Forces in Light Rods and Joints

1 (3 marks)

A light rod with forces at its ends only: the two forces are equal and opposite along the rod — **B1**
The force on the rod at A is 45 N directed along the rod away from B — **B1**
The ends are being pulled outwards, so the rod is in tension — **B1**
Answer: 45 N away from B; tension — **3 marks**

2 (2 marks)

By symmetry of the two equal and opposite forces, the rod exerts 36 N on the pin at Y, directed towards X — **B1**
The rod pulls both pins towards each other, so the rod is in tension — **B1**
Answer: 36 N towards X; tension — **2 marks**

3 (2 marks)

Resolving vertically at B: the rod must pull B upwards with force equal to the weight — **M1**
Force = 60 N, and the rod is in tension — **A1**
Answer: 60 N, tension — **2 marks**

4 (3 marks)

The rod pushes B away from A, so the force acts up and to the right along AB — **B1**
Horizontal component = 80 cos 30° = 69.3 N, vertical component = 80 sin 30° = 40 N — **M1 A1**
Answer: Horizontal 69.3 N, vertical 40 N — **3 marks**

5 (3 marks)

Horizontally the only force at C is from BC, so the force in BC is zero — **B1**
Vertically the rod AC must support the weight: force in AC is 50 N — **M1**
AC pulls C upwards (A is above C), so AC is in tension — **A1**
Answer: AC: 50 N tension; BC: 0 N — **3 marks**

6 (3 marks)

BC is a strut: its vertical component must push C upwards to balance W — **B1**
AC is a tie: the horizontal component of the push of BC on C acts away from the wall, so AC must pull C towards the wall — **B1 E1**
Answer: AC is a tie (tension); BC is a strut (thrust) — **3 marks**

7 (5 marks)

By symmetry the forces in the two rods are equal; call each T — **B1**
Resolving vertically at C: $2T \sin 35° = 80$ — **M1 A1**
$T = 69.7$ N — **A1**
Both rods pull C upwards towards the supports, so both are in tension — **E1**
Answer: 69.7 N in each rod; tension — **5 marks**

8 (5 marks)

Resolving vertically at C: $P \sin 30° = 90$, where P is the force in BC — **M1 A1**
$P = 180$ N, a thrust (BC pushes C upwards and away from the wall) — **A1**
Resolving horizontally: $T = P \cos 30°$ — **M1**
$T = 156$ N, a tension in AC — **A1**
Answer: AC: 156 N tension; BC: 180 N thrust — **5 marks**

9 (5 marks)

$\tan \alpha = \frac{3}{4}$, so $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$ — **B1**
Resolving vertically: $T_{BC} \sin \alpha = 120$, so $T_{BC} = 200$ N — **M1 A1**
Resolving horizontally: $T_{AC} = 200 \cos \alpha = 160$ N — **M1**
Both rods pull on C (BC upwards to the right, AC to the left), so both are in tension — **A1**
Answer: BC: 200 N tension; AC: 160 N tension — **5 marks**

10 (4 marks)

Horizontally at C: $T \cos \alpha = 60$, where T is the tension in AC — **M1**

$\cos \alpha = \frac{12}{13}$, so $T = 65$ N — **A1**

Vertically: $W = T \sin \alpha = 65 \times \frac{5}{13}$ — **M1**

$W = 25$ N — **A1**

Answer: AC: 65 N (tension); W = 25 N — **4 marks**

11 (3 marks)

Resolving along AB: the force in DB is equal to the force in AD, so DB is in tension, 40 N — **M1 A1**

Resolving perpendicular to AB: the only force with a component in that direction is from DC, so the force in DC is zero — **B1**

Answer: DB: 40 N tension; DC: 0 N — **3 marks**

12 (5 marks)

Resolving horizontally: force in EF = $100 \cos 40° = 76.6$ N — **M1 A1**

Resolving vertically: force in EG = $100 \sin 40° = 64.3$ N — **M1 A1**

Each rod pulls E (towards F and towards G respectively), so both are in tension — **A1**

Answer: EF: 76.6 N tension; EG: 64.3 N tension — **5 marks**

13 (4 marks)

The rod exerts a force on B equal to 40 N directed away from C, since the value of T is negative — **M1 A1**

The rod pushes the joint, so it is in thrust — **B1**

A tension of -40 N means a thrust of 40 N because the assumed direction was reversed — **E1**

Answer: 40 N thrust — **4 marks**

14 (6 marks)

The rod must balance the resultant of the other two forces, which is $(30, -40)$ in components — **M1**

Magnitude = $\sqrt{30^2 + 40^2} = 50$ N — **M1 A1**

The rod must exert $(-30, +40)$ on P, directed up and to the left towards Q, so PQ is in tension and Q is above and to the left of P — **B1**

Angle with the horizontal = $\arctan \frac{40}{30} = 53.1°$ — **M1 A1**

Answer: 50 N tension; 53.1° to the horizontal — **6 marks**

15 (6 marks)

Horizontally: $T_A \cos 30° = T_B \cos 60°$ — **M1 A1**

Vertically: $T_A \sin 30° + T_B \sin 60° = 100$ — **M1 A1**

$T_B = T_A \sqrt{3}$, so $T_A(\frac{1}{2} + \frac{3}{2}) = 100$ — **M1**

$T_A = 50$ N and $T_B = 86.6$ N, both rods in tension — **A1**

Answer: AC: 50 N tension; BC: 86.6 N tension — **6 marks**

16 (6 marks)

Half of AB = 2 m, so $\cos \theta = \frac{2}{2.5} = 0.8$ and $\sin \theta = 0.6$, where θ is the angle each rod makes with the horizontal — **M1 A1**

By symmetry both forces are equal, T say: resolving vertically $2T \sin \theta = 120$ — **B1 M1**

$T = 100$ N — **A1**

The rods push C upwards, so both are in thrust — **A1**

Answer: 100 N thrust in each rod — **6 marks**

17 (6 marks)

Resolving vertically at B (upwards positive): $40 \sin 30° - T_{BD} = 0$ — M1 A1

$T_{BD} = 20$ N, and BD pulls B downwards, so it is in tension — A1

Resolving horizontally (towards C positive): $T_{BC} - 50 + 40 \cos 30° = 0$ — M1 A1

$T_{BC} = 50 - 34.6 = 15.4$ N, a tension — A1

Answer: BC: 15.4 N tension; BD: 20 N tension — 6 marks

18 (6 marks)

Weight of lamp = $6 \times 9.8 = 58.8$ N — B1

Angle of BC to horizontal: $\sin \theta = \frac{0.9}{1.5} = 0.6$, $\cos \theta = 0.8$ — M1

Vertically at C: $T_{BC} \sin \theta = 58.8$, so $T_{BC} = 98$ N (tension) — M1 A1

Horizontally: $P = T_{BC} \cos \theta = 78.4$ N, and AC pushes C away from the wall so it is in thrust — M1 A1

Answer: BC: 98 N tension; AC: 78.4 N thrust — 6 marks

19 (5 marks)

(a) Resolving vertically at C: $P \sin 60° = W$ — M1 A1

$P = \frac{W}{\sin 60°} = \frac{2W}{\sqrt{3}}$; BC pushes C up and away from the wall, so thrust — A1

Answer: The force in BC is a thrust of $\frac{2W}{\sqrt{3}}$ (shown) — 3 marks

(b) Resolving horizontally: $T = P \cos 60°$ — M1

$T = \frac{2W}{\sqrt{3}} \times \frac{1}{2} = \frac{W}{\sqrt{3}}$, a tension — A1

Answer: $\frac{W}{\sqrt{3}}$ tension — 2 marks

20 (5 marks)

Vertically at C: $3W \sin \theta = W$, so $\sin \theta = \frac{1}{3}$ — M1 A1

$\theta = 19.5°$ — A1

Horizontally: force in BC = $3W \cos \theta = 3W \times \frac{2\sqrt{2}}{3}$ — M1

$= 2\sqrt{2}W \approx 2.83W$, and BC is in thrust — A1

Answer: $\theta = 19.5°$; BC: $2\sqrt{2}W$ thrust — 5 marks

21 (8 marks)

(a) The student has used the 30° as the angle to the vertical; it is the angle to the horizontal, so the vertical component is P sin 30° (and the horizontal component P cos 30°) — E1 E1

Answer: Components of P interchanged — 2 marks

(b) Vertically: $P \sin 30° = 90$, so $P = 180$ N — M1 A1

Horizontally: $T = 180 \cos 30° = 156$ N — M1 A1

BC is a thrust and AC is a tension — A1 A1

Answer: BC: 180 N thrust; AC: 156 N tension — 6 marks

22 (8 marks)

(a) By symmetry the tensions are equal, T — B1

Resolving vertically at C: $2T \sin \theta = W$ — M1 A1

$T = \frac{W}{(2 \sin \theta)}$ — A1

Answer: $T = \frac{W}{(2 \sin \theta)}$ in each rod (shown) — 4 marks

(b) $\frac{W}{(2 \sin \theta)} = W$, so $\sin \theta = \frac{1}{2}$ — M1

$\theta = 30°$ — A1

Answer: 30° — 2 marks

(c) If $\theta = 0$ then $\sin \theta = 0$ and the vertical components $2T \sin \theta = 0$ for any T — E1

so the weight could never be balanced; the tension tends to infinity as θ tends to 0 — E1

Answer: No vertical component when horizontal — **2 marks**

23 (8 marks)

$\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$, $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$ — **B1**

