

Work Done by Forces

1 (1 mark)

$$W = Fd = 20 \times 5 = 100 \text{ J}$$

Answer: 100 J

B1

1 mark

2 (2 marks)

$$W = Fd \cos \theta = 15 \times 4 \times \cos 60^\circ$$

$$= 30 \text{ J}$$

Answer: 30 J

M1

A1

2 marks

3 (2 marks)

$$\text{Force} = 3 \times 9.8 = 29.4 \text{ N}$$

$$W = 29.4 \times 2 = 58.8 \text{ J}$$

Answer: 58.8 J

M1

A1

2 marks

4 (1 mark)

$$W = 12 \times 7 = 84 \text{ J}$$

Answer: 84 J

B1

1 mark

5 (2 marks)

Work done = 0

$W = Fd \cos 90^\circ = 0$, as there is no displacement in the direction of the force

Answer: 0 J

B1

E1

2 marks

6 (2 marks)

$$W = 200 \times 15 = 3000 \text{ J}$$

$$= 3 \text{ kJ}$$

Answer: 3 kJ

M1

A1

2 marks

7 (3 marks)

$$W = T d \cos \theta$$

$$= 40 \times 10 \times \cos 30^\circ$$

$$= 346 \text{ J}$$

Answer: 346 J

M1

A1

A1

3 marks

8 (4 marks)

$$\text{Force} = 5 \times 9.8 \times \sin 30^\circ = 24.5 \text{ N}$$

$$W = 24.5 \times 4$$

$$= 98 \text{ J}$$

Answer: 98 J

M1 A1

M1

A1

4 marks

9 (4 marks)

$$R = 10 \times 9.8 = 98 \text{ N}$$

$$F = 0.3 \times 98 = 29.4 \text{ N}$$

$$W = 29.4 \times 6 = 176.4 \text{ J}$$

Answer: 176.4 J

B1

M1 A1

A1

4 marks

10 (3 marks)

$$\text{Tension} = 80 \times 9.8 = 784 \text{ N}$$

$$W = 784 \times 12 = 9408 \text{ J}$$

$$= 9.41 \text{ kJ}$$

Answer: 9.41 kJ

M1

A1

A1

3 marks

11 (3 marks)

$$W = F \cdot d$$

M1

$$= 3(2) + 4(-1)$$

M1

$$= 2 \text{ J}$$

A1

Answer: 2 J

3 marks

12 (2 marks)

$$W = mgh = 0.4 \times 9.8 \times 5$$

M1

$$= 19.6 \text{ J}$$

A1

Answer: 19.6 J

2 marks

13 (4 marks)

$$\text{Work by pulling force} = 50 \times 8 \times \cos 40^\circ = 306 \text{ J}$$

M1 A1

$$\text{Work by resistance} = -20 \times 8 = -160 \text{ J}$$

B1

$$\text{Total} = 146 \text{ J}$$

A1

Answer: 146 J

4 marks

14 (4 marks)

$$\text{Displacement} = (5 - 1)\mathbf{i} + (5 - 2)\mathbf{j} = (4\mathbf{i} + 3\mathbf{j}) \text{ m}$$

M1 A1

$$W = F \cdot d = 16 + 9$$

M1

$$= 25 \text{ J}$$

A1

Answer: 25 J

4 marks

15 (6 marks)

$$R = 8 \times 9.8 \times \cos 25^\circ = 71.1 \text{ N}$$

M1 A1

$$F = 0.2R = 14.2 \text{ N}$$

A1

$$\text{Equilibrium along slope: } T = 8 \times 9.8 \times \sin 25^\circ + F = 47.3 \text{ N}$$

M1 A1

$$W = 47.3 \times 6 = 284 \text{ J}$$

A1

Answer: 284 J

6 marks

16 (6 marks)

$$(a) W = 30 \times 5 \times \cos 20^\circ$$

M1

$$= 141 \text{ J}$$

A1

Answer: 141 J

2 marks

$$(b) R + 30\sin 20^\circ = 12 \times 9.8, R = 107 \text{ N}$$

M1 A1

$$F = 30\cos 20^\circ = 28.2 \text{ N}$$

M1

$$\mu = \frac{F}{R} = 0.263$$

A1

Answer: 0.263

4 marks

17 (4 marks)

$$(a) W = mgh = 0.5 \times 9.8 \times 1.2$$

M1

$$= 5.88 \text{ J}$$

A1

Answer: 5.88 J

2 marks

(b) Zero

B1

The tension is always perpendicular to the direction of motion

E1

Answer: 0 J

2 marks

18 (5 marks)

$$\text{Level road: } W = 15 \times 2000 = 30\,000 \text{ J}$$

M1 A1

$$\text{Hill: } W = mgh = 80 \times 9.8 \times 40$$

M1

$$= 31\,360 \text{ J}$$

A1

$$\text{Total} = 61\,360 \text{ J}$$

A1

Answer: 61 360 J

5 marks

19 (4 marks)

$$\text{Displacement AB} = (3\mathbf{i} + (a - 2)\mathbf{j} + (5 - a)\mathbf{k}) \text{ m}$$

M1 A1

$$\text{Work} = (2)(3) + (-1)(a - 2) + (3)(5 - a) = 23 - 4a$$

M1

$$23 - 4a = 7, \text{ so } a = 4$$

A1

Answer: $a = 4$

4 marks

20 (5 marks)

$$\text{Displacement AB} = (2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \text{ m}$$

B1

$$\text{Work done} = (4)(2) + (2)(2) + (-4)(-2) = 20 \text{ J}$$

M1 A1

$$20 = \frac{1}{2}(0.5)v^2$$

M1

$$v = \sqrt{80} = 8.94 \text{ m s}^{-1}$$

A1

Answer: 8.94 m s^{-1}

5 marks

21 (6 marks)**(a)** On the horizontal part the weight is perpendicular to the displacement

E1

so gravity does no work and no work is done against it

E1

Answer: No work is done against gravity horizontally

2 marks

(b) Only the stairs contribute: $20 \times 9.8 \times 3$

M1

$$= 588 \text{ J}$$

A1

Answer: 588 J

2 marks

(c) Muscles exert force to hold the case and to overcome resistance such as air resistance and the small changes in speed

E1

but the work done by the force that holds the case up is zero, since the force is perpendicular to the motion

E1

Answer: Muscular effort is not the same as mechanical work

2 marks

22 (6 marks)

$$W = \mathbf{F} \cdot \mathbf{d} = 8 + 3p$$

M1

$$8 + 3p = 14, p = 2$$

M1 A1

$$\mathbf{F} = (2\mathbf{i} + 2\mathbf{j}), |\mathbf{F}| = \sqrt{8}, |\mathbf{d}| = 5$$

B1

$$\cos \theta = \frac{14}{5\sqrt{8}}$$

M1

$$\theta = 8.13^\circ$$

A1

Answer: $p = 2, \theta = 8.13^\circ$

6 marks

23 (6 marks)**(a)** Resolve along the ramp: $D = 1200 \times 9.8 \times \sin 8^\circ + 300$

M1 A1

$$= 1940 \text{ N}$$

A1

Answer: 1940 N

3 marks

(b) Work by driving force $= 1940 \times 40 = 77500 \text{ J}$

M1 A1

$$\text{Work against resistance} = 300 \times 40 = 12\,000 \text{ J}$$

A1

Answer: 77500 J, 12 000 J

3 marks

24 (6 marks)**(a)** Components perpendicular to the centre line cancel

E1

$$R = 2 \times 400 \cos 25^\circ = 725 \text{ N}$$

A1

Answer: 725 N

2 marks

(b) Total $= 725 \times 100 = 72500 \text{ J}$

M1 A1

$$\text{Each: } 400 \times 100 \times \cos 25^\circ = 36250 \text{ J}$$

A1

Answer: 72500 J total, 36250 J each

3 marks

(c) Work is a scalar product, $\mathbf{F} \cdot \mathbf{d}$, and is additive over forces: $(\mathbf{F}_1 + \mathbf{F}_2) \cdot \mathbf{d} = \mathbf{F}_1 \cdot \mathbf{d} + \mathbf{F}_2 \cdot \mathbf{d}$

E1

Answer: scalar product is distributive

1 mark

25 (6 marks)Path OA: $d = (3\mathbf{i} + 4\mathbf{j})$, $W = 6 \times 3 + 0 \times 4$

M1 A1

 $W = 18 \text{ J}$

A1

Path OBA: $OB = 3\mathbf{i}$; $W = 18 \text{ J}$

M1

 $BA = 4\mathbf{j}$; $W = 0 \text{ J}$ (force perpendicular)

M1

Total 18 J on both paths

A1

Answer: 18 J

6 marks

26 (7 marks)(a) Vertical: $R + P \sin \theta = 20 \times 9.8$

M1

Horizontal: $P \cos \theta = 0.5R$

M1 A1

 $P \cos \theta = 0.5(196 - P \sin \theta)$, so $P(\cos \theta + 0.5 \sin \theta) = 98$

A1

Answer: $P = \frac{98}{\cos \theta + 0.5 \sin \theta}$

4 marks

(b) $\cos \theta + 0.5 \sin \theta$ has maximum $\sqrt{1 + 0.25} = \sqrt{1.25}$, when $\tan \theta = 0.5$

M1 A1

 $P_{\min} = \frac{98}{\sqrt{1.25}} = 87.7 \text{ N}$ at $\theta = 26.6^\circ$

A1

Answer: 87.7 N at 26.6°

3 marks

27 (11 marks)(a) B: $5g - T = 5a$, A: $T - 3g = 3a$

M1 M1

 $a = \frac{2 \times 9.8}{8} = 2.45 \text{ m s}^{-2}$

A1

 $T = 3(9.8 + 2.45) = 36.75 \text{ N}$

A1

Answer: 2.45 m s^{-2} , 36.75 N

4 marks

(b) On A: $+36.75 \times 2 = 73.5 \text{ J}$ (upwards displacement)

M1 A1

On B: $-36.75 \times 2 = -73.5 \text{ J}$

A1

Answer: $+73.5 \text{ J}$ on A, -73.5 J on B

3 marks

(c) $73.5 - 73.5 = 0$

B1

The string is inextensible so both particles move equal distances and the tension is the same throughout, so no net work is done

