

Straight Lines

1 (2 marks)

$$m = \frac{11 - 3}{6 - 2} = \frac{8}{4} = 2$$

M1 A1

Answer: 2

2 marks

2 (2 marks)

$$\text{Midpoint} = \left(\frac{-2 + 6}{2}, \frac{5 + 1}{2} \right)$$

M1

$$= (2, 3)$$

A1

Answer: (2, 3)

2 marks

3 (2 marks)

$$d^2 = (7 - 1)^2 + (10 - 2)^2 = 36 + 64 = 100$$

M1

$$d = 10$$

A1

Answer: 10

2 marks

4 (2 marks)

$$m = 3, \text{ y-intercept } -5$$

B1

$$y = 2x + 3, \text{ so } m = 2, \text{ y-intercept } 3$$

B1

Answer: (a) 3, -5 (b) 2, 3

2 marks

5 (2 marks)

$$y - 5 = 2(x - 1)$$

M1

$$y = 2x + 3$$

A1

Answer: $y = 2x + 3$

2 marks

6 (2 marks)

$$x = 3: y = 7, \text{ so } (3, 7) \text{ lies on the line}$$

B1

$$x = -1: y = -1 \neq 4, \text{ so } (-1, 4) \text{ does not}$$

B1

Answer: (a) yes (b) no

2 marks

7 (4 marks)

$$m = \frac{-4 - 4}{3 - (-1)} = -2$$

M1 A1

$$y - 4 = -2(x + 1)$$

M1

$$2x + y - 2 = 0$$

A1

Answer: $2x + y - 2 = 0$

4 marks

8 (4 marks)

$$4y = -3x + 12, \text{ so } m = -\frac{3}{4}$$

M1 A1

$$x = 0: y = 3, \text{ so } (0, 3); y = 0: x = 4, \text{ so } (4, 0)$$

B1 B1

Answer: $m = -\frac{3}{4}$; (0, 3) and (4, 0)

4 marks

9 (3 marks)

$$x + 2x - 1 = 8, \text{ so } 3x = 9$$

M1

$$x = 3, y = 5$$

M1 A1

Answer: (3, 5)

3 marks

10 (3 marks)

$$\frac{-3 + x_B}{2} = 1 \text{ and } \frac{2 + y_B}{2} = 5$$

M1

$$x_B = 5, y_B = 8: B(5, 8)$$

A1 A1

Answer: B(5, 8)

3 marks

11 (5 marks)

$$AB^2 = 4^2 + 2^2 = 20, BC^2 = 2^2 + 4^2 = 20$$

M1 A1

So $AB = BC$ (isosceles)

E1

$$AC^2 = 2^2 + 6^2 = 40 = AB^2 + BC^2, \text{ so the angle at B is } 90^\circ \text{ (Pythagoras)}$$

M1 A1

Answer: $AB = BC$ and $AB^2 + BC^2 = AC^2$

5 marks

12 (3 marks)

$$k = 2(4) + 3(-1) = 5$$

M1 A1

$$3y = -2x + 5, \text{ so } m = -\frac{2}{3}$$

B1

Answer: $k = 5$; $m = -\frac{2}{3}$

3 marks

13 (3 marks)

$$m = \frac{-3 - 0}{0 - 4} = \frac{3}{4}$$

M1

$$y = \frac{3}{4}x - 3$$

A1

$$3x - 4y - 12 = 0$$

A1

Answer: $3x - 4y - 12 = 0$

3 marks

14 (6 marks)

$$(a) y = 0: x = 6; x = 0: y = 4$$

M1

$$A(6, 0), B(0, 4)$$

A1

Answer: $A(6, 0), B(0, 4)$

2 marks

$$(b) \text{Area} = \frac{1}{2} \times 6 \times 4$$

M1

$$= 12$$

A1

Answer: 12

2 marks

$$(c) AB^2 = 6^2 + 4^2 = 52$$

M1

$$AB = 2\sqrt{13}$$

A1

Answer: $2\sqrt{13}$

2 marks

15 (4 marks)

$$(a) \text{Gradient } AB = \frac{1+5}{2+1} = 2$$

M1

$$\text{Gradient } BC = \frac{7-1}{5-2} = 2, \text{ the same, so collinear}$$

A1

Answer: shown

2 marks

$$(b) \text{Line: } y - 1 = 2(x - 2), \text{ so } y = 2x - 3$$

M1

$$13 = 2k - 3, \text{ so } k = 8$$

A1

Answer: $k = 8$

2 marks

16 (7 marks)

$$(a) x + 1 = -2x + 7, \text{ so } x = 2, y = 3: P(2, 3)$$

M1 A1

$$A: y = 0 \text{ gives } x = -1; B: -2x + 7 = 0 \text{ gives } x = \frac{7}{2}$$

B1 B1

Answer: $P(2, 3), A(-1, 0), B(\frac{7}{2}, 0)$

4 marks

$$(b) AB = \frac{7}{2} - (-1) = \frac{9}{2}$$

M1

$$\text{Area} = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$$

M1 A1

Answer: $\frac{27}{4}$

3 marks

17 (5 marks)

(a) Gradient = $\frac{5 - (-7)}{2 - (-2)} = 3$

M1

$y - 5 = 3(x - 2)$, so $y = 3x - 1$

A1

Answer: $y = 3x - 1$

2 marks

(b) At $x = -50$ the line has $y = -151$, and $-152 < -151$, so C is below l

M1 A1

At $x = 40$ the line has $y = 119$, and $120 > 119$, so D is above l

A1

Answer: C below l, D above l

3 marks

18 (7 marks)

(a) Gradient = $\frac{22 - 11.5}{12 - 5} = 1.5$

M1 A1

$C - 11.5 = 1.5(d - 5)$, so $C = 1.5d + 4$

M1 A1

Answer: $C = 1.5d + 4$

4 marks

(b) The gradient is the cost per km, £1.50

B1

The constant is the fixed fee of £4

B1

Answer: £1.50 per km; fixed fee £4

2 marks

(c) $C = 1.5(20) + 4 = 34$, so £34.00

B1

Answer: £34.00

1 mark

19 (4 marks)

(a) AC: $(\frac{1+8}{2}, \frac{2+9}{2}) = (\frac{9}{2}, \frac{11}{2})$

M1 A1

BD: $(\frac{5+4}{2}, \frac{4+7}{2}) = (\frac{9}{2}, \frac{11}{2})$

A1

Answer: both $(\frac{9}{2}, \frac{11}{2})$

3 marks

(b) The diagonals bisect each other, so ABCD is a parallelogram

E1

Answer: parallelogram

1 mark

20 (5 marks)

(a) $P = A + \frac{1}{3}(B - A)$

M1

$= (1 + 2, 2 + 3) = (3, 5)$

M1 A1

Answer: (3, 5)

3 marks

(b) Gradient AB = $\frac{9}{6} = \frac{3}{2}$; gradient AP = $\frac{3}{2}$

M1 A1

Answer: verified

2 marks

21 (8 marks)

(a) $M = (\frac{5+3}{2}, \frac{1+7}{2})$

M1

$= (4, 4)$

A1

Answer: (4, 4)

2 marks

(b) Gradient AM = $\frac{4-3}{4+1} = \frac{1}{5}$

M1 A1

$y - 3 = \frac{1}{5}(x + 1)$, so $x - 5y + 16 = 0$

M1 A1

Answer: $x - 5y + 16 = 0$

4 marks

(c) $\frac{7}{3} - 5(\frac{11}{3}) + 16 = \frac{7 - 55 + 48}{3}$

M1

$= 0$, so the centroid lies on AM

A1

Answer: shown

2 marks

22 (12 marks)

(a) $x = 0$ gives $y = -1$, so $A(0, -1)$

B1

$x^2 - 1 = \frac{3x}{2}$, so $2x^2 - 3x - 2 = 0$

M1

$(2x + 1)(x - 2) = 0$, so $x = 2$ or $x = -\frac{1}{2}$

M1 A1

$P(2, 3)$ and $Q(-\frac{1}{2}, -\frac{3}{4})$

A1

Answer: $A(0, -1), P(2, 3), Q(-\frac{1}{2}, -\frac{3}{4})$

5 marks

(b) Gradient $AP = \frac{3 - (-1)}{2 - 0} = 2$

M1

Gradient $AQ = \frac{1}{4} \div (-\frac{1}{2}) = -\frac{1}{2}$

M1

$2 \times (-\frac{1}{2}) = -1$, so AP is perpendicular to AQ

E1

Answer: shown

3 marks

(c) $AP = \sqrt{4 + 16} = 2\sqrt{5}$

M1

$AQ^2 = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$, so $AQ = \frac{1}{4}\sqrt{5}$

M1

Area = $\frac{1}{2} \times 2\sqrt{5} \times \frac{1}{4}\sqrt{5} = \frac{5}{4}$

M1 A1

Answer: $\frac{5}{4}$

4 marks

23 (8 marks)

(a) $3k + 2(k + 1) = 12$

M1

$5k = 10$, so $k = 2$

A1

Answer: $k = 2$

2 marks

(b) $2x + 3y = 12$, so gradient = $-\frac{2}{3}$

M1 A1

Axes: $(6, 0)$ and $(0, 4)$

B1 B1

Answer: $-\frac{2}{3}; (6, 0), (0, 4)$

4 marks

(c) $\frac{1}{2} \times 6 \times 4 = 12$

M1 A1

Answer: 12

2 marks

24 (10 marks)

