve $(x - 2)(x^2 + 4x - 3) = 2(x - 2)$. [4]
- 13** Show that $(x + 1)$ is a factor of $x^3 + 2x^2 - 5x - 6$ and hence solve $x^3 + 2x^2 - 5x - 6 = 0$. [4]
- 14** Solve $x^4 + x^3 - 7x^2 - x + 6 = 0$. [6]
- 15** Solve $x^3 - 3x^2 + 4 = 0$, identifying any repeated root. [5]
- 16** The diagram shows a cubic curve which crosses the x -axis at $x = -2$, $x = 1$ and $x = 3$ and the y -axis at $(0, -12)$. Find the equation of the curve in the form $y = ax^3 + bx^2 + cx + d$.



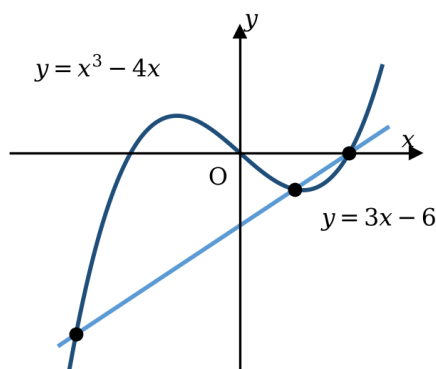
- [5]
- 17** Solve the equation $x^3 - 2x^2 - (x + 1)(x - 2)(x - 3) = 9$. [5]
- 18** Solve $x^3 - 3x^2 - 4x + 12 = 0$ by factorising in pairs. [4]
- 19** Solve $x^3 - 5x^2 + 5x - 1 = 0$, giving exact answers. [6]
- 20** Find the real solutions of $x^6 - 7x^3 - 8 = 0$. [5]

- 21** A square piece of card has side 24 cm. A square of side x cm is cut from each corner and the sides are folded up to make an open box, as shown.



- (a) Show that the volume of the box is $V = 4x^3 - 96x^2 + 576x$ cm³. [3]
 (b) Show that $V = 1024$ when $x = 4$, and solve $V = 1024$ completely. [5]
 (c) Explain why $x = 16$ is not a valid solution, and what the repeated root at $x = 4$ tells you about the volume. [2]

- 22** The diagram shows the curve $y = x^3 - 4x$ and the line $y = 3x - 6$. Find the coordinates of the three points where the line meets the curve.



[6]

- 23** This question is about the real roots of simple polynomial equations.

- (a) Show that the equation $x^3 - 1 = 0$ has only one real root. [3]
 (b) Hence solve $x^6 - 1 = 0$ for real x . [3]

- 24** $f(x) = x^4 - 8x^2 + 16$.

- (a) Show that $f(x) = (x^2 - 4)^2$, and hence solve $f(x) = 0$, stating which roots are repeated. [3]
 (b) Solve $f(x) = 9$, giving exact answers. [4]

- 25** The equation $x^3 - 6x^2 + qx - 6 = 0$ has three real roots which are in arithmetic progression.

- (a) Show that one of the roots is 2. [3]
 (b) Find the value of q and the other two roots. [4]

- 26** Consider the equation $(x - 1)(x + 1)(x + 3)(x + 5) = 9$.

- (a) Show that, with $m = x^2 + 4x$, the equation can be written $(m - 5)(m + 3) = 9$. [3]
 (b) Hence solve the equation, giving exact answers. [4]

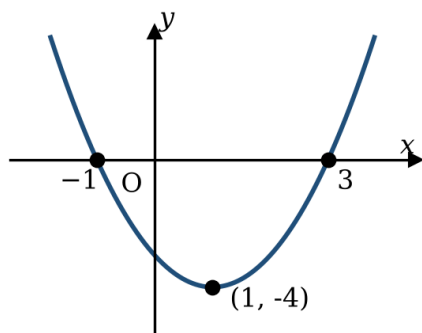
- 27** $f(x) = x^3 - 3x + k$, where k is a constant.

