

Hooke's Law

- 1 A light elastic string has natural length 0.8 m and modulus of elasticity 40 N. Find the tension in the string when its length is 1.0 m. [2]
- 2 A light elastic string has natural length 1.5 m and modulus of elasticity 60 N. Find its extension when the tension is 24 N. [2]
- 3 A light elastic string has natural length 0.5 m. When it is stretched to a length of 0.7 m the tension in it is 16 N. Find the modulus of elasticity of the string. [2]
- 4 A light elastic string has modulus of elasticity 50 N. When it is extended by 0.3 m the tension in it is 12 N. Find the natural length of the string. [2]
- 5 A light elastic spring has natural length 0.6 m and modulus of elasticity 30 N.
 - (a) Find the stiffness of the spring, $\frac{\lambda}{l}$, in N m^{-1} . [1]
 - (b) Find the tension when the spring is extended by 0.04 m. [2]
- 6 A particle of mass 0.5 kg hangs in equilibrium from a fixed point by a light elastic string of natural length 2 m and modulus of elasticity 98 N. Find the length of the string. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 7 A particle of mass 0.8 kg hangs in equilibrium at the end of a light elastic string of natural length 1.2 m. The other end of the string is fixed, and the string is extended by 0.3 m. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 8 A light spring has natural length 0.5 m and modulus of elasticity 20 N. It is compressed to a length of 0.42 m. Find the thrust in the spring. [3]
- 9 A light elastic string of natural length 1.2 m has one end fixed. When a particle of mass 0.3 kg hangs in equilibrium from the other end, the string is extended by 0.06 m. Take $g = 9.8 \text{ m s}^{-2}$.
 - (a) Find the modulus of elasticity of the string. [2]
 - (b) Find the extension when a particle of mass 0.75 kg hangs from the string. [2]
- 10 A light elastic string of natural length 0.9 m and modulus of elasticity 45 N will break if it is stretched to more than twice its natural length. Find the greatest tension the string can carry, and the greatest mass that can hang in equilibrium from it, giving your answer to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 11 A particle of mass 2 kg hangs in equilibrium from a ceiling by two identical vertical light elastic strings, each of natural length 0.6 m and modulus of elasticity 49 N. Find the length of each string. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 12 A light elastic string of natural length 1.2 m and modulus of elasticity 72 N has its ends attached to two fixed points A and B which are 1.5 m apart on a smooth horizontal table. Find the tension in the string, and the magnitude of the force the string exerts on the point A. [4]
- 13 Two light elastic strings P and Q have the same modulus of elasticity, 30 N, but natural lengths 1 m and 2 m respectively. Each is stretched by a tension of 15 N. Find the extension of each string, and explain why the extension of Q is larger than that of P. [4]
- 14 A light elastic string is stretched between two fixed points. When the distance between them is 0.8 m the tension is 6 N, and when the distance is 1.0 m the tension is 14 N. Find the natural length and the modulus of elasticity of the string. [5]

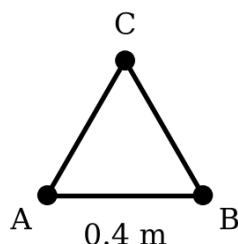
- 15** Two light elastic strings are joined end to end. String 1 has natural length 0.6 m and modulus of elasticity 30 N. String 2 has natural length 0.4 m and modulus of elasticity 20 N. The composite string is stretched by a tension of 10 N.
- (a) Find the total length of the composite string. [3]
- (b) Find the modulus of elasticity of a single string of natural length 1.0 m that has the same extension as the composite string when the tension is 10 N. [2]

- 16** A light elastic string of natural length 1.2 m and modulus of elasticity 36 N is cut into two pieces, one of which is twice as long as the other.
- (a) State the natural length of each piece and, with a reason, the modulus of elasticity of each piece. [2]
- (b) Each piece is stretched by a tension of 9 N. Find the extension of each piece. [2]

- 17** A light elastic string has one end fixed. When a particle of mass 0.5 kg hangs in equilibrium from the other end the length of the string is 1.5 m, and when a particle of mass 0.8 kg hangs from it the length is 1.8 m. Find the natural length and the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 18** A particle P rests in equilibrium on a smooth horizontal table. It is attached to a fixed point A on the table by a light elastic string of natural length 0.8 m and modulus 24 N, and to a fixed point B on the table by a light elastic string of natural length 1.0 m and modulus λ N. The points A, P and B lie in a straight line with $AB = 3.0 \text{ m}$ and $AP = 1.2 \text{ m}$. Find λ . [5]

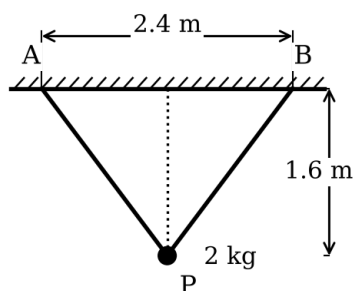
- 19** A closed loop of light elastic string has natural length 0.6 m and modulus of elasticity 30 N. It is stretched round three smooth fixed pegs A, B and C, which lie at the vertices of an equilateral triangle of side 0.4 m on a smooth horizontal table, as shown. Find the tension in the string, and the magnitude of the force the string exerts on the peg A, giving your answer to 3 significant figures.



[4]

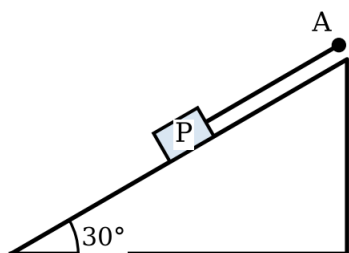
- 20** A student writes: 'A light elastic string has natural length 1 m and modulus of elasticity 20 N. When its length is 1.5 m the tension is 10 N, so when its length is doubled to 3 m the tension is doubled to 20 N.' Explain the mistake and find the correct tension when the length is 3 m. [4]

- 21** A particle P of mass 2 kg hangs in equilibrium from two identical light elastic strings AP and BP, each of natural length 1.5 m and modulus of elasticity λ N. The points A and B are fixed at the same horizontal level, 2.4 m apart, and P is 1.6 m vertically below AB, as shown. Find λ . Take $g = 9.8 \text{ m s}^{-2}$.



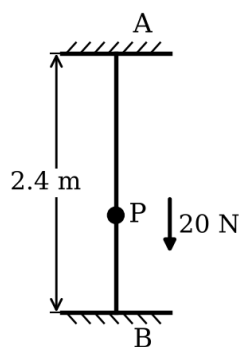
[7]

- 22** A particle P of mass 2 kg is on a rough plane inclined at 30° to the horizontal. The coefficient of friction between P and the plane is 0.3. P is attached to a fixed point A at the top of the plane by a light elastic string of natural length 0.7 m and modulus of elasticity 29.4 N. The string lies along a line of greatest slope, as shown. Find the range of lengths of the string for which P can rest in equilibrium. Give your answers to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[8]

- 23** A light elastic string AP, of natural length 0.8 m and modulus of elasticity 40 N, and a light elastic string PB, of natural length 0.6 m and modulus of elasticity 30 N, are attached to a particle P of weight 20 N. The point A is vertically above the point B with $AB = 2.4 \text{ m}$, and P is in equilibrium on the line AB, as shown.