(a) AB and DC are equal vectors: $C - D = B - A = (6, 2)$

M1

$D = (10 - 6, 8 - 2) = (4, 6)$

A1

Answer: $D(4, 6)$

2 marks

(b) Gradient $BD = \frac{6 - 3}{4 - 7} = -1$, so $y - 3 = -(x - 7)$, $x + y = 10$

M1 A1

Midpoint of $AC = (\frac{11}{2}, \frac{9}{2})$: $\frac{11}{2} + \frac{9}{2} = 10$

M1 A1

Answer: $x + y = 10$

4 marks

(c) Area = $2 \times$ area of triangle ABC

M1

Triangle ABC : $\frac{1}{2}|(6)(7) - (2)(9)| = \frac{1}{2}(24) = 12$

M1 A1

Area = 24

A1

Answer: 24

4 marks

25 (9 marks)

(a) Gradient = $\frac{-2 - 7}{5 + 1} = -\frac{3}{2}$

M1 A1

$y - 7 = -\frac{3}{2}(x + 1)$, so $3x + 2y = 11$

M1 A1

Answer: $3x + 2y = 11$

4 marks

(b) $\tan \theta = \frac{3}{2}$

M1

$$\theta = 56.3^\circ$$

Answer: 56.3°

A1

2 marks

(c) Intercepts $(\frac{11}{3}, 0)$ and $(0, \frac{11}{2})$

M1 A1

$$\text{Area} = \frac{1}{2} \times \frac{11}{3} \times \frac{11}{2} = \frac{121}{12}$$

A1

Answer: $\frac{121}{12}$

3 marks

26 (11 marks)

(a) Gradient AB = $\frac{2}{8} = \frac{1}{4}$, gradient DC = $\frac{1}{4}$: parallel

M1 A1

Gradient BC = $-\frac{4}{8} = -\frac{1}{2}$, gradient AD = $-\frac{5}{-4}$: not parallel

M1

ABCD is a trapezium

A1

Answer: trapezium

4 marks

(b) AB = $\sqrt{68} = 2\sqrt{17}$, DC = $\sqrt{16 + 1} = \sqrt{17}$

M1 A1

Answer: shown

2 marks

(c) Split into triangles ABC and ACD with common diagonal AC along x = 0 (length 6)

M1

$$\text{Area ABC} = \frac{1}{2} \times 6 \times 8 = 24, \text{ area ACD} = \frac{1}{2} \times 6 \times 4 = 12$$

M1 A1 A1

Total 36

A1

Answer: 36

5 marks

27 (9 marks)

(a) Expand: $kx + x + 2y - ky = 3k + 4$

M1

Collect terms in k: $k(x - y - 3) + (x + 2y - 4) = 0$

A1

Answer: shown

2 marks

(b) The equation holds for all k when $x - y - 3 = 0$ and $x + 2y - 4 = 0$

M1 E1

Subtracting: $3y = 1$, so $y = \frac{1}{3}$

M1

$$x = 3 + \frac{1}{3} = \frac{10}{3}: \text{fixed point } (\frac{10}{3}, \frac{1}{3})$$

A1

Answer: $(\frac{10}{3}, \frac{1}{3})$

4 marks

(c) Coefficient of x is zero: $k + 1 = 0$, so $k = -1$

M1 A1

$$3y = 1, y = \frac{1}{3}$$

A1

Answer: $k = -1$; $y = \frac{1}{3}$

3 marks

28 (10 marks)

(a) Gradient PA = gradient PQ: $\frac{1}{2-p} = \frac{-q}{p}$

M1

$$p = q(p - 2), \text{ i.e. } pq = 2q + p$$

M1

$$\text{Divide by } pq: 1 = \frac{2}{p} + \frac{1}{q}$$

A1

Answer: shown

3 marks

$$(b) q = \frac{p}{p-2}$$

M1

$$S = \frac{1}{2}pq = \frac{p^2}{2(p-2)}$$

A1

Answer: shown

2 marks

$$(c) \frac{dS}{dp} = \frac{2p(p-2) - p^2}{2(p-2)^2} = \frac{p(p-4)}{2(p-2)^2}$$

M1 A1

$$\frac{dS}{dp} = 0 \text{ gives } p = 4$$

A1

$$S = \frac{16}{4} = 4 \text{ (a minimum, since } \frac{dS}{dp} \text{ changes from negative to positive)}$$

M1 A1

Answer: $p = 4$, minimum area 4

5 marks

29 (7 marks)

$$(a) M = \left(\frac{2a+0}{2}, \frac{0+2b}{2} \right) = (a, b)$$

M1 A1

Answer: (a, b)

2 marks

$$(b) OM^2 = a^2 + b^2$$

B1

$$AM^2 = (2a - a)^2 + (0 - b)^2 = a^2 + b^2$$

M1

$$BM^2 = (0 - a)^2 + (2b - b)^2 = a^2 + b^2$$

M1

So $OM = AM = BM$

E1

Answer: proved

4 marks

(c) The midpoint of the hypotenuse is the centre of the circle through all three vertices (circumcentre)

E1

Answer: the circumcentre

1 mark

30 (10 marks)

$$(a) AB^2 = 4^2 + (-2)^2 = 20$$

M1 A1

$$\text{Gradient } AB = -\frac{1}{2}$$

B1

$$\text{Answer: } AB^2 = 20; \text{ gradient } -\frac{1}{2}$$

3 marks

(b) AC is perpendicular to AB, so its gradient is 2

M1 E1

Through (0, 2): $y = 2x + 2$

A1

Answer: shown

3 marks

$$(c) C(x, 2x + 2): AC^2 = x^2 + (2x)^2 = 5x^2 = 20$$

M1 A1

 $x = \pm 2$: C(2, 6) or C(-2, -2)

A1 A1

Answer: C(2, 6) or C(-2, -2)

4 marks

Parallel and Perpendicular Lines**1 (2 marks)**

Parallel lines have equal gradients: 3

B1

$$\text{Perpendicular gradient} = -\frac{1}{3}$$

B1

$$\text{Answer: (a) 3 (b) } -\frac{1}{3}$$

2 marks

2 (2 marks)

Negative reciprocal

M1

$$\frac{5}{2}$$

A1

$$\text{Answer: } \frac{5}{2}$$

2 marks

3 (2 marks)

$$4x - 2y = 7 \text{ gives } y = 2x - \frac{7}{2}$$

M1

Both have gradient 2, so they are parallel

A1

Answer: both gradient 2

2 marks

4 (2 marks)

Gradient -1, y-intercept 4

M1

$$y = -x + 4$$

A1

$$\text{Answer: } y = -x + 4$$

2 marks

5 (2 marks)

$$x + 2y = 10 \text{ gives } y = -\frac{1}{2}x + 5, \text{ gradient } -\frac{1}{2}$$

M1

$$2 \times \left(-\frac{1}{2}\right) = -1, \text{ so they are perpendicular}$$

A1

$$\text{Answer: } m_1 m_2 = -1$$

2 marks

6 (2 marks)

$$4y = -3x + 8, \text{ gradient } -\frac{3}{4}$$

M1

$$\text{Perpendicular gradient } \frac{4}{3}$$

A1

$$\text{Answer: } \frac{4}{3}$$

2 marks

7 (2 marks)

$$m = 3: y - 9 = 3(x - 2)$$

M1

$$y = 3x + 3$$

A1

$$\text{Answer: } y = 3x + 3$$

2 marks

8 (3 marks)

$$\text{Gradient} = -\frac{1}{2}$$

B1

$$y + 2 = -\frac{1}{2}(x - 1)$$

M1

$$x + 2y + 3 = 0$$

A1

$$\text{Answer: } x + 2y + 3 = 0$$

3 marks

9 (4 marks)

$$\text{Gradients: a: 2, b: } -\frac{1}{2}, \text{ c: 2, d: } -\frac{1}{2}$$

M1 A1

$$\text{Parallel: a and c; b and d}$$

B1

$$\text{Perpendicular: a and b, a and d, c and b, c and d}$$

B1

$$\text{Answer: } a \parallel c, b \parallel d; a, c \perp b, d$$

4 marks

10 (4 marks)

$$\text{Midpoint (3, 4); gradient AB} = 1$$

M1 A1

$$\text{Perpendicular gradient } -1: y - 4 = -(x - 3)$$

M1

$$x + y = 7$$

A1

$$\text{Answer: } x + y = 7$$

4 marks

11 (5 marks)

$$\text{Gradients } -\frac{k}{3} \text{ and } -2$$

B1

$$\text{Parallel: } -\frac{k}{3} = -2, \text{ so } k = 6$$

M1 A1

$$\text{Perpendicular: } \left(-\frac{k}{3}\right)(-2) = -1, \text{ so } k = -\frac{3}{2}$$

M1 A1

$$\text{Answer: (a) } k = 6 \text{ (b) } k = -\frac{3}{2}$$

5 marks

12 (4 marks)

$$\text{Gradient AB} = \frac{1-5}{3-1} = -2$$

M1 A1

$$\text{Gradient BC} = \frac{3-1}{7-3} = \frac{1}{2}$$

A1

$$-2 \times \frac{1}{2} = -1, \text{ so } AB \perp BC$$

E1

$$\text{Answer: shown}$$

4 marks

13 (4 marks)

Gradient of $3x - y = 2$ is 3, so gradient of l is $-\frac{1}{3}$

M1 A1

$$y + 1 = -\frac{1}{3}(x - 4); y = 0: 1 = -\frac{1}{3}(x - 4) \text{ so } x = 1$$

M1 A1

Answer: (1, 0)

4 marks

14 (6 marks)

