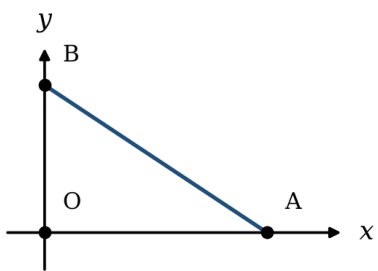
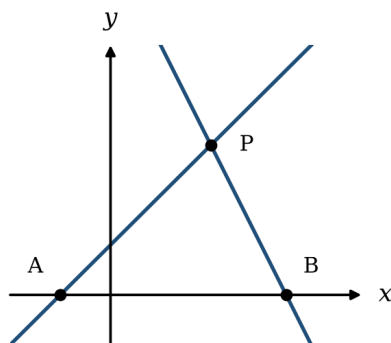


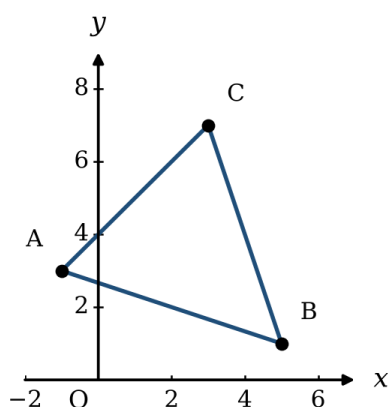
Straight Lines

- 1 Find the gradient of the line through A(2, 3) and B(6, 11). [2]
- 2 Find the coordinates of the midpoint of the line segment joining P(-2, 5) and Q(6, 1). [2]
- 3 Find the distance between the points (1, 2) and (7, 10). [2]
- 4 Write down the gradient and the y-intercept of each line.
(a) $y = 3x - 5$, (b) $2y - 4x = 6$. [2]
- 5 Find an equation of the line with gradient 2 that passes through the point (1, 5). Give your answer in the form $y = mx + c$. [2]
- 6 Determine whether each point lies on the line $y = 2x + 1$.
(a) (3, 7), (b) (-1, 4). [2]
- 7 The line l passes through the points A(-1, 4) and B(3, -4). Find an equation of l , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [4]
- 8 The line l has equation $3x + 4y = 12$. Find its gradient, and the coordinates of the points where it meets the coordinate axes. [4]
- 9 Find the coordinates of the point of intersection of the lines $y = 2x - 1$ and $x + y = 8$. [3]
- 10 The point M(1, 5) is the midpoint of the line segment AB, where A has coordinates (-3, 2). Find the coordinates of B. [3]
- 11 The points A, B and C have coordinates (1, 1), (5, 3) and (3, 7). Show that triangle ABC is isosceles and right-angled. [5]
- 12 The line with equation $2x + 3y = k$ passes through the point (4, -1). Find the value of k , and the gradient of the line. [3]
- 13 A line crosses the x-axis at (4, 0) and the y-axis at (0, -3). Find its equation in the form $ax + by + c = 0$, where a , b and c are integers. [3]
- 14 The line l has equation $2x + 3y = 12$. It meets the x-axis at A and the y-axis at B, as shown, and O is the origin.

 - (a) Find the coordinates of A and B. [2]
 - (b) Find the area of triangle OAB. [2]
 - (c) Find the length AB, giving your answer as a surd in its simplest form. [2]
- 15 The points A(-1, -5), B(2, 1) and C(5, 7) are given.
 - (a) Show that A, B and C are collinear. [2]
 - (b) The point (k, 13) lies on the same line. Find the value of k . [2]

- 16** The line l_1 has equation $y = x + 1$ and the line l_2 has equation $y = -2x + 7$. The lines meet at the point P. The line l_1 meets the x-axis at A and the line l_2 meets the x-axis at B.



- (a) Find the coordinates of P, A and B. [4]
 (b) Find the area of triangle ABP. [3]
-
- 17** The points $A(-2, -7)$ and $B(2, 5)$ lie on the line l .
 (a) Find an equation of l in the form $y = mx + c$. [2]
 (b) Determine, by calculation, whether each of the points $C(-50, -152)$ and $D(40, 120)$ lies above or below l . [3]
-
- 18** A taxi company charges a fixed fee plus a cost per kilometre. A journey of 5 km costs £11.50 and a journey of 12 km costs £22.00. The cost £C of a journey of d km is modelled by a linear equation.
 (a) Find C in terms of d . [4]
 (b) Interpret the gradient and the constant in the model. [2]
 (c) Find the cost of a 20 km journey. [1]
-
- 19** The points $A(1, 2)$, $B(5, 4)$, $C(8, 9)$ and $D(4, 7)$ are the vertices of a quadrilateral ABCD.
 (a) Find the coordinates of the midpoints of the diagonals AC and BD. [3]
 (b) Hence state, with a reason, what type of quadrilateral ABCD is. [1]
-
- 20** The points $A(1, 2)$ and $B(7, 11)$ are given. The point P lies on AB such that $AP : PB = 1 : 2$.
 (a) Find the coordinates of P. [3]
 (b) Verify that P lies on the line through A and B. [2]
-
- 21** The triangle ABC has vertices $A(-1, 3)$, $B(5, 1)$ and $C(3, 7)$. The point M is the midpoint of BC.

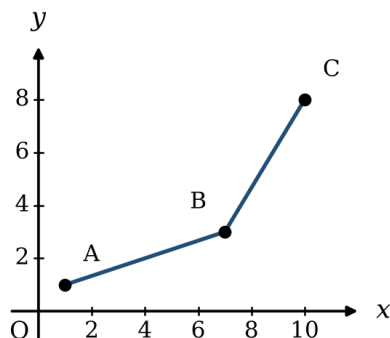


- (a) Find the coordinates of M. [2]
 (b) Find an equation of the median AM in the form $ax + by + c = 0$, where a , b and c are integers. [4]
 (c) The centroid of the triangle has coordinates $(\frac{7}{3}, \frac{11}{3})$. Show that it lies on AM. [2]

- 22** The curve C has equation $y = x^2 - 1$ and the line l has equation $2y = 3x$. The curve meets the y -axis at A and meets the line at P and Q , where P has the larger x -coordinate.
- (a) Find the coordinates of A , P and Q . [5]
- (b) Show that angle $PAQ = 90^\circ$. [3]
- (c) Find the area of triangle APQ . [4]

- 23** The line l has equation $kx + (k + 1)y = 12$, where k is a constant. The point $(3, 2)$ lies on l .
- (a) Find the value of k . [2]
- (b) Find the gradient of l and the coordinates of the points where l meets the axes. [4]
- (c) Find the area of the triangle bounded by l and the two coordinate axes. [2]

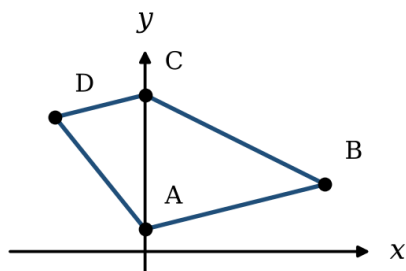
- 24** The points $A(1, 1)$, $B(7, 3)$ and $C(10, 8)$ are three vertices of the parallelogram $ABCD$.



