

Linear Momentum

- 1 A particle of mass 4 kg moves in a straight line with speed 7 m s^{-1} . Find the magnitude of its momentum. [1]
- 2 A ball of mass 0.35 kg travels at 12 m s^{-1} . Find its momentum. [1]
- 3 A car of mass 1500 kg travels at 72 km per hour. Find its momentum in N s. [2]
- 4 A particle of mass 9 kg has momentum of magnitude 36 N s. Find its speed. [1]
- 5 A particle P of mass 3 kg moves along a straight line with velocity -5 m s^{-1} , where the positive direction is to the right. Write down the momentum of P, stating its direction. [1]
- 6 Two particles move along a straight line. A has mass 2 kg and moves right at 3 m s^{-1} ; B has mass 5 kg and moves left at 2 m s^{-1} . Taking right as positive, find the total momentum of the system. [2]
- 7 A particle of mass 2.5 kg has velocity $(4\mathbf{i} - 6\mathbf{j}) \text{ m s}^{-1}$. Find the momentum of the particle and the magnitude of its momentum, giving the magnitude to 3 significant figures. [3]
- 8 A cricket ball of mass 0.16 kg is travelling at 20 m s^{-1} when it is struck by a bat. It leaves the bat at 30 m s^{-1} in exactly the opposite direction. Find the magnitude of the change in momentum of the ball. [3]
- 9 A particle of mass 6 kg increases its speed in a straight line from 5 m s^{-1} to 8 m s^{-1} without changing direction. Find the increase in its momentum. [2]
- 10 A cyclist and bicycle have a combined mass of 90 kg and travel at 8 m s^{-1} .
 - (a) Find the momentum of the cyclist and bicycle. [1]
 - (b) A runner of mass 60 kg has the same momentum as the cyclist. Find the speed of the runner. [2]
- 11 A particle of mass 0.6 kg moves with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$.
 - (a) Find the speed of the particle. [1]
 - (b) Find the momentum of the particle as a vector, and hence the magnitude of the momentum. [2]
- 12 Three particles of masses 2 kg, 3 kg and 5 kg move along a straight line with velocities 4 m s^{-1} , -2 m s^{-1} and 1 m s^{-1} respectively.
 - (a) Find the total momentum of the three particles. [2]
 - (b) A single particle of mass 10 kg has the same momentum as the three particles together. Find its velocity. [1]
- 13 A particle of mass 2 kg moves along the x-axis. At time t seconds its velocity is $(3t^2 - 4t) \text{ m s}^{-1}$ in the positive x direction.
 - (a) Find the momentum of the particle when $t = 2$. [2]
 - (b) Find the non-zero time at which the momentum of the particle is zero. [2]
- 14 A particle of mass 1.5 kg moves so that its position vector at time t seconds is $\mathbf{r} = (t^3\mathbf{i} + 2t^2\mathbf{j}) \text{ m}$.
 - (a) Find the momentum of the particle when $t = 2$. [4]
 - (b) Find the magnitude of this momentum. [2]

- 15** The momentum of a particle at time t seconds is $\mathbf{p} = (6t^2\mathbf{i} + 4t\mathbf{j}) \text{ N s}$.
- (a) Find the force acting on the particle at time t , using the fact that force is the rate of change of momentum. [2]
- (b) Find the magnitude of the force when $t = 3$. [2]
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- 16** A particle has mass m , speed v and momentum of magnitude p .
- (a) Show that the kinetic energy of the particle is $\frac{p^2}{2m}$. [2]
- (b) A particle of mass 5 kg has kinetic energy 250 J. Find the magnitude of its momentum. [2]
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- 17** Particle A has mass m and particle B has mass $3m$. The two particles have momenta of equal magnitude.
Show that the kinetic energy of A is three times the kinetic energy of B. [3]
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- 18** Grain falls vertically onto a horizontal conveyor belt at a rate of 12 kg per second. The belt moves horizontally at a constant speed of 1.5 m s^{-1} and the grain is carried along with it without slipping.
Find the horizontal force that the belt exerts on the grain in the steady state. [3]
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- 19** A particle of mass 4 kg moves with velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$. A force changes its momentum to $(14\mathbf{i} + 6\mathbf{j}) \text{ N s}$.
- (a) Find the new velocity of the particle. [2]
- (b) Find the magnitude of the change in momentum. [3]
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- 20** A ball of mass 2 kg is projected from level ground with speed 10 m s^{-1} at 30° above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the horizontal and vertical components of the initial momentum of the ball. [2]
- (b) Find the time after projection at which the ball's vertical momentum is zero. [3]
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- 21** Particle P has mass 2 kg and moves due west at 3 m s^{-1} . Particle Q has mass 3 kg and moves due east at 2 m s^{-1} . A student writes: "Total momentum = $2 \times 3 + 3 \times 2 = 12 \text{ N s}$."
- (a) Explain the mistake in the student's working. [2]
- (b) Find the correct total momentum of the two particles. [2]
- (c) A third particle R of mass 5 kg is added so that the total momentum of the three particles is 20 N s due east. Find the velocity of R. [2]
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- 22** A particle of mass 0.8 kg moves in a straight line. At time t seconds its velocity is $6e^{-t/2} \text{ m s}^{-1}$.
- (a) Find the initial momentum of the particle. [1]
- (b) Find the time at which the momentum of the particle is 2.4 N s . [3]
- (c) Find the rate of change of momentum when $t = 2$. [3]
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- 23** Particle A has mass 2 kg and velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Particle B has mass 3 kg and velocity $(-\mathbf{i} + k\mathbf{j}) \text{ m s}^{-1}$, where k is a constant. The total momentum of A and B is parallel to $\mathbf{i}$.
- (a) Find the value of k . [3]
- (b) Find the total momentum of A and B. [2]
- (c) Find the speed of B. [2]
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- 24** Particle P has mass 2 kg and moves due east at 6 m s^{-1} . Particle Q has mass 5 kg and moves due north at 3 m s^{-1} . Find the magnitude and direction of the total momentum of the two particles, giving the direction as a bearing to the nearest 0.1° . [6]
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- 25** The kinetic energy of a particle of constant mass increases by 44%.
- (a) Find the percentage increase in the magnitude of the momentum of the particle. [4]
- (b) A different particle has its mass multiplied by 4 while its momentum is unchanged. Find the factor by which its kinetic energy changes. [2]

- 26** Particle A has mass 3 kg and velocity $(2\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$. Particle B has mass 4 kg. The total momentum of A and B is $(14\mathbf{i} - 10\mathbf{j}) \text{ N s}$. Find the speed of B. [7]
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- 27** A particle of mass 2 kg moves in a plane. At time t seconds its velocity is $\mathbf{v} = (4\cos 2t \mathbf{i} + 4\sin 2t \mathbf{j}) \text{ m s}^{-1}$.
- (a) Find the magnitude of the momentum of the particle at time t . [2]
 - (b) Find the force acting on the particle at time t . [2]
 - (c) Show that the force is perpendicular to the velocity at every instant. [2]
 - (d) Find the magnitude of the force. [1]
 - (e) Explain, in terms of the work done by the force, why the speed of the particle remains constant. [2]
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- 28** Sand falls vertically from a hopper onto a horizontal conveyor belt at a constant rate of $k \text{ kg per second}$. The belt is driven by a motor at a constant speed V . The sand is carried along by the belt without slipping, and the sand has no horizontal motion before landing on the belt.
- (a) Show that the horizontal force needed to keep the sand moving with the belt is kV . [3]
 - (b) Hence show that the power which the motor must supply to keep the belt moving at constant speed is kV^2 . [2]
 - (c) Find the rate at which the kinetic energy of the sand increases, and hence find the fraction of the motor's power which is not used to increase the kinetic energy of the sand. [3]
 - (d) Suggest what happens to this lost energy. [1]
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- 29** A particle of mass 2 kg moves in a plane with velocity $\mathbf{v} = ((t - 3)\mathbf{i} + (t - 4)\mathbf{j}) \text{ m s}^{-1}$ at time t seconds, for $t \geq 0$.
- (a) Show that the square of the magnitude of the momentum of the particle is $(8t^2 - 56t + 100) \text{ N}^2 \text{ s}^2$. [3]
 - (b) Find the least magnitude of the momentum and the time at which this occurs. [4]
 - (c) Find the angle between the direction of motion and the direction of $\mathbf{i}$ at the instant of least momentum. [2]
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- 30** A hose directs a horizontal jet of water, of cross-sectional area $4.0 \times 10^{-4} \text{ m}^2$, at a speed of 12 m s^{-1} towards a smooth vertical wall. The density of water is 1000 kg m^{-3} .
- (a) Find the mass of water leaving the hose per second. [2]
 - (b) The jet strikes the wall at right angles and the water stops dead. Find the force exerted by the water on the wall. [2]
 - (c) The hose is turned so that the jet strikes the wall with its direction making an angle of 60° with the normal to the wall. The wall is smooth, so the water then flows away along the wall. Find the force exerted by the water on the wall. [3]
 - (d) Explain why there is no force on the wall parallel to the wall in part (c). [1]

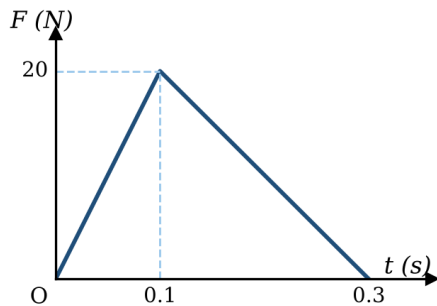
Impulse

- 1** A constant force of 12 N acts on a particle for 0.5 s. Find the magnitude of the impulse of the force. [1]
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- 2** The speed of a particle of mass 2 kg changes from 3 m s^{-1} to 8 m s^{-1} in a straight line without change of direction. Find the magnitude of the impulse acting on the particle. [2]
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- 3** A particle of mass 3 kg is at rest on a smooth horizontal surface. It receives a horizontal impulse of magnitude 15 N s. Find the speed of the particle immediately afterwards. [2]

