

A LEVEL FURTHER MATHS • EXTREME PAPERS

MATHS



EXAM PAPER

EXTREME PAPER: FURTHER MECHANICS

SCORE

/90

PERCENTAGE

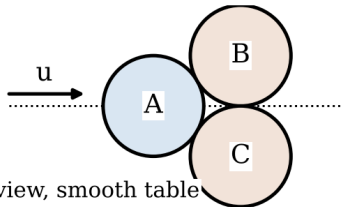
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STEMUP

Paper: Further Mechanics (90 marks)

1 Edexcel | OCR A

Three identical smooth uniform spheres A, B and C, each of radius r and mass m , lie on a smooth horizontal table. Spheres B and C are at rest and touching one another. Sphere A moves with speed u along the line perpendicular to BC through the midpoint of BC, and strikes B and C simultaneously. At the instant of impact the centres of A, B and C form an equilateral triangle. The coefficient of restitution between any two of the spheres is e , where $0 < e \leq 1$.



plan view, smooth table

- Find, in terms of u and e , the velocity of A (positive in its original direction) and the speed of B immediately after the impact. You must justify that there is no impulse between B and C. [5]
- Hence find the range of values of e for which A rebounds. Find the fraction of the initial kinetic energy of the system that is lost when A is brought to rest. [3]
- B and C are now placed with their centres further apart, still symmetrically about the path of A, and A strikes both simultaneously with the line of centres from A to B making an angle φ with the path of A. Show that the speed of A after impact, in its original direction, is $\frac{(1 - 2e \cos^2 \varphi)}{(1 + 2\cos^2 \varphi)} u$. Hence show that A can be brought to rest only if $e \geq \frac{2}{3}$, and find the value of $\cos^2 \varphi$ for which A is brought to rest when $e = \frac{4}{5}$. [3]

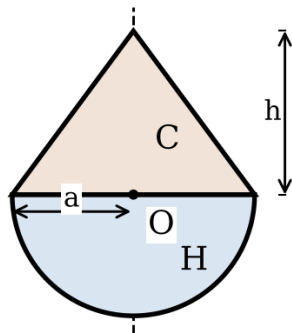
2 Edexcel | AQA | OCR A | OCR MEI | CIE

A particle of mass m is projected vertically upwards from the ground with speed U . The medium through which it moves exerts a resistance of magnitude $\frac{m g v^2}{V^2}$, where v is the speed of the particle and V is a positive constant. No other forces act apart from gravity, and g is the acceleration due to gravity.

- Show that the greatest height of the particle above the ground is $H = \frac{V^2}{(2g)} \ln\left(1 + \frac{U^2}{V^2}\right)$. [3]
- Show that the speed with which the particle returns to the ground is $w = \frac{U V}{\sqrt{U^2 + V^2}}$. [2]
- Let θ be the acute angle with $\tan \theta = \frac{U}{V}$. Show that the time of ascent is $\frac{V}{g} \theta$ and that the time of descent is $\frac{V}{g} \ln(\sec \theta + \tan \theta)$. [4]
- Hence prove that, whatever the value of U , the descent takes longer than the ascent. [2]

3 Edexcel | AQA | OCR A | OCR MEI | CIE

A uniform solid hemisphere H has radius a , and the centre of its plane face is O .

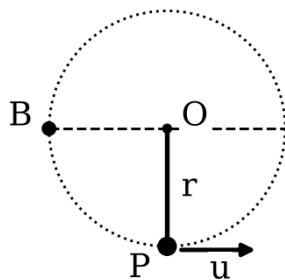


- (a) Use integration to show that the centre of mass of H lies on its axis of symmetry at a distance $\frac{3a}{8}$ from O . [4]
- (b) A uniform solid right circular cone C , made of the same material as H , has base radius a and height h . The base of C is joined to the plane face of H , with the centres of the two faces coinciding, to form a solid S . Show that the centre of mass of S lies on the axis of symmetry at a distance $\frac{(3a^2 - h^2)}{4(2a + h)}$ from O , on the same side of O as H . [3]
- (c) Now let $h = a$. The solid S rests in equilibrium with the curved surface of H in contact with a plane inclined at an angle θ to the horizontal. The coefficient of friction between S and the plane is $\frac{1}{5}$. Let φ be the acute angle between the axis of symmetry of S and the upward vertical. Show that $\sin \varphi = 6 \sin \theta$, and find the greatest value of θ for which this equilibrium is possible. [4]

4 Edexcel | AQA | OCR A | OCR MEI | CIE

A particle P of mass m is attached to one end of a light inextensible string of length r . The other end is fixed at O . P hangs at rest vertically below O and is projected horizontally with speed u . It moves in part of a vertical circle until the string goes slack when OP makes an angle α above the horizontal through O , where $0 < \alpha < 90^\circ$.

After the string goes slack, P moves freely under gravity and passes through the point B , which is level with O and at distance r from O , on the opposite side of the vertical through O from the point where the string went slack. Air resistance may be ignored.