(a) Midpoint of AB: $M(1, 3)$; gradient $AB = \frac{1}{2}$

M1 A1

$$y - 3 = -2(x - 1)$$

M1

$$y = -2x + 5$$

A1

Answer: $y = -2x + 5$

4 marks

(b) $P(\frac{5}{2}, 0)$, $Q(0, 5)$

M1

$$\text{Area} = \frac{1}{2} \times \frac{5}{2} \times 5 = \frac{25}{4}$$

A1

Answer: $\frac{25}{4}$

2 marks

15 (4 marks)

(a) $\frac{3 - 1}{4 + 2} = \frac{1}{3}$

B1

Answer: $\frac{1}{3}$

1 mark

(b) Perpendicular gradient -3

B1

$$y - 7 = -3(x - 1)$$

M1

$$3x + y = 10$$

A1

Answer: $3x + y = 10$

3 marks

16 (11 marks)

(a) Midpoint of AC = $(4, 2)$

B1

Gradient AC = $\frac{-2 - 6}{7 - 1} = -\frac{4}{3}$, so the gradient of BD is $\frac{3}{4}$

M1 A1

$$y - 2 = \frac{3}{4}(x - 4), \text{ so } 3x - 4y = 4$$

A1

Answer: $3x - 4y = 4$

4 marks

(b) $x = 0$ gives $-4y = 4$, so $y = -1$

M1

$B(0, -1)$

A1

Answer: $B(0, -1)$

2 marks

(c) $3(12) - 4y = 4$, so $y = 8$ and $D(12, 8)$

M1 A1

$$AC = \sqrt{36 + 64} = 10 \text{ and } BD = \sqrt{144 + 81} = 15$$

M1 A1

The diagonals are perpendicular, so area = $\frac{1}{2} \times 10 \times 15 = 75$

A1

Answer: $D(12, 8)$; area 75

5 marks

17 (6 marks)

(a) Gradient $-\frac{1}{2}$: $y = -\frac{1}{2}x + 1$

B1 B1

Answer: $y = -\frac{1}{2}x + 1$

2 marks

(b) $2x - 9 = -\frac{1}{2}x + 1$, so $\frac{5}{2}x = 10$

M1

$x = 4$, $y = -1$: $Q(4, -1)$

A1

Answer: $Q(4, -1)$

2 marks

(c) Distance from $(0, 1)$ to $(4, -1) = \sqrt{16 + 4}$

M1

$$= 2\sqrt{5}$$

A1

Answer: $2\sqrt{5}$

2 marks

18 (6 marks)

(a) $AB = \sqrt{16 + 9} = 5$, $BC = 5$, $CD = \sqrt{16 + 9} = 5$, $DA = 5$

M1 A1

A rhombus

A1

Answer: rhombus

3 marks

(b) Gradient $AC = \frac{3}{9} = \frac{1}{3}$

B1

Gradient $BD = \frac{0 - 3}{5 - 4} = -3$

B1

$\frac{1}{3} \times (-3) = -1$, so perpendicular

E1

Answer: shown

3 marks

19 (6 marks)

(a) Add: $3x = 9$, $x = 3$, $y = 3$

M1 A1

Answer: P(3, 3)

2 marks

(b) Gradient of $3x + 2y = 1$ is $-\frac{3}{2}$, so required gradient $\frac{2}{3}$

M1 A1

$y - 3 = \frac{2}{3}(x - 3)$

M1

$2x - 3y + 3 = 0$

A1

Answer: $2x - 3y + 3 = 0$

4 marks

20 (6 marks)

(a) $A(-6, 0)$, $B(0, 3)$

B1 B1

Answer: A(-6, 0), B(0, 3)

2 marks

(b) Midpoint $(-3, \frac{3}{2})$; gradient $AB = \frac{1}{2}$

M1 A1

$y - \frac{3}{2} = -2(x + 3)$

M1

$4x + 2y + 9 = 0$

A1

Answer: $4x + 2y + 9 = 0$

4 marks

21 (9 marks)

(a) Gradient $AB = \frac{1}{2}$, so gradient $BC = -2$

M1 A1

$y - 4 = -2(x - 3)$, so $2x + y = 10$

A1

Answer: $2x + y = 10$

3 marks

(b) $y = 0$: $x = 5$, so $C(5, 0)$

M1 A1

$D = A + (C - B) = (-1 + 2, 2 - 4) = (1, -2)$

M1 A1

Answer: C(5, 0), D(1, -2)

4 marks

(c) $AB = \sqrt{20}$, $BC = \sqrt{20}$

M1

Area = 20

A1

Answer: 20

2 marks

22 (9 marks)

(a) Gradient $-\frac{1}{2}$: $y - 2 = -\frac{1}{2}(x - 3)$

M1 A1

$x + 2y = 7$

A1

Answer: $x + 2y = 7$

3 marks

(b) $x + 2(2x + 1) = 7$, so $5x = 5$

M1

$F(1, 3)$

A1 A1

Answer: F(1, 3)

3 marks

(c) F is the midpoint of P and its image P'

M1

$P' = (2(1) - 3, 2(3) - 2) = (-1, 4)$

A1 A1

Answer: (-1, 4)

3 marks

23 (8 marks)

(a) $OA = \sqrt{16 + 9} = 5$

M1

$OB = 5$, so $OA = OB$

A1

Answer: shown

2 marks

(b) $M = \left(\frac{4+5}{2}, \frac{3+0}{2}\right) = \left(\frac{9}{2}, \frac{3}{2}\right)$

M1 A1

 $OA = OB$, so triangle OAB is isosceles and OM bisects angle AOB

E1

Gradient of $OM = \frac{1}{3}$, so $y = \frac{1}{3}x$

A1

Answer: $y = \frac{1}{3}x$

4 marks

(c) The two bisectors are perpendicular

M1

Gradient = -3 , so $y = -3x$

A1

Answer: $y = -3x$

2 marks

24 (8 marks)(a) AB is horizontal, so the altitude is vertical: $x = 2$

M1 A1

Answer: $x = 2$

2 marks

(b) Gradient $BC = \frac{4-0}{2-6} = -1$, so altitude gradient 1

M1 A1

Through the origin: $y = x$

A1

Answer: $y = x$

3 marks

(c) $H(2, 2)$

B1

Gradient $AC = 2$, so altitude from B has gradient $-\frac{1}{2}$: $y = -\frac{1}{2}(x - 6)$

M1

At $x = 2$: $y = 2$, so it passes through H

A1

Answer: $H(2, 2)$

3 marks

25 (7 marks)(a) C is on the perpendicular bisector of AB

M1

Midpoint $(3, 0)$, gradient $AB = -1$, perpendicular gradient 1: $y = x - 3$

M1 A1

Answer: shown

3 marks

(b) $x = 0$ gives $y = -3$: $C(0, -3)$

B1

Answer: $C(0, -3)$

1 mark

(c) $CA^2 = 1 + 25 = 26$, $CB^2 = 25 + 1 = 26$

M1 A1

$CA = CB = \sqrt{26}$

A1

Answer: $\sqrt{26}$

3 marks

26 (9 marks)(a) BD is the perpendicular bisector of AC ; midpoint $(2, 2)$, gradient $AC = \frac{2}{3}$

M1 A1

$y - 2 = -\frac{3}{2}(x - 2)$, so $3x + 2y = 10$

A1

Answer: $3x + 2y = 10$

3 marks

(b) B : $x = 0$, $y = 5$, so $B(0, 5)$

M1 A1

$D = 2M - B = (4, -1)$

M1 A1

Answer: $B(0, 5)$, $D(4, -1)$

4 marks

(c) $AC = \sqrt{36 + 16} = \sqrt{52}$, $BD = \sqrt{16 + 36} = \sqrt{52}$

M1

Area = $\frac{1}{2} \times AC \times BD = 26$

A1

Answer: 26

2 marks

27 (8 marks)

(a) $OP^2 = 1^2 + m_1^2$

M1

$OQ^2 = 1 + m_2^2$

A1

Answer: shown

2 marks

(b) $PQ^2 = (1 - 1)^2 + (m_1 - m_2)^2$

M1 A1

Answer: shown**2 marks****(c)** Right angle at O iff $OP^2 + OQ^2 = PQ^2$ **M1**

$$2 + m_1^2 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

M1 A1

$$2 = -2m_1m_2, \text{ so } m_1m_2 = -1$$

A1**Answer: proved****4 marks****28 (6 marks)****(a)** $M = (b, c)$ **B1**

$$N = \left(\frac{2a + 2b}{2}, \frac{2c}{2} \right) = (a + b, c)$$

B1**Answer: M(b, c), N(a + b, c)****2 marks****(b)** MN is horizontal (both have y-coordinate c)**M1**OA is horizontal ($y = 0$), so $MN \parallel OA$ **E1****Answer: proved****2 marks****(c)** $MN = (a + b) - b = a$ **M1**

$$OA = 2a, \text{ so } MN = \frac{1}{2}OA$$

E1**Answer: proved****2 marks****29 (10 marks)****(a)** Midpoint (4, 0); AB is horizontal, so the bisector is vertical: $x = 4$ **M1 A1****Answer: $x = 4$** **2 marks****(b)** Midpoint (1, 3), gradient AC = 3**M1 A1**

$$y - 3 = -\frac{1}{3}(x - 1), \text{ so } x + 3y = 10$$

A1**Answer: $x + 3y = 10$** **3 marks****(c)** $x = 4$ into $x + 3y = 10$: $y = 2$ **M1**

O(4, 2)