- (a) Find the coordinates of D . [2]
- (b) Find the equation of the diagonal BD , and show that it passes through the midpoint of AC . [4]
- (c) Find the area of the parallelogram. [4]

- 25** The line l passes through the points $A(-1, 7)$ and $B(5, -2)$.
- (a) Find an equation for l in the form $ax + by = c$, where a , b and c are integers. [4]
- (b) Find the acute angle that l makes with the x -axis, correct to 1 decimal place. [2]
- (c) Find the area of the triangle formed by l and the coordinate axes. [3]

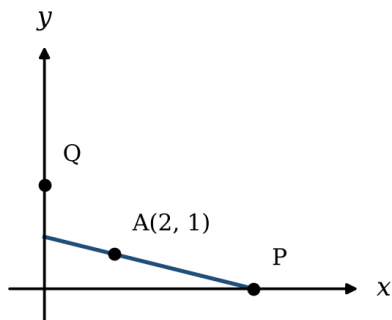
- 26** The quadrilateral $ABCD$ has vertices $A(0, 1)$, $B(8, 3)$, $C(0, 7)$ and $D(-4, 6)$.



- (a) Show that AB is parallel to DC , but that BC is not parallel to AD . Hence name the quadrilateral. [4]
- (b) Show that $AB = 2DC$. [2]
- (c) Find the area of $ABCD$. [5]

- 27** The line l_k has equation $(k + 1)x + (2 - k)y = 3k + 4$, where k is a constant.
- (a) Show that the equation can be written in the form $k(x - y - 3) + (x + 2y - 4) = 0$. [2]
- (b) Hence show that all the lines l_k pass through a fixed point, and find its coordinates. [4]
- (c) Find the value of k for which l_k is parallel to the x -axis, and state its equation. [3]

- 28** A line passes through the point $A(2, 1)$ and meets the positive x -axis at $P(p, 0)$ and the positive y -axis at $Q(0, q)$, where $p > 2$.

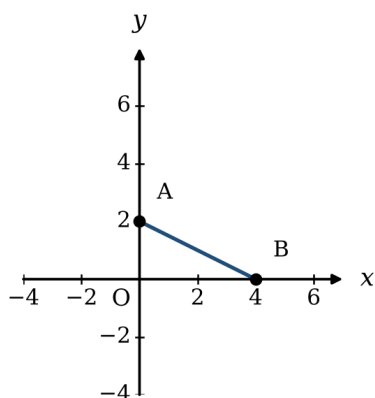


- (a) Show that $\frac{2}{p} + \frac{1}{q} = 1$. [3]
- (b) Hence show that the area S of triangle OPQ is $S = \frac{p^2}{2(p-2)}$. [2]
- (c) Use differentiation to find the value of p for which S is a minimum, and find this minimum area. [5]

- 29** The triangle OAB has a right angle at O , where O is the origin, and A and B have coordinates $(2a, 0)$ and $(0, 2b)$, where a and b are positive constants. The point M is the midpoint of AB .

- (a) Find the coordinates of M . [2]
- (b) Prove that M is equidistant from O , A and B . [4]
- (c) Hence state a geometric property of the midpoint of the hypotenuse of a right-angled triangle. [1]

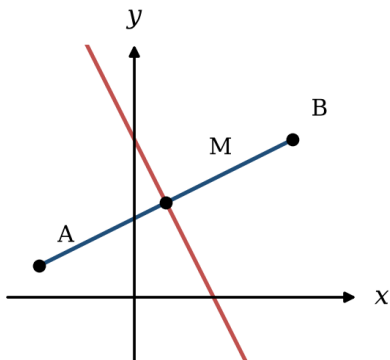
- 30** The points $A(0, 2)$ and $B(4, 0)$ are given. The point C is such that triangle ABC is right-angled at A and $AC = AB$. There are two possible positions for C .



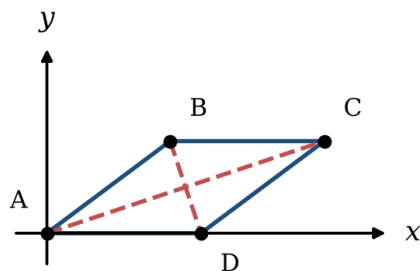
- (a) Show that $AB^2 = 20$ and find the gradient of AB . [3]
- (b) Show that AC lies on the line $y = 2x + 2$. [3]
- (c) Hence find the coordinates of the two possible positions of C . [4]

Parallel and Perpendicular Lines

- 1** A line has equation $y = 3x - 2$. Write down the gradient of (a) a line parallel to it, (b) a line perpendicular to it. [2]
- 2** A line has gradient $-\frac{2}{5}$. Write down the gradient of a line perpendicular to it. [2]
- 3** Show that the lines $y = 2x + 1$ and $4x - 2y = 7$ are parallel. [2]
- 4** Find an equation of the line through $(0, 4)$ that is parallel to $y = -x + 7$. [2]

- 5** Show that the lines $y = 2x + 3$ and $x + 2y = 10$ are perpendicular. [2]
- 6** Find the gradient of a line perpendicular to the line $3x + 4y = 8$. [2]
- 7** Find an equation of the line through $(2, 9)$ that is parallel to the line $y = 3x - 1$. Give your answer in the form $y = mx + c$. [2]
- 8** Find an equation of the line through $(1, -2)$ that is perpendicular to the line $y = 2x + 5$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. [3]
- 9** Four lines have equations
a: $y = 2x + 3$, b: $x + 2y = 4$, c: $6x - 3y = 5$, d: $4x + 8y = 3$.
State which pairs of lines are parallel and which pairs are perpendicular. [4]
- 10** The points $A(1, 2)$ and $B(5, 6)$ are given. Find an equation of the perpendicular bisector of AB , in the form $ax + by = c$. [4]
- 11** The lines $kx + 3y = 7$ and $2x + y = 5$ are given, where k is a constant. Find the value of k for which the lines are (a) parallel, (b) perpendicular. [5]
- 12** The points $A(1, 5)$, $B(3, 1)$ and $C(7, 3)$ are given. Show that angle ABC is a right angle. [4]
- 13** The line l passes through $(4, -1)$ and is perpendicular to the line $3x - y = 2$. Find the coordinates of the point where l crosses the x -axis. [4]
- 14** The points $A(-3, 1)$ and $B(5, 5)$ are given. The line l is the perpendicular bisector of AB .

