

Hooke's Law

1 (2 marks)

$$x = 1.0 - 0.8 = 0.2 \text{ m}$$

M1

$$T = \frac{40 \times 0.2}{0.8} = 10 \text{ N}$$

A1

Answer: 10 N

2 marks

2 (2 marks)

$$24 = \frac{60x}{1.5}$$

M1

$$x = 0.6 \text{ m}$$

A1

Answer: 0.6 m

2 marks

3 (2 marks)

$$x = 0.7 - 0.5 = 0.2 \text{ m}, 16 = \frac{\lambda \times 0.2}{0.5}$$

M1

$$\lambda = 40 \text{ N}$$

A1

Answer: 40 N

2 marks

4 (2 marks)

$$12 = \frac{50 \times 0.3}{l}$$

M1

$$l = 1.25 \text{ m}$$

A1

Answer: 1.25 m

2 marks

5 (3 marks)

$$(a) \frac{\lambda}{l} = \frac{30}{0.6} = 50 \text{ N m}^{-1}$$

B1

Answer: 50 N m⁻¹

1 mark

$$(b) T = 50 \times 0.04$$

M1

$$= 2 \text{ N}$$

A1

Answer: 2 N

2 marks

6 (3 marks)

$$T = 0.5 \times 9.8 = 4.9 \text{ N}, x = \frac{4.9 \times 2}{98} = 0.1 \text{ m}$$

M1 A1

$$\text{Length} = 2.1 \text{ m}$$

A1

Answer: 2.1 m

3 marks

7 (3 marks)

$$T = 0.8 \times 9.8 = 7.84 \text{ N}$$

B1

$$7.84 = \frac{\lambda \times 0.3}{1.2}$$

M1

$$\lambda = 31.36 \text{ N}$$

A1

Answer: 31.36 N

3 marks

8 (3 marks)

$$\text{Compression} = 0.5 - 0.42 = 0.08 \text{ m}$$

B1

$$\text{Thrust} = \frac{20 \times 0.08}{0.5}$$

M1

$$= 3.2 \text{ N}$$

A1

Answer: 3.2 N

3 marks

9 (4 marks)

$$(a) 0.3 \times 9.8 = \frac{\lambda \times 0.06}{1.2}$$

M1

$$\lambda = 58.8 \text{ N}$$

A1

Answer: 58.8 N

2 marks

$$(b) 0.75 \times 9.8 = \frac{58.8x}{1.2}$$

M1

$$x = 0.15 \text{ m}$$

A1

Answer: 0.15 m

2 marks

10 (4 marks)

$$\text{Greatest extension} = 0.9 \text{ m, so } T = \frac{45 \times 0.9}{0.9} = 45 \text{ N}$$

M1 A1

$$m = \frac{45}{9.8}$$

M1

$$= 4.59 \text{ kg}$$

A1

Answer: 45 N; 4.59 kg

4 marks

11 (4 marks)By symmetry the tensions are equal: $2T = 2 \times 9.8$, so $T = 9.8 \text{ N}$

M1 A1

$$9.8 = \frac{49x}{0.6}, x = 0.12 \text{ m}$$

M1

$$\text{Length} = 0.72 \text{ m}$$

A1

Answer: 0.72 m

4 marks

12 (4 marks)

$$x = 1.5 - 1.2 = 0.3 \text{ m}$$

B1

$$T = \frac{72 \times 0.3}{1.2} = 18 \text{ N}$$

M1 A1

The force on A has magnitude equal to the tension, 18 N

B1

Answer: T = 18 N; force on A 18 N

4 marks

13 (4 marks)

$$P: 15 = \frac{30x}{1}, x = 0.5 \text{ m}$$

M1 A1

$$Q: 15 = \frac{30x}{2}, x = 1 \text{ m}$$

A1

$$T = \frac{\lambda x}{l}, \text{ so for the same } T \text{ and } \lambda \text{ the extension } x \text{ is proportional to the natural length } l$$

E1

Answer: P: 0.5 m; Q: 1 m; x is proportional to l

4 marks

14 (5 marks)

$$6 = \frac{\lambda(0.8 - l)}{1}, 14 = \frac{\lambda(1.0 - l)}{1}$$

M1

$$\text{Subtract: } 8 = \frac{0.2\lambda}{1}, \text{ so } \frac{\lambda}{1} = 40$$

M1 A1

$$6 = 40(0.8 - l), \text{ so } l = 0.65 \text{ m}$$

A1

$$\lambda = 40 \times 0.65 = 26 \text{ N}$$

A1

Answer: l = 0.65 m, λ = 26 N

5 marks

15 (5 marks)

$$(a) \text{ Tension is the same in both strings: } x_1 = \frac{10 \times 0.6}{30} = 0.2 \text{ m, } x_2 = \frac{10 \times 0.4}{20} = 0.2 \text{ m}$$

M1 A1

$$\text{Total length} = 0.6 + 0.4 + 0.2 + 0.2 = 1.4 \text{ m}$$

A1

Answer: 1.4 m

3 marks

$$(b) 10 = \frac{\lambda' \times 0.4}{1.0}$$

M1

$$\lambda' = 25 \text{ N}$$

A1

Answer: 25 N

2 marks

16 (4 marks)

(a) Natural lengths 0.4 m and 0.8 m

B1

Each piece carries the same tension as the original at the same value of $\frac{x}{l}$, so each has modulus 36 N

E1

Answer: 0.4 m and 0.8 m; each has modulus 36 N

2 marks

$$(b) x_1 = \frac{9 \times 0.4}{36} = 0.1 \text{ m}, x_2 = \frac{9 \times 0.8}{36} = 0.2 \text{ m}$$

M1 A1

Answer: 0.1 m and 0.2 m

2 marks

17 (5 marks)

$$4.9 = \frac{\lambda(1.5 - l)}{1}, 7.84 = \frac{\lambda(1.8 - l)}{1}$$

M1 A1

$$\text{Subtract: } 2.94 = \frac{0.3\lambda}{1}, \text{ so } \frac{\lambda}{1} = 9.8$$

M1

$$4.9 = 9.8(1.5 - l), \text{ so } l = 1.0 \text{ m}$$

A1

$$\lambda = 9.8 \times 1.0 = 9.8 \text{ N}$$

A1

Answer: l = 1.0 m, λ = 9.8 N

5 marks

18 (5 marks)

$$\text{AP extension} = 1.2 - 0.8 = 0.4 \text{ m}, T = \frac{24 \times 0.4}{0.8} = 12 \text{ N}$$

M1 A1

$$\text{BP} = 3.0 - 1.2 = 1.8 \text{ m}, \text{ extension } 0.8 \text{ m}$$

B1

$$\text{Equilibrium: the tensions are equal, } 12 = \frac{\lambda \times 0.8}{1.0}$$

M1

$$\lambda = 15 \text{ N}$$

A1

Answer: λ = 15 N

5 marks

19 (4 marks)

$$\text{Stretched length} = 3 \times 0.4 = 1.2 \text{ m}, \text{ extension } 0.6 \text{ m}$$

M1

$$T = \frac{30 \times 0.6}{0.6} = 30 \text{ N}$$

A1

The two parts of string at A each pull with 30 N, at 60° to each other

M1

$$\text{Resultant} = 2 \times 30 \times \cos 30^\circ = 52.0 \text{ N}$$

A1

Answer: T = 30 N; 52.0 N

4 marks

20 (4 marks)

Tension is proportional to the extension, not to the length. Doubling the length does not double the extension

E1

$$\text{At length 1.5 m: } x = 0.5 \text{ m}, T = \frac{20 \times 0.5}{1} = 10 \text{ N, which agrees}$$

B1

$$\text{At length 3 m: } x = 2 \text{ m}, T = \frac{20 \times 2}{1}$$

M1

$$T = 40 \text{ N}$$

A1

Answer: T = 40 N

4 marks

21 (7 marks)

$$\text{Each string has length } \sqrt{1.2^2 + 1.6^2} = 2.0 \text{ m}$$

M1 A1

$$\text{Resolving vertically: } 2T \times \frac{1.6}{2.0} = 2 \times 9.8$$

M1 A1

$$T = 12.25 \text{ N}$$

A1

$$\text{Extension} = 2.0 - 1.5 = 0.5 \text{ m, so } 12.25 = \frac{\lambda \times 0.5}{1.5}$$

M1

$$\lambda = 36.75 \text{ N}$$

A1

Answer: λ = 36.75 N

7 marks

22 (8 marks)

$$R = 2 \times 9.8 \times \cos 30^\circ = 17.0 \text{ N}$$

B1

$$\text{Limiting friction } F = 0.3R = 5.09 \text{ N}$$

M1

$$\text{About to slide down: } T + F = 2 \times 9.8 \times \sin 30^\circ = 9.8, \text{ so } T = 4.71 \text{ N}$$

M1 A1

$$\text{About to slide up: } T = 9.8 + F = 14.9 \text{ N}$$

A1

$$x = \frac{Tl}{\lambda} = \frac{T \times 0.7}{29.4}: x = 0.112 \text{ m and } 0.355 \text{ m}$$

M1 A1

The length of the string lies between 0.812 m and 1.05 m

A1

Answer: 0.812 m to 1.05 m

8 marks

23 (7 marks)**(a)** Let $AP = y$, so $PB = 2.4 - y$ **B1**

$$T_A = \frac{40(y - 0.8)}{0.8} = 50(y - 0.8), T_B = \frac{30(2.4 - y - 0.6)}{0.6} = 50(1.8 - y)$$

M1 A1

$$\text{Equilibrium: } T_A = T_B + 20$$

M1

$$50y - 40 = 90 - 50y + 20, \text{ so } y = 1.5 \text{ m}$$

A1**Answer: AP = 1.5 m****5 marks****(b)** $T_A = 50 \times 0.7 = 35 \text{ N}$, $T_B = 50 \times 0.3 = 15 \text{ N}$ **B1 B1****Answer: $T_A = 35 \text{ N}$, $T_B = 15 \text{ N}$** **2 marks****24 (8 marks)****(a)** The tension is the same in both springs**B1**

$$x_1 = \frac{T}{k_1}, x_2 = \frac{T}{k_2}, \text{ so } x = T\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

M1 A1

$$k = \frac{T}{x} = \frac{1}{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} = \frac{k_1 k_2}{(k_1 + k_2)}$$

A1

$$\text{Answer: } k = \frac{k_1 k_2}{(k_1 + k_2)}$$

4 marks**(b)** Total tension $= k_1 x + k_2 x$ **M1**

$$= (k_1 + k_2)x, \text{ so the stiffness is } k_1 + k_2$$

A1**Answer: $k_1 + k_2$** **2 marks****(c)** Series: $\frac{40 \times 60}{100} = 24 \text{ N m}^{-1}$; side by side: 100 N m^{-1} **B1 B1****Answer: 24 N m^{-1} ; 100 N m^{-1}** **2 marks****25 (8 marks)**The ring is smooth, so the tension T is the same in both parts of the string**B1**

$$AR = RB = \sqrt{1.2^2 + 0.9^2} = 1.5 \text{ m}, \text{ so the string has length } 3.0 \text{ m and extension } 1.2 \text{ m}$$

M1 A1

$$\text{Resolving vertically: } 2T \times \frac{0.9}{1.5} = 12$$

M1 A1

$$T = 10 \text{ N}$$

A1

$$10 = \frac{\lambda \times 1.2}{1.8}, \text{ so } \lambda = 15 \text{ N}$$

M1 A1**Answer: $\lambda = 15 \text{ N}$** **8 marks****26 (10 marks)**

$$\text{(a) } mg = \frac{\lambda(a - l)}{l} \text{ and } 3mg = \frac{\lambda(b - l)}{l}$$

M1 A1

$$\text{Dividing: } 3(a - l) = b - l, \text{ so } l = \frac{(3a - b)}{2}$$

M1 A1

$$a - l = \frac{(b - a)}{2}, \text{ so } \lambda = \frac{mg l}{a - l} = \frac{mg(3a - b)}{b - a}$$

