

Linear Momentum

1 (1 mark)

$$p = mv = 4 \times 7 = 28 \text{ N s}$$

Answer: 28 N s

B1

1 mark

2 (1 mark)

$$p = 0.35 \times 12 = 4.2 \text{ N s}$$

Answer: 4.2 N s

B1

1 mark

3 (2 marks)

$$72 \text{ km per hour} = 72 \times \frac{1000}{3600} = 20 \text{ m s}^{-1}$$

M1

$$p = 1500 \times 20 = 30\,000 \text{ N s}$$

A1

Answer: 30 000 N s

2 marks

4 (1 mark)

$$v = \frac{p}{m} = \frac{36}{9} = 4 \text{ m s}^{-1}$$

B1

Answer: 4 m s⁻¹

1 mark

5 (1 mark)

$$p = 3 \times (-5) = -15 \text{ N s, so 15 N s to the left}$$

B1

Answer: 15 N s to the left

1 mark

6 (2 marks)

$$\text{Total} = 2 \times 3 + 5 \times (-2)$$

M1

$$= 6 - 10 = -4 \text{ N s (4 N s to the left)}$$

A1

Answer: -4 N s

2 marks

7 (3 marks)

$$p = 2.5(4\mathbf{i} - 6\mathbf{j}) = (10\mathbf{i} - 15\mathbf{j}) \text{ N s}$$

M1 A1

$$|p| = \sqrt{10^2 + 15^2} = \sqrt{325} = 18.0 \text{ N s}$$

A1

Answer: (10i - 15j) N s, magnitude 18.0 N s

3 marks

8 (3 marks)

$$\text{Take the initial direction as positive. Before: } 0.16 \times 20 = 3.2 \text{ N s}$$

B1

$$\text{After: } 0.16 \times (-30) = -4.8 \text{ N s}$$

B1

$$\text{Change} = -4.8 - 3.2 = -8, \text{ so magnitude } 8 \text{ N s}$$

A1

Answer: 8 N s

3 marks

9 (2 marks)

$$\text{Change} = 6 \times 8 - 6 \times 5 = 6 \times 3$$

M1

$$= 18 \text{ N s}$$

A1

Answer: 18 N s

2 marks

10 (3 marks)

$$(a) p = 90 \times 8 = 720 \text{ N s}$$

B1

Answer: 720 N s

1 mark

$$(b) 60v = 720$$

M1

$$v = 12 \text{ m s}^{-1}$$

A1

Answer: 12 m s⁻¹

2 marks

11 (3 marks)

$$(a) \text{Speed} = \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-1}$$

B1

Answer: 5 m s⁻¹

1 mark

$$(b) p = 0.6(3\mathbf{i} + 4\mathbf{j}) = (1.8\mathbf{i} + 2.4\mathbf{j}) \text{ N s}$$

M1

$$|p| = 0.6 \times 5 = 3 \text{ N s}$$

Answer: (1.8i + 2.4j) N s, 3 N s

A1

2 marks

12 (3 marks)

$$(a) 2 \times 4 + 3 \times (-2) + 5 \times 1$$

$$= 8 - 6 + 5 = 7 \text{ N s}$$

Answer: 7 N s

M1

A1

2 marks

$$(b) v = \frac{7}{10} = 0.7 \text{ m s}^{-1}$$

B1

Answer: 0.7 m s⁻¹

1 mark

13 (4 marks)

$$(a) v = 3 \times 4 - 8 = 4$$

$$p = 2 \times 4 = 8 \text{ N s}$$

Answer: 8 N s

M1

A1

2 marks

$$(b) 3t^2 - 4t = 0, t(3t - 4) = 0$$

$$t = \frac{4}{3} \text{ s}$$

M1

A1

Answer: t = $\frac{4}{3}$ s

2 marks

14 (6 marks)

$$(a) \mathbf{v} = \frac{d\mathbf{r}}{dt} = (3t^2\mathbf{i} + 4t\mathbf{j})$$

$$\text{At } t = 2: \mathbf{v} = (12\mathbf{i} + 8\mathbf{j})$$

$$\mathbf{p} = 1.5\mathbf{v} = (18\mathbf{i} + 12\mathbf{j}) \text{ N s}$$

Answer: (18i + 12j) N s

M1 A1

A1

A1

4 marks

$$(b) |p| = \sqrt{18^2 + 12^2} = \sqrt{468}$$

$$= 21.6 \text{ N s}$$

M1

A1

Answer: 21.6 N s

2 marks

15 (4 marks)

$$(a) \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$= (12t\mathbf{i} + 4\mathbf{j}) \text{ N}$$

Answer: (12ti + 4j) N

$$(b) \mathbf{F} = 36\mathbf{i} + 4\mathbf{j}$$

$$|\mathbf{F}| = \sqrt{36^2 + 4^2} = \sqrt{1312} = 36.2 \text{ N}$$

Answer: 36.2 N

M1

A1

2 marks

M1

A1

2 marks

16 (4 marks)

$$(a) p = mv \text{ so } v = \frac{p}{m}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{p^2}{m^2} = \frac{p^2}{2m}$$

Answer: The kinetic energy of the particle is $\frac{p^2}{2m}$

B1

M1

2 marks

$$(b) 250 = \frac{p^2}{10}$$

$$p^2 = 2500, p = 50 \text{ N s}$$

Answer: 50 N s

M1

A1

2 marks

17 (3 marks)

Let each momentum be p . Using $KE = \frac{p^2}{2m}$

M1

$$KE \text{ of A} = \frac{p^2}{2m}, KE \text{ of B} = \frac{p^2}{6m}$$

A1

$$\frac{KE \text{ of A}}{KE \text{ of B}} = \frac{\frac{p^2}{2m}}{\frac{p^2}{6m}} = 3$$

A1

Answer: The kinetic energy of A is three times the kinetic energy of B

3 marks

18 (3 marks)

Each second 12 kg gains horizontal speed 1.5 from 0

M1

Horizontal momentum gained per second = $12 \times 1.5 = 18 \text{ N s per s}$

A1

Force = rate of change of momentum = 18 N

A1

Answer: 18 N

3 marks

19 (5 marks)

(a) $\mathbf{v} = \frac{1}{4}(14\mathbf{i} + 6\mathbf{j})$

M1

$$= (3.5\mathbf{i} + 1.5\mathbf{j}) \text{ m s}^{-1}$$

A1

Answer: $(3.5\mathbf{i} + 1.5\mathbf{j}) \text{ m s}^{-1}$

2 marks

(b) Initial momentum = $(8\mathbf{i} - 12\mathbf{j})$

B1

$$\text{Change} = (6\mathbf{i} + 18\mathbf{j})$$

M1

$$\text{Magnitude} = \sqrt{360} = 19.0 \text{ N s}$$

A1

Answer: 19.0 N s

3 marks

20 (5 marks)

(a) Horizontal: $2 \times 10 \cos 30^\circ = 17.3 \text{ N s}$

B1

$$\text{Vertical: } 2 \times 10 \sin 30^\circ = 10 \text{ N s}$$

B1

Answer: 17.3 N s horizontal, 10 N s vertical

2 marks

(b) Vertical speed = $10 \sin 30^\circ - 9.8t = 0$

M1 A1

$$t = \frac{5}{9.8} = 0.510 \text{ s}$$

A1

Answer: 0.510 s

3 marks

21 (6 marks)

(a) Momentum is a vector, so directions must be accounted for

E1

P and Q move in opposite directions so the momenta must be subtracted, not added

E1

Answer: Momenta in opposite directions must have opposite signs

2 marks

(b) Taking east as positive: $-6 + 6$

M1

$$= 0 \text{ N s}$$

A1

Answer: 0 N s

2 marks

(c) $5v = 20$

M1

$$v = 4 \text{ m s}^{-1} \text{ due east}$$

A1

Answer: 4 m s⁻¹ due east

2 marks

22 (7 marks)

(a) $p = 0.8 \times 6 = 4.8 \text{ N s}$

B1

Answer: 4.8 N s

1 mark

(b) $4.8e^{-t/2} = 2.4$

M1

$$e^{-t/2} = \frac{1}{2}, \text{ so } -\frac{t}{2} = -\ln 2$$

M1

$$t = 2 \ln 2 = 1.39 \text{ s}$$

A1

Answer: $t = 2 \ln 2 = 1.39 \text{ s}$

3 marks

(c) $p = 4.8e^{-t/2}, \frac{dp}{dt} = -2.4e^{-t/2}$

M1 A1

$$\text{At } t = 2: -2.4e^{-1} = -0.883 \text{ N}$$

A1

Answer: -0.883 N **3 marks****23 (7 marks)****(a)** Total **j** momentum: $2 \times 4 + 3k$ **M1**

$$8 + 3k = 0$$

M1

$$k = -\frac{8}{3}$$

A1**Answer: $k = -\frac{8}{3}$** **3 marks****(b)** **i** momentum: $2 \times 3 + 3 \times (-1) = 3$ **M1**Total momentum = **3i** N s**A1****Answer: **3i** N s****2 marks**

(c) $\text{Speed}^2 = 1 + \frac{64}{9} = \frac{73}{9}$

M1

$$\text{Speed} = \frac{\sqrt{73}}{3} = 2.85 \text{ m s}^{-1}$$

A1**Answer: 2.85 m s^{-1}** **2 marks****24 (6 marks)**

Momentum of P = 12 N s east, momentum of Q = 15 N s north

B1 B1

$$\text{Magnitude} = \sqrt{12^2 + 15^2}$$

M1

$$= \sqrt{369} = 19.2 \text{ N s}$$

A1

$$\text{Angle east of north} = \tan^{-1}\left(\frac{12}{15}\right)$$

M1

$$\text{Bearing} = 038.7^\circ$$

A1**Answer: 19.2 N s on a bearing of 038.7°** **6 marks****25 (6 marks)****(a)** $\text{KE} = \frac{1}{2}mv^2$, so KE is proportional to v^2 **M1** v^2 is multiplied by 1.44, so v is multiplied by 1.2**M1 A1**Momentum is proportional to v : increase of 20%**A1****Answer: 20%****4 marks**

(b) $\text{KE} = \frac{p^2}{2m}$

M1 m multiplied by 4, so KE is multiplied by $\frac{1}{4}$ **A1****Answer: multiplied by $\frac{1}{4}$** **2 marks****26 (7 marks)**Momentum of A = **(6i + 15j)****B1**Momentum of B = **(14 - 6)i + (-10 - 15)j****M1**= **(8i - 25j)** N s**A1**

Velocity of B = $\frac{1}{4}$ **(8i - 25j)**

M1

$$= (2\mathbf{i} - 6.25\mathbf{j}) \text{ m s}^{-1}$$

A1

$$\text{Speed} = \sqrt{2^2 + 6.25^2}$$

M1

$$= 6.56 \text{ m s}^{-1}$$

A1**Answer: 6.56 m s^{-1}** **7 marks****27 (9 marks)****(a)** $\mathbf{p} = 2\mathbf{v} = (8\cos 2t \mathbf{i} + 8\sin 2t \mathbf{j})$ **M1**

$$|\mathbf{p}| = 8\sqrt{\cos^2 2t + \sin^2 2t} = 8 \text{ N s}$$

A1**Answer: 8 N s (constant)****2 marks**

(b) $\mathbf{F} = \frac{d\mathbf{p}}{dt}$

M1

$$= (-16 \sin 2t \mathbf{i} + 16 \cos 2t \mathbf{j}) \text{ N}$$

A1

Answer: $(-16 \sin 2t \mathbf{i} + 16 \cos 2t \mathbf{j}) \text{ N}$ 2 marks

(c) $\mathbf{F} \cdot \mathbf{v} \propto (-16 \sin 2t)(4 \cos 2t) + (16 \cos 2t)(4 \sin 2t)$ M1

$= 0$, so $\mathbf{F}$ is perpendicular to $\mathbf{v}$ A1

Answer: The force is perpendicular to the velocity at every instant 2 marks

(d) $|\mathbf{F}| = 16\sqrt{\sin^2 2t + \cos^2 2t} = 16 \text{ N}$ B1

Answer: 16 N 1 mark

(e) The force is always perpendicular to the direction of motion E1

so it does no work and the kinetic energy (hence speed) does not change E1

Answer: Force does no work 2 marks

28 (9 marks)

(a) In one second a mass k of sand changes horizontal speed from 0 to V M1

Rate of change of horizontal momentum $= kV$ A1

Force on sand $=$ rate of change of momentum $= kV$ A1

Answer: The horizontal force needed to keep the sand moving with the belt is kV 3 marks

(b) The belt exerts kV on the sand, so the sand exerts kV back on the belt, which the motor must overcome M1

Power $=$ force $\times$ speed $= kV \times V = kV^2$ A1

Answer: The power which the motor must supply to keep the belt moving at constant speed is kV^2 2 marks

(c) Rate of KE gain $= \frac{1}{2}kV^2$ M1 A1

Fraction not used $= \frac{\frac{1}{2}kV^2}{kV^2} = \frac{1}{2}$ A1

Answer: $\frac{1}{2}$ 3 marks

(d) It is dissipated as heat and sound by friction as the sand slips on the belt before reaching belt speed E1

Answer: Heat and sound 1 mark

29 (9 marks)

(a) $\mathbf{p} = 2\mathbf{v} = (2(t - 3)\mathbf{i} + 2(t - 4)\mathbf{j})$ M1

$|\mathbf{p}|^2 = 4(t - 3)^2 + 4(t - 4)^2$ M1

$= 4(2t^2 - 14t + 25) = 8t^2 - 56t + 100$ A1

Answer: The square of the magnitude of the momentum of the particle is $(8t^2 - 56t + 100) \text{ N}^2 \text{ s}^2$ 3 marks

(b) $8t^2 - 56t + 100 = 8(t - 3.5)^2 + 2$ M1 A1

Least $|\mathbf{p}|^2 = 2$ when $t = 3.5 \text{ s}$ A1

Least $|\mathbf{p}| = \sqrt{2} = 1.41 \text{ N s}$ A1

Answer: $\sqrt{2} \text{ N s}$ at $t = 3.5 \text{ s}$ 4 marks

(c) At $t = 3.5$: $\mathbf{v} = (0.5\mathbf{i} - 0.5\mathbf{j})$ M1

Direction is 45° below the direction of $\mathbf{i}$ A1

Answer: 45° below $\mathbf{i}$ 2 marks

30 (8 marks)

(a) Volume per second $= 4.0 \times 10^{-4} \times 12$ M1

Mass per second $= 1000 \times 4.8 \times 10^{-3} = 4.8 \text{ kg}$ A1

Answer: 4.8 kg per second 2 marks

(b) Force $=$ rate of change of momentum $= 4.8 \times 12$ M1

$= 57.6 \text{ N}$ A1

Answer: 57.6 N 2 marks

(c) Only the component of velocity normal to the wall is destroyed E1

Normal component $= 12 \cos 60^\circ = 6 \text{ m s}^{-1}$ B1

Force $= 4.8 \times 6 = 28.8 \text{ N}$ A1

Answer: 28.8 N 3 marks

(d) The wall is smooth, so it can exert no force parallel to its surface E1

Answer: Wall is smooth 1 mark

Impulse

1 (1 mark)

$$I = Ft = 12 \times 0.5 = 6 \text{ N s}$$

Answer: 6 N s

B1

1 mark

2 (2 marks)

$$I = mv - mu = 2 \times 8 - 2 \times 3$$

$$= 10 \text{ N s}$$

Answer: 10 N s

M1

A1

2 marks

3 (2 marks)

$$I = mv: 15 = 3v$$

$$v = 5 \text{ m s}^{-1}$$

Answer: 5 m s⁻¹

M1

A1

2 marks

4 (2 marks)

$$\text{Impulse} = 30 \times 0.2 = 6 \text{ N s}$$

$$0.4v = 6, v = 15 \text{ m s}^{-1}$$

Answer: 15 m s⁻¹

M1

A1

2 marks

5 (1 mark)

$$I = 0.5 \times 14 = 7 \text{ N s}$$

Answer: 7 N s

B1

1 mark

6 (2 marks)

$$8 = 4v - 4 \times 6$$

$$v = 8 \text{ m s}^{-1}$$

Answer: 8 m s⁻¹

M1

A1

2 marks

7 (3 marks)

$$\text{Take the initial direction as positive: } I = 0.25(-8) - 0.25(12)$$

$$I = -5, \text{ so magnitude } 5 \text{ N s}$$

Answer: 5 N s

M1 A1

A1

3 marks

8 (3 marks)

$$3\mathbf{v} = 3(2\mathbf{i} + \mathbf{j}) + (6\mathbf{i} - 9\mathbf{j}) = 12\mathbf{i} - 6\mathbf{j}$$

$$\mathbf{v} = (4\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}, \text{ speed} = \sqrt{20} = 4.47 \text{ m s}^{-1}$$