- 4** A force of 30 N acts for 0.2 s on a stationary ball of mass 0.4 kg. Find the speed of the ball after the force has acted. [2]
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- 5** A stationary ball of mass 0.5 kg is struck and moves off at 14 m s^{-1} . Find the magnitude of the impulse given to the ball. [1]
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- 6** A particle of mass 4 kg is moving at 6 m s^{-1} . It receives an impulse of magnitude 8 N s in the direction of its motion. Find its new speed. [2]
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- 7** A ball of mass 0.25 kg is moving at 12 m s^{-1} when it is hit by a bat. It moves off in the opposite direction at 8 m s^{-1} . Find the magnitude of the impulse the bat exerts on the ball. [3]
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- 8** A particle of mass 3 kg has velocity $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. It receives an impulse $(6\mathbf{i} - 9\mathbf{j}) \text{ N s}$. Find the new velocity of the particle and its speed to 3 significant figures. [3]
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- 9** A particle of mass 2 kg is initially at rest. A force $(4\mathbf{i} - 2\mathbf{j}) \text{ N}$ acts on it for 3 s. Find the impulse of the force, and the speed of the particle at the end of the 3 s. [4]
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- 10** A ball of mass 0.4 kg hits a horizontal floor with a vertical speed of 6 m s^{-1} and rebounds vertically with speed 4.5 m s^{-1} .
 (a) Find the magnitude and direction of the impulse that the floor exerts on the ball. [3]
 (b) The ball is in contact with the floor for 0.02 s. Find the average force exerted by the floor on the ball. [2]
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- 11** A particle of mass 3 kg is moving on a smooth horizontal plane with speed 2.5 m s^{-1} when it receives an impulse of magnitude $I \text{ N s}$ in a direction perpendicular to its direction of motion. Immediately afterwards the speed of the particle is 6.5 m s^{-1} .
 (a) Find the value of I . [3]
 (b) Find the angle through which the direction of motion of the particle is turned. [2]
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- 12** A car of mass 1200 kg is travelling at 15 m s^{-1} on a straight horizontal road. The brakes apply a constant resistive force of 3000 N until the car stops.
 (a) Find the magnitude of the impulse of the braking force. [2]
 (b) Find the time taken for the car to stop. [2]
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- 13** A particle of mass 3 kg is at rest on a smooth horizontal surface. A horizontal force of magnitude $6t$ newtons acts on it for 4 s, where t is the time in seconds from the start of the motion.
 (a) Find the magnitude of the impulse of the force. [3]
 (b) Find the speed of the particle at the end of the 4 s. [2]
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- 14** A particle of mass 4 kg is moving in a straight line at 2 m s^{-1} . A force of magnitude $(12t - 3t^2) \text{ N}$ then acts on it in the direction of motion, for $0 \leq t \leq 4$, where t is the time in seconds after the force starts to act.
 (a) Show that the force is never negative during this time. [2]
 (b) Find the speed of the particle when $t = 4$. [4]
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- 15** A tennis ball of mass 0.15 kg is at rest when it is hit by a racket. The graph shows the force F newtons exerted on the ball by the racket for the 0.3 s that they are in contact. The graph consists of two straight lines.

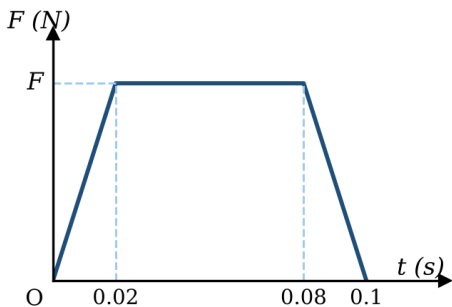
Assume that gravity can be ignored during the contact.



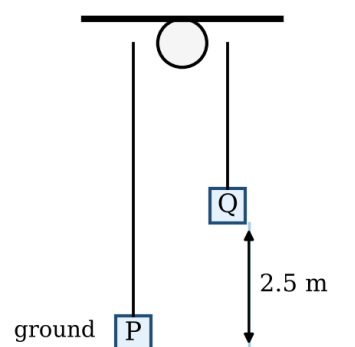
- (a) Find the magnitude of the impulse exerted by the racket on the ball. [2]
 (b) Find the speed of the ball as it leaves the racket. [2]
 (c) Find the average force exerted by the racket over the 0.3 s . [1]
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- 16** A cricket ball of mass 0.16 kg is moving horizontally at 30 m s^{-1} when it is struck by a bat. Immediately afterwards the ball is moving horizontally at 20 m s^{-1} in a direction that makes an angle of 100° with its original direction of motion. Ignore the weight of the ball.
- (a) Find the magnitude of the impulse that the bat exerts on the ball. [3]
 (b) Find the acute angle between the line of the impulse and the original direction of motion of the ball. [2]
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- 17** A particle of mass 2 kg moves with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. It is struck by a blow which changes its velocity to $(-\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$.
- (a) Find the impulse of the blow as a vector. [3]
 (b) Find the magnitude of the impulse. [2]
 (c) Find the angle between the impulse and the original direction of motion. [2]
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- 18** A particle of mass 2 kg is moving with velocity $5\mathbf{i} \text{ m s}^{-1}$ on a smooth horizontal plane. It receives an impulse of magnitude 12 N s in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$. Find the speed of the particle immediately after the impulse. [5]
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- 19** A ball of mass 0.3 kg is released from rest 4.9 m above a horizontal floor. It rebounds to a height of 2.5 m . Take $g = 9.8 \text{ m s}^{-2}$. Find the magnitude of the impulse exerted by the floor on the ball. [5]
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- 20** A bullet of mass 0.02 kg travelling horizontally at 300 m s^{-1} hits a fixed block of wood and penetrates a distance of 0.15 m before coming to rest. Assume that the resistive force of the wood is constant.
- (a) Find the magnitude of the impulse exerted by the wood on the bullet. [2]
 (b) Find the resistive force exerted by the wood on the bullet. [3]
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- 21** Particles A and B, of masses 2 kg and 3 kg , are connected by a light inextensible string. They lie on a smooth horizontal table with the string slack. A is projected directly away from B at 9 m s^{-1} while B is at rest. The string becomes taut.



- (a) Find the common speed of the particles immediately after the string becomes taut. [4]
 (b) Find the magnitude of the impulse in the string. [2]

- 22** A particle of mass 2 kg moves with velocity $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ on a smooth horizontal plane. It receives an impulse $(4\mathbf{i} + 3\mathbf{j}) \text{ N s}$, and later receives a second impulse $\mathbf{J} \text{ N s}$, after which its velocity is $(6\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$.
- Find the velocity of the particle after the first impulse. [2]
 - Find $\mathbf{J}$. [3]
 - Find the magnitude of the total impulse received and the angle it makes with $\mathbf{i}$. [3]
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- 23** Particles A and B, of masses 3 kg and 2 kg, are joined by a light inextensible string of length 2 m. They lie at rest on a smooth horizontal table with $AB = 1 \text{ m}$, so the string is slack. B is projected with speed 10 m s^{-1} in a direction perpendicular to AB. The string becomes taut at the instant when the particles are 2 m apart.
- Show that, at the instant the string becomes taut, the string makes an angle of 60° with the line AB, and find the components of the velocity of B along the string and perpendicular to the string just before the string becomes taut. [3]
 - Find the magnitude of the impulsive tension in the string and the speed of B immediately after the string becomes taut. [5]
 - Find the kinetic energy lost when the string becomes taut. [2]
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- 24** A particle P of mass 0.5 kg is moving with velocity $6\mathbf{i} \text{ m s}^{-1}$ on a smooth horizontal plane when it receives an impulse $\lambda(-\mathbf{i} + \mathbf{j}) \text{ N s}$, where λ is a positive constant. Immediately after receiving the impulse, the speed of P is $\sqrt{20} \text{ m s}^{-1}$.
- Find the two possible values of λ . [4]
 - For each value of λ find the angle through which the direction of motion of P is turned by the impulse. [3]
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- 25** A ball of mass 0.5 kg hits a smooth wall at right angles at 8 m s^{-1} and rebounds at 6 m s^{-1} . A student writes: "Impulse = $0.5 \times 8 - 0.5 \times 6 = 1 \text{ N s}$."
- Explain the error in the student's working. [2]
 - Find the correct magnitude of the impulse. [2]
 - The ball is in contact with the wall for 0.01 s. Find the average force exerted by the wall on the ball. [2]
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- 26** A ball of mass 0.4 kg is travelling horizontally at 10 m s^{-1} towards a bat. The graph shows the horizontal force F newtons exerted by the bat on the ball. The force rises uniformly to a maximum value F in 0.02 s, remains constant for 0.06 s, then falls uniformly to zero in 0.02 s. The ball leaves the bat at 20 m s^{-1} in the opposite direction. Ignore the weight of the ball.
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- Find the magnitude of the impulse exerted on the ball. [2]
 - Show that the area under the graph is $0.08F$ and hence find the value of F . [4]
 - Find the average force exerted by the bat on the ball over the 0.1 s of contact. [2]

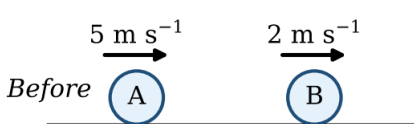
- 27 Particle P of mass 2 kg lies on horizontal ground. Particle Q of mass 3 kg is 2.5 m above the ground. P and Q are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The string is initially slack, with a length of 0.9 m of slack. Q is released from rest and falls vertically. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the speed of Q at the instant before the string becomes taut. [2]
 (b) Find the speed of the particles immediately after the string becomes taut. [3]
 (c) Find the magnitude of the impulsive tension in the string. [2]
 (d) Find the speed of Q when it hits the ground. [3]
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- 28 A particle of mass 4 kg moves on a smooth horizontal surface with speed 2 m s^{-1} . At time $t = 0$ a horizontal force of magnitude $kt(6 - t)$ newtons, where k is a constant, begins to act on the particle in the direction opposite to its initial motion. The force acts for $0 \leq t \leq 6$ and then ceases. When the force ceases the particle is moving at 5 m s^{-1} in the direction of the force.
- (a) Show that the magnitude of the impulse of the force is $36k$ newton seconds. [3]
 (b) Find the value of k . [3]
 (c) Find the time at which the particle is momentarily at rest, giving your answer to 3 significant figures. [4]
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- 29 A particle of mass 2 kg is at rest on a smooth horizontal table. It receives an impulse of magnitude 20 N s directed at 30° below the horizontal, so that the particle moves along the table.
- (a) Find the speed with which the particle begins to move along the table. [3]
 (b) Find the magnitude of the impulse exerted on the particle by the table. [3]
 (c) The same impulse is now applied at 30° above the horizontal. Explain what happens to the particle and find its initial speed. [3]
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- 30 A golf ball of mass 0.4 kg is at rest on level ground. It is struck and receives an impulse of magnitude 5 N s directed at an angle θ above the horizontal, where $0^\circ < \theta < 90^\circ$. The ball lands on the ground at a horizontal distance of 10 m from the point where it was struck. Model the ball as a particle moving freely under gravity, and take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the speed of projection. [2]
 (b) Show that $\sin 2\theta = 0.627$, correct to 3 significant figures. [3]
 (c) Find the two possible values of θ , and the corresponding times of flight. [4]