M1 A1

$$\text{Answer: } l = \frac{(3a - b)}{2}; \lambda = \frac{mg(3a - b)}{b - a}$$

6 marks**(b)** The string stretches more under the larger load, so $b > a$ **B1**The natural length is positive, so $3a - b > 0$ and $b < 3a$ **B1****Answer: $a < b < 3a$** **2 marks**

$$\text{(c) } l = \frac{2.7 - 1.5}{2} = 0.6 \text{ m}$$

A1

$$\lambda = \frac{0.4 \times 9.8 \times 1.2}{0.6} = 7.84 \text{ N}$$

A1**Answer: 0.6 m ; 7.84 N** **2 marks**

27 (11 marks)

$$(a) x_{BC} = \frac{a}{2}, \text{ so } T_{BC} = \frac{4mg \times a}{2/2a} = mg$$

M1 A1

Resolving vertically at B: $T_{AB} = T_{BC} + mg = 2mg$

M1 A1

Answer: $T_{BC} = mg, T_{AB} = 2mg$

4 marks

$$(b) x_{AB} = \frac{2mg \times a}{3mg} = \frac{2a}{3}, \text{ so } AB = \frac{5a}{3}$$

M1 A1

$$AC = \frac{5a}{3} + \frac{5a}{2} = \frac{25a}{6}, \text{ so } k = \frac{25}{6}$$

A1

$$\text{Answer: } k = \frac{25}{6}$$

3 marks

$$(c) \text{ Tension } T' \text{ is the same in both strings: } AB = a + \frac{T'a}{3mg}, BC = 2a + \frac{2aT'}{4mg}$$

M1

$$\text{Sum: } 3a + \frac{5aT'}{6mg} = \frac{25a}{6}, \text{ so } T' = \frac{7mg}{5}$$

M1 A1

$$AB = a + \frac{7a}{15} = \frac{22a}{15}$$

A1

$$\text{Answer: } AB = \frac{22a}{15}$$

4 marks

28 (10 marks)

$$(a) T = \frac{\lambda(L - 0.8)}{0.8}$$

B1

Each string makes an angle ϕ with the horizontal, where $\sin \phi = \frac{h}{L}$

B1

Resolving vertically: $W = 2T \sin \phi$

M1

$$W = 2 \times \frac{\lambda(L - 0.8)}{0.8} \times \frac{h}{L} = \frac{2\lambda h(L - 0.8)}{0.8L}$$

A1

$$\text{Answer: } W = \frac{2\lambda h(L - 0.8)}{0.8L}$$

4 marks

$$(b) L = \sqrt{0.6^2 + 0.8^2} = 1.0$$

M1 A1

$$W = \frac{2 \times 50 \times 0.8 \times 0.2}{0.8 \times 1.0} = 20 \text{ N}$$

M1 A1

Answer: $L = 1.0, W = 20 \text{ N}$

4 marks

$$(c) \tan \phi = \frac{0.8}{0.6}$$

M1

$$\phi = 53.1^\circ$$

A1

Answer: 53.1°

2 marks

29 (10 marks)(a) Each half of the string has natural length d and modulus λ

B1

Each half has length $\sqrt{d^2 + y^2}$, so its extension is $\sqrt{d^2 + y^2} - d$

M1

$$T = \frac{\lambda(\sqrt{d^2 + y^2} - d)}{d}$$

A1

$$\text{Answer: } T = \frac{\lambda(\sqrt{d^2 + y^2} - d)}{d}$$

3 marks

$$(b) \text{ Resolving perpendicular to AB: } F = 2T \times \frac{y}{\sqrt{d^2 + y^2}}$$

M1

$$F = \frac{2\lambda y}{d} \times \frac{(\sqrt{d^2 + y^2} - d)}{\sqrt{d^2 + y^2}} = \frac{2\lambda y}{d} \left(1 - \frac{d}{\sqrt{d^2 + y^2}}\right)$$

A1

$$\text{Answer: } F = \frac{2\lambda y}{d} \left(1 - \frac{d}{\sqrt{d^2 + y^2}}\right)$$

2 marks

$$(c) \frac{d}{\sqrt{d^2 + y^2}} = \left(1 + \frac{y^2}{d^2}\right)^{-1/2}$$

M1

$$\approx 1 - \frac{y^2}{2d^2}$$

A1

$$F \approx \frac{2\lambda y}{d} \times \frac{y^2}{2d^2} = \frac{\lambda y^3}{d^3}$$

A1

Answer: $F \approx \frac{\lambda y^3}{d^3}$

3 marks

(d) Exact: 0.155 N

M1

Approximation: 0.160 N

A1

Answer: exact 0.155 N; approximate 0.160 N

2 marks

30 (11 marks)

(a) Lowest position: friction acts down the plane, so $T_2 = 2g \sin 30^\circ + F$

M1 A1

Highest position: friction acts up the plane, so $T_1 + F = 2g \sin 30^\circ$

M1 A1

Adding: $T_1 + T_2 = 4g \sin 30^\circ = 19.6$, and $T_1 + T_2 = \frac{\lambda \times 0.2}{1.2} + \frac{\lambda \times 0.6}{1.2}$

M1

$\frac{0.8\lambda}{1.2} = 19.6$, so $\lambda = 29.4$

A1

Answer: $\lambda = 29.4$

6 marks

(b) $T_1 = \frac{29.4 \times 0.2}{1.2} = 4.9$ N, $T_2 = \frac{29.4 \times 0.6}{1.2} = 14.7$ N

M1 A1

$T_2 - T_1 = 2F$, so $F = 4.9$ N

M1

$R = 2g \cos 30^\circ = 16.97$, and $F = \mu R$ at the extremes, so $\mu = \frac{4.9}{16.97} = 0.289$

M1 A1

Answer: $\mu = 0.289$

5 marks

Elastic Strings

1 (2 marks)

Tension = weight: $mg = \frac{3mg \times x}{1}$

M1

$x = \frac{1}{3}$

A1

Answer: $x = \frac{1}{3}$

2 marks

2 (3 marks)

$T = \frac{49 \times 0.5}{1} = 24.5$ N

M1

$2a = 24.5 - 2 \times 9.8$

M1

$a = 2.45 \text{ m s}^{-2}$ upwards

A1

Answer: 2.45 m s^{-2} upwards

3 marks

3 (3 marks)

$F = 0.4 \times 3 \times 9.8 = 11.76$ N

B1

$11.76 = \frac{60x}{1.0}$

M1

$x = 0.196$ m

A1

Answer: 0.196 m

3 marks

4 (3 marks)

$T = 2 \times 9.8 \times \sin 30^\circ = 9.8$ N

M1 A1

$9.8 = \frac{49x}{0.5}$, so $x = 0.1$ m

A1

Answer: 0.1 m

3 marks

5 (3 marks)

$T = \frac{4mg \times a}{2/a} = 2mg$

M1 A1

$ma = 2mg - mg$, so $a = g$ upwards

A1

Answer: $T = 2mg$; g upwards

3 marks

6 (2 marks)

$$\frac{\lambda x}{l} = \frac{\lambda}{4} = 1.5 \times 9.8$$

M1

$$\lambda = 58.8 \text{ N}$$

A1

Answer: 58.8 N

2 marks

7 (4 marks)

$$\text{Limiting friction } F = 0.5 \times 2 \times 9.8 = 9.8 \text{ N}$$

B1

$$9.8 = \frac{24x}{0.6}$$

M1

$$x = 0.245 \text{ m}$$

A1

$$\text{Greatest distance} = 0.6 + 0.245 = 0.845 \text{ m}$$

A1

Answer: 0.845 m

4 marks

8 (4 marks)

$$\sin \alpha = \frac{3}{5}, \text{ so } T = 2 \times 9.8 \times \frac{3}{5} = 11.76 \text{ N}$$

M1 A1

$$11.76 = \frac{58.8x}{0.7}, \text{ so } x = 0.14 \text{ m}$$

M1

$$\text{Length} = 0.84 \text{ m}$$

A1

Answer: 0.84 m

4 marks

9 (5 marks)

$$T_A = \frac{30 \times 0.4}{0.6} = 20 \text{ N upwards}$$

M1 A1

$$T_B = \frac{20 \times 0.6}{0.4} = 30 \text{ N downwards (PB} = 1.0 \text{ m)}$$

M1 A1

$$1 \times a = 9.8 + 30 - 20, \text{ so } a = 19.8 \text{ m s}^{-2} \text{ downwards}$$

A1

Answer: 19.8 m s⁻² downwards

5 marks

10 (4 marks)

$$\text{(a) } 3mg = \frac{6mg \times x}{l}$$

M1

$$x = \frac{l}{2}$$

A1

$$\text{Answer: } \frac{l}{2}$$

2 marks

$$\text{(b) Extension} = l, \text{ so } T = \frac{6mg \times l}{l} = 6mg$$

M1

$$3ma = 6mg - 3mg, \text{ so } a = g \text{ upwards}$$

A1

Answer: g upwards

2 marks

11 (5 marks)

$$R = 4 \times 9.8 \times \cos \alpha = 31.36 \text{ N}, F = 0.25R = 7.84 \text{ N}$$

B1 M1

$$T = 4 \times 9.8 \times \sin \alpha + F = 23.52 + 7.84 = 31.36 \text{ N}$$

M1

$$31.36 = \frac{\lambda \times 0.6}{1.2}$$

M1

$$\lambda = 62.72 \text{ N}$$

A1

Answer: 62.72 N

5 marks

12 (4 marks)

$$\text{(a) } mg = \frac{2mg \times x}{a}, \text{ so } x = \frac{a}{2}$$

M1

$$\text{Length} = \frac{3a}{2}$$

A1

$$\text{Answer: } \frac{3a}{2}$$

2 marks

$$\text{(b) } 2mg = \frac{2mg \times x}{a}, \text{ so } x = a$$

M1

$$\text{Length} = 2a$$

A1

Answer: 2a

2 marks

13 (5 marks)

$$(a) \text{ Tension in string 1} = \frac{40(L - 1)}{1}$$

M1

$$\text{Tension in string 2} = \frac{60(L - 1.2)}{1.2} = 50(L - 1.2)$$

M1

$$40(L - 1) + 50(L - 1.2) = 49, \text{ so } 90L - 100 = 49 \text{ and } 90L = 149$$

A1

$$\text{Answer: } 90L = 149$$

3 marks

$$(b) L = \frac{149}{90} = 1.656 \text{ m}$$

B1

$$T_1 = 40 \times 0.656 = 26.2 \text{ N}, T_2 = 50 \times 0.456 = 22.8 \text{ N}$$

A1

$$\text{Answer: } 26.2 \text{ N and } 22.8 \text{ N}$$

2 marks

14 (6 marks)

$$T = \frac{35 \times 0.8}{0.7} = 40 \text{ N}$$

M1 A1

$$R = 2.5 \times 9.8 \times 0.8 = 19.6 \text{ N}, F = 0.5 \times 19.6 = 9.8 \text{ N}$$

B1

Weight component down the plane = $2.5 \times 9.8 \times 0.6 = 14.7 \text{ N}$; $40 > 14.7 + 9.8$ so P moves up the plane

E1

$$2.5a = 40 - 14.7 - 9.8$$

M1

$$a = 6.2 \text{ m s}^{-2} \text{ up the plane}$$

A1

$$\text{Answer: } 6.2 \text{ m s}^{-2} \text{ up the plane}$$

6 marks

15 (6 marks)

$$\text{Vertically: } T \cos 30^\circ = 1.5 \times 9.8$$

M1

$$T = 17.0 \text{ N}$$

A1

$$\text{Horizontally: } F = T \sin 30^\circ = 8.49 \text{ N}$$

M1 A1

$$x = \frac{Tl}{\lambda} = \frac{17.0 \times 0.8}{49} = 0.277 \text{ m}$$

M1

$$\text{Length} = 0.8 + 0.277 = 1.08 \text{ m}$$

A1

$$\text{Answer: } F = 8.49 \text{ N; length } 1.08 \text{ m}$$

6 marks

16 (6 marks)

$$BD = \sqrt{2^2 + 1.5^2} = 2.5 \text{ m}$$

B1

$$\text{Moments about A: } T \times (\text{perpendicular distance from A to BD}) = 6 \times 9.8 \times 1, \text{ with distance } \frac{2 \times 1.5}{2.5}$$

M1 A1

$$= 1.2 \text{ m}$$

$$T = 49 \text{ N}$$

A1

$$\text{Extension} = 2.5 - 1.25 = 1.25 \text{ m, so } 49 = \frac{\lambda \times 1.25}{1.25}$$

M1

$$\lambda = 49 \text{ N}$$

A1

$$\text{Answer: } \lambda = 49 \text{ N}$$

6 marks

17 (5 marks)