E1

Answer: zero

2 marks

(d) B: $5 \times 9.8 \times 2 = 98 \text{ J}$, A: $-3 \times 9.8 \times 2 = -58.8 \text{ J}$

M1

Total = 39.2 J

A1

Answer: 39.2 J

2 marks

28 (11 marks)(a) Perpendicular to the plane: $R = 4 \times 9.8 \cos 30^\circ + P \sin 30^\circ$

M1 A1

Along the plane: $P \cos 30^\circ = 4 \times 9.8 \sin 30^\circ + 0.3R$

M1 A1

 $P(\cos 30^\circ - 0.3 \sin 30^\circ) = 4 \times 9.8(\sin 30^\circ + 0.3 \cos 30^\circ)$

M1

 $P = 41.6$

A1

Answer: $P = 41.6 \text{ N}$

6 marks

(b) Component of P along the plane = $P \cos 30^\circ$; displacement along the plane 5 m

M1

 $W = 41.6 \times \cos 30^\circ \times 5 = 180 \text{ J}$

A1

Answer: 180 J

2 marks

(c) Friction: $0.3R \times 5 = 82.1 \text{ J}$

M1

Gravity: $4 \times 9.8 \times 5 \times \sin 30^\circ = 98 \text{ J}$

B1

Sum = 180 J, equal to the work by P (constant speed)

A1

Answer: 82.1 J and 98 J

3 marks

29 (8 marks)(a) Distance = $2\pi \times 2 = 4\pi \text{ m}$

M1

 $W = 5 \times 4\pi = 20\pi = 62.8 \text{ J}$

A1

Answer: 62.8 J

2 marks

(b) Net displacement over a complete revolution is zero

M1

 $W = F \cdot d = 0$

A1

Answer: Work done in one complete revolution = 0 J

2 marks

(c) $d = -4\mathbf{i}$

M1

$$W = 3(-4) = -12 \text{ J}$$

A1

Answer: -12 J

2 marks

(d) Zero

B1

The tension acts towards O, perpendicular to the velocity

E1

Answer: zero

2 marks

30 (9 marks)

(a) Constant force: $W = (3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 3\mathbf{j}) = 12 - 12 = 0$

M1 A1

Friction: $-2 \times 5 = -10 \text{ J}$

M1

Total = -10 J

A1

Answer: 0 J, -10 J, total -10 J

4 marks

(b) Constant force: $W = \mathbf{F} \cdot \mathbf{d}$ is path independent = 0

M1 A1

Friction: $-2(4 + 3) = -14 \text{ J}$

M1

Total = -14 J

A1

Answer: 0 J, -14 J, total -14 J

4 marks

(c) The constant force does the same work on any path (conservative) but friction's work depends on the path length

E1

Answer: Friction is path dependent

1 mark

Kinetic Energy

1 (1 mark)

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 25 = 50 \text{ J}$$

B1

Answer: 50 J

1 mark

2 (2 marks)

$$KE = \frac{1}{2} \times 1000 \times 400 = 200\,000 \text{ J}$$

M1

= 200 kJ

A1

Answer: 200 kJ

2 marks

3 (2 marks)

$$9 = \frac{1}{2}(0.2)v^2$$

M1

$$v^2 = 90, v = 9.49 \text{ m s}^{-1}$$

A1

Answer: 9.49 m s⁻¹

2 marks

4 (2 marks)

$$\frac{1}{2}(3)(36) - \frac{1}{2}(3)(4)$$

M1

$$= 54 - 6 = 48 \text{ J}$$

A1

Answer: 48 J

2 marks

5 (2 marks)

$$\text{Work done} = \text{gain in KE: } 40 = \frac{1}{2}(5)v^2$$

M1

$$v = 4 \text{ m s}^{-1}$$

A1

Answer: 4 m s⁻¹

2 marks

6 (2 marks)

$$\text{Speed}^2 = 9 + 16 = 25$$

M1

$$KE = \frac{1}{2}(2)(25) = 25 \text{ J}$$

A1

Answer: 25 J

2 marks

7 (2 marks)

$$KE = \frac{1}{2}mv^2 \text{ is proportional to } v^2$$

M1

Doubling v multiplies KE by 4

A1

Answer: $\times 4$

2 marks

8 (3 marks)

$$\begin{aligned}\text{Work} &= \text{change in KE} \\ &= \frac{1}{2}(900)(625 - 100) \\ &= 236\,250 \text{ J}\end{aligned}$$

M1

M1

A1

Answer: 236 250 J

3 marks

9 (3 marks)

Work done by braking force = loss of KE

M1

$$F \times 90 = \frac{1}{2}(1200)(225)$$

M1

$$F = 1500 \text{ N}$$

A1

Answer: 1500 N

3 marks

10 (3 marks)

$$\text{Loss of KE} = \frac{1}{2}(0.02)(250^2) = 625 \text{ J}$$

M1

$$F \times 0.1 = 625$$

M1

$$F = 6250 \text{ N}$$

A1

Answer: 6250 N

3 marks

11 (4 marks)

$$\text{KE}_A = \frac{1}{2}(2)(36) = 36 \text{ J}$$

B1

$$\text{KE}_B = \frac{1}{2}(8)(9) = 36 \text{ J, ratio } 1 : 1$$

B1

$$p_A = 12 \text{ N s, } p_B = 24 \text{ N s}$$

M1

B has the greater momentum

A1

Answer: 1 : 1; B

4 marks

12 (3 marks)

$$\text{Work done} = 6 \times 4 = 24 \text{ J}$$

M1

$$24 = \frac{1}{2}(3)v^2$$

M1

$$v = 4 \text{ m s}^{-1}$$

A1

Answer: 4 m s⁻¹

3 marks

13 (3 marks)

$$250\,000 = \frac{1}{2}(5000)v^2, v = 10 \text{ m s}^{-1}$$

M1 A1

$$= 10 \times 3.6 = 36 \text{ km per hour}$$

A1

Answer: 36 km per hour

3 marks

14 (4 marks)

$$\text{Net work} = (12 - 4) \times 5 = 40 \text{ J}$$

M1 A1

$$40 = \frac{1}{2}(2)v^2 - \frac{1}{2}(2)(64)$$

M1

$$v = 10.2 \text{ m s}^{-1}$$

A1

Answer: 10.2 m s⁻¹

4 marks

15 (5 marks)

$$\text{(a) Work} = \text{loss of KE: } 200s = \frac{1}{2}(80)(100)$$

M1 A1

$$s = 20 \text{ m}$$

A1

Answer: 20 m

3 marks

$$\text{(b) Distance is proportional to } v^2: 20 \times (1.5)^2$$

M1

$$= 45 \text{ m}$$

A1

Answer: 45 m

2 marks

16 (4 marks)

$$\text{Work by force} = 240 \text{ J, work by gravity} = -10 \times 9.8 \times 8 \times \sin 15^\circ = -203 \text{ J}$$

M1 A1

$$\text{Net work} = \frac{1}{2}(10)v^2$$

M1

$$v = 2.72 \text{ m s}^{-1}$$

A1

Answer: 2.72 m s⁻¹

4 marks

17 (6 marks)**(a)** Resistive forces on the way up: $0.5 \times 9.8 + 1.5 = 6.4 \text{ N}$

B1

$$6.4h = \frac{1}{2}(0.5)(24^2) = 144$$

M1

$$h = 22.5 \text{ m}$$

A1

Answer: 22.5 m

3 marks

(b) On the way down the resultant force is $0.5 \times 9.8 - 1.5 = 3.4 \text{ N}$

M1

$$\frac{1}{2}(0.5)v^2 = 3.4 \times 22.5 = 76.5$$

M1

$$v = \sqrt{306} = 17.5 \text{ m s}^{-1}$$

A1

Answer: 17.5 m s⁻¹

3 marks

18 (4 marks)

$$\frac{1}{2}(4m)v_1^2 = \frac{1}{2}mv_2^2$$

M1

$$v_1^2 : v_2^2 = 1 : 4, \text{ so speeds in ratio } 1 : 2$$

A1

$$p_1 : p_2 = 4m v_1 : m v_2 = 4 \times 1 : 2$$

M1

$$= 2 : 1$$

A1

Answer: speeds 1 : 2, momenta 2 : 1

4 marks

19 (4 marks)**(a)** KE initial = 18 J, so $18 + 24 = \frac{1}{2}(4)v^2$

M1

$$v = 4.58 \text{ m s}^{-1}$$

A1

Answer: 4.58 m s⁻¹

2 marks

(b) $18 - 10 = \frac{1}{2}(4)v^2$

M1

$$v = 2 \text{ m s}^{-1}$$

A1

Answer: 2 m s⁻¹

2 marks

20 (4 marks)

$$\text{Work-energy: } W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

M1

$$v^2 = u^2 + \frac{2W}{m}$$

A1

$$v^2 = 16 + 36 = 52$$

M1

$$v = 7.21 \text{ m s}^{-1}$$

A1

Answer: 7.21 m s⁻¹

4 marks

21 (6 marks)

(a) $F \times 40 = \frac{1}{2}(1500)(400)$

M1 A1

$$F = 7500 \text{ N}$$

A1

Answer: 7500 N

3 marks

(b) $7500s = \frac{1}{2}(1000)(625)$

M1 A1

$$s = 41.7 \text{ m}$$

A1

Answer: 41.7 m

3 marks

22 (5 marks)**(a)** KE depends on speed squared; the speed is the same before and after

E1

KE change is $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$, not $\frac{1}{2}m(v - u)^2$