Conservation of Momentum

- 1 A particle of mass 3 kg moving at 4 m s^{-1} collides with a stationary particle of mass 2 kg. The particles coalesce. Find their common speed. [2]
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- 2 Particle A of mass 2 kg moves right at 5 m s^{-1} and particle B of mass 3 kg moves left at 5 m s^{-1} along the same straight line. They collide and coalesce. Find the velocity of the combined particle, stating its direction. [3]

- 3** A gun of mass 4 kg fires a bullet of mass 0.05 kg horizontally at 400 m s^{-1} . The gun is free to move on a smooth horizontal surface. Find the speed of recoil of the gun. [2]
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- 4** A railway truck of mass 12 000 kg moving at 3 m s^{-1} couples to a stationary truck of mass 8000 kg. Find the speed of the two trucks after coupling. [2]
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- 5** A particle of mass $2m$ moving at speed u collides with a stationary particle of mass m and they coalesce. Find the speed of the combined particle in terms of u . [2]
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- 6** Particle A of mass 4 kg moving at 3 m s^{-1} collides directly with particle B of mass 2 kg moving in the opposite direction at 2 m s^{-1} . After the collision A moves at 1 m s^{-1} in its original direction. Find the velocity of B after the collision, in the original direction of A. [3]
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- 7** A skater of mass 60 kg stands at rest on smooth ice and throws a ball of mass 2 kg horizontally at 9 m s^{-1} . Find the speed at which the skater moves backwards. [2]
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- 8** Two railway wagons A and B, of masses 800 kg and 1200 kg, move in the same direction along a straight horizontal track at 5 m s^{-1} and 2 m s^{-1} respectively. They collide and couple together.
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- 5 m s^{-1} 2 m s^{-1}
 $\xrightarrow{\hspace{1cm}}$ $\xrightarrow{\hspace{1cm}}$
Before (A) (B)
- (a) Find the speed of the wagons after the collision. [2]
 (b) Find the kinetic energy lost in the collision. [3]
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- 9** A shell of mass 8 kg is at rest when it explodes into two parts of masses 3 kg and 5 kg. The 3 kg part moves off with speed 20 m s^{-1} . Find the speed of the 5 kg part. [3]
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- 10** A bullet of mass 0.02 kg moving horizontally at 250 m s^{-1} embeds itself in a block of mass 4.98 kg which is at rest on a smooth horizontal surface. Find the speed of the block immediately afterwards. [2]
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- 11** A particle A of mass 2 kg moving at 6 m s^{-1} collides with a stationary particle B of mass m kg and they coalesce. The speed of the combined particle is 4 m s^{-1} . Find m . [3]
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- 12** Particles A and B have masses 3 kg and 5 kg and move towards each other along a straight line with speeds 4 m s^{-1} and 2 m s^{-1} . They collide and coalesce. Find the speed of the combined particle and the direction in which it moves relative to the original direction of A. [3]
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- 13** A particle of mass 4 kg moving at 5 m s^{-1} collides with a particle of mass 6 kg moving at 1 m s^{-1} in the same direction. After the collision the 4 kg particle moves at 2 m s^{-1} in the same direction. Find the speed of the 6 kg particle after the collision. [3]
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- 14** Particle A of mass 2 kg has velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ and particle B of mass 3 kg has velocity $(-2\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ on a smooth horizontal plane. They collide and coalesce.
 (a) Find the velocity of the combined particle. [3]
 (b) Find the loss of kinetic energy in the collision. [3]
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- 15** Three railway trucks A, B and C have masses 2000 kg, 3000 kg and 5000 kg. Truck A moves at 6 m s^{-1} and couples to the stationary truck B. The coupled pair then collides with the stationary truck C and couples to it.
 (a) Find the speed of A and B immediately after they couple. [2]
 (b) Find the speed of all three trucks after C is coupled. [2]
 (c) Find the total loss of kinetic energy in the two couplings. [3]
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- 16** A man of mass 70 kg stands at the stern of a stationary boat of mass 210 kg on still water. He jumps horizontally off the boat so that his speed relative to the boat is 4 m s^{-1} . Ignore water resistance. [5]
- 17** A firework of mass 6 kg is moving horizontally at 4 m s^{-1} when it explodes into two parts of masses 2 kg and 4 kg. The 2 kg part continues in the original direction of motion at 10 m s^{-1} .
 (a) Find the velocity of the 4 kg part immediately after the explosion. [3]
 (b) Find the kinetic energy gained in the explosion. [3]
- 18** A cannon of mass 200 kg stands on smooth horizontal ground. It fires a shell of mass 5 kg with speed 80 m s^{-1} relative to the ground, at an angle of 30° above the horizontal. Find the initial speed of recoil of the cannon. [4]
- 19** Particle A of mass m and particle B of mass $2m$ move towards each other along a straight line on a smooth horizontal surface with speeds $3u$ and u respectively. After the collision A moves with speed u in the direction opposite to its original direction of motion.
 (a) Find the speed and direction of B after the collision. [3]
 (b) Show that the fraction of the total kinetic energy lost in the collision is $\frac{8}{11}$. [3]
- 20** A man of mass 80 kg stands on a stationary sledge of mass 20 kg on smooth ice. He holds two rocks, each of mass 5 kg. He throws the rocks one after the other in the same horizontal direction, each with speed 6 m s^{-1} relative to the ice. Find the speed of the man and the sledge after both rocks have been thrown. [6]
- 21** Particle A of mass m moving at speed $5u$ collides directly with a stationary particle B of mass $2m$ on a smooth horizontal surface. After the collision A moves in the opposite direction with speed u . B then collides with a stationary particle C of mass $3m$, and B and C coalesce.
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- Before
- (a) Find the speed of B immediately after its collision with A. [3]
 (b) Find the speed of the combined particle formed from B and C. [2]
 (c) Find the fraction of the initial kinetic energy of the system that has been lost by the end of both collisions. [4]
- 22** Particle A of mass $2m$ is moving with speed ku and particle B of mass m is moving with speed u in the opposite direction, along a straight line on a smooth horizontal surface, where k is a positive constant. The particles collide directly. Immediately after the collision the direction of motion of B is reversed and the speed of A is v . The magnitude of the impulse of A on B is $\frac{5}{2}mu$, and B has speed $3v$ after the collision. The coefficient of restitution between A and B is e , where $0 \leq e \leq 1$.
 (a) Find v in terms of u . [3]
 (b) Find the two possible values of k . [3]
 (c) Show that one of these values of k is not possible, and state which. [2]
- 23** A firework of mass 6 kg moves with velocity $(4\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$ when it explodes into two parts of masses 2 kg and 4 kg. Immediately after the explosion the 2 kg part has velocity $(10\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$.
 (a) Find the velocity of the 4 kg part immediately after the explosion. [3]
 (b) Find the speed of the 4 kg part. [1]
 (c) Find the kinetic energy gained in the explosion. [3]

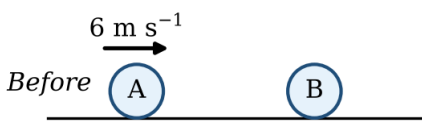
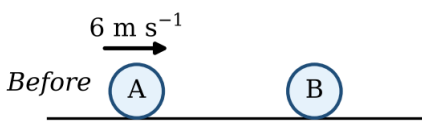
- 24** A plank of mass 40 kg lies at rest on smooth horizontal ice. A man of mass 60 kg stands at one end of the plank and walks along it with constant speed 1.5 m s^{-1} relative to the plank.
- Find the speed of the plank relative to the ice. [4]
 - Find the speed of the man relative to the ice. [1]
 - The man walks a distance of 4 m along the plank. Find the distance that the plank moves relative to the ice during this time. [3]
 - Explain what happens to the plank when the man stops walking. [1]
-
- 25** Two particles of masses 3 kg and 1 kg collide directly and coalesce. Before the collision the 3 kg particle moves at 4 m s^{-1} and the 1 kg particle is at rest. A student says: "Momentum is conserved in the collision, so kinetic energy must also be conserved, and therefore the final speed is found from $\frac{1}{2}(3)(16) = \frac{1}{2}(4)v^2$."
- Explain why the student's reasoning is wrong. [2]
 - Find the correct speed of the combined particle. [2]
 - Find the kinetic energy lost, and the percentage of the initial kinetic energy that this represents. [3]
-
- 26** A bullet of mass 0.03 kg travelling horizontally at 400 m s^{-1} passes through a block of mass 3 kg which is at rest on a smooth horizontal surface. The bullet emerges with speed 100 m s^{-1} in the same direction as before.
- Find the speed of the block immediately after the bullet has passed through it. [3]
 - Find the loss of kinetic energy. [3]
 - The bullet is in the block for 0.0005 s. Find the average force that the bullet exerts on the block. [2]
-
- 27** A pile driver consists of a hammer of mass 500 kg which falls from rest through a vertical distance of 2 m onto the top of a stationary pile of mass 300 kg. The hammer does not rebound: the hammer and pile move down together immediately after the impact. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the speed of the hammer just before it hits the pile. [2]
 - Find the speed of the hammer and pile immediately after impact. [2]
 - Find the kinetic energy lost in the impact. [3]
 - The pile is driven a distance of 0.1 m into the ground before coming to rest. Assuming that the ground exerts a constant resistive force R on the pile, find R . [4]
-
- 28** A shell of mass 6 kg is moving with velocity $6\mathbf{i} \text{ m s}^{-1}$ when it explodes into three fragments A, B and C of masses 1 kg, 2 kg and 3 kg respectively. Immediately after the explosion A has velocity $(20\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$ and B has velocity $(-5\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$.
- Find the velocity of C immediately after the explosion. [4]
 - Find the speed of C, and the angle its direction makes with the direction of the original motion of the shell. [3]
 - Find the kinetic energy gained in the explosion. [5]
-
- 29** Particle A of mass m moves with speed $4u$ on a smooth horizontal surface and collides directly with a stationary particle B of mass km , where k is a positive constant. The particles coalesce.
- Show that the speed of the combined particle is $\frac{4u}{(1+k)}$. [2]
 - Show that the fraction of the initial kinetic energy that is lost is $\frac{k}{(1+k)}$. [4]
 - Given that 60% of the initial kinetic energy is lost, find k . [2]
 - Explain why it is impossible for more than 100% of the kinetic energy to be lost, and describe what happens to the fraction as k becomes very large. [2]