A1**Answer: O(4, 2)****2 marks****(d)** $OA^2 = 16 + 4 = 20$, $OB^2 = 16 + 4 = 20$, $OC^2 = 4 + 16 = 20$ **M1 A1**

$$(x - 4)^2 + (y - 2)^2 = 20$$

A1**Answer: $(x - 4)^2 + (y - 2)^2 = 20$** **3 marks****30 (11 marks)**

$$\text{(a) Gradient AB} = \frac{7 - 3}{4 + 2} = \frac{2}{3}$$

M1 A1

$$\text{Gradient BC} = \frac{a + 1 - 7}{a - 4} = \frac{a - 6}{a - 4}, \text{ and } \frac{2}{3} \times \frac{a - 6}{a - 4} = -1$$

M1

$$2a - 12 = -3a + 12, \text{ so } a = \frac{24}{5}$$

A1 A1**Answer: $a = \frac{24}{5}$** **5 marks**

$$\text{(b) Gradient AC} = \frac{a - 2}{a + 2} = \frac{2}{3}$$

M1

$$3a - 6 = 2a + 4, \text{ so } a = 10$$

A1**Answer: $a = 10$** **2 marks****(c)** C lies on the perpendicular bisector of AB: midpoint (1, 5), gradient $\frac{2}{3}$, so $y - 5 = -\frac{3}{2}(x - 1)$ **M1 A1**

$$a + 1 - 5 = -\frac{3}{2}(a - 1), \text{ so } 2a - 8 = -3a + 3$$

M1

$$a = \frac{11}{5}$$

A1**Answer: $a = \frac{11}{5}$** **4 marks**

Circles

1 (2 marks)

Centre $(3, -2)$, radius 5

B1

Centre $(0, 0)$, radius 7

B1

Answer: (a) $(3, -2)$, 5 (b) $(0, 0)$, 7

2 marks

2 (2 marks)

$$(x - 2)^2 + (y - 5)^2 = 3^2$$

M1

$$(x - 2)^2 + (y - 5)^2 = 9$$

A1

Answer: $(x - 2)^2 + (y - 5)^2 = 9$

2 marks

3 (2 marks)

$$6^2 + 8^2 = 36 + 64 = 100$$

M1 A1

Answer: shown

2 marks

4 (2 marks)

$$(x + 1)^2 + (y - 4)^2 = 7$$

M1 A1

Answer: $(x + 1)^2 + (y - 4)^2 = 7$

2 marks

5 (2 marks)

$$r^2 = (4 - 1)^2 + (6 - 2)^2 = 25$$

M1

$$r = 5$$

A1

Answer: 5

2 marks

6 (3 marks)

$(1, 6)$: $0 + 16 = 16$, on the circle

B1

$(3, 5)$: $4 + 9 = 13 < 16$, inside

B1

$(5, 5)$: $16 + 9 = 25 > 16$, outside

B1

Answer: (a) on (b) inside (c) outside

3 marks

7 (4 marks)

$$(x - 3)^2 - 9 + (y + 2)^2 - 4 - 12 = 0$$

M1 A1

$$(x - 3)^2 + (y + 2)^2 = 25$$

A1

Centre $(3, -2)$, radius 5

A1

Answer: centre $(3, -2)$, radius 5

4 marks

8 (4 marks)

Centre = midpoint of AB = $(4, 6)$

M1 A1

$$r^2 = (7 - 4)^2 + (10 - 6)^2 = 25$$

M1

$$(x - 4)^2 + (y - 6)^2 = 25$$

A1

Answer: $(x - 4)^2 + (y - 6)^2 = 25$

4 marks

9 (4 marks)

Divide by 2: $x^2 + y^2 + 4x - 6y - 7 = 0$

M1

$$(x + 2)^2 + (y - 3)^2 = 7 + 4 + 9 = 20$$

M1 A1

Centre $(-2, 3)$, radius $\sqrt{20} = 2\sqrt{5}$

A1

Answer: centre $(-2, 3)$; radius $2\sqrt{5}$

4 marks

10 (3 marks)

$$r^2 = 3^2 + (-1)^2 = 10$$

M1 A1

$$(x - 3)^2 + (y + 1)^2 = 10$$

A1

Answer: $(x - 3)^2 + (y + 1)^2 = 10$

3 marks

11 (4 marks)

Angle AOB = 90° and the angle in a semicircle is 90° , so AB is a diameter

E1

Centre = midpoint of AB = (4, 3); radius = $\frac{1}{2}AB = 5$

M1 A1

$$(x - 4)^2 + (y - 3)^2 = 25$$

A1

Answer: $(x - 4)^2 + (y - 3)^2 = 25$

4 marks

12 (8 marks)

(a) $(x - 2)^2 + (y - 5)^2 = -20 + 4 + 25 = 9$

M1 A1

Centre (2, 5), radius 3

A1

Answer: centre (2, 5); radius 3

3 marks

(b) The lowest point of C has $y = 5 - 3 = 2$

M1

$2 > 0$, so every point of C is above the x-axis

E1

Answer: shown

2 marks

(c) $(4 - 2)^2 + (k - 5)^2 > 9$

M1

$$(k - 5)^2 > 9$$

A1

$$k < 5 - \sqrt{5} \text{ or } k > 5 + \sqrt{5}$$

A1

Answer: $k < 5 - \sqrt{5}$ or $k > 5 + \sqrt{5}$

3 marks

13 (8 marks)

(a) $(x - 2)^2 + (y - 3)^2 = 25$

B1

$$(x - 2)^2 + (x - 1)^2 = 25, \text{ so } 2x^2 - 6x + 5 = 25$$

M1

$$x^2 - 3x - 10 = 0$$

A1

Answer: shown

3 marks

(b) $(x - 5)(x + 2) = 0$

M1

A(-2, 0), B(5, 7)

A1 A1

Answer: A(-2, 0), B(5, 7)

3 marks

(c) $AB^2 = 7^2 + 7^2 = 98$

M1

$$AB = 7\sqrt{2}$$

A1

Answer: $7\sqrt{2}$

2 marks

14 (7 marks)

(a) $M = \left(\frac{-2+5}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

M1 A1

Answer: $\left(\frac{3}{2}, \frac{7}{2}\right)$

2 marks

(b) Gradient PM = $\frac{7}{2 - 3/2 - 2} = -1$

M1 A1

$$\text{Gradient AB} = \frac{7-0}{5+2} = 1, \text{ and } 1 \times (-1) = -1$$

M1 A1

Answer: shown

4 marks

(c) The perpendicular from the centre to a chord bisects the chord

E1

Answer: perpendicular from the centre bisects a chord

1 mark

15 (6 marks)

(a) AB is a chord and the centre lies on its perpendicular bisector

M1

AB is horizontal with midpoint (3, 4), so the bisector is $x = 3$

A1

Answer: shown

2 marks

(b) Centre (3, k): distance to A equals distance to C: $16 + (4 - k)^2 = (k + 4)^2$

M1

$$32 - 8k = 8k + 16, \text{ so } k = 1$$

M1 A1

$$r^2 = 16 + 9 = 25: (x - 3)^2 + (y - 1)^2 = 25$$

A1

Answer: $(x - 3)^2 + (y - 1)^2 = 25$

4 marks

16 (5 marks)

Touches the x-axis, centre height 5, so radius 5

B1

$$(1 - a)^2 + (1 - 5)^2 = 25$$

M1

$$(a - 1)^2 = 9, \text{ so } a = 4 \text{ or } a = -2$$

M1 A1

$$(x - 4)^2 + (y - 5)^2 = 25 \text{ and } (x + 2)^2 + (y - 5)^2 = 25$$

A1

Answer: $a = 4$ or -2

5 marks

17 (8 marks)

$$(a) (x - 5)^2 + (y + 3)^2 = 25 + 9 - 9 = 25$$

M1 A1

Centre (5, -3), radius 5

A1

Answer: centre (5, -3), radius 5

3 marks

$$(b) x = 0: y^2 + 6y + 9 = 0, \text{ so } (y + 3)^2 = 0$$

M1 A1

One repeated root $y = -3$, so C touches the y-axis at (0, -3)

A1

Answer: (0, -3)

3 marks

$$(c) y = 0: x^2 - 10x + 9 = 0, \text{ so } (x - 1)(x - 9) = 0$$

M1

(1, 0) and (9, 0)

A1

Answer: (1, 0), (9, 0)

2 marks

18 (5 marks)

$$(a) (x - 3)^2 + (y + 2)^2 = 144$$

B1

Answer: $(x - 3)^2 + (y + 2)^2 = 144$

1 mark

$$(b) MP^2 = 49 + 25 = 74 < 144: \text{ receives}$$

M1 A1

$$MQ^2 = 81 + 16 = 97 < 144: \text{ receives; } MR^2 = 144 + 16 = 160 > 144: \text{ does not}$$

M1 A1

Answer: P and Q receive; R does not

4 marks

19 (6 marks)

$$(a) (x - 1)^2 + (y + 2)^2 = 5 - k$$

M1 A1

$$r = \sqrt{5 - k}$$

A1

Answer: $r = \sqrt{5 - k}$

3 marks

$$(b) 5 - k > 0, \text{ so } k < 5$$

B1

Answer: $k < 5$

1 mark

$$(c) 5 - k = 9, k = -4$$

M1 A1

Answer: $k = -4$

2 marks

20 (8 marks)

$$(a) \text{ Gradient PQ} = \frac{5 - (-1)}{5 - 1} = \frac{3}{2}$$

M1 A1

$$\text{Gradient QR} = -\frac{2}{3}, \text{ so } \frac{k - 5}{11 - 5} = -\frac{2}{3}$$

M1

$$k - 5 = -4, \text{ so } k = 1$$

A1

Answer: $k = 1$

4 marks

(b) Angle in a semicircle is 90° , so PR is a diameter

M1

Centre = midpoint of PR = (6, 0)

