

Algorithms and Flowcharts

1 (2 marks)

(a) $m = 4 \times 5 - 3 = 17$

B1

Answer: 17

1 mark

(b) $m = 4 \times (-2) - 3 = -11$

B1

Answer: -11

1 mark

2 (2 marks)

(a) $c = 22, d = 11$

B1

Answer: 11

1 mark

(b) The mean of a and b

B1

Answer: The mean of a and b

1 mark

3 (2 marks)

(a) $y = 25 - 3 = 22$ and $22 > 10$, so the output is Large

B1

Answer: Large

1 mark

(b) $y = 4 - 3 = 1$ and 1 is not greater than 10, so the output is Small

B1

Answer: Small

1 mark

4 (2 marks)

(a) 14 is even, so $r = 7$

B1

Answer: 7

1 mark

(b) 7 is odd, so $r = 3 \times 7 + 1 = 22$

B1

Answer: 22

1 mark

5 (3 marks)

$47 \div 6 = 7.83\dots$, so $q = 7$

M1

$r = 47 - 6 \times 7 = 5$

A1

r is the remainder when a is divided by b

E1

Answer: 5, the remainder on dividing 47 by 6

3 marks

6 (2 marks)

M changes $7 \rightarrow 12 \rightarrow 15$, other numbers do not change M, so the output is 15

B1

It finds the largest number in the list

B1

Answer: 15, the largest number in the list

2 marks

7 (4 marks)

(a) t after each pass: 1, 4, 9, 16, 25 (x is 1, 3, 5, 7, 9 when it is added)

M1 A1

When x becomes 11 the test $x > 9$ is true, so the output is 25

A1

Answer: 25

3 marks

(b) The sum of the first five odd numbers ($1 + 3 + 5 + 7 + 9$)

B1

Answer: Sum of the first five odd numbers

1 mark

8 (4 marks)

$k = 1$: test true, $A = 1, k = 2$

M1

$k = 2$: $A = 1 + 4 = 5, k = 3$

A1

$k = 3$: $A = 5 + 9 = 14$; $k = 4$: $A = 14 + 16 = 30, k = 5$

A1

$k = 5 > 4$, test false, so the output is 30

A1

Answer: 30

4 marks

9 (4 marks)

$84 = 2 \times 36 + 12$, so $r = 12$

M1

$a = 36, b = 12$: $36 = 3 \times 12 + 0$, so $r = 0$

A1

Output is $b = 12$

A1

12 is the highest common factor of 84 and 36

E1

Answer: 12, the highest common factor of 84 and 36

4 marks

10 (4 marks)

Remainders: $45 \rightarrow 1$; $22 \rightarrow 0$; $11 \rightarrow 1$; $5 \rightarrow 1$; $2 \rightarrow 0$; $1 \rightarrow 1$, and then $n = 0$ so stop

M1 A1

Reverse order gives 101101

A1

This is 45 written in base 2 (binary)

E1

Answer: 101101, the binary form of 45

4 marks

11 (4 marks)

(a) Pass 1: $c = 2$, $a = 1$, $b = 2$, $k = 3$; pass 2: $c = 3$, $a = 2$, $b = 3$, $k = 4$

M1 A1

Pass 3: $c = 5$, $a = 3$, $b = 5$, $k = 5$; pass 4: $c = 8$, $a = 5$, $b = 8$, $k = 6$; now $k < n$ is false so the output is 8

A1

Answer: 8

3 marks

(b) The Fibonacci sequence; 8 is the 6th term, so the n th term is printed

B1

Answer: Fibonacci sequence; the 6th term

1 mark

12 (4 marks)

(a) T: $1 + 1 = 2$, then $2 + 2 = 4$, then $4 + 3 = 7$

M1

Output 7 (the correct sum is 6)

A1

Answer: 7

2 marks

(b) T starts at 1 so the first value, $k = 1$, is counted twice

E1

Step 2 should be: Let $T = 0$ and $k = 1$

B1

Answer: Start with $T = 0$

2 marks

13 (3 marks)

Multiples of 3: 3, 6, 9; multiples of 4: 4, 8; multiple of 12: 12

M1

Output for $k = 1$ to 6: 1, 2, A, B, 5, A

A1

Output for $k = 7$ to 12: 7, B, A, 10, 11, AB

A1

Answer: 1, 2, A, B, 5, A, 7, B, A, 10, 11, AB

3 marks

14 (5 marks)

(a) (a, b): $(48, 18) \rightarrow (30, 18) \rightarrow (12, 18) \rightarrow (12, 6) \rightarrow (6, 6)$

M1 A1 A1

Now $a = b = 6$, so the output is 6

A1

Answer: 6

4 marks

(b) The highest common factor of the two inputs

B1

Answer: Highest common factor of a and b

1 mark

15 (6 marks)

(a) $d = 2, 3, 4, 5, 6$: 91 is not divisible by any of these

M1

$d = 7$: $91 = 7 \times 13$, so the output is Not prime

A1

Answer: Not prime

2 marks

(b) $d = 2, 3, \dots, 9$: 97 is not divisible by any of these

M1 A1

When $d = 10$, $10 \times 10 > 97$ so the output is Prime and the final value of d is 10

A1

Answer: Prime, final $d = 10$

3 marks

(c) If $n = pq$ with $p \leq q$ then $p \times p \leq pq = n$, so any composite n has a factor not exceeding $\sqrt{n}$

E1

Answer: Any composite n has a factor at most $\sqrt{n}$

1 mark

16 (4 marks)

1 Input n and the n numbers

B1

2 Let $T = 0$ and $k = 1$

B1

3 Let $T = T + (k\text{-th number})$, then Let $k = k + 1$; if $k \leq n$ repeat step 3

M1

4 Print $T \div n$

A1

Answer: Total the n numbers in a loop, then print the total divided by n

4 marks

17 (4 marks)

Pass 1: $x = (1 + 10) \div 2 = 5.5$

B1

Pass 2: $x = (5.5 + 10 \div 5.5) \div 2 = 3.6591$

M1 A1

Pass 3: $x = (3.6591 + 10 \div 3.6591) \div 2 = 3.1960$, so the output is 3.196

A1

Answer: 3.196, an approximation to $\sqrt{10} = 3.162$ (3 d.p.)