- 30** A truck of mass m moving at speed u along a straight horizontal track collides with the first of a line of n identical stationary trucks, each of mass m . The trucks couple together on collision: the first truck couples to the moving truck, and the coupled trucks then collide with, and couple to, the second truck, and so on. There are no resistances to motion.
- (a) Show that after the moving truck has coupled to r of the stationary trucks, the speed of the coupled trucks is $\frac{u}{(r+1)}$. [3]
- (b) Find the kinetic energy of the trucks after all n stationary trucks have been coupled. [2]
- (c) Show that the fraction of the initial kinetic energy that is lost is $\frac{n}{(n+1)}$. [2]
- (d) Find the least value of n for which more than 95% of the initial kinetic energy is lost. [2]

Direct Collisions

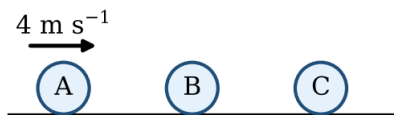
- 1** Two particles approach each other along a line with a speed of approach of 8 m s^{-1} . The coefficient of restitution between them is 0.5. Find their speed of separation after the collision. [1]
- 2** Particle A moves at 6 m s^{-1} and particle B moves at 2 m s^{-1} in the same direction along a straight line, and A collides with B. The coefficient of restitution is $\frac{1}{4}$. Find the speed of separation of the particles after the collision. [2]
- 3** Particles A and B move towards each other with speeds 5 m s^{-1} and 3 m s^{-1} . After the collision they move apart with a relative speed of 6 m s^{-1} . Find the coefficient of restitution. [2]
- 4** Two identical smooth spheres A and B lie on a smooth horizontal table. A is projected at 6 m s^{-1} directly towards B, which is at rest. The coefficient of restitution between the spheres is 1. Find the velocities of A and B after the collision. [3]
- 5** A particle of mass m moving at speed u collides directly with an identical stationary particle. The coefficient of restitution is 0. Find the speed of each particle after the collision. [2]
- 6** Particle A of mass 3 kg moving at 4 m s^{-1} collides directly with a stationary particle B. Immediately after the collision, A has speed 1 m s^{-1} and B has speed 3 m s^{-1} , both in the original direction of A. Find the coefficient of restitution. [2]
- 7** A particle A of mass 2 kg moving at 6 m s^{-1} collides directly with a stationary particle B of mass 1 kg. The coefficient of restitution is 0.5. Find the speeds of A and B after the collision. [4]
- 8** A particle A of mass 2 kg moving at 5 m s^{-1} collides directly with a particle B of mass 3 kg moving in the opposite direction at 2 m s^{-1} . The coefficient of restitution is 0.6. Find the magnitude of the impulse exerted by A on B. [4]
- 9** Particle A of mass 4 kg moves at 6 m s^{-1} towards particle B of mass 2 kg moving at 3 m s^{-1} in the opposite direction, as shown. The coefficient of restitution is 0.5. Find the velocities of A and B after the collision, taking the direction of A's initial motion as positive. [5]



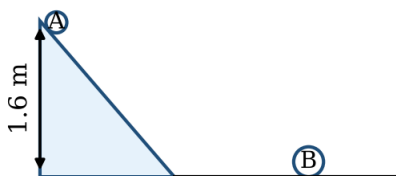
- 10** Particles A and B, of masses 5 kg and 3 kg, move in the same direction along a straight line with speeds 4 m s^{-1} and 1 m s^{-1} respectively, with A behind B. They collide, and immediately afterwards B moves at 4 m s^{-1} . Find the speed of A after the collision and the coefficient of restitution. [5]
-
- 11** A particle A of mass $m \text{ kg}$ moving at 6 m s^{-1} collides directly with a stationary particle B of mass 3 kg. The coefficient of restitution is 0.5 and A is brought to rest by the collision. Find m . [4]
-
- 12** A particle of mass 1 kg moving at 8 m s^{-1} collides directly with a stationary particle of mass 1 kg. The coefficient of restitution is 0.5.
 (a) Find the speeds of the two particles after the collision. [3]
 (b) Find the kinetic energy lost in the collision. [2]
-
- 13** A particle A of mass m moving at $3u$ collides directly with a stationary particle B of mass $2m$. The collision is perfectly elastic. Find the velocities of A and B after the collision. [4]
-
- 14** A particle A of mass m moving at speed u collides directly with a stationary particle B of mass $2m$. The coefficient of restitution is e .
 (a) Show that the speed of A after the collision is $\frac{u(1-2e)}{3}$ in the original direction of A, and find the corresponding expression for B. [4]
 (b) Find the range of values of e for which the direction of A is reversed by the collision. [2]
-
- 15** Particle A of mass m moving at $4u$ and particle B of mass $3m$ moving at u in the opposite direction collide directly. The coefficient of restitution is 0.5.
 (a) Find the velocities of A and B after the collision. [4]
 (b) Find the loss of kinetic energy in the collision in terms of m and u . [3]
-
- 16** Particles A and B, of masses 2 kg and 1 kg, are on a smooth horizontal surface. A is projected at 6 m s^{-1} towards B, which is at rest. The coefficient of restitution between A and B is 0.5.
- 
- Before 
- (a) Find the speeds of A and B immediately after their collision. [4]
 (b) B then collides with a stationary particle C of mass 2 kg, with coefficient of restitution 0.5. Find the velocities of B and C after this collision. [4]
-
- 17** Particle A of mass 3 kg moving at 4 m s^{-1} catches up with particle B of mass 1 kg moving in the same direction at 2 m s^{-1} and collides directly with it. The coefficient of restitution is 0.5. Find the speeds of A and B after the collision. [5]
-
- 18** Particle A of mass $2m$ is moving with speed $3u$ and particle B of mass $3m$ is moving with speed u in the opposite direction along a straight line on a smooth horizontal surface. The particles collide directly and the magnitude of the impulse exerted by A on B is $6mu$.
 (a) Find the coefficient of restitution between A and B. [4]
 (b) Find the loss of kinetic energy in the collision. [2]
-
- 19** A particle A of mass 3 kg moving at 5 m s^{-1} collides directly with a stationary particle B of mass 2 kg. The coefficient of restitution is 0.6. The collision lasts 0.02 s.
 (a) Find the velocity of B immediately after the collision. [3]
 (b) Find the magnitude of the impulse exerted by A on B. [2]
 (c) Find the average force exerted by A on B during the collision. [1]

- 20** A particle A of mass m moving at speed u collides directly with a stationary particle B of mass m . The coefficient of restitution is e .
Given that 32% of the kinetic energy of the system is lost in the collision, find the value of e . [7]

- 21** Three identical smooth spheres A, B and C, each of mass m , lie in a straight line on a smooth horizontal surface with B and C at rest. A is projected towards B with speed 4 m s^{-1} . The coefficient of restitution between any two spheres is 0.5. All collisions are direct.



- (a) Find the velocities of A and B after the first collision. [3]
 (b) B then collides with C. Show that there is a second collision between A and B, and find the speeds of A and B immediately after it. [6]
 (c) Show that there are no further collisions. [2]
- 22** Three particles A, B and C, of masses $2m$, $3m$ and $2m$ respectively, lie in a straight line on a smooth horizontal surface with B between A and C. C is at rest. Particle A moves with speed $5u$ and particle B moves with speed u , both towards C, and A collides directly with B. The coefficient of restitution between A and B is $\frac{1}{2}$. Particle B then collides directly with C, and the coefficient of restitution between B and C is e .
 (a) Find the speeds of A and B immediately after their collision. [4]
 (b) Find the speed of B immediately after its collision with C, in terms of u and e . [3]
 (c) Given that there are no further collisions, find the range of possible values of e . [3]
- 23** Particle A of mass $2m$ is moving with speed $3u$ and particle B of mass m is moving with speed $2u$ in the opposite direction, along a straight line on a smooth horizontal surface. The particles collide directly. The coefficient of restitution between A and B is e .
 (a) Show that, after the collision, the velocity of A is $\frac{u(4 - 5e)}{3}$ in its original direction of motion, and find the velocity of B in the same direction. [4]
 (b) Given that both particles move in the original direction of A after the collision, find the range of possible values of e . [2]
 (c) Given that the speed of B after the collision is twice the speed of A, find e and the kinetic energy lost in the collision. [3]
- 24** Particle A of mass 2 kg is released from rest at the top of a smooth ramp, at a height of 1.6 m above a smooth horizontal surface, and slides down onto the horizontal surface. It then collides directly with a stationary particle B of mass 1 kg on the surface. The coefficient of restitution is 0.5 . Take $g = 9.8 \text{ m s}^{-2}$ and assume there is no loss of speed where the ramp meets the surface.