E1

Answer: KE is a scalar; the change is zero

2 marks

(b) Change in KE = 0

B1

Impulse = $2(-3 - 3)$

M1

Magnitude = 12 N s

A1

Answer: 0 J, 12 N s

3 marks

23 (7 marks)

$$R + 20\sin 30^\circ = 4 \times 9.8, R = 29.2 \text{ N}$$

M1 A1

$$F = 0.2R = 5.84 \text{ N}$$

A1

$$\text{Net work} = (20\cos 30^\circ - 5.84) \times 6 = 68.9 \text{ J}$$

M1 A1

$$\frac{1}{2}(4)v^2 - \frac{1}{2}(4)(49) = 68.9$$

M1

$$v = 9.13 \text{ m s}^{-1}$$

A1

Answer: 9.13 m s⁻¹

7 marks

24 (9 marks)

$$(a) R = 2 \times 9.8\cos 20^\circ = 18.4 \text{ N}, F = 5.53 \text{ N}$$

M1 A1

Work against gravity and friction = loss of KE

M1

$$s(2 \times 9.8 \sin 20^\circ + 5.53) = \frac{1}{2}(2)(100)$$

M1 A1

$$s = 8.18 \text{ m}$$

A1

Answer: 8.18 m

6 marks

(b) $\tan 20^\circ = 0.364 > 0.3$, so gravity component exceeds limiting friction and it slides down

E1

$$\frac{1}{2}(2)v^2 = s(2 \times 9.8 \sin 20^\circ - 5.53)$$

M1

$$v = 3.10 \text{ m s}^{-1}$$

A1

Answer: 3.10 m s⁻¹

3 marks

25 (5 marks)

$$\mu u = 4mv, v = \frac{u}{4}$$

M1 A1

$$\text{KE after} = \frac{1}{2}(4m)\frac{u^2}{16} = \frac{\mu u^2}{8}$$

M1

$$\text{KE before} = \frac{1}{2}\mu u^2$$

B1

$$\text{Fraction lost} = 1 - \frac{1}{4} = \frac{3}{4}$$

A1

Answer: The fraction of the initial kinetic energy that is lost is $\frac{3}{4}$

5 marks

26 (7 marks)

$$(a) 6t^2 + 24 = \frac{1}{2}(3)v^2$$

M1

$$v = 2\sqrt{t^2 + 4} \text{ m s}^{-1}$$

A1

$$\text{Answer: } 2\sqrt{t^2 + 4}$$

2 marks

$$(b) \frac{dE}{dt} = 12t$$

M1

$$= 36 \text{ W}$$

A1

Answer: 36 W

2 marks

$$(c) t = 0: v = 4 \text{ m s}^{-1}$$

B1

$$100 = 4(t^2 + 4)$$

M1

$$t = 3 \text{ s}$$

A1

Answer: 4 m s⁻¹, 3 s

3 marks

27 (10 marks)

(a) Work done by net force = KE gained

M1

$$(D - R)s = \frac{1}{2}mv^2$$

A1

$$v^2 = \frac{2(D - R)s}{m}$$

A1

Answer: The speed v after distance s satisfies $v^2 = \frac{2(D - R)s}{m}$

3 marks

(b) Loss of KE = work against resistance

M1

$$\frac{1}{2}mV^2 = Rs'$$

A1

$$s' = \frac{mV^2}{2R}$$

A1

Answer: The car then travels a further distance $\frac{mV^2}{2R}$

3 marks

(c) From (a), $mV^2 = 2(2R)s_1 = 4Rs_1$

M1 A1

$$s' = \frac{4Rs_1}{2R} = 2s_1$$

M1

$$s_1 + 2s_1 = 600, s_1 = 200 \text{ m}$$

A1

Answer: 200 m

4 marks

28 (8 marks)

(a) $R = 0.5 \times 9.8 \times \cos 30^\circ$

M1

$$F = \frac{\sqrt{3}}{3} \times 0.5 \times 9.8 \times \frac{\sqrt{3}}{2} = 2.45 \text{ N}$$

A1

Answer: The friction force is $0.5 \times 9.8 \times \frac{\sqrt{3}}{3} \times \cos 30^\circ = 2.45 \text{ N}$

2 marks

(b) Resistive force along slope $= 0.5 \times 9.8 \times \sin 30^\circ + 2.45 = 4.9 \text{ N}$

M1 A1

$$4.9s = \frac{1}{2}(0.5)(36) = 9$$

M1

$$s = 1.84 \text{ m}$$

A1

Answer: 1.84 m

4 marks

(c) Weight component down the slope $= 2.45 \text{ N}$ equals the limiting friction 2.45 N ($\tan 30^\circ = \mu$, so the particle is in limiting equilibrium)

E1

so friction can hold the particle in equilibrium and it remains at rest

E1

Answer: It remains at rest

2 marks

29 (9 marks)

(a) Work done $= \int^x (6 - 2x) \, dx = 6x - x^2$

M1 A1

$$6x - x^2 = \frac{1}{2}(2)v^2 - \frac{1}{2}(2)(16)$$

M1

$$v^2 = 16 + 6x - x^2$$

A1

Answer: $v^2 = 16 + 6x - x^2$

4 marks

(b) $v^2 = 25 - (x - 3)^2$

M1

$$\text{Greatest } v^2 = 25 \text{ when } x = 3$$

A1

$$\text{Greatest speed} = 5 \text{ m s}^{-1}$$

A1

Answer: 5 m s^{-1} at $x = 3$

3 marks

(c) $v = 0: x^2 - 6x - 16 = 0$

M1

$$x = 8$$

A1

Answer: 8 m

2 marks

30 (10 marks)

(a) $\frac{1}{2}(2)v^2 = 9$

M1

$$v = 3 \text{ m s}^{-1}$$

A1

Answer: 3 m s^{-1}

2 marks

(b) Work done by the resultant force F from 0 to $x = E - 9$

M1

$$\int_0^x F \, dx = 4x^2 + 8x, \text{ so } F = \frac{d}{dx}(4x^2 + 8x)$$

M1

$$F = 8x + 8$$

A1

Answer: $F = 8x + 8$

3 marks

(c) $\frac{1}{2}(2)(25) = 25 = 4x^2 + 8x + 9$

M1

$$x^2 + 2x - 4 = 0$$

A1

$$x = -1 + \sqrt{5} = 1.24 \text{ m}$$

A1

Answer: $x = -1 + \sqrt{5} = 1.24$

3 marks

(d) $\int_1^3 (8x + 8) \, dx = [4x^2 + 8x]_1^3$

M1

$$= 60 - 12 = 48 \text{ J}$$

A1

Answer: 48 J

2 marks

Gravitational Potential Energy

1 (1 mark)

$$\text{GPE} = mgh = 6 \times 9.8 \times 4 = 235.2 \text{ J}$$

Answer: 235.2 J

B1
1 mark

2 (1 mark)

$$\text{GPE} = 1.5 \times 9.8 \times 2 = 29.4 \text{ J}$$

Answer: 29.4 J

B1
1 mark

3 (2 marks)

$$\text{Loss} = mgh = 3 \times 9.8 \times 5$$

$$= 147 \text{ J}$$

Answer: 147 J

M1
A1
2 marks

4 (2 marks)

$$58.8 = 2 \times 9.8 \times h$$

$$h = 3 \text{ m}$$

Answer: 3 m

M1
A1
2 marks

5 (2 marks)

$$70 \times 9.8 \times 100 = 68\,600 \text{ J}$$

$$= 68.6 \text{ kJ}$$

Answer: 68.6 kJ

M1
A1
2 marks

6 (2 marks)

$$\text{Height gained} = 10 \sin 30^\circ = 5 \text{ m}$$

$$\text{GPE gain} = 4 \times 9.8 \times 5 = 196 \text{ J}$$

Answer: 196 J

M1
A1
2 marks

7 (3 marks)

$$\text{Loss of KE} = \text{gain of GPE: } \frac{1}{2}(0.4)(14^2) = 0.4(9.8)h$$

$$39.2 = 3.92h, \text{ so } h = 10 \text{ m}$$

Answer: 10 m

M1 A1
A1
3 marks

8 (2 marks)

$$800 \times 9.8 \times 15 = 117\,600 \text{ J}$$

$$= 117.6 \text{ kJ}$$

Answer: 117.6 kJ

M1
A1
2 marks

9 (4 marks)

$$\text{(a) } 0.5 \times 9.8 \times 8 = 39.2 \text{ J}$$

Answer: 39.2 J

M1 A1
2 marks

$$\text{(b) } \frac{1}{2}(0.5)v^2 = 39.2$$

$$v = 12.5 \text{ m s}^{-1}$$

Answer: 12.5 m s⁻¹

M1
A1
2 marks

10 (3 marks)

$$\text{(a) } 2 \times 9.8 \times 3 = 58.8 \text{ J}$$

Answer: 58.8 J

B1
1 mark

$$\text{(b) Height above floor} = 3.8 \text{ m}$$

$$\text{GPE} = 2 \times 9.8 \times 3.8 = 74.48 \text{ J}$$

Answer: 74.48 J

M1
A1
2 marks

11 (3 marks)

Centre of mass rises from 0 to 1.5 m

M1

$$\text{GPE} = 6 \times 9.8 \times 1.5$$

M1

$$= 88.2 \text{ J}$$

A1

Answer: 88.2 J

3 marks

12 (4 marks)

$$\text{Height} = 1.5 - 1.5\cos 60^\circ$$

M1 A1

$$= 0.75 \text{ m}$$

A1

$$\text{GPE} = 0.8 \times 9.8 \times 0.75 = 5.88 \text{ J}$$

A1

Answer: 5.88 J

4 marks

13 (3 marks)

$$\text{Height} = 400 \times \frac{1}{20} = 20 \text{ m}$$

M1

$$\text{GPE} = 85 \times 9.8 \times 20$$

M1

$$= 16\,660 \text{ J}$$

A1

Answer: 16 660 J

3 marks

14 (3 marks)

Centre of mass rises 1 m

M1

$$\text{GPE} = 4 \times 9.8 \times 1$$

M1

$$= 39.2 \text{ J}$$

A1

Answer: 39.2 J

3 marks

15 (5 marks)

$$\text{(a)} \quad 2 \times 9.8 \times 3 + 5 \times 9.8 \times 1.2$$

M1

$$= 58.8 + 58.8 = 117.6 \text{ J}$$

A1

Answer: 117.6 J

2 marks

$$\text{(b)} \quad \bar{h} = \frac{2 \times 3 + 5 \times 1.2}{7} = \frac{12}{7} = 1.714 \text{ m}$$

