

Introduction to Complex Numbers

- 1 Simplify (a) $\sqrt{-25}$, (b) $\sqrt{-18}$. [2]
- 2 Write down the real part and the imaginary part of (a) $4 - 7i$, (b) $3i$. [2]
- 3 Write i^3 , i^4 and i^5 in their simplest form. [3]
- 4 Solve the equation $x^2 + 16 = 0$. [2]
- 5 Express $\sqrt{-36} + \sqrt{-9}$ in the form bi . [2]
- 6 Given that x and y are real and $x + yi = 5 - 3i$, write down x and y . [2]
- 7 Find the values of i^{27} and i^{-1} . [3]
- 8 Solve the equation $x^2 + 2x + 5 = 0$, giving your answers in the form $a + bi$. [3]
- 9 Given that x and y are real and $(x + 3) + (y - 2)i = 7 + 5i$, find x and y . [2]
- 10 Simplify (a) $(2i)^3$, (b) $(-3i)^2$. [2]
- 11 A student writes: " $\sqrt{-4} \times \sqrt{-9} = \sqrt{36} = 6$ ". Explain the error and state the correct value. [3]
- 12 Given that p and q are real and $p + qi = 2i(3 - i)$, find p and q . [3]
- 13 Solve $2x^2 + x + 3 = 0$, giving your answers in the form $a + bi$. [3]
- 14 Evaluate $i^{101} + i^{102} + i^{103} + i^{104}$. [3]
- 15 Given that x and y are real and $(2x + y) + (x - 3y)i = 8 - 3i$, find the values of x and y . [4]
- 16 Show that $(1 + i)^2 = 2i$ and hence evaluate $(1 + i)^8$. [4]
- 17 Solve $x^2 - 6x + 13 = 0$. Verify that the sum of the roots is 6 and that their product is 13. [4]
- 18 Simplify $i^{15} - i^{10} + 3i^{-3}$, giving your answer in the form $a + bi$. [4]
- 19 The equation $x^2 + kx + 4 = 0$ has no real roots. Find the set of possible values of k . For $k = 2$, solve the equation, giving the roots in the form $a + bi$. [5]
- 20 Find the real values of x for which $(x + 2i)^2$ is (a) real, (b) purely imaginary. [4]
- 21 Given that x and y are real and $(x + yi)^2 = 5 + 12i$, find the possible values of x and y . [6]
- 22 Find the value of (a) $\sum_{k=1}^{40} i^k$, (b) $\sum_{k=1}^{42} i^k$. [5]
- 23 The roots of $z^2 - 4z + k = 0$, where $k > 4$ is real, are α and β . Given that the imaginary parts of α and β differ by 6, find the value of k , and write down α and β . [5]
- 24 The equation $z^4 + 4 = 0$ has four complex roots.
 - (a) Show that $1 + i$ is a root of $z^4 + 4 = 0$. [2]
 - (b) Find all four roots. [4]

- 25** Find the real values of a for which $(a + i)^3$ is purely imaginary. [5]
- 26** Find all the possible values of $i^n + i^{-n}$, where n is an integer, and state for which values of n each occurs. [5]
- 27** Find the real values of a for which $(a + i)^4$ is (a) real, (b) purely imaginary. Give exact values. [9]
- 28** Let x and y be real with $(x + yi)^2 = p + qi$, where p and q are real and $q \neq 0$.
 (a) Show that $x^2 - y^2 = p$ and $2xy = q$. [2]
 (b) Hence show that $4x^4 - 4px^2 - q^2 = 0$, and that $x^2 = \frac{p + \sqrt{p^2 + q^2}}{2}$. [5]
 (c) Use these results to find the square roots of $21 + 20i$. [4]
- 29** Let a and b be real numbers.
 (a) Show that $(a + bi)^2 = (a - bi)^2$ if and only if the number $a + bi$ is either real or purely imaginary. [6]
 (b) Hence find all the numbers $a + bi$ that satisfy $(a + bi)^2 = (a - bi)^2$ and $a^2 + b^2 = 25$. [3]
- 30** The sequence u_n is defined by $u_n = i^n + (-i)^n$ for positive integers n .
 (a) Find u_1, u_2, u_3 and u_4 . [2]
 (b) Show that $u_n = 0$ when n is odd and that $u_n = 2(-1)^{\frac{n}{2}}$ when n is even. [4]
 (c) Hence find $\sum_{n=1}^{100} u_n$. [3]