Horizontally: $T_A \times \frac{4}{5} = T_B \times \frac{12}{13}$ — **M1 A1**

so $T_A = \frac{15}{13}T_B$ — **A1**

Vertically: $\frac{3}{5}T_A + \frac{5}{13}T_B = W$ — **M1 A1**

$\frac{9}{13}T_B + \frac{5}{13}T_B = W$, so $T_B = \frac{13W}{14}$ — **M1**

$T_A = \frac{15W}{14}$; both are in tension — **A1**

Answer: AC: $\frac{15W}{14}$ tension; BC: $\frac{13W}{14}$ tension — **8 marks**

24 (7 marks)

Vertically: $P\sin 30° = W$, so the thrust in BC is $P = 2W$ — **M1 A1**

Horizontally: $T = P\cos 30° = \sqrt{3}W$ for AC — **M1 A1**

BC: $2W \leq 800$ gives $W \leq 400$; AC: $\sqrt{3}W \leq 1200$ gives $W \leq 693$ — **M1 A1**

Greatest W is the smaller value, 400 N — **A1**

Answer: 400 N — **7 marks**

25 (8 marks)

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\sin\varphi = \frac{5}{13}$, $\cos\varphi = \frac{12}{13}$ — **B1**

Second force: components 24 N away from the wall and 10 N downwards — **M1 A1**

Vertically at C: $T\sin\theta = 80 + 10 = 90$, so $T = 150$ N, a tension in AC — **M1 A1**

Horizontally at C (away from the wall positive): $S - T\cos\theta + 24 = 0$ — **M1 A1**

$S = 120 - 24 = 96$ N, and BC pushes C away from the wall so it is in thrust — **A1**

Answer: AC: 150 N tension; BC: 96 N thrust — **8 marks**

26 (7 marks)

(a) Only two forces act on the rod, one at each end — **E1**

For zero moment about one end, the force at the other end must pass through the first end, so both lie along the rod — **E1**

Answer: Two-force member — **2 marks**

(b) BC $= \sqrt{1.6^2 + 1.2^2} = 2.0$ m, so $\sin\theta = 0.6$ and $\cos\theta = 0.8$ — **B1**

Vertically: $T\sin\theta = 300$, so $T = 500$ N (tension in BC) — **M1 A1**

Horizontally: $P = 500\cos\theta = 400$ N (thrust in AC) — **M1 A1**

Answer: BC: 500 N tension; AC: 400 N thrust — **5 marks**

27 (12 marks)

(a) Let the tensions be T_1 (AC) and T_2 (DB). Horizontally: $T_1\cos 60° = T_2\cos 45°$ — **M1 A1**

Vertically: $T_1\sin 60° + T_2\sin 45° = 20 + 30 = 50$ — **M1 A1**

$T_2 = T_1\cos 60° / \cos 45°$; substituting, $T_1(\frac{\sqrt{3}}{2} + \frac{1}{2}) = 50$, so $T_1 = 36.6$ N and $T_2 = 25.9$ N — **M1 A1**

Answer: AC: 36.6 N; DB: 25.9 N — **6 marks**

(b) At C: horizontal component of AC on C $= T_1\cos 60° = 18.3$ N, directed towards A — **M1**

Vertical component $= T_1\sin 60° = 31.7$ N upwards, weight 20 N down, net 11.7 N upwards — **M1 A1**

CD must exert 18.3 N horizontally away from A and 11.7 N downwards on C, so it pulls C towards D: tension, magnitude $\sqrt{18.3^2 + 11.7^2} = 21.7$ N — **M1 A1**

$\tan\varphi = \frac{11.7}{18.3}$, so CD is inclined at 32.6° below the horizontal (D lower than C) — **A1**

Answer: CD: 21.7 N tension; 32.6° below the horizontal, D below C — **6 marks**

28 (9 marks)

(a) Horizontally: $T_A \cos 30° = T_B \cos 45°$, so $T_B = T_A \times \frac{\sqrt{3}}{\sqrt{2}}$ **M1 A1**

Vertically: $T_A \sin 30° + T_B \sin 45° = W$ **M1 A1**

$T_A(\frac{1}{2} + \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{\sqrt{2}}) = W$, so $T_A = \frac{2W}{(1+\sqrt{3})}$ **M1**

Multiplying numerator and denominator by $(\sqrt{3} - 1)$: $T_A = \frac{2W(\sqrt{3}-1)}{2} = (\sqrt{3} - 1)W$ **A1**

Answer: $T_{AC} = (\sqrt{3} - 1)W$ (shown) **6 marks**

(b) $T_B = (\sqrt{3} - 1)W \times \frac{\sqrt{3}}{\sqrt{2}}$ **M1**

$= \frac{(3-\sqrt{3})}{\sqrt{2}}W = \frac{(3\sqrt{2}-\sqrt{6})}{2}W$ **A1 A1**

Answer: $\frac{(3-\sqrt{3})W}{\sqrt{2}}$ **3 marks**

29 (12 marks)

(a) Resolve along CA: the force from CB is perpendicular to CA, so it has no component in this direction **M1 A1**

$T_A = W \cos \alpha$ **A1**

Resolve along CB, perpendicular to CA: $T_B = W \sin \alpha$ **M1 A1**

Answer: $T_{CB} = W \sin \alpha$ **5 marks**

(b) $W \cos \alpha = 2W \sin \alpha$, so $\tan \alpha = \frac{1}{2}$ **M1 A1**

$\alpha = 26.6°$ **A1**

$\cos \alpha = \frac{2}{\sqrt{5}}$ and $\sin \alpha = \frac{1}{\sqrt{5}}$, so $T_A = \frac{2W}{\sqrt{5}}$ and $T_B = \frac{W}{\sqrt{5}}$ **M1 A1**

Answer: $\alpha = 26.6°$; CA: $\frac{2W}{\sqrt{5}}$, CB: $\frac{W}{\sqrt{5}}$ **5 marks**

(c) The tensions $\frac{2W}{\sqrt{5}} + \frac{W}{\sqrt{5}} = \frac{3W}{\sqrt{5}} \approx 1.34W$ **M1**

The rods are not vertical, so the tensions must be larger than W to give the necessary vertical components **E1**

Answer: Rods are inclined, so tensions exceed W **2 marks**

30 (10 marks)

(a) Horizontally: $T_A \cos 30° = T_B \cos \varphi$ **M1 A1**

Vertically: $T_A \sin 30° + T_B \sin \varphi = W$ **M1 A1**

Eliminating T_A: $T_B(\cos \varphi \tan 30° + \sin \varphi) = W$, so $T_B = \frac{W \cos 30°}{(\sin \varphi \cos 30° + \cos \varphi \sin 30°)} = \frac{W \cos 30°}{\sin(\varphi + 30°)}$ **M1 A1**

Answer: $T_{BC} = \frac{W \cos 30°}{\sin(\varphi + 30°)}$ (shown) **6 marks**

(b) T_B is least when $\sin(\varphi + 30°)$ is greatest, that is equal to 1 **M1 A1**

$\varphi + 30° = 90°$, so $\varphi = 60°$, and least tension $= W \cos 30° = \frac{\sqrt{3}}{2}W \approx 0.866W$ **A1 A1**

Answer: $\varphi = 60°$; least tension $\frac{\sqrt{3}W}{2}$ **4 marks**

Frameworks and Support Reactions

1 (3 marks)

Moments about A: $R_B \times 6 = 80 \times 3$, so $R_B = 40$ N — **M1 A1**

Resolving vertically: $R_A = 80 - 40 = 40$ N — **A1**

Answer: $R_A = 40$ N, $R_B = 40$ N — **3 marks**

2 (3 marks)

Moments about A: $R_B \times 5 = 90 \times 1.5$ — **M1 A1**

$R_B = 27$ N, and vertically $R_A = 90 - 27 = 63$ N (no horizontal reaction as no horizontal force acts) — **A1**

Answer: $R_A = 63$ N, $R_B = 27$ N — **3 marks**

3 (3 marks)

(a) The surface is smooth, so the reaction is perpendicular to it, that is vertical (upwards) — **B1**

Answer: Vertical, upwards — **1 mark**

(b) Two components at A (horizontal and vertical) — **B1**

Three equations: resolving in two directions and taking moments — **B1**

Answer: 2 components; 3 equations — **2 marks**

4 (3 marks)

Magnitude $= \sqrt{24^2 + 7^2} = 25$ N — **M1 A1**

Angle $= \arctan \frac{7}{24} = 16.3°$ above the horizontal — **A1**

Answer: 25 N at 16.3° above the horizontal — **3 marks**

5 (3 marks)

Moments about A: $R_B \times 4 = 40 \times 3$ — **M1 A1**

$R_B = 30$ N upwards — **A1**

Answer: 30 N upwards — **3 marks**

6 (3 marks)

Moments about B: $X_A \times 2 = 60 \times 1.5$ — **M1 A1**

$X_A = 45$ N (directed towards the wall) — **A1**

Answer: 45 N — **3 marks**

7 (5 marks)

Moments about A: $R_B \times 4 = 30 \times 3$, so $R_B = 22.5$ N upwards — **M1 A1**

Horizontally: $X_A = 30$ N to the left; vertically: $Y_A = 22.5$ N downwards — **M1 A1**

Magnitude $\sqrt{30^2 + 22.5^2} = 37.5$ N, at $\arctan \frac{22.5}{30} = 36.9°$ below the horizontal — **A1**

Answer: $R_B = 22.5$ N up; $R_A = 37.5$ N at 36.9° below the horizontal, pointing left and down — **5 marks**

8 (5 marks)

Moments about A: $R_B \times 6 = 36 \times 2 + 54 \times 4$ — **M1 A1**

$R_B = \frac{288}{6} = 48$ N — **A1**

Vertically: $R_A = 36 + 54 - 48 = 42$ N — **M1 A1**

Answer: $R_A = 42$ N, $R_B = 48$ N — **5 marks**

9 (5 marks)