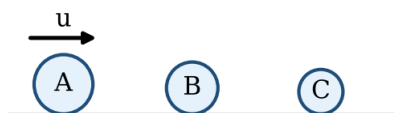


- (a) Find the speed of A just before the collision. [2]
 (b) Find the speeds of A and B just after the collision. [4]
 (c) B now moves up a second smooth ramp. Find the greatest height above the horizontal surface that B reaches. [3]

- 25** Particle A of mass m moving at $5u$ catches up with particle B of mass $2m$ moving in the same direction at u , and they collide directly. After the collision both particles continue in the original direction of motion and the speed of A is half the speed of B.
- (a) Find the speeds of A and B after the collision. [3]
 (b) Find the coefficient of restitution. [2]
 (c) Find the kinetic energy lost in the collision. [3]

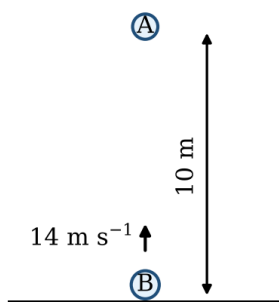
- 26** Particle A of mass 4 kg moves at 6 m s^{-1} towards particle B of mass 2 kg , which moves at 3 m s^{-1} in the opposite direction. The coefficient of restitution between them is 0.5 . A student solves the problem as follows.
- "Speed of approach $= 6 - 3 = 3\text{ m s}^{-1}$, so $v_B - v_A = 0.5 \times 3 = 1.5$. Momentum: $24 + 6 = 4v_A + 2v_B$."
- (a) Identify two errors in the student's working. [2]
 (b) Find the correct velocities of A and B after the collision, taking A's initial direction as positive. [4]
 (c) Find the kinetic energy lost in the collision. [2]

- 27** Three smooth spheres A, B and C have masses $3m$, $2m$ and m respectively. They lie in a straight line on a smooth horizontal surface, with B between A and C. B and C are at rest and A is projected with speed u directly towards B. The coefficient of restitution between any two spheres is 0.5 .



- (a) Find the speeds of A and B after the first collision. [4]
 (b) B then collides with C. Find the speeds of B and C after this collision. [4]
 (c) Show that there are no further collisions. [2]
 (d) Find the total kinetic energy lost in the two collisions as a fraction of the initial kinetic energy. [2]
- 28** Three identical smooth spheres A, B and C, each of mass m , lie in a straight line on a smooth horizontal surface, with B between A and C. B and C are at rest. A is projected directly towards B with speed u . The coefficient of restitution between any two spheres is e , where $0 < e < 1$.
- (a) Show that after the first collision the speed of A is $\frac{u(1-e)}{2}$ and the speed of B is $\frac{u(1+e)}{2}$. [3]
 (b) Show that after B collides with C, the speed of B is $\frac{u(1-e^2)}{4}$. [4]
 (c) Hence show that A and B must collide a second time, whatever the value of e . [3]

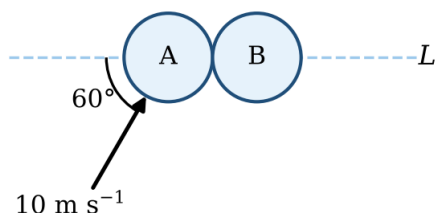
- 29** Particle A of mass 1 kg is released from rest at a point 10 m vertically above a point on horizontal ground. At the same instant a particle B of mass 3 kg is projected vertically upwards from that point on the ground with speed 14 m s^{-1} . The particles collide directly. The coefficient of restitution between A and B is 0.5. Take $g = 9.8 \text{ m s}^{-2}$ and ignore air resistance.



- Find the time at which the particles collide, and show that their speeds immediately before the collision are both 7 m s^{-1} . [3]
 - Find the velocities of A and B immediately after the collision. [3]
 - Find the greatest height above the ground reached by B. [3]
 - Find the greatest height above the ground reached by A, and comment on your answer in relation to the height from which A was released. [3]
- 30** A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with a particle B of mass $3m$ which is at rest. The coefficient of restitution between A and B is e . The direction of motion of A is reversed by the collision. Subsequently B hits a smooth fixed vertical wall, which is perpendicular to the direction of motion of B, and rebounds. The coefficient of restitution between B and the wall is $\frac{3}{5}$. A and B then collide again.
- Find the velocities of A and B immediately after their first collision, taking the direction of motion of B towards the wall as positive. [3]
 - Show that $e > \frac{1}{3}$. [2]
 - Show that $e < \frac{2}{3}$, and hence state the range of possible values of e . [4]

Oblique Collisions

- 1** A smooth sphere A is moving with speed 10 m s^{-1} when it collides with a second sphere. At the instant of impact the direction of motion of A makes an angle of 60° with the line of centres L, as shown. Find the components of the velocity of A along L and perpendicular to L, just before the impact.



- 2** Two smooth spheres collide obliquely. Explain why the component of the velocity of each sphere perpendicular to the line of centres is unchanged by the collision. [2]

3 A smooth sphere is moving at 12 m s^{-1} at an angle of 30° to the line of centres when it collides with another smooth sphere. Write down the component of its velocity perpendicular to the line of centres immediately after the collision. [1]

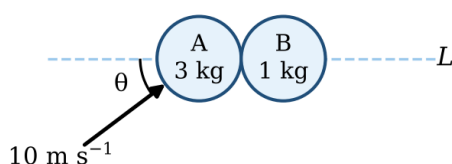
4 After a collision a smooth sphere has components of velocity 3 m s^{-1} along the line of centres and 4 m s^{-1} perpendicular to it. Find the speed of the sphere. [2]

5 Before a collision a smooth sphere has components of velocity 5 m s^{-1} along the line of centres and 12 m s^{-1} perpendicular to it. Find the angle between the direction of motion of the sphere and the line of centres, to 1 decimal place. [2]

6 A smooth sphere A of mass m is moving along the line of centres at 6 m s^{-1} when it collides with an identical sphere B at rest. The coefficient of restitution is 0.5. Find the speed of B after the collision. [3]

7 Two identical smooth spheres A and B each have mass m . B is at rest. A is moving with a component of velocity 8 m s^{-1} along the line of centres and a component 6 m s^{-1} perpendicular to it when it strikes B. The coefficient of restitution is 0.5. Find the speed of each sphere after the collision. [4]

8 Smooth spheres A and B have masses 3 kg and 1 kg. B is at rest. A strikes B with speed 10 m s^{-1} , in a direction making an angle θ with the line of centres, where $\tan \theta = \frac{3}{4}$. The coefficient of restitution is 0.5.



(a) Find the speed of A after the collision. [4]

(b) Find the magnitude of the impulse exerted by A on B. [1]

9 Two identical smooth spheres A and B collide with their velocities, just before impact, having components along the line of centres (from A to B) of 6 m s^{-1} for A and -2 m s^{-1} for B, and components perpendicular to it of 2 m s^{-1} and -2 m s^{-1} respectively. The collision is perfectly elastic. Find the speeds of A and B after the collision. [4]

10 A smooth sphere A of mass 3 kg is moving with components of velocity 16 m s^{-1} along the line of centres and 12 m s^{-1} perpendicular to it when it collides with another smooth sphere. Immediately after the collision, the component of A's velocity along the line of centres is 6 m s^{-1} in the same direction.

(a) Find the magnitude of the impulse exerted on A. [2]

(b) Find the speed of A after the collision. [1]

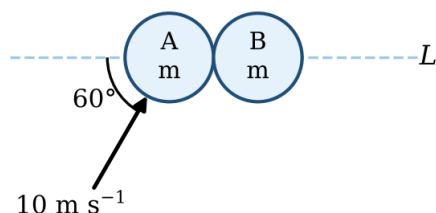
11 Two identical smooth spheres collide obliquely. Sphere A has components of velocity 4 m s^{-1} along the line of centres and 3 m s^{-1} perpendicular to it; sphere B is at rest. After the collision, the speed of B is 3 m s^{-1} . Find the coefficient of restitution. [3]

12 A smooth sphere A of mass 2 kg is moving with components of velocity 6 m s^{-1} along the line of centres and 8 m s^{-1} perpendicular to it when it collides with an identical stationary sphere B. The coefficient of restitution is 0.5. Find the kinetic energy lost in the collision. [5]

13 A smooth sphere A is moving at 8 m s^{-1} at an angle of 60° to the line of centres when it collides with an identical stationary sphere B. The coefficient of restitution is 1. Find the velocity of each sphere after the collision, giving the direction relative to the line of centres. [4]

- 14** Two identical smooth spheres A and B collide. B is initially at rest, and the coefficient of restitution is 1. Before the collision A moves with speed u at an angle θ to the line of centres. Show that after the collision the spheres move in directions that are at right angles to each other. [5]

- 15** Two identical smooth spheres A and B each have mass m . B is at rest. A strikes B with speed 10 m s^{-1} at an angle of 60° to the line of centres, as shown. The coefficient of restitution is 0.5.



- (a) Find the speed of each sphere after the collision. [4]
 (b) Find the angle between the direction of motion of A after the collision and the line of centres. [2]

- 16** Two smooth spheres A and B of masses 2 kg and 3 kg are moving on a smooth horizontal plane. Just before they collide, the velocity of A is $(4\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$ and the velocity of B is $(-3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. At the instant of impact the line of centres is parallel to $\mathbf{i}$. The coefficient of restitution is 0.5.