M1 A1

$$7 \times 9.8 \times \frac{12}{7} = 117.6 \text{ J}$$

A1

Answer: 1.71 m

3 marks

16 (4 marks)

KE lost = GPE gained

M1

$$\frac{1}{2}(2)(49) = 2 \times 9.8 \times s \sin 30^\circ$$

M1 A1

$$s = 10 \text{ m}$$

A1

Answer: 5 m

4 marks

17 (4 marks)

Initially the centre of mass is 0.2 m high

B1

Finally it is 0.6 m high

B1

$$\text{Gain} = 12 \times 9.8 \times 0.4$$

M1

$$= 47.04 \text{ J}$$

A1

Answer: 47.04 J

4 marks

18 (4 marks)

Hanging part has mass 4 kg with centre of mass 1 m below the table

M1 A1

$$\text{Work} = \text{GPE gained} = 4 \times 9.8 \times 1$$

M1

$$= 39.2 \text{ J}$$

A1

Answer: 39.2 J

4 marks

19 (4 marks)

$$\text{(a)} \quad h = 300 \sin 12^\circ = 62.4 \text{ m}$$

M1

$$\text{Loss} = 65 \times 9.8 \times 62.4 = 39700 \text{ J}$$

A1

Answer: 39700 J

2 marks

$$\text{(b)} \quad 90 \times 300$$

M1

$= 27\,000\text{ J}$	A1
Answer: 27 000 J	2 marks
20 (5 marks)	
Vertical height gained $= s \sin \alpha$	B1
GPE gain $= mg \times s \sin \alpha$	M1
$98 = 4 \times 9.8 \times 5 \sin \alpha$	M1
$\sin \alpha = 0.5$	A1
$\alpha = 30^\circ$	A1
Answer: $\alpha = 30^\circ$	5 marks
21 (4 marks)	
(a) Centre of mass rises 2 m	M1
GPE $= 8 \times 9.8 \times 2 = 156.8\text{ J}$	A1
Answer: 156.8 J	2 marks
(b) Every part rises 1 m	M1
$8 \times 9.8 \times 1 = 78.4\text{ J}$	A1
Answer: 78.4 J	2 marks
22 (5 marks)	
B loses GPE: $5 \times 9.8 \times 2 = 98\text{ J}$	M1
A gains GPE: $3 \times 9.8 \times 2 = 58.8\text{ J}$	M1
Net loss $= 39.2\text{ J}$	A1
No energy is lost (smooth pulley), so KE gain $= 39.2\text{ J}$	M1 A1
Answer: 39.2 J, 39.2 J	5 marks
23 (5 marks)	
(a) GPE depends on vertical height, $s \sin 30^\circ = 2\text{ m}$	E1
$\cos 30^\circ$ gives the component perpendicular to the slope	E1
Answer: should be $\sin 30^\circ$	2 marks
(b) $5 \times 9.8 \times 4 \sin 30^\circ$	M1
$= 98\text{ J}$	A1
Answer: 98 J	2 marks
(c) No: GPE depends only on the vertical height, not on friction or path	E1
Answer: No	1 mark
24 (5 marks)	
Initial height of centre of mass $= 0.3\text{ m}$	B1
Final height $= \text{half the face diagonal} = \frac{0.6\sqrt{2}}{2} = 0.4243\text{ m}$	M1 A1
Gain in height $= 0.1243\text{ m}$	A1
GPE gain $= 30 \times 9.8 \times 0.1243 = 36.5\text{ J}$	A1
Answer: 36.5 J	5 marks
25 (5 marks)	
(a) $3 \times 9.8 \times (2 - 5)$	M1
$= -88.2\text{ J}$	A1
Answer: -88.2 J (a loss of 88.2 J)	2 marks
(b) No, it depends only on the change in vertical height	E1
Answer: No	1 mark
(c) $\frac{1}{2}(3)v^2 = 88.2$	M1
$v = 7.67\text{ m s}^{-1}$	A1
Answer: 7.67 m s⁻¹	2 marks

26 (5 marks)

Centre of mass is 0.75 m from A

B1

Initial depth below A = 0.75 m

B1

Final depth = $0.75\cos 50^\circ$

M1

Height gained = $0.75(1 - \cos 50^\circ) = 0.268$ m

A1

GPE = $6 \times 9.8 \times 0.268 = 15.8$ J

A1

Answer: 15.8 J

5 marks

27 (9 marks)(a) Centre of mass at height $L/2$

B1

$$\text{GPE} = \frac{MgL}{2}$$

A1

Answer: $\frac{MgL}{2}$

2 marks

(b) Final GPE ≈ 0

B1

$$\text{Loss} = \frac{MgL}{2}$$

A1

Answer: $\frac{MgL}{2}$

2 marks

(c) All the GPE lost is dissipated: $\frac{1}{2} \times 6 \times 9.8 \times 2 = 58.8$ J

M1

A1

Answer: 58.8 J

2 marks

(d) It is converted by inelastic impacts into internal energy (heat, sound)

E1

Answer: dissipated as internal energy

1 mark

(e) Top end falls 2 m: $v^2 = 2 \times 9.8 \times 2$

M1

$$v = 6.26 \text{ m s}^{-1}$$

A1

Answer: 6.26 m s^{-1}

2 marks

28 (10 marks)(a) Initial height above lowest point = $L - L \cos \alpha$

M1

Loss = $mgL(1 - \cos \alpha)$

A1

Answer: The loss in GPE when the string is vertical is $mgL(1 - \cos \alpha)$

2 marks

(b) Energy conservation: $\frac{1}{2}mv^2 = mgL(1 - \cos \alpha)$

M1

$$v = \sqrt{2gL(1 - \cos \alpha)}$$

A1

Answer: The speed at the lowest point is $\sqrt{2gL(1 - \cos \alpha)}$

2 marks

(c) $2(1 - \cos \alpha) = 1$

M1

 $\cos \alpha = \frac{1}{2}$, $\alpha = 60^\circ$

A1

Answer: 60°

2 marks

(d) The tension is perpendicular to the velocity

E1

Answer: perpendicular to motion

1 mark

(e) $T - mg = \frac{mv^2}{L}$ with $v^2 = gL$

M1 A1

$$T = 2mg$$

A1

Answer: $2mg$

3 marks

29 (10 marks)(a) Longer part: mass 8 kg, centre of mass 2 m below peg: $-8 \times 9.8 \times 2 = -156.8$ J

M1 A1

Shorter part: mass 4 kg, centre of mass 1 m below: $-4 \times 9.8 \times 1 = -39.2$ J

M1

Total = -196 J

A1

Answer: -196 J

4 marks

(b) 5 m side: mass 10 kg, centre of mass 2.5 m below peg: $-10 \times 9.8 \times 2.5 = -245$ J

M1 A1

1 m side: mass 2 kg, centre of mass 0.5 m below: $-2 \times 9.8 \times 0.5 = -9.8$ J, total = -254.8 J

A1

Answer: -254.8 J

3 marks

(c) Loss in GPE = $254.8 - 196 = 58.8$ J

M1

$$58.8 = \frac{1}{2}(12)v^2$$

M1

$$v = 3.13 \text{ m s}^{-1}$$

A1

Answer: 3.13 m s^{-1} **3 marks****30 (10 marks)****(a)** Mass hanging = $2x$ kg, centre of mass $\frac{x}{2}$ below the table**M1 A1**

$$\text{GPE} = -(2x)(9.8)\left(\frac{x}{2}\right) = -9.8x^2$$

A1**Answer: When a length x metres hangs over the edge, the GPE of the chain relative to the table top is $-9.8x^2$** **(b)** Initially $\text{GPE} = -9.8(0.25) = -2.45 \text{ J}$ **B1**Finally $x = 2$: $\text{GPE} = -9.8(4) = -39.2 \text{ J}$ **M1**Loss = 36.75 J **A1****Answer: 36.75 J** **3 marks****(c)** Energy conservation (table smooth): $\frac{1}{2}(4)v^2 = 36.75$ **M1 A1**

$$v = 4.29 \text{ m s}^{-1}$$

A1**Answer: 4.29 m s^{-1}** **3 marks****(d)** The weight of the hanging part, $2xg$, increases as x increases, and this is the driving force**E1****Answer: driving force depends on x** **1 mark**

Conservation of Mechanical Energy

1 (2 marks)

$$\frac{1}{2}mv^2 = mgh$$

M1

$$v = \sqrt{2 \times 9.8 \times 5} = 9.90 \text{ m s}^{-1}$$

A1**Answer: 9.90 m s^{-1}** **2 marks****2 (2 marks)**

$$\frac{1}{2}m(64) = mgh$$

M1

$$h = \frac{32}{9.8} = 3.27 \text{ m}$$

A1**Answer: 3.27 m** **2 marks****3 (2 marks)**

$$\frac{1}{2}v^2 = 9.8 \times 2.5$$

M1

$$v = 7 \text{ m s}^{-1}$$

A1**Answer: 7 m s^{-1}** **2 marks****4 (2 marks)**

$$v = \sqrt{2 \times 9.8 \times 0.45}$$

M1

$$= 2.97 \text{ m s}^{-1}$$

A1**Answer: 2.97 m s^{-1}** **2 marks****5 (2 marks)**

$$\text{KE} = \frac{1}{2}(0.3)(36) = 5.4 \text{ J}$$

B1

$$\text{GPE} = 0.3 \times 9.8 \times 2 = 5.88; \text{ total} = 11.28 \text{ J}$$

A1**Answer: 11.28 J** **2 marks****6 (1 mark)**

Only conservative forces (gravity, elastic) do work; no friction, air resistance or other non-conservative forces

E1**Answer: Only conservative forces do work****1 mark****7 (3 marks)**

$$\frac{1}{2}mv^2 = \frac{1}{2}m(100) + mg(8)$$

M1 A1

$$v = 16.0 \text{ m s}^{-1}$$

A1**Answer: 16.0 m s^{-1}** **3 marks**

8 (4 marks)

$$\text{Height fallen} = 3(1 - \cos 60^\circ) = 1.5 \text{ m}$$

M1 A1

$$\frac{1}{2}v^2 = 9.8 \times 1.5$$

M1

$$v = 5.42 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 5.42 \text{ m s}^{-1}$$