- (a) Find the length AP. [5]
 (b) Find the tension in each string. [2]

- 24** A light spring of natural length l and modulus of elasticity λ has stiffness $k = \frac{\lambda}{l}$, so that its tension at extension x is kx . Two light springs, of stiffnesses k_1 and k_2 , are considered.

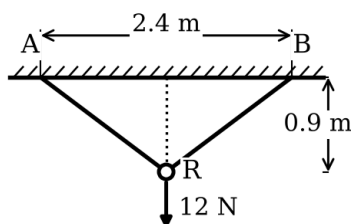
- (a) The springs are joined end to end and the combination is stretched by a tension T . Show that the total extension is $T(\frac{1}{k_1} + \frac{1}{k_2})$, and hence that the combination has stiffness

$$\frac{k_1 k_2}{(k_1 + k_2)}. \quad [4]$$

- (b) The springs are instead placed side by side with their ends attached to the same two fixed points, so that both have the same extension x . Show that the combination has stiffness $k_1 + k_2$. [2]

- (c) Find the stiffness of each combination when $k_1 = 40 \text{ N m}^{-1}$ and $k_2 = 60 \text{ N m}^{-1}$. [2]

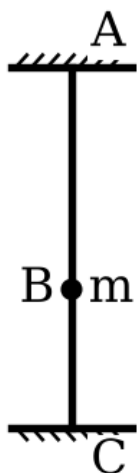
- 25** A smooth ring R of weight 12 N is threaded on a light elastic string of natural length 1.8 m and modulus of elasticity λ N. The ends of the string are fixed to points A and B at the same horizontal level, 2.4 m apart. The ring rests in equilibrium at a point 0.9 m vertically below the midpoint of AB, as shown. Find λ .



[8]

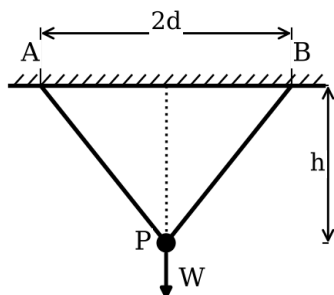
- 26** A light elastic string has one end fixed. When a particle of mass m hangs in equilibrium from the other end the length of the string is a . When the particle is replaced by one of mass $3m$, the length in equilibrium is b .
- Show that the natural length of the string is $\frac{(3a - b)}{2}$, and find its modulus of elasticity in terms of m , g , a and b . [6]
 - Deduce that $a < b < 3a$. [2]
 - Given that $a = 0.9$ m, $b = 1.5$ m and $m = 0.4$ kg, find the natural length and the modulus of elasticity, taking $g = 9.8$ m s⁻². [2]

- 27** Two light elastic strings AB and BC are fastened together at B. String AB has natural length a and modulus of elasticity $3mg$. String BC has natural length $2a$ and modulus of elasticity $4mg$. The end A is attached to a fixed ceiling and the end C to a fixed floor, with A, B and C in a vertical line. A particle of mass m is attached to the strings at B and rests in equilibrium with $BC = \frac{5a}{2}$, as shown.



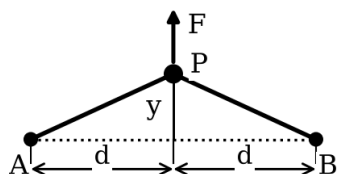
- Find the tension in BC and the tension in AB. [4]
- Given that $AC = ka$, find the value of k . [3]
- The particle is removed, with A and C remaining fixed. Find the new length of AB. [4]

- 28** Two identical light elastic strings AP and BP, each of natural length 0.8 m and modulus of elasticity λ N, support a particle P of weight W N. The points A and B are at the same horizontal level, $2d$ metres apart, where $d = 0.6$, and P hangs in equilibrium at a distance h metres vertically below the midpoint of AB, as shown. The length of each string is L metres.



- Show that $W = \frac{2\lambda h(L - 0.8)}{0.8L}$. [4]
- Given that $\lambda = 50$ and $h = 0.8$, find L and the value of W . [4]
- Find the angle that each string makes with the horizontal, giving your answer to the nearest tenth of a degree. [2]

- 29** A light elastic string of natural length $2d$ and modulus of elasticity λ has its ends fixed to two points A and B, a distance $2d$ apart, on a smooth horizontal table. A particle P is attached to the midpoint of the string. A horizontal force F , perpendicular to AB, holds P in equilibrium at a small distance y from the line AB, as shown. The tension in the string is T .



- Show that $T = \frac{\lambda(\sqrt{d^2 + y^2} - d)}{d}$. [3]
- Show that $F = \frac{2\lambda y}{d} \left(1 - \frac{d}{\sqrt{d^2 + y^2}}\right)$. [2]
- Use the binomial expansion to show that, for small y , $F \approx \frac{\lambda y^3}{d^3}$. [3]
- Taking $d = 0.5$ m, $\lambda = 20$ N and $y = 0.1$ m, calculate F from the exact expression, and from the approximation in part (c). Give your answers to 3 significant figures. [2]

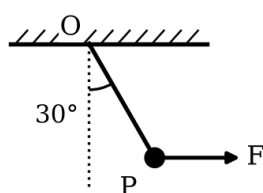
- 30** A particle P of mass 2 kg lies on a rough plane inclined at 30° to the horizontal. P is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity λ N. The other end of the string is fixed to the point A at the top of the plane, and the string lies along a line of greatest slope. The coefficient of friction between P and the plane is μ . P can rest in equilibrium at any point where the length of the string is between 1.4 m and 1.8 m inclusive, and at no other point. Take $g = 9.8 \text{ m s}^{-2}$.
- By considering the two extreme positions, show that $\lambda = 29.4$. [6]
 - Find the value of μ , to 3 significant figures. [5]

Elastic Strings

- 1** A particle P of mass m hangs in equilibrium from a fixed point O by a light elastic string of natural length l and modulus of elasticity $3mg$. Find the extension of the string in terms of l . [2]