Complex Arithmetic and Conjugates

- 1** Given $z = 3 + 2i$ and $w = 1 - 4i$, find $z + w$ and $z - w$. [2]
- 2** Given $z = 2 - i$ and $w = 4 + 3i$, find $2z - w$ and $z + 3w$. [3]
- 3** Write down the complex conjugate of (a) $5 - 2i$, (b) $7i$, (c) -3 . [3]
- 4** Expand and simplify $(2 + i)(3 - i)$. [2]
- 5** Find the value of $(3 + 4i)(3 - 4i)$. [2]
- 6** Simplify $\frac{1}{i}$, giving your answer in the form bi . [2]
- 7** Express $(1 + 3i)^2$ in the form $a + bi$. [2]
- 8** Express $\frac{3 + i}{1 - i}$ in the form $a + bi$. [3]
- 9** Given $z = 4 - 3i$, find $z + z^*$ and zz^* . [3]
- 10** Solve $(1 + 2i)z = 4 + 3i$, giving z in the form $a + bi$. [3]
- 11** Simplify $(2 + 3i)^2 - (2 - 3i)^2$. [3]
- 12** Given that $z = 3 - i$, find (a) z^2 , (b) $(z^*)^2$. Comment on your answers. [4]
- 13** Given that x and y are real and $(x + iy)(2 + i) = 3 + 4i$, find x and y . [4]

- 14** Express $\frac{2+i}{3-i}$ in the form $a + bi$, and hence find its square. [4]
- 15** Find the complex number z that satisfies $3z + 2z^* = 10 - i$. [4]
- 16** Given that a is real, find the value of a for which $\frac{a+2i}{1-i}$ is (a) purely imaginary, (b) real. [5]
- 17** Solve the simultaneous equations $z + w = 4 + i$ and $z - 2w = 1 + 4i$ for the complex numbers z and w . [4]
- 18** Given $z = 2 + i$, show that $z^2 - 4z + 5 = 0$. State, with a reason, another root of this equation. [4]
- 19** Show that $\frac{2+i}{1-i} + \frac{2-i}{1+i}$ is real, and find its value. [4]
- 20** The complex number $z = a + bi$, where a and b are real. Show that $z + z^* = 2a$ and $zz^* = a^2 + b^2$, and explain why both are real. [3]
- 21** Find the square roots of $-8 + 6i$, giving your answers in the form $a + bi$. [6]
- 22** Find the complex numbers z that satisfy $zz^* + 3(z - z^*) = 13 + 12i$. [5]
- 23** Let $z = a + bi$ and $w = c + di$, where a, b, c, d are real. Prove that $(zw)^* = z^*w^*$. [5]
- 24** Two components in an electric circuit have impedances $Z_1 = 2 + i$ and $Z_2 = 1 - i$ (in ohms). When they are connected in parallel, the total impedance Z satisfies $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$. Find Z in the form $a + bi$. [5]
- 25** The complex number $z = x + iy$ and $w = \frac{z+1}{z-1}$.
 (a) Find the real and imaginary parts of w in terms of x and y . [4]
 (b) Show that if $x^2 + y^2 = 1$ then w is purely imaginary ($z \neq 1$). [2]
- 26** Solve the simultaneous equations $z + iw = 3 + 2i$ and $iz + w = i$ for the complex numbers z and w . [5]
- 27** Let $z = a + bi$ be a non-zero complex number with a and b real.
 (a) Show that $z + \frac{1}{z}$ is real if and only if $b = 0$ or $a^2 + b^2 = 1$. [5]
 (b) Find all the complex numbers z such that $z + \frac{1}{z} = 1$. [4]
 (c) Show that there is no non-real z with $z + \frac{1}{z} = \frac{5}{2}$, and find the real solutions. [3]
- 28** For complex numbers z_1 and z_2 define the conjugate $z^* = a - bi$ when $z = a + bi$.
 (a) Prove that $(z_1 + z_2)^* = z_1^* + z_2^*$ and $(z_1 z_2)^* = z_1^* z_2^*$. [4]
 (b) Deduce that $(z^n)^* = (z^*)^n$ for every positive integer n . [2]
 (c) Let $p(z) = pz^2 + qz + r$, where p, q and r are real. Show that $p(z)^* = p(z^*)$, and deduce that if $p(\alpha) = 0$ then $p(\alpha^*) = 0$. [4]
- 29** Find all the complex numbers z such that $z^2 = z^*$. [9]