The peg is smooth, so the tension is the same throughout, and $T = 2 \times 9.8 = 19.6 \text{ N}$

M1 A1

$$19.6 = \frac{39.2x}{1.5}, \text{ so } x = 0.75 \text{ m}$$

M1 A1

$$AB + BP = 1.5 + 0.75 = 2.25, \text{ so } BP = 1.05 \text{ m}$$

A1

$$\text{Answer: } BP = 1.05 \text{ m}$$

5 marks

18 (6 marks)

$$\text{Limiting friction: A: } 0.4 \times 2 \times 9.8 = 7.84 \text{ N, B: } 0.4 \times 3 \times 9.8 = 11.76 \text{ N}$$

M1 A1

A would slip first, so $T \leq 7.84 \text{ N}$

E1

$$7.84 = \frac{39.2x}{0.5}, \text{ so } x = 0.1 \text{ m}$$

M1 A1

$$\text{Greatest distance AB} = 0.5 + 0.1 = 0.6 \text{ m}$$

A1

$$\text{Answer: } 0.6 \text{ m}$$

6 marks

19 (7 marks)

The rod is uniform and horizontal, so each string has tension $T = 2 \times 9.8 = 19.6 \text{ N}$

M1 A1

String at A: $x = \frac{19.6 \times 0.8}{58.8} = \frac{4}{15} \text{ m}$, so its length is $\frac{16}{15} \text{ m}$

M1 A1

The rod is horizontal, so the string at B also has length $\frac{16}{15} \text{ m}$ and extension $\frac{16}{15} - 0.6 = \frac{7}{15} \text{ m}$

B1

$$19.6 = \frac{\lambda \times 7}{15/0.6}$$

M1

$$\lambda = 25.2 \text{ N}$$

A1

Answer: $\lambda = 25.2 \text{ N}$

7 marks

20 (8 marks)

(a) Tension in the elastic string $= \frac{49 \times 0.1}{0.5} = 9.8 \text{ N}$

M1 A1

Tension in the inextensible string $= 1.8g = 17.64 \text{ N}$

B1

Limiting friction $F = 17.64 - 9.8 = 7.84 \text{ N}$, $R = 2g = 19.6 \text{ N}$

M1

$$\mu = \frac{7.84}{19.6} = 0.4$$

A1

Answer: $\mu = 0.4$

5 marks

(b) Friction now acts towards the pulley: $T = 17.64 + 7.84 = 25.48 \text{ N}$

M1 A1

$$25.48 = \frac{49x}{0.5}, \text{ so } x = 0.26 \text{ m}$$

A1

Answer: 0.26 m

3 marks

21 (7 marks)

$AC = AD$ and angle $DAC = 60^\circ$, so triangle ACD is equilateral and $CD = 2 \text{ m}$

B1

Moments about A: $T \times 2 \sin 60^\circ = 8 \times 9.8 \times 1.5 \sin 60^\circ$

M1 A1 A1

$$T = 58.8 \text{ N}$$

A1

$$\text{Extension} = 2 - 1.5 = 0.5 \text{ m, so } 58.8 = \frac{\lambda \times 0.5}{1.5}$$

M1

$$\lambda = 176.4 \text{ N}$$

A1

Answer: $\lambda = 176.4 \text{ N}$

7 marks

22 (7 marks)

The student has interchanged $\sin \alpha$ and $\cos \alpha$: $R = 3g \cos \alpha$ and the component of weight down the plane is $3g \sin \alpha$

E1

$$R = 3 \times 9.8 \times 0.6 = 17.64 \text{ N}, F = 0.5R = 8.82 \text{ N}$$

M1 A1

$$T + F = 3 \times 9.8 \times 0.8, \text{ so } T = 23.52 - 8.82 = 14.7 \text{ N}$$

M1 A1

$$14.7 = \frac{49x}{0.9}, \text{ so } x = 0.27 \text{ m}$$

M1

$$\text{Length} = 1.17 \text{ m}$$

A1

Answer: 1.17 m

7 marks

23 (7 marks)

(a) Component of weight down the plane $= 20 \times 0.6 = 12 \text{ N}$

B1

$$T_A = \frac{30(y - 0.5)}{0.5} = 60(y - 0.5), T_B = \frac{36(3 - y - 0.6)}{0.6} = 60(2.4 - y), \text{ with } AP = y$$

M1 A1

$$T_A = T_B + 12$$

M1

$$60y - 30 = 144 - 60y + 12, \text{ so } y = 1.55 \text{ m}$$

A1

Answer: $AP = 1.55 \text{ m}$

5 marks

(b) $T_A = 60 \times 1.05 = 63 \text{ N}$, $T_B = 60 \times 0.85 = 51 \text{ N}$

B1 B1

Answer: 63 N and 51 N

2 marks

24 (6 marks)

String length = $\sqrt{0.6^2 + 0.8^2} = 1.0$ m, extension 0.2 m M1

$$T = \frac{40 \times 0.2}{0.8} = 10 \text{ N} \quad \text{A1}$$

Horizontally: $P = T \times \frac{0.6}{1.0} = 6 \text{ N}$ M1 A1

Vertically: $R = 0.5 \times 9.8 + T \times \frac{0.8}{1.0}$ M1

$$R = 12.9 \text{ N} \quad \text{A1}$$

Answer: $P = 6 \text{ N}$, $R = 12.9 \text{ N}$ 6 marks

25 (7 marks)

Let $AP = y$, so $PB = 2.0 - y$ B1

$$T_A = \frac{20(y - 0.5)}{0.5} = 40(y - 0.5), T_B = \frac{24(1.6 - y)}{0.4} = 60(1.6 - y) \quad \text{M1 A1}$$

Limiting friction = $0.5 \times 3 \times 9.8 = 14.7 \text{ N}$ B1

$$|T_A - T_B| = |100y - 116| \leq 14.7 \quad \text{M1 A1}$$

$$1.01 \leq y \leq 1.31 \quad \text{A1}$$

Answer: $1.01 \text{ m} \leq AP \leq 1.31 \text{ m}$ 7 marks

26 (7 marks)

The height of B is $\sqrt{5^2 - 3^2} = 4 \text{ m}$ B1

Moments about A: $S \times 4 = 20 \times 9.8 \times 1.5$ M1 A1

$$S = 73.5 \text{ N} \quad \text{A1}$$

Horizontally: $T = S = 73.5 \text{ N}$ M1

Extension = $3 - 1.2 = 1.8 \text{ m}$, so $73.5 = \frac{\lambda \times 1.8}{1.2}$ M1

$$\lambda = 49 \text{ N} \quad \text{A1}$$

Answer: $\lambda = 49 \text{ N}$ 7 marks

27 (11 marks)

(a) $R = 2.5 \times 9.8 \times 0.6 = 14.7 \text{ N}$ B1

$$F = 0.25 \times 14.7 = 3.675 \text{ N} \quad \text{M1}$$

$T + F = 2.5 \times 9.8 \times 0.8 = 19.6$, so $T = 15.925 \text{ N}$ M1 A1

$$x = \frac{15.925 \times 0.7}{31.85} = 0.35 \text{ m, so } AP = 1.05 \text{ m} \quad \text{A1}$$

Answer: $AP = 1.05 \text{ m}$ 5 marks

(b) $T = 19.6 + 3.675 = 23.275 \text{ N}$, $x = \frac{23.275 \times 0.7}{31.85} = 0.512 \text{ m}$ M1 A1

$$AP = 1.21 \text{ m} \quad \text{A1}$$

Answer: $AP = 1.21 \text{ m}$ 3 marks

(c) $T = \frac{31.85 \times 0.8}{0.7} = 36.4 \text{ N}$ M1

$$36.4 - 19.6 - 3.675 = 2.5a \quad \text{M1}$$

$$a = 5.25 \text{ m s}^{-2} \text{ up the plane} \quad \text{A1}$$

Answer: 5.25 m s^{-2} up the plane 3 marks

28 (10 marks)

(a) $OA = OP = a$, so triangle OAP is isosceles and $AP = 2a \sin \frac{\theta}{2}$ M1

Angle $OPA = 90^\circ - \frac{\theta}{2}$ and the tangent is perpendicular to OP, so the angle between PA and the tangent is $\frac{\theta}{2}$ A1

Answer: $AP = 2a \sin \frac{\theta}{2}$; the string makes angle $\frac{\theta}{2}$ with the tangent 2 marks

(b) The weight makes an angle θ with the radius OP produced, so its component along the tangent is $mg \sin \theta$ M1 A1

$$T \cos \frac{\theta}{2} = mg \sin \theta = 2mg \sin \frac{\theta}{2} \cos \frac{\theta}{2} \quad \text{M1}$$

$$T = 2mg \sin \frac{\theta}{2} \quad \text{A1}$$

Answer: $T = 2mg \sin \frac{\theta}{2}$ 4 marks

(c) $T = \frac{3mg(AP - a)}{a}$ and $T = 2mg \sin \frac{\theta}{2} = \frac{mg AP}{a}$ M1

$$3(AP - a) = AP, \text{ so } AP = \frac{3a}{2} \quad \text{A1}$$

$$\sin \frac{\theta}{2} = \frac{3}{4} \quad \text{M1}$$

$$\theta = 97.2^\circ \quad \text{A1}$$

Answer: $AP = \frac{3a}{2}$, $\theta = 97.2^\circ$ 4 marks

29 (10 marks)

(a) $AP = BP = \sqrt{0.6^2 + 0.8^2} = 1.0 \text{ m}$ B1

$$T_A = \frac{24 \times 0.4}{0.6} = 16 \text{ N}, T_B = \frac{20 \times 0.2}{0.8} = 5 \text{ N} \quad \text{M1 A1}$$

Answer: 16 N and 5 N 3 marks

(b) Parallel to AB: $F_1 = 16 \times 0.6 - 5 \times 0.6 = 6.6 \text{ N}$ M1 A1

Perpendicular to AB: $F_2 = 16 \times 0.8 + 5 \times 0.8 = 16.8 \text{ N}$ A1

$$F = \sqrt{6.6^2 + 16.8^2} = 18.0 \text{ N} \quad \text{M1}$$

$$\tan \theta = \frac{16.8}{6.6}, \theta = 68.6^\circ \quad \text{A1}$$

Answer: $F = 18.0 \text{ N}$ at 68.6° to AB 5 marks

(c) Resultant force on P = F in the opposite direction M1

$$a = \frac{18.0}{0.8} = 22.6 \text{ m s}^{-2} \quad \text{A1}$$

Answer: 22.6 m s^{-2} 2 marks

30 (11 marks)

(a) The ring is smooth, so the tension T is the same in both parts B1

The horizontal components $T \cos \alpha_1$ and $T \cos \alpha_2$ must balance, so $\alpha_1 = \alpha_2$ E1

Answer: equal tensions, so equal horizontal components, so equal angles 2 marks

(b) The horizontal span is $2.4 = 3.0 \cos \alpha$, so $\cos \alpha = 0.8$ and $\sin \alpha = 0.6$ M1 A1

Vertically: $2T \sin \alpha = 12$, so $T = 10 \text{ N}$ M1 A1

$$\text{Extension} = 3.0 - 2.2 = 0.8 \text{ m, so } 10 = \frac{\lambda \times 0.8}{2.2} \quad \text{M1}$$

$$\lambda = 27.5 \text{ N} \quad \text{A1}$$

Answer: $T = 10 \text{ N}$, $\lambda = 27.5 \text{ N}$ 6 marks

(c) Let the horizontal distances of R from A and B be x_1 and x_2 : $x_1 + x_2 = 2.4$ and $(x_1 - x_2) \tan \alpha = 0.9$ M1 A1

$$x_1 = 1.8, \text{ so the depth below A} = 1.8 \times 0.75 = 1.35 \text{ m} \quad \text{A1}$$

Answer: 1.35 m 3 marks

Elastic Springs

1 (2 marks)

$$\text{Compression} = 0.1 \text{ m, thrust} = \frac{20 \times 0.1}{0.4} \quad \text{M1}$$

$$= 5 \text{ N} \quad \text{A1}$$

Answer: 5 N 2 marks

2 (2 marks)

$$\text{Thrust} = 0.5 \times 9.8 = 4.9 \text{ N, so } 4.9 = \frac{14.7x}{0.6}$$

M1

$$x = 0.2 \text{ m, so the length is } 0.4 \text{ m}$$

A1

Answer: 0.4 m

2 marks

3 (2 marks)

$$0.4 \times 9.8 = \frac{\lambda \times 0.098}{0.5}$$

M1

$$\lambda = 20 \text{ N}$$

A1

Answer: 20 N

2 marks

4 (3 marks)

$$\text{Stiffness} = \frac{2 \times 9.8}{0.05} = 392 \text{ N m}^{-1}$$

M1 A1

$$5 \times 9.8 = 392x, \text{ so } x = 0.125 \text{ m}$$

A1

Answer: 0.125 m

3 marks

5 (3 marks)

$$\text{Thrust is proportional to compression: } 30 = k \times 0.15, \text{ so } k = 200 \text{ N m}^{-1}$$