Answer: (4i - 2j) m s⁻¹, 4.47 m s⁻¹

M1 A1

A1

3 marks

9 (4 marks)

$$\text{Impulse} = 3(4\mathbf{i} - 2\mathbf{j}) = (12\mathbf{i} - 6\mathbf{j}) \text{ N s}$$

$$2\mathbf{v} = 12\mathbf{i} - 6\mathbf{j}, \mathbf{v} = 6\mathbf{i} - 3\mathbf{j}$$

$$\text{Speed} = \sqrt{45} = 6.71 \text{ m s}^{-1}$$

Answer: (12i - 6j) N s; 6.71 m s⁻¹

M1 A1

M1

A1

4 marks

10 (5 marks)

$$\text{(a) Take upward as positive: } I = 0.4(4.5) - 0.4(-6)$$

$$I = 4.20 \text{ N s upwards}$$

Answer: 4.20 N s upwards

M1 A1

A1

3 marks

$$\text{(b) } F = \frac{I}{t} = \frac{4.2}{0.02}$$

$$= 210 \text{ N}$$

Answer: 210 N

M1

A1

2 marks

11 (5 marks)

(a) Component of velocity along the impulse = $\sqrt{6.5^2 - 2.5^2} = 6 \text{ m s}^{-1}$

M1 A1

$$I = 3 \times 6 = 18 \text{ N s}$$

A1

Answer: I = 18

3 marks

(b) $\tan \theta = \frac{6}{2.5}$

M1

$$\theta = 67.4^\circ$$

A1

Answer: 67.4°

2 marks

12 (4 marks)

(a) $I = mv = 1200 \times 15$

M1

$$= 18\,000 \text{ N s}$$

A1

Answer: 18 000 N s

2 marks

(b) $3000t = 18\,000$

M1

$$t = 6 \text{ s}$$

A1

Answer: 6 s

2 marks

13 (5 marks)

(a) $I = \int_0^4 6t \, dt$

M1

$$= [3t^2]_0^4 = 48 \text{ N s}$$

M1 A1

Answer: 48 N s

3 marks

(b) $3v = 48$

M1

$$v = 16 \text{ m s}^{-1}$$

A1

Answer: 16 m s⁻¹

2 marks

14 (6 marks)

(a) $12t - 3t^2 = 3t(4 - t)$

M1

$$t \geq 0 \text{ and } 4 - t \geq 0 \text{ for } 0 \leq t \leq 4, \text{ so } F \geq 0$$

E1

Answer: The force is never negative during this time

2 marks

(b) $I = \int_0^4 (12t - 3t^2) \, dt$

M1

$$= [6t^2 - t^3]_0^4 = 96 - 64 = 32 \text{ N s}$$

A1

$$4v - 4 \times 2 = 32$$

M1

$$v = 10 \text{ m s}^{-1}$$

A1

Answer: 10 m s⁻¹

4 marks

15 (5 marks)

(a) Impulse = area under graph

M1

$$= \frac{1}{2} \times 0.3 \times 20 = 3 \text{ N s}$$

A1

Answer: 3 N s

2 marks

(b) $0.15v = 3$

M1

$$v = 20 \text{ m s}^{-1}$$

A1

Answer: 20 m s⁻¹

2 marks

(c) Average force = $\frac{3}{0.3} = 10 \text{ N}$

B1

Answer: 10 N

1 mark

16 (5 marks)

(a) Take the original direction as the x direction: before (30, 0), after (20cos100°, 20sin100°) = (-3.473, 19.70)

B1

$$\text{Change in velocity} = (-33.47, 19.70), \text{ magnitude } 38.84 \text{ m s}^{-1}$$

M1

$$I = 0.16 \times 38.84 = 6.21 \text{ N s}$$

A1

Answer: 6.21 N s

3 marks

(b) $\tan \alpha = \frac{19.70}{33.47}$

M1

$$\alpha = 30.5^\circ$$

Answer: 30.5°

A1

2 marks

17 (7 marks)

$$(a) \mathbf{I} = 2(-\mathbf{i} + 2\mathbf{j}) - 2(3\mathbf{i} + 4\mathbf{j})$$

M1 A1

$$= (-8\mathbf{i} - 4\mathbf{j}) \text{ N s}$$

A1

Answer: (-8i - 4j) N s

3 marks

$$(b) |\mathbf{I}| = \sqrt{8^2 + 4^2} = \sqrt{80}$$

M1

$$= 8.94 \text{ N s}$$

A1

Answer: 8.94 N s

2 marks

$$(c) \mathbf{I} \cdot \mathbf{u} = (-8)(3) + (-4)(4) = -40, |\mathbf{I}||\mathbf{u}| = \sqrt{80} \times 5$$

M1

$$\cos \theta = \frac{-40}{(5\sqrt{80})} \text{ so } \theta = 153.4^\circ$$

A1

Answer: 153.4°

2 marks

18 (5 marks)

$$\text{Impulse} = 12 \times \frac{1}{5}(3\mathbf{i} + 4\mathbf{j}) = (7.2\mathbf{i} + 9.6\mathbf{j})$$

M1 A1

$$2\mathbf{v} = 10\mathbf{i} + (7.2\mathbf{i} + 9.6\mathbf{j})$$

M1

$$\mathbf{v} = (8.6\mathbf{i} + 4.8\mathbf{j})$$

A1

$$\text{Speed} = \sqrt{8.6^2 + 4.8^2} = 9.85 \text{ m s}^{-1}$$

A1

Answer: 9.85 m s⁻¹

5 marks

19 (5 marks)

$$\text{Speed before impact: } v^2 = 2 \times 9.8 \times 4.9, v = 9.8 \text{ m s}^{-1}$$

M1 A1

$$\text{Speed after impact: } v^2 = 2 \times 9.8 \times 2.5, v = 7 \text{ m s}^{-1}$$

M1 A1

$$\mathbf{I} = 0.3(7 + 9.8) = 5.04 \text{ N s}$$

A1

Answer: 5.04 N s

5 marks

20 (5 marks)

$$(a) \mathbf{I} = 0.02 \times 300$$

M1

$$= 6 \text{ N s}$$

A1

Answer: 6 N s

2 marks

$$(b) 0 = 300^2 - 2a(0.15), a = 300\,000 \text{ m s}^{-2}$$

M1 A1

$$F = 0.02 \times 300\,000 = 6000 \text{ N}$$

A1

Answer: 6000 N

3 marks

21 (6 marks)

$$(a) \text{ Let } J \text{ be the impulsive tension. A: } 2v - 18 = -J$$

M1 A1

$$\text{B: } 3v = J$$

M1

$$5v = 18, v = 3.6 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 3.6 \text{ m s}^{-1}$$

4 marks

$$(b) J = 3v = 3 \times 3.6$$

M1

$$= 10.8 \text{ N s}$$

A1

Answer: 10.8 N s

2 marks

22 (8 marks)

$$(a) 2\mathbf{v} = 2(\mathbf{i} - 2\mathbf{j}) + (4\mathbf{i} + 3\mathbf{j}) = 6\mathbf{i} - \mathbf{j}$$

M1

$$\mathbf{v} = (3\mathbf{i} - 0.5\mathbf{j}) \text{ m s}^{-1}$$

A1

$$\text{Answer: } (3\mathbf{i} - 0.5\mathbf{j}) \text{ m s}^{-1}$$

2 marks

$$(b) \mathbf{J} = 2(6\mathbf{i} + \mathbf{j}) - 2(3\mathbf{i} - 0.5\mathbf{j})$$

M1 A1

$$= (6\mathbf{i} + 3\mathbf{j}) \text{ N s}$$

A1

Answer: (6i + 3j) N s

3 marks

$$(c) \text{ Total} = (4 + 6)\mathbf{i} + (3 + 3)\mathbf{j} = 10\mathbf{i} + 6\mathbf{j}$$

M1

$$|\text{total}| = \sqrt{136} = 11.7 \text{ N s}$$

A1

$$\text{Angle} = \tan^{-1}\left(\frac{6}{10}\right) = 31.0^\circ$$

A1

Answer: 11.7 N s, 31.0° to i

3 marks

23 (10 marks)

(a) $\cos \theta = \frac{1}{2}$, so the string makes 60° with AB

M1 A1

Along the string: $10 \sin 60^\circ = 5\sqrt{3} = 8.66 \text{ m s}^{-1}$; perpendicular: $10 \cos 60^\circ = 5 \text{ m s}^{-1}$

A1

Answer: Along the string $5\sqrt{3} = 8.66 \text{ m s}^{-1}$, perpendicular 5 m s^{-1}

3 marks

(b) Perpendicular component of B is unchanged

B1

Common speed w along the string: $3w = J$ and $2(5\sqrt{3} - w) = J$

M1 A1

$5w = 10\sqrt{3}$, $w = 2\sqrt{3} = 3.46 \text{ m s}^{-1}$, $J = 6\sqrt{3} = 10.4 \text{ N s}$

A1

Speed of B = $\sqrt{(2\sqrt{3})^2 + 5^2} = \sqrt{37} = 6.08 \text{ m s}^{-1}$

A1

Answer: J = 10.4 N s, speed of B = 6.08 m s⁻¹

5 marks

(c) Before: $\frac{1}{2}(2)(100) = 100 \text{ J}$; after: $\frac{1}{2}(3)(12) + \frac{1}{2}(2)(37) = 55 \text{ J}$

M1

Loss = 45 J

A1

Answer: 45 J

2 marks

24 (7 marks)

(a) New velocity = $6\mathbf{i} + 2\lambda(-\mathbf{i} + \mathbf{j}) = (6 - 2\lambda)\mathbf{i} + 2\lambda\mathbf{j}$

M1 A1

$(6 - 2\lambda)^2 + (2\lambda)^2 = 20$, so $8\lambda^2 - 24\lambda + 16 = 0$

M1

$\lambda^2 - 3\lambda + 2 = 0$, $\lambda = 1$ or $\lambda = 2$

A1

Answer: $\lambda = 1$ or $\lambda = 2$

4 marks

(b) $\lambda = 1$: new velocity (4, 2), angle with $\mathbf{i} = \arctan\left(\frac{2}{4}\right) = 26.6^\circ$

M1 A1

$\lambda = 2$: new velocity (2, 4), angle with $\mathbf{i} = \arctan 2 = 63.4^\circ$

A1

Answer: 26.6° ($\lambda = 1$) and 63.4° ($\lambda = 2$)

3 marks

25 (6 marks)

(a) The velocity reverses, so the final velocity is -6 (not $+6$) relative to the initial direction

E1

Change in momentum is $0.5(6 + 8)$, the speeds should be added

E1

Answer: Velocity reverses; speeds add

2 marks

(b) $I = 0.5(6 + 8)$

M1

$= 7 \text{ N s}$

A1

Answer: 7 N s

2 marks

(c) $F = \frac{7}{0.01}$

M1

$= 700 \text{ N}$

A1

Answer: 700 N

2 marks

26 (8 marks)

(a) $I = 0.4(20 + 10)$

M1

$= 12 \text{ N s}$

A1

Answer: 12 N s

2 marks

(b) Area = $\frac{1}{2}(0.02)F + 0.06F + \frac{1}{2}(0.02)F$

M1

$= 0.01F + 0.06F + 0.01F = 0.08F$

A1

$0.08F = 12$

M1

$F = 150 \text{ N}$

A1

Answer: F = 150

4 marks

(c) $\frac{12}{0.1}$

M1

$= 120 \text{ N}$

A1

Answer: 120 N

2 marks

27 (10 marks)

(a) $v^2 = 2 \times 9.8 \times 0.9$

M1

$v = 4.2 \text{ m s}^{-1}$

A1

Answer: 4.2 m s^{-1}

2 marks

(b) Impulsive tension J: Q: $3v - 3 \times 4.2 = -J$, P: $2v = J$

M1 A1

$5v = 12.6$, $v = 2.52 \text{ m s}^{-1}$

A1

Answer: 2.52 m s^{-1}

3 marks

(c) $J = 2v = 2 \times 2.52$

M1

$= 5.04 \text{ N s}$

A1

Answer: 5.04 N s

2 marks

(d) After the jerk: $a = \frac{(3 - 2)(9.8)}{5} = 1.96 \text{ m s}^{-2}$

M1 A1

$v^2 = 2.52^2 + 2(1.96)(1.6)$, $v = 3.55 \text{ m s}^{-1}$

A1

Answer: 3.55 m s^{-1}

3 marks

28 (10 marks)

(a) $I = \int_0^6 kt(6 - t) dt$

M1

$= k[3t^2 - \frac{t^3}{3}]_0^6$

M1

$= k(108 - 72) = 36k$

A1

Answer: The magnitude of the impulse of the force is $36k$ newton seconds

3 marks

(b) Take the direction of the force as positive: initial velocity -2

B1

$36k = 4(5) - 4(-2) = 28$

M1

$k = \frac{7}{9}$

A1

Answer: $k = \frac{7}{9}$

3 marks

(c) At rest when the impulse so far equals $4 \times 2 = 8 \text{ N s}$

B1

$\frac{7}{9}(3t^2 - \frac{t^3}{3}) = 8$

M1

$t^3 - 9t^2 + \frac{216}{7} = 0$, so $t = 2.12$

M1 A1

Answer: 2.12 s

4 marks

29 (9 marks)

(a) Horizontal component of impulse $= 20\cos 30^\circ$

M1 A1

$2v = 10\sqrt{3}$, $v = 8.66 \text{ m s}^{-1}$

A1

Answer: 8.66 m s^{-1}

3 marks

(b) Vertical momentum remains zero

B1

Impulse from table $= 20\sin 30^\circ$

M1

$= 10 \text{ N s}$, acting upwards

A1

Answer: 10 N s upwards

3 marks

(c) Vertical component 10 N s upwards, nothing prevents it moving up, so the particle leaves the table

E1

Horizontal speed $= 8.66 \text{ m s}^{-1}$ as before

B1

Vertical speed $= \frac{10}{2} = 5 \text{ m s}^{-1}$, so initial speed $= \sqrt{(5\sqrt{3})^2 + 5^2} = 10 \text{ m s}^{-1}$

B1

Answer: Leaves the table, 10 m s^{-1}

3 marks

30 (9 marks)

(a) $5 = 0.4v$

M1

$v = 12.5 \text{ m s}^{-1}$

A1

Answer: 12.5 m s^{-1}

2 marks

(b) Time of flight $T = \frac{2v \sin \theta}{g}$, so $R = v \cos \theta \times T = \frac{v^2 \sin 2\theta}{g}$

M1 A1

$$10 = \frac{156.25 \sin 2\theta}{9.8}, \text{ so } \sin 2\theta = \frac{98}{156.25} = 0.627$$

A1

Answer: $\sin 2\theta = 0.627$ (3 s.f.)