4 marks

9 (4 marks)

$$\text{Height lost} = 4\sin 30^\circ = 2 \text{ m}$$

M1 A1

$$\frac{1}{2}v^2 = \frac{1}{2}(36) + 9.8 \times 2$$

M1

$$v = 8.67 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 8.67 \text{ m s}^{-1}$$

4 marks

10 (3 marks)

$$\frac{1}{2}(144) = \frac{1}{2}(36) + 9.8h$$

M1 A1

$$h = \frac{54}{9.8} = 5.51 \text{ m}$$

A1

$$\text{Answer: } 5.51 \text{ m}$$

3 marks

11 (4 marks)

$$\frac{1}{2}(400)v^2 = \frac{1}{2}(400)(16) + 400 \times 9.8 \times 30$$

M1 A1

$$v^2 = 16 + 588 = 604$$

A1

$$v = 24.6 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 24.6 \text{ m s}^{-1}$$

4 marks

12 (4 marks)

$$\text{Energy at the bottom} = 70 \times 9.8 \times 25 = 17\,150 \text{ J}$$

M1 A1

$$\text{Work against friction} = 140s$$

M1

$$s = 122.5 \text{ m}$$

A1

$$\text{Answer: } 122.5 \text{ m}$$

4 marks

13 (3 marks)

Normal reaction is perpendicular to the motion, so no work: energy conserved

E1

$$\frac{1}{2}v^2 = 9.8 \times 4$$

M1

$$v = 8.85 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 8.85 \text{ m s}^{-1}$$

3 marks

14 (6 marks)

$$R = 2 \times 9.8\cos 30^\circ = 17.0 \text{ N}, F = 3.39 \text{ N}$$

M1 A1

$$\text{Work against friction} = 3.39 \times 10 = 33.9 \text{ J}$$

A1

$$\text{GPE lost} = 2 \times 9.8 \times 10\sin 30^\circ = 98 \text{ J}$$

B1

$$98 - 33.9 = \frac{1}{2}(2)v^2$$

M1

$$v = 8.00 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 8.00 \text{ m s}^{-1}$$

6 marks

15 (5 marks)

$$\text{Loss of PE} = 5 \times 9.8 \times 1.6 - 3 \times 9.8 \times 1.6 = 31.36 \text{ J}$$

M1 A1

$$\text{Gain in KE} = \frac{1}{2}(3 + 5)v^2$$

M1 A1

$$v = 2.80 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 2.80 \text{ m s}^{-1}$$

5 marks

16 (6 marks)

(a) Drop of 0.8 m: $\frac{1}{2}v^2 = 9.8 \times 0.8$

M1 A1

$$v = 3.96 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 3.96 \text{ m s}^{-1}$$

3 marks

(b) KE at B = $\frac{1}{2}(0.5)(15.68) = 3.92 \text{ J}$

B1

$F \times 4 = 3.92$	M1
$F = 0.98 \text{ N}$	A1
Answer: 0.98 N	3 marks
17 (4 marks)	
Total energy conserved; the horizontal velocity changes only through gravity's vertical effect	E1
$\frac{1}{2}(225) = \frac{1}{2}v^2 + 9.8 \times 3$	M1 A1
$v = 12.9 \text{ m s}^{-1}$	A1
Answer: 12.9 m s⁻¹	4 marks
18 (5 marks)	
(a) $\frac{1}{2}(4)(81) = 4 \times 9.8 \times s \sin 20^\circ$	M1 A1
$s = 12.1 \text{ m}$	A1
Answer: 12.1 m	3 marks
(b) Speed = 9 m s^{-1}	B1
Energy is conserved, GPE returns to its initial value so KE does too	E1
Answer: 9 m s⁻¹	2 marks
19 (5 marks)	
(a) Height fallen = $1.5 \cos 30^\circ = 1.30 \text{ m}$	M1 A1
$v = \sqrt{2 \times 9.8 \times 1.30} = 5.05 \text{ m s}^{-1}$	A1
Answer: 5.05 m s⁻¹	3 marks
(b) KE = $mgh = 0.4 \times 9.8 \times 1.5$	M1
$= 5.88 \text{ J}$	A1
Answer: 5.88 J	2 marks
20 (5 marks)	
GPE lost = $800 \times 9.8 \times 12 = 94\,080 \text{ J}$	B1
KE gained = $\frac{1}{2}(800)(144 - 25) = 47\,600 \text{ J}$	B1
Work against resistance = $94\,080 - 47\,600 = 46\,480 \text{ J}$	M1 A1
$R = \frac{46\,480}{200} = 232 \text{ N}$	A1
Answer: 232 N	5 marks
21 (9 marks)	
(a) Height fallen = $r - r \cos \theta$	M1 A1
$\frac{1}{2}mv^2 = mgr(1 - \cos \theta)$, so $v = \sqrt{2gr(1 - \cos \theta)}$	A1
Answer: The speed of P when OP makes angle θ with the vertical is $\sqrt{2gr(1 - \cos \theta)}$	3 marks
(b) Radial equation: $mg \cos \theta - R = \frac{mv^2}{r}$	M1 A1
$R = 0$: $g \cos \theta = 2g(1 - \cos \theta)$	M1
$3 \cos \theta = 2$, $\cos \theta = \frac{2}{3}$	A1
Answer: $\cos \theta = \frac{2}{3}$	4 marks
(c) $v^2 = 2gr(1/3) = 2 \times 9.8 \times 0.6/3$	M1
$v = 1.98 \text{ m s}^{-1}$	A1
Answer: 1.98 m s⁻¹	2 marks
22 (9 marks)	
(a) Resistive forces on the way up: $mg + \frac{mg}{2} = \frac{3mg}{2}$	B1
$\frac{1}{2}mv^2 = \frac{1}{2}m(8gH) - \frac{3mg}{2} \times H$	M1 A1
$v^2 = 8gH - 3gH = 5gH$, so $v = \sqrt{5gH}$	A1
Answer: $\sqrt{5gH}$	4 marks

(b) Speed after the ceiling = $\frac{1}{2}\sqrt{5gH}$

B1

On the way down the resultant force is $mg - \frac{mg}{2} = \frac{mg}{2}$

B1

$\frac{1}{2}mw^2 = \frac{1}{2}m\left(\frac{5gH}{4}\right) + \frac{mg}{2} \times H$

M1

$w^2 = \frac{5gH}{4} + gH = \frac{9gH}{4}, w = \frac{3}{2}\sqrt{gH}$

A1

Answer: $\frac{3}{2}\sqrt{gH}$

4 marks

(c) $\frac{mg}{2} \times 2H = mgH$

B1

Answer: mgH

1 mark

23 (4 marks)

(a) Energy conserved: $\frac{1}{2}v^2 = \frac{1}{2}(100) + 9.8 \times 20$

M1

Independent of direction, since only KE and height matter

E1

$v = 22.2 \text{ m s}^{-1}$ in each case

A1

Answer: Same speed 22.2 m s^{-1}

3 marks

(b) The time of flight (or the horizontal distance)

E1

Answer: time of flight

1 mark

24 (5 marks)

(a) At the top $KE \geq 0$: $\frac{1}{2}u^2 \geq g \times 1$

M1 A1

$u^2 > 2 \times 9.8 = 19.6$

A1

Answer: The bead reaches the top if $u^2 > 19.6$

3 marks

(b) $\frac{1}{2}(36) = \frac{1}{2}v^2 + 9.8 \times 1$

M1

$v = 4.05 \text{ m s}^{-1}$

A1

Answer: 4.05 m s^{-1}

2 marks

25 (7 marks)

(a) $\frac{1}{2}mu^2 = \frac{1}{2}mu^2 \cos^2 \alpha + mgH$

M1 A1

$mgH = \frac{1}{2}mu^2(1 - \cos^2 \alpha)$

M1

$H = \frac{u^2 \sin^2 \alpha}{2g}$

A1

Answer: $H = \frac{u^2 \sin^2 \alpha}{2g}$

4 marks

(b) $H = \frac{400 \times 0.25}{19.6} = 5.102 \text{ m}$

M1

$v^2 = 400 - 2 \times 9.8 \times 2.551$

M1

$v = 18.7 \text{ m s}^{-1}$

A1

Answer: 18.7 m s^{-1}

3 marks

26 (7 marks)

(a) Energy: loss of PE of B = $3 \times 9.8 \times 1 = 29.4 \text{ J}$

B1

gain in PE of A = $2 \times 9.8 \times 1 \times \sin 30^\circ = 9.8 \text{ J}$

M1 A1

$19.6 = \frac{1}{2}(5)v^2, v = 2.80 \text{ m s}^{-1}$

M1 A1

Answer: 2.80 m s^{-1}

5 marks

(b) A then moves up as a particle: $\frac{1}{2}(2)v^2 = 2 \times 9.8 \times s \sin 30^\circ$

M1

$s = 0.800 \text{ m}$

A1

Answer: 0.800 m

2 marks

27 (10 marks)

(a) Height fallen = $r \cos \theta$

M1

$v = \sqrt{2gr \cos \theta}$

A1

Answer: The speed of the particle is $\sqrt{2gr \cos \theta}$

2 marks

$$(b) \theta = 0: v = \sqrt{2 \times 9.8 \times 1.2} = 4.85 \text{ m s}^{-1}$$

M1 A1

Answer: 4.85 m s⁻¹

2 marks

$$(c) R - mg = \frac{mv^2}{r}$$

M1

$$v^2 = 2gr$$

B1

$$R = mg + 2mg = 3mg$$

A1

Answer: The normal reaction at the lowest point is 3mg

3 marks

$$(d) R - mg \cos \theta = \frac{mv^2}{r} = 2mg \cos \theta$$

M1 A1

$$R = 3mg \cos \theta = 2mg, \cos \theta = \frac{2}{3}, \theta = 48.2^\circ$$

A1

Answer: $\theta = 48.2^\circ$

3 marks

28 (9 marks)

$$(a) \frac{1}{2}v^2 = \frac{1}{2}(9) + 9.8 \times 1$$

M1 A1

$$v = 5.35 \text{ m s}^{-1}$$

A1

Answer: 5.35 m s⁻¹

3 marks

$$(b) T - 2 \times 9.8 = 2 \times \frac{v^2}{1}$$

M1 A1

$$T = 19.6 + 2 \times 28.6 = 76.8 \text{ N}$$

A1

Answer: 76.8 N

3 marks

$$(c) \text{Energy: } \frac{1}{2}v_g^2 = \frac{1}{2}(28.6) + 9.8 \times 1.5$$

M1 A1

$$v_g = 7.62 \text{ m s}^{-1}$$

A1

Answer: 7.62 m s⁻¹

3 marks

29 (9 marks)

$$(a) \text{Loss of GPE of B} = 3 \times 9.8 \times 1.5 = 44.1 \text{ J}$$

B1

$$\text{Work against friction} = 0.4 \times 2 \times 9.8 \times 1.5 = 11.76 \text{ J}$$