M1 A1

$$\text{Thrust} = 200 \times 0.3 = 60 \text{ N}$$

A1

Answer: 60 N

3 marks

6 (3 marks)

$$\text{(a) } \frac{\lambda}{l} = \frac{48}{0.6} = 80 \text{ N m}^{-1}$$

B1

Answer: 80 N m⁻¹

1 mark

(b) Each piece has natural length 0.3 m and modulus of elasticity 48 N

M1

$$\text{Stiffness} = \frac{48}{0.3} = 160 \text{ N m}^{-1}, \text{ twice that of the original spring}$$

A1

Answer: 160 N m⁻¹

2 marks

7 (4 marks)

$$\text{Compression} = 0.5 - 0.3 = 0.2 \text{ m}$$

B1

$$\text{Thrust} = \frac{15 \times 0.2}{0.5} = 6 \text{ N}$$

M1 A1

The force on each point is 6 N

B1

Answer: thrust 6 N; force on each point 6 N

4 marks

8 (4 marks)

$$\text{Thrust} = 2 \times 9.8 \times \sin 30^\circ = 9.8 \text{ N}$$

M1 A1

$$9.8 = \frac{49x}{0.8}, \text{ so } x = 0.16 \text{ m}$$

M1

$$\text{Length} = 0.64 \text{ m}$$

A1

Answer: 0.64 m

4 marks

9 (4 marks)

$$\text{Compression} = 0.3 \text{ m, so thrust} = \frac{58.8 \times 0.3}{0.4} = 44.1 \text{ N}$$

M1 A1

$$3a = 44.1 - 3 \times 9.8$$

M1

$$a = 4.9 \text{ m s}^{-2} \text{ upwards}$$

A1

Answer: 4.9 m s⁻² upwards

4 marks

10 (6 marks)**(a)** Length less than natural length, so thrust

B1

$$\text{Thrust} = \frac{18 \times 0.15}{0.6} = 4.5 \text{ N}$$

M1 A1

Answer: thrust of 4.5 N

3 marks

(b) Length greater than natural length, so tension

B1

$$\text{Tension} = \frac{18 \times 0.18}{0.6} = 5.4 \text{ N}$$

M1 A1

Answer: tension of 5.4 N

3 marks

11 (4 marks)

Both springs have the same extension x : tension total = $\frac{20x}{0.5} + \frac{30x}{0.5} = 100x$

M1 A1

$$100x = 1.5 \times 9.8, \text{ so } x = 0.147 \text{ m}$$

M1

$$\text{Length} = 0.647 \text{ m}$$

A1

Answer: 0.647 m

4 marks

12 (4 marks)

Extension = $0.9 - 0.6 = 0.3 \text{ m}$, so tension = $\frac{24 \times 0.3}{0.6} = 12 \text{ N}$

M1 A1

$$12 = 1.5a$$

M1

$$a = 8 \text{ m s}^{-2} \text{ towards the wall}$$

A1

Answer: 8 m s^{-2}

4 marks

13 (5 marks)

Let the natural length be l and the stiffness k : $2g = k(l - 0.40)$ and $3g = k(l - 0.35)$

M1 A1

Dividing: $3(l - 0.40) = 2(l - 0.35)$, so $l = 0.5 \text{ m}$

M1 A1

$$k = \frac{19.6}{0.1} = 196 \text{ N m}^{-1}, \text{ so } \lambda = kl = 98 \text{ N}$$

A1

Answer: natural length 0.5 m; modulus 98 N

5 marks

14 (5 marks)

Let $AP = y$, so $PB = 1.8 - y$

B1

$$T_A = \frac{40(y - 0.5)}{0.5} = 80(y - 0.5), T_B = \frac{24(1.4 - y)}{0.4} = 60(1.4 - y)$$

M1 A1

$$T_A = T_B + 1.5 \times 9.8$$

M1

$$80y - 40 = 84 - 60y + 14.7, \text{ so } y = 0.991 \text{ m}$$

A1

Answer: AP = 0.991 m

5 marks

15 (5 marks)

(a) Extension 0.08 m , tension $2 \times 9.8 = 19.6 \text{ N}$

B1

$$\text{Stiffness} = \frac{19.6}{0.08} = 245 \text{ N m}^{-1}$$

M1 A1

Answer: 245 N m^{-1}

3 marks

(b) Extension = 0.115 m , so tension = $245 \times 0.115 = 28.175 \text{ N}$

M1

$$m = \frac{28.175}{9.8} = 2.875 \text{ kg}$$

A1

Answer: 2.875 kg

2 marks

16 (5 marks)

Compression = 0.2 m , so thrust = $\frac{30 \times 0.2}{0.5} = 12 \text{ N}$

M1 A1

A: $2a_A = 12$, so $a_A = 6 \text{ m s}^{-2}$, away from B

M1 A1

B: $3a_B = 12$, so $a_B = 4 \text{ m s}^{-2}$, away from A

A1

Answer: A: 6 m s^{-2} ; B: 4 m s^{-2} , in opposite directions

5 marks

17 (5 marks)

(a) $4 \times \frac{\lambda \times 0.05}{0.5} = 8 \times 9.8$

M1 A1

$$\lambda = 196 \text{ N}$$

A1

Answer: $\lambda = 196 \text{ N}$

3 marks

(b) $4 \times \frac{196x}{0.5} = 20 \times 9.8$

M1

$$x = 0.125 \text{ m, so the length is } 0.375 \text{ m}$$

A1

Answer: 0.375 m

2 marks

18 (4 marks)

$$T - 2 \times 9.8 = 2 \times 2.45$$

M1

$$T = 24.5 \text{ N}$$

A1

$$24.5 = \frac{49x}{0.5}, \text{ so } x = 0.25 \text{ m}$$

M1

$$\text{Length} = 0.75 \text{ m}$$

A1

Answer: 0.75 m

4 marks

19 (3 marks)

The student has used the length 0.4 m in place of the compression

E1

$$\text{Compression} = 0.5 - 0.4 = 0.1 \text{ m}$$

M1

$$\text{Thrust} = \frac{20 \times 0.1}{0.5} = 4 \text{ N}$$

A1

Answer: 4 N

3 marks

20 (8 marks)

$$\text{(a) Tension in AP} = \frac{20(x - 0.5)}{0.5} = 40x - 20$$

M1 A1

$$\text{Tension in PB} = \frac{30(1.1 - x)}{0.5} = 66 - 60x$$

M1 A1

$$\text{Resultant towards B} = (66 - 60x) - (40x - 20) = 86 - 100x$$

A1

Answer: 86 - 100x

5 marks

$$\text{(b) Limiting friction} = 0.25 \times 2 \times 9.8 = 4.9 \text{ N}$$

B1

$$|86 - 100x| \leq 4.9$$

M1

$$0.811 \leq x \leq 0.909$$

A1

Answer: 0.811 ≤ AP ≤ 0.909

3 marks

21 (6 marks)

$$\text{(a) Spring length} = 1.5 - 0.6 = 0.9 \text{ m, so compression} = 0.1 \text{ m}$$

M1

$$\text{Thrust} = \frac{39.2 \times 0.1}{1.0} = 3.92 \text{ N}$$

A1

$$T + 3.92 = 2 \times 9.8$$

M1

$$T = 15.68 \text{ N}$$

A1

Answer: 15.68 N

4 marks

$$\text{(b) } 2a = 2 \times 9.8 - 3.92$$

M1

$$a = 7.84 \text{ m s}^{-2} \text{ downwards}$$

A1

Answer: 7.84 m s⁻² downwards

2 marks

22 (7 marks)

$$\text{(a) } T = 3mg, \text{ so } 3mg = \frac{4mg \times x}{1}$$

M1

$$x = \frac{3l}{4}$$

A1

$$\text{Answer: } \frac{3l}{4}$$

2 marks

$$\text{(b) P: } 2ma = 3mg - 2mg$$

M1

$$a = \frac{g}{2} \text{ upwards}$$

A1

Q falls freely with acceleration g downwards

B1

Answer: P: $\frac{g}{2}$ upwards; Q: g downwards

3 marks

$$\text{(c) } 2mg = \frac{4mg \times x}{1}, \text{ so } x = \frac{l}{2}$$

M1

$$\text{Length} = \frac{3l}{2}$$

A1

$$\text{Answer: } \frac{3l}{2}$$

2 marks

23 (7 marks)

(a) B is in equilibrium, so the tension in the string is $2 \times 9.8 = 19.6 \text{ N}$

B1

A is in equilibrium, so the tension in the spring is 19.6 N

M1

$$19.6 = \frac{49x}{0.8}, \text{ so } x = 0.32 \text{ m}$$

M1 A1

Length = 1.12 m

A1

Answer: 19.6 N; 1.12 m

5 marks

(b) $3a = 19.6$

M1

$a = 6.53 \text{ m s}^{-2}$ towards W

A1

Answer: 6.53 m s⁻² towards W

2 marks

24 (8 marks)

$R = 2 \times 9.8 \times 0.8 = 15.68 \text{ N}$, $F = 0.5R = 7.84 \text{ N}$

B1 M1

About to slide down: $S + F = 2 \times 9.8 \times 0.6 = 11.76$, so $S = 3.92 \text{ N}$

M1 A1

About to slide up: $S = 11.76 + 7.84 = 19.6 \text{ N}$

A1

$$\text{Compression} = \frac{Sl}{\lambda} = \frac{S \times 0.9}{49}; 0.072 \text{ m and } 0.36 \text{ m}$$

M1 A1

The length lies between 0.54 m and 0.828 m

A1

Answer: 0.54 m to 0.828 m

8 marks

25 (7 marks)

The length of the spring is $\sqrt{0.4^2 + 0.3^2} = 0.5 \text{ m}$, so the extension is 0.2 m

B1

$$\text{Vertically: } T \times \frac{0.3}{0.5} = 0.6 \times 9.8$$

M1

$T = 9.8 \text{ N}$

A1

$$9.8 = \frac{\lambda \times 0.2}{0.3}, \text{ so } \lambda = 14.7 \text{ N}$$

M1 A1

$$\text{Horizontally: } R = T \times \frac{0.4}{0.5} = 7.84 \text{ N}$$

M1 A1

Answer: $\lambda = 14.7 \text{ N}$, $R = 7.84 \text{ N}$

7 marks

26 (6 marks)

(a) The stiffnesses $50 + 50$ apply to springs side by side, which share the load

E1

In series each spring carries the whole tension and the extensions add

E1

Answer: stiffnesses add only for springs in parallel

2 marks

(b) Each spring has extension $\frac{4.9}{50} = 0.098 \text{ m}$

M1

Total extension = 0.196 m

A1

Answer: 0.196 m

2 marks

(c) 3×0.098

M1

= 0.294 m

A1

Answer: 0.294 m

2 marks

27 (10 marks)

(a) S_2 : $T_2 = 9.8 \text{ N}$, so $x_2 = \frac{9.8 \times 0.4}{39.2} = 0.1 \text{ m}$

M1 A1

S_1 : $T_1 = 3 \times 9.8 = 29.4 \text{ N}$, so $x_1 = \frac{29.4 \times 0.5}{24.5} = 0.6 \text{ m}$

M1

Lengths: S_1 is 1.1 m and S_2 is 0.5 m

A1

Answer: S_1 : 1.1 m, S_2 : 0.5 m

4 marks

(b) Only S_1 acts on P: $2a = 29.4 - 2 \times 9.8$

M1

$a = 4.9 \text{ m s}^{-2}$ upwards

A1

Answer: 4.9 m s⁻² upwards

2 marks

(c) $T_1 = 19.6 \text{ N}$, so $x = \frac{19.6 \times 0.5}{24.5} = 0.4 \text{ m}$