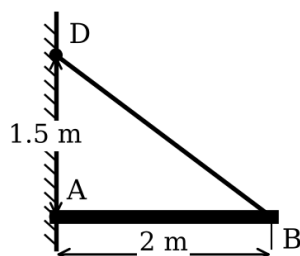
- 2** A particle of mass 2 kg is attached to the lower end of a light elastic string of natural length 1 m and modulus of elasticity 49 N. The upper end of the string is fixed at O. The particle is held at rest at a point 1.5 m vertically below O and then released. Find the initial acceleration of the particle. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 3** A particle of mass 3 kg rests on a rough horizontal table, with coefficient of friction 0.4. It is attached to a fixed point O on the table by a light elastic string of natural length 1.0 m and modulus of elasticity 60 N. The particle is on the point of slipping directly away from O. Find the extension of the string. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 4** A particle of mass 2 kg rests in equilibrium on a smooth plane inclined at 30° to the horizontal. It is attached to a point A at the top of the plane by a light elastic string, of natural length 0.5 m and modulus of elasticity 49 N, lying along a line of greatest slope. Find the extension of the string. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 5** A particle of mass m is attached to a fixed point O by a light elastic string of natural length a and modulus of elasticity $4mg$. The particle is at rest at a point vertically below O where the extension of the string is $\frac{a}{2}$. Find the tension in the string, and the magnitude and direction of the acceleration of the particle. [3]
- 6** A particle of mass 1.5 kg hangs in equilibrium from a fixed point by a light elastic string. The extension of the string is one quarter of its natural length. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 7** A particle P of mass 2 kg rests on a rough horizontal table, with coefficient of friction 0.5. It is attached to a fixed point O on the table by a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. Find the greatest distance from O at which P can rest in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 8** A particle of mass 2 kg rests in equilibrium on a smooth plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. It is attached to a point A at the top of the plane by a light elastic string of natural length 0.7 m and modulus of elasticity 58.8 N, lying along a line of greatest slope. Find the length of the string. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 9** A particle P of mass 1 kg is attached to two light elastic strings. The string AP has natural length 0.6 m and modulus of elasticity 30 N, and the string PB has natural length 0.4 m and modulus of elasticity 20 N. The point A is vertically above the point B with $AB = 2.0 \text{ m}$. The particle is held at rest on AB with $AP = 1.0 \text{ m}$, and then released. Find the initial acceleration of P. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 10** A particle of mass $3m$ is attached to one end of a light elastic string of natural length l and modulus of elasticity $6mg$. The other end of the string is fixed at O, and the particle hangs in equilibrium.
- (a) Find the extension of the string. [2]
- (b) The particle is pulled down a further distance $\frac{l}{2}$ and released from rest. Find its initial acceleration. [2]
- 11** A particle of mass 4 kg is on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.25. The particle is attached to a point A at the top of the plane by a light elastic string of natural length 1.2 m, lying along a line of greatest slope. The particle is on the point of slipping up the plane when the extension of the string is 0.6 m. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 12** A light elastic string has natural length a and modulus of elasticity $2mg$. One end is fixed at O and a particle of mass m is attached to the other end.
- (a) Find the length of the string when the particle hangs in equilibrium. [2]
- (b) A second particle, also of mass m , is attached to the first so that both hang from the string. Find the new length of the string in equilibrium. [2]
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- 13** Two light elastic strings are both attached at one end to a fixed point O . String 1 has natural length 1.0 m and modulus of elasticity 40 N. String 2 has natural length 1.2 m and modulus of elasticity 60 N. Their other ends are attached to a particle of weight 49 N, which hangs in equilibrium below O with both strings vertical and taut. Take the length of each string to be L metres.
- (a) Show that $90L = 149$. [3]
- (b) Find the tension in each string, to 3 significant figures. [2]
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- 14** A particle P of mass 2.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. The coefficient of friction between P and the plane is 0.5 . P is attached to a point A at the top of the plane by a light elastic string of natural length 0.7 m and modulus of elasticity 35 N, which lies along a line of greatest slope. P is held at rest with $AP = 1.5$ m and released. Find the initial acceleration of P , stating its direction. Take $g = 9.8 \text{ m s}^{-2}$. [6]
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- 15** A particle P of mass 1.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 49 N. The other end of the string is attached to a fixed point O . A horizontal force F holds P in equilibrium with the string making an angle of 30° with the vertical, as shown. Find the magnitude of F , and the length of the string, giving your answers to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



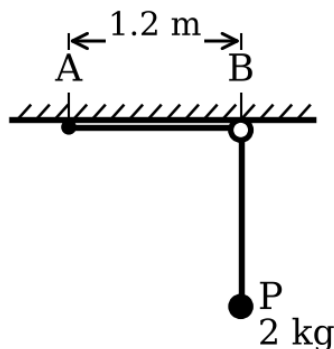
[6]

- 16** A uniform rod AB , of length 2 m and mass 6 kg, is smoothly hinged at A to a vertical wall and held horizontal by a light elastic string of natural length 1.25 m. One end of the string is attached to B and the other end to a point D on the wall, 1.5 m vertically above A , as shown. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

- 17** A light elastic string of natural length 1.5 m and modulus of elasticity 39.2 N has one end attached to a fixed point A on a horizontal ceiling. The string passes over a smooth peg B on the ceiling, where $AB = 1.2$ m. A particle P of mass 2 kg is attached to the other end of the string and hangs in equilibrium, as shown. Find the length BP. Take $g = 9.8 \text{ m s}^{-2}$.

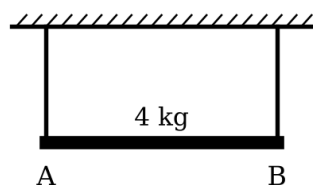


[5]

- 18** Two particles A and B, of masses 2 kg and 3 kg, rest on a rough horizontal table. The coefficient of friction between each particle and the table is 0.4. The particles are connected by a light elastic string of natural length 0.5 m and modulus of elasticity 39.2 N. Find the greatest distance AB for which both particles can remain at rest. Take $g = 9.8 \text{ m s}^{-2}$.

[6]

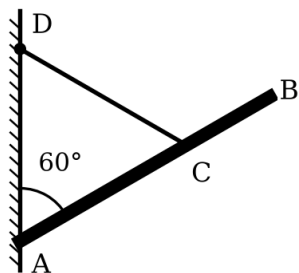
- 19** A uniform rod AB of mass 4 kg hangs in equilibrium, horizontal, from a horizontal ceiling by two vertical light elastic strings attached at A and B, as shown. The string at A has natural length 0.8 m and modulus of elasticity 58.8 N. The string at B has natural length 0.6 m and modulus of elasticity λ N. Find λ . Take $g = 9.8 \text{ m s}^{-2}$.



[7]

- 20** A particle P of mass 2 kg lies on a rough horizontal table. P is attached to a fixed point O on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 49 N. P is also attached to one end of a light inextensible string which passes over a smooth pulley at the edge of the table and carries a particle of mass 1.8 kg hanging freely. The string between P and the pulley is horizontal and in the same straight line as OP. The system is in limiting equilibrium, about to move towards the pulley, when the extension of the elastic string is 0.1 m. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the coefficient of friction between P and the table. [5]
- (b) The system is instead about to move away from the pulley. Find the extension of the elastic string at that instant. [3]

- 21** A uniform rod AB, of length 3 m and mass 8 kg, is smoothly hinged at A to a vertical wall. A light elastic string of natural length 1.5 m joins a point C on the rod, where $AC = 2$ m, to a point D on the wall, where D is vertically above A and $AD = 2$ m. The rod rests in equilibrium with angle $DAB = 60^\circ$, as shown. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$.



[7]

- 22** A particle P of mass 3 kg is on a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.8$. The coefficient of friction is 0.5. P is attached to a point A at the top of the plane by a light elastic string, of natural length 0.9 m and modulus of elasticity 49 N, along a line of greatest slope. P is on the point of slipping down the plane. A student's working to find the length of the string is: ' $R = 3g \sin \alpha = 23.52 \text{ N}$, $F = 0.5R = 11.76 \text{ N}$, $T = 3g \cos \alpha - F = 17.64 - 11.76 = 5.88 \text{ N}$.' Explain the error in the student's working, and find the correct length of the string. Take $g = 9.8 \text{ m s}^{-2}$.

