

Work Done by Forces

- 1 A constant force of 20 N moves a box 5 m in the direction of the force. Find the work done by the force. [1]
- 2 A force of 15 N acts at 60° to the direction of motion of a particle, which moves 4 m. Find the work done by the force. [2]
- 3 A student lifts a 3 kg book vertically through 2 m at constant speed. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done against gravity. [2]
- 4 A constant frictional force of 12 N opposes the motion of a sledge, which moves 7 m. Find the work done against friction. [1]
- 5 A force of 8 N acts on a particle at right angles to its displacement. State the work done by this force and give a reason. [2]
- 6 A worker pushes a crate with a constant horizontal force of 200 N for a distance of 15 m. Find the work done in kilojoules. [2]
- 7 A rope inclined at 30° to the horizontal pulls a sledge 10 m along level ground with a constant tension of 40 N. Find the work done by the tension. [3]
- 8 A block of mass 5 kg is pushed 4 m up a smooth slope inclined at 30° to the horizontal at constant speed, by a force parallel to the slope. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done by the force. [4]
- 9 A box of mass 10 kg is pulled at constant speed along a rough horizontal floor through 6 m. The coefficient of friction is 0.3. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done against friction. [4]
- 10 A lift raises a man of mass 80 kg through 12 m at constant speed. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done by the lift cable on the man, giving your answer in kilojoules to 3 significant figures. [3]
- 11 A particle moves with displacement $(2\mathbf{i} - \mathbf{j})$ m under the action of a constant force $(3\mathbf{i} + 4\mathbf{j})$ N. Find the work done by the force. [3]
- 12 A ball of mass 0.4 kg falls vertically through 5 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done by gravity. [2]
- 13 A force of 50 N, acting at 40° above the horizontal, pulls a trolley 8 m along a horizontal track. A constant horizontal resistance of 20 N opposes the motion. Find the total work done on the trolley by the two forces. [4]
- 14 A particle moves in a straight line from the point A with position vector $(\mathbf{i} + 2\mathbf{j})$ m to the point B with position vector $(5\mathbf{i} + 5\mathbf{j})$ m under the action of a constant force $(4\mathbf{i} + 3\mathbf{j})$ N. Find the work done by the force. [4]
- 15 A block of mass 8 kg is pulled at constant speed 6 m up a rough slope inclined at 25° to the horizontal by a rope parallel to the slope. The coefficient of friction is 0.2. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done by the tension in the rope. [6]

- 16** A box of mass 12 kg is dragged 5 m along rough horizontal ground at constant speed by a rope inclined at 20° above the horizontal with tension 30 N. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the work done by the tension. [2]
 (b) Find the coefficient of friction between the box and the ground. [4]
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- 17** A pendulum bob of mass 0.5 kg is attached to a light inextensible string of length 1.2 m. The bob is released from rest with the string horizontal and swings to its lowest point. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the work done by gravity as the bob moves to its lowest point. [2]
 (b) State the work done by the tension in the string, giving a reason. [2]
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- 18** A cyclist and bicycle have a total mass of 80 kg. The cyclist rides at constant speed along a straight horizontal road for 2 km against a constant resistance of 15 N, and then up a hill, rising through a vertical height of 40 m, with the resistance negligible on the hill. Take $g = 9.8 \text{ m s}^{-2}$. Find the total work done by the cyclist. [5]
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- 19** A particle moves in a straight line from the point A with position vector $(\mathbf{i} + 2\mathbf{j} + a\mathbf{k})$ m to the point B with position vector $(4\mathbf{i} + a\mathbf{j} + 5\mathbf{k})$ m, where a is a constant, under the action of a constant force $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ N. The work done by the force is 7 J. Find the value of a . [4]
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- 20** A particle of mass 0.5 kg is at rest at the point A with position vector $(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ m. A single constant force $(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ N then moves the particle in a straight line to the point B with position vector $(3\mathbf{i} + 2\mathbf{k})$ m. Find the speed of the particle at B. [5]
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- 21** A student carries a suitcase of mass 20 kg horizontally at constant speed for 30 m and then up a flight of stairs, rising through 3 m. Take $g = 9.8 \text{ m s}^{-2}$. The student calculates the work done against gravity as " $20 \times 9.8 \times 30 = 5880$ J for the horizontal part and $20 \times 9.8 \times 3 = 588$ J for the stairs, a total of 6468 J".
- (a) Explain the mistake in the student's calculation. [2]
 (b) Find the correct work done against gravity. [2]
 (c) Explain why the student may still be considered to be doing work during the horizontal part. [2]
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- 22** A particle moves from O to the point with position vector $(4\mathbf{i} + 3\mathbf{j})$ m under the action of a constant force $(2\mathbf{i} + p\mathbf{j})$ N, where p is a constant. The work done by the force is 14 J. Find the value of p , and the angle between the force and the displacement. [6]
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- 23** A car of mass 1200 kg drives 40 m up a straight ramp inclined at 8° to the horizontal at constant speed. The total resistance to motion is a constant 300 N. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the driving force of the engine. [3]
 (b) Find the work done by the driving force, and the work done against resistance. [3]
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- 24** Two tugs pull a barge along a canal. Each tug exerts a force of 400 N, one on each side of the canal, at an angle of 25° to the centre line of the canal. The barge moves 100 m along the centre line.
- (a) Find the resultant force of the tugs along the centre line. [2]
 (b) Find the total work done by the tugs, and the work done by each tug. [3]
 (c) Explain why the work done is the same whether it is found from the resultant or from the two tugs separately. [1]
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- 25** A constant force $\mathbf{F} = 6\mathbf{i}$ N acts on a particle which moves from O(0, 0) to A(3, 4). The particle can travel along the straight line OA, or along the path O to B(3, 0) and then B to A. Show that the work done by $\mathbf{F}$ is the same along both paths, and state the value. [6]