M1

Length = 0.9 m

A1

Answer: 0.9 m

2 marks

$$(d) T_2 = 29.4 \text{ N}, x_2 = \frac{29.4 \times 0.4}{39.2} = 0.3 \text{ m}; T_1 = 49 \text{ N}, x_1 = \frac{49 \times 0.5}{24.5} = 1.0 \text{ m}$$

M1

$$\text{Total length} = 0.5 + 1.0 + 0.4 + 0.3 = 2.2 \text{ m}$$

A1

Answer: 2.2 m

2 marks

28 (10 marks)

$$(a) \text{ Moments about A: } S \times \cos \theta = 3 \times 9.8 \times 0.5 \cos \theta$$

M1 A1

$$S = 14.7 \text{ N, independent of } \theta$$

A1

Answer: thrust = 14.7 N for every θ

3 marks

$$(b) 14.7 = \frac{88.2x}{1.2}, \text{ so } x = 0.2 \text{ m}$$

M1

$$\text{Length} = 1.0 \text{ m}$$

A1

Answer: 1.0 m

2 marks

$$(c) \text{ Height of B above the ground} = 0.4 + \sin \theta = 1.0$$

M1 A1

$$\theta = 36.9^\circ$$

A1

Answer: $\theta = 36.9^\circ$

3 marks

$$(d) x = \frac{14.7 \times 1.2}{176.4} = 0.1 \text{ m, so the spring has length 1.1 m}$$

M1

$$0.4 + \sin \theta = 1.1, \text{ so } \theta = 44.4^\circ$$

A1

Answer: $\theta = 44.4^\circ$

2 marks

29 (10 marks)

$$(a) \text{ Spring length} = \sqrt{a^2 + h^2}, \text{ so } T = \frac{\lambda(\sqrt{a^2 + h^2} - a)}{a}$$

M1

$$\text{Vertically: } T \times \frac{h}{\sqrt{a^2 + h^2}} = mg$$

M1

$$\frac{\lambda h}{a} \left(1 - \frac{a}{\sqrt{a^2 + h^2}}\right) = mg, \text{ with } \lambda = \frac{15mg}{8}$$

A1

$$\text{Answer: } \frac{15mg}{8} \times \frac{h}{a} \left(1 - \frac{a}{\sqrt{a^2 + h^2}}\right) = mg$$

3 marks

$$(b) \text{ Spring length} = \frac{5a}{3}, \text{ extension } \frac{2a}{3}, \text{ so } T = \frac{15mg}{8} \times \frac{2}{3} = \frac{5mg}{4}$$

M1 A1

$$\text{Vertical component} = \frac{5mg}{4} \times \frac{4a}{3/5a/3} = mg$$

A1

$$\text{Answer: equilibrium at } h = \frac{4a}{3}, \text{ where } T = \frac{5mg}{4}$$

3 marks

$$(c) \text{ As } h \text{ increases, both } h \text{ and } 1 - \frac{a}{\sqrt{a^2 + h^2}} \text{ increase, so the vertical component of the tension}$$

E1

increases with h It therefore equals mg for only one value of h

E1

Answer: the vertical component increases with h

2 marks

$$(d) R = T \times \frac{a}{\frac{5a}{3}} = \frac{5mg}{4} \times \frac{3}{5}$$

M1

$$R = \frac{3mg}{4}$$

A1

$$\text{Answer: } \frac{3mg}{4}$$

2 marks

30 (10 marks)

$$(a) \text{ Natural lengths } 0.25 \text{ m and } 0.75 \text{ m}$$

B1

A part of the spring has the same tension at the same value of $\frac{x}{l}$, so each part has modulus 40 N

E1

Answer: 0.25 m and 0.75 m, each with modulus 40 N

2 marks

$$(b) \text{ Equal tensions: } \frac{40x_1}{0.25} = \frac{40x_2}{0.75}, \text{ so } x_2 = 3x_1$$

M1

$$x_1 + x_2 = 1.4 - 1.0 = 0.4, \text{ so } x_1 = 0.1 \text{ m and } x_2 = 0.3 \text{ m}$$

M1 A1

$$AP = 0.35 \text{ m, PB} = 1.05 \text{ m, } T = \frac{40 \times 0.1}{0.25} = 16 \text{ N}$$

A1

Answer: AP = 0.35 m, PB = 1.05 m, T = 16 N**4 marks****(c) Stiffnesses:** $\frac{40}{0.25} = 160 \text{ N m}^{-1}$ and $\frac{40}{0.75} = \frac{160}{3} \text{ N m}^{-1}$ **B1**

$$160x_1 = \frac{160}{3}(0.4 - x_1) + 8$$

M1 A1

$$x_1 = 0.1375, \text{ so AP} = 0.25 + 0.1375 = 0.388 \text{ m}$$

A1**Answer: AP = 0.388 m****4 marks**

Elastic Potential Energy

1 (2 marks)

$$\text{EPE} = \frac{\lambda x^2}{2l} = \frac{40 \times 0.2^2}{2 \times 0.8}$$

M1

$$= 1 \text{ J}$$

A1**Answer: 1 J****2 marks****2 (2 marks)**

$$\text{Work done} = \text{EPE gained} = \frac{50 \times 0.4^2}{2 \times 2}$$

M1

$$= 2 \text{ J}$$

A1**Answer: 2 J****2 marks****3 (2 marks)**

$$\text{Compression} = 0.1 \text{ m, EPE} = \frac{20 \times 0.1^2}{2 \times 0.4}$$

M1

$$= 0.25 \text{ J}$$

A1**Answer: 0.25 J****2 marks****4 (2 marks)**

$$1.44 = \frac{16x^2}{2 \times 0.5}$$

M1

$$x^2 = 0.09, \text{ so } x = 0.3 \text{ m}$$

A1**Answer: 0.3 m****2 marks****5 (2 marks)**

$$\text{EPE at } 0.2 \text{ m} = \frac{20 \times 0.2^2}{2} = 0.4 \text{ J, EPE at } 0.4 \text{ m} = \frac{20 \times 0.4^2}{2} = 1.6 \text{ J}$$

M1

$$\text{Work done} = 1.2 \text{ J}$$

A1**Answer: 1.2 J****2 marks****6 (3 marks)**

$$0.75 = \frac{\lambda \times 0.2^2}{2 \times 0.8}$$

M1 A1

$$\lambda = \frac{0.75 \times 1.6}{0.04} = 30 \text{ N}$$

A1**Answer: 30 N****3 marks****7 (4 marks)**

$$\text{Extension} = 0.6 \text{ m, EPE} = \frac{29.4 \times 0.6^2}{2 \times 1} = 5.292 \text{ J}$$

M1 A1

$$\text{Energy: } \frac{1}{2} \times 0.5v^2 = 0.5 \times 9.8 \times 1.6 - 5.292$$

M1

$$v^2 = 10.192, \text{ so } v = 3.19 \text{ m s}^{-1}$$

A1**Answer: 3.19 m s⁻¹****4 marks**

8 (4 marks)

$$\text{Initial EPE} = \frac{24 \times 0.3^2}{2 \times 0.6} = 1.8 \text{ J}$$

M1 A1

$$\frac{1}{2} \times 0.4v^2 = 1.8$$

M1

$$v = 3 \text{ m s}^{-1}$$

A1

Answer: 3 m s⁻¹

4 marks

9 (5 marks)

$$\text{Initial EPE} = \frac{39.2 \times 0.2^2}{2 \times 0.5} = 1.568 \text{ J}$$

M1 A1

$$\text{Work done against friction} = 0.25 \times 2 \times 9.8 \times 0.2 = 0.98 \text{ J}$$

B1

$$\frac{1}{2} \times 2v^2 = 1.568 - 0.98$$

M1

$$v = 0.767 \text{ m s}^{-1}$$

A1

Answer: 0.767 m s⁻¹

5 marks

10 (4 marks)

$$\text{Let the greatest extension be } x. \text{ Energy: } mg(l + x) = \frac{4mg x^2}{2l}$$

M1 A1

$$2x^2 - lx - l^2 = 0, \text{ so } (2x + l)(x - l) = 0$$

M1

$$x = l$$

A1

Answer: x = l

4 marks

11 (4 marks)

$$\text{Energy: } 1 \times 9.8(0.8 + x) = \frac{196x^2}{2 \times 0.4}$$

M1 A1

$$25x^2 - x - 0.8 = 0$$

M1

$$x = 0.2 \text{ m}$$

A1

Answer: 0.2 m

4 marks

12 (5 marks)

Let the greatest extension be x. P moves a distance 0.5 + x down the plane

B1

$$2 \times 9.8 \times \sin 30^\circ \times (0.5 + x) = \frac{39.2x^2}{2 \times 0.5}$$

M1 A1

$$8x^2 - 2x - 1 = 0$$

M1

$$x = 0.5 \text{ m}$$

A1

Answer: 0.5 m

5 marks

13 (5 marks)

$$\text{Elastic energy lost} = \frac{49 \times 0.2^2}{2 \times 0.5} = 1.96 \text{ J}$$

M1 A1

$$\text{Work against friction} = \mu \times 2 \times 9.8 \times 0.2 = 3.92\mu$$

M1 A1

$$3.92\mu = 1.96, \text{ so } \mu = 0.5$$

A1

Answer: $\mu = 0.5$

5 marks

14 (5 marks)

$$\text{(a) When the extension is } s, \text{ the tension is } T = \frac{\lambda s}{l}$$

B1

$$\text{Work done} = \int_0^x \frac{\lambda s}{l} \text{ d } s$$

M1

$$= \left[\frac{\lambda s^2}{2l} \right]_0^x = \frac{\lambda x^2}{2l}$$

A1

$$\text{Answer: work done} = \frac{\lambda x^2}{2l}$$

3 marks

$$\text{(b) Work done} = \frac{\lambda b^2}{2l} - \frac{\lambda a^2}{2l} = \frac{\lambda(b^2 - a^2)}{2l}$$

M1

$$= \frac{30(0.81 - 0.09)}{3} = 7.2 \text{ J}$$

A1

Answer: 7.2 J

2 marks

15 (4 marks)

Initially AP = 0.8 (extension 0.3 m) and BP = 1.2 (extension 0.7 m): $EPE = \frac{20 \times 0.3^2}{2 \times 0.5} + \frac{20 \times 0.7^2}{2 \times 0.5}$
 $= 11.6 \text{ J}$

M1 A1

At the midpoint both extensions are 0.5 m: $EPE = 2 \times \frac{20 \times 0.5^2}{2 \times 0.5} = 10 \text{ J}$

M1

$$\frac{1}{2} \times 0.8v^2 = 11.6 - 10 = 1.6, \text{ so } v = 2 \text{ m s}^{-1}$$

A1

Answer: 2 m s⁻¹

4 marks

16 (6 marks)

Compression = 0.1 m, $EPE = \frac{29.4 \times 0.1^2}{2 \times 0.3} = 0.49 \text{ J}$

M1 A1

$$\frac{1}{2} \times 0.2v^2 = 0.49 - 0.2 \times 9.8 \times 0.1$$

M1

$$v = 1.71 \text{ m s}^{-1}$$

A1

After leaving the spring: $\frac{1}{2} \times 0.2v^2 = 0.2 \times 9.8h$, so $h = \frac{v^2}{2g} = 0.15 \text{ m}$

M1 A1

Answer: v = 1.71 m s⁻¹; h = 0.15 m

6 marks

17 (6 marks)

(a) $T = \frac{\lambda x}{l}$, so $\frac{\lambda x^2}{2l} = \frac{1}{2} \times \frac{\lambda x}{l} \times x$

M1

$$= \frac{1}{2}Tx$$

A1

Answer: elastic potential energy = $\frac{1}{2}Tx$

2 marks

(b) $T = 3 \times 9.8 = 29.4 \text{ N}$, $x = \frac{29.4 \times 1.2}{58.8} = 0.6 \text{ m}$

M1

$$EPE = \frac{1}{2} \times 29.4 \times 0.6 = 8.82 \text{ J}$$

A1

Answer: 8.82 J

2 marks

(c) PE lost = $29.4 \times 0.6 = 17.64 \text{ J}$

B1

The tension increases from 0 to 29.4 N, while the weight is constant; the extra energy is the work done by the force which lowers the particle gently (or becomes kinetic energy if the particle is released)

E1

Answer: 17.64 J

2 marks

18 (3 marks)