[7]

- 23** A particle P of weight 20 N rests in equilibrium on a smooth plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. The particle is attached to two light elastic strings. The string AP, of natural length 0.5 m and modulus of elasticity 30 N, is attached to a point A on the plane. The string PB, of natural length 0.6 m and modulus of elasticity 36 N, is attached to a point B on the plane, below A. The points A, P and B lie on a line of greatest slope, with $AB = 3.0 \text{ m}$.
- (a) Find the length AP. [5]
- (b) Find the tension in each string. [2]

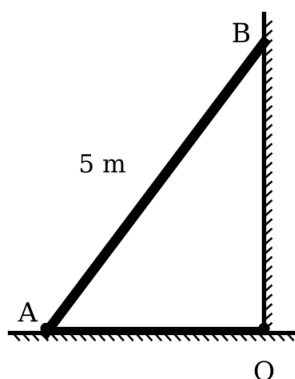
- 24** A small smooth ring of mass 0.5 kg is threaded on a fixed smooth horizontal rod. The ring is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 40 N. The other end of the string is fixed to a point O which is 0.8 m vertically below the rod. A horizontal force P, acting along the rod, holds the ring at rest at a point on the rod which is 0.6 m horizontally from the point of the rod vertically above O. Find the magnitude of P, and the magnitude of the normal reaction of the rod on the ring. Take $g = 9.8 \text{ m s}^{-2}$.

[6]

- 25** A particle P of mass 3 kg rests on a rough horizontal table, with coefficient of friction 0.5. It is attached to a light elastic string AP, of natural length 0.5 m and modulus of elasticity 20 N, and to a light elastic string PB, of natural length 0.4 m and modulus of elasticity 24 N. The points A and B are fixed on the table, 2.0 m apart, and A, P and B lie in a straight line. Find the range of values of AP for which P can rest in equilibrium. Give your answers to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

[7]

- 26** A uniform ladder AB, of length 5 m and mass 20 kg, rests with A on smooth horizontal ground and B against a smooth vertical wall, as shown. The foot A is 3 m from the wall. A light elastic string of natural length 1.2 m joins A to the point O at the foot of the wall, and lies along the ground. The ladder is in equilibrium in a vertical plane perpendicular to the wall. Find the modulus of elasticity of the string. Take $g = 9.8 \text{ m s}^{-2}$.

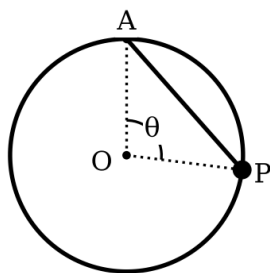


[7]

- 27** A particle P of mass 2.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.8$. The coefficient of friction between P and the plane is 0.25. P is attached to a point A at the top of the plane by a light elastic string of natural length 0.7 m and modulus of elasticity 31.85 N, which lies along a line of greatest slope. Take $g = 9.8 \text{ m s}^{-2}$.

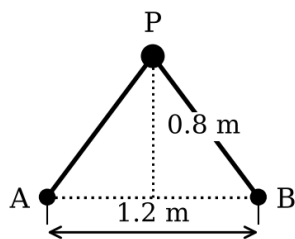
- Find AP when P is on the point of slipping down the plane. [5]
- Find the greatest value of AP for which P can rest in equilibrium. Give your answer to 3 significant figures. [3]
- P is released from rest with AP = 1.5 m. Find its initial acceleration. [3]

- 28** A smooth bead P of mass m is threaded on a smooth fixed circular wire, of centre O and radius a, in a vertical plane. A light elastic string of natural length a and modulus of elasticity $3mg$ joins P to the highest point A of the wire. The bead rests in equilibrium at a point other than the lowest point of the wire, with angle $AOP = \theta$, as shown.



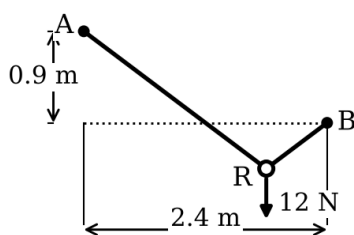
- Show that $AP = 2a \sin \frac{\theta}{2}$, and that the string makes an angle $\frac{\theta}{2}$ with the tangent to the wire at P. [2]
- By resolving along the tangent at P, show that the tension in the string is $2mg \sin \frac{\theta}{2}$. [4]
- Find AP in terms of a, and find θ to the nearest tenth of a degree. [4]

- 29** A particle P rests on a smooth horizontal table. It is attached to a point A on the table by a light elastic string of natural length 0.6 m and modulus of elasticity 24 N, and to a point B on the table by a light elastic string of natural length 0.8 m and modulus of elasticity 20 N. The distance AB is 1.2 m, and P is 0.8 m from the line AB, equidistant from A and B, as shown. P is held in equilibrium by a horizontal force F.



- Find the tension in each string. [3]
- Find the magnitude of F, and the angle it makes with the direction AB. Give your answers to 3 significant figures. [5]
- The force F is removed. Find the initial acceleration of P, given that its mass is 0.8 kg. Give your answer to 3 significant figures. [2]

- 30** A smooth ring R of weight 12 N is threaded on a light elastic string of natural length 2.2 m and modulus of elasticity λ N. The ends of the string are fixed to points A and B, where B is 0.9 m below A and the horizontal distance between A and B is 2.4 m, as shown. The ring rests in equilibrium and the total length of the string is 3.0 m.

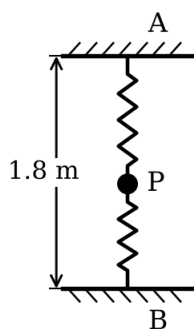


- Explain why the two parts of the string make equal angles with the horizontal. [2]
- Find the tension in the string and the value of λ . [6]
- Find the vertical distance of R below A. [3]

Elastic Springs

- A light spring has natural length 0.4 m and modulus of elasticity 20 N. It is compressed to a length of 0.3 m. Find the thrust in the spring. [2]
- A particle of mass 0.5 kg rests in equilibrium on top of a vertical light spring, of natural length 0.6 m and modulus of elasticity 14.7 N, whose lower end is fixed to the ground. Find the length of the spring. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- A particle of mass 0.4 kg hangs in equilibrium from a light spring of natural length 0.5 m, the extension of the spring being 0.098 m. Find the modulus of elasticity of the spring. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- A particle of mass 2 kg is placed on top of a light vertical spring, whose lower end is fixed to the ground, and compresses the spring by 0.05 m in equilibrium. Find the compression of the spring when a particle of mass 5 kg is placed on top of it instead. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- A light spring of natural length 0.8 m is compressed to a length of 0.65 m by a thrust of 30 N. Find the thrust in the spring when its length is 0.5 m. [3]