3 marks

(c) $2\theta = 38.84^\circ$ or 141.2°

M1

$\theta = 19.42^\circ$ or 70.58°

A1

$T = \frac{2 \times 12.5 \sin \theta}{9.8}$: 0.848 s and 2.41 s

M1 A1

Answer: $\theta = 19.42^\circ$ (T = 0.848 s) or $\theta = 70.58^\circ$ (T = 2.41 s)

4 marks

Conservation of Momentum

1 (2 marks)

$$3 \times 4 = (3 + 2)v$$

M1

$$v = 2.4 \text{ m s}^{-1}$$

A1

Answer: 2.4 m s^{-1}

2 marks

2 (3 marks)

Take right as positive: $2 \times 5 + 3 \times (-5) = 5v$

M1 A1

$v = -1$, so 1 m s^{-1} to the left

A1

Answer: 1 m s^{-1} to the left

3 marks

3 (2 marks)

$$0 = 0.05 \times 400 + 4V$$

M1

$V = -5$, so recoil speed 5 m s^{-1}

A1

Answer: 5 m s^{-1}

2 marks

4 (2 marks)

$$12\,000 \times 3 = 20\,000v$$

M1

$$v = 1.8 \text{ m s}^{-1}$$

A1

Answer: 1.8 m s^{-1}

2 marks

5 (2 marks)

$$2mu = 3mv$$

M1

$$v = \frac{2u}{3}$$

A1

Answer: $\frac{2u}{3}$

2 marks

6 (3 marks)

$$4 \times 3 + 2 \times (-2) = 4 \times 1 + 2v$$

M1 A1

$v = 2 \text{ m s}^{-1}$ in the original direction of A

A1

Answer: 2 m s^{-1}

3 marks

7 (2 marks)

$$0 = 2 \times 9 + 60V$$

M1

$V = -0.3$, speed 0.3 m s^{-1}

A1

Answer: 0.3 m s^{-1}

2 marks

8 (5 marks)

(a) $800 \times 5 + 1200 \times 2 = 2000v$

M1

$$v = 3.2 \text{ m s}^{-1}$$

A1

Answer: 3.2 m s^{-1}

2 marks

(b) KE before = $\frac{1}{2}(800)(5^2) + \frac{1}{2}(1200)(2^2) = 12\,400 \text{ J}$

M1

KE after = $\frac{1}{2}(2000)(3.2^2) = 10\,240 \text{ J}$

M1

Loss = 2160 J

A1

Answer: 2160 J**3 marks****9 (3 marks)**

Momentum before = 0

B1

$$0 = 3 \times 20 + 5V$$

M1
 $V = -12$, speed 12 m s^{-1} in the opposite direction
A1**Answer: 12 m s^{-1}** **3 marks****10 (2 marks)**

$$0.02 \times 250 = (0.02 + 4.98)v$$

M1

$$v = 1 \text{ m s}^{-1}$$

A1**Answer: 1 m s^{-1}** **2 marks****11 (3 marks)**

$$2 \times 6 = (2 + m) \times 4$$

M1 A1

$$m = 1$$

A1**Answer: $m = 1$** **3 marks****12 (3 marks)**Take A's direction as positive: $3 \times 4 - 5 \times 2 = 8v$ **M1 A1**
 $v = 0.25 \text{ m s}^{-1}$ in the original direction of A
A1**Answer: 0.25 m s^{-1} in the direction of A****3 marks****13 (3 marks)**

$$4 \times 5 + 6 \times 1 = 4 \times 2 + 6v$$

M1 A1

$$v = 3 \text{ m s}^{-1}$$

A1**Answer: 3 m s^{-1}** **3 marks****14 (6 marks)**

$$(a) 2(3i + 2j) + 3(-2i + 4j) = 5v$$

M1

$$16j = 5v$$

A1

$$v = 3.2j \text{ m s}^{-1}$$

A1**Answer: $3.2j \text{ m s}^{-1}$** **3 marks**

$$(b) \text{ KE before} = \frac{1}{2}(2)(13) + \frac{1}{2}(3)(20) = 43 \text{ J}$$

M1

$$\text{KE after} = \frac{1}{2}(5)(3.2^2) = 25.6 \text{ J}$$

M1

$$\text{Loss} = 17.4 \text{ J}$$

A1**Answer: 17.4 J****3 marks****15 (7 marks)**

$$(a) 2000 \times 6 = 5000v$$

M1

$$v = 2.4 \text{ m s}^{-1}$$

A1**Answer: 2.4 m s^{-1}** **2 marks**

$$(b) 5000 \times 2.4 = 10\,000w$$

M1

$$w = 1.2 \text{ m s}^{-1}$$

A1**Answer: 1.2 m s^{-1}** **2 marks**

$$(c) \text{ Initial KE} = \frac{1}{2}(2000)(36) = 36\,000 \text{ J}$$

M1

$$\text{Final KE} = \frac{1}{2}(10\,000)(1.44) = 7200 \text{ J}$$

M1

$$\text{Loss} = 28\,800 \text{ J}$$

A1**Answer: 28 800 J****3 marks****16 (5 marks)**Let the boat's speed be V backwards; the man's speed relative to the water is $(4 - V)$ forwards**B1 M1**

$$0 = 70(4 - V) - 210V$$

M1 A1

$$280 = 280V, \text{ so } V = 1 \text{ m s}^{-1}$$

A1**Answer: Boat moves backwards at 1 m s^{-1} (man at 3 m s^{-1})****5 marks**

17 (6 marks)

(a) $6 \times 4 = 2 \times 10 + 4v$

M1 A1

$v = 1 \text{ m s}^{-1}$ in the original direction

A1

Answer: 1 m s^{-1} in the original direction

3 marks

(b) KE before = $\frac{1}{2}(6)(16) = 48 \text{ J}$

M1

KE after = $\frac{1}{2}(2)(100) + \frac{1}{2}(4)(1) = 102 \text{ J}$

M1

Gain = 54 J

A1

Answer: 54 J

3 marks

18 (4 marks)

Horizontal momentum is conserved (the ground exerts an impulsive vertical force)

E1

$0 = 5 \times 80 \cos 30^\circ - 200V$

M1 A1

$V = 1.73 \text{ m s}^{-1}$

A1

Answer: 1.73 m s^{-1}

4 marks

19 (6 marks)

(a) Take A's original direction as positive: $m(3u) + 2m(-u) = m(-u) + 2mv$

M1 A1

$3u - 2u = -u + 2v$, so $v = u$ in the original direction of A

A1

Answer: u in the direction of A's original motion

3 marks

(b) KE before = $\frac{1}{2}m(9u^2) + \frac{1}{2}(2m)(u^2) = \frac{11}{2}mu^2$

M1

KE after = $\frac{1}{2}mu^2 + \frac{1}{2}(2m)u^2 = \frac{3}{2}mu^2$

M1

Loss = $4mu^2$, fraction = $\frac{4mu^2}{\frac{11}{2}mu^2} = \frac{8}{11}$

A1

Answer: The fraction of the total kinetic energy lost in the collision is $\frac{8}{11}$

3 marks

20 (6 marks)

After the first throw: $0 = 5 \times 6 + 105V_1$

M1 A1

$V_1 = -\frac{2}{7}$ (backwards)

A1

Second throw: $105V_1 = 5 \times 6 + 100V_2$

M1 A1

$-30 = 30 + 100V_2$, $V_2 = -0.6$, speed 0.6 m s^{-1}

A1

Answer: 0.6 m s^{-1}

6 marks

21 (9 marks)

(a) $m(5u) = m(-u) + 2mv$

M1 A1

$v = 3u$

A1

Answer: $3u$

3 marks

(b) $2m(3u) = 5mV$

M1

$V = \frac{6u}{5}$

A1

Answer: $\frac{6u}{5}$

2 marks

(c) Initial KE = $\frac{1}{2}m(25u^2) = 12.5mu^2$

B1

Final KE = $\frac{1}{2}mu^2 + \frac{1}{2}(5m)(\frac{6u}{5})^2 = 4.1mu^2$

M1 A1

Fraction lost = $\frac{8.4}{12.5} = 0.672$

A1

Answer: 0.672

4 marks

22 (8 marks)

(a) Take the direction of A as positive. Impulse on B = $m(3v - (-u)) = \frac{5}{2}mu$ M1 A1

$$3v + u = \frac{5}{2}u, \text{ so } v = \frac{u}{2} \quad \text{A1}$$

Answer: $v = \frac{u}{2}$ 3 marks

(b) Momentum: $2m(ku) - mu = 2mV_A + m(3v)$, with B's velocity $3v = \frac{3u}{2}$ M1

$$V_A = \pm \frac{u}{2}: 2k - 1 = \pm 1 + \frac{3}{2}, \text{ so } 2k = \frac{5}{2} \pm 1 \quad \text{M1}$$

$$k = \frac{7}{4} \text{ or } k = \frac{3}{4} \quad \text{A1}$$

Answer: $k = \frac{7}{4}$ or $k = \frac{3}{4}$ 3 marks

(c) $k = \frac{3}{4}$: speed of approach = $\frac{3u}{4} + u = \frac{7u}{4}$, A has velocity $-\frac{u}{2}$, B has $\frac{3u}{2}$, so speed of separation = $2u$ M1

$$e = 2u \div \frac{7u}{4} = \frac{8}{7} > 1, \text{ which is impossible, so } k = \frac{3}{4} \text{ is rejected} \quad \text{E1}$$

Answer: $k = \frac{3}{4}$ is not possible, because it gives $e = \frac{8}{7} > 1$ 2 marks

23 (7 marks)

(a) $6(4\mathbf{i} + 3\mathbf{j}) = 2(10\mathbf{i} - 3\mathbf{j}) + 4\mathbf{v}$ M1 A1

$$\mathbf{v} = (\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1} \quad \text{A1}$$

Answer: $(\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$ 3 marks

(b) $\sqrt{1^2 + 6^2} = \sqrt{37} = 6.08 \text{ m s}^{-1}$ B1

Answer: 6.08 m s^{-1} 1 mark

(c) KE before = $\frac{1}{2}(6)(25) = 75 \text{ J}$ M1

KE after = $\frac{1}{2}(2)(109) + \frac{1}{2}(4)(37) = 183 \text{ J}$ M1

Gain = 108 J A1

Answer: 108 J 3 marks

24 (9 marks)

(a) Let the plank move at V backwards: man's speed over the ice = $1.5 - V$ B1

Momentum conserved: $0 = 60(1.5 - V) - 40V$ M1 A1

$$V = 0.9 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 0.9 m s^{-1} 4 marks

(b) $1.5 - 0.9 = 0.6 \text{ m s}^{-1}$ B1

Answer: 0.6 m s^{-1} 1 mark

(c) Time = $\frac{4}{1.5} = \frac{8}{3} \text{ s}$ M1

$$\text{Distance} = 0.9 \times \frac{8}{3} \quad \text{M1}$$

$$= 2.4 \text{ m} \quad \text{A1}$$

Answer: 2.4 m 3 marks

(d) Total momentum is zero, so when the man stops relative to the plank both are at rest E1

Answer: Both come to rest 1 mark

25 (7 marks)

(a) Momentum is conserved in any collision with no external impulse E1

but KE is only conserved if the collision is perfectly elastic: when particles coalesce KE is lost E1

Answer: KE is not conserved when particles coalesce 2 marks

(b) $3 \times 4 = 4v$ M1

$$v = 3 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 3 m s^{-1} 2 marks

(c) KE before = 24 J , KE after = $\frac{1}{2}(4)(9) = 18 \text{ J}$ M1

$$\text{Loss} = 6 \text{ J}$$

A1

$$\text{Percentage} = \frac{6}{24} \times 100 = 25\%$$

A1

Answer: 6 J, 25%

3 marks

26 (8 marks)

$$(a) 0.03 \times 400 = 0.03 \times 100 + 3V$$

M1 A1

$$V = 3 \text{ m s}^{-1}$$

A1

Answer: 3 m s⁻¹

3 marks

$$(b) \text{ KE before} = \frac{1}{2}(0.03)(400^2) = 2400 \text{ J}$$

M1

$$\text{KE after} = \frac{1}{2}(0.03)(100^2) + \frac{1}{2}(3)(3^2) = 163.5 \text{ J}$$

M1

$$\text{Loss} = 2236.5 \text{ J}$$

A1

Answer: 2236.5 J

3 marks

$$(c) \text{ Impulse on block} = 3 \times 3 = 9 \text{ N s}$$

M1

$$F = \frac{9}{0.0005} = 18\,000 \text{ N}$$

A1

Answer: 18 000 N

2 marks

27 (11 marks)

$$(a) v^2 = 2 \times 9.8 \times 2$$

M1

$$v = 6.26 \text{ m s}^{-1}$$

A1

Answer: 6.26 m s⁻¹

2 marks

$$(b) 500 \times 6.261 = 800V$$

M1

$$V = 3.91 \text{ m s}^{-1}$$

A1

Answer: 3.91 m s⁻¹

2 marks

$$(c) \text{ KE before} = \frac{1}{2}(500)(39.2) = 9800 \text{ J}$$

M1

$$\text{KE after} = \frac{1}{2}(800)(15.3125) = 6125 \text{ J}$$

M1

$$\text{Loss} = 3675 \text{ J}$$

A1

Answer: 3675 J

3 marks

$$(d) 0 = V^2 - 2a(0.1) \text{ with } V^2 = 15.3125, a = 76.5625 \text{ m s}^{-2}$$

M1 A1

$$R - 800 \times 9.8 = 800a$$

M1

$$R = 69090 \text{ N}$$

A1

Answer: R = 69090 N

4 marks

28 (12 marks)

$$(a) 6(6\mathbf{i}) = 1(20\mathbf{i} + 10\mathbf{j}) + 2(-5\mathbf{i} + 8\mathbf{j}) + 3\mathbf{v}$$

M1 A1

$$36\mathbf{i} = 10\mathbf{i} + 26\mathbf{j} + 3\mathbf{v}$$

M1

$$\mathbf{v} = \left(\frac{26}{3}\mathbf{i} - \frac{26}{3}\mathbf{j}\right) \text{ m s}^{-1}$$

A1

Answer: $\frac{26}{3}(\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$

4 marks

$$(b) \text{ Speed} = \frac{26}{3}\sqrt{2} = 12.3 \text{ m s}^{-1}$$

M1 A1

$$\text{Angle} = 45^\circ \text{ (below the direction of } \mathbf{i} \text{)}$$

A1

Answer: 12.3 m s⁻¹ at 45° to i

3 marks

$$(c) \text{ KE before} = \frac{1}{2}(6)(36) = 108 \text{ J}$$

B1

$$\text{KE of A} = \frac{1}{2}(1)(500) = 250 \text{ J, KE of B} = \frac{1}{2}(2)(89) = 89 \text{ J}$$

M1 A1

$$\text{KE of C} = \frac{1}{2}(3)(2)\left(\frac{26}{3}\right)^2 = 225.3 \text{ J}$$

M1

$$\text{Gain} = 250 + 89 + 225.3 - 108 = 456.3 \text{ J}$$

A1

Answer: 456.3 J

5 marks

29 (10 marks)

(a) $m(4u) = (m + km)V$

M1

$$V = \frac{4u}{(1 + k)}$$

A1

Answer: The speed of the combined particle is $\frac{4u}{(1 + k)}$

2 marks

(b) KE before = $\frac{1}{2}m(16u^2) = 8mu^2$

B1

KE after = $\frac{1}{2}(1 + k)m\frac{16u^2}{(1 + k)^2} = \frac{8mu^2}{(1 + k)}$

M1 A1

Fraction lost = $1 - \frac{1}{(1 + k)} = \frac{k}{(1 + k)}$

A1

Answer: The fraction of the initial kinetic energy that is lost is $\frac{k}{(1 + k)}$