30 For real t , let $z = \frac{1}{1 + it}$, where $z = x + iy$ with x and y real.

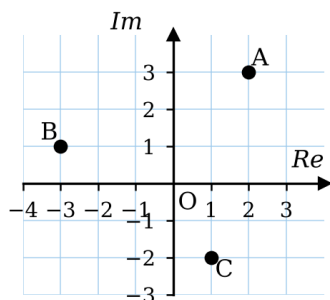
(a) Show that $x = \frac{1}{1 + t^2}$ and $y = -\frac{t}{1 + t^2}$. [3]

(b) Show that $x^2 + y^2 = x$. [3]

(c) Hence show that as t varies the point (x, y) lies on a circle, and give its centre and radius. [3]

Argand Diagrams

1 Write down the complex numbers represented by the points A, B and C on the Argand diagram.



[3]

2 The point P represents $z = 2 + 3i$. Write down the complex number represented by the reflection of P in the real axis, and state its relation to z .

[2]

3 State the coordinates of the points on an Argand diagram that represent $4 - i$, $-2i$ and 5 .

[3]

4 The point P represents the complex number $3 + 4i$. Find the distance OP, where O is the origin.

[2]

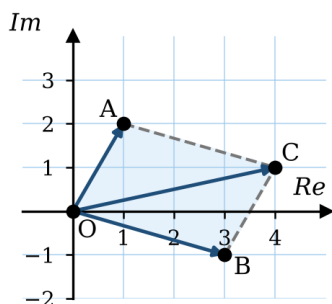
5 Write down $-z$ for $z = 2 - 3i$. Describe the geometrical relationship between the points representing z and $-z$.

[2]

6 Find the complex number represented by the midpoint of the line joining the points representing $2 + i$ and $6 + 5i$.

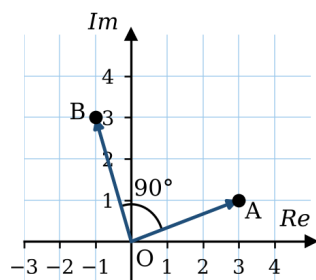
[2]

7 The points A and B represent $z_1 = 1 + 2i$ and $z_2 = 3 - i$. Find $z_1 + z_2$, represented by C, and explain why OACB is a parallelogram (as drawn).



[3]

- 8** The point A represents $z = 3 + i$ and B represents iz . Find iz , and describe the geometrical relationship between OA and OB.



[3]

- 9** Find the distance between the points representing $2 + 3i$ and $5 + 7i$.

[3]

- 10** Describe and sketch the locus of points z on an Argand diagram satisfying $|z - (1 + 2i)| = 3$.

[3]

- 11** Sketch the locus of points z satisfying $\arg z = \frac{\pi}{4}$, and state its Cartesian equation.

[3]

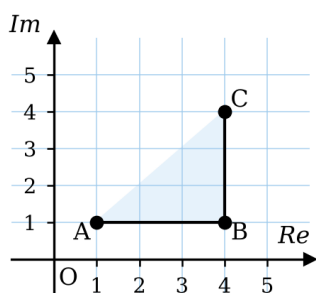
- 12** The complex number $z = t + (2t + 1)i$, where t is real, is represented by a point P on an Argand diagram. Show that P lies on a straight line as t varies, and find its Cartesian equation.

[3]

- 13** Describe the locus of points z for which $|z - 3| = |z + 1|$, and find its Cartesian equation.

[3]

- 14** The points A, B and C represent $1 + i$, $4 + i$ and $4 + 4i$. Given that ABCD is a square, find the complex number represented by D, and the length of the diagonal AC.



[4]

- 15** Shade on an Argand diagram the region in which both $|z - 1| \leq 2$ and $\text{Im } z \geq 0$ hold, and find its area.

[4]

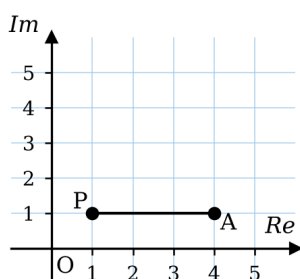
- 16** The complex number z satisfies $|z - 3 - 4i| = 2$. Find the greatest and least possible values of $|z|$.

[4]

- 17** Shade on an Argand diagram the region defined by $1 \leq |z - 2| \leq 3$, and find its area.