- (a) Find the velocities of A and B immediately after the collision. [5]
 (b) Find the magnitude of the impulse exerted by A on B. [2]

- 17** A smooth sphere A of mass m moving at speed u collides with an identical stationary sphere B. Before the collision the direction of motion of A makes an angle of 60° with the line of centres. The coefficient of restitution is 0.5. Show that the kinetic energy lost in the collision is $\frac{3}{64}mu^2$. [5]

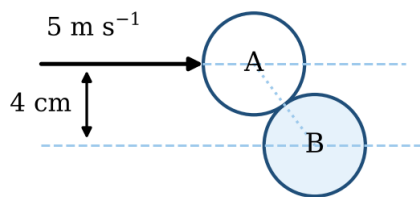
- 18** A smooth sphere A of mass m moving at speed u strikes an identical stationary sphere B, with the direction of motion of A making an angle θ with the line of centres, where $\tan \theta = \frac{1}{5}$. After the collision the direction of motion of A makes an angle of 45° with the line of centres. Find the coefficient of restitution. [5]

- 19** A smooth sphere A of mass 0.4 kg moving at 15 m s^{-1} collides with a stationary smooth sphere B of mass 0.6 kg. Just before the collision A is moving at an angle α to the line of centres, where $\cos \alpha = 0.8$. The coefficient of restitution is 0.5.
- (a) Find the speed of B after the collision, and the magnitude of the impulse exerted on B. [4]
 (b) Find the speed of A after the collision and the angle between its new direction of motion and the line of centres. [3]

- 20** Two smooth spheres A and B, of masses 2 kg and 3 kg, collide on a smooth horizontal plane. Just before the collision the speed of A is 10 m s^{-1} at an acute angle α to the line of centres, where $\tan \alpha = \frac{3}{4}$, and the speed of B is 5 m s^{-1} at an acute angle β to the line of centres, where $\tan \beta = \frac{4}{3}$. The spheres are moving towards each other along the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.
- (a) Find the components of the velocities of A and B along the line of centres just after the collision. [4]
 (b) Find the speed of each sphere after the collision. [2]

- 21** A smooth sphere A of mass m moving with speed u collides with a stationary smooth sphere B of mass $4m$. Just before the collision the direction of motion of A makes an angle θ with the line of centres, where $0^\circ < \theta < 90^\circ$. Immediately after the collision the directions of motion of A and B are at right angles. Show that the coefficient of restitution is $\frac{1}{4}$. [7]
-
- 22** Two identical smooth spheres A and B are moving on a smooth horizontal plane. B is initially at rest. A strikes B with speed 10 m s^{-1} at an angle θ to the line of centres. After the collision B moves with speed 6 m s^{-1} and A moves with speed 8 m s^{-1} . Find θ and the coefficient of restitution. [8]
-
- 23** A smooth sphere A of mass $2m$ moving at 13 m s^{-1} collides with a stationary smooth sphere B of mass m . Before the collision A is moving at an angle α to the line of centres, where $\cos \alpha = \frac{5}{13}$. The coefficient of restitution is 0.5.
- (a) Find the velocities of A and B along the line of centres after the collision, and the speed of A. [5]
- (b) Find the angle through which the direction of motion of A is turned by the collision. [3]
-
- 24** Two smooth uniform spheres P and Q have equal radii and masses 0.3 kg and 0.2 kg . They are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(4\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ and the velocity of Q is $(-3\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. At the instant of collision the line joining the centres of the spheres is parallel to $\mathbf{i}$. The coefficient of restitution between the spheres is e . The kinetic energy of Q immediately after the collision is half of its kinetic energy immediately before the collision.
- (a) Show that the velocity of Q immediately after the collision is $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. [3]
- (b) Find the velocity of P immediately after the collision. [2]
- (c) Find the value of e . [2]
- (d) Find the magnitude of the impulse exerted by Q on P. [2]
-
- 25** A smooth sphere A of mass m moving with speed 10 m s^{-1} strikes a stationary smooth sphere B of mass $2m$. After the collision B moves in a direction making an angle of 30° with the original direction of motion of A. The coefficient of restitution is 0.8.
- (a) Write down the angle between the original direction of A and the line of centres, and find the speed of B after the collision. [4]
- (b) Find the speed of A after the collision and the angle through which its direction of motion is turned. [4]
-
- 26** A smooth sphere A moving with speed 10 m s^{-1} at an angle of 60° to the line of centres strikes an identical stationary sphere B. The coefficient of restitution is 0.5. A student writes: "Speed of approach = 10 m s^{-1} , so speed of separation = $0.5 \times 10 = 5 \text{ m s}^{-1}$. Hence A moves at 2.5 m s^{-1} and B at 7.5 m s^{-1} ."
- (a) Identify two mistakes in the student's working. [2]
- (b) Find the correct speeds of A and B after the collision. [4]
-
- 27** Two identical smooth spheres A and B are on a smooth horizontal plane. B is at rest and A moves with speed u at an angle θ to the line of centres, where $0^\circ < \theta < 90^\circ$. The coefficient of restitution is e , where $0 < e \leq 1$.
- (a) Show that after the collision the velocity components of A along and perpendicular to the line of centres are $\frac{(1-e)}{2}u \cos \theta$ and $u \sin \theta$, and find the velocity of B. [4]
- (b) Show that the angle between the directions of motion of A and B after the collision is acute when $e < 1$, and a right angle when $e = 1$. [4]

- 28** Two identical smooth balls A and B, each of radius 2.5 cm and mass m , are on a smooth horizontal table. B is at rest. A moves with speed 5 m s^{-1} along a straight line which is parallel to, and at a perpendicular distance of 4 cm from, the line through the centre of B in the direction of A's motion, as shown. A strikes B. The coefficient of restitution between the balls is 0.5.



- Show that at the moment of impact the line of centres makes an angle θ with the direction of motion of A, where $\sin \theta = 0.8$. [2]
 - Find the speed of each ball immediately after the collision. [5]
 - Find the time after the collision at which the distance between the centres of the balls is 1.2 m. [3]
- 29** A smooth sphere A of mass m moving with speed u strikes an identical stationary smooth sphere B. Before the collision, A moves at an angle θ to the line of centres. The coefficient of restitution is e . The fraction of the initial kinetic energy that is lost in the collision is denoted by f .
- Show that $f = \frac{(1 - e^2)}{4} \cos^2 \theta$. [5]
 - Given that $e = 0.6$, find the value of θ for which $f = 0.1$. [3]
 - Find the greatest possible value of f when $e = 0.6$, and state the value of θ for which this occurs. [2]
- 30** A smooth sphere A moving with speed u strikes an identical stationary smooth sphere B. Before the collision A moves at an angle θ to the line of centres, where $0^\circ < \theta < 90^\circ$. The coefficient of restitution is e , where $0 \leq e < 1$. After the collision A moves at an angle α to the line of centres.
- Show that $\tan \alpha = \frac{2 \tan \theta}{(1 - e)}$. [4]
 - Given that $e = 0.5$ and $\theta = 30^\circ$, find the angle through which the direction of motion of A is turned by the collision. [3]
 - Show that, for any θ and any e with $0 \leq e < 1$, the direction of motion of A is turned away from its original direction, and describe what happens to α as e approaches 1. [3]

Coefficient of Restitution

- The coefficient of restitution between two colliding particles is denoted by e .
 - State the range of possible values of e . [1]
 - Describe what happens in a collision when $e = 1$. [1]
 - Describe what happens in a collision when $e = 0$. [1]
- A ball hits a fixed vertical wall at right angles with speed 10 m s^{-1} and rebounds with speed 6 m s^{-1} . Find the coefficient of restitution between the ball and the wall. [1]
- A ball hits a fixed wall at right angles with speed 8 m s^{-1} . The coefficient of restitution between the ball and the wall is 0.75. Find the speed with which the ball rebounds. [1]
- A ball hits horizontal ground with a speed of 14 m s^{-1} and rebounds vertically with speed 9.8 m s^{-1} . Find the coefficient of restitution between the ball and the ground. [1]

- 5** Two particles approach each other with speed of approach 5 m s^{-1} and separate with speed of separation 2 m s^{-1} . Find the coefficient of restitution between the particles. [1]
-
- 6** A ball hits the floor at 12 m s^{-1} and rebounds at 6 m s^{-1} . Find the coefficient of restitution, and the fraction of the ball's kinetic energy that is retained in the impact. [3]
-
- 7** A ball is dropped from rest from a height of 2.5 m above a horizontal floor and rebounds to a height of 1.6 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the coefficient of restitution between the ball and the floor. [3]
-
- 8** A ball is dropped from rest from a height of 4 m above a horizontal floor. The coefficient of restitution between the ball and the floor is 0.5. Find the height to which the ball first rebounds. [4]
-
- 9** A ball hits a fixed smooth wall at right angles. Of the kinetic energy of the ball just before impact, 36% is lost in the impact.
 (a) Find the coefficient of restitution between the ball and the wall. [3]
 (b) The ball hits the wall at 15 m s^{-1} . Find the speed with which it rebounds. [1]
-
- 10** A ball is dropped from rest from a height of 4.9 m above horizontal ground. The coefficient of restitution between the ball and the ground is 0.6. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the speed of the ball just after the first impact. [3]
 (b) Find the time between the first and second impacts. [2]
-
- 11** A ball is dropped from rest from a height of 5 m onto a horizontal floor. After the second impact the ball rises to a height of 0.8 m. Find the coefficient of restitution between the ball and the floor. [4]
-
- 12** A particle A of mass 2 kg moving at 5 m s^{-1} collides directly with a stationary particle B of mass 3 kg. After the collision the speed of B is 3 m s^{-1} . Find the coefficient of restitution between A and B. [4]
-
- 13** A ball is released from rest at a height h above a horizontal floor. The coefficient of restitution between the ball and the floor is e . Show that the ball rises to a height $e^2 h$ after the first bounce. [3]
-
- 14** A ball is dropped from rest from a height h above a horizontal floor. The coefficient of restitution between the ball and the floor is e , where $0 < e < 1$. The ball bounces repeatedly.
 (a) Show that the total distance travelled by the ball before it comes to rest is $h \frac{(1 + e^2)}{(1 - e^2)}$. [4]
 (b) Find the total distance when $h = 2 \text{ m}$ and $e = 0.5$. [1]
-
- 15** A ball is dropped from rest from a height of 4.9 m above a horizontal floor and bounces repeatedly. The coefficient of restitution between the ball and the floor is 0.5. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Show that the ball hits the floor for the first time after 1 s and find its speed at this instant. [2]
 (b) Show that the time between the n th and $(n + 1)$ th impacts is $2(0.5)^n$ seconds. [3]
 (c) Find the total time from release until the ball comes to rest. [2]
-
- 16** A ball is released from rest at a height of 19.6 m above a horizontal floor. The ball hits the floor and rebounds. The second impact with the floor occurs 4.4 s after the ball is released. Take $g = 9.8 \text{ m s}^{-2}$. Find the coefficient of restitution between the ball and the floor. [5]
-
- 17** A ball of mass 0.2 kg hits a horizontal floor at 12 m s^{-1} . The coefficient of restitution between the ball and the floor is 0.75. Find the kinetic energy lost in the impact. [4]
-