- 26** A box of mass 20 kg is pulled at constant speed along rough horizontal ground by a rope inclined at an angle θ above the horizontal. The coefficient of friction is 0.5. Take $g = 9.8 \text{ m s}^{-2}$. The tension in the rope is P N.
- (a) Show that $P = \frac{98}{\cos \theta + 0.5 \sin \theta}$. [4]
- (b) Find the least possible value of P and the value of θ for which it occurs. [3]
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- 27** Particles A and B, of masses 3 kg and 5 kg, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with both particles at the same height and the string taut. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the acceleration of the particles and the tension in the string. [4]
- (b) Find the work done by the tension on A and on B while B descends 2 m. [3]
- (c) Show that the total work done by the tension is zero, and explain why this is expected for a light inextensible string over a smooth pulley. [2]
- (d) Find the total work done by gravity on the two particles during this motion. [2]
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- 28** A particle of mass 4 kg is pulled at constant speed 5 m up a rough plane inclined at 30° to the horizontal by a horizontal force of magnitude P N, acting in the vertical plane containing a line of greatest slope. The coefficient of friction is 0.3. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the value of P. [6]
- (b) Find the work done by the force P. [2]
- (c) Find the work done against friction and the work done against gravity, and verify that the work done by P equals their sum. [3]
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- 29** A particle moves around a horizontal circle of radius 2 m, centre O, on a smooth horizontal table, under the action of the forces described in each part.
- (a) A force of constant magnitude 5 N acts at all times in the direction of motion (tangential). Find the work done in one complete revolution. [2]
- (b) A constant force of $3\mathbf{i}$ N acts on the particle. Show that the work done in one complete revolution is zero. [2]
- (c) For the force in (b), find the work done as the particle moves from (2, 0) to (−2, 0) along the semicircle through (0, 2). [2]
- (d) The particle is held on its path by a string from O. State the work done by the tension in the string in each case above and explain. [2]
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- 30** A particle moves from A(0, 0) to B(4, 3) (distances in metres). Take $\mathbf{i}$ and $\mathbf{j}$ as unit vectors along the axes. A force $(3\mathbf{i} - 4\mathbf{j})$ N acts throughout, and a frictional force of constant magnitude 2 N always opposes the motion.
- (a) The particle travels along the straight line AB. Find the work done by each of the two forces and the total work. [4]
- (b) The particle instead travels from A to C(4, 0) and then from C to B. Find the work done by each force and the total. [4]
- (c) Explain what your answers show about the two forces. [1]

Kinetic Energy

- 1** Find the kinetic energy of a particle of mass 4 kg moving at 5 m s^{-1} . [1]
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- 2** A car of mass 1000 kg travels at 20 m s^{-1} . Find its kinetic energy in kilojoules. [2]
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- 3** A ball of mass 0.2 kg has kinetic energy 9 J. Find its speed. [2]
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- 4** A particle of mass 3 kg has its speed increased from 2 m s^{-1} to 6 m s^{-1} . Find the increase in its kinetic energy. [2]

- 5 The work done on a stationary particle of mass 5 kg is 40 J, and no other forces act. Find the speed it acquires. [2]
- 6 A particle of mass 2 kg moves with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Find its kinetic energy. [2]
- 7 A particle's speed is doubled. By what factor does its kinetic energy change? Justify your answer. [2]
- 8 A car of mass 900 kg accelerates from 10 m s^{-1} to 25 m s^{-1} . Find the work done by the engine's net force. [3]
- 9 A car of mass 1200 kg travelling at 15 m s^{-1} is brought to rest by a constant braking force over 90 m. Find the magnitude of the braking force. [3]
- 10 A bullet of mass 0.02 kg moving at 250 m s^{-1} embeds in a wooden block and is brought to rest after penetrating 0.1 m. Find the average resistive force. [3]
- 11 Particle A has mass 2 kg and speed 6 m s^{-1} . Particle B has mass 8 kg and speed 3 m s^{-1} . Find the ratio of the kinetic energy of A to that of B, and state which has greater momentum. [4]
- 12 A particle of mass 3 kg is at rest on a smooth horizontal surface. A horizontal force of 6 N acts on it over a distance of 4 m. Find its final speed using the work-energy principle. [3]
- 13 A lorry of mass 5000 kg has kinetic energy 250 kJ. Find its speed in kilometres per hour. [3]
- 14 A box of mass 2 kg slides on a rough horizontal surface. A horizontal force of 12 N acts on it over a distance of 5 m, while a constant friction force of 4 N opposes the motion. The initial speed is 8 m s^{-1} . Find the final speed. [4]
- 15 A cyclist and bicycle of total mass 80 kg travelling at 10 m s^{-1} stops by braking with a constant resistive force of 200 N.
 - (a) Find the distance the cyclist travels while stopping. [3]
 - (b) Find the stopping distance if the initial speed is 15 m s^{-1} . [2]
- 16 A block of mass 10 kg is pulled from rest up a smooth slope inclined at 15° to the horizontal by a constant force of 30 N parallel to the slope, over 8 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the block after 8 m. [4]
- 17 A particle of mass 0.5 kg is projected vertically upwards from ground level with speed 24 m s^{-1} . The particle is subject to a constant air resistance of magnitude 1.5 N. Take $g = 9.8 \text{ m s}^{-2}$.
 - (a) Use the work-energy principle to find the greatest height reached by the particle. [3]
 - (b) Find the speed with which the particle returns to the ground. [3]
- 18 Two particles have equal kinetic energies. The mass of the first is four times the mass of the second. Find the ratio of their speeds and the ratio of their momenta (first to second). [4]
- 19 A particle of mass 4 kg is moving at 3 m s^{-1} along a straight line when a force acts on it.
 - (a) The force does 24 J of positive work on the particle. Find its final speed. [2]
 - (b) Instead the force does -10 J of work on the particle (that is, 10 J of negative work). Find its final speed. [2]
- 20 A particle of mass m moving with speed u is acted upon by a force that does work W . Show that the final speed v is given by $v^2 = u^2 + \frac{2W}{m}$. Hence find v when $m = 0.5 \text{ kg}$, $u = 4 \text{ m s}^{-1}$ and $W = 9 \text{ J}$. [4]