(a) Find an equation of l . [4]
(b) The line l meets the axes at P and Q . Find the area of triangle OPQ . [2]
- 15** The triangle PQR has vertices $P(-2, 1)$, $Q(4, 3)$ and $R(1, 7)$.
(a) Find the gradient of PQ . [1]
(b) Find an equation of the altitude from R to PQ (the line through R perpendicular to PQ). [3]
- 16** The kite $ABCD$ has $A(1, 6)$ and $C(7, -2)$. The diagonal BD is a line of symmetry of the kite, and B lies on the y -axis.
(a) Find an equation of BD . [4]
(b) Find the coordinates of B . [2]
(c) The point D has x -coordinate 12. Find the coordinates of D , and the area of the kite. [5]
- 17** The lines $l_1: y = 2x + 1$ and $l_2: y = 2x - 9$ are parallel.
(a) Find an equation of the line through $(0, 1)$ perpendicular to both lines. [2]
(b) This line meets l_2 at the point Q . Find the coordinates of Q . [2]
(c) Hence find the perpendicular distance between l_1 and l_2 , as a surd in simplest form. [2]

- 18** The diagram shows the quadrilateral ABCD with A(0, 0), B(4, 3), C(9, 3) and D(5, 0).



- (a) Show that all four sides of ABCD have the same length. Name the quadrilateral. [3]
 (b) Show that the diagonals AC and BD are perpendicular. [3]

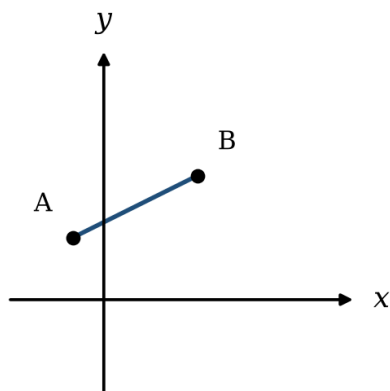
- 19** The lines $x + y = 6$ and $2x - y = 3$ meet at the point P.

- (a) Find the coordinates of P. [2]
 (b) Find an equation of the line through P perpendicular to the line $3x + 2y = 1$, giving your answer in the form $ax + by + c = 0$ with integer coefficients. [4]

- 20** The line l has equation $2y = x + 6$. It meets the x-axis at A and the y-axis at B.

- (a) Find the coordinates of A and B. [2]
 (b) Find an equation of the perpendicular bisector of AB, in the form $ax + by + c = 0$, where a , b and c are integers. [4]

- 21** The rectangle ABCD has vertices A(−1, 2) and B(3, 4). The vertex C lies on the x-axis.



- (a) Find an equation of the line BC. [3]
 (b) Find the coordinates of C and of D. [4]
 (c) Find the area of the rectangle. [2]

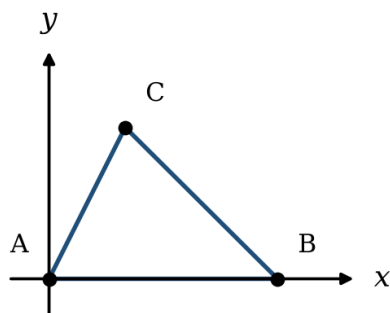
- 22** The point P(3, 2) and the line $l: y = 2x + 1$ are given. The point F is the foot of the perpendicular from P to l .

- (a) Find an equation of the line through P perpendicular to l . [3]
 (b) Find the coordinates of F. [3]
 (c) Hence find the coordinates of the image of P when it is reflected in l . [3]

- 23** The line l_1 has equation $4y = 3x$ and the line l_2 is the x-axis. They meet at the origin O. The point A(4, 3) lies on l_1 and the point B(5, 0) lies on l_2 .

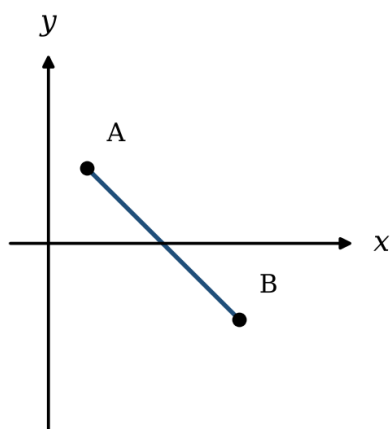
- (a) Show that $OA = OB$. [2]
 (b) Find the coordinates of the midpoint M of AB, and hence find an equation of the line through O that bisects angle AOB. [4]
 (c) Find an equation of the other line through O that bisects an angle between l_1 and l_2 . [2]

- 24** The triangle ABC has vertices A(0, 0), B(6, 0) and C(2, 4). The altitudes of a triangle are the lines through each vertex perpendicular to the opposite side.



- Find an equation of the altitude from C to AB. [2]
- Find an equation of the altitude from A to BC. [3]
- Find the coordinates of the point H where these two altitudes meet, and show that the altitude from B to AC also passes through H. [3]

- 25** The points A(1, 2) and B(5, -2) are given. The point C lies on the y-axis and $CA = CB$.



- Show that C lies on the line $y = x - 3$. [3]
- Find the coordinates of C. [1]
- Verify that $CA = CB$, and find this length. [3]

- 26** A(-1, 0) and C(5, 4) are opposite vertices of the rhombus ABCD. The vertex B lies on the y-axis.

- Find an equation of the diagonal BD. [3]
- Find the coordinates of B and of D. [4]
- Find the area of the rhombus. [2]

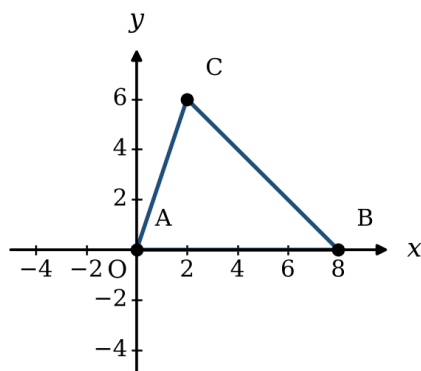
- 27** Two lines through the origin have equations $y = m_1x$ and $y = m_2x$, where m_1 and m_2 are non-zero constants. The points P(1, m_1) and Q(1, m_2) lie on the lines respectively.