4 marks

(c) $\frac{k}{(1 + k)} = 0.6$

M1

$k = 1.5$

A1

Answer: $k = 1.5$

2 marks

(d) $\frac{k}{(1 + k)} < 1$ for all $k > 0$ (the particles' final KE cannot be negative)

E1

The fraction tends to 1; almost all of the kinetic energy is lost

E1

Answer: Fraction tends to 1 but never exceeds it

2 marks

30 (9 marks)

(a) Total mass after r couplings = $(r + 1)m$

B1

Momentum is conserved throughout: $mu = (r + 1)mv$

M1

$$v = \frac{u}{(r + 1)}$$

A1

Answer: After the moving truck has coupled to r of the stationary trucks, the speed of the coupled trucks is $\frac{u}{(r + 1)}$

(b) Final speed = $\frac{u}{(n + 1)}$, mass $(n + 1)m$

M1

KE = $\frac{1}{2}(n + 1)m\frac{u^2}{(n + 1)^2} = \frac{mu^2}{2(n + 1)}$

A1

Answer: $\frac{mu^2}{2(n + 1)}$

2 marks

(c) Fraction remaining = $\frac{1}{(n + 1)}$, since the initial KE is $\frac{1}{2}mu^2$

M1

Fraction lost = $1 - \frac{1}{(n + 1)} = \frac{n}{(n + 1)}$

A1

Answer: The fraction of the initial kinetic energy that is lost is $\frac{n}{(n + 1)}$

2 marks

(d) $\frac{n}{(n + 1)} > 0.95$, so $n > 19$

M1

Least $n = 20$

A1

Answer: $n = 20$

2 marks

Direct Collisions**1 (1 mark)**

Speed of separation = $e \times \text{speed of approach} = 0.5 \times 8 = 4 \text{ m s}^{-1}$

B1

Answer: 4 m s^{-1}

1 mark

2 (2 marks)

Speed of approach = $6 - 2 = 4 \text{ m s}^{-1}$

M1

Speed of separation = $\frac{1}{4} \times 4 = 1 \text{ m s}^{-1}$

A1

Answer: 1 m s^{-1}

2 marks

3 (2 marks)

$$\text{Speed of approach} = 5 + 3 = 8 \text{ m s}^{-1}$$

B1

$$e = \frac{6}{8} = 0.75$$

A1

Answer: 0.75

2 marks

4 (3 marks)

$$\text{Equal masses: } 6m = m(v_A + v_B), \text{ so } v_A + v_B = 6$$

M1

$$v_B - v_A = 1 \times 6 = 6$$

M1

$$v_A = 0, v_B = 6 \text{ m s}^{-1}$$

A1

Answer: A at rest, B moves at 6 m s⁻¹

3 marks

5 (2 marks)

$$e = 0 \text{ means the speed of separation is zero, so both move with the same velocity } v$$

M1

$$mu = 2mv, v = \frac{u}{2}$$

A1

Answer: both $\frac{u}{2}$

2 marks

6 (2 marks)

$$\text{Speed of approach} = 4, \text{ speed of separation} = 3 - 1 = 2$$

M1

$$e = \frac{2}{4} = 0.5$$

A1

Answer: 0.5

2 marks

7 (4 marks)

$$\text{Momentum: } 2 \times 6 = 2v_A + v_B$$

M1

$$\text{Restitution: } v_B - v_A = 0.5 \times 6 = 3$$

M1

$$12 = 3v_A + 3, \text{ so } v_A = 3 \text{ m s}^{-1}$$

A1

$$v_B = 6 \text{ m s}^{-1}$$

A1

Answer: A: 3 m s⁻¹, B: 6 m s⁻¹

4 marks

8 (4 marks)

$$\text{Take the direction of A as positive: } 2(5) + 3(-2) = 2v_A + 3v_B, \text{ so } 2v_A + 3v_B = 4$$

M1

$$v_B - v_A = 0.6 \times 7 = 4.2$$

M1

$$v_A = -1.72, v_B = 2.48$$

A1

$$\text{Impulse} = 3(2.48 - (-2)) = 13.44 \text{ N s}$$

A1

Answer: 13.44 N s

4 marks

9 (5 marks)

$$\text{Momentum: } 4 \times 6 + 2 \times (-3) = 4v_A + 2v_B, \text{ so } 18 = 4v_A + 2v_B$$

M1 A1

$$\text{Restitution: } v_B - v_A = 0.5(6 + 3) = 4.5$$

M1

$$18 = 4v_A + 2v_A + 9, v_A = 1.5 \text{ m s}^{-1}$$

A1

$$v_B = 6 \text{ m s}^{-1}$$

A1

Answer: $v_A = 1.5 \text{ m s}^{-1}$, $v_B = 6 \text{ m s}^{-1}$

5 marks

10 (5 marks)

$$5 \times 4 + 3 \times 1 = 5v_A + 3 \times 4$$

M1 A1

$$v_A = 2.2 \text{ m s}^{-1}$$

A1

$$e = \frac{(4 - 2.2)}{(4 - 1)}$$

M1

$$e = 0.6$$

A1

Answer: $v_A = 2.2 \text{ m s}^{-1}$, $e = 0.6$

5 marks

11 (4 marks)

$$B \text{ moves off at } v_B: v_B - 0 = 0.5 \times 6 = 3 \text{ m s}^{-1}$$

M1 A1

$$6m = 3 \times 3$$

M1

$$m = 1.5$$

A1

Answer: $m = 1.5$

4 marks

12 (5 marks)

$$(a) v_A + v_B = 8 \text{ and } v_B - v_A = 4$$

M1 M1

$$v_A = 2 \text{ m s}^{-1}, v_B = 6 \text{ m s}^{-1}$$

A1

Answer: 2 m s^{-1} and 6 m s^{-1}

3 marks

$$(b) \text{ KE before} = \frac{1}{2}(1)(64) = 32 \text{ J, KE after} = \frac{1}{2}(4) + \frac{1}{2}(36) = 20 \text{ J}$$

M1

$$\text{Loss} = 12 \text{ J}$$

A1

Answer: 12 J

2 marks

13 (4 marks)

$$m(3u) = mv_A + 2mv_B$$

M1

$$v_B - v_A = 3u$$

M1

$$3u = v_A + 2(v_A + 3u), \text{ so } v_A = -u$$

A1

$$v_B = 2u$$

A1

Answer: A: $-u$ (reversed), B: $2u$

4 marks

14 (6 marks)

$$(a) mu = mv_A + 2mv_B, v_B - v_A = eu$$

M1 M1

$$u = v_A + 2(v_A + eu), \text{ so } v_A = \frac{u(1 - 2e)}{3}$$

A1

$$v_B = v_A + eu = \frac{u(1 + e)}{3}$$

A1

$$\text{Answer: } v_B = \frac{u(1 + e)}{3}$$

4 marks

$$(b) v_A < 0, \text{ so } 1 - 2e < 0$$

M1

$$e > \frac{1}{2} \text{ (and } e \leq 1)$$

A1

$$\text{Answer: } \frac{1}{2} < e \leq 1$$

2 marks

15 (7 marks)

$$(a) \text{ Taking A's direction as positive: } 4mu - 3mu = mv_A + 3mv_B$$

M1 A1

$$v_B - v_A = 0.5(5u) = 2.5u$$

M1

$$u = v_A + 3(v_A + 2.5u), v_A = -\frac{13u}{8}, v_B = \frac{7u}{8}$$

A1

$$\text{Answer: } v_A = -\frac{13u}{8}, v_B = \frac{7u}{8}$$

4 marks

$$(b) \text{ KE before} = \frac{1}{2}m(16u^2) + \frac{1}{2}(3m)u^2 = \frac{19}{2}mu^2$$

M1

$$\text{KE after} = \frac{1}{2}m\left(\frac{169}{64}\right)u^2 + \frac{1}{2}(3m)\left(\frac{49}{64}\right)u^2 = \frac{316}{128}mu^2$$

M1

$$\text{Loss} = \frac{19}{2}mu^2 - \frac{79}{32}mu^2 = \frac{225}{32}mu^2$$

A1

$$\text{Answer: } \frac{225}{32}mu^2$$

3 marks

16 (8 marks)

$$(a) 12 = 2v_A + v_B, v_B - v_A = 3$$

M1 M1

$$v_A = 3 \text{ m s}^{-1}$$

A1

$$v_B = 6 \text{ m s}^{-1}$$

A1

Answer: A: 3 m s^{-1} , B: 6 m s^{-1}

4 marks

$$(b) 6 = v_B + 2v_C$$

M1

$$v_C - v_B = 3$$

M1

$$6 = 3v_B + 6, \text{ so } v_B = 0 \text{ m s}^{-1}$$

A1

$$v_C = 3 \text{ m s}^{-1}$$

A1

Answer: B: 0 (at rest), C: 3 m s⁻¹

4 marks

17 (5 marks)

$$12 + 2 = 3v_A + v_B$$

M1 A1

$$v_B - v_A = 0.5(4 - 2) = 1$$

M1

$$14 = 4v_A + 1, v_A = 3.25 \text{ m s}^{-1}$$

A1

$$v_B = 4.25 \text{ m s}^{-1}$$

A1

Answer: A: 3.25 m s⁻¹, B: 4.25 m s⁻¹

5 marks

18 (6 marks)

(a) Take the direction of A as positive. B's velocity changes from $-u$ to v_B : $3m(v_B + u) = 6mu$, so

M1 A1

$$v_B = u$$

$$\text{Momentum: } 2m(3u) + 3m(-u) = 2mv_A + 3m(u), \text{ so } v_A = 0$$

M1

$$e = \frac{u - 0}{3u + u} = \frac{1}{4}$$

A1

Answer: $e = \frac{1}{4}$

4 marks

(b) Before: $\frac{1}{2}(2m)(9u^2) + \frac{1}{2}(3m)(u^2) = \frac{21}{2}mu^2$; after: $\frac{1}{2}(3m)(u^2) = \frac{3}{2}mu^2$

M1

$$\text{Loss} = 9mu^2$$

A1

Answer: $9mu^2$

2 marks

19 (6 marks)

(a) $15 = 3v_A + 2v_B, v_B - v_A = 3$

M1 M1

$$v_B = 4.8 \text{ m s}^{-1} \text{ (and } v_A = 1.8 \text{ m s}^{-1})$$

A1

Answer: 4.8 m s⁻¹

3 marks

(b) $I = 2 \times 4.8 - 0$

M1

$$= 9.6 \text{ N s}$$

A1

Answer: 9.6 N s

2 marks

(c) $F = \frac{9.6}{0.02} = 480 \text{ N}$

B1

Answer: 480 N

1 mark

20 (7 marks)

$$v_A + v_B = u, v_B - v_A = eu$$

M1 M1

$$v_A = \frac{u(1 - e)}{2}, v_B = \frac{u(1 + e)}{2}$$

A1

$$\text{KE after} = \frac{1}{2}m\frac{u^2}{4}[(1 - e)^2 + (1 + e)^2]$$

M1

$$= \frac{1}{2}mu^2\frac{(1 + e^2)}{2}$$

A1

$$\frac{(1 + e^2)}{2} = 0.68, \text{ so } e^2 = 0.36$$

M1

$$e = 0.6$$

A1

Answer: $e = 0.6$

7 marks

21 (11 marks)

(a) $v_A + v_B = 4, v_B - v_A = 2$

M1 M1

$$v_A = 1 \text{ m s}^{-1}, v_B = 3 \text{ m s}^{-1}$$

A1

Answer: A: 1 m s⁻¹, B: 3 m s⁻¹

3 marks

(b) B and C: $v_B' + v_C = 3, v_C - v_B' = 1.5$

M1 A1

$$v_B' = 0.75 \text{ m s}^{-1}, v_C = 2.25 \text{ m s}^{-1}$$

A1

A (1 m s⁻¹) is moving faster than B (0.75 m s⁻¹) in the same direction so A catches B

E1

$$v_A' + v_B'' = 1.75, v_B'' - v_A' = 0.5 \times 0.25 = 0.125 \quad \text{M1}$$

$$v_A' = 0.8125 \text{ m s}^{-1}, v_B'' = 0.9375 \text{ m s}^{-1} \quad \text{A1}$$

Answer: A: 0.8125 m s⁻¹, B: 0.9375 m s⁻¹ 6 marks

(c) Final speeds in order: A 0.8125 < B 0.9375 < C 2.25 M1

Each sphere is moving slower than the one ahead, so no more collisions occur E1

Answer: There are no further collisions: A and B are moving apart and B and C are moving apart 2 marks

22 (10 marks)

(a) Momentum: $2m(5u) + 3m(u) = 2mv_A + 3mv_B$, so $13u = 2v_A + 3v_B$ M1 A1

$$v_B - v_A = \frac{1}{2}(5u - u) = 2u \quad \text{M1}$$

$$v_A = \frac{7u}{5}, v_B = \frac{17u}{5} \quad \text{A1}$$

Answer: A: $\frac{7u}{5}$, B: $\frac{17u}{5}$ 4 marks

(b) $3m(\frac{17u}{5}) = 3mv_B' + 2mv_C, v_C - v_B' = e(\frac{17u}{5})$ M1 A1

$$v_B' = \frac{17u(3 - 2e)}{25} \quad \text{A1}$$

Answer: $\frac{17u(3 - 2e)}{25}$ 3 marks

(c) No further collision requires $v_B' \geq v_A$: $\frac{17u(3 - 2e)}{25} \geq \frac{7u}{5}$ M1

$$17(3 - 2e) \geq 35, \text{ so } e \leq \frac{8}{17} \quad \text{A1}$$

$$\frac{0 \leq e \leq 8}{17} \quad \text{A1}$$

Answer: $0 \leq e \leq \frac{8}{17}$ 3 marks

23 (9 marks)

(a) Take the direction of A as positive: $2m(3u) + m(-2u) = 2mv_A + mv_B$, so $4u = 2v_A + v_B$ M1

$$v_B - v_A = e(3u + 2u) = 5eu \quad \text{M1}$$

$$3v_A = 4u - 5eu, \text{ so } v_A = \frac{u(4 - 5e)}{3} \quad \text{A1}$$

$$v_B = \frac{u(4 + 10e)}{3} \quad \text{A1}$$

Answer: $v_B = \frac{u(4 + 10e)}{3}$ 4 marks

(b) $v_A > 0$: $4 - 5e > 0$ M1

$$0 \leq e < \frac{4}{5} \quad \text{A1}$$

Answer: $0 \leq e < \frac{4}{5}$ 2 marks

(c) $4 + 10e = 2(4 - 5e)$, so $e = \frac{1}{5}$ M1 A1

Speeds after: $v_A = u, v_B = 2u$; KE before = $9mu^2 + 2mu^2 = 11mu^2$, KE after = $mu^2 + 2mu^2 = 3mu^2$, loss = $8mu^2$ A1

Answer: $e = \frac{1}{5}$, loss = $8mu^2$ 3 marks

24 (9 marks)

(a) $\frac{1}{2}(2)v^2 = 2 \times 9.8 \times 1.6$ M1

$$v = 5.6 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 5.6 m s⁻¹ 2 marks