- 21** A car of mass 1500 kg travels at 20 m s^{-1} on a horizontal road. The driver brakes, producing a constant braking force which brings the car to rest in 40 m. Another car of mass 1000 kg travelling at 25 m s^{-1} brakes with the same braking force.
- (a) Find the braking force on the first car. [3]
- (b) Find the distance in which the second car stops. [3]
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- 22** A student claims: "A particle of mass 2 kg changes velocity from 3 m s^{-1} to -3 m s^{-1} , so its kinetic energy has changed by $\frac{1}{2}(2)(-3 - 3)^2 = 36 \text{ J}$."
- (a) Explain why this is wrong. [2]
- (b) Find the correct change in kinetic energy and the magnitude of the impulse on the particle. [3]
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- 23** A block of mass 4 kg moving at 7 m s^{-1} on a rough horizontal surface is pulled by a force of 20 N at 30° above the horizontal. The coefficient of friction is 0.2. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the block after it has moved 6 m. [7]
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- 24** A particle of mass 2 kg is projected at 10 m s^{-1} up a rough slope inclined at 20° to the horizontal. The coefficient of friction is 0.3. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the distance up the slope that the particle travels before coming to rest. [6]
- (b) Show that the particle then slides back down, and find its speed when it returns to its starting point. [3]
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- 25** A particle of mass m moving with speed u collides with and sticks to a stationary particle of mass $3m$. Show that the fraction of the initial kinetic energy that is lost is $\frac{3}{4}$. [5]
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- 26** The kinetic energy of a particle of mass 3 kg at time t seconds is $E = 6t^2 + 24$ joules for $t \geq 0$, where the particle moves in a straight line.
- (a) Find the speed of the particle at time t . [2]
- (b) Find the rate of change of kinetic energy when $t = 3$. [2]
- (c) Find the speed when $t = 0$ and the time when the speed is 10 m s^{-1} . [3]
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- 27** A car of mass m moves along a straight horizontal road against a constant resistance R . The driving force is constant at D , where $D > R$, and the car starts from rest.
- (a) Show that the speed v after distance s satisfies $v^2 = \frac{2(D - R)s}{m}$. [3]
- (b) The driving force is removed when $v = V$. Show that the car then travels a further distance $\frac{mV^2}{2R}$. [3]
- (c) Given that the total distance travelled is 600 m and $D = 3R$, find the distance over which the driving force acted. [4]
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- 28** A particle of mass 0.5 kg is projected up a rough slope inclined at 30° to the horizontal with speed 6 m s^{-1} . The coefficient of friction is $\frac{\sqrt{3}}{3}$. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the friction force is $0.5 \times 9.8 \times \frac{\sqrt{3}}{3} \times \cos 30^\circ = 2.45 \text{ N}$. [2]
- (b) Find the distance up the slope travelled before the particle first comes to rest. [4]
- (c) Describe what happens to the particle after it first comes to rest, giving a reason. [2]
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- 29** A particle P of mass 2 kg moves in a straight line. The only force acting on it is F newtons directed along the line, where $F = 6 - 2x$ and x metres is its displacement from O. At $x = 0$ the speed of P is 4 m s^{-1} .
- (a) Show that $v^2 = 16 + 6x - x^2$. [4]
- (b) Find the greatest speed of P and the value of x when this occurs. [3]
- (c) Find the greatest value of x reached. [2]

- 30** A particle of mass 2 kg moves in a straight line. When the particle is at displacement x metres from a fixed point O on the line, where $x \geq 0$, its kinetic energy is E joules, where $E = 4x^2 + 8x + 9$.
- Find the speed of the particle when $x = 0$. [2]
 - Use the work-energy principle to show that the resultant force on the particle at displacement x is $(8x + 8)$ N, in the direction of increasing x . [3]
 - Find the value of x at which the speed of the particle is 5 m s^{-1} . [3]
 - Find the work done by the resultant force as the particle moves from $x = 1$ to $x = 3$. [2]

Gravitational Potential Energy

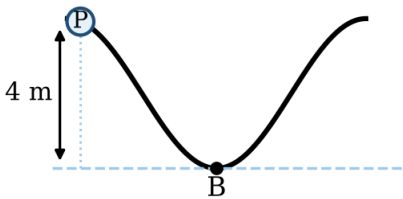
- Find the gravitational potential energy gained by a 6 kg mass raised through 4 m. Take $g = 9.8 \text{ m s}^{-2}$. [1]
- A book of mass 1.5 kg is on a shelf 2 m above the floor. Find its GPE relative to the floor. Take $g = 9.8 \text{ m s}^{-2}$. [1]
- A particle of mass 3 kg falls through 5 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the loss in its GPE. [2]
- The GPE of a 2 kg object increases by 58.8 J when it is lifted. Take $g = 9.8 \text{ m s}^{-2}$. Find the height through which it is lifted. [2]
- A skier of mass 70 kg descends a vertical height of 100 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the loss in GPE in kilojoules. [2]
- A particle of mass 4 kg moves 10 m up a smooth slope inclined at 30° to the horizontal. Take $g = 9.8 \text{ m s}^{-2}$. Find the gain in GPE. [2]
- A ball of mass 0.4 kg is thrown vertically upwards with speed 14 m s^{-1} . Ignoring air resistance and taking $g = 9.8 \text{ m s}^{-2}$, use conservation of energy to find the greatest height reached above the point of projection. [3]
- A lift of mass 800 kg rises 15 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done against gravity, in kilojoules. [2]
- A ball of mass 0.5 kg is thrown vertically upwards from ground level and rises 8 m. Take $g = 9.8 \text{ m s}^{-2}$.
 - Find the gain in GPE. [2]
 - Hence find the speed of projection, ignoring air resistance. [2]
- A particle of mass 2 kg is at a height of 3 m above a table and the table is 0.8 m above the floor. Take $g = 9.8 \text{ m s}^{-2}$.
 - Find the GPE of the particle relative to the table. [1]
 - Find the GPE of the particle relative to the floor. [2]
- A uniform chain of mass 6 kg and length 3 m lies on the ground. One end is lifted vertically until the chain hangs just clear of the ground. Take $g = 9.8 \text{ m s}^{-2}$. Find the GPE gained. [3]
- A pendulum bob of mass 0.8 kg is on a string of length 1.5 m. It is raised from its lowest point until the string makes an angle of 60° with the vertical. Take $g = 9.8 \text{ m s}^{-2}$. Find its gain in GPE. [4]
- A cyclist of total mass 85 kg climbs a hill of constant gradient $\sin \theta = \frac{1}{20}$ for 400 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the gain in GPE. [3]