Moments about A: $45 \times 6 = W \times 2$ — **M1 A1**

$W = 135$ N — **A1**

Vertical reaction at A $= 135 - 45 = 90$ N — **M1 A1**

Answer: $W = 135$ N; $R_A = 90$ N — **5 marks**

10 (5 marks)

Moments about A (anticlockwise positive): $R_B \times 3 - 36 \times 3 + 24 \times 2 = 0$ **M1 A1**

$R_B = 20$ N **A1**

Horizontally: $X_A = 24$ N, to the right; vertically: $Y_A = 36 - 20 = 16$ N upwards **M1**

Magnitude $= \sqrt{24^2 + 16^2} = 28.8$ N at 33.7° above the horizontal **A1**

Answer: 28.8 N at 33.7° above the horizontal, pointing right and up **5 marks**

11 (5 marks)

The plane is smooth, so the reaction R at B is perpendicular to the plane, at 30° to the vertical: vertical component R cos 30° **B1**

Moments about A: R cos 30° × 4 = 50 × 2, so R = 28.9 N **M1 A1**

Vertically: Y_A + R cos 30° = 50, so $Y_A = 50 - 25 = 25$ N **M1 A1**

Answer: R = 28.9 N, Y_A = 25 N **5 marks**

12 (5 marks)

Moments about A: $X_B \times 3 = 40 \times 2 + 60 \times 2 = 200$ (clockwise loads), so $X_B = 66.7$ N, directed towards the wall **M1 A1**

Horizontally: $X_A = 66.7$ N away from the wall; vertically $Y_A = 100$ N upwards **M1 A1**

Magnitude $\sqrt{66.7^2 + 100^2} = 120$ N at 56.3° to the horizontal **A1**

Answer: R_B = 66.7 N towards the wall; R_A = 120 N at 56.3° above the horizontal **5 marks**

13 (4 marks)

(a) $2j - 3 = 2 \times 7 - 3 = 11$ **M1**

which equals the number of rods, so the framework is simply rigid **A1**

Answer: 2 × 7 − 3 = 11 rods, so it is simple rigid **2 marks**

(b) A rigid framework needs $2 \times 6 - 3 = 9$ rods, but there are only 8 **M1**

so it is not rigid: it has one degree of freedom (a mechanism) and could deform **E1**

Answer: Not rigid (needs 9 rods) **2 marks**

14 (6 marks)

Moments about A (anticlockwise positive): $8R_C - 12 \times 2 - 18 \times 6 + 6 \times 2 = 0$ **M1 A1 A1**

$R_C = \frac{120}{8} = 15$ kN upwards **A1**

Horizontally: $X_A = 6$ kN (to the right); vertically: $Y_A = 30 - 15 = 15$ kN upwards **M1**

Magnitude $\sqrt{6^2 + 15^2} = 16.2$ kN at 68.2° above the horizontal **A1**

Answer: R_C = 15 kN; R_A = 16.2 kN at 68.2° above the horizontal, pointing right and up **6 marks**

15 (7 marks)

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$: the force has components 25 N to the right and 60 N downwards **B1 B1**

Moments about A: $6R_B = 60 \times 2 + 25 \times 3$ **M1 A1**

$R_B = 32.5$ N **A1**

$X_A = 25$ N to the left, $Y_A = 60 - 32.5 = 27.5$ N upwards **M1**

Magnitude 37.2 N at 47.7° above the horizontal, pointing left and up **A1**

Answer: R_B = 32.5 N; R_A = 37.2 N at 47.7° above the horizontal, pointing left and up **7 marks**

16 (7 marks)

The wall is smooth so the reaction at C is horizontal, X_C, pointing away from the wall **B1**

Moments about A: $3X_C = 50 \times 3 + 30 \times 4$ **M1 A1 A1**

$X_C = \frac{270}{3} = 90$ N **A1**

$X_A = 90$ N (towards the wall), $Y_A = 50 + 30 = 80$ N upwards **M1**

Magnitude $\sqrt{90^2 + 80^2} = 120$ N at 41.6° above the horizontal **A1**

Answer: R_C = 90 N horizontal; R_A = 120 N at 41.6° above the horizontal **7 marks**

17 (6 marks)

(a) Moments about A: $9R_B = Wx$, so $R_B = \frac{Wx}{9}$ **M1 A1**

$R_A = W - \frac{Wx}{9} = \frac{W(9 - x)}{9}$ **M1 A1**

Answer: $R_B = \frac{Wx}{9}$, $R_A = \frac{W(9 - x)}{9}$ **4 marks**

(b) $\frac{W(9 - x)}{9} = \frac{5}{4} \times \frac{Wx}{9}$, so $4(9 - x) = 5x$ **M1**

$x = 4$ **A1**

Answer: x = 4 **2 marks**

18 (6 marks)

Moments about A: $4R_B = 60 \times 2 + 3H$, so $R_B = 30 + \frac{3H}{4}$ **M1 A1**

Vertical component at A: $Y_A = 60 - R_B = 30 - \frac{3H}{4}$; horizontal component $X_A = H$ **M1 A1**

At 45°: $30 - \frac{3H}{4} = H$ **M1**

$H = \frac{120}{7} \approx 17.1$ N **A1**

Answer: $H = \frac{120}{7} \approx 17.1$ N **6 marks**

19 (6 marks)

Moments about A: $3|X_B| = 2 \times 50 \sin 40° + 3 \times 50 \cos 40° + 2 \times 30$ **M1 A1 A1**

$X_B = 79.7$ N, directed towards the wall **A1**

Horizontally: $X_A = 79.7 - 50 \cos 40° = 41.4$ N (away from the wall); vertically: $Y_A = 50 \sin 40° + 30 = 62.1$ N **M1**

$|R_A| = \sqrt{41.4^2 + 62.1^2} = 74.7$ N **A1**

Answer: $R_B = 79.7$ N towards the wall; $R_A = 74.7$ N **6 marks**

20 (7 marks)

(a) j = 5 joints, so $2j - 3 = 7$ **M1**

There are 7 rods, equal to $2j - 3$, so it is simply rigid **A1**

Answer: 7 rods = 2 × 5 − 3, so the framework is simple rigid (shown) **2 marks**

(b) Moments about A: $6R_B = 48 \times 3 + 24 \times 5$ **M1 A1**

$R_B = 44$ N upwards **A1**

$X_A = 24$ N to the left, $Y_A = 48 - 44 = 4$ N upwards **M1**

Magnitude $\sqrt{24^2 + 4^2} = 24.3$ N at 9.46° to the horizontal **A1**

Answer: $R_B = 44$ N; $R_A = 24.3$ N **5 marks**

21 (9 marks)

(a) Moments about A (anticlockwise positive): $4R_B - 120 \times 4 + 3P = 0$ **M1 A1**

$R_B = 120 - \frac{3P}{4}$ **A1**

Answer: $R_B = 120 - \frac{3P}{4}$ N (shown) **3 marks**

(b) Contact requires $R_B \geq 0$: $120 - \frac{3P}{4} \geq 0$ **M1**

$P \leq 160$, so the greatest value is 160 N **A1**

Answer: 160 N **2 marks**

(c) $R_B = 120 - 75 = 45$ N **B1**

$X_A = 100$ N to the right, $Y_A = 120 - 45 = 75$ N upwards **M1 A1**

Magnitude $\sqrt{100^2 + 75^2} = 125$ N at $\arctan \frac{75}{100} = 36.9°$ above the horizontal **A1**

Answer: 125 N at 36.9° above the horizontal (right and up) **4 marks**

22 (6 marks)

(a) The student used the height of C (4 m) as the moment arm of the weight — **E1**

The moment arm is the horizontal distance of the line of action from A, namely 3 m — **E1**

Answer: Wrong distance used (4 m instead of 3 m) — **2 marks**

(b) $8R_B = 80 \times 3$, so $R_B = 30$ N — **M1 A1**

$R_A = 80 - 30 = 50$ N (vertical, upwards) — **A1 A1**

Answer: $R_B = 30$ N, $R_A = 50$ N — **4 marks**

23 (9 marks)

The reaction R at B is perpendicular to the plane: components $(-R\sin\theta, R\cos\theta) = (-\frac{3}{5}R, \frac{4}{5}R)$ — **B1 B1**

Moments about A: $5 \times \frac{4}{5}R - 40 \times 2 - 15 \times 3 = 0$ — **M1 A1 A1**

$4R = 125$, $R = 31.25$ N — **A1**

Horizontally: $X_A - \frac{3}{5}(31.25) + 15 = 0$, so $X_A = 3.75$ N — **M1**

Vertically: $Y_A + \frac{4}{5}(31.25) = 40$, so $Y_A = 15$ N; $|R_A| = \sqrt{3.75^2 + 15^2} = 15.5$ N at 76.0° above the horizontal — **M1 A1**

Answer: $R_B = 31.25$ N; $R_A = 15.5$ N at 76.0° above the horizontal (pointing right and up) — **9 marks**

24 (9 marks)

(a) Moments about A: $4aR_B - W \times a - 3W \times 3a - H \times h = 0$ — **M1 A1 A1**

$R_B = \frac{(10Wa + Hh)}{(4a)} = \frac{5W}{2} + \frac{Hh}{(4a)}$ — **A1**

$Y_A = 4W - R_B = \frac{3W}{2} - \frac{Hh}{(4a)}$ — **M1 A1**

Answer: $Y_A = \frac{3W}{2} - \frac{Hh}{(4a)}$ — **6 marks**

(b) $\frac{3W}{2} = \frac{H \times (3a}{2)/(4a)} = \frac{3H}{8}$, so $H = 4W$ — **M1 A1**

The horizontal reaction at A balances H: it is 4W, directed to the left (opposite to H) — **B1**

Answer: $H = 4W$; $X_A = 4W$ to the left — **3 marks**

25 (7 marks)