M1 A1

$$44.1 - 11.76 = \frac{1}{2}(5)v^2$$

M1

$$v = 3.60 \text{ m s}^{-1}$$

A1

Answer: 3.60 m s⁻¹

5 marks

$$(b) \text{The string goes slack; A decelerates under friction: } 0.4 \times 9.8 = 3.92 \text{ m s}^{-2}$$

M1 A1

$$v^2 = 2 \times 3.92 s$$

M1

$$s = 1.65 \text{ m}$$

A1

Answer: 1.65 m

4 marks

30 (12 marks)

$$(a) \text{Drop of 3.2 m: } \frac{1}{2}v^2 = 9.8 \times 3.2$$

M1 A1

$$v = 7.92 \text{ m s}^{-1}$$

A1

Answer: 7.92 m s⁻¹

3 marks

$$(b) 1.8 = \frac{1}{2}(9.8)t^2$$

M1

$$t = 0.606 \text{ s}$$

A1

$$\text{Distance} = 7.920 \times 0.6061$$

M1

$$= 4.80 \text{ m}$$

A1

Answer: 0.606 s, 4.80 m

4 marks

$$(c) \frac{1}{2}v^2 = 9.8 \times 5, v = 9.90 \text{ m s}^{-1}$$

M1 A1

$$\text{Horizontal } 7.920, \text{ vertical } 9.8t = 5.940$$

B1

$$\sqrt{7.920^2 + 5.940^2} = 9.90$$

A1

Answer: 9.90 m s⁻¹

4 marks

(d) No air resistance (or the particle is treated as a point, the ramp is smooth)

E1

Answer: No air resistance

1 mark

Power

1 (1 mark)

$$P = Fv = 500 \times 4 = 2000 \text{ W}$$

Answer: 2000 W

B1

1 mark

2 (1 mark)

$$P = \frac{12\,000}{8} = 1500 \text{ W}$$

Answer: 1500 W

B1

1 mark

3 (2 marks)

$$P = 1800 \times 25 = 45\,000 \text{ W}$$

$$= 45 \text{ kW}$$

Answer: 45 kW

M1

A1

2 marks

4 (2 marks)

$$\text{Work} = 3000 \times 120$$

$$= 360\,000 \text{ J}$$

Answer: 360 kJ

M1

A1

2 marks

5 (2 marks)

$$T = 200 \times 9.8 = 1960 \text{ N}$$

$$P = 1960 \times 0.5 = 980 \text{ W}$$

Answer: 980 W

M1

A1

2 marks

6 (2 marks)

$$F = \frac{P}{v} = \frac{40\,000}{20}$$

$$= 2000 \text{ N}$$

Answer: 2000 N

M1

A1

2 marks

7 (3 marks)

At maximum speed driving force = resistance = 800 N

$$P = 800 \times 40 = 32\,000 \text{ W} = 32 \text{ kW}$$

Answer: 32 kW

M1 A1

A1

3 marks

8 (4 marks)

$$\text{Driving force} = \frac{30\,000}{15} = 2000 \text{ N}$$

$$2000 - 900 = 1200 \text{ a}$$

$$a = 0.917 \text{ m s}^{-2}$$

Answer: 0.917 m s⁻²

M1 A1

M1

A1

4 marks

9 (4 marks)

$$\text{Driving force} = 800 \times 9.8 \sin 5^\circ + 300$$

$$= 983 \text{ N}$$

$$P = 983 \times 12 = 11800 \text{ W}$$

Answer: 11800 W

M1 A1

A1

A1

4 marks

10 (3 marks)

$$T = 600 \times 9.8 = 5880 \text{ N}$$

$$P = 5880 \times 2 = 11\,760 \text{ W}$$

$$= 11.76 \text{ kW}$$

Answer: 11.76 kW

M1

A1

A1

3 marks

11 (3 marks)

$$\text{Work per second} = 50 \times 9.8 \times 6$$

M1 A1

$$= 2940 \text{ W}$$

A1

Answer: 2940 W

3 marks

12 (4 marks)

$$F = \frac{250}{6} = 41.7 \text{ N}$$

M1 A1

$$\text{Resistance } 2F = 83.3 \text{ N}$$

B1

$$v = \frac{250}{83.3} = 3 \text{ m s}^{-1}$$

A1

Answer: 41.7 N, 3 m s⁻¹

4 marks

13 (3 marks)

$$\text{At max speed } F = 15\,000 \text{ N}$$

M1

$$v = \frac{600\,000}{15\,000}$$

M1

$$= 40 \text{ m s}^{-1}$$

A1

Answer: 40 m s⁻¹

3 marks

14 (6 marks)

$$(a) v = \frac{45\,000}{1200}$$

M1

$$= 37.5 \text{ m s}^{-1}$$

A1

Answer: 37.5 m s⁻¹

2 marks

$$(b) \text{ Driving force} = 1200 + 1500 \times 9.8 \sin 4^\circ$$

M1 A1

$$= 2230 \text{ N}$$

A1

$$v = \frac{45\,000}{2230} = 20.2 \text{ m s}^{-1}$$

A1

Answer: 20.2 m s⁻¹

4 marks

15 (5 marks)

$$(a) F = \frac{36\,000}{10} = 3600 \text{ N}$$

M1

$$3600 - 600 = 900a$$

M1

$$a = 3.33 \text{ m s}^{-2}$$

A1

Answer: 3.33 m s⁻²

3 marks

$$(b) \text{ At max speed } a = 0: F = 600$$

M1

$$v = \frac{36\,000}{600} = 60 \text{ m s}^{-1}$$

A1

Answer: 60 m s⁻¹

2 marks

16 (5 marks)

$$\text{Driving force} = \frac{24\,000}{8} = 3000 \text{ N}$$

B1

$$N2L \text{ along slope: } 3000 - 400 - 1000 \times 9.8 \sin 3^\circ = 1000a$$

M1 A1 A1

$$a = 2.09 \text{ m s}^{-2}$$

A1

Answer: 2.09 m s⁻²

5 marks

17 (4 marks)

$$T - 500 \times 9.8 = 500 \times 0.4$$

M1

$$T = 5100 \text{ N}$$

A1

$$P = Tv = 5100 \times 1.5$$

M1

$$= 7650 \text{ W}$$

A1

Answer: 5100 N, 7650 W

4 marks

18 (4 marks)Resistance = kv ; at steady speed $P_w = kv^2$ M1

$$15\,000 = 36k, k = 416.7$$
 A1

$$P = kv^2 = 416.7 \times 81$$
 M1

$$= 33\,750 \text{ W}$$
 A1

Answer: 33.75 kW 4 marks**19 (5 marks)**Level road: $P = 30R$ B1Hill: driving force = $R + 1000 \times 9.8 \times \frac{1}{10} = R + 980$, so $P = 20(R + 980)$ M1 A1

$$30R = 20R + 19\,600, \text{ so } R = 1960 \text{ N}$$
 A1

$$P = 30 \times 1960 = 58\,800 \text{ W}$$
 A1

Answer: R = 1960 N, P = 58 800 W 5 marks**20 (5 marks)**(a) At steady speed $\frac{18\,400}{v} = 300 + 4v$ M1

$$4v^2 + 300v - 18\,400 = 0, \text{ that is } (v - 40)(v + 115) = 0$$
 M1

$$v = 40 \text{ m s}^{-1}$$
 A1

Answer: 40 m s⁻¹ 3 marks(b) Driving force = $\frac{18\,400}{20} = 920 \text{ N}$, resistance = $300 + 80 = 380 \text{ N}$ M1

$$800a = 540, a = 0.675 \text{ m s}^{-2}$$
 A1

Answer: 0.675 m s⁻² 2 marks**21 (8 marks)**(a) $v = \frac{36\,000}{500} = 72 \text{ m s}^{-1}$ M1 A1**Answer: 72 m s⁻¹** 2 marks(b) Driving force = $500 + 1200 \times 9.8 \times \frac{1}{20}$ M1 A1

$$= 1088 \text{ N}, v = \frac{36\,000}{1088} = 33.1 \text{ m s}^{-1}$$
 A1

Answer: Maximum speed up the hill = 33.1 m s⁻¹ (3 s.f.) 3 marks(c) $D = \frac{36\,000}{20} = 1800 \text{ N}$ B1

$$1800 - 1088 = 1200a$$
 M1

$$a = 0.593 \text{ m s}^{-2}$$
 A1

Answer: 0.593 m s⁻² 3 marks**22 (8 marks)**(a) $v = \frac{30\,000}{600} = 50 \text{ m s}^{-1}$ M1 A1**Answer: 50 m s⁻¹** 2 marks(b) Work = power $\times$ time B1**Answer: The work done by the engine while the car accelerates is given by 30 000t, where t is the time taken**(c) Work by engine = $30\,000 \times 65.5 = 1\,965\,000 \text{ J}$ B1

$$\text{Gain in KE} = \frac{1}{2}(1000)(1600 - 100) = 750\,000 \text{ J}$$
 M1 A1

$$1\,965\,000 - 750\,000 = 600s$$
 M1

$$s = 2025 \text{ m}$$
 A1

Answer: 2025 m 5 marks**23 (6 marks)**(a) $P = Fv$ so $F = \frac{P}{v} = \frac{20\,000}{10}$ M1The student forgot the kilo: $P = 20\,000 \text{ W}$, $F = 2000 \text{ N}$ A1**Answer: F = 2000 N** 2 marks