4 marks

18 (4 marks)

98765 is printed; $9 + 8 + 7 + 6 + 5 = 35$, so 35 is printed

M1 A1

$3 + 5 = 8$, so 8 is printed and the algorithm stops, with output 98765, 35, 8

A1

The digit sum of a multiple of 9 is again a multiple of 9, so the final single digit printed is 9

E1

Answer: 98765, 35, 8; for a multiple of 9 the last number printed is 9

4 marks

19 (5 marks)

(a) Pass 1: $n = 307$, $c = 1$; pass 2: $n = 30$, $c = 2$; pass 3: $n = 3$, $c = 3$; pass 4: $n = 0$, $c = 4$

M1 A1

Test $n > 0$ is false, so output 4

A1

Answer: 4

3 marks

(b) $n = 0$ has one digit, but the test is false at once

E1

The output is 0 (not 1)

B1

Answer: $n = 0$ gives 0

2 marks

20 (4 marks)

(a) $5 + 12 > 13$ and $25 + 144 = 169$

B1

Answer: Right-angled

1 mark

(b) $6 + 8 > 11$, so a triangle exists

M1

$36 + 64 = 100 \neq 121$, so the output is Other

A1

Answer: Other

2 marks

(c) $3 + 4 = 7 \leq 8$

B1

Answer: Not a triangle

1 mark

21 (7 marks)

(a) $H = 0.5$ and $S = 1 + 1/3 = 4/3$ at the start

B1

$X = 1.5$: $S = 4/3 + 2/1.5 = 8/3$; $X = 2$: $S = 8/3 + 1 = 11/3$

M1 A1

$X = 2.5$: $S = 11/3 + 0.8 = 67/15$; then $X = 3 \geq B$, so $T = 0.5 \times 67/15 \div 2 = 67/60$

A1

Answer: 67/60

4 marks

(b) Exact area = $\ln 3 = 1.0986\dots$; $T = 67/60 = 1.1167$

B1

Percentage error = $(1.1167 - 1.0986) \div 1.0986 \times 100 = 1.65\%$

M1 A1

Answer: 1.65%, the trapezium rule with 4 strips

3 marks

22 (8 marks)

(a) $s = 0, 1, 2, \dots, 7$

M1 A1

$(7 + 1)^2 = 64 > 50$ so the output is 7

A1

Answer: 7

3 marks

(b) The loop stops at the first s with $(s + 1)^2 > N$, so each earlier value s satisfied $(s + 1)^2 \leq N$, hence every value passed satisfies $s^2 \leq N$

M1 A1

So $s^2 \leq N < (s + 1)^2$, that is $s = \text{INT}(\sqrt{N})$

A1

Answer: $s = \text{INT}(\sqrt{N})$

3 marks

(c) Step 3 is carried out $\text{INT}(\sqrt{N})$ times

B1

For $N = 10^{12}$ that is 10^6 passes; the number of passes grows with $\sqrt{N}$, so a quicker method (such as halving the interval) is better

E1

Answer: $\text{INT}(\sqrt{N})$ times

2 marks

23 (9 marks)

(a) Pass 1: $i = 1$, $j = 6$, $s = 3 + 25 = 28 < 37$, so $i = 2$

M1 A1

Pass 2: $i = 2$, $j = 6$, $s = 8 + 25 = 33 < 37$, so $i = 3$

A1

Pass 3: $i = 3$, $j = 6$, $s = 12 + 25 = 37$, so output 12 and 25

A1

Answer: 12 and 25

4 marks

(b) $28 < 30$: $i = 2$; $33 > 30$: $j = 5$; $8 + 20 = 28 < 30$: $i = 3$; $12 + 20 = 32 > 30$: $j = 4$

M1 A1

$12 + 17 = 29 < 30$: $i = 4$, then $i = j$ so the output is None

A1

Answer: None

3 marks

(c) The list is in increasing order, so the largest possible partner of position i is the number in position j

E1

Since that sum is already less than T , every other partner gives an even smaller sum

E1

Answer: Even the largest remaining partner gives too small a sum**2 marks****24 (6 marks)****(a)** Pass 1: $a = 3$, $n = 13$, $r = 1$; pass 2: $a = 9$, $n = 6$, $r = 3$ **M1 A1**Pass 3: $a = 81$, $n = 3$, $r = 3$; pass 4: $a = 6561$, $n = 1$, $r = 243$ **A1**After pass 4: $r = 243 \times 6561 = 1594323$, $n = 0$ **A1****Answer: 1594323****4 marks****(b)** The output is a^n , here $3^{13} = 1594323$ **B1** $13 = 1101$ in binary and r is multiplied exactly when the binary digit (from the right) is 1**E1****Answer: a^n ; the 1s in the binary form of n** **2 marks****25 (7 marks)****(a)** $S = 0 + 27 + 0 + 42 + 24 + 0 + 18 + 3 + 10$ **M1 A1** $S = 130$; the next multiple of 11 is 132, so $c = 2$ **A1****Answer: 2****3 marks****(b)** Contribution of d_2 and d_3 was $9 \times 3 + 8 \times 0 = 27$ and is now $9 \times 0 + 8 \times 3 = 24$, so S falls by 3**M1**New $S = 127$, next multiple of 11 is 132, so the new check digit is 5**A1**Swapping adjacent digits p and q changes S by (weight difference 1) $\times (p - q)$, that is by $\pm(p - q)$ **M1** $0 < |p - q| \leq 9 < 11$, so the change is never a multiple of 11 and the check digit always changes**E1****Answer: New check digit 5; the change in S is $p - q$, never a multiple of 11****4 marks****26 (7 marks)****(a)** $n = 28$: divisors 1, 2, 4, 7, 14, so $s = 28$ and the output is 28, Perfect**M1 A1** $n = 12$: divisors 1, 2, 3, 4, 6, $s = 16$, output 16 only**A1****Answer: 28 and Perfect; 16****3 marks****(b)** d runs from 1 to 495, so 495 tests**B1****Answer: 495****1 mark****(c)** No divisor of n other than n itself is larger than $n \div 2$ **E1**