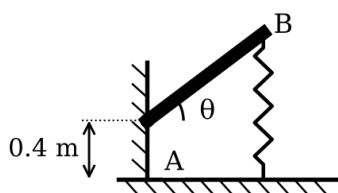
- 6** A light spring has natural length 0.6 m and modulus of elasticity 48 N.
 (a) Find the stiffness of the spring. [1]
 (b) The spring is cut into two equal pieces. Find the stiffness of each piece, giving a reason. [2]
- 7** A light spring of natural length 0.5 m and modulus of elasticity 15 N is held between two fixed points which are 0.3 m apart. Find the thrust in the spring and the magnitude of the force it exerts on each point. [4]
- 8** A light spring, of natural length 0.8 m and modulus of elasticity 49 N, has its lower end fixed to the foot of a smooth plane inclined at 30° to the horizontal. A particle of mass 2 kg rests in equilibrium on the upper end of the spring, the spring lying along a line of greatest slope. Find the length of the spring. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 9** A block of mass 3 kg rests on top of a vertical light spring of natural length 0.4 m and modulus of elasticity 58.8 N, whose lower end is fixed to the ground. The block is pressed down until the length of the spring is 0.1 m and is then released. Find the initial acceleration of the block. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 10** A light spring has natural length 0.6 m and modulus of elasticity 18 N. State whether the force in the spring is a tension or a thrust, and find its magnitude, when the length of the spring is
 (a) 0.45 m, [3]
 (b) 0.78 m. [3]
- 11** Two light springs, each of natural length 0.5 m, have moduli of elasticity 20 N and 30 N. The upper ends are attached to the same point of a ceiling and the lower ends are attached to a particle of mass 1.5 kg, which hangs in equilibrium. Find the length of the springs. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 12** A block of mass 1.5 kg lies on a smooth horizontal table. It is attached to one end of a light horizontal spring of natural length 0.6 m and modulus of elasticity 24 N. The other end of the spring is fixed to a vertical wall. The block is held at a distance 0.9 m from the wall and released from rest. Find the initial acceleration of the block. [4]
- 13** A light vertical spring has its lower end fixed to the ground. When a particle of mass 2 kg rests on top of the spring, the length of the spring is 0.40 m. When this particle is replaced by one of mass 3 kg, the length of the spring is 0.35 m. Find the natural length of the spring and its modulus of elasticity. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 14** A light spring AP, of natural length 0.5 m and modulus of elasticity 40 N, and a light spring PB, of natural length 0.4 m and modulus of elasticity 24 N, are attached to a particle P of mass 1.5 kg. The point A is fixed vertically above the fixed point B, with $AB = 1.8 \text{ m}$, and P rests in equilibrium on the line AB, as shown. Find the length AP, giving your answer to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

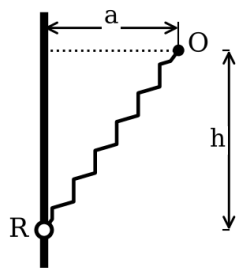
- 15** The pointer of a spring balance is 20 cm from the top of its scale when the balance carries no load, and 28 cm from the top when a 2 kg mass hangs from it. Assume that the spring obeys Hooke's law. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the stiffness of the spring, in N m^{-1} . [3]
- (b) Find the mass of an object that makes the pointer read 31.5 cm. [2]
-
- 16** Two particles A and B, of masses 2 kg and 3 kg, lie on a smooth horizontal table and are connected by a light spring of natural length 0.5 m and modulus of elasticity 30 N. The particles are held at rest with $AB = 0.3 \text{ m}$ and then released simultaneously. Find the initial acceleration of each particle. [5]
-
- 17** A platform of mass 8 kg is supported by four identical vertical light springs, each of natural length 0.5 m and modulus of elasticity $\lambda \text{ N}$, whose lower ends are fixed to the ground. With only the platform on them, the springs have length 0.45 m. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find λ . [3]
- (b) A child of mass 12 kg then stands on the centre of the platform. Find the new length of the springs. [2]
-
- 18** A particle of mass 2 kg hangs from the lower end of a light spring of natural length 0.5 m and modulus of elasticity 49 N, inside a lift. The upper end of the spring is fixed to the ceiling of the lift. The lift accelerates upwards at 2.45 m s^{-2} and the particle is at rest relative to the lift. Find the length of the spring. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 19** A student writes: 'A light spring has natural length 0.5 m and modulus of elasticity 20 N. When it is compressed to a length of 0.4 m the thrust is $T = \frac{20 \times 0.4}{0.5} = 16 \text{ N}$.' Explain the mistake and find the correct thrust. [3]
-
- 20** A block P of mass 2 kg lies on a rough horizontal table, the coefficient of friction between P and the table being 0.25. Two light springs, AP and PB, are attached to P and to fixed points A and B on the table, where A, P and B lie in a straight line and $AB = 1.6 \text{ m}$. Spring AP has natural length 0.5 m and modulus of elasticity 20 N. Spring PB has natural length 0.5 m and modulus of elasticity 30 N. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that when $AP = x$ metres ($0.5 < x < 1.1$) the resultant force of the springs on P is $(86 - 100x) \text{ N}$, directed towards B. [5]
- (b) Find the set of values of x for which P can rest in equilibrium, giving the end values to 3 significant figures. [3]
-
- 21** A particle P of mass 2 kg is attached to a point on a horizontal ceiling by a light inextensible string of length 0.6 m. The particle rests in equilibrium directly above a light spring, of natural length 1.0 m and modulus of elasticity 39.2 N, which is attached to P and to a point on the horizontal floor, 1.5 m below the ceiling. The string is taut and the spring is vertical. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the tension in the string. [4]
- (b) The string is cut. Find the initial acceleration of P. [2]
-
- 22** A light spring, of natural length l and modulus of elasticity $4mg$, has its upper end fixed to a ceiling. A particle P of mass $2m$ is attached to the lower end of the spring. A particle Q of mass m hangs from P by a light inextensible string, and the system hangs in equilibrium.
- (a) Find the extension of the spring. [2]
- (b) The string between P and Q is cut. Find the initial acceleration of P and of Q. [3]
- (c) Find the length of the spring when P comes to rest in its new equilibrium position. [2]

- 23** A particle A of mass 3 kg lies on a smooth horizontal table. It is attached to a fixed point W on the table by a light spring of natural length 0.8 m and modulus of elasticity 49 N. A light inextensible string joins A to a particle B of mass 2 kg, passing over a smooth pulley at the edge of the table on the side of A opposite to W, so that B hangs freely and the system is at rest with the spring in line with the string. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the tension in the string and the length of the spring. [5]
 (b) The string is cut. Find the initial acceleration of A. [2]
-
- 24** A light spring, of natural length 0.9 m and modulus of elasticity 49 N, has its lower end fixed at the foot of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. The coefficient of friction between the plane and a particle P of mass 2 kg is 0.5. P rests on the upper end of the spring, which lies along a line of greatest slope. Find the range of lengths of the spring for which P can rest in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$. [8]
-
- 25** A smooth bead of mass 0.6 kg is threaded on a fixed smooth vertical rod. It is attached to one end of a light spring of natural length 0.3 m and modulus of elasticity λ N. The other end of the spring is fixed to a point O which is at a horizontal distance 0.4 m from the rod. The bead rests in equilibrium at a point 0.3 m below the level of O. Find λ , and the magnitude of the force exerted by the rod on the bead. Take $g = 9.8 \text{ m s}^{-2}$. [7]
-
- 26** Two identical light springs, each of natural length 0.4 m and modulus of elasticity 20 N, are joined end to end. A particle of mass 0.5 kg hangs from the lower end of the combination, and the upper end is fixed. A student calculates the extension as follows: 'Each spring has stiffness $\frac{20}{0.4} = 50 \text{ N m}^{-1}$, so the combined stiffness is $50 + 50 = 100 \text{ N m}^{-1}$, and $x = \frac{0.5 \times 9.8}{100} = 0.049 \text{ m}$.' Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Explain the error in the student's working. [2]
 (b) Find the correct total extension of the combination. [2]
 (c) Find the total extension when three such springs are joined end to end in the same way. [2]
-
- 27** A light spring S_1 , of natural length 0.5 m and modulus of elasticity 24.5 N, has its upper end fixed to a ceiling. Its lower end is attached to a particle P of mass 2 kg. A second light spring S_2 , of natural length 0.4 m and modulus of elasticity 39.2 N, is attached to P and to a particle Q of mass 1 kg, which hangs below P. The system hangs in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the lengths of the two springs. [4]
 (b) Q becomes detached. Find the initial acceleration of P. [2]
 (c) Find the length of S_1 when P is again in equilibrium. [2]
 (d) Q is replaced by a particle of mass 3 kg, with P still attached to S_1 . Find the total length of S_1 and S_2 in equilibrium. [2]
-
- 28** A uniform rod AB, of length 1 m and mass 3 kg, is smoothly hinged at A to a fixed support, 0.4 m above horizontal ground. A light vertical spring, of natural length 1.2 m and modulus of elasticity 88.2 N, joins B to the ground directly below B. The rod rests in equilibrium inclined at an angle θ above the horizontal, as shown. Take $g = 9.8 \text{ m s}^{-2}$.