[4]

- 18** The point A represents $z = 4 + i$. The point B is obtained by rotating A through 90° anticlockwise about the point P representing $1 + i$. Find the complex number represented by B.

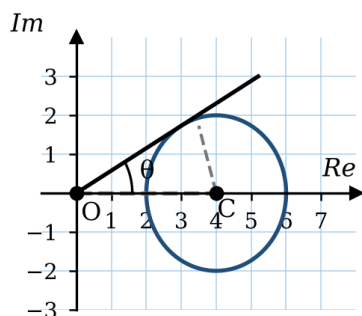


[4]

19 Find the Cartesian equation of (a) the locus $|z - 2 - i| = 3$, (b) the locus $\arg(z - 2) = \frac{\pi}{3}$. [4]

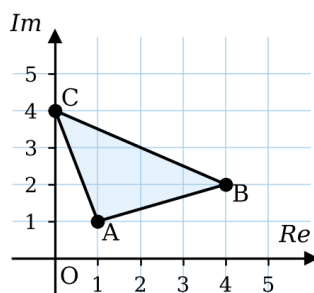
20 Find the complex numbers z which satisfy both $|z| = 5$ and $\operatorname{Im} z = 3$. [4]

21 The complex number z satisfies $|z - 4| = 2$. Find the greatest possible value of $\arg z$, giving your answer as an exact multiple of π .



[4]

22 The points A, B and C represent $1 + i$, $4 + 2i$ and $4i$. Show that triangle ABC is isosceles with a right angle at A, and find its area.

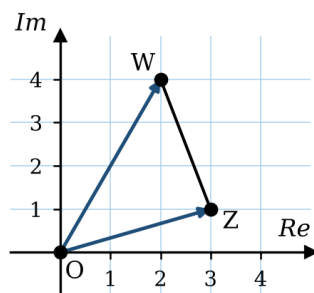


[6]

23 On an Argand diagram sketch the loci $|z - 4| = |z - 4i|$ and $|z - 2| = 2$ on the same axes, and find the complex numbers represented by their points of intersection. [5]

24 The locus L_1 is $\arg(z - 2) = \frac{\pi}{3}$ and the locus L_2 is $|z - 2| = 4$. Sketch both loci on one Argand diagram and find the complex number at which they intersect, giving it in the form $a + bi$. [5]

25 The point Z represents $z = 3 + i$ and the point W represents $w = (1 + i)z$. Find w , and show that $w - z = iz$. Hence show that triangle OZW is isosceles and right-angled at Z.

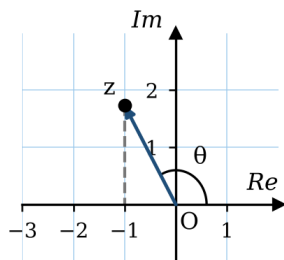


[5]

26 A square ABCD has opposite vertices A and C representing $1 + 2i$ and $5 + 4i$. Find the complex numbers represented by B and D, and the area of the square. [6]

- 27** The complex number z satisfies $|z - 4| = 2|z - 1|$.
- (a) Show that the locus of z is a circle, and state its centre and radius. [4]
- (b) Find the complex number z on this locus with $\arg z = \frac{\pi}{3}$, and verify that it satisfies the original equation. [4]
- (c) Find the greatest value of $\operatorname{Re} z$ on the locus, and the value of $|z - 4|$ there. [2]
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- 28** The region R of the Argand diagram is defined by $|z - 2| \leq 2$ and $|z| \geq |z - 2|$.
- (a) Show that $|z| \geq |z - 2|$ is equivalent to $\operatorname{Re} z \geq 1$, and sketch and shade R . [4]
- (b) Find the complex numbers where the line $\operatorname{Re} z = 1$ meets the circle $|z - 2| = 2$. [2]
- (c) Find the exact area of R . [5]
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- 29** The complex number $z \neq 0$ satisfies $|z - 1| = 1$.
- (a) Show that $zz^* = z + z^*$. [3]
- (b) Hence show that the real part of $\frac{1}{z}$ is $\frac{1}{2}$. [3]
- (c) Hence describe the locus of $w = \frac{1}{z}$ as z varies over $|z - 1| = 1$ ($z \neq 0$). [2]
-
- 30** Let $z_1 = a + bi$ and $z_2 = c + di$, with a, b, c, d real, be represented by the points A and B on an Argand diagram.
- (a) Show that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2(ac + bd)$. [3]
- (b) Use the inequality $(ac + bd)^2 \leq (a^2 + b^2)(c^2 + d^2)$ to prove the triangle inequality $|z_1 + z_2| \leq |z_1| + |z_2|$. [3]
- (c) State the geometrical meaning of this inequality, and state when equality holds. [2]