- (a) Find the x -coordinates of the stationary points of the curve $y = f(x)$, and the corresponding y -coordinates in terms of k . [4]
 (b) Hence show that $f(x) = 0$ has three distinct real roots if and only if $-2 < k < 2$. [4]

- 28** This question is about a quartic that is a perfect square.
- (a) Show that $(x^2 + 2x - 1)^2 \equiv x^4 + 4x^3 + 2x^2 - 4x + 1$. [3]
- (b) Hence solve $x^4 + 4x^3 + 2x^2 - 4x + 1 = 0$, giving exact answers. [3]
- (c) Explain why each root in (b) is a repeated root and what this means for the graph of $y = x^4 + 4x^3 + 2x^2 - 4x + 1$. [2]
-
- 29** Consider the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.
- (a) Show that, with $t = x + \frac{1}{x}$, the equation can be written $6t^2 - 35t + 50 = 0$. [4]
- (b) Hence solve the original equation. [4]
-
- 30** The curve $y = x^3 + ax^2 + bx - 4$ touches the x-axis at the point where $x = 2$.
- (a) Find the values of a and b . [5]
- (b) Factorise $x^3 + ax^2 + bx - 4$ fully, and sketch the curve showing all intercepts. [4]

Linear and Quadratic Inequalities

- 1** Solve $3x - 5 > 10$. [2]
- 2** Solve $4 - 2x \leq 10$. [2]
- 3** Solve $x^2 - 9 < 0$. [2]
- 4** Solve $x^2 - 5x + 6 \geq 0$. [3]
- 5** Solve $2(x + 3) < x + 10$. [2]
- 6** Solve $-3 < 2x + 1 \leq 9$. [3]
- 7** Solve $x^2 + 2x - 15 < 0$. [3]
- 8** Solve $2x^2 - x - 6 > 0$. [4]
- 9** Find the set of values of x for which both $2x + 1 > 9$ and $3x - 4 < 17$. [3]
- 10** Solve $x^2 \geq 4x$. [4]
- 11** Solve $x(x - 3) < 10$. [4]
- 12** The diagram shows the curve $y = x^2 - 2x - 3$. Use the diagram to write down the solution of (a) $x^2 - 2x - 3 < 0$ (b) $x^2 - 2x - 3 \geq 0$.



- (a) $x^2 - 2x - 3 < 0$ [1]
- (b) $x^2 - 2x - 3 \geq 0$ [2]

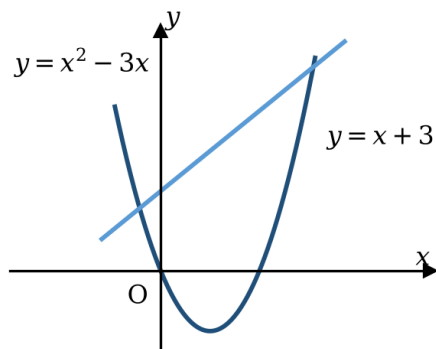
13 Find the number of integers n for which $n^2 - 10n + 13 < 0$. [4]

14 Find the set of values of x for which both $x^2 - x - 6 < 0$ and $2x + 1 > 3$. [5]

15 Find the set of values of k for which $x^2 - 2kx + k + 6 > 0$ for all real values of x . [5]

16 Solve $x(x - 2)(x + 3) > 0$. [5]

17 The diagram shows the curve $y = x^2 - 3x$ and the line $y = x + 3$. Find the exact set of values of x for which the line lies above the curve.