- 14** A uniform rod of mass 4 kg and length 2 m lies on horizontal ground. It is raised to a vertical position, standing on one end. Take $g = 9.8 \text{ m s}^{-2}$. Find the gain in GPE. [3]
- 15** Particles of masses 2 kg and 5 kg are at heights 3 m and 1.2 m above the ground. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the total GPE relative to the ground. [2]
 (b) Find the height of the centre of mass of the two particles above the ground and verify that the total GPE equals Mgh . [3]
- 16** A particle of mass 2 kg is projected up a smooth slope of angle 30° with speed 7 m s^{-1} from the bottom. Take $g = 9.8 \text{ m s}^{-2}$. Use energy to find how far up the slope it travels. [4]
- 17** A uniform rectangular block of mass 12 kg has a square cross-section of side 0.4 m and length 1.2 m. It lies on horizontal ground on its longest face and is turned about an edge until it stands on one end. Take $g = 9.8 \text{ m s}^{-2}$. Find the gain in GPE. [4]
- 18** A rope of mass 10 kg and length 5 m hangs over the edge of a table with 2 m hanging vertically. Take $g = 9.8 \text{ m s}^{-2}$. Find the work done in pulling the rope back onto the table, assuming the table is smooth and ignoring friction. [4]
- 19** A skier of mass 65 kg skis 300 m down a slope of constant angle 12° . Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the loss in GPE. [2]
 (b) The skier starts from rest and the resistance has constant magnitude 90 N. Find the work done against resistance. [2]
- 20** Show that a particle of mass m moved a distance s up a smooth plane inclined at angle α to the horizontal gains GPE of $mgs \sin \alpha$, and use this to find the angle α for which a gain of 98 J is made by a 4 kg mass moved 5 m up the plane. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 21** A uniform chain of length 4 m and mass 8 kg is coiled in a heap on the ground. One end is raised vertically at constant speed until the chain is just clear of the ground. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the increase in the GPE of the chain. [2]
 (b) The chain is lifted a further 1 m. Find the further gain in GPE. [2]
- 22** Particle A of mass 3 kg and particle B of mass 5 kg are connected by a light inextensible string passing over a smooth fixed pulley. They are released from rest with B descending. Take $g = 9.8 \text{ m s}^{-2}$. Find the loss in the total GPE of the system when B has descended 2 m, and the gain in KE of the system. [5]
- 23** A student calculates the GPE gained by a 5 kg box pushed 4 m up a rough slope at 30° to the horizontal as " $5 \times 9.8 \times 4 \times \cos 30^\circ = 169.7 \text{ J}$ ". Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Identify the error. [2]
 (b) Find the correct GPE gain. [2]
 (c) Explain whether the roughness of the slope affects the GPE gain. [1]
- 24** A uniform cube of side 0.6 m and mass 30 kg rests on horizontal ground. It is tipped about one edge until its diagonal through that edge is vertical. Take $g = 9.8 \text{ m s}^{-2}$. Find the gain in GPE. [5]
- 25** A particle moves along a smooth curved track from A to B. A is 5 m and B is 2 m above a horizontal datum. The particle has mass 3 kg. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the change in GPE from A to B. [2]
 (b) State, with a reason, whether the change in GPE depends on the shape of the track. [1]
 (c) The particle starts from rest at A. Find its speed at B. [2]

- 26** A rod AB of mass 6 kg and length 1.5 m is smoothly hinged at A and can swing in a vertical plane. It is raised from the hanging position until it makes 50° with the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$. The rod is uniform. Find the gain in GPE. [5]
- 27** A uniform chain of mass M and length L is held vertically with its lower end touching a horizontal table. It is released and falls onto the table in a heap. [2]
 (a) Find the initial GPE of the chain relative to the table. [2]
 (b) Find the final GPE and the loss in GPE, assuming the heap has negligible height. [2]
 (c) The chain is 2 m long with mass 6 kg. Find the energy dissipated as heat and sound, given that no kinetic energy remains. Take $g = 9.8 \text{ m s}^{-2}$. [2]
 (d) Explain why this energy cannot be recovered as GPE. [1]
 (e) For the chain in part (c), find the speed with which the top end hits the table, assuming the chain falls freely as a rigid body through the distance its top end travels. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 28** A particle P of mass m is attached to one end of a light inextensible string of length L . The other end is fixed at O. P is held with the string taut making angle α to the downward vertical and released from rest. [2]
 (a) Show that the loss in GPE when the string is vertical is $mgL(1 - \cos \alpha)$. [2]
 (b) Hence show that the speed at the lowest point is $\sqrt{2gL(1 - \cos \alpha)}$. [2]
 (c) Find α for which the lowest-point speed equals $\sqrt{gL}$. [2]
 (d) Explain why the tension does no work. [1]
 (e) Taking $\alpha = 60^\circ$, find the tension in the string at the lowest point in terms of m and g . [3]
- 29** A uniform rope of mass 12 kg and length 6 m hangs over a smooth horizontal peg so that 4 m hangs on one side and 2 m on the other. Take $g = 9.8 \text{ m s}^{-2}$. [4]
 (a) Find the GPE of the rope relative to the level of the peg. [4]
 (b) The rope is released from rest and the longer side moves downwards. Find the GPE of the rope when 5 m hangs on one side of the peg and 1 m on the other. [3]
 (c) Find the speed of the rope at this instant. [3]
- 30** A uniform chain of mass 4 kg and length 2 m lies in a straight line on a smooth horizontal table with a short length of 0.5 m hanging over the edge, held at rest. The chain is released. Take $g = 9.8 \text{ m s}^{-2}$. [3]
 (a) Show that when a length x metres hangs over the edge, the GPE of the chain relative to the table top is $-9.8x^2$. [3]
 (b) Find the loss in GPE as the chain slides until the last link leaves the table. [3]
 (c) Hence find the speed of the chain as the last link leaves the table. [3]
 (d) Explain why the resultant force on the chain is not constant. [1]

Conservation of Mechanical Energy

- 1** A ball is dropped from rest from a height of 5 m. Take $g = 9.8 \text{ m s}^{-2}$. Use conservation of energy to find its speed on reaching the ground. [2]
- 2** A stone is thrown vertically upwards with speed 8 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Use energy to find the greatest height reached. [2]
- 3** A particle slides from rest down a smooth slope from a height of 2.5 m. Take $g = 9.8 \text{ m s}^{-2}$. Find its speed at the bottom. [2]
- 4** A pendulum bob is released from rest at a height of 0.45 m above its lowest point. Take $g = 9.8 \text{ m s}^{-2}$. Find its speed at the lowest point. [2]