- Show that $OP^2 = 1 + m_1^2$ and write down a similar expression for OQ^2 . [2]
- Show that $PQ^2 = (m_1 - m_2)^2$. [2]
- Use Pythagoras' theorem to prove that angle POQ is a right angle if and only if $m_1m_2 = -1$. [4]

- 28** The triangle OAB has vertices O(0, 0), A(2a, 0) and B(2b, 2c), where a, b and c are positive constants. The points M and N are the midpoints of OB and AB respectively.

- Find the coordinates of M and N. [2]
- Prove that MN is parallel to OA. [2]
- Prove that $MN = \frac{1}{2}OA$. [2]

- 29** The triangle ABC has vertices A(0, 0), B(8, 0) and C(2, 6).



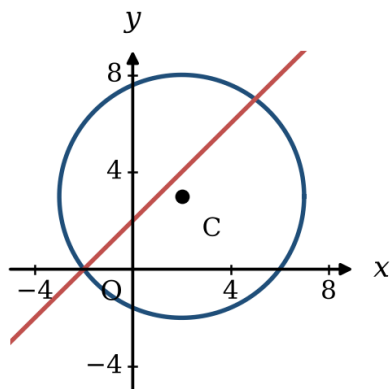
- Find an equation of the perpendicular bisector of AB. [2]
 - Find an equation of the perpendicular bisector of AC. [3]
 - Hence find the coordinates of the point O equidistant from A, B and C. [2]
 - Show that $OA = OB = OC$, and write down an equation of the circle through A, B and C. [3]
- 30** The points A(−2, 3), B(4, 7) and C(a, a + 1) are given, where a is a constant.
- Given that angle $ABC = 90^\circ$, find the value of a. [5]
 - Given instead that the points A, B and C are collinear, find a. [2]
 - Given instead that $AC = BC$, find a. [4]

Circles

- Write down the coordinates of the centre and the radius of each circle.
(a) $(x - 3)^2 + (y + 2)^2 = 25$, (b) $x^2 + y^2 = 49$. [2]
- Write down an equation of the circle with centre (2, 5) and radius 3. [2]
- Show that the point (6, 8) lies on the circle $x^2 + y^2 = 100$. [2]
- Write down an equation of the circle with centre (−1, 4) and radius $\sqrt{7}$. [2]
- A circle has centre (1, 2) and passes through the point (4, 6). Find its radius. [2]
- The circle C has equation $(x - 1)^2 + (y - 2)^2 = 16$. Determine whether each point lies on C, inside C or outside C.
(a) (1, 6), (b) (3, 5), (c) (5, 5). [3]
- The circle C has equation $x^2 + y^2 - 6x + 4y - 12 = 0$. Find the coordinates of the centre of C and the radius of C. [4]
- The points A(1, 2) and B(7, 10) are the ends of a diameter of a circle. Find an equation of the circle. [4]
- The circle C has equation $2x^2 + 2y^2 + 8x - 12y - 14 = 0$. Find the coordinates of the centre of C and the radius of C, giving the radius as a surd in its simplest form. [4]
- A circle has centre (3, −1) and passes through the origin. Find an equation of the circle. [3]
- The points O(0, 0), A(8, 0) and B(0, 6) lie on a circle. Explain why AB is a diameter, and find an equation of the circle. [4]

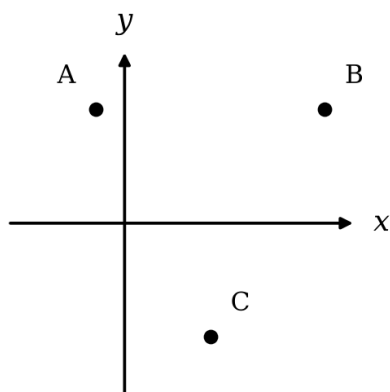
- 12** The circle C has equation $x^2 + y^2 - 4x - 10y + 20 = 0$.
- (a) Find the coordinates of the centre of C and the radius of C . [3]
- (b) Show that C lies entirely above the x -axis. [2]
- (c) The point $P(4, k)$ lies outside C . Find the set of possible values of k . [3]

- 13** The circle C has centre $(2, 3)$ and radius 5. The line l has equation $y = x + 2$. The line meets the circle at the points A and B .



- (a) Write down an equation of C , and show that the x -coordinates of A and B satisfy $x^2 - 3x - 10 = 0$. [3]
- (b) Find the coordinates of A and B . [3]
- (c) Find the length of the chord AB , as a surd in simplest form. [2]
- 14** The circle C has centre $P(2, 3)$ and radius 5. The chord AB of C has endpoints $A(-2, 0)$ and $B(5, 7)$.
- (a) Find the coordinates of the midpoint M of AB . [2]
- (b) Show that PM is perpendicular to AB . [4]
- (c) State the circle theorem illustrated by this result. [1]

- 15** The points $A(-1, 4)$, $B(7, 4)$ and $C(3, -4)$ lie on a circle.



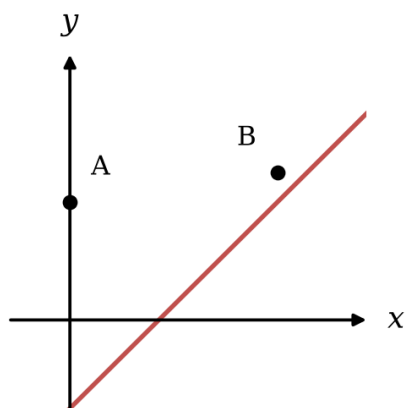
- (a) Show that the centre of the circle lies on the line $x = 3$. [2]
- (b) Hence find an equation of the circle. [4]
- 16** A circle has centre $(a, 5)$, touches the x -axis and passes through the point $(1, 1)$. Find the two possible values of a , and an equation of each circle. [5]
- 17** The circle C has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.
- (a) Find the centre and radius of C . [3]
- (b) Show that C touches the y -axis, and find the coordinates of the point of contact. [3]
- (c) Find the coordinates of the points where C meets the x -axis. [2]

- 18** A radio mast is positioned at the point $M(3, -2)$ on a map, where the units are kilometres. Its signal can be received at any point within 12 km of M . The towns $P(10, 3)$, $Q(-6, -6)$ and $R(15, 2)$ are plotted on the map.
- (a) Write down an equation of the circle that marks the boundary of the signal. [1]
 (b) Determine which of the towns can receive the signal. [4]

- 19** The equation $x^2 + y^2 - 2x + 4y + k = 0$ represents a circle, where k is a constant.
- (a) Find, in terms of k , the radius of the circle. [3]
 (b) State the range of values of k for which the equation represents a circle. [1]
 (c) Find k when the radius is 3. [2]

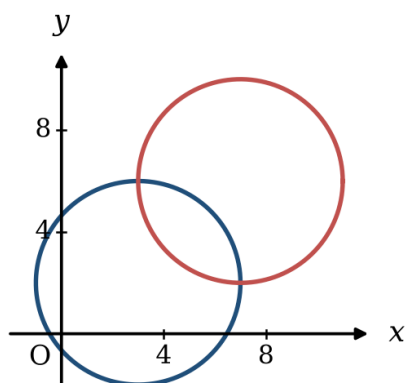
- 20** The points $P(1, -1)$, $Q(5, 5)$ and $R(11, k)$ are given, where k is a constant. Angle $PQR = 90^\circ$.
- (a) Find the value of k . [4]
 (b) Find an equation of the circle through P , Q and R . [4]

- 21** The points $A(0, 4)$ and $B(7, 5)$ lie on a circle whose centre lies on the line $y = x - 3$.