Tests become 248 instead of 495

M1A saving of $495 - 248 = 247$ tests**A1****Answer: 247 fewer tests****3 marks****27 (9 marks)****(a)** The output is the number of steps taken to reach 1, so work backwards from 1. The only input that reaches 1 in one step is 2 ($3n + 1 = 1$ has no positive solution)**M1**A value m can be reached from $2m$, or from $(m - 1) \div 3$ if that is an odd integer. Output 2: 4;**A1**output 3: 8 ($(4 - 1) \div 3 = 1$ does not count, since $n = 1$ stops at once)Output 4: 16, since $(8 - 1) \div 3$ is not an integer. So 16 is the only input with output 4**A1****Answer: 16****3 marks****(b)** A value m is reached from $2m$, and also from $(m - 1) \div 3$ if that is an odd integer (and not 1)**M1**Output 5: 32 and 5 (since $(16 - 1) \div 3 = 5$)**A1**

Output 6: 64 and 10 (from 5; 32 gives no integer)

A1Output 7: 128, 21, 20 and 3 (from 64 and 10: $(64 - 1) \div 3 = 21$, $(10 - 1) \div 3 = 3$)**M1 A1**Output 8: 256, 42, 40 and 6 (the other predecessors $(128 - 1) \div 3$, $(21 - 1) \div 3$, $(20 - 1) \div 3$, $(3 - 1) \div 3$ are not integers)**A1****Answer: 6, 40, 42 and 256****6 marks****28 (9 marks)****(a)** $1071 = 2 \times 462 + 147$; $462 = 3 \times 147 + 21$ **M1 A1** $147 = 7 \times 21 + 0$, so $r = 0$ and the output is 21**A1****Answer: 21****3 marks****(b)** If d divides a and b then d divides $a - qb = r$ **M1**If d divides b and r then d divides $qb + r = a$ **A1**So the pairs (a, b) and (b, r) have the same common divisors and hence the same highest common factor**E1**

At the end $r = 0$ and the highest common factor of $(b, 0)$ is b

E1

Answer: Pairs (a, b) and (b, r) have the same common divisors

4 marks

(c) The remainder r satisfies $0 \leq r < b$, so b strictly decreases at each pass

E1

A strictly decreasing sequence of non-negative integers must reach 0 after finitely many passes

E1

Answer: b strictly decreases through the non-negative integers

2 marks

29 (10 marks)

(a) After n passes $s = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$

M1

The algorithm stops at the first n with $\frac{n(n+1)}{2} > N$

A1

That is the smallest n with $n(n+1) > 2N$

A1

Answer: Smallest n with $n(n+1) > 2N$

3 marks

(b) $N = 500$: $31 \times 32 = 992 \leq 1000$ and $32 \times 33 = 1056 > 1000$, so the output is 32

M1 A1

$N = 10^6$: $n(n+1) > 2 \times 10^6$; $1414 \times 1415 = 2000810 > 2000000$ and $1413 \times 1414 = 1997982 < 2000000$

M1

The output is 1414

A1

Answer: 32; 1414

4 marks

(c) $n(n+1) \approx n^2$ so $n^2 \approx 2N$ and $n \approx \sqrt{2N}$

M1 A1

Solve $n^2 + n - 2N > 0$ directly: n is the smallest integer greater than $\frac{-1 + \sqrt{1 + 8N}}{2}$

E1

Answer: $n \approx \sqrt{2N}$; solve the quadratic directly

3 marks

30 (10 marks)

(a) $87 + 78 = 165$; $165 + 561 = 726$

M1

$726 + 627 = 1353$; $1353 + 3531 = 4884$

A1

4884 reads the same backwards, so the output is 4884 and 4 (four additions)

A1

Answer: 4884 and 4

3 marks

(b) $R = 021 = 21$ (leading zero dropped), $21 \neq 120$

M1

$120 + 21 = 141$, and 141 is the same reversed, so the output is 141 and 1

A1

The reversed number has fewer digits because of the trailing zero in 120

E1

Answer: 141 and 1

3 marks

(c) $n = 10a + b$, $R = 10b + a$, $n + R = 11(a + b)$

M1

If $a + b \leq 9$ then $n + R = 11(a + b) = 10(a + b) + (a + b)$ is the two-digit number with equal digits, a palindrome

A1

If $a + b = 10$ then $n + R = 110$, which is not a palindrome (reverse 011 = 11)

M1

For example $n = 19$: $19 + 91 = 110$, so more additions are needed

A1

Answer: $11(a + b)$ is a palindrome when $a + b \leq 9$

4 marks

Sorting Algorithms

1 (2 marks)

Compare adjacent pairs from the left, swapping when the left item is larger: 8 and 3 swap, 8 and 5 swap, 8 and 9 do not, 9 and 2 swap, 9 and 6 swap

M1

3, 5, 8, 2, 6, 9

A1

Answer: 3, 5, 8, 2, 6, 9

2 marks

2 (2 marks)

The list has 7 items so the pivot is the 4th item, 5

B1

Items smaller than 5 go to the left, the rest (in order) to the right: 3 | 5 | 14, 9, 20, 11, 17

B1

Answer: Pivot 5; list 3, 5, 14, 9, 20, 11, 17

2 marks

3 (2 marks)

Bin 1: 4, 3, 2; Bin 2: 7; Bin 3: 5; Bin 4: 6

M1 A1

Answer: 4 bins: Bin 1: 4, 3, 2; Bin 2: 7; Bin 3: 5; Bin 4: 6

2 marks

4 (2 marks)

Middle item of the 7 numbers is 18; $31 > 18$ so keep 23, 31, 40

M1

Middle of 23, 31, 40 is 31, found

A1

Answer: 18 then 31 (found)

2 marks

5 (2 marks)

(a) $9 - 1 = 8$

B1

Answer: 8

1 mark

(b) Each pass fixes the largest remaining item, so at most 8 passes

B1

Answer: 8

1 mark

6 (4 marks)

(a) Compare the first number with the second and swap them if the second is larger than the first

B1

Move on one place and repeat (second with third, third with fourth, ...) until the end of the list is reached

B1

Answer: Compare adjacent pairs from the left, swapping when the right-hand number is larger

2 marks

(b) 5, 8: swap; 5, 3: no swap; 3, 9: swap; 3, 4: swap (three swaps in four comparisons)

M1

8, 5, 9, 4, 3

A1

Answer: 8, 5, 9, 4, 3

2 marks

7 (5 marks)

Pass 1: 5, 9, 3, 8, 12 (comparisons 4, swaps 4)

M1

Pass 2: 5, 3, 8, 9, 12 (comparisons 3, swaps 2)

A1

Pass 3: 3, 5, 8, 9, 12 (comparisons 2, swaps 1)