- 5** A ball of mass 0.3 kg has speed 6 m s^{-1} at a point 2 m above the ground. Take $g = 9.8 \text{ m s}^{-2}$. Find its total mechanical energy relative to the ground. [2]
- 6** State the condition under which the total mechanical energy of a particle is conserved. [1]
- 7** A ball is thrown from a point 8 m above level ground with speed 10 m s^{-1} in any direction. Take $g = 9.8 \text{ m s}^{-2}$. Find its speed on reaching the ground. [3]
- 8** A bob on a light string of length 3 m is released from rest with the string at 60° to the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$. Find its speed at the lowest point. [4]
- 9** A particle moves down a smooth slope inclined at 30° to the horizontal. At one point its speed is 6 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Find its speed after a further 4 m down the slope. [4]
- 10** A ball is projected vertically upwards with speed 12 m s^{-1} from ground level. Take $g = 9.8 \text{ m s}^{-2}$. Find the height at which its speed is 6 m s^{-1} . [3]
- 11** A roller coaster car of mass 400 kg passes the top of a hill, 30 m above the lowest point, at speed 4 m s^{-1} . Ignore resistance. Take $g = 9.8 \text{ m s}^{-2}$. Find its speed at the lowest point. [4]
- 12** A skier of mass 70 kg starts from rest and descends a smooth slope losing 25 m in height. She then moves onto a horizontal rough patch with friction 140 N until she stops. Take $g = 9.8 \text{ m s}^{-2}$. Find the length of the rough patch. [4]
- 13** A small bead P of mass 0.05 kg is threaded on a smooth curved wire. It is released from rest at a point 4 m above the point B, as shown. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the bead at B. [3]
- 
- 14** A particle of mass 2 kg is released from rest at the top of a rough slope of length 10 m inclined at 30° to the horizontal. The coefficient of friction is 0.2 . Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of the particle at the bottom. [6]
- 15** Particles A (3 kg) and B (5 kg) are connected by a light inextensible string over a smooth fixed pulley. The system is released from rest with B 1.6 m above the floor. Take $g = 9.8 \text{ m s}^{-2}$. Use energy to find the speed of B as it reaches the floor. [5]
- 16** A smooth track in a vertical plane consists of a quarter circle of radius 0.8 m from the top T (at the level of the centre) down to the lowest point B, followed by a rough horizontal section. A particle of mass 0.5 kg is released from rest at T. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- (a) Find the speed at B. [3]
- (b) The particle comes to rest after sliding 4 m along the horizontal section. Find the friction force. [3]
- 17** A ball of mass 0.2 kg is projected from ground level at 15 m s^{-1} at 40° above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$. Use energy conservation to find its speed when it is 3 m above the ground. [4]

- 18** A particle of mass 4 kg is projected up a smooth slope of angle 20° with speed 9 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find how far up the slope it travels. [3]
- (b) Find its speed when it returns to the starting point and explain your answer. [2]
-
- 19** A pendulum of length 1.5 m with a bob of mass 0.4 kg is released from rest with the string horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the speed of the bob when the string makes 30° with the downward vertical. [3]
- (b) Find the kinetic energy at the lowest point. [2]
-
- 20** A car of mass 800 kg freewheels (engine off) down a straight road of length 200 m which falls through a vertical height of 12 m. Its speed at the top is 5 m s^{-1} and at the bottom is 12 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Find the constant resistance to motion. [5]
-
- 21** A particle P of mass m is released from rest at the top A of a smooth hemisphere of radius r which is fixed with its flat face on horizontal ground. P slides down the surface. Let θ be the angle between the radius to P and the upward vertical.
- (a) Show that the speed of P when OP makes angle θ with the vertical is $\sqrt{2gr(1 - \cos \theta)}$. [3]
- (b) P leaves the surface when the normal reaction is zero. Show that $\cos \theta = \frac{2}{3}$ at that instant. [4]
- (c) Find the speed at which P leaves the surface, for $r = 0.6 \text{ m}$. Take $g = 9.8 \text{ m s}^{-2}$. [2]
-
- 22** A small ball of mass m is thrown vertically upwards with speed $\sqrt{8gH}$ from a point O on a smooth horizontal floor. The ball moves towards a smooth horizontal ceiling which is at a vertical distance H above O. The ball is subject to air resistance of constant magnitude $\frac{mg}{2}$ at all times while it is moving. The coefficient of restitution between the ball and the ceiling is $\frac{1}{2}$.
- (a) Use the work-energy principle to show that the speed of the ball immediately before it hits the ceiling is $\sqrt{5gH}$. [4]
- (b) Find, in terms of g and H , the speed of the ball immediately before it hits the floor. [4]
- (c) Find the total work done against air resistance from the instant of projection until the ball hits the floor. [1]
-
- 23** A student says: "A ball is thrown at 10 m s^{-1} from a cliff 20 m high. Thrown upwards it hits the sea faster than if thrown downwards, since it travels further." Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Use energy conservation to show that the student is wrong. [3]
- (b) State one aspect of the motion that does depend on the direction of projection. [1]
-
- 24** A smooth bead of mass 0.1 kg is threaded onto a circular wire of radius 0.5 m in a vertical plane, centre O. The bead is projected from the lowest point with speed $u \text{ m s}^{-1}$. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the bead reaches the top of the wire if $u^2 > 19.6$. [3]
- (b) When $u = 6$, find the speed at the top. [2]
-
- 25** A particle of mass m is projected from ground level with speed u at angle α above the horizontal. At its greatest height H its speed is $u \cos \alpha$.
- (a) Use energy conservation to show that $H = \frac{u^2 \sin^2 \alpha}{2g}$. [4]
- (b) Given $u = 20$ and $\alpha = 30^\circ$, find the speed when the height is half the maximum. Take $g = 9.8 \text{ m s}^{-2}$. [3]
-

- 26** Particle A of mass 2 kg lies on a smooth slope inclined at 30° to the horizontal and is connected by a light inextensible string over a smooth pulley at the top of the slope to particle B of mass 3 kg hanging vertically. The system is released from rest with the string taut. Take $g = 9.8 \text{ m s}^{-2}$. When B has descended 1 m it hits the ground and stops, and the string goes slack.
- Find the speed of B just before it hits the ground. [5]
 - Find the further distance that A moves up the slope. [2]
-
- 27** A smooth hemispherical bowl of radius 1.2 m is fixed with its rim horizontal. A particle is released from rest on the inner surface at the rim. Take $g = 9.8 \text{ m s}^{-2}$. Let θ be the angle that the radius to the particle makes with the downward vertical.
- Show that the speed of the particle is $\sqrt{2gr \cos \theta}$. [2]
 - Find the speed at the lowest point. [2]
 - Show that the normal reaction at the lowest point is $3mg$. [3]
 - Find the angle θ at which the normal reaction equals $2mg$. [3]
-
- 28** A particle of mass 2 kg is attached to one end of a light inextensible string of length 1 m. The other end is fixed at O. The particle is held with the string taut and horizontal, and then projected vertically downwards with speed 3 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- Find the speed of the particle when the string is vertical. [3]
 - Find the tension in the string at this instant. [3]
 - The string breaks at this instant. The particle is then 1.5 m above the ground. Find the speed with which it hits the ground. [3]
-
- 29** Particle A of mass 2 kg lies on a rough horizontal table. It is attached by a light inextensible string passing over a smooth pulley at the table edge to particle B of mass 3 kg hanging freely. The coefficient of friction between A and the table is 0.4. The system is released from rest with B 1.5 m above the floor. Take $g = 9.8 \text{ m s}^{-2}$.
- Use energy to find the speed of B just before it hits the floor. [5]
 - After B hits the floor it does not rebound. Find how much further A slides along the table before stopping, assuming the table is long enough. [4]
-
- 30** A particle slides from rest down a smooth curved ramp, starting 5 m above horizontal ground. It leaves the end of the ramp horizontally at a point 1.8 m above the ground and then moves freely under gravity. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the speed of the particle as it leaves the ramp. [3]
 - Find the time of flight and the horizontal distance from the end of the ramp to the landing point. [4]
 - Use energy conservation to find the speed of the particle as it lands, and verify your answer by finding the horizontal and vertical components. [4]
 - State one modelling assumption you have made. [1]