(b) $2 \times 5.6 = 2v_A + v_B, v_B - v_A = 0.5 \times 5.6 = 2.8$ M1 M1

$$v_A = 2.8 \text{ m s}^{-1} \quad \text{A1}$$

$$v_B = 5.6 \text{ m s}^{-1} \quad \text{A1}$$

Answer: A: 2.8 m s^{-1} , B: 5.6 m s^{-1}

4 marks

(c) $\frac{1}{2}(1)(5.6^2) = 1 \times 9.8 \times h$

M1 A1

$h = 1.6 \text{ m}$

A1

Answer: 1.6 m

3 marks

25 (8 marks)**(a)** Let B move at v : A moves at $\frac{v}{2}$

B1

$m(5u) + 2m(u) = m\frac{v}{2} + 2mv$

M1

$7u = \frac{5v}{2}, v = 2.8u, \text{ so A: } 1.4u, \text{ B: } 2.8u$

A1

Answer: A: 1.4u, B: 2.8u

3 marks

(b) $e = \frac{(2.8 - 1.4)u}{(5 - 1)u}$

M1

$= 0.35$

A1

Answer: 0.35

2 marks

(c) KE before = $\frac{1}{2}m(25u^2) + \frac{1}{2}(2m)(u^2) = 13.5mu^2$

M1

KE after = $\frac{1}{2}m(1.96u^2) + \frac{1}{2}(2m)(7.84u^2) = 8.82mu^2$

M1

Loss = $4.68mu^2$

A1

Answer: $4.68mu^2$

3 marks

26 (8 marks)**(a)** The particles move in opposite directions, so the speed of approach is $6 + 3 = 9 \text{ m s}^{-1}$

E1

B's momentum is negative in the chosen direction: $24 - 6$, not $24 + 6$

E1

Answer: Approach speed 9; B's momentum should be -6

2 marks

(b) $4v_A + 2v_B = 18$

M1 A1

$v_B - v_A = 4.5$

M1

$v_A = 1.5 \text{ m s}^{-1}, v_B = 6 \text{ m s}^{-1}$

A1

Answer: A: 1.5 m s^{-1} , B: 6 m s^{-1}

4 marks

(c) KE before = $\frac{1}{2}(4)(36) + \frac{1}{2}(2)(9) = 81 \text{ J}$, KE after = $\frac{1}{2}(4)(2.25) + \frac{1}{2}(2)(36) = 40.5 \text{ J}$

M1

Loss = 40.5 J

A1

Answer: 40.5 J

2 marks

27 (12 marks)

(a) $3mu = 3mv_A + 2mv_B, v_B - v_A = 0.5u$

M1 M1

$3u = 3v_A + 2v_B + u$

M1

$v_A = 0.4u, v_B = 0.9u$

A1

Answer: A: 0.4u, B: 0.9u

4 marks

(b) $2m(0.9u) = 2mv_B' + mv_C, v_C - v_B' = 0.45u$

M1 M1

$1.8u = 2v_B' + v_B' + 0.45u$

M1

$v_B' = 0.45u, v_C = 0.9u$

A1

Answer: B: 0.45u, C: 0.9u

4 marks

(c) A moves at $0.4u$, B at $0.45u$, C at $0.9u$, all in the same direction

M1

Each sphere is moving slower than the sphere ahead of it, so there are no further collisions

E1

Answer: There are no further collisions: $0.4u < 0.45u < 0.9u$

2 marks

(d) Final KE = $\frac{1}{2}(3m)(0.16u^2) + \frac{1}{2}(2m)(0.2025u^2) + \frac{1}{2}(m)(0.81u^2) = 0.8475mu^2$

M1

Initial KE = $1.5mu^2$, fraction lost = $\frac{0.6525}{1.5} = 0.435$

A1

Answer: 0.435

2 marks

28 (10 marks)

(a) $v_A + v_B = u$ M1

$v_B - v_A = eu$ M1

Solving: $v_A = \frac{u(1-e)}{2}$, $v_B = \frac{u(1+e)}{2}$ A1

Answer: After the first collision the speed of A is $\frac{u(1-e)}{2}$ and the speed of B is $\frac{u(1+e)}{2}$ 3 marks

(b) B and C: $v_B' + v_C = \frac{u(1+e)}{2}$, $v_C - v_B' = e\frac{u(1+e)}{2}$ M1 A1

Subtracting: $2v_B' = \frac{u(1+e)}{2}(1-e)$ M1

$v_B' = \frac{u(1-e^2)}{4}$ A1

Answer: After B collides with C, the speed of B is $\frac{u(1-e^2)}{4}$ 4 marks

(c) A catches B if $v_A > v_B'$: $\frac{u(1-e)}{2} - \frac{u(1-e)(1+e)}{4} > 0$ M1

$= \frac{u(1-e)}{4}[2 - (1+e)] = \frac{u(1-e)^2}{4}$ M1

This is positive for $0 < e < 1$, so A is faster than B and the second collision occurs E1**Answer: A and B must collide a second time, whatever the value of e** 3 marks**29 (12 marks)**

(a) Relative to B, A approaches at a constant 14 m s^{-1} (both have acceleration g), so $t = \frac{10}{14} = \frac{5}{7} \text{ s}$ M1 A1

Speed of A = $9.8 \times \frac{5}{7} = 7 \text{ m s}^{-1}$, speed of B = $14 - 7 = 7 \text{ m s}^{-1}$ A1

Answer: $t = \frac{5}{7} \text{ s}$ 3 marks

(b) Take upwards as positive: $3(7) + 1(-7) = 3v_B + v_A$ and $v_A - v_B = 0.5(7 + 7) = 7$ M1 M1

$v_B = \frac{7}{4}$, $v_A = \frac{35}{4}$: B moves up at 1.75 m s^{-1} and A moves up at 8.75 m s^{-1} A1

Answer: B: 1.75 m s^{-1} upwards, A: 8.75 m s^{-1} upwards 3 marks

(c) Height at collision = $14 \times \frac{5}{7} - \frac{1}{2}(9.8)(\frac{5}{7})^2 = 7.5 \text{ m}$ M1

Further rise = $\frac{1.75^2}{2(9.8)} = 0.156 \text{ m}$ M1

Greatest height = 7.66 m A1

Answer: 7.66 m 3 marks

(d) Further rise = $\frac{8.75^2}{2(9.8)} = 3.91 \text{ m}$ M1

Greatest height = 11.4 m A1

This is more than 10 m : A gains energy from B, which loses much more kinetic energy in the collision E1**Answer: 11.4 m , above its release height** 3 marks**30 (9 marks)**

(a) $mu = mv_A + 3mv_B$, $v_B - v_A = eu$ M1 A1

$v_A = \frac{u(1-3e)}{4}$, $v_B = \frac{u(1+e)}{4}$ A1

Answer: $v_A = \frac{u(1-3e)}{4}$, $v_B = \frac{u(1+e)}{4}$ 3 marks

(b) The direction of A is reversed, so $v_A < 0$: $1 - 3e < 0$ M1

$e > \frac{1}{3}$ A1

Answer: $e > \frac{1}{3}$ 2 marks

(c) B rebounds from the wall with speed $\frac{3}{5} \times \frac{u(1+e)}{4} = \frac{3u(1+e)}{20}$ M1 A1

For a second collision B must be moving away from the wall faster than A: $\frac{3u(1+e)}{20} > \frac{u(3e-1)}{4}$	M1
$3 + 3e > 15e - 5$, so $e < \frac{2}{3}$; hence $\frac{1}{3} < e < \frac{2}{3}$	A1
Answer: $\frac{1}{3} < e < \frac{2}{3}$	4 marks

Oblique Collisions

1 (2 marks)

Along L: $10\cos 60^\circ = 5 \text{ m s}^{-1}$	B1
Perpendicular to L: $10\sin 60^\circ = 5\sqrt{3} = 8.66 \text{ m s}^{-1}$	B1
Answer: 5 m s^{-1} along L, 8.66 m s^{-1} perpendicular	2 marks

2 (2 marks)

The spheres are smooth so the only force (impulse) between them acts along the line of centres	E1
There is no impulse perpendicular to L, so the component of momentum in that direction does not change	E1
Answer: Impulse acts along the line of centres only	2 marks

3 (1 mark)

Unchanged: $12\sin 30^\circ = 6 \text{ m s}^{-1}$	B1
Answer: 6 m s^{-1}	1 mark

4 (2 marks)

Speed = $\sqrt{3^2 + 4^2}$	M1
$= 5 \text{ m s}^{-1}$	A1
Answer: 5 m s^{-1}	2 marks

5 (2 marks)

$\tan \theta = \frac{12}{5}$	M1
$\theta = 67.4^\circ$	A1
Answer: 67.4°	2 marks

6 (3 marks)

$6m = mv_A + mv_B$, $v_B - v_A = 0.5 \times 6 = 3$	M1 M1
$v_B = 4.5 \text{ m s}^{-1}$	A1
Answer: 4.5 m s^{-1}	3 marks

7 (4 marks)

Along L: $8m = mv_A + mv_B$, $v_B - v_A = 4$	M1 M1
$v_A = 2 \text{ m s}^{-1}$, $v_B = 6 \text{ m s}^{-1}$ along L	A1
Speed of A = $\sqrt{2^2 + 6^2} = 6.32 \text{ m s}^{-1}$, speed of B = 6 m s^{-1}	A1
Answer: A: 6.32 m s^{-1}, B: 6 m s^{-1}	4 marks

8 (5 marks)

(a) Components of A: 8 along L, 6 perpendicular	B1
$24 = 3v_A + v_B$, $v_B - v_A = 4$	M1 A1
$v_A = 5$ along L, speed = $\sqrt{5^2 + 6^2} = 7.81 \text{ m s}^{-1}$	A1
Answer: 7.81 m s^{-1}	4 marks
(b) $I = 1 \times 9 = 9 \text{ N s}$ ($v_B = 9 \text{ m s}^{-1}$)	B1
Answer: 9 N s	1 mark

9 (4 marks)

Along L: equal masses and $e = 1$, so the components are exchanged: A: -2 , B: 6

M1 A1

$$\text{A: speed} = \sqrt{(-2)^2 + 2^2} = 2.83 \text{ m s}^{-1}$$

A1

$$\text{B: speed} = \sqrt{6^2 + (-2)^2} = 6.32 \text{ m s}^{-1}$$

A1

Answer: A: 2.83 m s^{-1} , B: 6.32 m s^{-1}

4 marks

10 (3 marks)

$$\text{(a) } I = 3(16 - 6)$$

M1

$$= 30 \text{ N s}$$

A1

Answer: 30 N s

2 marks

$$\text{(b) } \sqrt{6^2 + 12^2} = 13.4 \text{ m s}^{-1}$$

B1

Answer: 13.4 m s^{-1}

1 mark

11 (3 marks)

B moves along L: $v_B = 3$

B1

$$v_A + v_B = 4, \text{ so } v_A = 1, v_B - v_A = 2 = e \times 4$$

M1

$$e = 0.5$$

A1

Answer: 0.5

3 marks

12 (5 marks)

$$v_A + v_B = 6, v_B - v_A = 3$$

M1 M1

$$v_A = 1.5 \text{ m s}^{-1}, v_B = 4.5 \text{ m s}^{-1} \text{ along L}$$

A1

$$\text{KE before} = \frac{1}{2}(2)(6^2 + 8^2) = 100 \text{ J, KE after} = \frac{1}{2}(2)(1.5^2 + 8^2) + \frac{1}{2}(2)(4.5^2) = 86.5 \text{ J}$$

M1

$$\text{Loss} = 13.5 \text{ J}$$

A1

Answer: 13.5 J

5 marks

13 (4 marks)

$$\text{Along L: } 8\cos 60^\circ = 4; \text{ perpendicular: } 8\sin 60^\circ = 4\sqrt{3}$$

B1

$e = 1$ and equal masses: the components along L are exchanged, so $v_A = 0$, $v_B = 4$

M1 A1

$$\text{A moves perpendicular to L at } 4\sqrt{3} = 6.93 \text{ m s}^{-1}; \text{ B moves along L at } 4 \text{ m s}^{-1}$$

A1

Answer: A: 6.93 m s^{-1} perpendicular to L, B: 4 m s^{-1} along L

4 marks

14 (5 marks)

$$\text{Along L: } u \cos \theta = v_A + v_B, v_B - v_A = u \cos \theta$$

M1 M1

$$v_A = 0, v_B = u \cos \theta$$

A1

A has no component along L, and its perpendicular component $u \sin \theta$ is unchanged, so A moves perpendicular to L

M1

B moves along L, so the paths are perpendicular

A1

Answer: After the collision the spheres move in directions that are at right angles to each other

5 marks

15 (6 marks)

$$\text{(a) Along L: } 5 = v_A + v_B, v_B - v_A = 2.5$$

M1 M1

$$v_A = 1.25, v_B = 3.75$$

A1

$$\text{Speed of A} = \sqrt{1.25^2 + (5\sqrt{3})^2} = 8.75 \text{ m s}^{-1}; \text{ speed of B} = 3.75 \text{ m s}^{-1}$$

A1

Answer: A: 8.75 m s^{-1} , B: 3.75 m s^{-1}

4 marks

$$\text{(b) } \tan \phi = \frac{5\sqrt{3}}{1.25} = 4\sqrt{3}$$

M1

$$\phi = 81.8^\circ$$

A1

Answer: 81.8°

2 marks

16 (7 marks)**(a) j** components are unchanged: 7 and 4

B1

i: $2(4) + 3(-3) = 2v_A + 3v_B$

M1

$v_B - v_A = 0.5(4 + 3) = 3.5$

M1

$v_A = -2.3, v_B = 1.2$

A1

A: $(-2.3\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$, **B:** $(1.2\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$

A1

Answer: A: $(-2.3\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$, B: $(1.2\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$

5 marks

(b) $I = 3(1.2 - (-3))$

M1

$= 12.6 \text{ N s}$

A1

Answer: 12.6 N s

2 marks

17 (5 marks)

Along L: $u \cos 60^\circ = \frac{u}{2}$

B1

$v_A + v_B = \frac{u}{2}, v_B - v_A = \frac{u}{4}, \text{ so } v_A = \frac{u}{8}, v_B = \frac{3u}{8}$

M1 A1

Perpendicular components are unchanged, so only the energy along L changes

E1

$\text{Loss} = \frac{1}{2}m\left(\frac{u}{2}\right)^2 - \frac{1}{2}m\left(\frac{u}{8}\right)^2 - \frac{1}{2}m\left(\frac{3u}{8}\right)^2 = \frac{3}{64}mu^2$

A1

Answer: The kinetic energy lost in the collision is $\frac{3}{64}mu^2$

5 marks

18 (5 marks)

$v_A + v_B = u \cos \theta, v_B - v_A = eu \cos \theta, \text{ so } v_A = \frac{(1-e)}{2}u \cos \theta$

M1 A1

A at 45° to L: $v_A = u \sin \theta$

M1

$\frac{(1-e)}{2} = \tan \theta = \frac{1}{5}$

M1

$e = \frac{3}{5}$

A1

Answer: $e = 0.6$

5 marks

19 (7 marks)

(a) Along L: $0.4(12) = 0.4v_A + 0.6v_B$

M1

$v_B - v_A = 6$

M1

$v_B = 7.2 \text{ m s}^{-1}$

A1

$I = 0.6 \times 7.2 = 4.32 \text{ N s}$

A1

Answer: 7.2 m s^{-1} , 4.32 N s

4 marks

(b) $v_A = 7.2 - 6 = 1.2$ along L, perpendicular component 9

M1

$\text{Speed} = \sqrt{1.2^2 + 9^2} = 9.08 \text{ m s}^{-1}$

A1

$\text{Angle} = \tan^{-1}\left(\frac{9}{1.2}\right) = 82.4^\circ$

A1

Answer: 9.08 m s^{-1} at 82.4° to L

3 marks

20 (6 marks)**(a)** Before: A has 8 m s^{-1} along L and 6 m s^{-1} perpendicular; B has 3 m s^{-1} along L (towards A) and 4 m s^{-1} perpendicular

