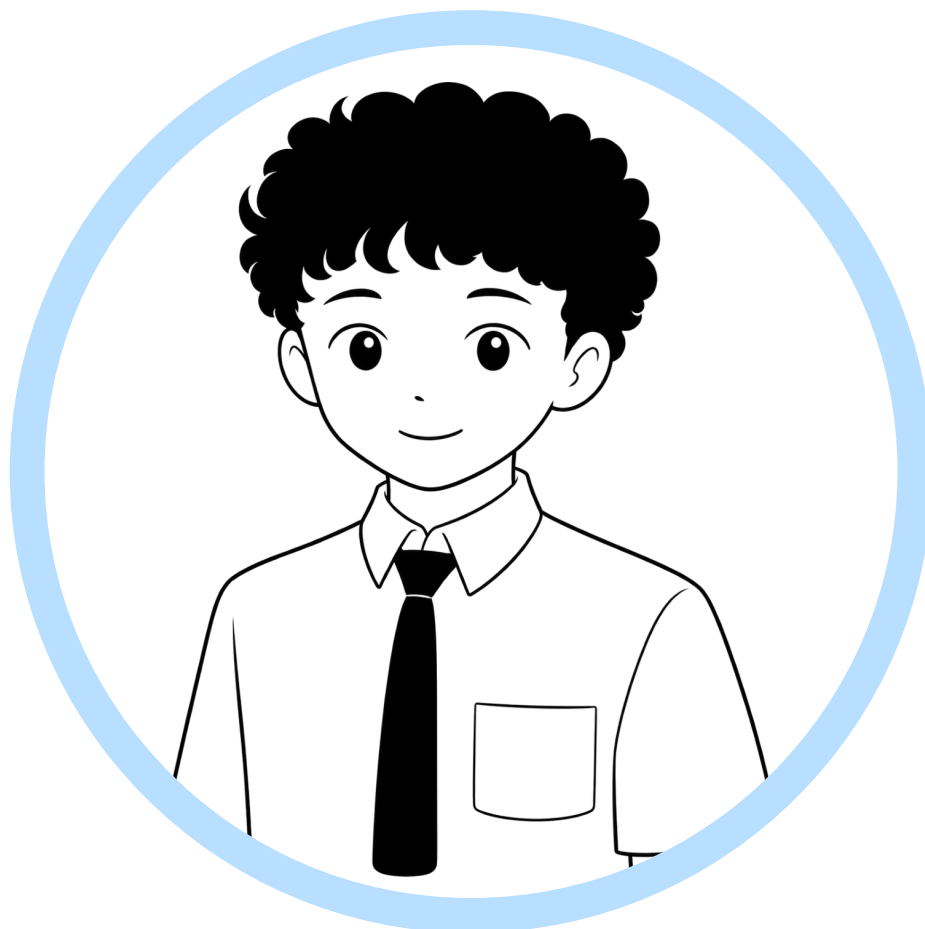


A LEVEL FURTHER MATHS • EXTREME PAPERS

MATHS



EXAM PAPER

EXTREME PAPER: FURTHER DECISION MATHEMATICS

SCORE

/80

PERCENTAGE

%

STEMUP

Paper: Further Decision Mathematics (80 marks)

1 Edexcel | OCR MEI

Five engineers, A, B, C, D and E, are to be given one task each from five tasks, 1 to 5, so that the total time taken, in hours, is as small as possible. The table shows the time each engineer takes on each task. The times for B on task 3 and for D on task 5 are both x hours, where x is a positive constant.

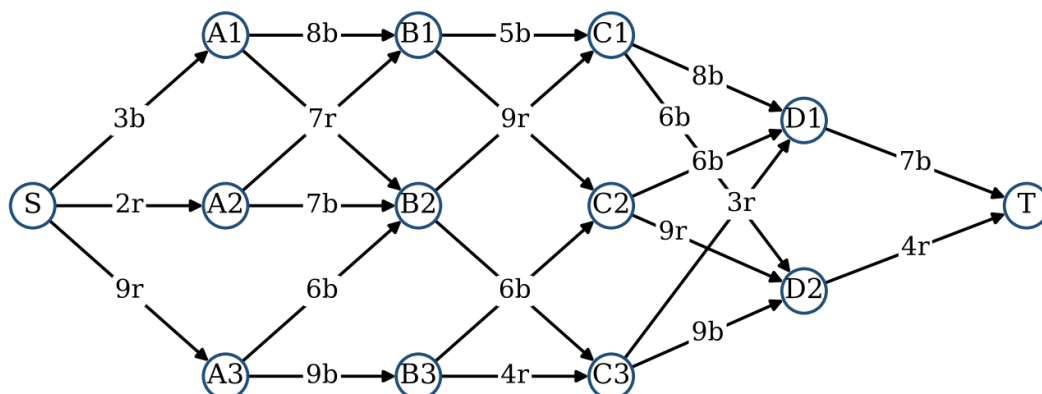
Engineer	Task 1	Task 2	Task 3	Task 4	Task 5
A	4	13	8	14	5
B	5	11	x	10	6
C	10	6	11	6	4
D	2	12	11	8	x
E	13	2	2	14	9