[5]

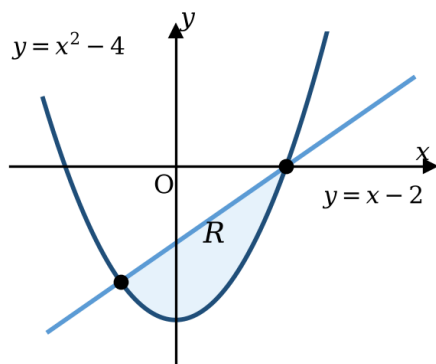
18 A rectangular garden has width x metres and length $(x + 5)$ metres. Its area is less than 84 m^2 and its perimeter is at least 26 m . Find the set of possible values of x . [6]

19 Solve the inequality $\frac{3x}{x+2} \leq 2$. [5]

20 Solve $x^2 - 4x + 1 \leq 0$, giving your answer in exact form. [4]

21 Find the set of values of x for which both $2x^2 - 7x + 3 \leq 0$ and $x^2 - x - 2 > 0$. [6]

22 The region R is bounded by the curve $y = x^2 - 4$ and the line $y = x - 2$, as shown shaded in the diagram. It is defined by the inequalities $y \geq x^2 - 4$ and $y \leq x - 2$.



(a) Find the coordinates of the points where the line meets the curve. [3]

(b) Write down all the points in R , including its boundary, whose coordinates are both integers. [4]

23 A student solves $x^2 > 4x$ as follows. 'Divide both sides by x to get $x > 4$.'

(a) Explain why this method is wrong. [2]

(b) Solve $x^2 > 4x$ correctly, and give one value of x that satisfies it but not the student's answer. [3]

- 24** This question is about proving an inequality.
- (a) Show that for all real a and b , $a^2 + b^2 \geq 2ab$. [2]
- (b) Hence show that $x^2 + \frac{4}{x^2} \geq 4$ for all real $x \neq 0$, and state the values of x for which equality holds. [4]
-
- 25** The line $y = kx - 5$ meets the curve $y = x^2 + 2x$ at two distinct points. Find the set of possible values of k . [6]
-
- 26** Solve $x^4 - 5x^2 + 4 < 0$. [6]
-
- 27** Solve $x^3 - 3x^2 - 4x + 12 \geq 0$. [7]
-
- 28** Solve the inequality $\frac{x}{x-2} > 3$. [8]
-
- 29** You may not use a calculator in this question.
- (a) Show that $3 + \sqrt{7} > 2 + \sqrt{10}$. [3]
- (b) a and b are positive real numbers. Show that $\sqrt{a} + \sqrt{b} \leq \sqrt{2(a+b)}$. [4]
- (c) Hence show that $\sqrt{5} + \sqrt{11} \leq 4\sqrt{2}$. [2]
-
- 30** a , b and c are real numbers.
- (a) Show that $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$. [4]
- (b) Hence prove that $a^2 + b^2 + c^2 \geq ab + bc + ca$. [2]
- (c) State when equality holds, and use the result to show that if $a + b + c = 6$ then $ab + bc + ca \leq 12$. [3]
-

Modulus Equations and Inequalities

- 1** Evaluate $|3 - 8| + |-2|$. [2]
-
- 2** Solve $|x - 3| = 5$. [2]
-
- 3** Solve $|2x + 1| = 7$. [3]
-
- 4** Solve $|x| < 4$. [2]
-
- 5** Solve $|x - 2| < 3$. [3]
-
- 6** Solve $|x + 1| \geq 2$. [3]
-
- 7** Sketch the graph of $y = |x - 2|$, showing the coordinates of any points where it meets the axes. [3]
-
- 8** Solve $|3x - 4| < 8$. [3]
-
- 9** Solve $|2x - 5| > 3$. [3]
-
- 10** Solve $|x + 3| = |2x - 1|$. [4]
-
- 11** Solve $|x^2 - 4| = 5$. [4]
-
- 12** Solve $|x| = 2x - 3$. [4]
-

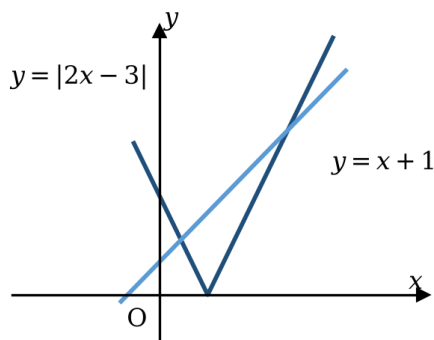
13 Solve $|x| = |2x - 3| - 3$. [5]

14 Solve $|2x + 3| = x + 6$. [5]

15 Solve $|x^2 - 5x| = 6$. [5]

16 Solve $|3x - 1| < |x + 5|$. [5]

- 17** The diagram shows the graphs of $y = |2x - 3|$ and $y = x + 1$. Find the x-coordinates of the two points of intersection.