B1

Momentum along L: $2(8) + 3(-3) = 2v_A + 3v_B$, so $7 = 2v_A + 3v_B$

M1

$v_B - v_A = \frac{1}{2}(8 + 3) = 5.5$

M1

$v_A = -1.9 \text{ m s}^{-1}, v_B = 3.6 \text{ m s}^{-1}$ along L

A1

Answer: A: -1.9 m s^{-1} , B: 3.6 m s^{-1} along L

4 marks

(b) A: $\sqrt{1.9^2 + 6^2} = 6.29 \text{ m s}^{-1}$

A1

B: $\sqrt{3.6^2 + 4^2} = 5.38 \text{ m s}^{-1}$

A1

Answer: A: 6.29 m s^{-1} , B: 5.38 m s^{-1}

2 marks

21 (7 marks)

Perpendicular to L: A keeps $u \sin \theta \neq 0$, B has no perpendicular component M1

B moves along L; for A's path to be perpendicular to B's, A must have no component along L: $v_A = 0$ M1 A1

Momentum along L: $mu \cos \theta = 4mv_B$, so $v_B = \frac{u \cos \theta}{4}$ M1 A1

Restitution: $v_B - 0 = e u \cos \theta$ M1

$$\frac{u \cos \theta}{4} = e u \cos \theta, \text{ so } e = \frac{1}{4}$$
 A1

Answer: The coefficient of restitution is $\frac{1}{4}$ 7 marks

22 (8 marks)

B moves along L: $v_B = 6$ B1

$$v_A + v_B = 10 \cos \theta$$
 M1

$$v_B - v_A = 10e \cos \theta$$
 M1

$$A: v_A^2 + (10 \sin \theta)^2 = 64, v_A = 10 \cos \theta - 6$$
 M1

$$(10 \cos \theta - 6)^2 + 100 - 100 \cos^2 \theta = 64$$
 M1

$$-120 \cos \theta + 36 + 100 = 64, \cos \theta = 0.6, \theta = 53.1^\circ$$
 M1 A1

$$v_A = 0, 6 = 10e(0.6), e = 1$$
 A1

Answer: $\theta = 53.1^\circ$, $e = 1$ 8 marks

23 (8 marks)

(a) Components of A: 5 along L, 12 perpendicular B1

$$2m(5) = 2mv_A + mv_B, v_B - v_A = 2.5$$
 M1 M1

$$v_A = 2.5, v_B = 5 \text{ m s}^{-1}$$
 A1

$$\text{Speed of A} = \sqrt{2.5^2 + 12^2} = 12.3 \text{ m s}^{-1}$$
 A1

Answer: $v_A = 2.5 \text{ m s}^{-1}$, $v_B = 5 \text{ m s}^{-1}$, speed of A = 12.3 m s^{-1} 5 marks

(b) Before: $\tan \alpha = \frac{12}{5}$, $\alpha = 67.38^\circ$ M1

After: $\tan \beta = \frac{12}{2.5}$, $\beta = 78.23^\circ$ M1

Turned through 10.9° A1

Answer: 10.9° 3 marks

24 (9 marks)

(a) KE of Q before = $\frac{1}{2}(0.2)(9 + 1) = 1 \text{ J}$, so after = 0.5 J B1

$$\frac{1}{2}(0.2)(q^2 + 1) = 0.5 \text{ where } q \text{ is the } \mathbf{i} \text{ component, so } q = \pm 2$$
 M1

Q's $\mathbf{j}$ component is unchanged. Momentum in the $\mathbf{i}$ direction: $0.3(4) + 0.2(-3) = 0.3p + 0.2q$; $q =$ A1

$$-2 \text{ gives } p = \frac{10}{3} \text{ and } e < 0, \text{ which is impossible, so } q = 2$$

Answer: $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$ 3 marks

(b) $0.6 = 0.3p + 0.2(2)$ M1

$$p = \frac{2}{3}, \text{ so the velocity of P is } \left(\frac{2}{3}\mathbf{i} + 2\mathbf{j}\right) \text{ m s}^{-1}$$
 A1

Answer: $\left(\frac{2}{3}\mathbf{i} + 2\mathbf{j}\right) \text{ m s}^{-1}$ 2 marks

(c) $2 - \frac{2}{3} = e(4 + 3)$ M1

$$e = \frac{4}{21}$$
 A1

Answer: $e = \frac{4}{21}$ 2 marks

(d) Impulse on P = $0.3\left(\frac{2}{3} - 4\right)\mathbf{i}$ M1

Magnitude = 1 N s A1

Answer: 1 N s 2 marks

25 (8 marks)

(a) B moves along L, so the angle is 30°

B1

Along L: A has $10\cos 30^\circ = 8.66$

M1

$$m(8.66) = mv_A + 2mv_B, v_B - v_A = 0.8(8.66)$$

M1

$$v_B = \frac{1.8}{3}(8.66) = 5.20 \text{ m s}^{-1}$$

A1

Answer: 30° , 5.20 m s^{-1}

4 marks

(b) $v_A = 5.20 - 0.8(8.66) = -1.73$ along L (reversed)

M1

Perpendicular component 5

B1

$$\text{Speed} = \sqrt{1.73^2 + 5^2} = 5.29 \text{ m s}^{-1}$$

A1

Direction 109.1° from the positive L direction, turned through 79.1°

A1

Answer: 5.29 m s^{-1} , turned through 79.1°

4 marks

26 (6 marks)

(a) The law of restitution applies only to components along the line of centres: the approach speed along L is $10\cos 60^\circ = 5$, not 10

E1

A has a perpendicular component of velocity, so its speed is not just the L component

E1

Answer: Restitution applies along L only; A keeps its perpendicular component

2 marks

(b) Along L: $5 = v_A + v_B$, $v_B - v_A = 2.5$

M1 M1

$$v_A = 1.25, v_B = 3.75$$

A1

$$\text{Speed of A} = \sqrt{1.25^2 + (5\sqrt{3})^2} = 8.75 \text{ m s}^{-1}, \text{ speed of B} = 3.75 \text{ m s}^{-1}$$

A1

Answer: A: 8.75 m s^{-1} , B: 3.75 m s^{-1}

4 marks

27 (8 marks)

(a) Along L: $u \cos \theta = v_A + v_B$, $v_B - v_A = eu \cos \theta$

M1 M1

$$v_A = \frac{(1-e)}{2}u \cos \theta, v_B = \frac{(1+e)}{2}u \cos \theta$$

A1

Perpendicular component of A unchanged at $u \sin \theta$; B has none

A1

Answer: B: $\frac{(1+e)}{2}u \cos \theta$ along L

4 marks

(b) Velocity of A = $(v_A, u \sin \theta)$, velocity of B = $(v_B, 0)$

M1

$$\text{Scalar product} = v_A v_B = \frac{(1-e^2)}{4}u^2 \cos^2 \theta$$

M1 A1

This is positive when $e < 1$ (acute) and zero when $e = 1$ (right angle)

E1

Answer: The angle between the directions of motion of A and B after the collision is acute when $e < 1$, and a right angle when $e = 1$.

28 (10 marks)

(a) Centres are $2 \times 2.5 = 5 \text{ cm}$ apart at impact

M1

$$\sin \theta = \frac{4}{5} = 0.8$$

A1

Answer: $\sin \theta = 0.8$

2 marks

(b) Components of A: $5\cos \theta = 3$ along L, $5\sin \theta = 4$ perpendicular

B1

$$3 = v_A + v_B, v_B - v_A = 1.5$$

M1 M1

$$v_A = 0.75, v_B = 2.25 \text{ along L}$$

A1

$$\text{Speed of A} = \sqrt{0.75^2 + 4^2} = 4.07 \text{ m s}^{-1}, \text{ speed of B} = 2.25 \text{ m s}^{-1}$$

A1

Answer: A: 4.07 m s^{-1} , B: 2.25 m s^{-1}

5 marks

(c) Relative to B, A moves 1.5 m s^{-1} further from B along L and 4 m s^{-1} perpendicular to L; the centres are initially 0.05 m apart along L

M1

$$\text{Separation}^2 = (0.05 + 1.5t)^2 + (4t)^2 = 1.2^2$$

M1

$$18.25t^2 + 0.15t - 1.4375 = 0, \text{ so } t = 0.277 \text{ s}$$

A1

Answer: 0.277 s

3 marks

29 (10 marks)

(a) Along L: $v_A = \frac{(1-e)}{2}u \cos \theta$, $v_B = \frac{(1+e)}{2}u \cos \theta$ M1 A1

Perpendicular components are unchanged, so only the L components lose energy E1

KE along L after = $\frac{1}{2}m \frac{u^2 \cos^2 \theta}{4} [(1-e)^2 + (1+e)^2] = \frac{1}{2}mu^2 \cos^2 \theta \times \frac{1+e^2}{2}$ M1

Loss = $\frac{1}{2}mu^2 \cos^2 \theta (1 - \frac{1+e^2}{2})$, so $f = \frac{(1-e^2)}{4} \cos^2 \theta$ A1

Answer: $f = \frac{(1-e^2)}{4} \cos^2 \theta$ 5 marks

(b) $\frac{0.64}{4} \cos^2 \theta = 0.1$ M1

$\cos^2 \theta = 0.625$ A1

$\theta = 37.8^\circ$ A1

Answer: $\theta = 37.8^\circ$ 3 marks

(c) f is greatest when $\cos^2 \theta = 1$, $\theta = 0^\circ$ M1

$f = \frac{0.64}{4} = 0.16$ A1

Answer: 0.16 when $\theta = 0^\circ$ 2 marks

30 (10 marks)

(a) Along L: $v_A = \frac{(1-e)}{2}u \cos \theta$ M1 A1

Perpendicular component of A is $u \sin \theta$ B1

$\tan \alpha = \frac{u \sin \theta}{\frac{(1-e)}{2}u \cos \theta} = \frac{2 \tan \theta}{(1-e)}$ A1

Answer: $\tan \alpha = \frac{2 \tan \theta}{(1-e)}$ 4 marks

(b) $\tan \alpha = \frac{2 \tan 30^\circ}{0.5} = 2.31$ M1

$\alpha = 66.6^\circ$ A1

Turned through $66.6 - 30 = 36.6^\circ$ A1

Answer: 36.6° 3 marks

(c) $\tan \alpha = \frac{2 \tan \theta}{(1-e)} > \tan \theta$ because $\frac{2}{(1-e)} > 1$ for $0 \leq e < 1$ M1

So $\alpha > \theta$: A is turned away from the line of centres E1

As $e \rightarrow 1$, $\tan \alpha \rightarrow \infty$, so $\alpha \rightarrow 90^\circ$ E1

Answer: The direction is always turned away, because $\tan \alpha = \frac{2 \tan \theta}{(1-e)} > \tan \theta$; as $e \rightarrow 1$, $\alpha \rightarrow 90^\circ$ 3 marks

Coefficient of Restitution**1 (3 marks)**

(a) $0 \leq e \leq 1$ B1

Answer: $0 \leq e \leq 1$ 1 mark

(b) Perfectly elastic: no kinetic energy is lost and the speed of separation equals the speed of approach E1

Answer: Perfectly elastic, no kinetic energy lost 1 mark

(c) The particles do not separate after impact (they coalesce, or the particle stops against a wall) E1

Answer: No separation after impact 1 mark

2 (1 mark)

$e = \frac{6}{10} = 0.6$ B1

Answer: 0.6 1 mark

3 (1 mark)

$$\text{Speed} = 0.75 \times 8 = 6 \text{ m s}^{-1}$$

B1**Answer: 6 m s⁻¹****1 mark****4 (1 mark)**

$$e = \frac{9.8}{14} = 0.7$$

B1**Answer: 0.7****1 mark****5 (1 mark)**

$$e = \frac{2}{5} = 0.4$$

B1**Answer: 0.4****1 mark****6 (3 marks)**

$$e = \frac{6}{12} = 0.5$$

B1

$$\text{KE after} / \text{KE before} = \frac{(6)^2}{(12)^2} = e^2$$

M1

$$= \frac{1}{4}$$

A1**Answer: e = 0.5, fraction $\frac{1}{4}$** **3 marks****7 (3 marks)**

$$\text{Speed on impact: } v^2 = 2 \times 9.8 \times 2.5, v = 7 \text{ m s}^{-1}$$

M1

$$\text{Speed of rebound: } u^2 = 2 \times 9.8 \times 1.6, u = 5.6 \text{ m s}^{-1}$$

M1

$$e = \frac{5.6}{7} = 0.8$$

A1**Answer: 0.8****3 marks****8 (4 marks)**

$$\text{Speed on impact } v = \sqrt{2g \times 4}$$

M1

$$\text{Speed after impact} = 0.5v$$

M1

$$\text{Height} = \frac{(0.5v)^2}{2g} = 0.25 \times 4$$

M1

$$= 1 \text{ m}$$

A1**Answer: 1 m****4 marks****9 (4 marks)**

$$\text{(a) Rebound speed } v = eu, \text{ so KE after} = e^2 \times \text{KE before}$$

M1

$$e^2 = 0.64$$

A1

$$e = 0.8$$

A1**Answer: e = 0.8****3 marks**

$$\text{(b) } 0.8 \times 15 = 12 \text{ m s}^{-1}$$

B1**Answer: 12 m s⁻¹****1 mark****10 (5 marks)**

$$\text{(a) } v^2 = 2 \times 9.8 \times 4.9, v = 9.8 \text{ m s}^{-1}$$

M1 A1

$$\text{Speed after} = 0.6 \times 9.8 = 5.88 \text{ m s}^{-1}$$

A1**Answer: 5.88 m s⁻¹****3 marks**

$$\text{(b) Time} = \frac{2 \times 5.88}{9.8}$$

M1

$$= 1.2 \text{ s}$$

A1**Answer: 1.2 s****2 marks**

11 (4 marks)

Each impact multiplies the height by e^2 , so after two impacts the height is $e^4 \times 5$

M1 A1

$$e^4 = \frac{0.8}{5} = 0.16$$

M1

$$e = \sqrt[4]{0.16} = 0.632$$

A1

Answer: 0.632

4 marks

12 (4 marks)

$$2 \times 5 = 2v_A + 3 \times 3$$

M1

$$v_A = 0.5 \text{ m s}^{-1}$$

A1

$$e = \frac{(3 - 0.5)}{5}$$

M1

$$e = 0.5$$

A1

Answer: 0.5

4 marks

13 (3 marks)

$$\text{Speed on impact } v = \sqrt{2gh}$$

B1

$$\text{Speed after impact} = ev, \text{ so height } H = \frac{(ev)^2}{2g}$$

M1

$$= \frac{e^2 \times 2gh}{2g} = e^2 h$$

A1

Answer: The ball rises to a height $e^2 h$ after the first bounce

3 marks

14 (5 marks)

(a) Rebound heights: $e^2 h$, $e^4 h$, $e^6 h$, ... each travelled twice (up and down)

M1 A1

$$\text{Total} = h + 2h(e^2 + e^4 + \dots) = h + 2h \frac{e^2}{(1 - e^2)}$$

M1

$$= h \frac{(1 - e^2 + 2e^2)}{(1 - e^2)} = h \frac{(1 + e^2)}{(1 - e^2)}$$

A1

Answer: The total distance travelled by the ball before it comes to rest is $h \frac{(1 + e^2)}{(1 - e^2)}$