Weights: 800g = 7840 N at D and Mg at E — **B1**

Moments about A: $8R_C = 7840 \times 2 + Mg \times 6$ — **M1 A1**

$R_C \leq 12000$: $96000 \geq 15680 + 58.8M$ — **M1 A1**

$M \leq \frac{80320}{58.8}$ — **M1**

The greatest value of M is 1360 kg (3 s.f., rounded down so that the limit is not exceeded) — **A1**

Answer: M = 1360 kg — **7 marks**

26 (8 marks)

(a) Height of C above AB = $a\tan\theta$ — **B1**

Moments about A: $2aR_B = P \times a\tan\theta$, so $R_B = \frac{P\tan\theta}{2}$ — **M1 A1**

Horizontal: $X_A = P$; vertical: $Y_A = \frac{P\tan\theta}{2}$ (downwards) — **M1**

$|R_A| = \sqrt{P^2 + \frac{P^2\tan^2\theta}{4}} = P\sqrt{1 + \frac{1}{4}\tan^2\theta}$ — **A1**

Answer: $|R_A| = P\sqrt{1 + \frac{1}{4}\tan^2\theta}$ (shown) — **5 marks**

(b) $\sqrt{1 + \frac{1}{4}\tan^2\theta} = \frac{5}{4}$, so $1 + \frac{1}{4}\tan^2\theta = \frac{25}{16}$ — **M1**

$\tan^2\theta = \frac{9}{4}$, so $\tan\theta = \frac{3}{2}$ — **A1**

$\theta = 56.3°$ — **A1**

Answer: $\theta = 56.3°$ — **3 marks**

27 (12 marks)

(a) The reaction at C is perpendicular to the plane: components $(-R\sin 30°, R\cos 30°)$ **B1**

Moments about A: $8R\cos 30° = 20 \times 2 + 40 \times 6$ **M1 A1 A1**

$R = \dfrac{280}{(8\cos 30°)} = 40.4$ N **A1**

Answer: 40.4 N **5 marks**

(b) Horizontally: $X_A = R\sin 30° = 20.2$ N (to the right) **M1 A1**

Vertically: $Y_A = 60 - R\cos 30° = 60 - 35 = 25$ N upwards **M1 A1**

Magnitude $\sqrt{20.2^2 + 25^2} = 32.1$ N at 51.1° above the horizontal **A1**

Answer: 32.1 N at 51.1° above the horizontal, right and up **5 marks**

(c) Only the vertical component of the reaction at C enters the moment equation, so R is unchanged **E1**

The horizontal component of R reverses, so X_A reverses; Y_A is unchanged, so the magnitude of the reaction at A is unchanged **E1**

Answer: Neither magnitude changes; the horizontal components reverse **2 marks**

28 (11 marks)

(a) Moments about A (anticlockwise positive): $4 \times 60 - 4P + 3Q = 0$ **M1 A1**

$4P - 3Q = 240$ **A1**

Horizontal component $X_A = Q$ (to the right), vertical component $Y_A = P - 60$ (upwards) **B1 B1**

Answer: $4P - 3Q = 240$; $X_A = Q$, $Y_A = P - 60$ **5 marks**

(b) From part (a), $P - 60 = \dfrac{3Q}{4}$, so $Y_A = \dfrac{3Q}{4}$ **M1 A1**

$|R_A|^2 = Q^2 + \dfrac{9Q^2}{16} = 75^2$, so $\dfrac{25Q^2}{16} = 5625$ **M1 A1**

$Q = 60$ and $P = 60 + 45 = 105$ **A1 A1**

Answer: P = 105, Q = 60 **6 marks**

29 (12 marks)

(a) Moments about A: $12R_C = 2W \times 3 + Wx$ **M1 A1**

$R_C = \dfrac{W(6 + x)}{12}$ **A1**

$R_A = 3W - R_C = \dfrac{W(30 - x)}{12}$ **M1 A1**

Answer: $R_C = \dfrac{W(6 + x)}{12}$, $R_A = \dfrac{W(30 - x)}{12}$ **5 marks**

(b) $R_C \leq 1.1W$: $6 + x \leq 13.2$, so $x \leq 7.2$ **M1 A1**

$R_A \leq 2.1W$: $30 - x \leq 25.2$, so $x \geq 4.8$ **M1 A1**

$4.8 \leq x \leq 7.2$ **A1**

Answer: $4.8 \leq x \leq 7.2$ **5 marks**

(c) $12R_C = 6W + 6W'$, so $R_C = \dfrac{W + W'}{2}$ **M1**

$\dfrac{W + W'}{2} \leq 1.1W$ gives $W' \leq 1.2W$ **A1**

Answer: $W' \leq 1.2W$ **2 marks**

30 (11 marks)

(a) C is at height $a\sqrt{3}$ above AB and directly above the midpoint, a from A

Moments about A: $2aR_B = W \times a + P \times a\sqrt{3}$, so $R_B = \dfrac{1}{2}(W + \sqrt{3}P)$ **M1 A1 A1**

Horizontal component of the reaction at A = P, directed against the force P; vertical component = $W - R_B = \dfrac{1}{2}(W - \sqrt{3}P)$ **M1 A1**

Answer: $R_B = \dfrac{1}{2}(W + \sqrt{3}P)$; $X_A = P$ (opposing P), $Y_A = \dfrac{1}{2}(W - \sqrt{3}P)$ **5 marks**

(b) $|R_A|^2 = P^2 + \dfrac{1}{4}(W - \sqrt{3}P)^2 = \dfrac{7}{4}P^2 - \dfrac{\sqrt{3}}{2}WP + \dfrac{1}{4}W^2$ **M1 A1**

Differentiating with respect to P: $\frac{7}{2}P - \frac{\sqrt{3}}{2}W = 0$, so $P = \frac{\sqrt{3}W}{7}$ **M1 A1**

Least $|R_A|^2 = \frac{3W^2}{28} - \frac{3W^2}{14} + \frac{W^2}{4} = \frac{W^2}{7}$, so least magnitude $\frac{W}{\sqrt{7}}$ **M1 A1**

Answer: P = $\frac{\sqrt{3}W}{7}$; least reaction $\frac{W}{\sqrt{7}}$ **6 marks**

Method of Joints

1 (3 marks)

Joint A, horizontally: $T_{AB} + 0.555T_{CA} = 0$ **M1**

Joint A, vertically: $0.832T_{CA} + 40 = 0$

$T_{AB} = 26.7$ N (tension), $T_{CA} = 48.1$ N (thrust) **A1 A1**

Answer: AB: 26.7 N tension; CA: 48.1 N thrust **3 marks**

2 (3 marks)

Joint C, horizontally: $\frac{1}{2}T_{BC} - \frac{1}{2}T_{CA} = 0$ **M1**

Joint C, vertically: $-0.866T_{BC} - 0.866T_{CA} - 60 = 0$

$T_{BC} = 34.6$ N (thrust), $T_{CA} = 34.6$ N (thrust) **A1 A1**

Answer: BC: 34.6 N thrust; CA: 34.6 N thrust **3 marks**

3 (2 marks)

At joint B the rods AB and BC are collinear and no external force acts, so resolving perpendicular to AB, the force in BD must be zero **B1 E1**

Answer: BD carries no force **2 marks**

4 (3 marks)

Joint C, horizontally: $-\frac{4}{5}T_{CA} = 0$ **M1**

Joint C, vertically: $-T_{BC} - \frac{3}{5}T_{CA} - 60 = 0$

$T_{BC} = 60$ N (thrust), $T_{CA} = 0$ **A1 A1**

Answer: BC: 60 N thrust; CA: 0 **3 marks**

5 (3 marks)

By symmetry about the vertical through C the forces in AC and BC are equal, T **E1**

Vertically at C: $2T \times \frac{4}{5} = 80$, so T = 50 N; the rods push C upwards, so thrust **M1 A1**

Answer: 50 N thrust in each **3 marks**

6 (3 marks)

Joint D, horizontally: $-0.894T_{BD} - T_{CD} = 0$ **M1**

Joint D, vertically: $0.447T_{BD} - 60 = 0$

$T_{BD} = 134$ N (tension), $T_{CD} = 120$ N (thrust) **A1 A1**

Answer: BD: 134 N tension; CD: 120 N thrust **3 marks**

7 (5 marks)

Joint A, horizontally: $T_{AB} + 0.555T_{CA} = 0$ **M1**

Joint A, vertically: $0.832T_{CA} + 40 = 0$

$T_{AB} = 26.7$ N (tension), $T_{CA} = 48.1$ N (thrust) **A1 A1**

Joint B, horizontally: $-\frac{4}{5}T_{BC} - 26.7 = 0$ **M1**

Joint B, vertically: $\frac{3}{5}T_{BC} + 20 = 0$

$T_{BC} = 33.3$ N (thrust) **A1**

Answer: AB: 26.7 N tension; BC: 33.3 N thrust; CA: 48.1 N thrust **5 marks**

8 (5 marks)

Joint C, horizontally: $-\frac{4}{5}T_{CA} + 40 = 0$ **M1**

Joint C, vertically: $-T_{BC} - \frac{3}{5}T_{CA} = 0$

$T_{BC} = 30$ N (thrust), $T_{CA} = 50$ N (tension) **A1 A1**

Joint B, horizontally: $-T_{AB} = 0$ **M1**

$T_{AB} = 0$ **A1**

Answer: AB: 0; BC: 30 N thrust; CA: 50 N tension **5 marks**

9 (3 marks)

Joint A, horizontally: $T_{AB} + 0.707T_{AD} = 0$ **M1**

Joint A, vertically: $0.707T_{AD} + 30 = 0$

$T_{AB} = 30$ N (tension), $T_{AD} = 42.4$ N (thrust) **A1 A1**

Answer: AB: 30 N tension; AD: 42.4 N thrust **3 marks**

10 (5 marks)