- (a) Find an equation of the perpendicular bisector of AB . [4]
 (b) Hence find the coordinates of the centre of the circle. [2]
 (c) Find an equation of the circle. [2]

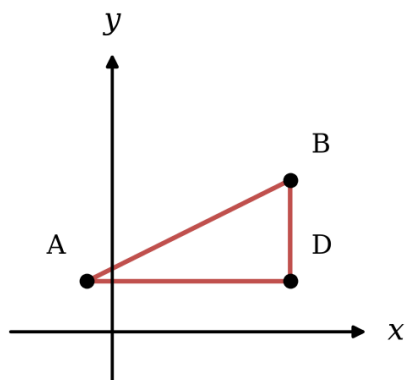
- 22** The circles C_1 and C_2 have equations $(x - 3)^2 + (y - 2)^2 = 16$ and $(x - 7)^2 + (y - 6)^2 = 16$ respectively. They intersect at the points P and Q .



- (a) Show that C_1 has equation $x^2 + y^2 - 6x - 4y - 3 = 0$ and find a similar expression for C_2 . [3]
 (b) Show that P and Q lie on the line $x + y = 9$. [3]
 (c) Find the coordinates of P and Q . [4]

- 23** The circles $C_1: x^2 + y^2 = 9$ and $C_2: (x - 8)^2 + (y - 6)^2 = 49$ are given.
- (a) Find the distance between the centres, and the sum of the radii. [3]
 (b) Hence show that the circles touch externally, and find the coordinates of the point of contact. [3]

- 24** The points $A(-1, 2)$ and $B(7, 6)$ are the ends of a diameter of the circle C . The point $D(7, 2)$ is also given.



- Find an equation of C . [4]
- Show that D lies on C . [2]
- Show that angle ADB is 90° , and explain why this is expected. [3]
- Find the area of triangle ABD . [2]

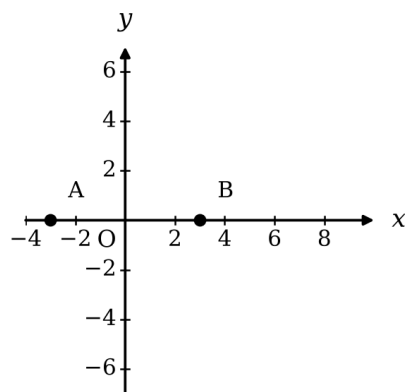
- 25** A circular pond is modelled by the circle with equation $x^2 + y^2 = 25$, where the units are metres. A small boat is at the point $P(9, 12)$.

- Find the distance from P to the centre of the pond. [2]
- Hence find the shortest and the longest distance from P to a point on the circumference of the pond. [3]
- Find the coordinates of the point on the circumference nearest to P . [2]

- 26** The circle C has equation $x^2 + y^2 - 8x + 2y - 8 = 0$, and the line l has equation $y = x + k$, where k is a constant.

- Show that the x -coordinates of the points where l meets C satisfy $2x^2 + (2k - 6)x + k^2 + 2k - 8 = 0$. [2]
- Find the set of values of k for which l meets C at two distinct points. [4]

- 27** The points $A(-3, 0)$ and $B(3, 0)$ are given. The point $P(x, y)$ moves so that $PA = 2PB$.



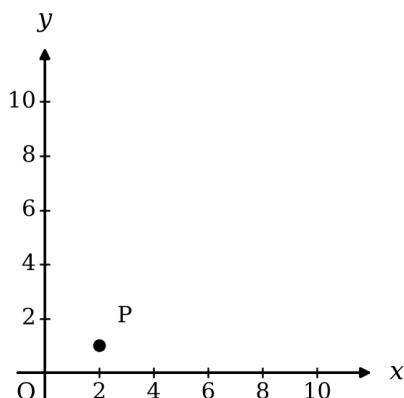
- Show that the locus of P has equation $3x^2 + 3y^2 - 30x + 27 = 0$. [4]
- Hence show that the locus is a circle, and find its centre and radius. [3]
- Find the coordinates of the points where the locus meets the x -axis, and verify that $PA = 2PB$ at each. [3]

- 28** The circle C has equation $x^2 + y^2 - 8x - 4y + 2 = 0$.

- Find the coordinates of the centre of C and its exact radius. [3]
- The line $x = 1$ meets C at the points P and Q . Find the coordinates of P and Q . [3]
- Show that PQ subtends a right angle at the centre C of the circle. [2]
- Find the exact area of the minor segment cut off by PQ . [4]

- 29** A circle passes through the points $A(-1, 3)$, $B(6, 2)$ and $C(-2, -4)$. Its equation is $x^2 + y^2 + 2gx + 2fy + c = 0$.
- Write down three equations connecting g , f and c . [3]
 - Solve them to find g , f and c . [4]
 - Find the centre and the radius of the circle. [2]
 - Show that $D(5, -5)$ lies on the circle, and state what this means for the quadrilateral $ABDC$. [3]

- 30** A circle C lies in the first quadrant, touches both coordinate axes, and passes through the point $P(2, 1)$.