4 marks

$$(b) 2 \times \frac{1.25}{0.75} = \frac{10}{3} = 3.33 \text{ m}$$

B1

Answer: $\frac{10}{3} \text{ m}$

1 mark

15 (7 marks)

$$(a) 4.9 = \frac{1}{2}(9.8)t^2, t = 1 \text{ s}$$

M1

$$v = 9.8 \times 1 = 9.8 \text{ m s}^{-1}$$

A1

Answer: 1 s, 9.8 m s^{-1}

2 marks

$$(b) \text{Speed after } n\text{th impact} = 9.8(0.5)^n$$

M1

$$\text{Time of flight} = \frac{2u}{g}$$

M1

$$= 2(0.5)^n \text{ s}$$

A1

Answer: The time between the n th and $(n + 1)$ th impacts is $2(0.5)^n$ seconds

3 marks

$$(c) T = 1 + 2(0.5 + 0.25 + \dots) = 1 + 2 \times \frac{0.5}{1 - 0.5}$$

M1

$$= 3 \text{ s}$$

A1

Answer: 3 s

2 marks

16 (5 marks)

$$19.6 = \frac{1}{2}(9.8)t^2, t = 2 \text{ s, speed on impact } 19.6 \text{ m s}^{-1}$$

M1 A1

$$\text{Time between impacts} = 4.4 - 2 = 2.4 \text{ s}$$

B1

$$\frac{2 \times 19.6e}{9.8} = 2.4, 4e = 2.4$$

M1

$$e = 0.6$$

A1

Answer: 0.6

5 marks

17 (4 marks)

$$\text{Speed after} = 0.75 \times 12 = 9 \text{ m s}^{-1}$$

M1 A1

$$\text{KE before} = \frac{1}{2}(0.2)(144) = 14.4 \text{ J, KE after} = \frac{1}{2}(0.2)(81) = 8.1 \text{ J}$$

M1

$$\text{Loss} = 6.3 \text{ J}$$

A1

Answer: 6.3 J

4 marks

18 (5 marks)

$$\text{(a) Rebound speed} = 0.6 \times 8 = 4.8 \text{ m s}^{-1}$$

M1

$$I = 0.5(4.8 + 8)$$

M1

$$= 6.4 \text{ N s}$$

A1

Answer: 6.4 N s

3 marks

$$\text{(b) } F = \frac{6.4}{0.01}$$

M1

$$= 640 \text{ N}$$

A1

Answer: 640 N

2 marks

19 (5 marks)

$$\text{Speed on impact: } v^2 = 7^2 + 2(9.8)(10) = 245$$

M1 A1

$$\text{Speed after impact: } (0.5v)^2 = \frac{245}{4} = 61.25$$

M1

$$\text{Height} = \frac{61.25}{2(9.8)}$$

M1

$$= 3.12 \text{ m}$$

A1

Answer: 3.12 m

5 marks

20 (5 marks)

$$\text{Let A have speed } u: u = v_A + v_B \text{ (equal masses)}$$

M1

$$v_B = 3v_A, \text{ so } u = 4v_A$$

M1 A1

$$v_B - v_A = eu: 2v_A = 4ev_A$$

M1

$$e = 0.5$$

A1

Answer: 0.5

5 marks

21 (9 marks)

$$\text{(a) } 2.5 = \frac{1}{2}(9.8)t^2$$

M1

$$t = 0.714 \text{ s}$$

A1

Answer: 0.714 s

2 marks

$$\text{(b) Impacts: time between } n\text{th and } (n+1)\text{th} = 2t_0e^n$$

M1

$$\text{Total} = t_0 + 2t_0(e + e^2 + \dots) = t_0 \frac{(1+e)}{(1-e)}$$

M1

$$= 0.7143 \times 9 = 6.43 \text{ s}$$

A1

Answer: Total time = 6.43 s (3 s.f.)

3 marks

$$\text{(c) } D = h \frac{(1+e^2)}{(1-e^2)} = 2.5 \times \frac{1.64}{0.36}$$

M1

$$= 11.4 \text{ m}$$

A1

Answer: 11.4 m

2 marks

$$\text{(d) } 2.5(0.64)^n < 0.01$$

M1

$$n > 12.37, \text{ so the 13th rebound}$$

A1

Answer: 13th rebound

2 marks

22 (8 marks)

$$\text{(a) } v^2 = 14^2 - 2(9.8)(6.4)$$

M1

$$v = 8.4 \text{ m s}^{-1}$$

A1

Answer: 8.4 m s⁻¹

2 marks

$$\text{(b) Speed after ceiling} = 0.5 \times 8.4 = 4.2 \text{ m s}^{-1}$$

B1

$$v^2 = 4.2^2 + 2(9.8)(6.4)$$

M1

$$v = 12.0 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 12.0 m s⁻¹ 3 marks

(c) Speed after floor impact = $0.6 \times 12.0 = 7.18 \text{ m s}^{-1}$ M1

$$\text{Height} = \frac{(7.18)^2}{2(9.8)} \quad \text{M1}$$

$$= 2.63 \text{ m} \quad \text{A1}$$

Answer: 2.63 m 3 marks

23 (7 marks)

$$mu = mv_A + 2mv_B, v_B - v_A = eu \quad \text{M1 M1}$$

$$v_A = \frac{u(1 - 2e)}{3} \quad \text{A1}$$

$$\text{KE of A after} = 0.04 \times \text{initial}, \text{ so } v_A^2 = 0.04u^2 \quad \text{M1}$$

$$|v_A| = 0.2u \quad \text{A1}$$

$$1 - 2e = 0.6 \text{ or } 1 - 2e = -0.6 \quad \text{M1}$$

$$e = 0.2 \text{ or } e = 0.8 \quad \text{A1}$$

Answer: e = 0.2 or e = 0.8 7 marks

24 (8 marks)

(a) Heights multiply by e^2 at each bounce M1

$$e^2 = \frac{2.025}{3.6} = 0.5625 \quad \text{M1}$$

$$e = 0.75 \quad \text{A1}$$

Answer: 0.75 3 marks

(b) $3.6 = 0.5625H$ M1

$$H = 6.4 \text{ m} \quad \text{A1}$$

Answer: 6.4 m 2 marks

$$\text{(c) Distance} = H \frac{(1 + e^2)}{(1 - e^2)} \quad \text{M1}$$

$$= 6.4 \times \frac{1.5625}{0.4375} \quad \text{M1}$$

$$= 22.9 \text{ m} \quad \text{A1}$$

Answer: 22.9 m 3 marks

25 (7 marks)

(a) Take A's direction as positive: $3mu - 2mu = 0 + mv_B$, so $v_B = u$ M1 A1

Separation speed = $u - 0 = u$, approach speed = $u + 2u = 3u$ M1

$$e = \frac{1}{3} \quad \text{A1}$$

Answer: e = $\frac{1}{3}$ 4 marks

$$\text{(b) KE before} = \frac{1}{2}(3m)u^2 + \frac{1}{2}m(4u^2) = \frac{7}{2}mu^2 \quad \text{M1}$$

$$\text{KE after} = \frac{1}{2}mu^2 \quad \text{M1}$$

$$\text{Fraction lost} = \frac{3}{7} = \frac{6}{7} \quad \text{A1}$$

Answer: $\frac{6}{7}$ 3 marks

26 (6 marks)

(a) e is the ratio of speeds, not heights E1

The heights are proportional to the squares of the speeds, so $e = \sqrt{\text{height ratio}}$ E1

Answer: e is a ratio of speeds, so e = $\sqrt{(1.28/2)}$ 2 marks

$$\text{(b) } e = \sqrt{\frac{1.28}{2}} = \sqrt{0.64} \quad \text{M1}$$

$$= 0.8 \quad \text{A1}$$

Answer: 0.8 2 marks

$$(c) \text{ Height} = e^2 \times 1.28 = 0.64 \times 1.28 \\ = 0.8192 \text{ m}$$

Answer: 0.819 m

M1

A1

2 marks

27 (11 marks)

(a) Each impact multiplies the speed by e

M1

so after n impacts the speed is $e^n v_0$

A1

Answer: The speed with which the ball leaves the floor after the n th impact is $e^n v_0$

2 marks

(b) Time of flight for projection speed u is $\frac{2u}{g}$

M1

with $u = e^n v_0$

A1

Answer: The time between the n th and the $(n + 1)$ th impacts is $\frac{2e^n v_0}{g}$

2 marks

$$(c) \text{ Total} = \frac{v_0}{g} + \frac{2v_0}{g}(e + e^2 + e^3 + \dots)$$

M1 A1

$$= \frac{v_0}{g} \left[1 + \frac{2e}{(1 - e)} \right]$$

M1

$$= \frac{v_0}{g} \frac{(1 + e)}{(1 - e)}$$

A1

Answer: The total time from release until the ball comes to rest is $\frac{v_0}{g} \frac{(1 + e)}{(1 - e)}$

4 marks

$$(d) \text{ Time to fall} = \frac{v_0}{g}$$

B1

$$\frac{(1 + e)}{(1 - e)} = 10$$

M1

$$e = \frac{9}{11}$$

A1

Answer: $e = \frac{9}{11}$

3 marks

28 (12 marks)

$$(a) m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

M1

$$v_2 - v_1 = e(u_1 - u_2)$$

M1

$$\text{Eliminate } v_1: (m_1 + m_2)v_2 = m_1 u_1 + m_2 u_2 + e m_1 (u_1 - u_2)$$

A1

$$v_2 = \frac{m_1 u_1 + m_2 u_2 - e m_2 (u_1 - u_2)}{m_1 + m_2}$$

A1

$$\text{Answer: } v_B = \frac{m_1 u_1 + m_2 u_2 + e m_1 (u_1 - u_2)}{m_1 + m_2}, v_A = \frac{m_1 u_1 + m_2 u_2 - e m_2 (u_1 - u_2)}{m_1 + m_2}$$

4 marks

(b) Let $w = u_1 - u_2$. Total momentum P is conserved

B1

$$\text{Using the centre of mass velocity } V = \frac{P}{m_1 + m_2}: v_1 = V - \frac{e m_2 w}{m_1 + m_2}, v_2 = V + \frac{e m_1 w}{m_1 + m_2}$$

M1 A1

KE after – KE before: terms in V cancel by momentum conservation

M1

$$\text{Loss} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} w^2 (1 - e^2)$$

A1

Answer: The kinetic energy lost in the collision is $\frac{m_1 m_2}{2(m_1 + m_2)} (1 - e^2) (u_1 - u_2)^2$

5 marks

$$(c) \text{ Loss} = \frac{1}{2} \times \frac{6}{5} \times (1 - 0.16) \times 25$$

M1

$$= 12.6 \text{ J}$$

A1

Answer: 12.6 J

2 marks

(d) $e = 0$, since the factor $(1 - e^2)$ is greatest

E1

Answer: $e = 0$

1 mark

29 (12 marks)

(a) $v^2 = 2(9.8)(4.9)$, $v = 9.8 \text{ m s}^{-1}$

M1 A1

$$u^2 = 2(9.8)(1.1) = 21.56$$

M1

$$u = 4.64 \text{ m s}^{-1}$$

A1

Answer: 9.8 m s^{-1} , 4.64 m s^{-1}

4 marks

(b) $e = \frac{4.643}{9.8}$

M1

$$= 0.474$$

A1

Answer: 0.474

2 marks

(c) $I = 0.4(9.8 + 4.643)$

M1 A1

$$= 5.78 \text{ N s}$$

A1

$$\text{KE loss} = \frac{1}{2}(0.4)(9.8^2 - 21.56) = 14.9 \text{ J}$$

A1

Answer: 5.78 N s , 14.9 J

4 marks

(d) Fall: $t_1 = \frac{9.8}{9.8} = 1 \text{ s}$; rise: $t_2 = \frac{4.643}{9.8} = 0.4738 \text{ s}$

M1

$$\text{Total} = 1.47 \text{ s}$$

A1

Answer: 1.47 s

2 marks

30 (10 marks)

(a) $mu = mv_A + kmv_B$, $v_B - v_A = eu$

M1 M1

$$u = v_A + k(v_A + eu), \text{ so } v_A = \frac{u(1 - ke)}{(1 + k)}$$

A1

Answer: The velocity of A after the collision, in the original direction of motion, is $\frac{u(1 - ke)}{(1 + k)}$

3 marks

(b) $\frac{u(1 - ke)}{(1 + k)} = u \frac{\frac{1}{k} - e}{\frac{1}{k} + 1}$

M1

$$\rightarrow -eu \text{ as } k \rightarrow \infty$$

A1

A very massive particle is barely moved by the collision and acts like a fixed wall, from which A rebounds with speed eu

E1

Answer: $-eu$

3 marks

(c) $\frac{u(1 - 0.8k)}{(1 + k)} = -0.6u$

M1

$$1 - 0.8k = -0.6 - 0.6k$$

M1

$$k = 8$$

A1

Answer: $k = 8$

3 marks

(d) $v_A > 0$ when $1 - ke > 0$, that is $k < \frac{1}{e}$

E1

Answer: For $k < \frac{1}{e}$, $1 - ke > 0$, so A continues in its original direction

1 mark

Collisions with Fixed Surfaces**1 (1 mark)**

$$\text{Speed} = \frac{2}{3} \times 9 = 6 \text{ m s}^{-1}$$

B1

Answer: 6 m s^{-1}

1 mark

2 (2 marks)

$$\text{Parallel: } 10\cos 30^\circ = 5\sqrt{3} = 8.66 \text{ m s}^{-1}$$

B1

$$\text{Perpendicular: } 10\sin 30^\circ = 5 \text{ m s}^{-1}$$

B1

Answer: 8.66 m s^{-1} parallel, 5 m s^{-1} perpendicular

2 marks

3 (3 marks)

Parallel component unchanged: 6

B1

Perpendicular component: $0.5 \times 8 = 4$

M1

Speed = $\sqrt{6^2 + 4^2} = 7.21 \text{ m s}^{-1}$

A1

Answer: 7.21 m s⁻¹

3 marks

4 (2 marks)Parallel component unchanged and, with $e = 1$, the perpendicular component only reverses (same magnitude)

E1

so the velocity components keep the same ratio and the angle to the wall is unchanged

E1

Answer: Parallel component unchanged and perpendicular component reversed

2 marks

5 (2 marks)Speed after = 3 m s^{-1} , KE before = $\frac{1}{2}(0.5)(36) = 9 \text{ J}$, KE after = $\frac{1}{2}(0.5)(9) = 2.25 \text{ J}$

M1

Loss = 6.75 J

A1

Answer: 6.75 J

2 marks

6 (2 marks) $I = 0.4(3 + 5)$

M1

 $= 3.2 \text{ N s}$

A1

Answer: 3.2 N s

2 marks

7 (5 marks)(a) Parallel component: $10\cos 60^\circ = 5$ (unchanged)

B1

Perpendicular: $10\sin 60^\circ = 5\sqrt{3}$, after impact $0.5 \times 5\sqrt{3} = 4.33$

M1

Speed = $\sqrt{5^2 + 4.33^2} = 6.61 \text{ m s}^{-1}$

A1

Answer: 6.61 m s⁻¹

3 marks

(b) $\tan \beta = \frac{4.33}{5}$

M1

 $\beta = 40.9^\circ$

A1

Answer: 40.9°

2 marks

8 (3 marks)Parallel component $u \cos 60^\circ$ unchanged; perpendicular component multiplied by e

B1

 $\tan 30^\circ = \frac{e u \sin 60^\circ}{u \cos 60^\circ} = e \tan 60^\circ$

M1

 $e = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1}{3}$

A1

Answer: $e = \frac{1}{3}$

3 marks

9 (4 marks)(a) Parallel component = $25\cos \alpha = 15 \text{ m s}^{-1}$, perpendicular component = $25\sin \alpha = 20 \text{ m s}^{-1}$

B1

Perpendicular component after = $\sqrt{17^2 - 15^2} = 8 \text{ m s}^{-1}$

M1

 $e = \frac{8}{20} = 0.4$

A1

Answer: $e = 0.4$

3 marks

(b) $\arctan\left(\frac{8}{15}\right) = 28.1^\circ$

B1

Answer: 28.1°

1 mark

10 (4 marks)Perpendicular component before = $12\sin 30^\circ = 6 \text{ m s}^{-1}$