Power

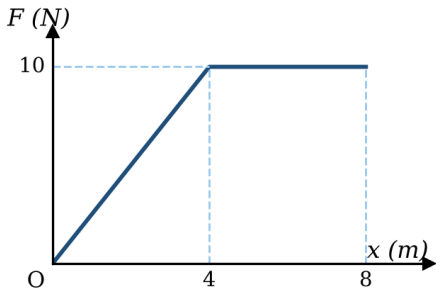
- A force of 500 N moves an object at a constant speed of 4 m s^{-1} in the direction of the force. Find the power. [1]
- An engine does 12 000 J of work in 8 s. Find its average power. [1]
- A car's engine produces a driving force of 1800 N at a speed of 25 m s^{-1} . Find the power in kilowatts. [2]
- A motor works at a rate of 3 kW. Find the work it does in 2 minutes. [2]

- 5** A crane lifts a load of 200 kg at constant speed 0.5 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Find the power of the crane. [2]
- 6** A car's engine has power 40 kW and the car is travelling at 20 m s^{-1} . Find the driving force. [2]
- 7** A car of mass 1000 kg travels on a level road at its maximum speed of 40 m s^{-1} . The resistance is a constant 800 N. Find the power of the engine. [3]
- 8** A car of mass 1200 kg is driven by an engine working at 30 kW along a level road against a constant resistance of 900 N. Find its acceleration when its speed is 15 m s^{-1} . [4]
- 9** A car of mass 800 kg travels at constant speed 12 m s^{-1} up a slope inclined at 5° to the horizontal. The resistance is 300 N. Take $g = 9.8 \text{ m s}^{-2}$. Find the power of the engine. [4]
- 10** A lift of mass 600 kg, including passengers, is raised at constant speed 2 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Find the power needed in kilowatts, ignoring resistance. [3]
- 11** A water pump raises 50 kg of water per second through a vertical height of 6 m and gives it no kinetic energy. Take $g = 9.8 \text{ m s}^{-2}$. Find the power of the pump. [3]
- 12** A cyclist produces a constant power of 250 W and moves at a steady 6 m s^{-1} on level ground. Find the resistance, and the speed if the resistance doubles and power is unchanged. [4]
- 13** A train of mass 200 000 kg has an engine of power 600 kW. The resistance is 15 000 N. Find the maximum speed on a level track. [3]
- 14** A car of mass 1500 kg has an engine of maximum power 45 kW. On a level road the resistance is 1200 N. On a hill inclined at 4° to the horizontal the resistance is the same. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the maximum steady speed on the level road. [2]
 (b) Find the maximum steady speed up the hill. [4]
- 15** A car of mass 900 kg travels along a level road with its engine working at a constant 36 kW. The resistance to motion is 600 N.
 (a) Find the acceleration when the speed is 10 m s^{-1} . [3]
 (b) Find the maximum speed. [2]
- 16** A van of mass 1000 kg travels up a road inclined at 3° to the horizontal against a resistance of 400 N. The engine power is 24 kW. Take $g = 9.8 \text{ m s}^{-2}$. Find the acceleration when the speed is 8 m s^{-1} . [5]
- 17** A lift of mass 500 kg accelerates upwards from rest at 0.4 m s^{-2} . Take $g = 9.8 \text{ m s}^{-2}$. Ignore resistance other than the weight, and let the tension in the cable be T. Find T, and the power the motor supplies when the speed of the lift is 1.5 m s^{-1} . [4]
- 18** A boat's engine works at 15 kW and the boat moves at a steady 6 m s^{-1} . The resistance is proportional to the speed. Find the power needed to move at 9 m s^{-1} . [4]
- 19** A car of mass 1000 kg has an engine working at a constant power P. The resistance to motion is a constant force R newtons on all roads. The maximum steady speed of the car on a horizontal road is 30 m s^{-1} and the maximum steady speed of the car up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{10}$, is 20 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$. Find R and P. [5]