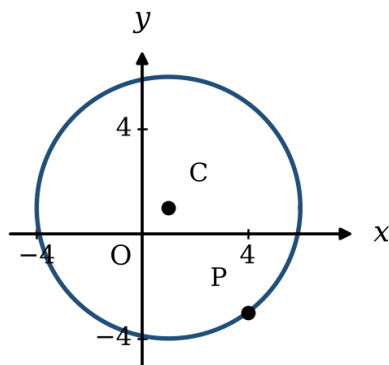


- Explain why the centre of C has coordinates (a, a) for some $a > 0$, and write down its radius. [2]
- Show that $a^2 - 6a + 5 = 0$ and hence find the two possible equations of C . [4]
- Show that the two circles intersect at two points. [3]

Tangents and Normals to Circles

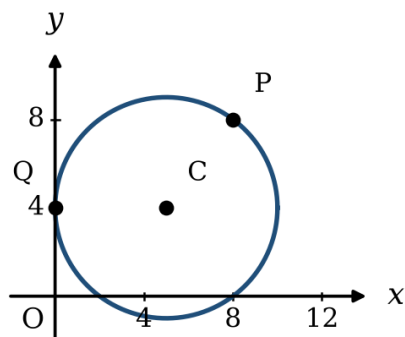
- A circle has centre $(2, 3)$. The point $P(5, 7)$ lies on the circle. Find the gradient of the radius CP , and hence the gradient of the tangent at P . [2]
- Show that the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ has equation $3x + 4y = 25$. [2]
- Find an equation of the tangent to the circle $x^2 + y^2 = 13$ at the point $(2, 3)$. [2]
- Show that the line $y = 5$ is a tangent to the circle $(x - 1)^2 + (y - 2)^2 = 9$. [2]
- The circle $(x - 1)^2 + y^2 = 16$ passes through the point $(5, 0)$. Write down an equation of the tangent at this point, explaining your answer. [2]
- State the geometrical relationship between a tangent to a circle and the radius drawn to the point of contact, and between a normal to a circle and its centre. [2]
- The circle C has equation $(x - 2)^2 + (y + 1)^2 = 25$. Show that the point $P(5, 3)$ lies on C , and find an equation of the tangent to C at P in the form $ax + by = c$. [4]
- The circle C has equation $x^2 + y^2 - 6x + 2y - 15 = 0$. Find equations of the tangent and the normal to C at the point $(6, 3)$. [5]
- The circle $x^2 + y^2 - 4x - 2y - 20 = 0$ passes through the point $P(5, 5)$. Find an equation of the tangent at P . [4]
- The tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ meets the coordinate axes at A and B . Find the coordinates of A and B , and the length AB . [5]

- 11** Show that the line $y = x + 4$ is a tangent to the circle $x^2 + y^2 = 8$, and find the coordinates of the point of contact. [4]
- 12** The circle C has equation $(x - 1)^2 + (y - 2)^2 = 10$ and passes through the point $P(4, 3)$. The normal to C at P meets the y -axis at Q . Find the coordinates of Q . [4]
- 13** The point $P(10, 0)$ lies outside the circle $x^2 + y^2 = 36$. A tangent from P touches the circle at T . Find the length PT . [3]
- 14** Find equations of the two tangents to the circle $x^2 + y^2 = 20$ that are parallel to the line $y = 2x$. [5]
- 15** The circle C has equation $(x - 1)^2 + (y - 1)^2 = 25$. The point $P(4, -3)$ lies on C .



- (a) Find an equation of the normal to C at P . [3]
 (b) Find an equation of the tangent to C at P . [2]

- 16** The circle C has centre $(5, 4)$ and radius 5. The point $P(8, 8)$ lies on C .

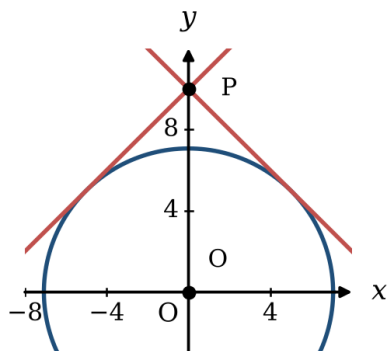


- (a) Write down an equation of C , and show that C touches the y -axis at $Q(0, 4)$. [3]
 (b) Find an equation of the tangent to C at P . [2]
 (c) Find the coordinates of the point where the tangent at P meets the y -axis. [1]

- 17** The circle C has equation $x^2 + y^2 - 6x - 2y + 6 = 0$, and the point $A(11, 7)$ lies outside C .
 (a) Find the coordinates of the centre of C and the radius of C . [3]
 (b) Find the shortest distance from A to C . [3]
 (c) A tangent from A touches C at the point B . Find the exact length of AB . [3]
- 18** The circle C has equation $x^2 + y^2 - 2x - 4y - 15 = 0$.
 (a) Find the centre and the radius of C . [2]
 (b) Show that the point $P(3, 6)$ lies on C and find an equation of the tangent at P . [3]
 (c) Verify that the line $x + 2y = 15$ meets C at exactly one point. [3]
- 19** Find an equation of the circle with centre $(2, 1)$ that touches the line $y = x + 7$. [5]

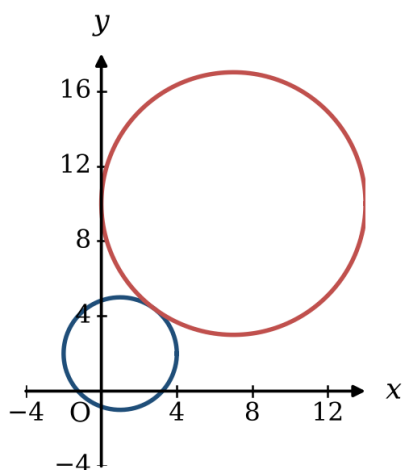
- 20** The circle C has equation $(x - 1)^2 + (y - 3)^2 = 20$ and passes through the point $P(5, 5)$.
- (a) Find an equation of the normal to C at P , in the form $ax + by + c = 0$ where a , b and c are integers. [3]
- (b) The normal meets C again at Q . Find the coordinates of Q . [3]
- (c) Find equations of the tangents to C at P and at Q , and hence find the exact distance between these tangents. [6]

- 21** The circle C has equation $x^2 + y^2 = 50$. The two tangents from the point $P(0, 10)$ touch C at A and B .



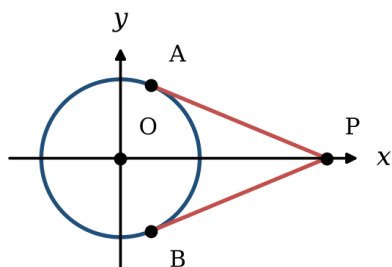
- (a) By considering $y = mx + 10$, show that the gradients of the tangents satisfy $m^2 = 1$. [4]
- (b) Hence find equations of the two tangents and the coordinates of A and B . [3]
- (c) Show that $OAPB$ is a square, and find its area. [3]
- 22** The circle C_1 has equation $x^2 + y^2 - 10x - 4y + 4 = 0$ and the circle C_2 has equation $x^2 + y^2 = a^2$, where $a > 0$.
- (a) Find the coordinates of the centre of C_1 and the radius of C_1 . [3]
- (b) C_2 touches C_1 externally. Find the exact value of a . [3]
- (c) Find the exact value of a for which C_1 lies inside C_2 and touches it. [2]
- 23** The circle C has equation $x^2 + y^2 - 2x + 4y = 0$. The line $y = 2x + k$ is a tangent to C .
- (a) Show that $5x^2 + (4k + 6)x + k^2 + 4k = 0$. [2]
- (b) Hence find the two possible values of k . [3]
- (c) Find the coordinates of the two points of contact. [3]

- 24** The circles $C_1: (x - 1)^2 + (y - 2)^2 = 9$ and $C_2: (x - 7)^2 + (y - 10)^2 = 49$ touch externally at the point T .