The student has used the length 1.5 m in place of the extension

E1

$$x = 1.5 - 1.2 = 0.3 \text{ m}$$

M1

$$EPE = \frac{36 \times 0.3^2}{2 \times 1.2} = 1.35 \text{ J}$$

A1

Answer: 1.35 J

3 marks

19 (10 marks)

(a) Elastic energy initially = $\frac{49 \times 0.3^2}{2 \times 0.5} = 4.41 \text{ J}$

M1 A1

$$\frac{1}{2} \times 2 \times v^2 = 4.41 - 2 \times 9.8 \times 0.5 \times 0.3 = 1.47$$

M1 A1

$$v = 1.21 \text{ m s}^{-1}$$

A1

Answer: 1.21 m s⁻¹

5 marks

(b) Greatest speed when the acceleration is zero: $2 \times 9.8 \times 0.5 = \frac{49x}{0.5}$, so $x = 0.1 \text{ m}$

M1 A1

$$\frac{1}{2} \times 2 \times v^2 = 49(0.3^2 - 0.1^2) - 9.8 \times 0.2 = 1.96$$

M1 A1

$$v = 1.4 \text{ m s}^{-1}$$

A1

Answer: 1.4 m s⁻¹

5 marks

20 (7 marks)

(a) Energy: $\frac{1}{2} \times 0.4 \times 6^2 = \frac{\lambda \times 0.3^2}{2 \times 0.5}$

M1 A1

$$7.2 = 0.09\lambda, \text{ so } \lambda = 80 \text{ N}$$

A1

Answer: $\lambda = 80 \text{ N}$

3 marks

(b) Elastic energy when the extension is 0.1 m = $\frac{80 \times 0.1^2}{2 \times 0.5} = 0.8 \text{ J}$

M1 A1

$$\frac{1}{2} \times 0.4 \times v^2 = 7.2 - 0.8, \text{ so } v^2 = 32 \text{ and } v = 5.66 \text{ m s}^{-1}$$

M1 A1

Answer: 5.66 m s⁻¹

4 marks

21 (9 marks)

(a) Energy: $0.5 \times 9.8(1.2 + x) = \frac{58.8x^2}{2 \times 1.2}$

M1 A1

$$5x^2 - x - 1.2 = 0$$

M1

$$x = 0.6 \text{ m}$$

A1

Answer: 0.6 m

4 marks

(b) $T = \frac{58.8 \times 0.6}{1.2} = 29.4 \text{ N}$

M1

$$0.5a = 29.4 - 0.5 \times 9.8, \text{ so } a = 49 \text{ m s}^{-2} \text{ upwards}$$

A1

Answer: 49 m s⁻² upwards

2 marks

(c) Greatest speed at equilibrium: $0.5 \times 9.8 = \frac{58.8x}{1.2}$, so $x = 0.1 \text{ m}$

M1

$$\frac{1}{2} \times 0.5v^2 = 0.5 \times 9.8 \times 1.3 - \frac{58.8 \times 0.1^2}{2 \times 1.2}$$

M1

$$v = 4.95 \text{ m s}^{-1}$$

A1

Answer: 4.95 m s⁻¹

3 marks

22 (9 marks)

(a) Initial EPE = $\frac{100 \times 0.3^2}{2 \times 0.5} = 9 \text{ J}$

M1 A1

$$\text{Friction } F = 0.25 \times 2 \times 9.8 \times 0.8 = 3.92 \text{ N}$$

M1

$$\frac{1}{2} \times 2v^2 = 9 - 2 \times 9.8 \times 0.6 \times 0.3 - 3.92 \times 0.3$$

M1

$$v = 2.07 \text{ m s}^{-1}$$

A1

Answer: 2.07 m s⁻¹

5 marks

(b) Deceleration: $2a = 2 \times 9.8 \times 0.6 + 3.92$, so $a = 7.84 \text{ m s}^{-2}$

M1 A1

$$s = \frac{v^2}{2a} = \frac{4.296}{15.68} = 0.274 \text{ m}$$

A1

Answer: 0.274 m

3 marks

(c) Component of weight down the plane = 11.76 N exceeds the maximum friction 3.92 N

E1

Answer: 11.76 N > 3.92 N, so P slides back

1 mark

23 (7 marks)

(a) Equilibrium: $0.5 \times 9.8 = \frac{19.6x}{1}$, so $x = 0.25 \text{ m}$; EPE = $\frac{19.6 \times 0.25^2}{2} = 0.6125 \text{ J}$

M1 A1

$$\text{At the natural length: } \frac{1}{2} \times 0.5v^2 = \frac{1}{2} \times 0.5 \times 4^2 + 0.6125 - 0.5 \times 9.8 \times 0.25$$

M1

$$v^2 = 13.55 > 0, \text{ so the string becomes slack, with } v = 3.68 \text{ m s}^{-1}$$

A1

Answer: $v = 3.68 \text{ m s}^{-1}$

4 marks

(b) After the string is slack: $h = \frac{v^2}{2g} = \frac{13.55}{19.6} = 0.691 \text{ m}$

M1 A1

Height above equilibrium = $0.25 + 0.691 = 0.941$ m

Answer: 0.941 m

A1

3 marks

24 (9 marks)

(a) Initial EPE = $\frac{39.2 \times 0.3^2}{2 \times 0.5} = 3.528$ J

B1

Friction $F = 0.2 \times 2 \times 9.8 = 3.92$ N; $3.528 = 39.2y^2 + 3.92(0.3 + y)$

M1 A1

$y^2 + 0.1y - 0.06 = 0$, so $y = 0.2$ m (compression); length 0.3 m

A1

Answer: 0.3 m

4 marks

(b) $39.2 \times 0.2^2 = 39.2z^2 + 3.92(0.2 + z)$

M1

$z^2 + 0.1z - 0.02 = 0$, so $z = 0.1$

M1 A1

Answer: $z = 0.1$ m

3 marks

(c) From an extension of 0.1 m: EPE = 0.392 J = 3.92×0.1 , so the block just reaches the natural length position with zero speed

E1

Total distance = $0.3 + 0.2 + 0.2 + 0.1 + 0.1 = 0.9$ m

A1

Answer: 0.9 m

2 marks

25 (8 marks)

(a) $T = 2 \times 9.8 = 19.6$ N, $x_{AB} = \frac{19.6 \times 0.5}{49} = 0.2$ m, $x_{BC} = \frac{19.6 \times 0.8}{78.4} = 0.2$ m

M1 A1

EPE = $\frac{49 \times 0.2^2}{2 \times 0.5} + \frac{78.4 \times 0.2^2}{2 \times 0.8} = 1.96 + 1.96 = 3.92$ J

M1 A1

Answer: 3.92 J

4 marks

(b) With tension T the extensions are $\frac{T}{98}$ each (stiffnesses $\frac{49}{0.5} = \frac{78.4}{0.8} = 98$ N m⁻¹), so $x = \frac{T}{49}$

M1 A1

EPE = $\frac{1}{2}Tx = \frac{1}{2} \times 49x \times x = 24.5x^2$

M1

$2 \times 9.8x = 24.5x^2$, so $x = 0.8$ m

A1

Answer: 0.8 m

4 marks

26 (10 marks)

(a) $\cos \alpha = 0.8$, so friction = $\mu \times 3g \times 0.8 = 23.52\mu$

B1

Elastic energy at AP = 1.0 m: $\frac{88.2 \times 0.4^2}{2 \times 0.6} = 11.76$ J

M1 A1

Energy: $3g \times 0.6 \times 1.0 - 23.52\mu \times 1.0 = 11.76$

M1 A1

$17.64 - 23.52\mu = 11.76$, so $\mu = 0.25$

A1

Answer: $\mu = 0.25$

6 marks

(b) Tension = $\frac{88.2 \times 0.4}{0.6} = 58.8$ N up the plane

M1 A1

Weight component down the plane = 17.64 N, maximum friction = 5.88 N, and $17.64 + 5.88 = 23.52$ N < 58.8 N

M1

The tension exceeds the greatest force that can hold P, so P moves back up the plane

A1

Answer: P moves up the plane

4 marks

27 (10 marks)

(a) $AP = 2a \sin \frac{\theta}{2}$, and $AP = a$ when $\sin \frac{\theta}{2} = \frac{1}{2}$, so $\theta = 60^\circ$

M1 A1

Energy: $\frac{1}{2}mv^2 = mga(1 - \cos 60^\circ)$, so $v = \sqrt{ga}$

A1

Answer: $v = \sqrt{ga}$

3 marks

(b) At the lowest point $AP = 2a$, so the extension is a

B1

EPE = $\frac{3mg a^2}{2a} = \frac{3}{2}mga$

M1

$\frac{1}{2}mv^2 = mg \times 2a - \frac{3}{2}mga$, so $v = \sqrt{ga}$

A1

Answer: $v = \sqrt{ga}$

3 marks

(c) $\cos \theta = 1 - 2\sin^2 \frac{\theta}{2} = -\frac{1}{8}$, so the drop in height is $a(1 + \frac{1}{8}) = \frac{9a}{8}$

M1 A1

$$AP = \frac{3a}{2}, \text{ so the extension is } \frac{a}{2} \text{ and } EPE = \frac{3mg}{2a} \times \frac{a^2}{4} = \frac{3mga}{8} \quad \text{M1}$$

$$\frac{1}{2}mv^2 = \frac{9mga}{8} - \frac{3mga}{8}, \text{ so } v = \sqrt{\frac{3ga}{2}} \quad \text{A1}$$

$$\text{Answer: } v = \sqrt{\frac{3ga}{2}} \quad \text{4 marks}$$

28 (10 marks)

$$\text{(a) Initial EPE} = \frac{147 \times 0.3^2}{2 \times 0.6} = 11.025 \text{ J} \quad \text{M1 A1}$$

$$\text{Weight component} = 1.5 \times 9.8 \times 0.6 = 8.82 \text{ N, friction} = 0.5 \times 1.5 \times 9.8 \times 0.8 = 5.88 \text{ N} \quad \text{B1}$$

$$\frac{1}{2} \times 1.5v^2 = 11.025 - (8.82 + 5.88) \times 0.3 \quad \text{M1}$$

$$v = 2.97 \text{ m s}^{-1} \quad \text{A1}$$

$$\text{Answer: } 2.97 \text{ m s}^{-1} \quad \text{5 marks}$$

$$\text{(b) Deceleration: } 1.5a = 8.82 + 5.88, \text{ so } a = 9.8 \text{ m s}^{-2} \quad \text{M1 A1}$$

$$s = \frac{v^2}{2a} = \frac{8.82}{19.6} = 0.45 \text{ m} \quad \text{A1}$$

$$\text{Answer: } 0.45 \text{ m} \quad \text{3 marks}$$

$$\text{(c) Component of weight down the plane } 8.82 \text{ N is greater than the maximum friction } 5.88 \text{ N} \quad \text{M1}$$

$$\text{so the block slides back down} \quad \text{E1}$$

$$\text{Answer: } 8.82 \text{ N} > 5.88 \text{ N} \quad \text{2 marks}$$

29 (10 marks)

$$\text{(a) Energy: } 70 \times 9.8(30 + x) = \frac{16\,464x^2}{2 \times 30} \quad \text{M1 A1}$$

$$2x^2 - 5x - 150 = 0 \quad \text{M1}$$

$$x = 10 \text{ m} \quad \text{A1}$$

$$\text{Answer: } 10 \text{ m} \quad \text{4 marks}$$

$$\text{(b) Greatest speed at equilibrium: } 70 \times 9.8 = \frac{16\,464x}{30}, \text{ so } x = 1.25 \text{ m} \quad \text{M1 A1}$$

$$\frac{1}{2} \times 70v^2 = 70 \times 9.8 \times 31.25 - \frac{16\,464 \times 1.25^2}{2 \times 30} \quad \text{M1}$$

$$v = 24.5 \text{ m s}^{-1} \quad \text{A1}$$

$$\text{Answer: } 24.5 \text{ m s}^{-1} \quad \text{4 marks}$$

$$\text{(c) At the lowest point } T = \frac{16\,464 \times 10}{30} = 5488 \text{ N} \quad \text{M1}$$

$$70a = 5488 - 686, \text{ so } a = 68.6 \text{ m s}^{-2} = 7g \text{ upwards} \quad \text{A1}$$

$$\text{Answer: } 68.6 \text{ m s}^{-2} \text{ (7g) upwards} \quad \text{2 marks}$$