- 20** A van of mass 800 kg moves along a straight horizontal road. At the instant when the speed of the van is $v \text{ m s}^{-1}$, the resistance to its motion is modelled as a force of magnitude $(300 + 4v) \text{ N}$. The engine of the van is working at a constant rate of 18.4 kW.
- Find the greatest steady speed of the van on this road. [3]
 - Find the acceleration of the van at the instant when $v = 20$. [2]
-
- 21** A car of mass 1200 kg has a maximum power of 36 kW. The resistance is 500 N on all roads. Take $g = 9.8 \text{ m s}^{-2}$.
- Find its maximum speed on a level road. [2]
 - Show that the maximum speed up a road of gradient $\sin \theta = \frac{1}{20}$ is 33.1 m s^{-1} to 3 s.f. [3]
 - Find the acceleration at 20 m s^{-1} on the hill when working at full power. [3]
-
- 22** A car of mass 1000 kg has an engine of constant power 30 kW. It travels on a level road against a constant resistance of 600 N. It accelerates from 10 m s^{-1} to 40 m s^{-1} .
- Find the maximum speed of the car. [2]
 - Show that the work done by the engine while the car accelerates is given by $30\,000t$, where t is the time taken. [1]
 - Given that the car takes 65.5 s to accelerate, use the work-energy principle to find the distance travelled. [5]
-
- 23** A student says: "The power of a car is 20 kW and it travels at 10 m s^{-1} , so the driving force is 200 N."
- Identify and correct the mistake. [2]
 - The car's mass is 1500 kg and the resistance is 1500 N on a level road. Find its acceleration. [2]
 - Explain why the acceleration decreases as the speed increases at constant power. [2]
-
- 24** A truck of mass 3000 kg tows a trailer of mass 1000 kg along a level road. The resistances are 400 N on the truck and 200 N on the trailer. The truck's engine power is 30 kW and they travel at 10 m s^{-1} . A tow bar connects the truck and trailer.
- Find the acceleration of the system. [3]
 - Find the tension in the tow bar. [3]
-
- 25** A car of mass 1500 kg descends a hill of angle θ where $\sin \theta = \frac{1}{15}$ at constant speed 20 m s^{-1} with the engine off. Take $g = 9.8 \text{ m s}^{-2}$. The resistance has the form kv where k is constant. Find k and the power needed to climb the same hill at 20 m s^{-1} . [6]
-
- 26** A pump raises water through a vertical height of 8 m and delivers it at a speed of 6 m s^{-1} . The pump is 80% efficient (80% of the input power is used to raise the water and give it kinetic energy). Take $g = 9.8 \text{ m s}^{-2}$.
- Find the input power needed to pump 20 kg of water each second. [3]
 - Find the mass of water pumped per second when the input power is 3000 W. [3]
-
- 27** A car of mass m moves up a hill inclined at θ to the horizontal with the engine at constant power P against a constant resistance R . The car reaches a steady speed V .
- Show that $P = V(R + mg \sin \theta)$. [2]
 - Show that when the speed is v the acceleration is $\frac{P}{mv} - \frac{R}{m} - g \sin \theta$. [3]
 - Given $P = 40 \text{ kW}$, $m = 1200 \text{ kg}$, $R = 800 \text{ N}$, $\sin \theta = \frac{1}{20}$, find the steady speed V . Take $g = 9.8 \text{ m s}^{-2}$. [3]
 - Find the acceleration at 10 m s^{-1} . [2]

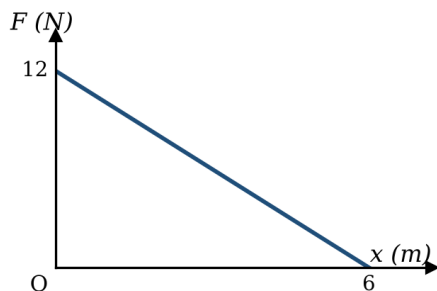
- 28** A car of mass 1000 kg has an engine working at constant power P . On a level road the car has maximum speed 50 m s^{-1} . The resistance is proportional to the speed: $R = kv$.
- Show that $P = 2500k$. [2]
 - Given that the car's acceleration is 0.5 m s^{-2} when $v = 20 \text{ m s}^{-1}$, find k and P . [5]
 - Show that the car cannot reach 55 m s^{-1} on this road. [2]
-
- 29** A car of mass 1000 kg moves along a straight horizontal road. When the speed of the car is $v \text{ m s}^{-1}$, the resistance to its motion is $c v^2$ newtons, where c is a constant. The maximum power that the engine of the car can develop is 54 kW. When the car is travelling at 54 km h^{-1} with the engine working at its maximum power, the acceleration of the car is 3.15 m s^{-2} . Take $g = 9.8 \text{ m s}^{-2}$.
- Show that $c = 2$. [3]
 - Find the maximum speed of the car on a horizontal road. [2]
 - Find the acceleration of the car at the instant when its speed is 25 m s^{-1} and the engine is working at maximum power. [3]
 - The car now travels up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{20}$, with the engine at maximum power and the same resistance. Show that the greatest steady speed v satisfies $2v^3 + 490v - 54\,000 = 0$, and verify that $v = 27.5$ correct to 3 significant figures. [3]
-
- 30** A car of mass 1200 kg travels along a straight horizontal road with the engine at constant power $P \text{ W}$. The resistance to motion is $25v \text{ N}$ when the speed is $v \text{ m s}^{-1}$. The car's maximum speed is 40 m s^{-1} .
- Find P . [2]
 - Find the acceleration when $v = 10 \text{ m s}^{-1}$. [3]
 - Show that the equation of motion can be written $1200v \frac{dv}{dx} = \frac{40\,000}{v} - 25v$. [2]
 - Find the distance travelled while accelerating from 10 m s^{-1} to 20 m s^{-1} , to the nearest metre. [4]

Variable Forces and Work

- 1** A force $F = 4x$ newtons acts on a particle moving along the x -axis. Find the work done as the particle moves from $x = 0$ to $x = 3$. [2]
-
- 2** The force acting on a particle is $F = (6 - x) \text{ N}$. Find the work done in moving the particle from $x = 0$ to $x = 4$. [2]
-
- 3** The graph shows the force F newtons acting on a particle against displacement x metres, for $0 \leq x \leq 8$. Find the work done by the force.
- 
- [2]
-
- 4** A force of $F = 3x^2 \text{ N}$ acts on a particle. Find the work done from $x = 1$ to $x = 2$. [2]
-
- 5** A spring exerts a force of $20x \text{ N}$ when stretched by $x \text{ m}$. Find the work done in stretching it from $x = 0$ to $x = 0.3 \text{ m}$. [2]

6 Write down the integral that gives the work done by a variable force $F(x)$ in moving a particle from $x = a$ to $x = b$, and state what the area under an F against x graph represents. [2]

7 A particle of mass 2 kg is acted on by a force which decreases uniformly from 12 N at $x = 0$ to 0 N at $x = 6$ m, as shown. The particle starts from rest. Find its speed when $x = 6$.



[4]

8 A particle moves along the x -axis under a force $F = (2x + 1)$ N. Find the work done from $x = 1$ to $x = 4$. [3]

9 A force $F = 6 \sin(\frac{\pi x}{4})$ N acts on a particle. Find the work done from $x = 0$ to $x = 4$. [3]

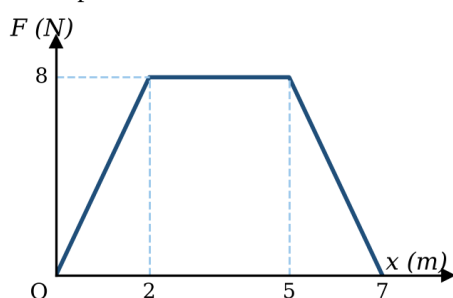
10 A particle of mass 3 kg, initially at rest at $x = 0$, moves under a force $F = (2x + 2)$ N. Find its speed when $x = 3$. [4]

11 A force $F = 8e^{-x}$ N acts on a particle. Find the work done from $x = 0$ to $x = \ln 2$. [3]

12 A spring has natural length 0.5 m and stiffness constant 40 N m^{-1} . Find the work done in stretching it from natural length to a length of 0.8 m. [3]

13 The force on a particle is $F = (x^2 - 4)$ N. Find the work done from $x = 0$ to $x = 2$, and explain the sign of your answer. [4]

14 A particle of mass 4 kg, at rest at $x = 0$ on a smooth horizontal surface, is acted on by a horizontal force F newtons, which varies with displacement x metres as shown. Find the speed of the particle at $x = 7$ and the work done between $x = 2$ and $x = 5$.