- (a) Take $x = 11$. Use the Hungarian algorithm, showing the matrix at each stage and the lines used each time, to find the allocation that minimises the total time, and state this time. [5]
- (b) Use your final matrix from part (a), without applying the algorithm again, to find the least total time $f(x)$ for every $x > 0$, and the optimal allocation or allocations in each range of x . State the values of x for which the optimal allocation is not unique. [6]

2 Edexcel | OCR MEI

A traveller goes from town S to town T in five legs, passing through one town in each of the stages A, B, C and D. Each leg is made by rail (r) or by bus (b). The number on each arc is the journey time in hours and the letter is the mode of transport, so $3r$ is a 3 hour rail leg.

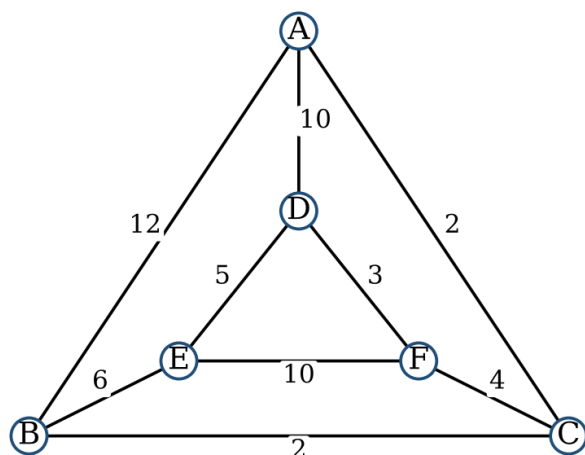
Company rules say that the traveller must change mode of transport after every leg: two consecutive legs may never use the same mode. The traveller wants the route of least total time that obeys this rule.



- (a) Ignore the rule for the moment. Use dynamic programming, working back from T, to find the shortest route from S to T, and show that it breaks the rule. [3]
- (b) Explain why the working in part (a) cannot be used to find the best route that obeys the rule, and show how to change the state so that dynamic programming does work. Hence find the route of least time that obeys the rule, and its time. [4]
- (c) Use dynamic programming again to find how many routes from S to T obey the rule. [2]

3 Edexcel

The network shows the roads, with their lengths in km, in a small town. A road inspector must travel along every road at least once. Every junction A to F is the end of an odd number of roads.



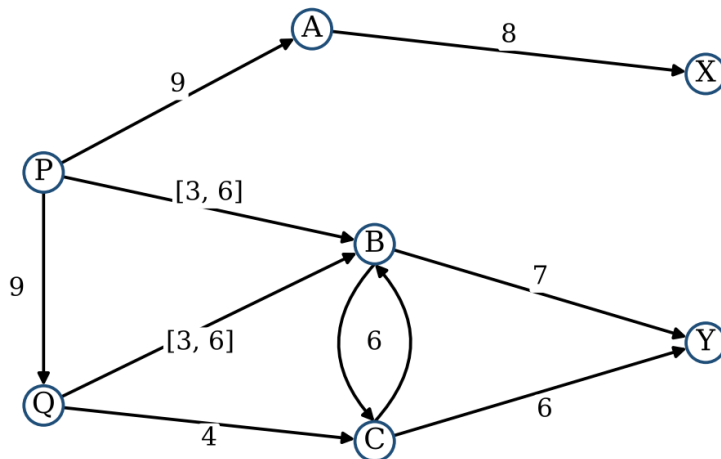
- Use Floyd's algorithm, with the pivots taken in the order A, B, C, D, E, F, to find the table of least distances between every pair of junctions. Show the working for each pivot. [4]
- The inspector starts and finishes at the same junction. Find the length of the shortest such route, and show that no other way of pairing the junctions does as well. [3]
- The inspector decides instead to start at A and to finish at a different junction. Find the junction at which to finish so that the route is as short as possible, and the length of the route. [3]

4 Edexcel | AQA | OCR MEI

Oil is pumped from two depots P and Q, through the pipe network shown, to two outlets X and Y. Depot P can supply at most 10 units per hour and depot Q at most 7 units per hour. Outlet X must receive between 2 and 11 units per hour, and outlet Y between 2 and 8 units per hour.

Each arc is labelled with its capacity. A single number is an upper capacity (the lower capacity is 0). A label $[l, u]$ means the pipe must carry at least l and at most u units per hour. The pipe between B and C can be used in either direction, as shown by the two arcs BC and CB, and each direction has capacity 6.

A feasible flow has PQ 0, PA 2, PB 5, QB 3, QC 0, BC 1, CB 0, BY 7, CY 1, AX 2.