- (a) Show that the circles touch externally. [3]
- (b) Find the coordinates of T . [2]
- (c) Find an equation of the common tangent at T . [3]

- 25** The tangents from the point $P(13, 0)$ to the circle $x^2 + y^2 = 25$ touch the circle at A and B.

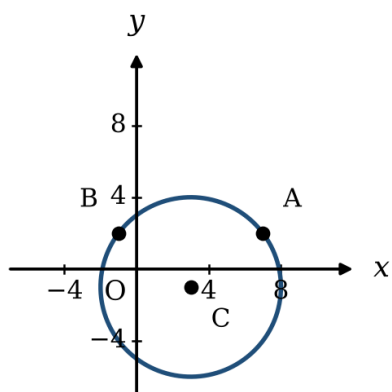


- Find the length of each tangent PA and PB. [2]
- Find the area of the quadrilateral OAPB. [2]
- Find the size of angle APB, correct to 1 decimal place. [3]

- 26** The circle C has equation $x^2 + y^2 = r^2$, where $r > 0$. The point $P(a, b)$, with $b \neq 0$, lies on C.

- Show that the gradient of the tangent at P is $-\frac{a}{b}$. [3]
- Hence show that the tangent at P has equation $ax + by = r^2$. [2]
- Use this result to find an equation of the tangent to the circle $x^2 + y^2 = 169$ at the point $(-5, 12)$. [2]

- 27** The circle C has equation $(x - 3)^2 + (y + 1)^2 = 25$. The points $A(7, 2)$ and $B(-1, 2)$ lie on C. The tangents to C at A and B meet at the point T.



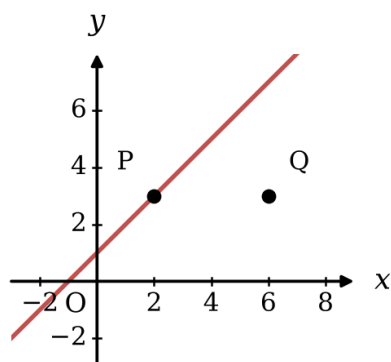
- Show that A and B lie on C. [2]
- Find equations of the tangents at A and at B. [4]
- Find the coordinates of T. [2]
- Show that $TA = \frac{20}{3}$, and find the area of the kite CATB. [4]

- 28** The points $P(x_1, y_1)$ and $Q(-x_1, -y_1)$ are the ends of a diameter of the circle $x^2 + y^2 = r^2$, where x_1 and y_1 are non-zero.

- Write down equations of the tangents at P and at Q. [2]
- Prove that the two tangents are parallel. [2]
- Prove that the perpendicular distance between the tangents is $2r$. [4]

- 29** (a) Prove that the line $y = mx + c$ is a tangent to the circle $x^2 + y^2 = r^2$ if and only if $c^2 = r^2(1 + m^2)$.
(b) Hence find equations of the tangents to the circle $x^2 + y^2 = 20$ that pass through the point $(0, 10)$.
(a) Prove the result. [5]
(b) Find the tangents through $(0, 10)$. [3]

- 30** A circle touches the line $y = x + 1$ at the point $P(2, 3)$, and passes through the point $Q(6, 3)$.



- (a) Show that the centre of the circle lies on the line $x + y = 5$. [3]
 (b) Let the centre be $(a, 5 - a)$. Show that $CP^2 = 2(a - 2)^2$ and find the value of a . [5]
 (c) Find an equation of the circle. [2]

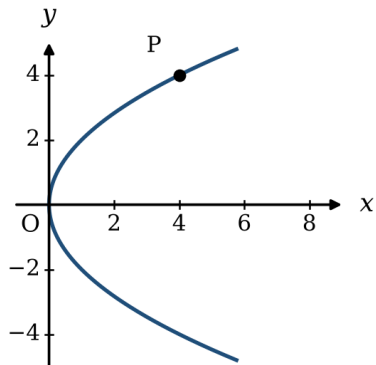
Parametric Equations

- 1** A curve has parametric equations $x = 3t - 1$, $y = t^2 + 2$. Find the coordinates of the points on the curve where $t = 0$, $t = 2$ and $t = -1$. [3]
- 2** A curve has parametric equations $x = 2t$, $y = t + 3$. Find a Cartesian equation of the curve. [2]
- 3** A curve has parametric equations $x = t^2$, $y = 2t$. Show that its Cartesian equation is $y^2 = 4x$. [2]
- 4** Determine whether the point $(5, 8)$ lies on the curve with parametric equations $x = 2t + 1$, $y = t^2 + 4$. [2]
- 5** A curve is given by $x = 4t$, $y = \frac{1}{t}$, $t \neq 0$. Find the value of t at the point where $x = 10$, and the y -coordinate there. [2]
- 6** A curve has parametric equations $x = 2t + a$, $y = t^2 - 3t$, where a is a constant. The curve passes through the point $(7, -2)$. Find the two possible values of a . [4]
- 7** A curve has parametric equations $x = 3t$, $y = t^2 - 1$. Find its Cartesian equation, and state the coordinates of its minimum point. [3]
- 8** A curve has parametric equations $x = \frac{1}{t}$, $y = t + 1$, $t \neq 0$. Show that $xy = x + 1$. [2]
- 9** A curve has parametric equations $x = 1 + 2\cos t$, $y = 3 + 2\sin t$. Find its Cartesian equation and describe the curve. [4]
- 10** The circle C has centre $(-2, 1)$ and radius 3.
 (a) Write down parametric equations for C , in the form $x = f(t)$, $y = g(t)$. [2]
 (b) Find the exact coordinates of the point on C where $t = \frac{\pi}{3}$. [3]
- 11** A curve has parametric equations $x = t + 2$, $y = t^2 - 4$. Find the coordinates of the points where the curve crosses the axes. [4]
- 12** A curve has parametric equations $x = t^2 + 1$, $y = t^3$. Find $\frac{dy}{dx}$ in terms of t , and its value when $t = 2$. [4]

- 13** A curve has parametric equations $x = 2t - 1$, $y = 4 - t^2$. Find the value of t at the point $(3, 0)$ and hence the gradient there. [4]

- 14** A curve has parametric equations $x = t^3$, $y = 3t$. Find a Cartesian equation of the curve in the form $x = f(y)$, and find $\frac{dy}{dx}$ when $t = 1$. [5]

- 15** The curve C has parametric equations $x = t^2$, $y = 2t$.