At D two non-collinear rods meet with no external force, so DA and DC carry no force **E1**

Joint C, horizontally (rightwards positive): $40 - T_{CA} \times \frac{3}{\sqrt{13}} = 0$, so $T_{CA} = \frac{40\sqrt{13}}{3} = 48.1$ N (tension) **M1 A1**

Vertically: $-T_{CA} \times \frac{2}{\sqrt{13}} - T_{CB} = 0$, so $T_{CB} = -\frac{80}{3}$ N: a thrust of 26.7 N **M1 A1**

Answer: DA, DC: 0; CA: 48.1 N tension; CB: 26.7 N thrust **5 marks**

11 (5 marks)

Joint D, horizontally: $-T_{DB} = 0$ **M1**

Joint D, vertically: $T_{CD} - 80 = 0$

$T_{CD} = 80$ N (tension), $T_{DB} = 0$ **A1 A1**

Joint C, vertically: $-T_{CD} - \frac{3}{5}T_{CB} = 0$, so $-80 - \frac{3}{5}T_{CB} = 0$ **M1**

$T_{CB} = -\frac{400}{3}$ N: a thrust of 133 N **A1**

Answer: DC: 80 N tension; DB: 0; BC: 133 N thrust **5 marks**

12 (5 marks)

(a) Joint A (or joint D) has only two unknown rod forces **B1**

Joint B has four unknown rod forces (AB, BC, BE and BF), which is more than two equations can determine **E1**

Answer: Start at A or D; joint B has 4 unknown forces **2 marks**

(b) Vertically at A: $30 + T_{AE} \times \frac{1}{\sqrt{2}} = 0$, so $T_{AE} = -30\sqrt{2} = -42.4$ N, a thrust of 42.4 N **M1 A1**

Horizontally: $T_{AB} + T_{AE} \times \frac{1}{\sqrt{2}} = 0$, so $T_{AB} = 30$ N, a tension **A1**

Answer: AB: 30 N tension; AE: 42.4 N thrust **3 marks**

13 (6 marks)

Joint C, horizontally: $0.707T_{BC} - T_{CD} = 0$ **M1**

Joint C, vertically: $-0.707T_{BC} - 30 = 0$

$T_{BC} = 42.4$ N (thrust), $T_{CD} = 30$ N (thrust) **A1 A1**

Joint D, horizontally: $-0.707T_{DA} + 0.894T_{DB} - 30 = 0$ **M1**

Joint D, vertically: $-0.707T_{DA} - 0.447T_{DB} - 60 = 0$

$T_{DA} = 70.7$ N (thrust), $T_{DB} = 22.4$ N (thrust) **A1 A1**

Answer: CB: 42.4 N thrust; CD: 30 N thrust; DA: 70.7 N thrust; DB: 22.4 N thrust **6 marks**

14 (6 marks)

Joint A, horizontally: $T_{AB} + 0.707T_{AD} = 0$ **M1**

Joint A, vertically: $0.707T_{AD} + 45 = 0$

$T_{AB} = 45$ N (tension), $T_{AD} = 63.6$ N (thrust) **A1 A1**

Joint D, horizontally: $0.707T_{DB} + 0.707T_{DE} + 45 = 0$ **M1**

Joint D, vertically: $-0.707T_{DB} + 0.707T_{DE} + 45 = 0$

$T_{DE} = 63.6$ N (thrust), $T_{DB} = 0$ **A1 A1**

Answer: AB: 45 N tension; AD: 63.6 N thrust; DE: 63.6 N thrust; DB: 0 **6 marks**

15 (6 marks)

Joint D, horizontally: $-T_{DA} = 0$ **M1**

Joint D, vertically: $T_{CD} - 40 = 0$

$T_{CD} = 40$ N (tension), $T_{DA} = 0$ **A1 A1**

Joint C, horizontally: $-0.832T_{AC} - T_{BC} = 0$ **M1**

Joint C, vertically: $-0.555T_{AC} - 100 = 0$

$T_{BC} = 150$ N (tension), $T_{AC} = 180$ N (thrust) **A1 A1**

Answer: DA: 0; CD: 40 N tension; BC: 150 N tension; CA: 180 N thrust **6 marks**

16 (6 marks)

Joint A, horizontally: $T_{AB} + \frac{3}{5}T_{AD} - 24 = 0$ **M1**

Joint A, vertically: $\frac{4}{5}T_{AD} + 12 = 0$

$T_{AB} = 33$ kN (tension), $T_{AD} = 15$ kN (thrust) **A1 A1**

Joint B, horizontally: $-0.914T_{DB} - \frac{3}{5}T_{EB} - 33 = 0$ **M1**

Joint B, vertically: $0.406T_{DB} + \frac{4}{5}T_{EB} + 28 = 0$

$T_{EB} = 25$ kN (thrust), $T_{DB} = 19.7$ kN (thrust) **A1 A1**

Answer: AB: 33 kN tension; AD: 15 kN thrust; BE: 25 kN thrust; BD: 19.7 kN thrust **6 marks**

17 (7 marks)

(a) At E the rods AE and ED are collinear and E is unloaded, so EB carries no force; similarly BF carries no force **E1 E1**

At B the rods AB and BC are collinear and EB, BF carry no force, so BD carries no force **E1**

Answer: EB, BF and BD **3 marks**

(b) Joint A, vertically: $T_{AE} \times \frac{3}{5} + 30 = 0$, so $T_{AE} = -50$ N, a thrust of 50 N **M1 A1**

Horizontally: $T_{AB} + T_{AE} \times \frac{4}{5} = 0$, so $T_{AB} = 40$ N, a tension **M1 A1**

Answer: AE: 50 N thrust; AB: 40 N tension **4 marks**

18 (7 marks)

BC is 5 m long, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, where θ is its angle with the horizontal — **B1**

Joint C, vertically: $T_{BC} \times \frac{3}{5} = 80$, so $T_{BC} = \frac{400}{3} = 133$ N, a tension — **M1 A1**

Horizontally (away from the wall positive): $T_{AC} - 133 \times \frac{4}{5} = 0$, since BC pulls C towards the wall; AC pushes C away from the wall with 107 N: a thrust — **M1 A1**

Joint B: BC pulls B down with vertical component 80 N, so the vertical reaction at B is 80 N upwards — **M1 A1**

Answer: BC: 133 N tension; AC: 107 N thrust; vertical reaction at B: 80 N upwards — **7 marks**

19 (6 marks)

By symmetry the reaction at A is $\frac{W}{2}$; AD has length 2.5 m, so its vertical component is $\frac{4}{5} \times 50 = 40$ N — **B1 B1**

Joint A, vertically: $40 = \frac{W}{2}$, so $W = 80$ N — **M1 A1**

Horizontally: $T_{AB} = 50 \times \frac{3}{5} = 30$ N, a tension — **M1 A1**

Answer: W = 80 N; AB: 30 N tension — **6 marks**

20 (7 marks)

At B the rods AB and BC are collinear and B is unloaded, so the force in BD is zero — **E1**

By symmetry the reactions are 30 N each; joint A, vertically: $T_{AD} \times \frac{3}{5} + 30 = 0$ — **B1 M1**

$T_{AD} = -50$ N, a thrust of 50 N — **A1**

Horizontally: $T_{AB} + T_{AD} \times \frac{4}{5} = 0$, so $T_{AB} = 40$ N, a tension — **M1 A1**

Joint C (or by symmetry): $T_{BC} = 40$ N, a tension — **A1**

Answer: BD: 0; AD: 50 N thrust; AB: 40 N tension; BC: 40 N tension — **7 marks**

21 (8 marks)

Moments about A (anticlockwise positive): $-24 - 108 + \frac{16}{2}R_C = 0$ — **M1 A1**

$R_C = 16.5$ kN — **A1**

Resolving: horizontal reaction at A is 0 kN (to the left), vertical reaction is 13.5 kN (upwards) — **M1 A1**

Joint A, horizontally: $T_{AB} + 0.707T_{AD} = 0$ — **M1**

Joint A, vertically: $0.707T_{AD} + 13.5 = 0$

$T_{AB} = 13.5$ kN (tension), $T_{AD} = 19.1$ kN (thrust) — **A1 A1**

Answer: AB: 13.5 kN tension; AD: 19.1 kN thrust — **8 marks**

22 (6 marks)

(a) The horizontal working is correct, but a tension in CA pulls C down and to the left, so CA has a downward component of 30 N on the joint — **E1**

This must be balanced by an upward force from CB, which pushes C upwards: CB is in thrust, not tension — **E1**

Answer: CB is a thrust, not a tension — **2 marks**

(b) CA: 50 N tension; CB: 30 N thrust — **B1**

Joint B: CB (thrust) pushes B downwards, balanced by the reaction (30 N upwards); horizontally $T_{AB} = 0$ since only AB acts horizontally on B — **M1 A1**

Answer: CA: 50 N tension; CB: 30 N thrust; AB: 0 N — **3 marks**

(c) CA is in tension, so it is a tie; CB is in thrust, so it is a strut — **B1**

Answer: CA is a tie; CB is a strut — **1 mark**

23 (8 marks)

Joint A, vertically: $T_{AD} \sin 60° + \frac{W}{4} = 0$, so $T_{AD} = -\frac{\sqrt{3}W}{6}$: a thrust of $\frac{\sqrt{3}W}{6}$ **M1 A1**

Horizontally: $T_{AB} + T_{AD} \cos 60° = 0$, so $T_{AB} = \frac{\sqrt{3}W}{12}$, a tension **M1 A1**

Joint D: AD pushes D up and to the right with vertical component $\frac{W}{4}$; DB runs down and to the right, so vertically $\frac{W}{4} - T_{DB} \sin 60° = 0$ **M1**