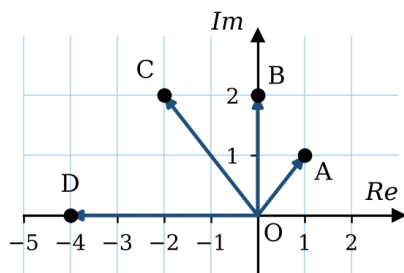
Modulus and Argument

- 1** Find the modulus of (a) $3 + 4i$, (b) $5 - 12i$. [2]
- 2** Find $|-7 + 24i|$. [2]
- 3** Write down the argument of (a) 3 , (b) -2 , (c) $4i$, (d) $-i$, in radians, in the range $-\pi < \theta \leq \pi$. [4]
- 4** Find the exact arguments of $1 + i$ and $1 - i$. [2]
- 5** Write $z = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ in the form $a + bi$. [2]
- 6** Given that $|z| = 5$ and $|w| = 2$, write down $|zw|$ and $|\frac{z}{w}|$. [2]
- 7** Find the modulus and argument of $z = -1 + \sqrt{3}i$, giving the argument as an exact multiple of π .
- 
- [3]
- 8** Find the exact argument of $-3 - 3i$, in the range $-\pi < \theta \leq \pi$. [3]

- 9** Express $1 - \sqrt{3}i$ in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]
- 10** Given $z = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$ and $w = 3(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$, find zw in the form $r(\cos \theta + i \sin \theta)$, and then in the form $a + bi$. [4]
- 11** Given $z = 8(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$ and $w = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$, find $\frac{z}{w}$ in the form $a + bi$. [4]
- 12** Find the modulus and the argument of $-4 + 3i$, giving the argument in radians to 3 significant figures. [3]
- 13** The complex number z is given by $z = 4(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4})$. Write z in the form $r(\cos \theta + i \sin \theta)$ with $r > 0$ and $-\pi < \theta \leq \pi$, and write down $\arg z^*$. [3]
- 14** Consider the complex numbers $1 + i$ and $\sqrt{3} - i$ and their product.
 (a) Find the modulus and argument of $(1 + i)(\sqrt{3} - i)$, using the moduli and arguments of the two factors. [3]
 (b) Hence show that $\tan \frac{\pi}{12} = 2 - \sqrt{3}$. [3]
- 15** Given $z = -1 + i$, find $\arg z$, and show that $\arg(z^2)$ is not equal to $2 \arg z$ when $\arg$ is taken in the range $-\pi < \theta \leq \pi$. Explain this. [4]
- 16** Find the exact value of $(1 + \sqrt{3}i)^6$. [4]
- 17** Express $\frac{1}{1+i}$ in modulus-argument form. [4]
- 18** The complex number z has modulus $2\sqrt{2}$ and argument $-\frac{\pi}{4}$. Find z in the form $a + bi$, and find z^2 . [4]
- 19** Let $z_1 = 1 + \sqrt{3}i$ and $z_2 = \sqrt{3} + i$. Find $\frac{z_1}{z_2}$ (a) by using moduli and arguments, (b) by rationalising the denominator, and check that the answers agree. [5]
- 20** Given that a is real, find the value of a for which $\arg(a + 3i) = \frac{2\pi}{3}$. Given that b is real, find the value of b for which $\arg(4 + bi) = -\frac{\pi}{6}$. [4]
- 21** Find the smallest positive integer n for which $(\sqrt{3} + i)^n$ is (a) real, (b) purely imaginary. Find the value of $(\sqrt{3} + i)^n$ in each case. [5]
- 22** Show that $(1 - i)^{10} = -32i$, using the modulus-argument form of $1 - i$. [5]
- 23** The complex numbers z and w satisfy $\arg z = \frac{3\pi}{4}$ and $\arg w = -\frac{\pi}{3}$. Find the principal value of $\arg(\frac{z}{w})$ (in the range $-\pi < \theta \leq \pi$), and of $\arg(zw)$. [5]
- 24** The complex number z satisfies $|z| = 1$. Show that $z^* = \frac{1}{z}$ and deduce that $\arg(z^*) = -\arg z$. Hence show that $z + \frac{1}{z} = 2 \cos \theta$, where $\theta = \arg z$. [5]

- 25 Evaluate $\frac{(1+i)^8}{(\sqrt{3}-i)^6}$, giving the exact value. [4]

- 26 The points A, B, C and D on an Argand diagram represent z , z^2 , z^3 and z^4 , where $z = 1 + i$, as shown. Find z^2 , z^3 and z^4 in the form $a + bi$, and describe how the modulus and argument of z^n change as n increases.