(b) $2000 - 1500 = 1500a$

M1

$$a = \frac{1}{3} \text{ m s}^{-2}$$

A1

Answer: $\frac{1}{3} \text{ m s}^{-2}$

2 marks

(c) Driving force $= \frac{P}{v}$ decreases as v increases

E1

so the resultant force (and hence acceleration) decreases

E1

Answer: $F = P/v$ falls

2 marks

24 (6 marks)

(a) Driving force $= 3000 \text{ N}$

B1

$$3000 - 600 = 4000a$$

M1

$$a = 0.6 \text{ m s}^{-2}$$

A1

Answer: 0.6 m s^{-2}

3 marks

(b) Trailer: $T - 200 = 1000(0.6)$

M1 A1

$$T = 800 \text{ N}$$

A1

Answer: 800 N

3 marks

25 (6 marks)

Down slope, engine off: $mg \sin \theta = kv$

M1

$$1500 \times 9.8/15 = 980 = 20k, k = 49$$

A1

Up: $D = 980 + 49 \times 20 = 1960 \text{ N}$

M1 A1

$$P = 1960 \times 20$$

M1

$$= 39\,200 \text{ W}$$

A1

Answer: $k = 49, 39.2 \text{ kW}$

6 marks

26 (6 marks)

(a) Useful power $= 20 \times 9.8 \times 8 + \frac{1}{2}(20)(36)$

M1 A1

$$= 1928 \text{ W}; \text{ input} = \frac{1928}{0.8} = 2410 \text{ W}$$

A1

Answer: 2410 W

3 marks

(b) Useful power $= 2400 \text{ W}$

B1

$$m(9.8 \times 8 + 18) = 2400$$

M1

$$m = 24.9 \text{ kg per second}$$

A1

Answer: $24.9 \text{ kg per second}$

3 marks

27 (10 marks)

(a) Steady speed: driving force $= R + mg \sin \theta$

M1

$$P = V(R + mg \sin \theta)$$

A1

Answer: $P = V(R + mg \sin \theta)$

2 marks

(b) Driving force $= \frac{P}{v}$

B1

$$\frac{P}{v} - R - mg \sin \theta = ma$$

M1

$$a = \frac{P}{mv} - \frac{R}{m} - g \sin \theta$$

A1

Answer: When the speed is v the acceleration is $\frac{P}{mv} - \frac{R}{m} - g \sin \theta$

3 marks

(c) $D = 800 + 1200 \times 9.8/20 = 1388 \text{ N}$

M1 A1

$$V = \frac{40\,000}{1388} = 28.8 \text{ m s}^{-1}$$

A1

Answer: 28.8 m s^{-1}

3 marks

(d) $a = \frac{40\,000}{1200 \times 10} - \frac{800}{1200} - 0.49$

M1

$$= 2.18 \text{ m s}^{-2}$$

A1

Answer: 2.18 m s^{-2}

2 marks

28 (9 marks)**(a)** At max speed $D = R = 50k$ M1

$$P = 50k \times 50 = 2500k$$
 A1

Answer: $P = 2500k$ 2 marks**(b)** Driving force = $\frac{2500k}{20} = 125k$ M1 A1

$$125k - 20k = 1000 \times 0.5$$
 M1 A1

$$k = \frac{500}{105} = 4.76, P = 11900 \text{ W}$$
 A1

Answer: $k = 4.76, P = 11900 \text{ W}$ 5 marks**(c)** Max speed where driving force equals resistance is 50 m s^{-1} M1which is less than 55; beyond it the resistance exceeds the driving force E1**Answer: Required driving force at 55 m s^{-1} is below the resistance, so 55 m s^{-1} cannot be reached** 2 marks**29 (11 marks)****(a)** $54 \text{ km h}^{-1} = 54 \times \frac{1000}{3600} = 15 \text{ m s}^{-1}$ B1

$$\text{Driving force} = \frac{54\,000}{15} = 3600 \text{ N, so } 3600 - 225c = 1000 \times 3.15$$
 M1

$$225c = 450, c = 2$$
 A1

Answer: $c = 2$ 3 marks**(b)** At steady speed $\frac{54\,000}{V} = 2V^2$, so $V^3 = 27\,000$ M1

$$V = 30 \text{ m s}^{-1}$$
 A1

Answer: 30 m s^{-1} 2 marks**(c)** Driving force = $\frac{54\,000}{25} = 2160 \text{ N}$, resistance = $2 \times 625 = 1250 \text{ N}$ M1 A1

$$1000a = 910, a = 0.91 \text{ m s}^{-2}$$
 A1

Answer: 0.91 m s^{-2} 3 marks**(d)** $\frac{54\,000}{v} = 2v^2 + 1000 \times 9.8 \times \frac{1}{20}$, so $2v^3 + 490v - 54\,000 = 0$ M1 A1

$$f(27.45) = 820 < 0 \text{ and } f(27.55) = 1300 > 0, \text{ so } 27.45 < v < 27.55, \text{ that is } v = 27.5 \text{ to } 3 \text{ s.f.}$$
 A1

Answer: $2v^3 + 490v - 54\,000 = 0, v = 27.5 \text{ m s}^{-1}$ 3 marks**30 (11 marks)****(a)** At $v = 40$: $D = 25 \times 40 = 1000 \text{ N}$ M1

$$P = 1000 \times 40 = 40\,000 \text{ W}$$
 A1

Answer: 40 kW 2 marks**(b)** $D = \frac{40\,000}{10} = 4000 \text{ N}$ B1

$$4000 - 250 = 1200a$$
 M1

$$a = 3.12 \text{ m s}^{-2}$$
 A1

Answer: 3.12 m s^{-2} 3 marks**(c)** $a = v \frac{dv}{dx}$ M1

$$1200v \frac{dv}{dx} = \frac{40\,000}{v} - 25v$$
 A1

Answer: The equation of motion can be written $1200v \frac{dv}{dx} = \frac{40\,000}{v} - 25v$ 2 marks

(d) $\frac{dx}{dv} = \frac{1200v^2}{40\,000 - 25v^2}$ M1

$$x = \int_{10}^{20} \frac{1200v^2}{40\,000 - 25v^2} dv$$
 M1

Write $v^2/(1600 - v^2) = -1 + 1600/(1600 - v^2)$ and integrate (partial fractions) M1

$$x = 84 \text{ m}$$
 A1

Answer: 84 m 4 marks

Variable Forces and Work

1 (2 marks)

$$W = \int_0^3 4x \, dx$$

M1

$$= [2x^2]_0^3 = 18 \, \text{J}$$

A1

Answer: 18 J

2 marks

2 (2 marks)

$$W = \int_0^4 (6 - x) \, dx = [6x - \frac{x^2}{2}]_0^4$$

M1

$$= 24 - 8 = 16 \, \text{J}$$

A1

Answer: 16 J

2 marks

3 (2 marks)

Work = area under the graph

M1

$$= \frac{1}{2} \times 4 \times 10 + 4 \times 10 = 60 \, \text{J}$$

A1

Answer: 60 J

2 marks

4 (2 marks)

$$W = [x^3]_1^2$$

M1

$$= 8 - 1 = 7 \, \text{J}$$

A1

Answer: 7 J

2 marks

5 (2 marks)

$$W = \int_0^{0.3} 20x \, dx = [10x^2]_0^{0.3}$$

M1

$$= 0.9 \, \text{J}$$

A1

Answer: 0.9 J

2 marks

6 (2 marks)

$$W = \int_a^b F \, dx$$

B1

The area under the graph is the work done

B1

Answer: $W = \int F \, dx$

2 marks

7 (4 marks)

$$\text{Work} = \text{area} = \frac{1}{2} \times 6 \times 12 = 36 \, \text{J}$$

M1 A1

$$36 = \frac{1}{2}(2)v^2$$

M1

$$v = 6 \, \text{m s}^{-1}$$

A1

Answer: $6 \, \text{m s}^{-1}$

4 marks

8 (3 marks)

$$W = \int_1^4 (2x + 1) \, dx = [x^2 + x]_1^4$$

M1 A1

$$= 20 - 2 = 18 \, \text{J}$$

A1

Answer: 18 J

3 marks

9 (3 marks)

$$W = \int_0^4 6 \sin\left(\frac{\pi x}{4}\right) \, dx$$

M1

$$= \left[-\frac{24}{\pi} \cos\left(\frac{\pi x}{4}\right)\right]_0^4$$

A1

$$= \frac{48}{\pi} = 15.3 \, \text{J}$$

A1

Answer: 15.3 J

3 marks

10 (4 marks)

$$W = \int_0^3 (2x + 2) \, dx = [x^2 + 2x]_0^3 = 15 \, \text{J}$$

M1 A1

$$15 = \frac{1}{2}(3)v^2$$

M1

$$v = 3.16 \, \text{m s}^{-1}$$

A1

Answer: 3.16 m s⁻¹

4 marks

11 (3 marks)

$$W = \int_0^{\ln 2} 8e^{-x} \, dx = [-8e^{-x}]_0^{\ln 2}$$

M1 A1

$$= -4 + 8 = 4 \, \text{J}$$

A1

Answer: 4 J

3 marks

12 (3 marks)

$$\text{Extension} = 0.3 \, \text{m}$$

B1

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(40)(0.09)$$

M1

$$= 1.8 \, \text{J}$$

A1

Answer: 1.8 J

3 marks

13 (4 marks)

$$W = \left[\frac{x^3}{3} - 4x \right]_0^2$$

M1 A1

$$= \frac{8}{3} - 8 = -\frac{16}{3} \, \text{J}$$

A1

Negative, since the force opposes the displacement for $0 < x < 2$

E1

Answer: $-\frac{16}{3} \, \text{J}$

4 marks

14 (6 marks)