$T_{DB} = \frac{W}{(2\sqrt{3})} = \frac{\sqrt{3}W}{6}$, a tension **A1**

Horizontally (right positive): $\frac{\sqrt{3}W}{6} \times \frac{1}{2} + T_{DB} \times \frac{1}{2} + T_{DE} = 0$ **M1**

$T_{DE} = -\frac{\sqrt{3}W}{6}$: a thrust of $\frac{\sqrt{3}W}{6}$ **A1**

Answer: AD: $\frac{\sqrt{3}W}{6}$ thrust; AB: $\frac{\sqrt{3}W}{12}$ tension; DB: $\frac{\sqrt{3}W}{6}$ tension; DE: $\frac{\sqrt{3}W}{6}$ thrust **8 marks**

24 (9 marks)

(a) Moments about A (anticlockwise positive), with X_B the force at B acting to the right: $-6X_B - 3 \times 48 - 4 \times 60 = 0$, so $X_B = -64$: the reaction at B is 64 N directed towards the wall **M1 A1**

Resolving horizontally: $X_A - 64 + 48 = 0$, so $X_A = 16$ N (away from the wall); vertically $Y_A = 60$ N upwards; $|R_A| = \sqrt{16^2 + 60^2} = 62.1$ N at 75.1° above the horizontal **A1**

Answer: RB = 64 N towards the wall; RA = 62.1 N at 75.1° above the horizontal **3 marks**

(b) Joint C, horizontally: $48 - \frac{4}{5}T_{AC} - \frac{4}{5}T_{BC} = 0$, so $T_{AC} + T_{BC} = 60$ **M1 A1**

Vertically: $-60 - \frac{3}{5}T_{AC} + \frac{3}{5}T_{BC} = 0$, so $T_{BC} - T_{AC} = 100$ **A1**

$T_{BC} = 80$ N (tension) and $T_{AC} = -20$ N (thrust of 20 N) **A1**

Joint B, vertically: $-T_{AB} - \frac{3}{5} \times 80 = 0$, so $T_{AB} = -48$ N: a thrust of 48 N **M1 A1**

Answer: AB: 48 N thrust; AC: 20 N thrust; BC: 80 N tension **6 marks**

25 (7 marks)

$\tan \theta = \frac{12}{5}$, so $\sin \theta = \frac{12}{13}$ and $\cos \theta = \frac{5}{13}$ **B1**

By symmetry the forces in AC and BC are equal; the reactions are $\frac{W}{2}$ each **B1**

Joint A, vertically: $T_{AC} \times \frac{12}{13} + \frac{W}{2} = 0$ **M1 A1**

$T_{AC} = -\frac{13W}{24}$: AC and BC are in thrust, each $\frac{13W}{24}$ **A1**

Horizontally: $T_{AB} + T_{AC} \times \frac{5}{13} = 0$, so $T_{AB} = \frac{5W}{24}$, a tension **M1 A1**

Answer: AC, BC: $\frac{13W}{24}$ thrust; AB: $\frac{5W}{24}$ tension **7 marks**

26 (7 marks)

By symmetry the reactions at A and C are each 20 kN upwards, and the forces in AD and FC, DE and EF, DB and BF are equal in pairs **B1**

Joint A, horizontally: $T_{AB} + 0.707T_{AD} = 0$ **M1**

Joint A, vertically: $0.707T_{AD} + 20 = 0$

$T_{AB} = 20$ kN (tension), $T_{AD} = 28.3$ kN (thrust) **A1 A1**

Joint D, horizontally: $0.707T_{DB} + 0.707T_{DE} + 20 = 0$ **M1**

Joint D, vertically: $-0.707T_{DB} + 0.707T_{DE} + 10 = 0$

$T_{DE} = 21.2$ kN (thrust), $T_{DB} = 7.07$ kN (thrust) **A1 A1**

Answer: AB 20 T, AD 28.3 S, DE 21.2 S, DB 7.07 S (kN; T tension, S thrust) **7 marks**

27 (11 marks)

Moments about A (anticlockwise positive): $-45 - 36 + \frac{12}{2}R_C = 0$ **M1 A1**

$R_C = 13.5$ kN **A1**

Resolving: horizontal reaction at A is 12 kN (to the left), vertical reaction is 16.5 kN (upwards) **M1 A1**

Joint A, horizontally: $T_{AB} + 0.707T_{AD} - 12 = 0$ **M1**

Joint A, vertically: $0.707T_{AD} + 16.5 = 0$

$T_{AB} = 28.5$ kN (tension), $T_{AD} = 23.3$ kN (thrust) **A1 A1**

Joint D, horizontally: $0.707T_{DB} + 0.707T_{DE} + 16.5 = 0$ **M1**

Joint D, vertically: $-0.707T_{DB} + 0.707T_{DE} - 13.5 = 0$

$T_{DE} = 2.12$ kN (thrust), $T_{DB} = 21.2$ kN (thrust) **A1 A1**

Answer: $R_C = 13.5$, $X_A = -12$, $Y_A = 16.5$ (kN); AB 28.5 T, AD 23.3 S, DB 21.2 S, DE 2.12 S **11 marks**

28 (10 marks)

By symmetry the reactions at A and D are each 15 kN upwards **B1**

Joint A, horizontally: $T_{AB} + 0.707T_{AE} = 0$ **M1**

Joint A, vertically: $0.707T_{AE} + 15 = 0$

$T_{AB} = 15$ kN (tension), $T_{AE} = 21.2$ kN (thrust) **A1 A1**

Joint E, horizontally: $0.707T_{EB} + T_{EF} + 15 = 0$ **M1**

Joint E, vertically: $-0.707T_{EB} + 5 = 0$

$T_{EB} = 7.07$ kN (tension), $T_{EF} = 20$ kN (thrust) **A1 A1**

Joint B, horizontally: $T_{BC} + 0.707T_{BF} - 20 = 0$ **M1**

Joint B, vertically: $0.707T_{BF} + 5 = 0$

$T_{BC} = 25$ kN (tension), $T_{BF} = 7.07$ kN (thrust) **A1 A1**

Answer: AB 15 T, AE 21.2 S, EB 7.07 T, EF 20 S, BF 7.07 S, BC 25 T (kN; T tension, S thrust) **10 marks**

29 (11 marks)

(a) Moments about A: $4aR_C = W \times a + 2W \times 3a$ **M1 A1**

$R_C = \frac{7W}{4}$ upwards, and $R_A = 3W - \frac{7W}{4} = \frac{5W}{4}$ upwards (no horizontal force) **A1 A1**

Answer: $R_C = \frac{7W}{4}$, $R_A = \frac{5W}{4}$ **4 marks**

(b) Joint A, vertically: $T_{AD} \sin 60° + \frac{5W}{4} = 0$, so $T_{AD} = -\frac{5\sqrt{3}W}{6}$: a thrust of $\frac{5\sqrt{3}W}{6}$ **M1 A1**

Joint D, vertically: $\frac{5\sqrt{3}W}{6} \sin 60° - W - T_{DB} \sin 60° = 0$, so $\frac{5W}{4} - W = \frac{\sqrt{3}}{2}T_{DB}$ **M1 A1**

$T_{DB} = \frac{\sqrt{3}W}{6}$, a tension **A1**

Horizontally: $\frac{5\sqrt{3}W}{6} \times \frac{1}{2} + \frac{\sqrt{3}W}{6} \times \frac{1}{2} + T_{DE} = 0$, so $T_{DE} = -\frac{\sqrt{3}W}{2}$: a thrust of $\frac{\sqrt{3}W}{2}$ **M1 A1**

Answer: AD: $\frac{5\sqrt{3}W}{6}$ thrust; DB: $\frac{\sqrt{3}W}{6}$ tension; DE: $\frac{\sqrt{3}W}{2}$ thrust **7 marks**

30 (11 marks)

(a) j = 5 joints, so a simple rigid framework needs $2j - 3 = 7$ rods; with AE there are 8 **M1**

There is one more rod than the minimum, so one force cannot be found from the equilibrium equations alone (the framework is redundant) **E1**

Answer: 8 rods > 7: redundant **2 marks**

(b) By symmetry the reactions at A and C are each 20 N upwards **B1**

Joint A, horizontally: $T_{AB} + 0.707T_{AD} = 0$ **M1**

Joint A, vertically: $0.707T_{AD} + 20 = 0$

$T_{AB} = 20$ N (tension), $T_{AD} = 28.3$ N (thrust) **A1 A1**

Joint D, horizontally: $0.707T_{DB} + T_{DE} + 20 = 0$ **M1**

Joint D, vertically: $-0.707T_{DB} = 0$

$T_{DB} = 0$, $T_{DE} = 20$ N (thrust) **A1 A1**

By symmetry $T_{EC} = T_{AD}$, $T_{BE} = T_{DB}$ (in thrust/tension as found) and $T_{BC} = T_{AB}$ **B1 B1**

Answer: AB 20 T, BC 20 T, AD 28.3 S, DB 0, DE 20 S, BE 0, EC 28.3 S (N; T tension, S thrust) **9 marks**

Method of Sections and Harder Frameworks

1 (3 marks)

Take a section through AB, DB, DE and consider the equilibrium of the part containing A, D **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} - 12 \times 4 + 8 \times 2 = 0$ **M1 A1**

$T_{DE} = 16$ kN (thrust)

Answer: DE: 16 kN thrust **3 marks**

2 (3 marks)

Take a section through AB, DB, DE and consider the equilibrium of the part containing A, D **B1**

Moments about D (left-hand part, anticlockwise positive): $2T_{AB} - 17.5 \times 2 = 0$ **M1 A1**

$T_{AB} = 17.5$ kN (tension)