- (a) Find the gradient of C at the point P where $t = 2$. [3]
 (b) Find equations of the tangent and the normal at P . [5]

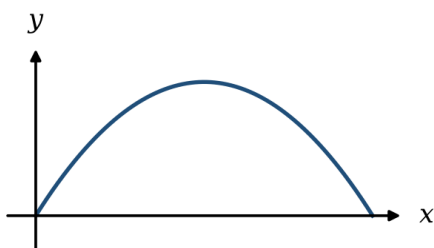
- 16** A curve has parametric equations $x = 4\cos t$, $y = 2\sin t$.

- (a) Find $\frac{dy}{dx}$ in terms of t . [2]
 (b) Show that the tangent to the curve at the point where $t = \frac{\pi}{3}$ has equation $x + 2\sqrt{3}y = 8$. [4]
 (c) The tangent meets the x -axis at A and the y -axis at B . Find the exact area of triangle OAB , where O is the origin. [4]

- 17** A curve has parametric equations $x = \sec t$, $y = \tan t$. Show that $x^2 - y^2 = 1$. [2]

- 18** A curve has parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$, $t \neq 0$. Show that $x^2 - y^2 = 4$. [3]

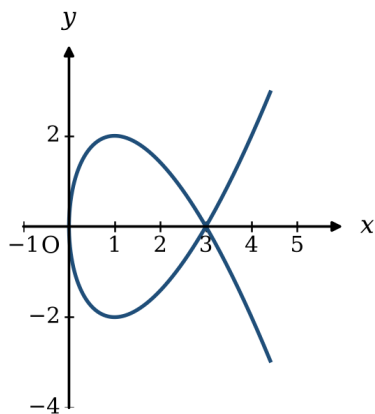
- 19** A ball is thrown so that its position t seconds after being thrown is $x = 20t$, $y = 15t - 5t^2$ (metres), where y is the height above the ground.



- (a) Show that the Cartesian equation of the path is $y = \frac{3x}{4} - \frac{x^2}{80}$. [3]
 (b) Find the maximum height reached. [2]
 (c) Find the horizontal distance travelled before the ball hits the ground. [2]

- 20** The curve C has parametric equations $x = t^2 - 1$, $y = 2t$. Show that the line $y = x + 2$ meets C at exactly one point, and find its coordinates. [4]

- 21** A curve has parametric equations $x = t^2$, $y = t^3 - 3t$.



- (a) Find $\frac{dy}{dx}$ in terms of t . [3]
 (b) Find the coordinates of the points where the tangent is horizontal. [2]
 (c) Find the point where the tangent is vertical. [2]

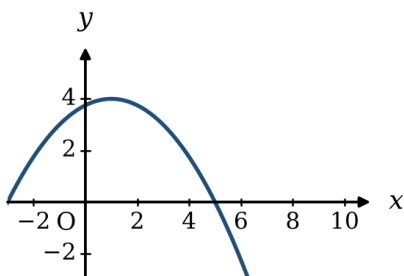
- 22** A curve has parametric equations $x = 2\sin t$, $y = \cos 2t$, for $0 \leq t \leq 2\pi$.

- (a) Show that $y = 1 - \frac{x^2}{2}$. [2]
 (b) State the domain and range of the Cartesian function. [2]

- 23** A curve has parametric equations $x = 3 + 5\cos t$, $y = -1 + 5\sin t$.

- (a) Find a Cartesian equation of the curve. [2]
 (b) Find the value of t at the point $P(6, 3)$, for $0 \leq t < 2\pi$, correct to 3 significant figures. [2]
 (c) Hence find an equation of the tangent at P . [3]

- 24** The curve C has parametric equations $x = 2t + 1$, $y = 4 - t^2$. The curve meets the x -axis at A and B , and V is its highest point.



- (a) Find the coordinates of A , B and V . [4]
 (b) Find the area of triangle ABV . [2]

- 25** A curve has parametric equations $x = t^2$, $y = t^3$.

- (a) Show that the tangent to the curve at the point where $t = p$, $p \neq 0$, has equation $3px - 2y = p^3$. [4]
 (b) The tangent at the point P where $t = 2$ meets the curve again at Q . Find the coordinates of Q . [5]

- 26** Particle A moves so that its position at time t is $(2t, 1 + t)$. Particle B moves so that its position at time s is $(10 - s, 2s - 4)$.

- (a) Find Cartesian equations of the paths of A and B . [3]
 (b) Show that the paths cross at $(6, 4)$, and show that the particles do not collide. [4]

- 27** A curve has parametric equations $x = 4\cos t$, $y = 4\sin t$.

- (a) Find the Cartesian equation of the curve. [2]
 (b) The line $y = x + k$ is a tangent to the curve. Find the possible values of k . [4]

- 28** A curve has parametric equations $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{2t}{1 + t^2}$.
- (a) Show that $x^2 + y^2 = 1$. [2]
 - (b) Find the coordinates of the point where $t = \frac{1}{2}$. [2]
 - (c) Show that the point $(-1, 0)$ is never reached for any real t . [2]
-
- 29** A projectile is launched from the origin with speed 25 m s^{-1} at an angle α above the horizontal. Taking $g = 9.8 \text{ m s}^{-2}$, its position after t seconds is $x = 25t \cos \alpha$, $y = 25t \sin \alpha - 4.9t^2$.
- (a) Show that the path satisfies $y = x \tan \alpha - \frac{4.9x^2}{625}(1 + \tan^2 \alpha)$. [4]
 - (b) The projectile passes through $(30, 10)$. Find the two possible angles of projection, correct to 1 decimal place. [5]
-
- 30** The curve C has parametric equations $x = t^2$, $y = 2t$. The normal to C at the point $P(p^2, 2p)$, $p \neq 0$, meets C again at $Q(q^2, 2q)$.
- (a) Show that the normal at P has equation $y + px = 2p + p^3$. [3]
 - (b) Show that $q = -p - \frac{2}{p}$. [4]
 - (c) Find the coordinates of Q when $p = 1$. [2]
-