A1

Pass 4: 3, 5, 8, 9, 12 (comparisons 1, swaps 0)

Final pass has no swaps so the sort is complete: 3, 5, 8, 9, 12

A1

Total swaps = 7

B1

Answer: 3, 5, 8, 9, 12; 7 swaps

5 marks

8 (4 marks)

Pass 1: 9, 4, 7, 6, 3, 2 (comparisons 5, swaps 4)

M1

Pass 2: 9, 7, 6, 4, 3, 2 (comparisons 4, swaps 2)

A1

Pass 3: 9, 7, 6, 4, 3, 2 (comparisons 3, swaps 0)

A1

Descending order: 9, 7, 6, 4, 3, 2

A1

Answer: 9, 7, 6, 4, 3, 2

4 marks

9 (4 marks)

Start: (23 8 15 4 19 11 30)

Pass 1: **4** (23 8 15 19 11 30)

M1

Pass 2: **4** (8 15 11) **19** (23 30)

A1

Pass 3: **4** (8 11) **15 19 23 30**

A1

Pass 4: **4 8 11 15 19 23 30**

Sorted list: 4, 8, 11, 15, 19, 23, 30 (the pivots are shown in bold)

A1

Answer: 4, 8, 11, 15, 19, 23, 30

4 marks

10 (4 marks)

(a) Bin 1: 6, 4, 2; Bin 2: 5, 3; Bin 3: 7, 5

M1 A1

Answer: 3 bins

2 marks

(b) Total = 32, $32 \div 12 = 2.67$, so at least 3 bins

M1

The packing uses 3 bins, equal to the bound, so the packing is optimal

A1

Answer: Lower bound 3; the packing is optimal

2 marks

11 (3 marks)

Sort into decreasing order: 9, 8, 7, 6, 5, 4, 3, 2

M1

Bin 1: 9, 3; Bin 2: 8, 4; Bin 3: 7, 5; Bin 4: 6, 2

M1 A1

Answer: 4 bins: Bin 1: 9, 3; Bin 2: 8, 4; Bin 3: 7, 5; Bin 4: 6, 2

3 marks

12 (3 marks)

Stage 1: items 3, 7, 12, 15, 21, 24, 28, 33, 40, 47, middle item 24

M1

Stage 2: items 28, 33, 40, 47, middle item 40

A1

Stage 3: items 28, 33, middle item 33

Stage 4: items 28, middle item 28

No items remain after comparing with 28, so 26 is not in the list

E1

Answer: 26 is not in the list

3 marks

13 (5 marks)

(a) Bin 1: 5, 4; Bin 2: 7, 3; Bin 3: 6, 2; Bin 4: 5; Bin 5: 8

M1 A1

Answer: 5 bins

2 marks

(b) Full bins: $5 + 5 = 10$, $7 + 3 = 10$, $4 + 6 = 10$ and $2 + 8 = 10$

M1

Four bins, all full; the total size is 40, so at least 4 bins are needed

A1 E1

Answer: 4 bins, each full

3 marks

14 (5 marks)

(a) Pass 1: 3, 11, 17, 6, 14, 9, 20, 25 (comparisons 7, swaps 6)

M1

Pass 2: 3, 11, 6, 14, 9, 17, 20, 25 (comparisons 6, swaps 3)

A1

Pass 3: 3, 6, 11, 9, 14, 17, 20, 25 (comparisons 5, swaps 2)

A1

Pass 4: 3, 6, 9, 11, 14, 17, 20, 25 (comparisons 4, swaps 1)

Pass 5: 3, 6, 9, 11, 14, 17, 20, 25 (comparisons 3, swaps 0)

Answer: 3, 6, 9, 11, 14, 17, 20, 25

3 marks

(b) Comparisons: $7 + 6 + 5 + 4 + 3 = 25$

B1

Swaps: $6 + 3 + 2 + 1 + 0 = 12$

B1

Answer: 25 comparisons, 12 swaps

2 marks

15 (5 marks)

Start: (18 7 25 12 30 4 21 15 9)

Pass 1: **30** (18 7 25 12 4 21 15 9)

M1

Pass 2: **30** (18 7 25 12 21 15 9) **4**

A1

Pass 3: **30** (18 25 21 15) **12** (7 9) **4**

A1

Pass 4: **30 25 21** (18 15) **12 9 7 4**Pass 5: **30 25 21 18 15 12 9 7 4**

Sorted list (descending): 30, 25, 21, 18, 15, 12, 9, 7, 4

A1

Number of passes: 5

B1

Answer: 30, 25, 21, 18, 15, 12, 9, 7, 4 in 5 passes

5 marks

16 (4 marks)

Pass 1: Abi, Kim, Tom, Eve, Ben, Zoe

M1

Pass 2: Abi, Kim, Eve, Ben, Tom, Zoe

A1

Pass 3: Abi, Eve, Ben, Kim, Tom, Zoe

A1

Pass 4: Abi, Ben, Eve, Kim, Tom, Zoe

Pass 5: Abi, Ben, Eve, Kim, Tom, Zoe

The sort stops after pass 5 (no swaps in that pass)

B1

Answer: Abi, Ben, Eve, Kim, Tom, Zoe; 5 passes

4 marks

17 (7 marks)

(a) Bin 1: 1.8, 2.4; Bin 2: 3.1, 0.9; Bin 3: 2.2, 1.5, 1.2; Bin 4: 2.7

M1 A1

Answer: 4 stock lengths

2 marks

(b) Decreasing order: 3.1, 2.7, 2.4, 2.2, 1.8, 1.5, 1.2, 0.9

M1

Bin 1: 3.1, 1.8; Bin 2: 2.7, 2.2; Bin 3: 2.4, 1.5, 0.9; Bin 4: 1.2

A1

4 stock lengths, no improvement

B1

Answer: 4 stock lengths

3 marks

(c) Total length = 15.8 m, and $15.8 \div 5 = 3.16$

M1

At least 4 stock lengths are needed

A1

Answer: At least 4 needed

2 marks

18 (5 marks)

Stage 1: items 5, 9, 14, 20, 26, 31, 37, 44, 52, 58, 63, 71, middle item 37

M1

Stage 2: items 5, 9, 14, 20, 26, 31, middle item 20

A1

Stage 3: items 5, 9, 14, middle item 9

A1

Stage 4: items 14, middle item 14

A1

4 items examined; 14 is found

B1

Answer: 14 found after examining 4 items

5 marks

19 (3 marks)