A1

$$\text{Radius}^2 = 5^2 + 1^2 = 26$$

M1

$$(x - 6)^2 + y^2 = 26$$

A1

Answer: $(x - 6)^2 + y^2 = 26$

4 marks

21 (8 marks)

$$(a) \text{ Midpoint } \left(\frac{7}{2}, \frac{9}{2}\right); \text{ gradient AB} = \frac{1}{7}$$

M1 A1

$$y - \frac{9}{2} = -7\left(x - \frac{7}{2}\right), \text{ so } y = -7x + 29$$

M1 A1

Answer: $y = -7x + 29$

4 marks

$$(b) x - 3 = -7x + 29, \text{ so } x = 4$$

M1

Centre (4, 1)

A1

Answer: (4, 1)

2 marks

(c) $r^2 = 16 + 9 = 25$

M1

$(x - 4)^2 + (y - 1)^2 = 25$

A1

Answer: $(x - 4)^2 + (y - 1)^2 = 25$

2 marks

22 (10 marks)

(a) $x^2 - 6x + 9 + y^2 - 4y + 4 = 16$, giving the result

M1 A1

$C_2: x^2 + y^2 - 14x - 12y + 69 = 0$

A1

Answer: $x^2 + y^2 - 14x - 12y + 69 = 0$

3 marks

(b) Subtract the equations: $8x + 8y - 72 = 0$

M1 A1

So $x + y = 9$

A1

Answer: shown

3 marks

(c) Substitute $y = 9 - x$ into $C_1: (x - 3)^2 + (7 - x)^2 = 16$

M1

$x^2 - 10x + 21 = 0$, so $x = 3$ or 7

M1 A1

$(3, 6)$ and $(7, 2)$

A1

Answer: $(3, 6)$ and $(7, 2)$

4 marks

23 (6 marks)

(a) Centres $(0, 0)$ and $(8, 6)$: distance $= \sqrt{64 + 36} = 10$

M1 A1

Radii 3 and 7, sum 10

B1

Answer: 10 and 10

3 marks

(b) Distance between centres = sum of radii, so they touch externally

E1

The point lies on the line of centres, $\frac{3}{10}$ of the way from $(0, 0)$ to $(8, 6)$

M1

$(\frac{12}{5}, \frac{9}{5})$

A1

Answer: $(\frac{12}{5}, \frac{9}{5})$

3 marks

24 (11 marks)

(a) Centre $(3, 4)$

M1 A1

$r^2 = 16 + 4 = 20: (x - 3)^2 + (y - 4)^2 = 20$

M1 A1

Answer: $(x - 3)^2 + (y - 4)^2 = 20$

4 marks

(b) $(7 - 3)^2 + (2 - 4)^2 = 16 + 4 = 20$

M1 A1

Answer: shown

2 marks

(c) Gradient AD = 0 (horizontal), gradient DB: $x = 7$ (vertical)

M1 A1

The angle in a semicircle is a right angle

E1

Answer: AD is horizontal, DB is vertical

3 marks

(d) AD = 8, DB = 4

M1

Area $= \frac{1}{2} \times 8 \times 4 = 16$

A1

Answer: 16

2 marks

25 (7 marks)

(a) $OP = \sqrt{81 + 144} = 15$

M1 A1

Answer: 15 m

2 marks

(b) Shortest = $15 - 5 = 10$ m

M1 A1

Longest = $15 + 5 = 20$ m

A1

Answer: 10 m and 20 m

3 marks

(c) It lies on OP, $\frac{5}{15} = \frac{1}{3}$ of the way from O to P

M1

$(3, 4)$

A1

Answer: $(3, 4)$

2 marks

26 (6 marks)

(a) Substitute $y = x + k$: $x^2 + (x + k)^2 - 8x + 2(x + k) - 8 = 0$ M1

$$2x^2 + (2k - 6)x + k^2 + 2k - 8 = 0$$
 A1

Answer: shown 2 marks

(b) Discriminant: $(2k - 6)^2 - 8(k^2 + 2k - 8) > 0$ M1

$$-4k^2 - 40k + 100 > 0, \text{ so } k^2 + 10k - 25 < 0$$
 A1

$$-5 - 5\sqrt{2} < k < -5 + 5\sqrt{2}$$
 M1 A1

Answer: $-5 - 5\sqrt{2} < k < -5 + 5\sqrt{2}$ 4 marks

27 (10 marks)

(a) $(x + 3)^2 + y^2 = 4[(x - 3)^2 + y^2]$ M1

$$x^2 + 6x + 9 + y^2 = 4x^2 - 24x + 36 + 4y^2$$
 M1 A1

$$3x^2 + 3y^2 - 30x + 27 = 0$$
 A1

Answer: shown 4 marks

(b) $x^2 + y^2 - 10x + 9 = 0$ M1

$$(x - 5)^2 + y^2 = 16$$
 M1

Circle, centre (5, 0), radius 4 A1

Answer: centre (5, 0), radius 4 3 marks

(c) $y = 0$: $x = 1$ or $x = 9$ M1 A1

At (1, 0): PA = 4, PB = 2; at (9, 0): PA = 12, PB = 6 A1

Answer: (1, 0) and (9, 0) 3 marks

28 (12 marks)

(a) $(x - 4)^2 + (y - 2)^2 = -2 + 16 + 4 = 18$ M1 A1

Centre (4, 2), radius $\sqrt{18} = 3\sqrt{2}$ A1

Answer: centre (4, 2); radius $3\sqrt{2}$ 3 marks

(b) Substitute $x = 1$: $9 + (y - 2)^2 = 18$ M1

$$(y - 2)^2 = 9, \text{ so } y = 5 \text{ or } y = -1$$
 M1

P(1, 5) and Q(1, -1) A1

Answer: P(1, 5), Q(1, -1) 3 marks

(c) $CP^2 + CQ^2 = 18 + 18 = 36$ M1

PQ = 6, so $PQ^2 = 36 = CP^2 + CQ^2$, and angle PCQ = 90° E1

Answer: shown 2 marks

(d) Sector PCQ has area $\frac{1}{2} \times 18 \times \frac{\pi}{2} = \frac{9\pi}{2}$ M1 A1

Triangle PCQ has area $\frac{1}{2} \times 18 \times \sin 90^\circ = 9$ M1

$$\text{Segment area} = \frac{9\pi}{2} - 9$$
 A1

Answer: $\frac{9\pi}{2} - 9$ 4 marks

29 (12 marks)

(a) A: $1 + 9 - 2g + 6f + c = 0$, i.e. $-2g + 6f + c = -10$ M1

B: $36 + 4 + 12g + 4f + c = 0$, i.e. $12g + 4f + c = -40$ A1

C: $4 + 16 - 4g - 8f + c = 0$, i.e. $-4g - 8f + c = -20$ A1

Answer: three equations 3 marks

(b) Subtract pairs to eliminate c: $14g - 2f = -30$ and $16g + 12f = -20$ M1 A1

$$g = -2, f = 1$$
 A1

$$c = -10 + 2g - 6f = -20$$
 A1

Answer: $g = -2, f = 1, c = -20$ 4 marks

(c) $(x - 2)^2 + (y + 1)^2 = 4 + 1 + 20 = 25$ M1

Centre (2, -1), radius 5 A1

Answer: centre (2, -1), radius 5 2 marks

(d) $(5 - 2)^2 + (-5 + 1)^2 = 9 + 16 = 25$ M1 A1

A, B, C, D are concyclic, so ABDC is a cyclic quadrilateral

Answer: D lies on the circle; ABDC is cyclic

E1

3 marks

30 (9 marks)

(a) The distances from the centre to both axes are equal, and each equals the radius

E1

Centre (a, a), radius a

B1

Answer: centre (a, a), radius a

2 marks

(b) $(2 - a)^2 + (1 - a)^2 = a^2$

M1

$$a^2 - 6a + 5 = 0$$

A1

$$a = 1 \text{ or } 5$$

M1

$$(x - 1)^2 + (y - 1)^2 = 1 \text{ and } (x - 5)^2 + (y - 5)^2 = 25$$

A1

Answer: a = 1 or a = 5

4 marks

(c) Distance between centres = $\sqrt{16 + 16} = 4\sqrt{2} \approx 5.66$

M1

Radii sum 6, difference 4: $4 < 5.66 < 6$, so they cross at two points

M1 E1

Answer: 2 intersection points

3 marks

Tangents and Normals to Circles

1 (2 marks)

$$\text{Gradient CP} = \frac{7 - 3}{5 - 2} = \frac{4}{3}$$

B1

$$\text{Tangent gradient} = -\frac{3}{4}$$

B1

$$\text{Answer: } \frac{4}{3}; -\frac{3}{4}$$

2 marks

2 (2 marks)

$$\text{Gradient of radius} = \frac{4}{3}, \text{ so tangent gradient } -\frac{3}{4}$$

M1

$$y - 4 = -\frac{3}{4}(x - 3), \text{ so } 3x + 4y = 25$$

A1

Answer: shown

2 marks

3 (2 marks)

$$\text{Radius gradient } \frac{3}{2}, \text{ tangent gradient } -\frac{2}{3}$$

M1

$$y - 3 = -\frac{2}{3}(x - 2), \text{ so } 2x + 3y = 13$$

A1

$$\text{Answer: } 2x + 3y = 13$$

2 marks

4 (2 marks)

Centre (1, 2), radius 3; $y = 5$ is 3 units above the centre

M1

Distance from centre to line = radius, so tangent

A1

Answer: tangent

2 marks

5 (2 marks)

The radius to (5, 0) is horizontal

B1

The tangent is perpendicular to it, so $x = 5$

B1

Answer: $x = 5$

2 marks

6 (2 marks)