- Add a supersource S and a supersink T, stating the lower and upper capacity of every new arc. Find the value of the given feasible flow. [2]
- Starting from this flow, use flow augmenting routes (taking care over the lower capacities) to find a maximum flow. List each route used and the increase it gives, and state the final flows on SP, SQ, XT and YT. [3]
- Find a cut that proves that your flow in part (b) is maximum. Explain carefully how its value is calculated when some arcs have lower capacities. [3]
- The lower capacity on PB is to be raised from 3 to m , the upper capacity staying at 6. Find the greatest value of m for which a feasible flow still exists, with full justification. [3]
- Return to the original network, with the lower capacity on PB equal to 3. A student claims that the smallest possible flow is 4 units per hour, because outlets X and Y each need at least 2. Find the smallest value that a feasible flow can have, and justify it fully. [3]

5 Edexcel | AQA | OCR A | OCR MEI

A project consists of nine activities, A to I. The table shows the duration of each activity, the number of workers it needs, and its immediate predecessors. Each activity needs the stated number of workers for the whole of its duration, and cannot be interrupted once started. At most 5 workers are available at any time, and activities start only at the beginning of a day.

Activity	Days	Workers	Predecessors
A	2	1	-
B	1	2	-
C	4	3	A
D	3	3	B
E	2	2	B
F	3	2	-
G	1	3	A, B
H	3	3	G
I	1	1	B, D

- (a) Ignoring the limit on workers, carry out a forward pass and a backward pass, and find the minimum completion time, the critical activities and the float on activity E. [2]
- (b) With at most 5 workers, show that the project cannot be completed in fewer than 11 days. Note that dividing the total number of worker-days needed by 5 is not enough. [2]
- (c) Prove that the project cannot be completed in fewer than 12 days. [2]
- (d) Find a schedule that completes the project in 12 days. Give the start day of each activity, where day 0 is the start of the project, and show that no more than 5 workers are ever needed. [2]

6 Edexcel | OCR MEI

A firm makes three products in amounts x , y and z (in hundreds of units, not necessarily whole numbers). The profit is $P = 7x + 4y + 6z$, in £1000, and is to be maximised subject to limits on three resources: $6x + 3y + 2z \leq 26$, $2x + 6y + 5z \leq 26$, $6x + 4y + 4z \leq 30$, $x, y, z \geq 0$.

Slack variables r , s and t are added to the three constraints in turn.

- (a) Use the simplex algorithm, always choosing the most negative entry in the profit row to select the pivot column, to find the maximum profit and the values of x , y , z , r , s and t . Show the tableau at each stage. [5]
- (b) The amount of the third resource is changed from 30 to $30 + \delta$, where δ is small. Using the final tableau, show that the maximum profit increases at a rate of $\frac{23}{22}$ per unit of δ , and find the range of values of δ for which the same three variables stay in the solution (so that this rate applies). [2]
- (c) The profit per hundred units of product Z becomes $6 + k$. Find the range of values of k for which the optimal values of x , y , z , r , s and t found in part (a) remain optimal. [2]
- (d) A new product W is being considered. Each hundred units of W uses 0 units of the first resource, 4 units of the second and 1 unit of the third, and gives a profit of q (in £1000). Using the final tableau of part (a), show that it is worth making some W if and only if $q > \frac{5}{2}$. When $q = 4$, find the new optimal production plan and the new maximum profit. [3]

7 Edexcel | OCR MEI

Rowan and Colin play a zero-sum game. The pay-off matrix shows the amount, in £, that Rowan wins from Colin for each pair of choices (a negative entry means Rowan pays Colin). Rowan chooses a row and Colin chooses a column.

$$\begin{pmatrix} 2 & -2 & 2 \\ 0 & 2 & 4 \\ 1 & 2 & -1 \end{pmatrix} \text{ (rows R1, R2, R3; columns C1, C2, C3)}$$

- (a) Find the play-safe strategy for each player and hence show that there is no stable solution. [2]
- (b) Show how to formulate Colin's problem as a linear programme in the form: maximise $P = y_1 + y_2 + y_3$, subject to three inequalities, with $y_1, y_2, y_3 \geq 0$. You should explain why 3 is added to every pay-off, define y_1, y_2, y_3 , and state how the value of the game and Colin's optimal strategy are obtained from the solution. [3]
- (c) Find the optimal mixed strategy for each player and the value of the game, and show that these strategies are optimal. [3]
- (d) Colin is offered a fourth choice C4, for which the pay-offs to Rowan against R1, R2, R3 are 3, k and -2 respectively. Find, with proof, the set of values of k for which the value of the game is still 1. [4]
- (e) Now suppose that $-\frac{1}{2} \leq k < 5$. Find, in terms of k , the value of the game with the four choices, and the optimal strategy for each player. Show that Colin never plays C3. [4]