The largest number moves right whenever it is compared with a smaller number to its right

E1

Once the largest number has been picked up it is swapped with every number it meets, because it is larger than all of them

E1

So it is carried to the end of the list in this pass (a later equal value does not stop it getting there)

E1

Answer: The largest number is carried right by successive swaps

3 marks

20 (5 marks)

(a) Bin 1: 5, 4, 3; Bin 2: 6, 3, 3; Bin 3: 8, 2; Bin 4: 9; Bin 5: 7

M1 A1

Unused space = $5 \times 12 - 50 = 10$

B1

Answer: 5 bins, 10 unused

3 marks

(b) Total size = 50 and $50 \div 12 = 4.17$

M1

Lower bound is 5 bins and the packing uses 5, so it is optimal

A1

Answer: Lower bound 5; the packing is optimal

2 marks

21 (5 marks)

(a) The student stopped carrying 9 along: 9 was not compared with 2 and then 8

E1

The end of the pass should have 9 in the last place

E1

Answer: 9 not compared with 2 and 8

2 marks

(b) After pass 1: 4, 7, 2, 8, 9

A1

After pass 2: 4, 2, 7, 8, 9

A1

Answer: 4, 7, 2, 8, 9; 4, 2, 7, 8, 9

2 marks

(c) The sort needs 4 passes, the last with no swaps

B1

Answer: 4

1 mark

22 (5 marks)

Start: (14 6 14 9 2 14 11 6 9 20 3)

Pass 1: **20 14** (14 6 14 9 2 11 6 9 3)

M1

Pass 2: **20 14** (14 6 14 9 11 6 9 3) 2

A1

Pass 3: **20 14** (14 14) **11** (6 9 6 9 3) 2

A1

Pass 4: **20 14 14 14 11** (9 9) 6 (6 3) 2

Pass 5: **20 14 14 14 11 9 9 6 6 3 2**

Descending order: 20, 14, 14, 14, 11, 9, 9, 6, 6, 3, 2

A1

5 passes in all

B1

Answer: 20, 14, 14, 14, 11, 9, 9, 6, 6, 3, 2; 5 passes

5 marks

23 (6 marks)

(a) x did not fit in bin 1 (which held $9 + 6 = 15$) or in bin 2 (which held 12), so x was put in a new bin

M1

7 did not fit in bin 3 alongside x, so $x + 7 > 20$ and $x > 13$

B1

x was placed in a bin of size 20, so $x \leq 20$

B1

Answer: $13 < x \leq 20$

3 marks

(b) Total of the sizes = $9 + 6 + 12 + 8 + 5 + 7 + 11 + x = 58 + x$

M1

Unused space = $4 \times 20 - (58 + x) = 22 - x$

M1

$22 - x = 5$, so $x = 17$ (this is greater than 13 and the other sizes, consistent with part (a))

A1

Answer: $x = 17$

3 marks

24 (6 marks)

(a) After each stage at most half of the names remain (rounded down), starting from 100 M1
 $100 \rightarrow 50 \rightarrow 25 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 1$, so at most 7 stages including the one that leaves a single name A1

Answer: At most 7 2 marks

(b) A list of n names needs at most 7 examinations if $n \leq 2^7 - 1$ M1
 $2^7 - 1 = 127$ A1

Answer: 127 2 marks

(c) $2^9 - 1 = 511 < 1000 \leq 1023 = 2^{10} - 1$ M1
 So at most 10 examinations are needed A1

Answer: 10 2 marks

25 (7 marks)

(a) Bin 1: 1.2, 0.8, 0.6; Bin 2: 2.0, 0.9; Bin 3: 1.5, 1.1; Bin 4: 1.4, 1.0, 0.5 M1 A1

Answer: 4 planks 2 marks

(b) Decreasing: 2.0, 1.5, 1.4, 1.2, 1.1, 1.0, 0.9, 0.8, 0.6, 0.5 M1

Bin 1: 2.0, 1.0; Bin 2: 1.5, 1.4; Bin 3: 1.2, 1.1, 0.6; Bin 4: 0.9, 0.8, 0.5 A1

Answer: 4 planks 2 marks

(c) Total length = 11.0 m, and $11.0 \div 3.0 = 3.67$, so at least 4 planks M1

The plan uses 4 planks, so it is optimal A1

Waste = $4 \times 3.0 - 11.0 = 1.0$ m B1

Answer: 1.0 m wasted 3 marks

26 (7 marks)

(a) Pass 1: 2, 6, 1, 5, 3, 4, 7 (swaps 5) M1

Pass 2: 2, 1, 5, 3, 4, 6, 7 (swaps 4) A1

Pass 3: 1, 2, 3, 4, 5, 6, 7 (swaps 3)

Pass 4: 1, 2, 3, 4, 5, 6, 7 (swaps 0)

Sorted: 1, 2, 3, 4, 5, 6, 7 A1

Answer: 1, 2, 3, 4, 5, 6, 7 3 marks

(b) Total swaps = $5 + 4 + 3 + 0 = 12$ B1

A swap exchanges two adjacent items that are in the wrong order, which corrects that pair and changes no other pair E1

Answer: 12 swaps 2 marks

(c) Every one of the $7 \times 6 \div 2 = 21$ pairs is in the wrong order M1

So 21 swaps are needed A1

Answer: 21 2 marks

27 (9 marks)

(a) Pass 1 has $n - 1$ comparisons, pass 2 has $n - 2$, and so on down to 1 M1

Sum = $(n - 1) + (n - 2) + \dots + 1 = \frac{n(n - 1)}{2}$ A1

Answer: $\frac{n(n - 1)}{2}$ 2 marks

(b) Each swap corrects exactly one wrong pair and creates none (adjacent items only) E1

The sort ends with no wrong pairs, so swaps = wrong pairs at the start E1

The most wrong pairs occurs when every pair is wrong, that is when the list is in exactly the reverse order A1

Answer: The reverse order 3 marks

(c) Swaps = $\frac{50 \times 49}{2} = 1225$ M1 A1

Comparisons = $\frac{50 \times 49}{2} = 1225$ A1

They are equal because every comparison in this list leads to a swap E1

Answer: 1225 swaps and 1225 comparisons 4 marks

28 (8 marks)**(a)** Pass 1: (3 2 1) **4** (5 6 7)