The tangent is perpendicular to the radius at the point of contact

B1

The normal at any point on a circle passes through the centre

B1

Answer: tangent $\perp$ radius; normal passes through the centre

2 marks

7 (4 marks)

$$(5 - 2)^2 + (3 + 1)^2 = 9 + 16 = 25$$

B1

$$\text{Radius gradient } \frac{4}{3}, \text{ tangent gradient } -\frac{3}{4}$$

M1 A1

$$y - 3 = -\frac{3}{4}(x - 5), \text{ so } 3x + 4y = 27$$

A1

$$\text{Answer: } 3x + 4y = 27$$

4 marks

8 (5 marks)

$$\text{Centre } (3, -1); \text{ check } (6, 3): 9 + 16 = 25 = r^2$$

B1

$$\text{Radius gradient } \frac{4}{3}; \text{ normal } 4x - 3y = 15$$

M1 A1

$$\text{Tangent gradient } -\frac{3}{4}; 3x + 4y = 30$$

M1 A1

$$\text{Answer: normal } 4x - 3y = 15; \text{ tangent } 3x + 4y = 30$$

5 marks

9 (4 marks)

$$\text{Centre } (2, 1)$$

B1

$$\text{Radius gradient } \frac{5 - 1}{5 - 2} = \frac{4}{3}; \text{ tangent gradient } -\frac{3}{4}$$

M1 A1

$$3x + 4y = 35$$

A1

$$\text{Answer: } 3x + 4y = 35$$

4 marks

10 (5 marks)

$$\text{Tangent: } 3x + 4y = 25$$

B1

$$A(\frac{25}{3}, 0), B(0, \frac{25}{4})$$

M1 A1

$$AB = \sqrt{(\frac{25}{3})^2 + (\frac{25}{4})^2} = \frac{125}{12}$$

M1 A1

$$\text{Answer: } A(\frac{25}{3}, 0), B(0, \frac{25}{4}); AB = \frac{125}{12}$$

5 marks

11 (4 marks)

$$x^2 + (x + 4)^2 = 8, \text{ so } 2x^2 + 8x + 8 = 0$$

M1

$$(x + 2)^2 = 0: \text{ a repeated root, so the line is a tangent}$$

M1 A1

$$x = -2, y = 2: (-2, 2)$$

A1

$$\text{Answer: } (-2, 2)$$

4 marks

12 (4 marks)

$$\text{The normal passes through the centre } (1, 2): \text{ gradient } \frac{1}{3}$$

M1 A1

$$y - 2 = \frac{1}{3}(x - 1); x = 0: y = \frac{5}{3}$$

M1 A1

$$\text{Answer: } Q(0, \frac{5}{3})$$

4 marks

13 (3 marks)

$$OT \perp PT, \text{ so triangle OTP is right-angled at T}$$

M1

$$PT^2 = OP^2 - OT^2 = 100 - 36 = 64$$

M1

$$PT = 8$$

A1

$$\text{Answer: } 8$$

3 marks

14 (5 marks)

$$\text{Tangent } y = 2x + c: x^2 + (2x + c)^2 = 20, \text{ so } 5x^2 + 4cx + c^2 - 20 = 0$$

M1 A1

$$\text{Tangent: discriminant } 16c^2 - 20(c^2 - 20) = 0$$

M1

$$c^2 = 100, c = \pm 10$$

A1

$$y = 2x + 10 \text{ and } y = 2x - 10$$

A1

$$\text{Answer: } y = 2x \pm 10$$

5 marks

15 (5 marks)**(a)** Normal passes through the centre (1, 1)

M1

$$\text{Gradient} = \frac{-3-1}{4-1} = -\frac{4}{3}; 4x + 3y = 7$$

A1 A1

Answer: $4x + 3y = 7$

3 marks

(b) Tangent gradient $\frac{3}{4}$

M1

$$y + 3 = \frac{3}{4}(x - 4), \text{ so } 3x - 4y = 24$$

A1

Answer: $3x - 4y = 24$

2 marks

16 (6 marks)**(a)** $(x - 5)^2 + (y - 4)^2 = 25$

B1

Centre is 5 from the y-axis, equal to the radius, so C touches it at (0, 4)

M1 A1

Answer: shown

3 marks

(b) Radius gradient $\frac{4}{3}$, tangent gradient $-\frac{3}{4}$

M1

$$y - 8 = -\frac{3}{4}(x - 8), \text{ so } 3x + 4y = 56$$

A1

Answer: $3x + 4y = 56$

2 marks

(c) $x = 0$: $y = 14$, so (0, 14)

B1

Answer: (0, 14)

1 mark

17 (9 marks)**(a)** $(x - 3)^2 + (y - 1)^2 = -6 + 9 + 1 = 4$

M1 A1

Centre (3, 1), radius 2

A1

Answer: centre (3, 1); radius 2

3 marks

(b) Distance from A to the centre = $\sqrt{8^2 + 6^2} = 10$

M1 A1

Shortest distance = $10 - 2 = 8$

A1

Answer: 8

3 marks

(c) The radius to B is perpendicular to AB

B1

$$AB^2 = 10^2 - 2^2 = 96$$

M1

$$AB = 4\sqrt{6}$$

A1

Answer: $4\sqrt{6}$

3 marks

18 (8 marks)**(a)** $(x - 1)^2 + (y - 2)^2 = 20$

M1

Centre (1, 2), radius $2\sqrt{5}$

A1

Answer: (1, 2), $2\sqrt{5}$

2 marks

(b) $4 + 16 = 20$

B1

Radius gradient 2, tangent gradient $-\frac{1}{2}$

M1

$$y - 6 = -\frac{1}{2}(x - 3), \text{ so } x + 2y = 15$$

A1

Answer: $x + 2y = 15$

3 marks

(c) $x = 15 - 2y$: $(14 - 2y)^2 + (y - 2)^2 = 20$

M1

$$5y^2 - 60y + 180 = 0, \text{ so } (y - 6)^2 = 0, \text{ one repeated root}$$

M1 A1

Answer: shown

3 marks

19 (5 marks)Perpendicular from the centre: $y - 1 = -(x - 2)$, so $y = 3 - x$

M1 A1

Meets $y = x + 7$ at $x = -2$, $y = 5$

M1

$$r^2 = 4^2 + 4^2 = 32$$

M1

$$(x - 2)^2 + (y - 1)^2 = 32$$

A1

Answer: $(x - 2)^2 + (y - 1)^2 = 32$

5 marks

20 (12 marks)

(a) Gradient of CP = $\frac{5-3}{5-1} = \frac{1}{2}$

M1 A1

$y - 5 = \frac{1}{2}(x - 5)$, so $x - 2y + 5 = 0$

A1

Answer: $x - 2y + 5 = 0$

3 marks

(b) The normal passes through the centre, so PQ is a diameter

M1

C is the midpoint of PQ, so $Q = (2 - 5, 6 - 5) = (-3, 1)$

M1 A1

Answer: $Q(-3, 1)$

3 marks

(c) Tangent gradient = -2

B1

At P: $y - 5 = -2(x - 5)$, so $2x + y = 15$

M1 A1

At Q: $y - 1 = -2(x + 3)$, so $2x + y = -5$

A1

The tangents are parallel and perpendicular to PQ, so the distance is $PQ = 2\sqrt{20} = 4\sqrt{5}$

M1 A1

Answer: $2x + y = 15$; $2x + y = -5$; $4\sqrt{5}$

6 marks

21 (10 marks)

(a) $x^2 + (mx + 10)^2 = 50$, so $(1 + m^2)x^2 + 20mx + 50 = 0$

M1 A1

Discriminant $400m^2 - 200(1 + m^2) = 0$

M1

$200m^2 = 200$, so $m^2 = 1$

A1

Answer: shown

4 marks

(b) $y = x + 10$ and $y = -x + 10$

B1

Repeated root $x = -\frac{20m}{2(1+m^2)} = -5m$, so $A(-5, 5)$ and $B(5, 5)$

M1 A1

Answer: $y = \pm x + 10$; $A(-5, 5)$, $B(5, 5)$

3 marks

(c) $OA = OB = \sqrt{50}$, $AP = BP = \sqrt{25 + 25} = \sqrt{50}$

M1

$OA \perp AP$, so OAPB is a square of side $\sqrt{50}$

E1

Area = 50

A1

Answer: 50

3 marks

22 (8 marks)

(a) $(x - 5)^2 + (y - 2)^2 = -4 + 25 + 4 = 25$

M1 A1

Centre (5, 2), radius 5

A1

Answer: centre (5, 2); radius 5

3 marks

(b) Distance between the centres = $\sqrt{25 + 4} = \sqrt{29}$

M1

Touching externally: $a + 5 = \sqrt{29}$

M1

$a = \sqrt{29} - 5$

A1

Answer: $a = \sqrt{29} - 5$

3 marks

(c) Touching internally: $a - 5 = \sqrt{29}$

M1

$a = 5 + \sqrt{29}$

A1

Answer: $a = 5 + \sqrt{29}$

2 marks

23 (8 marks)

(a) $x^2 + (2x + k)^2 - 2x + 4(2x + k) = 0$

M1

$5x^2 + 4kx + k^2 + 6x + 4k = 0$, giving the result

A1

Answer: shown

2 marks

(b) Discriminant $(4k + 6)^2 - 20(k^2 + 4k) = 0$

M1

$-4k^2 - 32k + 36 = 0$, so $k^2 + 8k - 9 = 0$

A1

$k = 1$ or $k = -9$

A1

Answer: $k = 1$ or $k = -9$

3 marks

(c) Repeated root $x = -\frac{4k + 6}{10}$

M1

$k = 1$: $x = -1$, $y = -1$, $(-1, -1)$

A1

$k = -9$: $x = 3$, $y = -3$, $(3, -3)$

A1

Answer: $(-1, -1)$ and $(3, -3)$

3 marks

24 (8 marks)