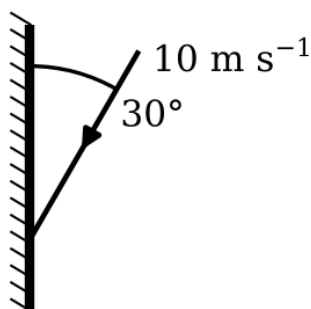
- 18** A ball of mass 0.5 kg hits a smooth fixed wall at right angles with speed 8 m s^{-1} . The coefficient of restitution between the ball and the wall is 0.6.
- (a) Find the magnitude of the impulse the wall exerts on the ball. [3]
- (b) The ball is in contact with the wall for 0.01 s. Find the average force exerted by the wall. [2]
-
- 19** A ball is projected vertically downwards with speed 7 m s^{-1} from a point 10 m above a horizontal floor. The coefficient of restitution between the ball and the floor is 0.5. Take $g = 9.8 \text{ m s}^{-2}$. Find the greatest height above the floor that the ball reaches after the first bounce. [5]
-
- 20** Two identical smooth spheres A and B lie on a smooth horizontal table, with B at rest. A is projected directly towards B and collides with it. After the collision the speed of B is three times the speed of A, and both spheres move in the same direction. Find the coefficient of restitution. [5]
-
- 21** A ball is dropped from rest from a height of 2.5 m above a horizontal floor. The coefficient of restitution between the ball and the floor is 0.8. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the time taken for the ball to reach the floor for the first time. [2]
- (b) Show that the total time for which the ball bounces before coming to rest is 6.43 seconds, correct to 3 significant figures. [3]
- (c) Find the total distance travelled by the ball. [2]
- (d) Find the number of the first rebound whose height is less than 1 cm. [2]
-
- 22** A ball is projected vertically upwards from a horizontal floor with speed 14 m s^{-1} . It strikes a horizontal ceiling which is 6.4 m above the floor. The coefficient of restitution between the ball and the ceiling is 0.5. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the speed of the ball just before it hits the ceiling. [2]
- (b) Find the speed of the ball just before it hits the floor after rebounding from the ceiling. [3]
- (c) The coefficient of restitution between the ball and the floor is 0.6. Find the greatest height above the floor reached by the ball after bouncing off the floor. [3]
-
- 23** A particle A of mass m moving with speed u collides directly with a stationary particle B of mass $2m$. The coefficient of restitution is e . After the collision, the kinetic energy of A is 4% of its original kinetic energy. Find the two possible values of e . [7]
-
- 24** A ball is dropped from rest from a height H above a horizontal floor. It rebounds to a height of 3.6 m after the first impact and to a height of 2.025 m after the second impact.
- (a) Find the coefficient of restitution between the ball and the floor. [3]
- (b) Find H . [2]
- (c) Find the total distance travelled by the ball, from release until it comes to rest. [3]
-
- 25** Particle A of mass $3m$ moving at speed u and particle B of mass m moving at speed $2u$ in the opposite direction collide directly. Immediately after the collision A is at rest.
- (a) Find the coefficient of restitution between A and B. [4]
- (b) Find the fraction of the total kinetic energy that is lost in the collision. [3]
-
- 26** A ball is dropped from rest from a height of 2 m onto a horizontal floor and rebounds to a height of 1.28 m. A student writes: " $e = \frac{1.28}{2} = 0.64$."
- (a) Explain the mistake in the student's working. [2]
- (b) Find the correct value of e . [2]
- (c) Find the height of the second rebound. [2]

- 27** A ball is dropped from rest from a height h above a horizontal floor. The coefficient of restitution between the ball and the floor is e , where $0 < e < 1$. Let v_0 be the speed with which the ball first hits the floor.
- Show that the speed with which the ball leaves the floor after the n th impact is $e^n v_0$. [2]
 - Show that the time between the n th and the $(n + 1)$ th impacts is $\frac{2e^n v_0}{g}$. [2]
 - Show that the total time from release until the ball comes to rest is $\frac{v_0}{g} \frac{(1 + e)}{(1 - e)}$. [4]
 - Given that the total time is 10 times the time taken to fall to the floor for the first time, find e . [3]
-
- 28** A particle A of mass m_1 moving with speed u_1 collides directly with a particle B of mass m_2 moving in the same direction with speed u_2 , where $u_1 > u_2$. The coefficient of restitution is e .
- Show that the velocity of B after the collision is $\frac{m_1 u_1 + m_2 u_2 + e m_1 (u_1 - u_2)}{m_1 + m_2}$, and write down the corresponding expression for A. [4]
 - Hence show that the kinetic energy lost in the collision is $\frac{m_1 m_2}{2(m_1 + m_2)} (1 - e^2) (u_1 - u_2)^2$. [5]
 - A particle of mass 2 kg moving at 6 m s^{-1} collides with a particle of mass 3 kg moving at 1 m s^{-1} in the same direction. The coefficient of restitution is 0.4. Use the result in part (b) to find the loss of kinetic energy. [2]
 - State, with a reason, the value of e for which the loss of kinetic energy is greatest. [1]
-
- 29** A ball of mass 0.4 kg is dropped from rest from a point 4.9 m above a horizontal floor. The coefficient of restitution between the ball and the floor is e . The ball strikes the floor, rebounds, and is caught at the highest point of its first bounce, 1.1 m above the floor. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the speed with which the ball hits the floor and the speed with which it leaves the floor. [4]
 - Find e . [2]
 - Find the magnitude of the impulse exerted by the floor on the ball and the loss of kinetic energy in the impact. [4]
 - Find the total time from the ball's release until it is caught. [2]
-
- 30** A particle A of mass m moving with speed u collides directly with a stationary particle B of mass km , where $k > 0$. The coefficient of restitution is e .
- Show that the velocity of A after the collision, in the original direction of motion, is $\frac{u(1 - ke)}{(1 + k)}$. [3]
 - Find the limit of the velocity of A as $k \rightarrow \infty$, and hence explain how a collision with a fixed wall is related to a collision with a very heavy particle. [3]
 - Given that $e = 0.8$, find the value of k for which A rebounds with speed $0.6u$. [3]
 - Show that, for $k < 1/e$, A continues in its original direction after the collision. [1]

Collisions with Fixed Surfaces

- 1** A ball hits a smooth fixed vertical wall at right angles with speed 9 m s^{-1} . The coefficient of restitution between the ball and the wall is $\frac{2}{3}$. Find the speed of the ball immediately after the impact. [1]

- 2 A particle hits a smooth fixed vertical wall with speed 10 m s^{-1} , moving at an angle of 30° to the wall, as shown. Find the components of the velocity of the particle parallel to the wall and perpendicular to the wall, just before the impact.



[2]

- 3 A ball hits a smooth fixed wall. Immediately before the impact the components of its velocity parallel and perpendicular to the wall are 6 m s^{-1} and 8 m s^{-1} respectively. The coefficient of restitution is 0.5. Find the speed of the ball immediately after the impact.

[3]

- 4 A ball hits a smooth fixed wall obliquely. Explain why, if the collision is perfectly elastic, the angle at which the ball rebounds from the wall equals the angle at which it hits the wall.

[2]

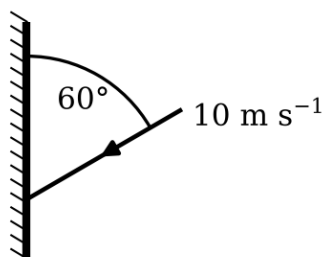
- 5 A ball of mass 0.5 kg hits a fixed smooth wall at right angles at 6 m s^{-1} . The coefficient of restitution is 0.5. Find the kinetic energy lost in the impact.

[2]

- 6 A ball of mass 0.4 kg hits a smooth fixed wall at right angles at 5 m s^{-1} and rebounds at 3 m s^{-1} . Find the magnitude of the impulse exerted by the wall on the ball.

[2]

- 7 A ball moving with speed 10 m s^{-1} hits a smooth fixed vertical wall. Its direction of motion makes an angle of 60° with the wall, as shown. The coefficient of restitution between the ball and the wall is 0.5.



- (a) Find the speed of the ball immediately after the impact.
- (b) Find the angle between the wall and the direction of motion of the ball after the impact.

[3]

[2]

- 8 A particle hits a smooth fixed wall with its direction of motion making an angle of 60° with the wall. It rebounds with its direction of motion making an angle of 30° with the wall. Find the coefficient of restitution between the particle and the wall.

[3]

- 9 A ball hits a smooth fixed vertical wall with speed 25 m s^{-1} . Its direction of motion before the impact makes an angle α with the wall, where $\tan \alpha = \frac{4}{3}$. The ball rebounds from the wall with speed 17 m s^{-1} .

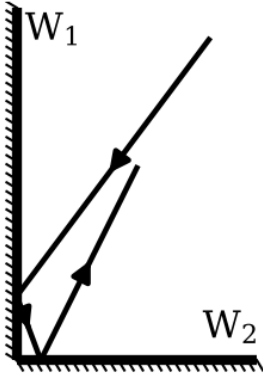
- (a) Find the coefficient of restitution between the ball and the wall.
- (b) Find the angle between the wall and the direction of motion of the ball after the impact.

[3]

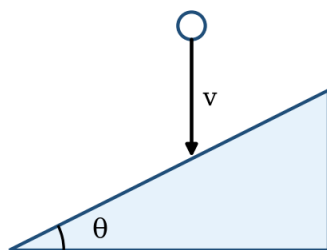
[1]

- 10 A ball of mass 0.3 kg hits a smooth fixed wall at 12 m s^{-1} , moving at an angle of 30° to the wall. The coefficient of restitution is 0.5. Find the magnitude of the impulse exerted by the wall on the ball.