M1

Pass 2: **1 2 3 4 5 6 7**

A1

The sort needs 2 passes

A1

Answer: 2 passes

3 marks

(b) After pass 1 each remaining sub-list must be fully sorted by pass 2

M1

In one pass a sub-list is fully sorted only if it has at most 3 items (a pivot with at most one item each side)

E1

Both sub-lists have at most 3 items and together they hold 6 items, so each has exactly 3, which makes the pivot the 4th smallest, that is 4

E1

Answer: First pivot must be 4

3 marks

(c) Pass 1: (1 3 5 6 4 2) **7**Pass 2: (1 3 5 4 2) **6 7**Pass 3: (1 3 4 2) **5 6 7**Pass 4: (1 3 2) **4 5 6 7**Pass 5: (1 2) **3 4 5 6 7**Pass 6: **1 2 3 4 5 6 7**

Each pass fixes at least one more item as a pivot, so after 6 passes at most one item is not yet fixed, and a sub-list of one item is sorted; so no list of 7 needs more than 6 passes

E1

This list needs exactly 6 passes

A1

Answer: 6 passes

2 marks

29 (6 marks)**(a)** Decreasing: 7, 7, 7, 6, 5, 5, 4, 3

M1

Bin 1: 7, 7; Bin 2: 7, 6; Bin 3: 5, 5, 4; Bin 4: 3

A1

Answer: 4 bins

2 marks

(b) Total size is 44, so 3 bins of size 15 (total 45) leave only 1 unit spare

M1

Bin 1: 7, 7 (14); Bin 2: 6, 5, 4 (15); Bin 3: 7, 5, 3 (15)

A1

Answer: Bins: (7, 7), (6, 5, 4), (7, 5, 3)

2 marks

(c) First-fit decreasing is not guaranteed to give the minimum number of bins (it gave 4 but 3 is possible)

E1

Total size $44 > 2 \times 15 = 30$, so 2 bins are not enough; 3 is the minimum

E1

Answer: FFD is not always optimal

2 marks

30 (10 marks)**(a)** The first examination is of the middle (8th) item, so 1 number is found there

B1

This leaves 7 numbers on each side; each has a middle item, so 2 numbers are found at stage 2

M1

Then 3 numbers remain on each side, each with a middle item: 4 numbers found at stage 3

A1

Then 1 number remains on each side: 8 numbers found at stage 4 ($1 + 2 + 4 + 8 = 15$)

A1

Answer: 1, 2, 4, 8

4 marks

(b) Total = $1 \times 1 + 2 \times 2 + 3 \times 4 + 4 \times 8 = 49$

M1 A1

Mean = $\frac{49}{15} = 3.27$ (3 s.f.)

A1

Answer: $\frac{49}{15} \approx 3.27$

3 marks

(c) Stage j finds 2^{j-1} numbers and $1 + 2 + \dots + 2^{k-1} = 2^k - 1$, so k examinations are enough and k are sometimes needed

M1 A1

 $1023 = 2^{10} - 1$, so at most 10 examinations

A1

Answer: k ; 10

3 marks

Algorithm Efficiency

1 (2 marks)

Passes have 9, 8, ..., 1 comparisons

M1

$$\text{Total} = \frac{10 \times 9}{2} = 45$$

A1

Answer: 45

2 marks

2 (2 marks)

Time $\propto n^2$, so doubling n multiplies the time by 2^2

M1

The time increases by a factor of 4

A1

Answer: 4

2 marks

3 (2 marks)

$$t = k \times 500^2 = 0.8 \text{ so for 1000 numbers } t = 0.8 \times \left(\frac{1000}{500}\right)^2$$

M1

= 3.2 seconds

A1

Answer: 3.2 seconds

2 marks

4 (2 marks)

$$\text{Number of tours} = \frac{(5-1)!}{2}$$

M1

$$= \frac{24}{2} = 12$$

A1

Answer: 12

2 marks

5 (2 marks)

Each examination halves the list, leaving at most 31, 15, 7, 3, 1 items after 1, 2, 3, 4, 5 examinations

M1

So the 6th examination checks the last remaining item: at most 6 ($63 = 2^6 - 1$)

E1

Answer: At most 6

2 marks

6 (2 marks)

$$\frac{20 \times 19}{2} = 190$$

B1

It is the greatest number of comparisons needed

B1

Answer: 190, the greatest number of comparisons

2 marks

7 (3 marks)

$$\text{Time} = c n \log n, \text{ so the time for 60 000 numbers is } 0.45 \times \frac{60000 \log 60000}{4000 \log 4000}$$

M1

$$\frac{60000}{4000} = 15 \text{ and } \log 60\,000 \div \log 4000 = 1.327 \text{ (any base), so the time is multiplied by 19.90}$$

A1

$$0.45 \times 19.90 = 8.95 \text{ seconds}$$

A1

Answer: 8.95 seconds

3 marks

8 (4 marks)

$$\text{(a) } n = 30: \frac{30 \times 29}{2} = 435$$

B1

$$n = 60: \frac{60 \times 59}{2} = 1770$$

B1

Answer: 435 and 1770

2 marks

$$\text{(b) } 1770 \div 435 = 4.07$$

M1

The number of comparisons is roughly proportional to n^2 , and doubling n multiplies n^2 by 4

E1

Answer: 4.07, close to 4

2 marks

9 (3 marks)

$$\text{Number of tours} = \frac{7!}{2}$$

M1

$$= 2520$$

A1

$$\text{Time} = 2520 \div 10^6 = 0.00252 \text{ seconds (about 2.5 milliseconds)}$$

A1

Answer: 2520 tours, 0.0025 s

3 marks

10 (4 marks)(a) $n = 4$: 16 and 16; $n = 5$: 25 and 32; $n = 10$: 100 and 1024

M1 A1

Answer: 16 and 16; 25 and 32; 100 and 1024

2 marks

(b) $2^n > n^2$ first happens (for $n \geq 3$) at $n = 5$ ($n = 3$ gives $8 < 9$ and $n = 4$ gives equal values)

B1

The exponential grows much faster than the polynomial, so 2^n stays larger (and the gap widens)

E1

Answer: $n = 5$

2 marks

11 (3 marks)

$$n = 100: \frac{100 \times 101}{2} = 5050$$

B1

$$n = 200: \frac{200 \times 201}{2} = 20100$$

B1

$$\text{Ratio} = \frac{20100}{5050} = 3.98 \text{ (about 4)}$$

B1

Answer: 5050, 20100, ratio 3.98

3 marks

12 (3 marks)