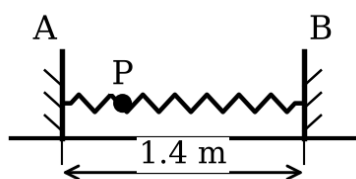


- (a) Show that the thrust in the spring is 14.7 N, whatever the value of θ . [3]
 (b) Find the length of the spring. [2]
 (c) Find θ , to the nearest tenth of a degree. [3]
 (d) The spring is replaced by one of the same natural length and modulus of elasticity 176.4 N. Find the new value of θ , to the nearest tenth of a degree. [2]

- 29** A smooth ring R of mass m is threaded on a fixed smooth vertical rod. R is attached to one end of a light spring of natural length a and modulus of elasticity $\frac{15mg}{8}$. The other end of the spring is fixed to a point O, which is at a horizontal distance a from the rod, as shown. The ring rests in equilibrium at a distance h below the level of O.



- Show that $\frac{15mg}{8} \times \frac{h}{a} \left(1 - \frac{a}{\sqrt{a^2 + h^2}}\right) = mg$. [3]
 - Show that the ring is in equilibrium when $h = \frac{4a}{3}$. [3]
 - Explain why there is only one equilibrium position. [2]
 - Find the magnitude of the force exerted by the rod on the ring. [2]
- 30** A uniform light spring, of natural length 1.0 m and modulus of elasticity 40 N, has its ends attached to two fixed points A and B on a smooth horizontal table, where $AB = 1.4$ m. A particle P is attached to the point of the spring at a distance 0.25 m from A when the spring is unstretched, as shown.

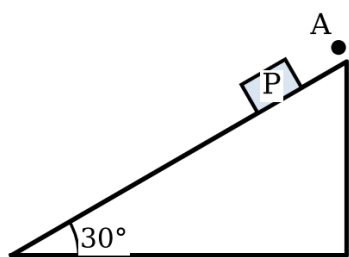


- State the natural length and modulus of elasticity of each of the two parts of the spring, AP and PB. [2]
- Find the lengths of AP and PB when P is in equilibrium, and the tension in the spring. [4]
- A horizontal force of 8 N, along AB and directed towards B, is now applied to P. Find the new value of AP, to 3 significant figures. [4]

Elastic Potential Energy

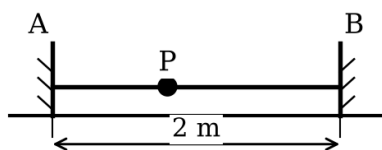
- A light elastic string has natural length 0.8 m and modulus of elasticity 40 N. Find the elastic potential energy stored in the string when it is extended by 0.2 m. [2]
- A light elastic string has natural length 2 m and modulus of elasticity 50 N. Find the work done in stretching the string slowly from its natural length to a length of 2.4 m. [2]
- A light spring has natural length 0.4 m and modulus of elasticity 20 N. Find the elastic potential energy stored in the spring when it is compressed to a length of 0.3 m. [2]
- A light elastic string has natural length 0.5 m and modulus of elasticity 16 N. The elastic potential energy stored in the string is 1.44 J. Find the extension of the string. [2]
- A light elastic string has natural length 1 m and modulus of elasticity 20 N. Find the work done in stretching the string from an extension of 0.2 m to an extension of 0.4 m. [2]

- 6** A light elastic string has natural length 0.8 m. When it is extended by 0.2 m the elastic potential energy stored in the string is 0.75 J. Find the modulus of elasticity of the string. [3]
- 7** A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 29.4 N. The other end of the string is fixed at O. The particle is released from rest at O. Find its speed when it is 1.6 m below O. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 8** A particle of mass 0.4 kg lies on a smooth horizontal table, attached to a fixed point O on the table by a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. The particle is held at a point 0.9 m from O and released. Find its speed when the string becomes slack. [4]
- 9** A block of mass 2 kg is on a rough horizontal table, with coefficient of friction 0.25. It is attached to a fixed point O on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 39.2 N. The block is released from rest at a point 0.7 m from O. Find the speed of the block when the string becomes slack. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 10** A particle of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $4mg$. The other end of the string is fixed at a point O. The particle is released from rest at O. Find the greatest extension of the string. [4]
- 11** A block of mass 1 kg is released from rest at a point 0.8 m vertically above the top end of a light vertical spring, of natural length 0.4 m and modulus of elasticity 196 N, whose lower end is fixed to the ground. Find the greatest compression of the spring. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 12** A particle P of mass 2 kg is released from rest at a point A at the top of a smooth plane inclined at 30° to the horizontal. P is attached to A by a light elastic string of natural length 0.5 m and modulus of elasticity 39.2 N. P slides down a line of greatest slope, as shown. Find the greatest extension of the string. Take $g = 9.8 \text{ m s}^{-2}$.



- [5]
- 13** A block of mass 2 kg lies on a rough horizontal table. It is attached to a fixed point O on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 49 N. The block is released from rest with the string extended by 0.2 m, and it comes to rest at the instant when the string reaches its natural length. Find the coefficient of friction between the block and the table. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 14** A light elastic string has natural length l and modulus of elasticity λ .
 (a) The string is stretched slowly from its natural length to an extension x . Show that the work done is $\frac{\lambda x^2}{2l}$. [3]
 (b) Hence show that the work done in stretching the string from an extension a to an extension b is $\frac{\lambda(b^2 - a^2)}{2l}$, and find its value when $\lambda = 30 \text{ N}$, $l = 1.5 \text{ m}$, $a = 0.3 \text{ m}$ and $b = 0.9 \text{ m}$. [2]

- 15** A particle P of mass 0.8 kg lies on a smooth horizontal table. It is attached to a point A on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 20 N, and to a point B on the table by another light elastic string of natural length 0.5 m and modulus of elasticity 20 N, where $AB = 2$ m and A, P and B lie in a straight line, as shown. P is released from rest at the point where $AP = 0.8$ m. Find the speed of P when it passes the midpoint of AB.