30 (11 marks)

$$\text{(a) At } 0.8 \text{ m below AB each string has length } \sqrt{0.6^2 + 0.8^2} = 1.0 \text{ m, extension } 0.4 \text{ m} \quad \text{B1}$$

$$EPE = 2 \times \frac{14.7 \times 0.4^2}{2 \times 0.6} = 3.92 \text{ J} \quad \text{M1 A1}$$

$$PE \text{ lost} = 0.5 \times 9.8 \times 0.8 = 3.92 \text{ J, so the speed is zero} \quad \text{A1}$$

$$\text{Answer: lowest point } 0.8 \text{ m below AB (EPE } 3.92 \text{ J} = \text{PE lost)} \quad \text{4 marks}$$

$$\text{(b) } T = \frac{14.7 \times 0.4}{0.6} = 9.8 \text{ N} \quad \text{M1}$$

$$0.5a = 2 \times 9.8 \times \frac{0.8}{1.0} - 0.5 \times 9.8 \quad \text{M1 A1}$$

$$a = 21.56 \text{ m s}^{-2} \text{ upwards} \quad \text{A1}$$

$$\text{Answer: } 21.56 \text{ m s}^{-2} \text{ upwards} \quad \text{4 marks}$$

$$\text{(c) Each string has length } \sqrt{0.6^2 + 0.45^2} = 0.75 \text{ m, extension } 0.15 \text{ m} \quad \text{B1}$$

$$\frac{1}{2} \times 0.5v^2 = 0.5 \times 9.8 \times 0.45 - 2 \times \frac{14.7 \times 0.15^2}{2 \times 0.6} \quad \text{M1}$$

$$v = 2.57 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 2.57 m s^{-1} **3 marks**

Motion Involving Elastic Forces

1 (3 marks)

$$\frac{d^2x}{dt^2} = -\frac{10}{0.2 \times 0.5}x = -100x, \text{ so } \omega = 10$$

M1 A1

$$\text{Period} = \frac{2\pi}{\omega} = 0.628 \text{ s}$$

A1**Answer: 0.628 s****3 marks**

2 (3 marks)

$$\text{(a) } \omega = 3, \text{ period} = \frac{2\pi}{3} = 2.09 \text{ s}$$

B1**Answer: 2.09 s****1 mark**

$$\text{(b) } v_{\max} = \omega A = 3 \times 0.4 = 1.2 \text{ m s}^{-1}$$

B1**Answer: 1.2 m s⁻¹****1 mark**

$$\text{(c) } a_{\max} = \omega^2 A = 9 \times 0.4 = 3.6 \text{ m s}^{-2}$$

B1**Answer: 3.6 m s⁻²****1 mark**

3 (2 marks)

$$v^2 = \omega^2(A^2 - x^2) = 16(0.25 - 0.09)$$

M1

$$v = 1.6 \text{ m s}^{-1}$$

A1**Answer: 1.6 m s⁻¹****2 marks**

4 (2 marks)

$$\omega^2 = \frac{k}{m} = \frac{200}{0.5} = 400, \text{ so } \omega = 20$$

M1

$$\text{Period} = \frac{2\pi}{\omega} = 0.314 \text{ s}$$

A1**Answer: 0.314 s****2 marks**

5 (3 marks)

$$mg = \frac{\lambda e}{l} \text{ gives } \frac{\lambda}{ml} = \frac{g}{e}, \text{ so } \omega^2 = \frac{9.8}{0.098} = 100$$

M1 A1

$$\text{Period} = \frac{2\pi}{\omega} = 0.628 \text{ s}$$

A1**Answer: 0.628 s****3 marks**

6 (5 marks)

$$\text{(a) } v = \frac{dx}{dt} = -1.2 \sin 4t$$

M1

$$\text{Maximum speed} = 1.2 \text{ m s}^{-1}$$

A1**Answer: 1.2 m s⁻¹****2 marks**

$$\text{(b) } a = \frac{dv}{dt} = -4.8 \cos 4t, \text{ so the maximum acceleration is } 4.8 \text{ m s}^{-2}$$

M1 A1

$$\text{Maximum force} = 0.5 \times 4.8 = 2.4 \text{ N}$$

A1**Answer: 4.8 m s⁻²; 2.4 N****3 marks**

7 (4 marks)

$$\text{Equilibrium: } mg = \frac{\lambda e}{l} \quad \text{B1}$$

$$\text{At displacement } y \text{ below equilibrium: } m \frac{d^2 y}{dt^2} = mg - \frac{\lambda(e + y)}{l} \quad \text{M1 A1}$$

$$= -\frac{\lambda y}{l}, \text{ so } \frac{d^2 y}{dt^2} = -\frac{\lambda}{ml} y; \text{ SHM with } \omega = \sqrt{\frac{\lambda}{ml}} \quad \text{A1}$$

$$\text{Answer: SHM with } \omega = \sqrt{\frac{\lambda}{ml}} \quad \text{4 marks}$$

8 (3 marks)

$$\omega^2 = \frac{\lambda}{ml} = \frac{30}{0.4 \times 0.6} = 125 \quad \text{M1}$$

$$v_{\max} = \omega A = \sqrt{125} \times 0.3 = 3.35 \text{ m s}^{-1} \quad \text{A1}$$

$$a_{\max} = \omega^2 A = 125 \times 0.3 = 37.5 \text{ m s}^{-2} \quad \text{A1}$$

$$\text{Answer: } 3.35 \text{ m s}^{-1}; 37.5 \text{ m s}^{-2} \quad \text{3 marks}$$

9 (6 marks)

$$\text{(a) Let } AP = y: T_A = \frac{20(y - 0.5)}{0.5} = 40(y - 0.5), T_B = \frac{30(1.5 - y)}{0.5} = 60(1.5 - y) \quad \text{M1 A1}$$

$$T_A = T_B: y = 1.1 \text{ m} \quad \text{A1}$$

$$\text{Answer: AP} = 1.1 \text{ m} \quad \text{3 marks}$$

$$\text{(b) } 0.5 \frac{d^2 x}{dt^2} = 60(0.4 - x) - 40(0.6 + x) \quad \text{M1 A1}$$

$$\frac{d^2 x}{dt^2} = -200x, \text{ so } \omega = \sqrt{200} \text{ and the period is } \frac{2\pi}{\sqrt{200}} = 0.444 \text{ s} \quad \text{A1}$$

$$\text{Answer: SHM, period } 0.444 \text{ s} \quad \text{3 marks}$$

10 (4 marks)

$$v = \frac{dx}{dt} = -2 \sin 5t \quad \text{M1 A1}$$

$$2 \sin 5t = 1, \text{ so } 5t = \frac{\pi}{6} \quad \text{M1}$$

$$t = 0.105 \text{ s} \quad \text{A1}$$

$$\text{Answer: } 0.105 \text{ s} \quad \text{4 marks}$$

11 (5 marks)

$$\text{Equilibrium extension } e = \frac{0.5 \times 9.8 \times 0.6}{29.4} = 0.1 \text{ m} \quad \text{M1 A1}$$

The amplitude 0.05 m is less than e, so the string never goes slack E1

$$\omega = \sqrt{\frac{29.4}{0.5 \times 0.6}} = 9.90, v_{\max} = \omega A = 0.495 \text{ m s}^{-1} \quad \text{M1 A1}$$

$$\text{Answer: taut since } 0.05 < 0.1; 0.495 \text{ m s}^{-1} \quad \text{5 marks}$$

12 (5 marks)

$$\frac{0.8 d^2 x}{dt^2} = -\frac{\lambda x}{0.6}, \text{ so } \omega^2 = \frac{\lambda}{0.8 \times 0.6} \quad \text{M1 A1}$$

$$\omega = \frac{2\pi}{0.5} = 4\pi, \text{ so } \frac{\lambda}{0.48} = 16\pi^2 \quad \text{M1 A1}$$

$$\lambda = 7.68\pi^2 = 75.8 \text{ N} \quad \text{A1}$$

$$\text{Answer: } \lambda = 75.8 \text{ N} \quad \text{5 marks}$$

13 (7 marks)

$$\text{(a) } x = 0.2 \sin \frac{2\pi t}{1.2} \quad \text{M1 A1}$$

$$\sin \frac{2\pi t}{1.2} = 0.6, \text{ so } \frac{2\pi t}{1.2} = 0.6435 \text{ and } t = 0.123 \text{ s} \quad \text{M1 A1}$$

$$\text{Answer: } 0.123 \text{ s} \quad \text{4 marks}$$

$$(b) \text{ Speed} = \omega \sqrt{A^2 - x^2} = \frac{2\pi}{1.2} \times 0.16 = 0.838 \text{ m s}^{-1}$$

M1 A1

$$\text{Time} = 2 \times 0.123 = 0.246 \text{ s}$$

A1

Answer: 0.838 m s⁻¹; 0.246 s

3 marks

14 (5 marks)

$$(a) \text{ Equilibrium: } 0.5 \times 9.8 = \frac{29.4e}{1.2}, \text{ so } e = 0.2 \text{ m}$$

M1 A1

The particle starts at rest 0.2 m above equilibrium, so the amplitude is 0.2 m and the greatest extension is $0.2 + 0.2 = 0.4 \text{ m}$

A1

Answer: e = 0.2 m; amplitude 0.2 m; greatest extension 0.4 m

3 marks

$$(b) 0.5 \times 9.8 \times x = \frac{29.4x^2}{2 \times 1.2}$$

M1

$$4.9x = 12.25x^2, \text{ so } x = 0.4 \text{ m}$$

A1

Answer: x = 0.4 m

2 marks

15 (5 marks)

$$(a) e = \frac{2 \times 9.8 \times 0.5}{39.2} = 0.25 \text{ m, and } \omega^2 = \frac{39.2}{2 \times 0.5} = 39.2$$

M1 A1

$$\text{The particle starts at the top of the motion, so } t = \frac{\pi}{2\omega} = 0.251 \text{ s}$$

A1

Answer: 0.251 s

3 marks

$$(b) \text{ Displacement below equilibrium } y = 0.125 = -0.25 \cos \omega t, \text{ so } \cos \omega t = -\frac{1}{2}$$

M1

$$\omega t = \frac{2\pi}{3}, t = 0.335 \text{ s}$$

A1

Answer: 0.335 s

2 marks

16 (6 marks)

$$(a) v^2 = \frac{k}{m}(A^2 - x^2)$$

M1

$$= \omega^2(A^2 - x^2)$$

A1

$$\text{Answer: } v^2 = \omega^2(A^2 - x^2) \text{ with } \omega^2 = \frac{k}{m}$$

2 marks

$$(b) v^2 = 100(0.09 - 0.04) = 5, \text{ so } v = 2.24 \text{ m s}^{-1}$$

M1 A1

$$2.25 = 100(0.09 - x^2), \text{ so } x^2 = 0.0675 \text{ and } x = 0.260 \text{ m}$$

M1 A1

Answer: 2.24 m s⁻¹; 0.260 m

4 marks

17 (6 marks)

$$(a) \text{ Thrust} = 1.5 \times 9.8 = 14.7 \text{ N} = \frac{30x}{0.5}, \text{ so } x = 0.245 \text{ m}$$

M1

$$\text{Length} = 0.255 \text{ m}$$

A1

Answer: 0.255 m

2 marks

$$(b) \text{ At displacement } y \text{ below equilibrium: } 1.5 \frac{d^2y}{dt^2} = 1.5 \times 9.8 - \frac{30(0.245 + y)}{0.5}$$

M1

$$= -60y, \text{ so } \omega^2 = 40 \text{ and the period is } \frac{2\pi}{\sqrt{40}} = 0.993 \text{ s}$$

A1

Answer: SHM, period 0.993 s

2 marks

$$(c) v_{\max} = \omega A = \sqrt{40} \times 0.1$$

M1

$$= 0.632 \text{ m s}^{-1}$$

A1

Answer: 0.632 m s⁻¹

2 marks

18 (5 marks)

$$\text{Equilibrium: } 2g \sin 30^\circ = \frac{49e}{0.7} \quad \text{B1}$$

$$\text{At displacement } y \text{ down the plane: } 2 \frac{d^2y}{dt^2} = 2g \sin 30^\circ - \frac{49(e+y)}{0.7} \quad \text{M1}$$

$$= -70y, \text{ so } \frac{d^2y}{dt^2} = -35y: \text{ SHM} \quad \text{A1}$$

$$\omega = \sqrt{35}, \text{ period } \frac{2\pi}{\sqrt{35}} \quad \text{M1}$$

$$= 1.06 \text{ s} \quad \text{A1}$$

Answer: SHM; period 1.06 s

5 marks

19 (3 marks)