[5]

18 Solve $|x^2 - 4x| < 5$. [6]

- 19** This question is about describing intervals with the modulus.

- (a) A bag of sugar has mass m grams and m satisfies $|m - 500| \leq 8$. Find the least and greatest possible values of m . [2]
 (b) Write the inequality $12 < x < 20$ in the form $|x - a| < b$. [2]

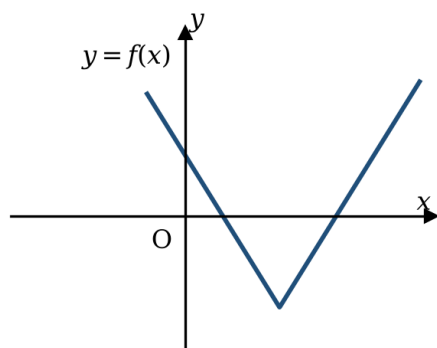
20 Solve $|x - 1| + |x + 2| = 5$. [6]

- 21** Sketch the graph of $y = |x^2 - x - 6|$, showing the coordinates of the points where it meets the axes, and hence solve $|x^2 - x - 6| = 6$. [7]

- 22** Consider the function $f(x) = |x + 4| - |x - 2|$.

- (a) Write $f(x)$ without modulus signs in each of the three cases $x < -4$, $-4 \leq x < 2$ and $x \geq 2$. [3]
 (b) Hence solve $f(x) > 2$. [3]

- 23** The function f is defined by $f(x) = |2x - 5| - 3$ for $x \in \mathbb{R}$. The diagram shows the graph of $y = f(x)$.



- (a) Find the coordinates of the vertex of the graph and state the range of f . [3]
 (b) Solve $f(x) = 0$. [2]
 (c) Solve $f(x) < 2$. [3]

- 24** A student's work on two questions contains errors.
 (i) 'Solve $|2x - 1| < 5$. $2x - 1 < 5$, so $x < 3$.'
 (ii) 'Solve $|x| = -3$. $x = -3$ or $x = 3$.'
 (a) Explain the error in (i) and give the correct solution. [3]
 (b) Explain the error in (ii) and state the solution. [2]
-
- 25** Solve $x^2 - 5|x| + 6 = 0$. [5]
-
- 26** By sketching the graph of $y = |x^2 - 4|$ and horizontal lines $y = k$, state, with reasons, the number of real solutions of $|x^2 - 4| = k$ when (a) $k < 0$ (b) $k = 0$ (c) $0 < k < 4$ (d) $k = 4$ (e) $k > 4$. [7]
-
- 27** Solve $|x + 1| + |x - 3| < 8$. [8]
-
- 28** Solve $|x - 3| < 2x - 1$. [6]
-
- 29** a and b are real numbers.
 (a) Show that $|a + b|^2 = a^2 + 2ab + b^2$, and that $ab \leq |a||b|$. [2]
 (b) Hence prove the triangle inequality $|a + b| \leq |a| + |b|$. [4]
 (c) By choosing a and b suitably in terms of x, show that $|x - 1| + |x - 4| \geq 3$ for all real x. [2]
-
- 30** Three houses lie on a straight road at positions 0 km, 4 km and 10 km measured from the first house. A depot is to be built at position x km (x may be negative). Let D be the sum of the distances from the depot to the three houses.
 (a) Write down an expression for D in terms of x using the modulus. [1]
 (b) Find D, without the modulus, for $4 \leq x < 10$, and hence find the least possible value of D and where it occurs. [4]
 (c) The total distance must not exceed 16 km. Find the set of possible values of x. [4]
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