(a) Centres (1, 2), (7, 10): distance = $\sqrt{36 + 64} = 10$

M1 A1

Radii $3 + 7 = 10$, equal, so they touch externally

A1

Answer: shown

3 marks

(b) T is $\frac{3}{10}$ of the way from (1, 2) to (7, 10)

M1

$$T = (1 + 1.8, 2 + 2.4) = \left(\frac{14}{5}, \frac{22}{5}\right)$$

A1

Answer: T $\left(\frac{14}{5}, \frac{22}{5}\right)$

2 marks

(c) The common tangent is perpendicular to the line of centres (gradient $\frac{4}{3}$), so its gradient is $-\frac{3}{4}$

M1 A1

$$y - \frac{22}{5} = -\frac{3}{4}\left(x - \frac{14}{5}\right), \text{ so } 3x + 4y = 26$$

A1

Answer: $3x + 4y = 26$

3 marks

25 (7 marks)

(a) $OA \perp AP$: $PA^2 = 13^2 - 5^2 = 144$

M1

$PA = PB = 12$

A1

Answer: 12

2 marks

(b) Two congruent right-angled triangles, each $\frac{1}{2} \times 5 \times 12 = 30$

M1

Area = 60

A1

Answer: 60

2 marks

(c) $\sin(\text{APO}) = \frac{5}{13}$, so $\text{APO} = 22.6^\circ$

M1 A1

$$\text{APB} = 2 \times 22.6^\circ = 45.2^\circ$$

A1

Answer: 45.2°

3 marks

26 (7 marks)

(a) Gradient of OP = $\frac{b}{a}$

B1

The tangent is perpendicular to OP, so its gradient is $-\frac{a}{b}$

M1 A1

Answer: shown

3 marks

(b) $y - b = -\frac{a}{b}(x - a)$, so $by - b^2 = -ax + a^2$

M1

$$ax + by = a^2 + b^2 = r^2$$

A1

Answer: shown

2 marks

(c) $-5x + 12y = 169$

M1 A1

Answer: $-5x + 12y = 169$

2 marks

27 (12 marks)

(a) A: $16 + 9 = 25$; B: $16 + 9 = 25$

M1 A1

Answer: shown

2 marks

(b) Centre (3, -1): radius gradient to A is $\frac{3}{4}$, tangent gradient $-\frac{4}{3}$: $4x + 3y = 34$

M1 A1

Radius gradient to B is $-\frac{3}{4}$, tangent gradient $\frac{4}{3}$: $-4x + 3y = 10$

M1 A1

Answer: $4x + 3y = 34$; $-4x + 3y = 10$

4 marks

(c) Add: $6y = 44$, so $y = \frac{22}{3}$

M1

$$4x = 34 - 22 = 12, \text{ so } x = 3: T\left(3, \frac{22}{3}\right)$$

A1

Answer: T $\left(3, \frac{22}{3}\right)$

2 marks

(d) $TA^2 = 16 + \left(\frac{16}{3}\right)^2 = \frac{400}{9}$, so $TA = \frac{20}{3}$

M1 A1

$$\text{Kite} = 2 \times \frac{1}{2} \times CA \times TA = 5 \times \frac{20}{3}$$

M1

$$= \frac{100}{3}$$

A1

Answer: $\frac{100}{3}$

4 marks

28 (8 marks)

(a) Tangent at P: $x_1x + y_1y = r^2$

B1

Tangent at Q: $-x_1x - y_1y = r^2$

B1

Answer: $x_1x + y_1y = \pm r^2$

2 marks

(b) Gradients: $-\frac{x_1}{y_1}$ for both

M1 E1

Answer: proved

2 marks

(c) The line through O perpendicular to both is $y = \frac{y_1}{x_1}x$

M1

It meets the tangents at P and Q, so the distance is PQ

M1 E1

PQ = diameter = $2r$

A1

Answer: $2r$

4 marks

29 (8 marks)

(a) Substitute: $x^2 + (mx + c)^2 = r^2$, so $(1 + m^2)x^2 + 2mcx + c^2 - r^2 = 0$

M1 A1

Tangent iff discriminant is zero: $4m^2c^2 - 4(1 + m^2)(c^2 - r^2) = 0$

M1

$-c^2 + r^2(1 + m^2) = 0$, i.e. $c^2 = r^2(1 + m^2)$

M1 A1

Answer: proved

5 marks

(b) Here $c = 10$: $100 = 20(1 + m^2)$

M1

$m^2 = 4$, so $m = \pm 2$

A1

$y = 2x + 10$ and $y = -2x + 10$

A1

Answer: $y = \pm 2x + 10$

3 marks

30 (10 marks)

(a) The radius at P is perpendicular to the tangent, so it has gradient -1

M1 E1

Line through P: $y - 3 = -(x - 2)$, i.e. $x + y = 5$

A1

Answer: shown

3 marks

(b) $CP^2 = (a - 2)^2 + (2 - a)^2 = 2(a - 2)^2$

M1 A1

$CQ^2 = (6 - a)^2 + (2 - a)^2$; set $CQ^2 = CP^2$: $40 - 16a = 8 - 8a$

M1 A1

$a = 4$

A1

Answer: $a = 4$

5 marks

(c) Centre $(4, 1)$, radius² = $2(2)^2 = 8$

M1

$(x - 4)^2 + (y - 1)^2 = 8$

A1

Answer: $(x - 4)^2 + (y - 1)^2 = 8$

2 marks

Parametric Equations

1 (3 marks)

$t = 0$: $(-1, 2)$

B1

$t = 2$: $(5, 6)$

B1

$t = -1$: $(-4, 3)$

B1

Answer: $(-1, 2)$, $(5, 6)$, $(-4, 3)$

3 marks

2 (2 marks)

$t = \frac{x}{2}$

M1

$y = \frac{x}{2} + 3$, i.e. $x - 2y + 6 = 0$

A1

Answer: $y = \frac{x}{2} + 3$

2 marks

3 (2 marks)

$$y^2 = 4t^2$$

$$= 4x$$

M1

A1

Answer: shown

2 marks

4 (2 marks)

$$x = 5 \text{ gives } t = 2$$

M1

$$y = 4 + 4 = 8, \text{ so it lies on the curve}$$

A1

Answer: yes

2 marks

5 (2 marks)

$$t = \frac{10}{4} = \frac{5}{2}$$

M1

$$y = \frac{2}{5}$$

A1

$$\text{Answer: } t = \frac{5}{2}, y = \frac{2}{5}$$

2 marks

6 (4 marks)

$$y = -2 \text{ gives } t^2 - 3t + 2 = 0$$

M1

$$(t - 1)(t - 2) = 0, \text{ so } t = 1 \text{ or } t = 2$$

A1

$$x = 7 \text{ gives } a = 7 - 2t$$

M1

$$a = 5 \text{ or } a = 3$$

A1

Answer: a = 5 or a = 3

4 marks

7 (3 marks)

$$t = \frac{x}{3}, \text{ so } y = \frac{x^2}{9} - 1$$

M1 A1

$$\text{Minimum at } (0, -1)$$

B1

$$\text{Answer: } y = \frac{x^2}{9} - 1; (0, -1)$$

3 marks

8 (2 marks)

$$t = \frac{1}{x}$$

M1

$$y = \frac{1}{x} + 1, \text{ so } xy = 1 + x$$

A1

Answer: shown

2 marks

9 (4 marks)

$$\cos t = \frac{x-1}{2}, \sin t = \frac{y-3}{2}$$

M1

$$(x-1)^2 + (y-3)^2 = 4$$

M1 A1

$$\text{Circle, centre } (1, 3), \text{ radius } 2$$

A1

$$\text{Answer: } (x-1)^2 + (y-3)^2 = 4$$

4 marks

10 (5 marks)

$$(a) x = -2 + 3\cos t$$

B1

$$y = 1 + 3\sin t$$

B1

$$\text{Answer: } x = -2 + 3\cos t, y = 1 + 3\sin t$$

2 marks

$$(b) x = -2 + 3 \times \frac{1}{2} = -\frac{1}{2}$$

M1

$$y = 1 + 3 \times \frac{\sqrt{3}}{2} = 1 + \frac{3\sqrt{3}}{2}$$

M1

$$(-\frac{1}{2}, 1 + \frac{3\sqrt{3}}{2})$$

A1

$$\text{Answer: } (-\frac{1}{2}, 1 + \frac{3\sqrt{3}}{2})$$

3 marks

11 (4 marks)

$$x = 0: t = -2, y = 0, \text{ so } (0, 0)$$

M1 A1

$$y = 0: t = \pm 2, \text{ so } x = 4 \text{ or } 0: (4, 0)$$

M1 A1

Answer: (0, 0) and (4, 0)