Linear search: up to 1000 items (if the target is last or absent)

B1

Binary search: $2^9 - 1 = 511 < 1000 \leq 1023 = 2^{10} - 1$, so at most 10

M1 A1

Answer: 1000 and 10

3 marks

13 (3 marks)

$$T = 3 \times 10^{-6} \times (2 \times 10^4)^2 = 3 \times 10^{-6} \times 4 \times 10^8$$

M1

$$= 1200 \text{ seconds}$$

A1

$$= 20.0 \text{ minutes}$$

A1

Answer: 20.0 minutes

3 marks

14 (5 marks)

$$(a) t = c n^2: c = \frac{3.2}{4000^2} = 2 \times 10^{-7}$$

M1

$$t = 2 \times 10^{-7} \times 7000^2 = 9.8 \text{ seconds}$$

A1

Answer: 9.8 seconds

2 marks

$$(b) 2 \times 10^{-7} n^2 = 60$$

M1

$$n^2 = 3 \times 10^8$$

A1

$$n = 17321 \text{ (to the nearest hundred, 17300)}$$

A1

Answer: About 17300 numbers

3 marks

15 (5 marks)

$$(a) 0.01 n^2 = 100 n$$

M1

$$n = \frac{100}{0.01} = 10000$$

A1

Answer: 10000

2 marks

(b) $n = 5000$: $A = 250\,000$ ms, $B = 500\,000$ ms, so A is faster

B1

 $n = 50\,000$: $A = 25\,000\,000$ ms, $B = 5\,000\,000$ ms, so B is faster

B1

Answer: A for 5000, B for 50 000

2 marks

(c) A grows like n^2 and B like n , so for large n the n^2 algorithm is slower whatever the multiplier

E1

Answer: Growth rate matters more than the multiplier for large n

1 mark

16 (4 marks)

(a) For $i = 1$ there are $n - 1$ comparisons, for $i = 2$ there are $n - 2$, ..., for $i = n - 1$ there is 1 M1

$$\text{Total} = (n - 1) + (n - 2) + \dots + 1 = \frac{n(n - 1)}{2} \quad \text{A1}$$

Answer: $\frac{n(n - 1)}{2}$ 2 marks

(b) Number of comparisons = $\frac{3000 \times 2999}{2} = 4498500$ M1

Time = $4498500 \div (2 \times 10^6) = 2.25$ seconds A1

Answer: 2.25 seconds 2 marks

17 (6 marks)

(a) Tours = $\frac{14!}{2} = 43589145600$ M1 A1

Time = $43589145600 \div 10^8 = 435.9$ seconds A1

Answer: 435.9 seconds 3 marks

(b) Tours = $\frac{19!}{2} = 6.08 \times 10^{16}$ M1

Time = $6.08 \times 10^{16} \div 10^8 = 6.08 \times 10^8$ seconds A1

= 19.3 years A1

Answer: 19.3 years 3 marks

18 (4 marks)

(a) Each doubling of n multiplies the time by $0.21 \div 0.05 = 4.2$ and $0.79 \div 0.21 = 3.76$ M1

Both ratios are close to $4 = 2^2$ E1

Answer: Ratios 4.2 and 3.76, close to 4 2 marks

(b) $t = c n^2$, $c = \frac{0.79}{400^2} \approx 4.94 \times 10^{-6}$ M1

$t \approx 4.94 \times 10^{-6} \times 1000^2 = 4.94$ seconds A1

Answer: 4.94 seconds 2 marks

19 (4 marks)

(a) Bubble sort: $\frac{1024 \times 1023}{2} = 523776$ B1

Quick sort: $1024 \times 10 = 10240$ (since $2^{10} = 1024$) B1

Answer: 523776 and 10240 2 marks

(b) $523776 \div 10240 = 51.1$ M1 A1

Answer: 51.1 times 2 marks

20 (4 marks)

(a) Item 1 needs 0 comparisons, item 2 needs 1, item 3 needs 2, ..., item n needs $n - 1$ M1

$$\text{Total} = 0 + 1 + \dots + (n - 1) = \frac{n(n - 1)}{2} \quad \text{A1}$$

Answer: $\frac{n(n - 1)}{2}$ 2 marks

(b) The order is n^2 (quadratic) B1

$$\frac{200 \times 199}{2} = 19900 \quad \text{B1}$$

Answer: n^2 ; 19900 2 marks

21 (6 marks)

(a) P: $1000 \times 100 = 100\,000$ B1

Q: $2^{10} = 1024$, which is fewer B1

Answer: Q is faster 2 marks

(b) $n = 18$: $2^{18} = 262144$, $1000n^2 = 324000$ (Q fewer) M1 A1

$n = 19$: $2^{19} = 524288$, $1000n^2 = 361000$ (Q more) A1

Answer: $n = 19$ 3 marks

(c) For large n the exponential algorithm needs far more steps, whatever the size of the multiplier E1

on the polynomial

Answer: Exponential algorithms are impractical for large inputs

1 mark

22 (8 marks)

(a) $t = 2.5 \times 10^{-7} \times (2 \times 10^4)^2$

M1

= 100 seconds

A1

Answer: 100 seconds

2 marks

(b) $600 = 2.5 \times 10^{-7} n^2$

M1

$n^2 = 2.4 \times 10^9$

A1

$n = 48990$, so about 49000 items

A1

Answer: About 49000 items

3 marks

(c) The new time is $t/25$ so the constant is divided by 25

M1

n^2 is multiplied by 25, so n is multiplied by $\sqrt{25} = 5$

A1

A faster machine helps much less than the speed-up factor because $t \propto n^2$

E1

Answer: Increases by a factor of 5

3 marks

23 (7 marks)

(a) $\frac{1\,000\,001}{2} = 500\,000.5$

M1 A1

Answer: 500 000.5

2 marks

(b) $2^{19} = 524\,288 \leq 1\,000\,000 < 1\,048\,576 = 2^{20}$

M1

$k = 20$

A1

Answer: 20

2 marks

(c) $2^{29} = 536\,870\,912 \leq 10^9 < 2^{30}$, so $k = 30$

M1 A1

Linear search averages about 5×10^8 examinations, over 10^7 times as many as binary search