The motion is simple harmonic with $\omega^2 = \frac{\lambda}{ml} = \frac{g}{e}$, which does not depend on the amplitude E1

$$\omega^2 = \frac{9.8}{0.2} = 49, \text{ so } \omega = 7 \quad \text{M1}$$

$$\text{Period} = \frac{2\pi}{\omega} = 0.898 \text{ s in both cases (the larger amplitude means a larger maximum speed)} \quad \text{A1}$$

Answer: period 0.898 s in both cases

3 marks

20 (8 marks)

$$\text{(a) A alone: } 0.5g = \frac{29.4e}{0.6}, \text{ so } e = 0.1 \text{ m} \quad \text{M1 A1}$$

$$\text{A and B: } 1.0g = \frac{29.4e}{0.6}, \text{ so } e = 0.2 \text{ m} \quad \text{A1}$$

Answer: 0.1 m; 0.2 m

3 marks

(b) New equilibrium is 0.1 m below the release point, so the amplitude is 0.1 m B1

$$1.0 \frac{d^2x}{dt^2} = 1.0g - \frac{29.4(0.2+x)}{0.6} = -49x, \text{ where } x \text{ is the displacement below the new equilibrium,} \quad \text{M1 A1}$$

so the motion is SHM with $\omega = 7$

$$\text{Period} = \frac{2\pi}{\omega} = 0.898 \text{ s} \quad \text{A1}$$

$$\text{Greatest speed} = \omega A = 7 \times 0.1 = 0.7 \text{ m s}^{-1} \quad \text{A1}$$

Answer: amplitude 0.1 m; period 0.898 s; greatest speed 0.7 m s⁻¹

5 marks

21 (11 marks)

$$\text{(a) } v^2 = 2 \times 9.8 \times 0.3 = 5.88 \quad \text{M1}$$

$$v = 2.42 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 2.42 m s⁻¹

2 marks

$$\text{(b) } e = \frac{0.5 \times 9.8 \times 0.8}{19.6} = 0.2 \text{ m}, \omega^2 = \frac{19.6}{0.5 \times 0.8} = 49, \text{ so } \omega = 7 \quad \text{M1 A1}$$

$$A^2 = 0.2^2 + \frac{5.88}{49} = 0.16, \text{ so } A = 0.4 \text{ m} \quad \text{M1 A1}$$

Answer: e = 0.2 m; A = 0.4 m

4 marks

$$\text{(c) } 0.2 + 0.4 = 0.6 \text{ m} \quad \text{B1}$$

Answer: 0.6 m

1 mark

$$\text{(d) Free fall: } t_1 = \sqrt{\frac{2 \times 0.3}{9.8}} = 0.247 \text{ s} \quad \text{M1}$$

SHM from 0.2 m above equilibrium to 0.4 m below: $y = 0.4 \sin \theta$, θ from $-\frac{\pi}{6}$ to $\frac{\pi}{2}$, so $t_2 =$ M1 A1

$$\frac{2\pi}{3 \times 7} = 0.299 \text{ s}$$

$$\text{Total time} = 0.547 \text{ s} \quad \text{A1}$$

Answer: 0.547 s

4 marks

22 (8 marks)**(a)** Friction $F = 0.2 \times 2 \times 9.8 = 3.92$ N, opposing the motion (away from O) **B1**

$$2 \frac{d^2x}{dt^2} = -\frac{39.2x}{0.5} + 3.92$$
M1

$$\frac{d^2x}{dt^2} = -39.2x + 1.96 = -39.2(x - 0.05)$$
A1

Answer: $\frac{d^2x}{dt^2} = -39.2(x - 0.05)$: SHM about $x = 0.05$ **3 marks****(b)** Amplitude $0.3 - 0.05 = 0.25$ m, so the block comes to rest at $x = 0.05 - 0.25 = -0.2$ m (compression 0.2 m) **M1 A1**

$$t = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{39.2}} = 0.502 \text{ s}$$
A1

Answer: compression 0.2 m after 0.502 s **3 marks**

(c) $v_{\max} = \omega A = \sqrt{39.2} \times 0.25$ **M1**

$= 1.57 \text{ m s}^{-1}$ **A1**

Answer: 1.57 m s⁻¹ **2 marks****23 (7 marks)****(a)** Let the tensions in equilibrium be T_A and T_B , so $T_A = T_B + 1.5g$ **B1**After a downward displacement y : tension in AP is $T_A + 80y$ and in PB is $T_B - 60y$ **M1**

$$1.5 \frac{d^2y}{dt^2} = 1.5g + (T_B - 60y) - (T_A + 80y) = -140y$$
M1 A1

$$\omega^2 = \frac{140}{1.5}, \text{ so period} = \frac{2\pi}{\omega} = 0.650 \text{ s}$$
A1

Answer: SHM, period 0.650 s **5 marks**

(b) $v_{\max} = \omega A = \sqrt{\frac{140}{1.5}} \times 0.05$ **M1**

$= 0.483 \text{ m s}^{-1}$ **A1**

Answer: 0.483 m s⁻¹ **2 marks****24 (6 marks)**

Period $T = \frac{2\pi}{\omega}$ with $\omega^2 = \frac{k}{m}$, so $T^2 = \frac{4\pi^2 m}{k}$ **B1**

$0.4^2 = \frac{4\pi^2 m}{k}$ and $0.5^2 = \frac{4\pi^2(m + 0.1)}{k}$ **M1**

Subtract: $0.25 - 0.16 = \frac{4\pi^2 \times 0.1}{k}$ **M1**

$k = 43.9 \text{ N m}^{-1}$ **A1**

$m = \frac{kT^2}{4\pi^2} = \frac{43.9 \times 0.16}{4\pi^2}$ **M1**

$m = 0.178 \text{ kg}$ **A1**

Answer: $k = 43.9 \text{ N m}^{-1}$, $m = 0.178 \text{ kg}$ **6 marks****25 (7 marks)**

(a) $mg = \frac{\lambda e}{l}$, so $\frac{\lambda}{ml} = \frac{g}{e}$ **M1**

The motion is SHM with $\omega^2 = \frac{\lambda}{ml} = \frac{g}{e}$ **A1**

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{e}{g}}$ **A1**

Answer: $T = 2\pi\sqrt{\frac{e}{g}}$ **3 marks**

(b) $e = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 0.81}{4\pi^2}$ **M1**

$= 0.201 \text{ m}$ **A1**

Answer: 0.201 m **2 marks**

$$(c) \lambda = \frac{mgl}{e} = \frac{0.4 \times 9.8 \times 0.5}{0.201}$$

M1

$$= 9.75 \text{ N}$$

A1

Answer: 9.75 N

2 marks

26 (10 marks)

(a) Tension in the string = $0.5g = 4.9 \text{ N}$, which equals the spring tension

M1

$$4.9 = \frac{49e}{0.5}, \text{ so } e = 0.05 \text{ m}$$

A1

Answer: 0.05 m

2 marks

$$(b) \text{ For the system: } (1.5 + 0.5) \frac{d^2x}{dt^2} = 0.5g - \frac{49(0.05 + x)}{0.5}$$

M1 A1

$$= -98x, \text{ so } \frac{d^2x}{dt^2} = -49x, \text{ which is SHM with } \omega = 7$$

M1 A1

$$\text{Period} = \frac{2\pi}{7} = 0.898 \text{ s}$$

A1

Answer: period 0.898 s

5 marks

(c) Q has a downward acceleration of at most g when its string is taut ($T \geq 0$): $\omega^2 A \leq g$

M1 A1

$49A \leq 9.8$, so the greatest amplitude is 0.2 m

A1

Answer: 0.2 m

3 marks

27 (9 marks)

(a) $BP = 0.1 \text{ m} < 0.5 \text{ m}$, so BP is slack

B1

While AP is taut: $\omega^2 = \frac{20}{0.5 \times 0.5} = 80$, amplitude 0.2 m

M1

$$v = \omega A = \sqrt{80} \times 0.2 = 1.79 \text{ m s}^{-1}$$

A1

Answer: slack; 1.79 m s⁻¹

3 marks

(b) Both are slack for AP between 0.3 m and 0.5 m , a distance of 0.2 m

M1

$$t = \frac{0.2}{1.789} = 0.112 \text{ s}$$

A1

Answer: 0.112 s

2 marks

(c) By symmetry, the string BP is taut for $AP < 0.3 \text{ m}$, with the same ω and amplitude

B1

Time taut: a quarter period for each of the two ends, plus a half period at the BP end: in total $\frac{2\pi}{\omega}$

M1

$$= \frac{2\pi}{\sqrt{80}} = 0.702 \text{ s}$$

$$\text{Total} = 0.702 + 2 \times 0.112 = 0.926 \text{ s}$$

M1 A1

Answer: 0.926 s

4 marks

28 (8 marks)

(a) Tension $T = \frac{30e}{0.5} = 60e$ acts on each particle towards the other

B1

Accelerations: $a_A = \frac{T}{2}$ towards B, $a_B = \frac{T}{3}$ towards A, so $\frac{d^2e}{dt^2} = -(a_A + a_B)$

M1

$$= -60e\left(\frac{1}{2} + \frac{1}{3}\right) = -50e$$

A1

Answer: $\frac{d^2e}{dt^2} = -50e$: SHM

3 marks

$$(b) \omega = \sqrt{50}$$

M1

$$\text{Period} = \frac{2\pi}{\sqrt{50}} = 0.889 \text{ s}$$

A1

Answer: 0.889 s

2 marks

(c) Greatest relative speed = $\omega A = \sqrt{50} \times 0.2 = 1.41 \text{ m s}^{-1}$

M1

Momentum is conserved, so $2v_A = 3v_B$ with $v_A + v_B = 1.41$

M1

$$v_A = 0.849 \text{ m s}^{-1}, v_B = 0.566 \text{ m s}^{-1}$$

A1

Answer: A: 0.849 m s⁻¹; B: 0.566 m s⁻¹

3 marks

29 (9 marks)

(a) Each half has natural length 0.5 m and length 0.6 m: $T_0 = \frac{40 \times 0.1}{0.5}$ M1

= 8 N A1

Answer: 8 N 2 marks

(b) Each half has length $\sqrt{0.36 + y^2} \approx 0.6$, so the tension is approximately $T_0 = 8$ N M1

Transverse component of each tension $\approx 8 \times \frac{y}{0.6}$ M1 A1

$0.25 \frac{d^2y}{dt^2} \approx -2 \times 8 \times \frac{y}{0.6}$, so $\frac{d^2y}{dt^2} \approx -\frac{320}{3}y$ A1

$\omega = \sqrt{\frac{320}{3}}$, period = 0.608 s A1

Answer: approximate SHM, period 0.608 s 5 marks

(c) Approximate: $2 \times 8 \times \frac{0.05}{0.6} = 1.33$ N M1

Exact: length 0.602 m, $T = 80(0.602 - 0.5)$, force = $2T \times \frac{0.05}{0.602} = 1.36$ N A1

Answer: approximate 1.33 N; exact 1.36 N 2 marks

30 (11 marks)

(a) Stiffness $k = \frac{100}{0.5} = 200$ N m⁻¹ B1

At displacement y below equilibrium: $2 \frac{d^2y}{dt^2} = -200y$, so $\omega^2 = 100$ M1

Period = $\frac{2\pi}{10} = \frac{\pi}{5}$ s A1

Answer: period $\frac{\pi}{5}$ s 3 marks

(b) At the lowest point: $R - 0.5g = 0.5\omega^2A$, so $R = 0.5(9.8 + 100 \times 0.06) = 7.9$ N M1 A1

At the highest point: $R = 0.5(9.8 - 6) = 1.9$ N A1

Answer: greatest 7.9 N; least 1.9 N 3 marks

(c) $R \geq 0$ at the highest point: $0.5(g - \omega^2A) \geq 0$ M1

$A \leq \frac{9.8}{100} = 0.098$ m A1

Answer: 0.098 m 2 marks

(d) The block leaves when the acceleration is g downwards, i.e. at 0.098 m above equilibrium M1

$v^2 = 100(0.15^2 - 0.098^2) = 1.29$ M1

Further rise = $\frac{v^2}{2g} = 0.0658$ m A1

Answer: 0.0658 m 3 marks