4 marks

12 (4 marks)

$$\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2$$

B1

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}$$

M1 A1

$$\text{When } t = 2: 3$$

A1

$$\text{Answer: } \frac{3t}{2}; 3$$

4 marks

13 (4 marks)

$$x = 3 \text{ gives } t = 2, \text{ and } y = 0 \text{ checks}$$

M1 A1

$$\frac{dy}{dx} = \frac{-2t}{2} = -t = -2$$

M1 A1

Answer: t = 2; -2

4 marks

14 (5 marks)

$$t = \frac{y}{3}, \text{ so } x = \frac{y^3}{27}$$

M1 A1

$$\frac{dy}{dx} = \frac{3}{3t^2} = \frac{1}{t^2}$$

M1 A1

$$\text{At } t = 1: 1$$

A1

$$\text{Answer: } x = \frac{y^3}{27}; 1$$

5 marks

15 (8 marks)

$$(a) \frac{dy}{dx} = \frac{2}{2t} = \frac{1}{t}$$

M1 A1

$$\text{At } t = 2: \frac{1}{2}$$

A1

$$\text{Answer: } \frac{1}{2}$$

3 marks

$$(b) P = (4, 4)$$

B1

$$\text{Tangent: } y - 4 = \frac{1}{2}(x - 4), \text{ so } x - 2y + 4 = 0$$

M1 A1

$$\text{Normal: } y - 4 = -2(x - 4), \text{ so } 2x + y = 12$$

M1 A1

$$\text{Answer: } x - 2y + 4 = 0; 2x + y = 12$$

5 marks

16 (10 marks)

$$(a) \frac{dx}{dt} = -4\sin t \text{ and } \frac{dy}{dt} = 2\cos t$$

M1

$$\frac{dy}{dx} = -\frac{\cos t}{2\sin t}$$

A1

$$\text{Answer: } -\frac{\cos t}{2\sin t}$$

2 marks

$$(b) \text{ At } t = \frac{\pi}{3} \text{ the point is } (2, \sqrt{3})$$

B1

$$\text{Gradient} = -\frac{1}{2} \div \sqrt{3}, \text{ so } 2\sqrt{3}(y - \sqrt{3}) = -(x - 2)$$

M1 A1

$$2\sqrt{3}y - 6 = -x + 2, \text{ so } x + 2\sqrt{3}y = 8$$

A1

Answer: shown

4 marks

$$(c) y = 0 \text{ gives } x = 8, \text{ and } x = 0 \text{ gives } y = \frac{4\sqrt{3}}{3}$$

M1 A1

$$\text{Area} = \frac{1}{2} \times 8 \times \frac{4\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}$$

M1 A1

Answer: $\frac{16\sqrt{3}}{3}$

4 marks

17 (2 marks)

Use $1 + \tan^2 t = \sec^2 t$

M1

$x^2 = 1 + y^2$

A1

Answer: shown

2 marks

18 (3 marks)

$x^2 = t^2 + 2 + \frac{1}{t^2}$

M1

$y^2 = t^2 - 2 + \frac{1}{t^2}$

M1

Difference = 4

A1

Answer: shown

3 marks

19 (7 marks)

(a) $t = \frac{x}{20}$

M1

$y = \frac{15x}{20} - \frac{5x^2}{400}$

M1 A1

Answer: shown

3 marks

(b) $\frac{dy}{dt} = 15 - 10t = 0$, $t = 1.5$

M1

$y = 22.5 - 11.25 = 11.25$ m

A1

Answer: 11.25 m

2 marks

(c) $y = 0$: $t = 3$

M1

$x = 60$ m

A1

Answer: 60 m

2 marks

20 (4 marks)

$2t = t^2 - 1 + 2$, so $t^2 - 2t + 1 = 0$

M1 A1

$(t - 1)^2 = 0$: repeated root, so one point (tangent)

M1

$t = 1$: (0, 2)

A1

Answer: (0, 2)

4 marks

21 (7 marks)

(a) $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2 - 3$

B1

$\frac{dy}{dx} = \frac{3(t^2 - 1)}{2t}$

M1 A1

Answer: $\frac{3(t^2 - 1)}{2t}$

3 marks

(b) $t^2 = 1$, $t = \pm 1$

M1

(1, -2) and (1, 2)

A1

Answer: (1, -2), (1, 2)

2 marks

(c) $t = 0$ makes $\frac{dx}{dt} = 0$ but $\frac{dy}{dt} \neq 0$

M1

(0, 0)

A1

Answer: (0, 0)

2 marks

22 (4 marks)

(a) $\cos 2t = 1 - 2\sin^2 t$

M1

$= 1 - 2\left(\frac{x}{2}\right)^2 = 1 - \frac{x^2}{2}$

A1

Answer: shown

2 marks

(b) $-2 \leq x \leq 2$

B1

$-1 \leq y \leq 1$

B1

Answer: $-2 \leq x \leq 2; -1 \leq y \leq 1$ **2 marks****23 (7 marks)**

(a) $(x - 3)^2 + (y + 1)^2 = 25$

M1 A1

Answer: $(x - 3)^2 + (y + 1)^2 = 25$

2 marks

(b) $\cos t = \frac{3}{5}, \sin t = \frac{4}{5}$

M1

$t = 0.927$

A1**Answer: 0.927****2 marks**

(c) $\frac{dy}{dx} = \frac{5\cos t}{-5\sin t} = -\frac{3}{4}$

M1 A1

$y - 3 = -\frac{3}{4}(x - 6), \text{ so } 3x + 4y = 30$

A1**Answer: $3x + 4y = 30$** **3 marks****24 (6 marks)**

(a) $y = 0: t = \pm 2$, so $A(-3, 0), B(5, 0)$

M1 A1

$\frac{dy}{dt} = -2t = 0, t = 0: V(1, 4)$

M1 A1**Answer: $A(-3, 0), B(5, 0), V(1, 4)$** **4 marks****(b)** Base 8, height 4**M1**

Area = 16

A1**Answer: 16****2 marks****25 (9 marks)**

(a) $\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}$

M1 A1

$y - p^3 = \frac{3p}{2}(x - p^2)$

M1

$2y - 2p^3 = 3px - 3p^3$, so $3px - 2y = p^3$

A1**Answer: shown****4 marks****(b)** Tangent at P: $6x - 2y = 8$, so $y = 3x - 4$ **B1**A point (t^2, t^3) on the tangent satisfies $t^3 = 3t^2 - 4$ **M1** $t^3 - 3t^2 + 4 = 0$ has a repeated root $t = 2$ (the point P)**M1**

$(t - 2)^2(t + 1) = 0$, so $t = -1$

A1

Q(1, -1)

A1**Answer: Q(1, -1)****5 marks****26 (7 marks)**

(a) A: $y = 1 + \frac{x}{2}$

B1

B: $s = 10 - x$, so $y = 16 - 2x$

M1 A1

Answer: $y = 1 + \frac{x}{2}; y = 16 - 2x$

3 marks

(b) $1 + 3 = 4$ and $16 - 12 = 4$

M1 A1A is at (6, 4) when $t = 3$; B is there when $s = 4$ **M1**

Different times, so no collision

E1**Answer: shown****4 marks****27 (6 marks)**

(a) $x^2 + y^2 = 16$

M1 A1

Answer: $x^2 + y^2 = 16$

2 marks

(b) $x^2 + (x + k)^2 = 16$, so $2x^2 + 2kx + k^2 - 16 = 0$

M1 A1

Discriminant $4k^2 - 8(k^2 - 16) = 0$

M1

$k^2 = 32, k = \pm 4\sqrt{2}$

A1

Answer: $k = \pm 4\sqrt{2}$

4 marks

28 (6 marks)

$$(a) x^2 + y^2 = \frac{(1 - t^2)^2 + 4t^2}{(1 + t^2)^2} \quad \text{M1}$$

$$= \frac{(1 + t^2)^2}{(1 + t^2)^2} = 1 \quad \text{A1}$$

Answer: shown 2 marks

$$(b) x = \frac{3}{4 \div 5/4} = \frac{3}{5} \quad \text{M1}$$

$$y = \frac{1 \div 5}{4} = \frac{4}{5} \quad \text{A1}$$

Answer: $(\frac{3}{5}, \frac{4}{5})$ 2 marks

$$(c) x = -1 \text{ gives } 1 - t^2 = -1 - t^2 \quad \text{M1}$$

$$1 = -1, \text{ impossible} \quad \text{E1}$$

Answer: shown 2 marks

29 (9 marks)

$$(a) t = \frac{x}{25 \cos \alpha} \quad \text{M1}$$

$$y = x \tan \alpha - \frac{4.9x^2}{625 \cos^2 \alpha} \quad \text{M1 A1}$$

$$\text{Use } \frac{1}{\cos^2 \alpha} = 1 + \tan^2 \alpha \quad \text{A1}$$

Answer: shown 4 marks

$$(b) 10 = 30T - 7.056(1 + T^2) \text{ where } T = \tan \alpha \quad \text{M1 A1}$$

$$7.056T^2 - 30T + 17.056 = 0 \quad \text{M1}$$

$$T = 0.676 \text{ or } T = 3.576 \quad \text{M1}$$

$$\alpha = 34.1^\circ \text{ or } 74.4^\circ \quad \text{A1}$$

Answer: 34.1° or 74.4° 5 marks

30 (9 marks)

$$(a) \frac{dy}{dx} = \frac{1}{p}, \text{ normal gradient } -p \quad \text{M1 A1}$$

$$y - 2p = -p(x - p^2), \text{ giving the result} \quad \text{A1}$$

Answer: shown 3 marks

$$(b) \text{ Substitute: } 2q + pq^2 = 2p + p^3 \quad \text{M1}$$

$$p(q^2 - p^2) + 2(q - p) = 0, \text{ so } (q - p)(p(q + p) + 2) = 0 \quad \text{M1 A1}$$

$$q \neq p, \text{ so } q = -p - \frac{2}{p} \quad \text{A1}$$

Answer: shown 4 marks

$$(c) q = -3 \quad \text{B1}$$

$$Q(9, -6) \quad \text{A1}$$

Answer: $Q(9, -6)$ 2 marks