[4]

- 11** Two smooth vertical walls W_1 and W_2 are parallel and 4 m apart. A particle is projected from a point on W_1 with speed 5 m s^{-1} perpendicular to W_1 , towards W_2 . The coefficient of restitution between the particle and each wall is 0.6. Find the time taken for the particle to return to W_1 . [4]
- 12** A ball hits a smooth horizontal floor with horizontal and vertical components of velocity 6 m s^{-1} and 8 m s^{-1} (downwards) respectively. The coefficient of restitution between the ball and the floor is 0.5. Find the speed of the ball immediately after the impact, and the angle its direction of motion then makes with the floor. [4]
- 13** A ball of mass 2 kg hits a smooth fixed wall at 10 m s^{-1} , moving at an angle of 60° to the wall. The coefficient of restitution is 0.5. Find the loss of kinetic energy. [3]
- 14** A ball moving on a smooth horizontal surface hits a smooth fixed vertical wall with speed u . Immediately before the impact the direction of motion of the ball makes an acute angle θ with the wall. The coefficient of restitution between the ball and the wall is e . The impact turns the direction of motion of the ball through exactly 90° .
- (a) Show that $\tan \theta = \frac{1}{\sqrt{e}}$. [3]
- (b) Hence find the range of possible values of θ . [2]
- (c) Given that $e = \frac{1}{4}$, find θ and the speed of the ball after the impact in terms of u . [3]
- 15** Two smooth fixed walls W_1 and W_2 meet at right angles. A smooth ball moving on a smooth horizontal surface hits W_1 and then W_2 , as shown. Take $\mathbf{i}$ perpendicular to W_1 , directed away from W_1 , and $\mathbf{j}$ perpendicular to W_2 , directed away from W_2 . The velocity of the ball before the first impact is $(-6\mathbf{i} - 8\mathbf{j}) \text{ m s}^{-1}$. The coefficient of restitution between the ball and W_1 is 0.5 and between the ball and W_2 is 0.75.
- 
- (a) Find the velocity of the ball after hitting W_1 , and after hitting W_2 . [4]
- (b) Find the speed of the ball after both impacts, and the fraction of its original kinetic energy that has been lost. [3]
- 16** A smooth ball moving at 13 m s^{-1} hits a smooth fixed wall. Immediately before the impact the direction of motion of the ball makes an angle α with the wall, where $\sin \alpha = \frac{12}{13}$. Immediately after the impact the speed of the ball is $\sqrt{61} \text{ m s}^{-1}$. Find the coefficient of restitution between the ball and the wall. [4]

- 17** A smooth ball falls vertically and strikes a smooth fixed plane which is inclined at an angle θ to the horizontal. Immediately after the impact the ball moves horizontally. The coefficient of restitution between the ball and the plane is 0.25. Find the value of θ .



[5]

- 18** A smooth wall is 6 m from a point O. A ball is projected from O at 8 m s^{-1} directly towards the wall. The coefficient of restitution between the ball and the wall is 0.75. Find the distance of the ball from the wall 1.5 s after it was projected.

[4]

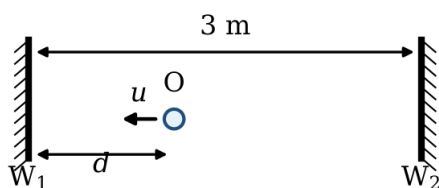
- 19** A ball is projected horizontally with speed 8 m s^{-1} from a point 1.5 m above smooth horizontal ground. The coefficient of restitution between the ball and the ground is 0.5. Take $g = 9.8 \text{ m s}^{-2}$. Find the horizontal distance from the point of projection to the point where the ball hits the ground for the second time.

[6]

- 20** A ball of mass m hits a smooth fixed wall with speed u , with its direction of motion making an angle α with the wall. The coefficient of restitution is e . Show that the magnitude of the impulse exerted by the wall on the ball is $mu(1 + e) \sin \alpha$, and state its direction. Hence find the magnitude of the impulse when $m = 0.25 \text{ kg}$, $u = 14 \text{ m s}^{-1}$, $\alpha = 30^\circ$ and $e = 0.4$.

[5]

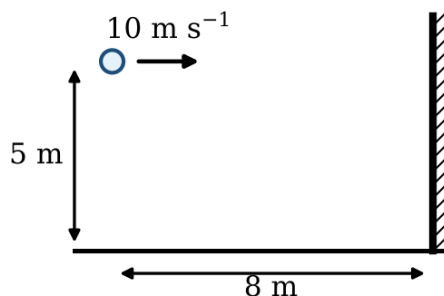
- 21** The diagram shows the plan view of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls 3 m apart. A particle lies at rest at a point O between the walls, at a distance d m from W_1 , where $0 < d < 3$. At time $t = 0$ the particle is projected from O towards W_1 with speed $u \text{ m s}^{-1}$ in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The particle returns to O at time T seconds, having bounced off each wall once.



- (a) Show that $T = \frac{45 - 5d}{4u}$. [4]
- (b) Given that $T = \frac{9}{u}$, find the value of d . [2]

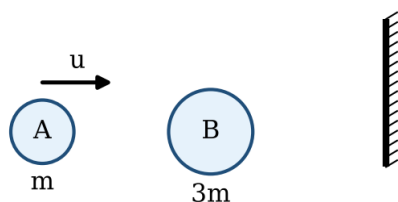
- 22** Two smooth fixed walls meet at right angles at a point C. A ball moving on a smooth horizontal surface hits one wall and then the other. The coefficient of restitution between the ball and each wall is e . Take $\mathbf{i}$ and $\mathbf{j}$ to be unit vectors perpendicular to the first and second walls respectively, both directed away from the walls. The velocity of the ball before the first impact is $(-12\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-1}$.
- (a) Show that the velocity of the ball after both impacts is $e(12\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$, and deduce that the ball then moves parallel to its original line of motion, in the opposite direction. [4]
- (b) Given that $e = 0.4$, find the speed of the ball after the two impacts and the percentage of its kinetic energy that has been lost. [3]
- (c) Find the value of e for which the final speed is 8.45 m s^{-1} . [1]

- 23** A ball is projected horizontally with speed 10 m s^{-1} from a point 5 m above smooth horizontal ground, directly towards a smooth vertical wall which is 8 m horizontally from the point of projection. The coefficient of restitution between the ball and the wall is 0.6 . Take $g = 9.8 \text{ m s}^{-2}$ and model the ball as a particle.

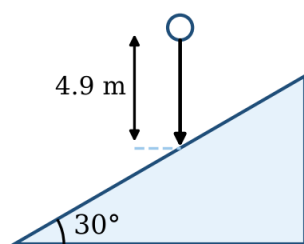


- (a) Find the height of the ball above the ground when it hits the wall. [3]
- (b) Find the horizontal distance from the wall of the point where the ball first hits the ground. [4]
-
- 24** Two smooth parallel vertical walls W_1 and W_2 are 12 m apart. A particle P is at a point O on a smooth horizontal floor between the walls, 4 m from W_1 . P is projected from O with components of velocity 8 m s^{-1} towards W_1 (perpendicular to the walls) and 6 m s^{-1} parallel to the walls. The coefficient of restitution between P and each wall is 0.5 .
- (a) Find the time taken for P to reach W_1 , and its distance, measured parallel to the walls, from O at that instant. [2]
- (b) Find the speed of P immediately after it hits W_1 . [2]
- (c) The wall W_2 is 12 m from W_1 . Find the total time from projection until P hits W_2 , and the total distance moved parallel to the walls during this time. [3]
-
- 25** A smooth ball hits a smooth fixed wall with its direction of motion making an angle of 40° with the wall. The coefficient of restitution is 0.6 . A student writes: "The wall is smooth, so the angle of rebound is 40° , and the speed is reduced by the factor 0.6 ."
- (a) Explain two errors in the student's statement. [2]
- (b) Find the correct angle between the wall and the direction of motion after the impact, and the ratio of the speed after impact to the speed before impact. [4]
-
- 26** A smooth ball hits a smooth fixed wall with speed u . Before the impact the direction of motion makes an angle of 60° with the wall. The coefficient of restitution between the ball and the wall is e . After the impact the direction of motion of the ball is at right angles to its direction of motion before the impact.
- (a) Write down the components of the velocity of the ball parallel and perpendicular to the wall before and after the impact. [2]
- (b) Use the scalar product of the two velocities to find e . [3]
- (c) Find the speed of the ball after the impact in terms of u . [2]

- 27** Particle A of mass m and particle B of mass $3m$ lie on a smooth horizontal floor in a straight line perpendicular to a smooth vertical wall, with B between A and the wall. B is at rest and A is projected towards B with speed u . The coefficient of restitution between A and B is $\frac{1}{2}$ and between B and the wall is $\frac{1}{2}$.



- Find the velocities of A and B immediately after their first collision, taking the direction towards the wall as positive. [4]
 - Find the speed of B after it hits the wall. [1]
 - Show that B and A collide again, and find the speeds of A and B immediately after this second collision. [5]
- 28** Two smooth vertical walls W_1 and W_2 are parallel and 3 m apart. A particle is projected from a point on W_1 with speed 6 m s^{-1} perpendicular to the walls, towards W_2 , and subsequently bounces between the walls. The coefficient of restitution between the particle and each wall is 0.5.
- Show that the time taken for the particle to move between the walls on its n th crossing is $0.5 \times 2^{n-1}$ seconds. [3]
 - Show that the total time for the first n crossings is $0.5(2^n - 1)$ seconds. [3]
 - Hence explain why the particle never comes to rest in a finite time, even though its speed tends to zero. [2]
 - Find the number of crossings after which the total time first exceeds 100 seconds. [2]
- 29** A smooth horizontal floor has coordinate axes with a smooth fixed wall along the y -axis, so that the floor occupies the region $x \geq 0$ (units in metres). A ball is at the point A(6, 0) and is projected with velocity $(-6\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$. The ball hits the wall at B, rebounds, and passes through the point C(3, 16). The coefficient of restitution between the ball and the wall is e .
- Find the coordinates of B and the time taken to reach B. [2]
 - Find the value of e . [3]
 - Find the speed of the ball at C. [2]
 - A second smooth wall lies along the line $x = 9$, with the coefficient of restitution between the ball and this wall equal to 0.8. Find the velocity of the ball immediately after it hits this wall, and the y -coordinate of the point where it hits. [4]
- 30** A smooth plane is inclined at 30° to the horizontal. A small smooth ball is released from rest at a point 4.9 m vertically above a point P on the plane, and falls freely to hit the plane at P. The coefficient of restitution between the ball and the plane is 0.5. Take $g = 9.8 \text{ m s}^{-2}$.



- Find the speed of the ball immediately before it hits the plane. [2]
- Find the components of the velocity of the ball, parallel and perpendicular to the plane, immediately after the impact. [3]
- Show that the ball returns to the plane after exactly 1 second. [3]
- Find the distance along the plane from P to the point where the ball next hits the plane. [3]