Answer: AB: 17.5 kN tension **3 marks**

3 (3 marks)

Take a section through AB, DB, DE and consider the equilibrium of the part containing A, D **B1**

Resolving vertically (left-hand part): $-0.707T_{DB} + 9 - 6 = 0$ **M1 A1**

$T_{DB} = 4.24$ kN (tension)

Answer: DB: 4.24 kN tension **3 marks**

4 (2 marks)

(a) Three independent equilibrium equations (two resolutions and one moment) are available for the part of the framework on one side of the section **E1**

so at most three unknown member forces can be found **E1**

Answer: Only 3 equations available **2 marks**

5 (3 marks)

Take a section through BC, EF, BF and consider the equilibrium of the part containing A, B, E **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{EF} + 10 \times 2 - 15 \times 4 = 0$ **M1 A1**

$T_{EF} = 20$ kN (thrust)

Answer: EF: 20 kN thrust **3 marks**

6 (3 marks)

Take a section through BC, EF, EC and consider the equilibrium of the part containing A, B, E **B1**

Moments about C (left-hand part, anticlockwise positive): $-3T_{EF} + 24 \times 3 - 20 \times 6 = 0$ **M1 A1**

$T_{EF} = 16$ kN (thrust)

Answer: EF: 16 kN thrust **3 marks**

7 (5 marks)

Take a section through BC, EF, BF and consider the equilibrium of the part containing A, B, E **B1**

Moments about B (left-hand part, anticlockwise positive): $-3T_{EF} - 28 \times 3 = 0$ **M1 A1**

$T_{EF} = 28$ kN (thrust)

Resolving vertically (left-hand part): $0.707T_{BF} - 24 + 28 = 0$ **M1 A1**

$T_{BF} = 5.66$ kN (thrust)

Answer: EF: 28 kN thrust; BF: 5.66 kN thrust **5 marks**

8 (5 marks)

Take a section through BC, EF, BF and consider the equilibrium of the part containing A, B, E **B1**

Resolving vertically (left-hand part): $0.707T_{BF} - 12 + 17 = 0$ **M1 A1**

$T_{BF} = 7.07$ kN (thrust)

Moments about F (left-hand part, anticlockwise positive): $2T_{BC} + 12 \times 4 - 17 \times 6 = 0$ **M1 A1**

$T_{BC} = 27$ kN (tension)

Answer: BF: 7.07 kN thrust; BC: 27 kN tension **5 marks**

9 (5 marks)

Take a section through CE, DF, DE and consider the equilibrium of the part containing E, F **B1**

Moments about D (right-hand part, anticlockwise positive): $2T_{CE} - 20 \times 2 - 30 \times 2 = 0$ **M1 A1**

$T_{CE} = 50$ kN (tension)

Resolving vertically (right-hand part): $-0.707T_{DE} - 20 - 30 = 0$ **M1 A1**

$T_{DE} = 70.7$ kN (thrust)

Answer: CE: 50 kN tension; DE: 70.7 kN thrust **5 marks**

10 (5 marks)

Moments about A (anticlockwise positive): $-180 + \frac{16}{2}R_C = 0$ **M1 A1**

$R_C = 22.5$ kN

Take a section through AB, DB, DE and consider the equilibrium of the part containing A, D **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} - 7.50 \times 4 = 0$ **M1 A1**

$T_{DE} = 15$ kN (thrust)

Answer: DE: 15 kN thrust **5 marks**

11 (5 marks)

Take a section through AB, DE, DB and consider the equilibrium of the part containing A, D **B1**

Moments about B (left-hand part, anticlockwise positive): $-2.12T_{DE} - 20 \times 3 + 10 \times 1.50 = 0$ **M1 A1**

$T_{DE} = 21.2$ kN (thrust)

Moments about D (left-hand part, anticlockwise positive): $1.50T_{AB} - 20 \times 1.50 = 0$ **M1 A1**

$T_{AB} = 20$ kN (tension)

Answer: DE: 21.2 kN thrust; AB: 20 kN tension **5 marks**

12 (3 marks)

Take a section through AB, DB, DE and consider the equilibrium of the part containing A, D **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} - 7.32 = 0$ **M1 A1**

$T_{DE} = 3.66$ kN (thrust)

Answer: DE: 3.66 kN thrust **3 marks**

13 (3 marks)

(a) AB and DB both pass through B, so take moments about B **B1**

Answer: B **1 mark**

(b) DB and DE both pass through D, so take moments about D **B1**

Answer: D **1 mark**

(c) Resolve vertically: AB and DE are horizontal so contribute nothing **B1**

Answer: Resolve vertically **1 mark**

14 (5 marks)

Take a section through BC, FG, FC and consider the equilibrium of the part containing A, B, F **B1**

Moments about C (left-hand part, anticlockwise positive): $-3T_{FG} - 35 \times 6 + 30 \times 3 = 0$ **M1 A1**

$T_{FG} = 40$ kN (thrust)

Moments about F (left-hand part, anticlockwise positive): $3T_{BC} - 35 \times 3 = 0$ **M1 A1**

$T_{BC} = 35$ kN (tension)

Answer: FG: 40 kN thrust; BC: 35 kN tension **5 marks**

15 (7 marks)

Take a section through EG, FH, FG and consider the equilibrium of the part containing G, H — **B1**

Moments about F (right-hand part, anticlockwise positive): $2T_{EG} - 18 \times 2 - 12 \times 2 = 0$ — **M1 A1**

$T_{EG} = 30$ kN (tension)

Moments about G (right-hand part, anticlockwise positive): $-2T_{FH} + 15 \times 2 = 0$ — **M1 A1**

$T_{FH} = 15$ kN (tension)

Resolving vertically (right-hand part): $-0.707T_{FG} - 18 - 12 = 0$ — **M1 A1**

$T_{FG} = 42.4$ kN (thrust)

Answer: EG: 30 kN tension; FH: 15 kN tension; FG: 42.4 kN thrust — **7 marks**

16 (7 marks)

Weights of the joints from half the weight of each rod meeting there: A: $\frac{1}{2}(20 + 20) = 20$ N, B: $\frac{1}{2}(20 \times 4) = 40$ N, C: 20 N, D: $\frac{1}{2}(20 \times 3) = 30$ N, E: 30 N — **B1 B1**

By symmetry the reactions at A and C are each $\frac{1}{2}(140) = 70$ N — **B1**

Take a section through AB, DB and DE, left-hand part — **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} + 20 \times 4 - 70 \times 4 + 30 \times 2 = 0$ — **M1 A1**

$T_{DE} = 70$ N (thrust) — **A1**

Answer: DE: 70 N S — **7 marks**

17 (5 marks)

Take a section through BC, EF, EC and consider the equilibrium of the part containing A, B, E — **B1**

Moments about C (left-hand part, anticlockwise positive): $-3T_{EF} - 43.9 - 10.7 = 0$ — **M1 A1**

$T_{EF} = 18.2$ kN (thrust)

Resolving vertically (left-hand part): $-0.707T_{EC} - 20 + 1.79 = 0$ — **M1 A1**

$T_{EC} = 25.8$ kN (thrust)

Answer: EF: 18.2 kN thrust; EC: 25.8 kN thrust — **5 marks**

18 (7 marks)

(a) Section through AB, DB, DE, left-hand part — **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} - 13.5 \times 4 + 12 \times 2 = 0$ — **M1 A1**

$T_{DE} = 15$ kN (thrust)

Answer: DE: 15 kN thrust — **3 marks**

(b) Joint E, horizontally: $-0.707T_{BE} + 0.707T_{EC} + 15 = 0$ — **M1**

Joint E, vertically: $-0.707T_{BE} - 0.707T_{EC} - 18 = 0$ — **M1**

$T_{BE} = 2.12$ kN (thrust), $T_{EC} = 23.3$ kN (thrust) — **A1 A1**

Answer: BE: 2.12 kN thrust; EC: 23.3 kN thrust — **4 marks**

19 (6 marks)

Moments about A: $8R_C = W \times 2 + 2W \times 6$, so $R_C = \frac{14W}{8} = \frac{7W}{4}$ and $R_A = \frac{5W}{4}$ — **M1 A1**

Section through AB, DB, DE; moments about B for the left-hand part: $-2T_{DE} + W \times 2 - \frac{5W}{4} \times 4 = 0$ — **M1 A1**

$T_{DE} = -\frac{3W}{2}$: a thrust of $\frac{3W}{2}$ — **A1**

$\frac{3W}{2} = 30$, so $W = 20$ — **A1**

Answer: W = 20 — **6 marks**

20 (7 marks)

Moments about A (anticlockwise positive): $-72 - 216 - 108 + \frac{24}{2}R_E = 0$ **M1 A1**

$R_E = 33$ kN

Take a section through CD, GH, HC and consider the equilibrium of the part containing D, E, H **B1**

Moments about C (right-hand part, anticlockwise positive): $3T_{GH} + 33 \times 6 - 12 \times 3 = 0$ **M1 A1**

$T_{GH} = 54$ kN (thrust)

Moments about H (right-hand part, anticlockwise positive): $-3T_{CD} + 33 \times 3 = 0$ **M1 A1**

$T_{CD} = 33$ kN (tension)

Answer: GH: 54 kN thrust; CD: 33 kN tension **7 marks**

21 (9 marks)

With W = 1: moments about A give $R_C = \frac{7}{4}$ and $R_A = \frac{5}{4}$ **M1 A1**

Joint A (W = 1): vertically $T_{AD} \sin 45° + \frac{5}{4} = 0$, so $T_{AD} = -1.768$; horizontally $T_{AB} = 1.250$ **M1 A1**

Joint D: vertically and horizontally give $T_{DB} = 0.3536$ and $T_{DE} = -1.500$; joint E (with symmetry) gives $T_{EC} = -2.475$ and $T_{BE} = -0.3536$ **M1 A1**