[6]

- 27 Let θ be an angle with $0 < \theta < \pi$.
- (a) Show that $1 + \cos \theta + i \sin \theta = 2\cos \frac{\theta}{2}(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2})$. [3]
- (b) State the modulus and argument of $1 + \cos \theta + i \sin \theta$. [2]
- (c) Hence find the modulus and argument of $(1 + \cos \theta + i \sin \theta)^3$, and for $\theta = \frac{\pi}{3}$ give its value in the form $a + bi$. [4]

- 28 Let $z_1 = r_1(\cos \alpha + i \sin \alpha)$ and $z_2 = r_2(\cos \beta + i \sin \beta)$, where $r_1, r_2 > 0$.
- (a) Prove that $z_1 z_2 = r_1 r_2(\cos(\alpha + \beta) + i \sin(\alpha + \beta))$. [4]
- (b) Hence prove that $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\alpha - \beta) + i \sin(\alpha - \beta))$. [3]
- (c) Deduce that $\arg(z^*) = -\arg z$ when $\arg z$ is not π . [2]

- 29 Consider the complex numbers $1 + \sqrt{3}i$ and $1 - i$.
- (a) Express $1 + \sqrt{3}i$ and $1 - i$ in modulus-argument form. [3]
- (b) Hence, by considering the product $(1 + \sqrt{3}i)(1 - i)$, find the exact values of $\cos \frac{\pi}{12}$ and $\sin \frac{\pi}{12}$. [6]

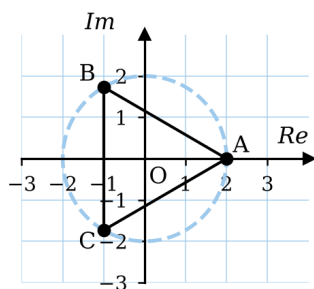
- 30 The complex numbers z_1 and z_2 are represented by points A and B on an Argand diagram.
- (a) Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$. [4]
- (b) Explain what this result says about the parallelogram with vertices at the origin, A, B and the point representing $z_1 + z_2$. [2]
- (c) Given that $|z_1| = 3$, $|z_2| = 5$ and $|z_1 + z_2| = 7$, find $|z_1 - z_2|$. [3]

Complex Roots of Equations

- 1 Solve $z^2 + 25 = 0$. [2]
- 2 The equation $z^2 + bz + c = 0$ has real coefficients b and c , and one root is $3 + 2i$. Write down the other root. [1]
- 3 Find the quadratic equation with real coefficients which has roots $1 + i$ and $1 - i$. [3]
- 4 Solve $z^2 - 2z + 5 = 0$. [2]