[4]

- 16** A light vertical spring, of natural length 0.3 m and modulus of elasticity 29.4 N, has its lower end fixed to the ground. A ball of mass 0.2 kg is placed on top of the spring and pressed down until the length of the spring is 0.2 m. The ball is then released from rest. Find the speed of the ball when the spring reaches its natural length, and the greatest height of the ball above this position. Give the speed to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

[6]

- 17** A light elastic string of natural length l and modulus of elasticity λ has tension T when its extension is x .
- Show that the elastic potential energy stored in the string is $\frac{1}{2}Tx$. [2]
 - A particle of mass 3 kg hangs in equilibrium from a string with $l = 1.2$ m and $\lambda = 58.8$ N. Find the elastic potential energy stored. Take $g = 9.8 \text{ m s}^{-2}$. [2]
 - Find the gravitational potential energy lost by the particle in moving from the natural length position to equilibrium, and explain why it is twice the elastic potential energy gained. [2]

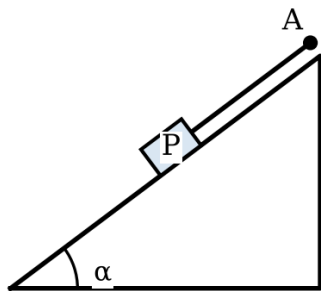
- 18** A student writes: 'A light elastic string has natural length 1.2 m and modulus of elasticity 36 N. When its length is 1.5 m, the elastic potential energy is $\frac{36 \times 1.5^2}{2 \times 1.2} = 33.75 \text{ J}$.' Explain the mistake and find the correct value. [3]

- 19** A block of mass 2 kg lies on a smooth plane inclined at 30° to the horizontal. A light spring, of natural length 0.5 m and modulus of elasticity 49 N, lies along a line of greatest slope with its lower end fixed to the bottom of the plane. The block is pushed down the plane against the upper end of the spring until the spring is compressed by 0.3 m, and is then released from rest. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the speed of the block when the spring reaches its natural length, to 3 significant figures. [5]
 - Find the greatest speed of the block. [5]

- 20** A particle of mass 0.4 kg lies on a smooth horizontal table. It is attached to a fixed point O on the table by a light elastic string of natural length 0.5 m and modulus of elasticity λ N. The particle is released from rest with $OP = 0.8$ m, and its speed when the string becomes slack is 6 m s^{-1} .
- Find the value of λ . [3]
 - Find the speed of the particle when $OP = 0.6$ m, to 3 significant figures. [4]

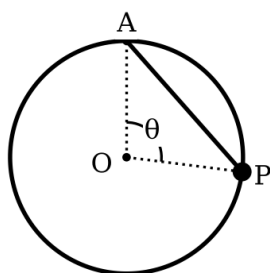
- 21** A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 58.8 N. The other end of the string is fixed at O. The particle is released from rest at O and falls vertically. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the greatest extension of the string. [4]
 - Find the acceleration of the particle when the string has its greatest extension. [2]
 - Find the greatest speed of the particle, giving your answer to 3 significant figures. [3]

- 22** A particle P of mass 2 kg is on a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. The coefficient of friction between P and the plane is 0.25. P is attached to a point A at the top of the plane by a light elastic string, of natural length 0.5 m and modulus of elasticity 100 N, along a line of greatest slope. P is released from rest with AP = 0.8 m, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



- Find the speed of P when the string becomes slack, giving your answer to 3 significant figures. [5]
 - Find the further distance P travels up the plane before first coming to rest, giving your answer to 3 significant figures. [3]
 - Show that P does not then remain at rest. [1]
-
- 23** A particle of mass 0.5 kg hangs in equilibrium from a fixed point O by a light elastic string of natural length 1 m and modulus of elasticity 19.6 N. The particle is projected vertically upwards from its equilibrium position with speed 4 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- Show that the string becomes slack, and find the speed of the particle at that instant, giving your answer to 3 significant figures. [4]
 - Find the greatest height reached by the particle above its equilibrium position. [3]
-
- 24** A block of mass 2 kg is on a rough horizontal table, with coefficient of friction 0.2. It is attached to a fixed point O on the table by a light spring of natural length 0.5 m and modulus of elasticity 39.2 N. The block is released from rest with the spring extended by 0.3 m. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the length of the spring when the block first comes to rest. [4]
 - Show that the block next comes to rest when the spring is extended by 0.1 m. [3]
 - Show that the block finally comes to rest with the spring at its natural length, and find the total distance travelled by the block. [2]
-
- 25** Two light elastic strings AB and BC are joined at B. String AB has natural length 0.5 m and modulus of elasticity 49 N. String BC has natural length 0.8 m and modulus of elasticity 78.4 N. The end A is fixed and a particle of mass 2 kg is attached at C. Take $g = 9.8 \text{ m s}^{-2}$.
- The particle hangs in equilibrium. Find the total elastic potential energy stored in the strings. [4]
 - The particle is instead released from rest with both strings just taut and unstretched. Find the greatest total extension of the strings. [4]
-
- 26** A particle P of mass 3 kg is released from rest at a point A at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. P is attached to A by a light elastic string of natural length 0.6 m and modulus of elasticity 88.2 N. P slides down the line of greatest slope and first comes to rest when AP = 1.0 m. The coefficient of friction between P and the plane is μ . Take $g = 9.8 \text{ m s}^{-2}$.
- Find the value of μ . [6]
 - Show that P does not remain at rest at this point, and state the direction in which it begins to move. [4]

- 27** A smooth bead P of mass m is threaded on a smooth fixed circular wire, of centre O and radius a , in a vertical plane. A light elastic string of natural length a and modulus of elasticity $3mg$ joins P to the highest point A of the wire. The bead is released from rest at A. The angle AOP is denoted by θ .



- Show that the string becomes taut when $\theta = 60^\circ$, and find the speed of the bead at that instant. [3]
- Find the speed of the bead when it reaches the lowest point of the wire. [3]
- The speed of the bead is greatest when $\sin \frac{\theta}{2} = \frac{3}{4}$. Find this greatest speed. [4]

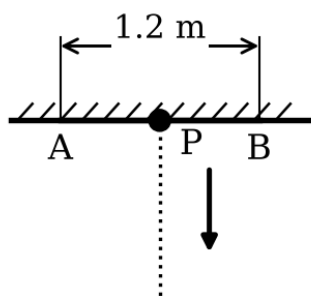
- 28** A light spring, of natural length 0.6 m and modulus of elasticity 147 N, has its lower end fixed at the foot of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 0.6$. The spring lies along a line of greatest slope. A block of mass 1.5 kg is placed on the upper end of the spring and pressed down until the length of the spring is 0.3 m. The coefficient of friction between the block and the plane is 0.5. The block is released from rest. Take $g = 9.8 \text{ m s}^{-2}$.

- Find the speed of the block when the spring reaches its natural length, giving your answer to 3 significant figures. [5]
- Find the further distance the block travels up the plane after leaving the spring. [3]
- Show that the block does not remain at rest at its highest point. [2]

- 29** A bungee jumper of mass 70 kg, modelled as a particle, is attached to one end of a light elastic rope of natural length 30 m and modulus of elasticity 16 464 N. The other end of the rope is fixed to a bridge. The jumper steps off the bridge at the fixed end and falls vertically from rest. Air resistance is ignored. Take $g = 9.8 \text{ m s}^{-2}$.