Largest tension per unit W is 1.750 in BC, so $W \leq \frac{500}{1.750} = 286$ **M1 A1**

Largest thrust per unit W is 2.475 in EC, so $W \leq \frac{300}{2.475} = 121$

The greatest W is the smaller, 121 N (3 s.f., rounded down so that no rod fails) **A1**

Answer: W = 121 N **9 marks**

22 (9 marks)

Joint weights: A 20 N, B 40 N, C 20 N, D 30 N + 40 N = 70 N, E 30 N (total 180 N) **B1**

Moments about A: $8R_C = 40 \times 4 + 70 \times 2 + 30 \times 6 + 20 \times 8 = 640$, so $R_C = 80$ N and $R_A = 100$ N **M1 A1 A1**

Take a section through AB, DB, DE (left-hand part) **B1**

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} + 20 \times 4 - 100 \times 4 + 70 \times 2 = 0$ **M1 A1**

$T_{DE} = 90$ N (thrust)

Moments about D (left-hand part, anticlockwise positive): $2T_{AB} + 20 \times 2 - 100 \times 2 = 0$ **M1**

$T_{AB} = 80$ N (tension) **A1**

Answer: DE: 90 N S; AB: 80 N T **9 marks**

23 (6 marks)

(a) AB passes through B, so its moment about B is zero: T_{AB} cannot appear in this moment equation **E1**

Moments about B would find T_{DE}, not T_{AB}; the point to use is D, where DB and DE meet **E1**

Answer: Moments about B eliminate AB; use D **2 marks**

(b) Moments about D: $2T_{AB} - 13.5 \times 2 = 0$, so $T_{AB} = 13.5$ kN (tension) **M1 A1**

Moments about B: $-2T_{DE} + 12 \times 2 - 13.5 \times 4 = 0$, so $T_{DE} = -15$: a thrust of 15 kN **M1 A1**

Answer: AB: 13.5 kN tension; DE: 15 kN thrust **4 marks**

24 (9 marks)

Moments about A (anticlockwise positive): $-168 - 228 + \frac{18}{2}R_D = 0$ **M1 A1**

$R_D = 44$ kN **A1**

Resolving: horizontal reaction at A is 24 kN (to the left), vertical reaction is 26 kN (upwards) **A1**

Take a section through BC, EF, BF and consider the equilibrium of the part containing A, B, E **B1**

Moments about B (left-hand part, anticlockwise positive): $-3T_{EF} - 72 - 78 = 0$ **M1 A1**

$T_{EF} = 50$ kN (thrust)

Resolving vertically (left-hand part): $0.707T_{BF} - 32 + 26 = 0$ **M1 A1**

$T_{BF} = 8.49$ kN (tension)

Answer: EF: 50 kN thrust; BF: 8.49 kN tension **9 marks**

25 (9 marks)

Moments about A (anticlockwise positive): $-18 - 90 - 60 + \frac{24}{2}R_D = 0$ **M1 A1**

$R_D = 14$ kN **A1**

Resolving: horizontal reaction at A is 0 kN (to the left), vertical reaction is 16 kN (upwards) **A1**

Take a section through FG, CG and CD (right-hand part) **B1**

Moments about C (right-hand part, anticlockwise positive): $2T_{FG} + 14 \times 4 - 6 \times 2 = 0$ **M1 A1**

$T_{FG} = 22$ kN (thrust)

Moments about G (right-hand part, anticlockwise positive): $-2T_{CD} + 14 \times 2 = 0$ **M1**

$T_{CD} = 14$ kN (tension) **A1**

Answer: $R_D = 14$ kN; FG: 22 kN S; CD: 14 kN T **9 marks**

26 (9 marks)

By symmetry $R_A = R_C = \frac{3W}{2}$ **B1**

Section through AB, DB, DE (left-hand part): moments about D: $T_{AB} \times 1.5 - \frac{3W}{2} \times 1.5 = 0$, so **M1 A1**

$T_{AB} = \frac{3W}{2}$ (tension)

Joint A: $T_{AD} \sin 45° + \frac{3W}{2} = 0$, so $T_{AD} = -\frac{3\sqrt{2}W}{2}$, a thrust of $2.121W$ **M1 A1**

The largest tension is $\frac{3W}{2}$ in AB, so $\frac{3W}{2} \leq 600$ gives $W \leq 400$ **M1 A1**

The largest thrust is $2.121W$ in AD, so $2.121W \leq 400$ gives $W \leq 189$ **M1**

Greatest W = 188 N (3 s.f., rounded down so that no rod fails), limited by the thrust in AD and FC **A1**

Answer: W = 188 N **9 marks**

27 (12 marks)

With W = 1: moments about A: $9R_D = 2 \times 3 + 1 \times 6 = 12$, so $R_D = \frac{4}{3}$ and $R_A = \frac{5}{3}$ **M1 A1**

Section through EF, EC, BC (left-hand part, W = 1): **B1**

Moments about C (left-hand part, anticlockwise positive): $-3T_{EF} + 2 \times 3 - 1.67 \times 6 = 0$ **M1**

$T_{EF} = -1.333$ **A1**

Resolving vertically (left-hand part): $-0.707T_{EC} - 2 + 1.67 = 0$ **M1**

$T_{EC} = -0.4714$ **A1**

Moments about E (left-hand part, anticlockwise positive): $3T_{BC} - 1.67 \times 3 = 0$ **M1**

$T_{BC} = 1.667$ **A1**

Joints A, B, F, D give the remaining forces: $T_{AB} = 1.667$, $T_{AE} = -2.357$, $T_{BE} = 0$, $T_{CF} = 0.3333$, **M1**

$T_{FD} = -1.886$, $T_{CD} = 1.333$ (per unit W)

Limits: tension 200 N, thrust 300 N: the smallest limiting W is 120 N, from the rods AB and BC (equal forces, so they fail together) **M1 A1**

Answer: W = 120 N; rods AB and BC fail first **12 marks**

28 (12 marks)

(a) Joint weights: A 30 N, B 60 N, C 30 N, D 45 N, E (45 + P) N **B1 B1**

Moments about A: $8R_C = 60 \times 4 + 45 \times 2 + (45 + P) \times 6 + 30 \times 8$ **M1 A1**

$8R_C = 840 + 6P$, so $R_C = 105 + \frac{3P}{4}$ **A1**

Answer: $R_C = 105 + \frac{3P}{4}$ **5 marks**

(b) $R_A = 210 + P - R_C = 105 + \frac{P}{4}$ **M1 A1**

$105 + \frac{3P}{4} = 2(105 + \frac{P}{4})$ gives $\frac{P}{4} = 105$, $P = 420$ **M1 A1**

Section through AB, DB, DE: moments about B for the left-hand part

Moments about B (left-hand part, anticlockwise positive): $-2T_{DE} + 30 \times 4 - 210 \times 4 + 45 \times 2 = 0$ **M1 A1**

$T_{DE} = 315$ N (thrust) **A1**

Answer: P = 420 N; DE: 315 N thrust **7 marks**

29 (11 marks)

(a) By symmetry the reactions at A and C are each $\frac{W}{2}$; D is at height a tan θ above AB and a horizontally from A; section through AB, DB, DE (left-hand part), moments about D **B1**

$T_{AB} \times a \tan\theta - \frac{W}{2} \times a = 0$ **M1 A1**

$T_{AB} = \frac{W}{(2 \tan\theta)}$, positive so a tension

Answer: $T_{AB} = \frac{W}{(2 \tan\theta)}$, a tension (shown) **3 marks**

(b) DE makes angle θ with the horizontal and acts at D; moment of T_{DE} about B is $-2aT_{DE} \sin\theta$ **M1 A1**

Moments about B: $-2aT_{DE} \sin\theta - \frac{W}{2} \times 2a = 0$ **M1 A1**

$T_{DE} = -\frac{W}{(2 \sin\theta)}$: a thrust of $\frac{W}{(2 \sin\theta)}$

Answer: DE: thrust $\frac{W}{(2 \sin\theta)}$ **4 marks**

(c) $\frac{W}{(2 \sin\theta)} = W$, so $\sin\theta = \frac{1}{2}$, $\theta = 30°$ **M1 A1**

$T_{AB} = \frac{W}{(2 \tan 30°)} = \frac{\sqrt{3}W}{2}$, a tension

Answer: θ = 30°; AB: $\frac{\sqrt{3}W}{2}$ tension **2 marks**

(d) $\frac{W}{(2 \tan\theta)}$ tends to infinity as θ tends to 0 **E1**

so flatter roofs need much stronger tie rods AB (or a smaller load) **E1**

Answer: AB tension increases without bound **2 marks**

30 (12 marks)

(a) Moments about A (anticlockwise positive): $36 - 108 + \frac{18}{2}R_D = 0$ **M1 A1**

$R_D = 8$ kN **A1**

Resolving: horizontal reaction at A is 12 kN (to the right), vertical reaction is 10 kN (upwards) **A1**

Magnitude $\sqrt{12^2 + 10^2} = 15.6$ kN at 39.8° to the horizontal **A1**

Answer: $R_D = 8$ kN; $R_A = 15.6$ kN **5 marks**

(b) Section through EF, EC, BC; left-hand part **B1**

Moments about C (left-hand part, anticlockwise positive): $-3T_{EF} + 12 \times 3 - 60 = 0$ **M1 A1**

$T_{EF} = 8$ kN (thrust) **A1**

Moments about E (left-hand part, anticlockwise positive): $3T_{BC} + 6 = 0$ **M1 A1**

$T_{BC} = 2$ kN (thrust) **A1**

Answer: EF: 8 kN S; BC: 2 kN S **7 marks**