- 5** The roots of $z^2 - 6z + 25 = 0$ are α and β . Write down $\alpha + \beta$ and $\alpha\beta$, and find the roots. [3]
- 6** Show that $z = i$ is a root of $z^3 + z^2 + z + 1 = 0$. [2]
- 7** Solve $z^2 + 4z + 13 = 0$, giving your answers in the form $a + bi$. [3]
- 8** The equation $z^2 + pz + q = 0$, where p and q are real, has a root $2 - 3i$. Find p and q . [3]
- 9** Show that $1 + 2i$ is a root of $z^2 - 2z + 5 = 0$ and hence factorise $z^2 - 2z + 5$ into linear factors. [4]
- 10** Solve $2z^2 - 2z + 5 = 0$. [3]
- 11** Factorise $z^3 - 1$ and hence solve $z^3 = 1$, giving the non-real roots in the form $a + bi$. [4]
- 12** Show that the equation $z^2 - 2az + a^2 + 1 = 0$, where a is a real constant, has non-real roots for every value of a , and find the roots in terms of a . [4]
- 13** Solve $z^4 = 16$ for complex z . [3]
- 14** Show that $z = 1$ is a root of $z^3 - 3z^2 + 4z - 2 = 0$ and find the other two roots. [5]
- 15** Show that $1 + i$ is a root of $z^3 - 4z^2 + 6z - 4 = 0$ and find the other roots. [5]
- 16** The equation $z^2 + kz + 9 = 0$ has non-real roots, where k is real. Find the possible values of k . For $k = 2$, solve the equation, giving exact roots. [5]
- 17** The real numbers p and q are such that $z = -1 + 2i$ is a root of $z^2 + pz + q = 0$. By substituting this root and equating real and imaginary parts, find p and q . [4]
- 18** Solve the equation $z^2 = -5 + 12i$, giving the roots in the form $a + bi$. [5]
- 19** Solve the quadratic equation $z^2 - 2iz - 5 = 0$, which has complex coefficients. [4]
- 20** The roots of $z^2 - 4z + 13 = 0$ are α and β . Write down $\alpha + \beta$ and $\alpha\beta$ and hence find $\alpha^2 + \beta^2$ and $\frac{1}{\alpha} + \frac{1}{\beta}$ without finding α and β . Check your first answer by solving the equation. [5]
- 21** A cubic equation with real coefficients has roots 3 and $1 - 2i$. Find the equation in the form $z^3 + az^2 + bz + c = 0$, and state the third root. [5]
- 22** Given that $1 + 2i$ is a root of $z^4 - 4z^3 + 11z^2 - 14z + 10 = 0$, find all four roots. [7]
- 23** Show that $z = 2 + i$ is a root of $2z^3 - 9z^2 + 14z - 5 = 0$ and find the other two roots. [6]
- 24** The real numbers a and b are such that $1 + i$ is a root of $z^3 + az + b = 0$. Find a and b , and find the other two roots. [6]

- 25** Solve $z^3 = 8$, giving the non-real roots in the form $a + bi$. The roots are represented by points A, B and C on an Argand diagram. Show that $|A| = |B| = |C| = 2$ and that triangle ABC is equilateral.



[6]

- 26** Solve the quadratic equation $z^2 - 3z + (3 + i) = 0$, which has complex coefficients.

[6]

- 27** The polynomial $p(z) = z^3 + az^2 + bz + c$ has real coefficients a , b and c .

- Show that if α is a root of $p(z) = 0$ then so is its conjugate α^* .
- Hence prove that $p(z) = 0$ always has at least one real root.
- Give an example of a cubic with real coefficients with exactly one real root, and state its roots.

[3]

[3]

[2]

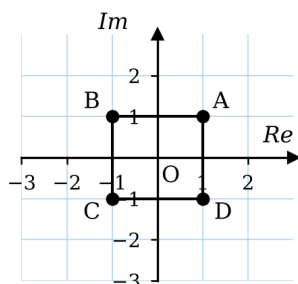
- 28** Given that $1 + 2i$ is a root of $z^3 + az^2 + bz + 10 = 0$, where a and b are real:

- find the values of a and b and the other two roots.
- find, without squaring each root separately, the value of the sum of the squares of the three roots, and check your answer.

[6]

[4]

- 29** The equation $z^4 + 4 = 0$ has four complex roots.



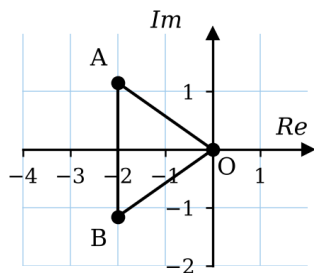
- Show that $z^4 + 4 = (z^2 + 2)^2 - 4z^2$, and hence factorise $z^4 + 4$ into two real quadratic factors.
- Hence solve $z^4 + 4 = 0$, and show that the four roots are the vertices of a square, finding its area.
- Find a quadratic equation with real coefficients whose roots are the squares of the roots of $z^4 + 4 = 0$.

[4]

[5]

[2]

- 30** The equation $z^2 + 4z + k = 0$, where $k > 4$ is real, has roots α and β , represented on an Argand diagram by the points A and B. The origin is O.



- (a) Show that $\alpha, \beta = -2 \pm i\sqrt{k-4}$, and find $|OA|$ and $|AB|$ in terms of k . [4]
- (b) Given that triangle OAB is equilateral, find the exact value of k . [4]
- (c) For this value of k , find the exact area of triangle OAB. [2]