$$\text{Area of trapezium} = \frac{1}{2}(3 + 7)(8)$$

M1 A1

$$= 40 \, \text{J}$$

A1

$$40 = \frac{1}{2}(4)v^2, v = 4.47 \, \text{m s}^{-1}$$

M1 A1

$$\text{Work from 2 to 5} = 3 \times 8 = 24 \, \text{J}$$

B1

Answer: 4.47 m s⁻¹, 24 J

6 marks

15 (5 marks)

$$W = \int_1^4 12x^{-2} \, dx = [-12x^{-1}]_1^4$$

M1 A1

$$= -3 + 12 = 9 \, \text{J}$$

A1

$$9 = \frac{1}{2}(2)v^2 - \frac{1}{2}(2)(9)$$

M1

$$v^2 = 18, v = 4.24 \, \text{m s}^{-1}$$

A1

Answer: 4.24 m s⁻¹

5 marks

16 (4 marks)

$$\text{Elastic energy} = \frac{1}{2}(50)(0.04) = 1 \, \text{J}$$

M1 A1

$$1 = \frac{1}{2}(0.4)v^2$$

M1

$$v = 2.24 \, \text{m s}^{-1}$$

A1

Answer: 2.24 m s⁻¹

4 marks

17 (5 marks)

$$W = [3x - x^2]_0^2$$

M1 A1

$$= 6 - 4 = 2 \, \text{J}$$

A1

$$2 = \frac{1}{2}(0.5)v^2 - \frac{1}{2}(0.5)(16)$$

M1

$$v = 4.90 \, \text{m s}^{-1}$$

A1

Answer: 4.90 m s⁻¹

5 marks

18 (5 marks)

$$\text{Power of resistance} = Fv = 6v^2 = 384e^{-4t}$$

M1 A1

$$\text{Work} = \int_0^2 384e^{-4t} dt$$

M1

$$= 96(1 - e^{-8})$$

A1

$$= 96.0 \text{ J}$$

A1

Answer: 96.0 J

5 marks

19 (5 marks)

$$\int_0^3 kx^2 dx = 9k$$

M1 A1

$$k = 4$$

A1

$$\int_3^5 4x^2 dx = \frac{4}{3}(125 - 27)$$

M1

$$= \frac{392}{3} = 131 \text{ J}$$

A1

Answer: k = 4, 131 J

5 marks

20 (5 marks)

$$\text{Acceleration} = 2t, \text{ so } F = 3 \times 2t = 6t$$

B1

$$\text{Work} = \int_0^2 F v dt = \int_0^2 6t(t^2 + 1) dt$$

M1

$$= \left[\frac{3t^4}{2} + 3t^2 \right]_0^2 = 24 + 12 = 36 \text{ J}$$

M1 A1

$$\text{Check: KE at } t = 2 \text{ is } \frac{1}{2}(3)(25) = 37.5 \text{ J, KE at } t = 0 \text{ is } \frac{1}{2}(3)(1) = 1.5 \text{ J, change} = 36 \text{ J}$$

A1

Answer: 36 J

5 marks

21 (8 marks)

$$(a) \text{ Work} = \left[12x - \frac{3x^2}{2} \right]_0^x$$

M1 A1

$$\frac{1}{2}(2)v^2 = 12x - \frac{3x^2}{2}$$

A1

$$\text{Answer: } v^2 = 12x - \frac{3x^2}{2}$$

3 marks

$$(b) \text{ Maximum where } F = 0: x = 4$$

M1

$$v^2 = 48 - 24 = 24$$

A1

$$v = 4.90 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 4.90 \text{ m s}^{-1} \text{ at } x = 4$$

3 marks

$$(c) 12x - 1.5x^2 = 0, x = 8$$

M1 A1

Answer: x = 8

2 marks

22 (6 marks)

$$(a) W = \int_R^{2R} \frac{mgR^2}{r^2} dr$$

M1

$$= mgR^2 \left[-\frac{1}{r} \right]_R^{2R}$$

A1

$$= mgR^2 \left(\frac{1}{R} - \frac{1}{2R} \right) = \frac{1}{2}mgR$$

A1

$$\text{Answer: The work done by gravity as the particle falls from rest at } r = 2R \text{ to } r = R \text{ is } \frac{1}{2}mgR$$

3 marks

$$(b) \frac{1}{2}mv^2 = \frac{1}{2}mgR$$

M1

$$v = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

A1

$$= 7920 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 7920 \text{ m s}^{-1}$$

3 marks

23 (6 marks)**(a)** $W = Fd$ applies only for a constant force

E1

Here F varies with x so the work must be found by integration

E1

Answer: F is not constant

2 marks

(b) $W = [x^3]_0^2$

M1

 $= 8 \text{ J}$

A1

Answer: 8 J

2 marks

(c) Student used the maximum value of F over the whole distance

E1

so it is an over-estimate

E1

Answer: over-estimate

2 marks

24 (5 marks)Work done against the force $= \int_0^s 0.5x^2 \, dx = \frac{s^3}{6}$

M1 A1

Loss of KE $= \frac{1}{2}(0.5)(36) = 9$

B1

 $\frac{s^3}{6} = 9, s^3 = 54$

M1

 $s = 3.78 \text{ m}$

A1

Answer: 3.78 m

5 marks

25 (5 marks)A small element of length dx of mass $\frac{6}{3}dx = 2dx$ lies at height x above the ground

B1

Raise it by 3 m: $dW = 2 \times 9.8 \times 3 \, dx$

M1

 $W = \int_0^3 58.8 \, dx$

M1 A1

 $= 176.4 \text{ J}$

A1

Answer: 176.4 J

5 marks

26 (6 marks)Work $= \int_0^x (6u - 2u^2) \, du = 3x^2 - \frac{2x^3}{3}$

M1 A1

 $\frac{1}{2}v^2 = 3x^2 - \frac{2x^3}{3}$

A1

 v is maximal when $F = 0$: $x = 3$

M1

 $\frac{1}{2}v^2 = 27 - 18 = 9$

A1

 $v = 4.24 \text{ m s}^{-1}$

A1

Answer: 4.24 m s⁻¹ at $x = 3$

6 marks

27 (10 marks)**(a)** N2L: $ma = -kv^2$

M1

 $a = v \frac{dv}{dx}$

A1

Answer: $mv \frac{dv}{dx} = -kv^2$

2 marks

(b) $m \frac{dv}{dx} = -kv$

M1

 $\int \frac{1}{v} \, dv = -\frac{k}{m} \int dx, \ln v = -\frac{kx}{m} + C$

M1

 $v = u e^{-kx/m}$

A1

Answer: $v = u e^{-kx/m}$

3 marks

(c) Loss of KE $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

M1

 $v^2 = u^2 e^{-2kx/m}$

A1

 $= \frac{1}{2}mu^2(1 - e^{-2kx/m})$

A1

Answer: The work done against the resistance in the first x metres is $\frac{1}{2}mu^2(1 - e^{-2kx/m})$

3 marks

(d) $e^{-2kx/m} = \frac{1}{4}$

M1

$$x = \frac{m \ln 2}{k}$$

A1

Answer: $x = \frac{m \ln 2}{k}$

2 marks

28 (12 marks)

(a) $2 \frac{dv}{dt} = 12t - 3t^2$

M1

$$v = 3t^2 - \frac{t^3}{2}$$

M1 A1

Answer: $v = 3t^2 - \frac{t^3}{2}$

3 marks

(b) $x = \int_0^t v \, dt$

M1

$$= t^3 - \frac{t^4}{8}$$

A1

Answer: $x = t^3 - \frac{t^4}{8}$

2 marks

(c) At $t = 4$: $v = 48 - 32 = 16 \text{ m s}^{-1}$, $x = 64 - 32 = 32 \text{ m}$

M1 A1

Work-energy: $\frac{1}{2}(2)(16^2) = 256 \text{ J}$

M1

Integral: $\int_0^4 (12t - 3t^2)(3t^2 - \frac{t^3}{2}) \, dt = 256 \text{ J}$

A1

Answer: 256 J

4 marks

(d) $P = Fv = (12t - 3t^2)(3t^2 - \frac{t^3}{2})$, evaluate over the interval

M1

$$P(t) = 3t^3(4 - t)(6 - t)/2$$

A1

Maximum value 127 W

A1

Answer: 127 W

3 marks

29 (10 marks)

(a) Work by gravity $= - \int_R^r \frac{mgR^2}{s^2} \, ds$

M1 A1

$$= -mgR^2(\frac{1}{R} - \frac{1}{r})$$

A1

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \text{this}$, giving the result

A1

Answer: The speed v at distance r satisfies $v^2 = u^2 - 2gR^2(\frac{1}{R} - \frac{1}{r})$

4 marks

(b) Need $v^2 \geq 0$ as $r \rightarrow \infty$: $u^2 \geq 2gR$

M1 A1

$$u = \sqrt{2gR}$$

A1

Answer: $\sqrt{2gR}$

3 marks

(c) $u = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$

M1

$$= 11200 \text{ m s}^{-1}$$

A1

Answer: 11200 m s⁻¹

2 marks

(d) With constant g the height would be $\frac{u^2}{2g}$, which is finite, but the real force weakens with distance

E1

Answer: g decreases with height

1 mark

30 (10 marks)

(a) $W = \int_0^{1500} (9000 - 4x) \, dx$

M1

$$= 9000 \times 1500 - 2 \times 1500^2$$

A1

$$= 9\,000\,000 \text{ J}$$

A1

Answer: 9 000 000 J

3 marks

(b) Net work $= \int_0^x (9000 - 4u - 6000) \, du = 3000x - 2x^2$

M1 A1

$$= \frac{1}{2}(5000)v^2$$

A1

Answer: The speed v at x satisfies $5000 \times \frac{v^2}{2} = 3000x - 2x^2$ for x within the range

3 marks

(c) Net force zero when $9000 - 4x = 6000$, $x = 750$

M1

$$\frac{1}{2}(5000)v^2 = 2\,250\,000 - 1\,125\,000 = 1\,125\,000$$

A1

$$v = 21.2 \text{ m s}^{-1}$$

A1

Answer: 21.2 m s^{-1} at $x = 750$

3 marks

(d) $3000x = 2x^2$, $x = 1500 \text{ m}$

B1

Answer: 1500 m

1 mark