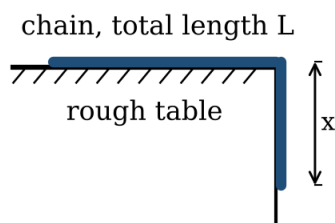


- (a) Show that $u^2 = g r (2 + 3 \sin \alpha)$. [3]
- (b) Show that $\cos \alpha = \frac{1}{2}$, and hence find u^2 in terms of g and r . [5]
- (c) Find the speed of P at B , and the angle that its velocity makes with the horizontal there. [2]

5 Edexcel | AQA | OCR A | OCR MEI

A uniform flexible chain of length L and mass M lies on a rough horizontal table, with a length $\frac{L}{2}$ hanging vertically over a smooth rounded edge of the table. The coefficient of friction between the chain and the table is $\frac{1}{2}$. The chain is released from rest and slides, remaining in contact with the table top and the edge as it does so, the hanging part remaining vertical.

At time t the length of chain hanging over the edge is x , where $\frac{L}{2} \leq x \leq L$.

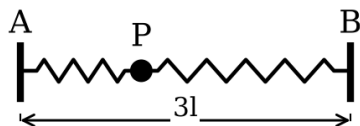


- Show that $\frac{d^2x}{dt^2} = \frac{3g}{(2L)}(x - \frac{L}{3})$. [3]
- Hence find the exact time taken for the last link of the chain to leave the table. [4]
- By considering the work done by the forces on the chain, find the speed of the chain when its last link leaves the table. [4]

6 Edexcel | AQA | OCR A | OCR MEI | CIE

Two fixed points A and B lie on a smooth horizontal table with $AB = 3l$. A particle P of mass m is attached to A by a light elastic string of natural length l and modulus of elasticity λ , and to B by a second light elastic string of natural length l and modulus of elasticity λ . P moves on the line AB . A string exerts no force when it is slack. Let $AP = x$, where $0 \leq x \leq 3l$, and let $\omega_1 = \sqrt{\lambda/(m l)}$.

P is released from rest at A .

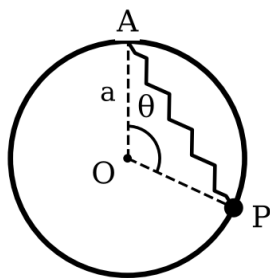


plan view, smooth horizontal table

- Show that the motion of P is simple harmonic in each of the three ranges $0 < x < l$, $l < x < 2l$ and $2l < x < 3l$. State the centre of oscillation and the angular speed in each range. [3]
- Find the speed of P when $x = l$, and show that the amplitude of the oscillation in the middle range $l < x < 2l$ is $\frac{\sqrt{7}}{2} l$. [4]
- Find the exact period of the motion of P in terms of m , l and λ . [5]

7 Edexcel | OCR A | OCR MEI | CIE

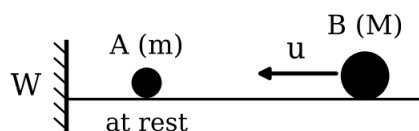
A smooth circular wire of centre O and radius a is fixed in a vertical plane. A is the highest point of the wire. A small bead P of mass m is threaded on the wire and is attached to A by a light elastic string of natural length a and modulus of elasticity $3mg$. Let $\angle AOP = \theta$, where $0 \leq \theta \leq \pi$. The string is taut only when $\theta > \frac{\pi}{3}$. Take the zero of gravitational potential energy at the level of O .



- Show that, for $\frac{\pi}{3} \leq \theta \leq \pi$, the total potential energy of the system is $V = mga \left[\frac{9}{2} - 6 \sin\left(\frac{\theta}{2}\right) - 2 \cos \theta \right]$. [3]
- Show that the bead has exactly two equilibrium positions with $0 < \theta \leq \pi$. Find the value of $\sin\left(\frac{\theta_0}{2}\right)$ for the one with $\theta_0 < \pi$, and determine whether each equilibrium is stable or unstable. [5]
- The bead is displaced slightly from $\theta = \theta_0$ and released. Show that it performs small oscillations that are approximately simple harmonic, and find their period. [4]

8 Edexcel | AQA | OCR A | OCR MEI | CIE

A fixed smooth vertical wall W is perpendicular to a smooth horizontal line along which two particles A and B move. A has mass m , B has mass M , and A is between W and B . All impacts, both between A and B and between A and W , are perfectly elastic. Initially A is at rest and B is moving towards W with speed u . Let v_A and v_B be the velocities of A and B , measured positive in the direction away from W , and let $X = \sqrt{m} v_A$ and $Y = \sqrt{M} v_B$. Let θ be the acute angle with $\tan \theta = \sqrt{m/M}$. Call a single A - B impact or a single A - W impact an impact. Let N be the total number of impacts that ever occur.



- Show that an impact between A and B leaves $X^2 + Y^2$ and $\sqrt{m} X + \sqrt{M} Y$ unchanged. Deduce that the effect on the point (X, Y) is reflection in the line through the origin parallel to the vector $(\sqrt{m}, \sqrt{M})$. [3]
- Show that the impacts alternate between A - B and A - W , and that no further impact occurs exactly when $X \geq 0$ and $\sqrt{M} X \leq \sqrt{m} Y$. Let ψ be the polar angle of (X, Y) , measured anticlockwise from the X -axis. Show that this condition is $90^\circ - \theta \leq \psi \leq 90^\circ$ (taking ψ modulo 360°). [3]
- Prove that N is the greatest integer for which $N\theta < \pi$. [4]
- Hence find, in exact form, the set of values of the ratio M/m for which there are at least 10 impacts. [4]

You may use $\cos 36^\circ = \frac{(1 + \sqrt{5})}{4}$.

[2]