E1

Answer: 30 examinations

3 marks

24 (8 marks)

(a) $k = 2: \frac{4!}{2^2 \times 2!} = \frac{24}{8} = 3$

B1

$k = 3: \frac{6!}{2^3 \times 3!} = \frac{720}{48} = 15$

B1

Answer: 3 and 15

2 marks

(b) $k = 5: \frac{10!}{2^5 \times 5!} = 945$

M1 A1

$k = 6: \frac{12!}{2^6 \times 6!} = 10395$

A1

Answer: 945 and 10395

3 marks

(c) Ratio = $\frac{(2k+2)!}{2^{k+1}(k+1)!} \div \frac{(2k)!}{2^k k!} = \frac{(2k+2)(2k+1)}{2(k+1)}$

M1

= $2k + 1$

A1

The number grows faster than exponentially (like a factorial), so for many odd vertices it becomes too large to check

E1

Answer: Multiplied by $2k + 1$

3 marks

25 (8 marks)

(a) $n = 100: 20 + 50 + 20 = 90$ ms

B1

$n = 10\,000: 200\,000 + 5000 + 20 = 205\,020$ ms

B1

Answer: 90 ms; 205 020 ms

2 marks

(b) $\frac{200\,000}{205\,020} \times 100 = 97.6$

M1 A1

Answer: 97.6%

2 marks

(c) $0.002n^2 > 0.5n + 20$

M1

$n^2 - 250n - 10\,000 > 0$, so $n > \frac{250 + \sqrt{62500 + 40000}}{2} = 285.1$

M1 A1

So the smallest integer n is 286

A1

Answer: $n = 286$

4 marks

26 (8 marks)**(a)** Pass 1: one sub-list of 7 items, 6 comparisons

M1

Pass 2: two sub-lists of 3 items, $2 \times 2 = 4$ comparisons

A1

Sub-lists of 1 item need no comparisons; total $6 + 4 = 10$

A1

Answer: 10

3 marks

(b) Pass 1: 14; pass 2: $2 \times 6 = 12$; pass 3: $4 \times 2 = 8$

M1 A1

Total = 34

A1

Answer: 34

3 marks

(c) $\frac{15 \times 14}{2} = 105$, about three times as many

M1 A1

Answer: 105 for bubble sort

2 marks

27 (9 marks)**(a)** Pass 1 has one list of $2^k - 1$ items; removing the pivot leaves two halves of $2^{k-1} - 1$ items

M1

Inductively pass j has 2^{j-1} sub-lists each of $2^{k-j+1} - 1$ items

A1

Each needs $2^{k-j+1} - 2$ comparisons, so pass j needs $2^{j-1}(2^{k-j+1} - 2)$

A1

Answer: $2^{j-1}(2^{k-j+1} - 2)$

3 marks

(b) Pass j gives $2^k - 2^j$, so the sum over $j = 1$ to $k - 1$ is $(k - 1)2^k - (2^k - 2)$

M1 A1

 $= k \times 2^k - 2 \times 2^k + 2 = k \times 2^k - 2(2^k - 1)$

A1

Answer: $(n + 1)k - 2n$

3 marks

(c) $k = 10$: $10 \times 1024 - 2 \times 1023 = 8194$

M1 A1

Bubble sort: $\frac{1023 \times 1022}{2} = 522753$, about 64 times as many

A1

Answer: 8194 comparisons; bubble sort 522753

3 marks

28 (8 marks)**(a)** One hour allows $3600 \times 10^9 = 3.6 \times 10^{12}$ tours

B1

 $n = 16$: $\frac{15!}{2} = 6.54 \times 10^{11} \leq 3.6 \times 10^{12}$

M1 A1

 $n = 17$: $\frac{16!}{2} = 1.05 \times 10^{13} > 3.6 \times 10^{12}$, so the greatest is 16 cities

A1

Answer: 16 cities

4 marks

(b) Now 3.6×10^{15} tours: $n = 19$: $\frac{18!}{2} = 3.2 \times 10^{15} \leq 3.6 \times 10^{15}$

M1 A1

 $n = 20$: $\frac{19!}{2} = 6.08 \times 10^{16}$ is too large, so the greatest is 19 cities: 3 more

A1

The number of tours grows like a factorial, multiplying by $n - 1$ for each extra city, so a 1000-fold speed-up buys only a few extra cities

E1

Answer: 3 more cities

4 marks

29 (9 marks)**(a)** The number of ways of choosing 3 items from n is the number of combinations $C(n, 3)$

M1

$$= \frac{n(n-1)(n-2)}{3!} = \frac{n(n-1)(n-2)}{6}$$

A1

Answer: $\frac{n(n-1)(n-2)}{6}$

2 marks

(b) $n = 200$: $\frac{200 \times 199 \times 198}{6} = 1313400$

M1 A1

 $n = 100$: 161700, ratio = 8.12

A1

Answer: 1313400; ratio 8.12

3 marks

(c) For large n , $n(n-1)(n-2) \approx n^3$, and doubling n multiplies n^3 by $2^3 = 8$

E1

The algorithm is of order n^3

B1

Answer: Order n^3

2 marks

(d) Time $\propto n^3$ approximately: $0.4 \times (\frac{500}{100})^3 = 0.4 \times 125$

M1

 $= 50$ seconds

A1

Answer: About 50 seconds

2 marks

30 (10 marks)

(a) Already in order: one pass of 5 comparisons, no swaps, then stop: 5 comparisons

M1

Reverse order: $5 + 4 + 3 + 2 + 1 = 15$ comparisons (all with swaps)

A1

Answer: 5 comparisons compared with 15

2 marks

(b) Worst case comparisons: $\frac{50 \times 49}{2} = 1225$ and $\frac{100 \times 99}{2} = 4950$

M1

$4950 \div 1225 = 4.04$, about 4 times, not 2 times

A1

Answer: About 4 times

2 marks

(c) $0.01 n^2 = 10n$ gives $n = 1000$

M1 A1

For $n > 1000$, $0.01n^2 > 10n$, so B is faster for larger inputs

A1

Answer: $n = 1000$; B faster beyond it

3 marks

(d) Best case $n - 1 = 999$ and worst case $\frac{n(n - 1)}{2} = 499\,500$

M1 A1

Ratio worst : best = $499\,500 \div 999 = 500$

A1

Answer: 999 and 499 500; ratio 500

3 marks