[6]

15 A particle of mass 2 kg moves along the x -axis. It passes $x = 1$ with speed 3 m s^{-1} under the action of a force $F = \frac{12}{x^2}$ N in the direction of increasing x . Find its speed at $x = 4$. [5]

16 A spring of natural length 0.6 m and stiffness 50 N m^{-1} lies along a smooth horizontal table, fixed at one end. A particle of mass 0.4 kg is attached to the free end and the spring is extended by 0.2 m and released from rest. Find the speed of the particle when the spring returns to its natural length. [4]

- 17** The force acting on a particle of mass 0.5 kg moving along the x-axis is $F = (3 - 2x)$ N. At $x = 0$ its speed is 4 m s^{-1} . Find its speed when $x = 2$. [5]
-
- 18** A particle P of mass 3 kg moves in a straight line against a resistive force of $6v$ N where $v \text{ m s}^{-1}$ is its speed. Its initial speed is 8 m s^{-1} . Find the work done against the resistance in the first 2 s if $v = 8e^{-2t}$. [5]
-
- 19** The work done by a force $F = kx^2$ N in moving a particle from $x = 0$ to $x = 3$ m is 36 J. Find k and the work done from $x = 3$ to $x = 5$. [5]
-
- 20** A particle of mass 3 kg moves in a straight line. At time t seconds its velocity is $(t^2 + 1) \text{ m s}^{-1}$. A force F newtons acts on the particle in the direction of motion. Use integration of $F v$ with respect to t to find the work done by the force between $t = 0$ and $t = 2$, and verify your answer using the work-energy principle. [5]
-
- 21** A particle of mass 2 kg moves along the x-axis under a force $F = (12 - 3x)$ N, starting from rest at $x = 0$.
 (a) Show that $v^2 = 12x - \frac{3x^2}{2}$. [3]
 (b) Find the maximum speed and the value of x at which it occurs. [3]
 (c) Find the value of x at which the particle first comes to rest again. [2]
-
- 22** The gravitational force on a particle of mass m at distance r from the centre of the Earth ($r \geq R$) is $F = \frac{mgR^2}{r^2}$ towards the centre, where $R = 6.4 \times 10^6 \text{ m}$ is the radius of the Earth. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Show that the work done by gravity as the particle falls from rest at $r = 2R$ to $r = R$ is $\frac{1}{2}mgR$. [3]
 (b) Hence find the speed with which the particle reaches the Earth's surface. [3]
-
- 23** A student calculates the work done by $F = 3x^2$ N moving a particle from $x = 0$ to $x = 2$ as " $F \times d = 3(2)^2 \times 2 = 24 \text{ J}$ ".
 (a) Explain why this is wrong. [2]
 (b) Find the correct work done. [2]
 (c) Is the student's value an over- or under-estimate? Explain. [2]
-
- 24** A particle of mass 0.5 kg is projected along a smooth horizontal surface. It moves with initial speed 6 m s^{-1} against a resistive force $F = 0.5x^2$ N, where x m is the distance moved. Find the distance moved before it comes to rest. [5]
-
- 25** A chain of mass 6 kg and length 3 m hangs vertically from one end, with its lower end just above the ground. It is lifted so that its lower end is raised 3 m, from the ground, by pulling the upper end up with constant speed. Use integration to find the work done. Take $g = 9.8 \text{ m s}^{-2}$. [5]
-
- 26** A particle of mass 1 kg moves along the x-axis under a force $F = 6x - 2x^2$ N, starting at rest at $x = 0$. Find the greatest speed attained and the position where it is attained. [6]
-

- 27** A particle P of mass m moves along the x -axis. Its resistance to motion has magnitude kv^2 , where v is its speed and k is a constant. At $x = 0$ the speed is u , and there are no other forces.
- Show that $mv \frac{dv}{dx} = -kv^2$. [2]
 - Hence show that $v = u e^{-kx/m}$. [3]
 - Show that the work done against the resistance in the first x metres is $\frac{1}{2}mu^2(1 - e^{-2kx/m})$. [3]
 - Find the distance at which the kinetic energy has fallen to a quarter of its initial value, in terms of m and k . [2]
-
- 28** A particle of mass 2 kg is acted on by a force $F = (12t - 3t^2) \text{ N}$ along a straight line for $0 \leq t \leq 4$. It starts from rest at the origin.
- Find the velocity at time t . [3]
 - Find the displacement at time t . [2]
 - Find the work done by the force in the first 4 s , by integrating F with respect to x , and confirm using the work-energy principle. [4]
 - Find the maximum power developed in this time. [3]
-
- 29** A particle of mass m is projected vertically upwards from the surface of the Earth with speed u . The gravitational force at distance r from the centre of the Earth is $\frac{mgR^2}{r^2}$, where R is the radius of the Earth.
- Show that the speed v at distance r satisfies $v^2 = u^2 - 2gR^2(\frac{1}{R} - \frac{1}{r})$. [4]
 - Find the least u (the escape speed) for which the particle never returns. [3]
 - Evaluate the escape speed for $R = 6.4 \times 10^6 \text{ m}$ and $g = 9.8 \text{ m s}^{-2}$. [2]
 - State why this model, with constant g , would give the wrong height for u close to this value. [1]
-
- 30** A lorry of mass 5000 kg is driven up a straight hill. The engine force F newtons depends on the distance x metres up the hill by $F = 9000 - 4x$ for $0 \leq x \leq 1500$, and the total resistance, including the component of weight, is a constant 6000 N . The lorry starts at rest at $x = 0$.
- Find the work done by the engine force from $x = 0$ to $x = 1500$. [3]
 - Show that the speed v at x satisfies $5000 \times \frac{v^2}{2} = 3000x - 2x^2$ for x within the range. [3]
 - Find the maximum speed and where it occurs. [3]
 - Find the distance at which the lorry stops. [1]