B1

Perpendicular component after = $0.5 \times 6 = 3 \text{ m s}^{-1}$ (reversed)

M1

 $I = 0.3(6 + 3)$

M1

 $= 2.7 \text{ N s}$

A1

Answer: 2.7 N s

4 marks

11 (4 marks)

$$\text{Time to reach } W_2 = \frac{4}{5} = 0.8 \text{ s}$$

B1

$$\text{Speed after } W_2 = 0.6 \times 5 = 3 \text{ m s}^{-1}$$

M1

$$\text{Time back to } W_1 = \frac{4}{3} \text{ s}$$

M1

$$\text{Total} = 2.13 \text{ s}$$

A1

Answer: 2.13 s

4 marks

12 (4 marks)

Horizontal component unchanged: 6

B1

Vertical component after = $0.5 \times 8 = 4$ upwards

M1

$$\text{Speed} = \sqrt{6^2 + 4^2} = 7.21 \text{ m s}^{-1}$$

A1

$$\tan \theta = \frac{4}{6}, \theta = 33.7^\circ$$

A1

Answer: 7.21 m s⁻¹ at 33.7° to the floor

4 marks

13 (3 marks)Perpendicular component $10\sin 60^\circ$: squared = 75; after impact squared = $0.25 \times 75 = 18.75$

M1 A1

$$\text{Loss} = \frac{1}{2}(2)(75 - 18.75) = 56.25 \text{ J}$$

A1

Answer: 56.25 J

3 marks

14 (8 marks)(a) Before: parallel $u \cos \theta$, towards the wall $u \sin \theta$. After: parallel $u \cos \theta$, away from the wall $eu \sin \theta$

B1

Velocities are perpendicular, so the scalar product is zero: $u^2 \cos^2 \theta - eu^2 \sin^2 \theta = 0$

M1

$$\tan^2 \theta = \frac{1}{e}, \text{ so } \tan \theta = \frac{1}{\sqrt{e}}$$

A1

$$\text{Answer: } \tan \theta = \frac{1}{\sqrt{e}}$$

3 marks

(b) $0 < e \leq 1$ gives $\tan \theta \geq 1$

M1

$$45^\circ \leq \theta < 90^\circ$$

A1

Answer: $45^\circ \leq \theta < 90^\circ$

2 marks

(c) $\tan \theta = 2$, $\theta = 63.4^\circ$

A1

$$\cos^2 \theta = \frac{1}{5}, \sin^2 \theta = \frac{4}{5}: \text{ speed after} = u \sqrt{\frac{1}{5} + \frac{1}{16} \times \frac{4}{5}}$$

M1

$$= u \sqrt{\frac{1}{4}} = \frac{u}{2}$$

A1

Answer: $\theta = 63.4^\circ$, speed $\frac{u}{2}$

3 marks

15 (7 marks)(a) W_1 : the **i** component is multiplied by -0.5 , the **j** component is unchanged: $(3\mathbf{i} - 8\mathbf{j})$

M1 A1

 W_2 : the **j** component is multiplied by -0.75 : $(3\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$

M1 A1

Answer: $(3\mathbf{i} - 8\mathbf{j}) \text{ m s}^{-1}$, then $(3\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$

4 marks

$$\text{(b) Speed} = \sqrt{9 + 36} = 6.71 \text{ m s}^{-1}$$

A1

$$\text{KE after / KE before} = \frac{45}{100}$$

M1

$$\text{Fraction lost} = 0.55$$

A1

Answer: 6.71 m s^{-1} , 0.55

3 marks

16 (4 marks)Parallel component $13\cos \alpha = 5$, perpendicular component 12

B1

$$\text{After: } 5^2 + (12e)^2 = 61$$

M1 A1

$$144e^2 = 36, e = 0.5$$

A1

Answer: $e = 0.5$

4 marks

17 (5 marks)

Let the speed before impact be v . Component along the plane: $v \sin \theta$ (unchanged) B1

Component perpendicular to the plane after impact: $0.25v \cos \theta$ M1

Vertical component after impact: $0.25v \cos \theta \cos \theta - v \sin \theta \sin \theta = 0$ M1 A1

$\tan^2 \theta = 0.25$, $\tan \theta = 0.5$, $\theta = 26.6^\circ$ A1

Answer: $\theta = 26.6^\circ$ 5 marks

18 (4 marks)

Time to reach the wall = $\frac{6}{8} = 0.75$ s M1 A1

Rebound speed = $0.75 \times 8 = 6$ m s⁻¹ B1

Distance after a further 0.75 s = $6 \times 0.75 = 4.5$ m A1

Answer: 4.5 m 4 marks

19 (6 marks)

Time to first impact: $1.5 = \frac{1}{2}(9.8)t^2$, $t = 0.5533$ s M1 A1

Vertical speed on impact = $9.8t = 5.422$ m s⁻¹ A1

Vertical speed after = $0.5 \times 5.422 = 2.711$ m s⁻¹ M1

Time between impacts = $\frac{2 \times 2.711}{9.8} = 0.5533$ s A1

Distance = $8(0.5533 + 0.5533) = 8.85$ m A1

Answer: 8.85 m 6 marks

20 (5 marks)

The wall is smooth so the impulse acts perpendicular to the wall E1

Perpendicular component before = $u \sin \alpha$, after = $eu \sin \alpha$ in the opposite direction M1

$I = m(eu \sin \alpha + u \sin \alpha) = mu(1 + e) \sin \alpha$ A1

$I = 0.25 \times 14 \times \sin 30^\circ \times 1.4$ M1

= 2.45 N s A1

Answer: $mu(1 + e) \sin \alpha$, perpendicular to the wall; 2.45 N s 5 marks

21 (6 marks)

(a) Time to $W_1 = \frac{d}{u}$; speed after = $\frac{2u}{3}$, time from W_1 to $W_2 = \frac{3}{\frac{2u}{3}} = \frac{9}{2u}$ M1 A1

Speed after $W_2 = \frac{4u}{9}$, time from W_2 to O = $\frac{3-d}{\frac{4u}{9}} = \frac{9(3-d)}{4u}$ M1

$T = \frac{d}{u} + \frac{9}{2u} + \frac{9(3-d)}{4u} = \frac{4d + 18 + 27 - 9d}{4u} = \frac{45 - 5d}{4u}$ A1

Answer: $T = \frac{45 - 5d}{4u}$ 4 marks

(b) $45 - 5d = 36$ M1

$d = 1.8$ A1

Answer: $d = 1.8$ 2 marks

22 (8 marks)

(a) First wall: the $\mathbf{i}$ component is reversed and multiplied by e : $(12\mathbf{i} - 5\mathbf{j})$ M1 A1

Second wall: the $\mathbf{j}$ component is reversed and multiplied by e : $(12e\mathbf{i} + 5e\mathbf{j})$ M1

= $e(12\mathbf{i} + 5\mathbf{j})$, which is parallel to $-(-12\mathbf{i} - 5\mathbf{j})$ A1

Answer: The velocity after both impacts is $e(12\mathbf{i} + 5\mathbf{j})$ m s⁻¹, which is parallel to the original velocity and in the opposite direction. 4 marks

(b) Initial speed 13, final speed $0.4 \times 13 = 5.2$ m s⁻¹ M1 A1

KE retained = $e^2 = 0.16$, so 84% lost A1

Answer: 5.2 m s⁻¹, 84% 3 marks

(c) $13e = 8.45$, $e = 0.65$ B1

Answer: 0.65 1 mark

23 (7 marks)

(a) Time to wall = $\frac{8}{10} = 0.8$ s

B1

Fall = $\frac{1}{2}(9.8)(0.8)^2 = 3.136$ m

M1

Height = $5 - 3.136 = 1.864$ m

A1

Answer: 1.864 m

3 marks

(b) Horizontal speed after = $0.6 \times 10 = 6 \text{ m s}^{-1}$; vertical speed at the wall = $9.8 \times 0.8 = 7.84 \text{ m s}^{-1}$

B1 B1

$1.864 = 7.84t + 4.9t^2$

M1

$t = 0.2102$ s, distance = $6t = 1.26$ m

A1

Answer: 1.26 m

4 marks

24 (7 marks)

(a) $t = \frac{4}{8} = 0.5$ s

M1

Parallel distance = $6 \times 0.5 = 3$ m

A1

Answer: 0.5 s, 3 m

2 marks

(b) Perpendicular speed after = $0.5 \times 8 = 4 \text{ m s}^{-1}$

M1

Speed = $\sqrt{6^2 + 4^2} = 7.21 \text{ m s}^{-1}$

A1

Answer: 7.21 m s⁻¹

2 marks

(c) Time from W_1 to $W_2 = \frac{12}{4} = 3$ s

M1

Total time = $0.5 + 3 = 3.5$ s

A1

Parallel distance = $6 \times 3.5 = 21$ m

A1

Answer: 3.5 s, 21 m

3 marks

25 (6 marks)

(a) Only the component perpendicular to the wall is multiplied by e ; the parallel component is unchanged

E1

So the speed is not simply multiplied by 0.6, and the angle changes because $\tan \beta = e \tan \alpha$

E1

Answer: Only the perpendicular component is reduced

2 marks

(b) $\tan \beta = 0.6 \tan 40^\circ$

M1

$\beta = 26.7^\circ$

A1

Speed ratio = $\sqrt{\cos^2 40^\circ + 0.36 \sin^2 40^\circ}$
= 0.858

M1

A1

Answer: 26.7°, ratio 0.858

4 marks

26 (7 marks)

(a) Before: parallel $u \cos 60^\circ = \frac{u}{2}$, perpendicular (towards the wall) $u \sin 60^\circ = \frac{\sqrt{3}u}{2}$

B1

After: parallel $\frac{u}{2}$, perpendicular (away from the wall) $\frac{\sqrt{3}eu}{2}$

B1

Answer: Before: parallel $\frac{u}{2}$, perpendicular $\frac{\sqrt{3}u}{2}$; after: parallel $\frac{u}{2}$, perpendicular $\frac{\sqrt{3}eu}{2}$ away from the wall

2 marks

(b) Taking the perpendicular to the wall as positive towards the wall: before $(\frac{u}{2}, \frac{\sqrt{3}u}{2})$, after $(\frac{u}{2}, -\frac{\sqrt{3}eu}{2})$

M1

Scalar product = $\frac{u^2}{4} - \frac{3eu^2}{4} = 0$

M1

$e = \frac{1}{3}$

A1

Answer: $e = \frac{1}{3}$

3 marks

(c) Speed = $u\sqrt{\frac{1}{4} + \frac{3}{4} \times \frac{1}{9}}$

M1

$$= u\sqrt{1/3} = 0.577u$$

Answer: 0.577u

A1

2 marks

27 (10 marks)

(a) $mu = mv_A + 3mv_B, v_B - v_A = \frac{u}{2}$

M1 M1

$$u = v_A + 3(v_A + \frac{u}{2})$$

M1

$$v_A = -\frac{u}{8}, v_B = \frac{3u}{8}$$

A1

Answer: A: $\frac{u}{8}$ away from the wall, B: $\frac{3u}{8}$ towards the wall

4 marks

(b) $\frac{1}{2} \times \frac{3u}{8} = \frac{3u}{16}$

B1

Answer: $\frac{3u}{16}$

1 mark

(c) B moves away from the wall at $\frac{3u}{16}$; A moves away from the wall at $\frac{u}{8} = \frac{2u}{16}$

B1

$$\frac{3u}{16} > \frac{2u}{16} \text{ so B catches A}$$

E1

Taking the direction away from the wall as positive: $3m(\frac{3u}{16}) + m(\frac{2u}{16}) = 3mv_B + mv_A$

M1

$$v_A - v_B = \frac{1}{2}(\frac{3u}{16} - \frac{2u}{16}) = \frac{u}{32}$$

M1

$$v_B = \frac{21u}{128}, v_A = \frac{25u}{128}, \text{ both away from the wall}$$

A1

Answer: A: $\frac{25u}{128}$, B: $\frac{21u}{128}$, both away from the wall

5 marks

28 (10 marks)

(a) Speed on the nth crossing $= 6 \times 0.5^{n-1} = 6 \times 2^{1-n}$

M1 A1

$$\text{Time} = \frac{3}{6 \times 2^{1-n}} = 0.5 \times 2^{n-1} \text{ s}$$

A1

Answer: The time taken for the particle to move between the walls on its nth crossing is $0.5 \times 2^{n-1}$ seconds

6 marks

(b) Geometric series with first term 0.5 and common ratio 2

M1

$$S_n = \frac{0.5(2^n - 1)}{2 - 1}$$

M1

$$= 0.5(2^n - 1)$$

A1

Answer: The total time for the first n crossings is $0.5(2^n - 1)$ seconds

3 marks

(c) The time for the nth crossing $0.5 \times 2^{n-1}$ increases without limit, so the total time tends to infinity

E1

The distance of each crossing is always 3 m, so the particle always has further to go

E1

Answer: Total time is unbounded

2 marks

(d) $0.5(2^n - 1) > 100, 2^n > 201$

M1

$$n = 8$$

A1

Answer: n = 8

2 marks

29 (11 marks)

(a) $x = 6 - 6t = 0, t = 1 \text{ s}$

M1

$$y = 8, \text{ so B is } (0, 8)$$

A1

Answer: B(0, 8), 1 s

2 marks

(b) After the impact the velocity is $(6\mathbf{i} + 8\mathbf{j})$

B1

$$\text{Time from B to C: } y: 16 = 8 + 8t, t = 1 \text{ s}$$

M1

$$x: 3 = 6e \times 1, e = 0.5$$

A1

Answer: e = 0.5

3 marks

(c) Velocity $(3\mathbf{i} + 8\mathbf{j})$

M1

$$\text{Speed} = \sqrt{9 + 64} = 8.54 \text{ m s}^{-1}$$

A1

Answer: 8.54 m s⁻¹

2 marks

(d) $x = 6e t$ gives $3t = 9$ from B, so $t = 3$ s after B

M1

$$y = 8 + 8 \times 3 = 32$$

A1

Velocity before $(3\mathbf{i} + 8\mathbf{j})$; the $\mathbf{i}$ component is reversed and multiplied by 0.8

M1

$$\text{Velocity after} = (-2.4\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$$

A1

Answer: $(-2.4\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$, $y = 32$

4 marks

30 (11 marks)

(a) $v^2 = 2 \times 9.8 \times 4.9$

M1

$$v = 9.8 \text{ m s}^{-1}$$

A1

Answer: 9.8 m s^{-1}

2 marks

(b) Parallel (down the plane) unchanged: $9.8\sin 30^\circ = 4.9 \text{ m s}^{-1}$

B1

Perpendicular before: $9.8\cos 30^\circ = 8.487$

M1

Perpendicular after $= 0.5 \times 8.487 = 4.244 \text{ m s}^{-1}$ away from the plane

A1

Answer: 4.9 m s^{-1} down the plane, 4.244 m s^{-1} perpendicular to the plane

3 marks

(c) Perpendicular to the plane the acceleration is $-g \cos 30^\circ$

B1

$$0 = 4.244 t - \frac{1}{2}(9.8\cos 30^\circ)t^2$$

M1

$$t = \frac{2 \times 4.244}{9.8\cos 30^\circ} = 1 \text{ s}$$

A1

Answer: The ball returns to the plane after exactly 1 second

3 marks

(d) Parallel to the plane: initial speed 4.9, acceleration $g \sin 30^\circ = 4.9$

M1

$$s = 4.9(1) + \frac{1}{2}(4.9)(1^2)$$

M1 A1

$$= 7.35 \text{ m}$$

Answer: 7.35 m

3 marks