- Find the greatest extension of the rope. [4]
- Find the greatest speed of the jumper. [4]
- Find the magnitude of the greatest acceleration of the jumper, in terms of g . [2]

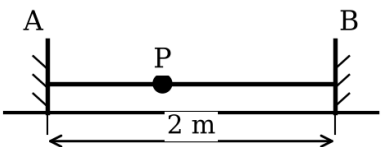
- 30** Two identical light elastic strings AP and BP, each of natural length 0.6 m and modulus of elasticity 14.7 N, are attached to a particle P of mass 0.5 kg. The points A and B are fixed at the same horizontal level, 1.2 m apart. P is released from rest at the midpoint of AB, with both strings straight and unstretched, and falls vertically, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



- Show that P first comes to instantaneous rest when it is 0.8 m below AB. [4]
- Find the acceleration of P at this lowest point. [4]
- Find the speed of P when it is 0.45 m below AB, giving your answer to 3 significant figures. [3]

Motion Involving Elastic Forces

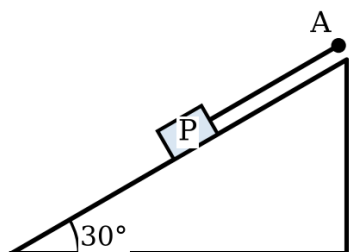
- 1 A particle of mass 0.2 kg lies on a smooth horizontal table, attached to a fixed point by a light spring of natural length 0.5 m and modulus of elasticity 10 N. The particle oscillates along the line of the spring, which stays taut, so that $m \frac{d^2x}{dt^2} = -\frac{\lambda}{l}x$, where x is the extension. Find the period of the oscillations. [3]
- 2 A particle moves in simple harmonic motion with $\frac{d^2x}{dt^2} = -9x$, where x is the displacement in metres from the centre, and the amplitude is 0.4 m.
 - (a) Find the period of the motion. [1]
 - (b) Find the maximum speed. [1]
 - (c) Find the maximum acceleration. [1]
- 3 A particle moves in simple harmonic motion with angular frequency 4 rad s^{-1} and amplitude 0.5 m. Find its speed when it is 0.3 m from the centre of the motion. [2]
- 4 A particle of mass 0.5 kg is attached to a light spring of stiffness 200 N m^{-1} , and oscillates in simple harmonic motion. Find the period of the oscillations. [2]
- 5 A particle hangs in equilibrium from a light spring, the extension of the spring being 0.098 m. The particle is pulled down a small distance and released, and then oscillates vertically in simple harmonic motion. Find the period of the oscillations. Take $g = 9.8 \text{ m s}^{-2}$. [3]
- 6 A particle P of mass 0.5 kg moves in simple harmonic motion along a straight line so that its displacement from the centre O at time t seconds is $x = 0.3 \cos 4t$ metres.
 - (a) Find the maximum speed of P. [2]
 - (b) Find the maximum acceleration of P, and the maximum resultant force acting on P. [3]
- 7 A particle of mass m hangs in equilibrium from a fixed point O by a light elastic string of natural length l and modulus of elasticity λ . The particle is pulled down a small distance and released. Show that, while the string is taut, the particle moves in simple harmonic motion with angular frequency $\sqrt{\frac{\lambda}{ml}}$. [4]
- 8 A particle of mass 0.4 kg is attached to a light elastic string of natural length 0.6 m and modulus of elasticity 30 N, and oscillates on a smooth horizontal table with the string always taut. The amplitude of the oscillation is 0.3 m. Find the greatest speed and the greatest acceleration of the particle. Give the speed to 3 significant figures. [3]
- 9 A particle P of mass 0.5 kg lies on a smooth horizontal table. It is attached to a point A on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 20 N, and to a point B on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 30 N. The points A, P and B lie in a straight line with $AB = 2 \text{ m}$, as shown.



 - (a) Find the distance AP when P is in equilibrium. [3]
 - (b) P is displaced a small distance x towards B and released. Show that P moves in simple harmonic motion, and find the period, giving your answer to 3 significant figures. [3]

- 10** A particle moves in a straight line so that its displacement from a fixed point O at time t seconds is $x = 0.4 \cos 5t$ metres. Find the first time at which the speed of the particle is 1 m s^{-1} . [4]
- 11** A particle of mass 0.5 kg hangs in equilibrium from a fixed point by a light elastic string of natural length 0.6 m and modulus of elasticity 29.4 N . The particle is pulled down 0.05 m from its equilibrium position and released. Show that the string remains taut throughout the motion, and find the greatest speed of the particle, giving your answer to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 12** A particle of mass 0.8 kg lies on a smooth horizontal table, attached to a fixed point on the table by a light elastic string of natural length 0.6 m and modulus of elasticity $\lambda \text{ N}$. The particle is pulled away from the point and released, and oscillates in simple harmonic motion with the string always taut. The period of the oscillations is 0.5 s . Find the value of λ , to 3 significant figures. [5]
- 13** A particle P moves in simple harmonic motion along a straight line with centre O, amplitude 0.2 m and period 1.2 s . The displacement of P from O is x metres at time t seconds, with $x = 0$ and P moving in the positive direction when $t = 0$.
 (a) Find the first time at which $x = 0.12$, to 3 significant figures. [4]
 (b) Find the speed of P when $x = 0.12$, and the time taken for P to move from $x = -0.12$ to $x = 0.12$. [3]
- 14** A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 29.4 N . The other end of the string is fixed at O. The particle is held at rest with the string just taut and then released. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the equilibrium extension of the string, and hence the amplitude of the motion, and show that the greatest extension of the string is 0.4 m . [3]
 (b) Verify the greatest extension using conservation of energy. [2]
- 15** A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 39.2 N . The other end of the string is fixed at O. The particle is held at rest with the string just taut and then released. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the time taken for the particle to reach its equilibrium position, giving your answer to 3 significant figures. [3]
 (b) Find the time taken for the extension of the string to first reach 0.375 m , giving your answer to 3 significant figures. [2]
- 16** A particle of mass m , attached to a light spring of stiffness k , moves on a smooth horizontal table so that its extension x satisfies the energy equation $\frac{1}{2}m v^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, where A is the amplitude.
 (a) Show that $v^2 = \omega^2(A^2 - x^2)$, where $\omega^2 = \frac{k}{m}$. [2]
 (b) Given that $m = 0.5 \text{ kg}$, $k = 50 \text{ N m}^{-1}$ and $A = 0.3 \text{ m}$, find the speed when the extension is 0.2 m , and the extension when the speed is 1.5 m s^{-1} . Give your answers to 3 significant figures. [4]
- 17** A block of mass 1.5 kg rests on top of a light vertical spring, of natural length 0.5 m and modulus of elasticity 30 N , whose lower end is fixed to the ground. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the length of the spring when the block rests in equilibrium. [2]
 (b) The block is pressed down a further 0.1 m and released. Show that the block moves in simple harmonic motion, and find the period, giving your answer to 3 significant figures. [2]
 (c) Find the greatest speed of the block, giving your answer to 3 significant figures. [2]

- 18** A particle of mass 2 kg is on a smooth plane inclined at 30° to the horizontal. It is attached to a point A at the top of the plane by a light elastic string of natural length 0.7 m and modulus of elasticity 49 N, which lies along a line of greatest slope. The particle hangs in equilibrium and is then pulled a small distance down the plane and released, as shown. Show that the particle moves in simple harmonic motion, and find the period of the motion, giving your answer to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 19** A particle hangs in equilibrium from a light elastic string, the extension being 0.2 m. The particle is pulled down 0.05 m and released, and the string stays taut. A student says: 'If I pull it down 0.10 m instead, it has further to travel, so the period will be twice as long.' Explain why the student is wrong, and find the period of the motion in both cases. Take $g = 9.8 \text{ m s}^{-2}$.

[3]

- 20** A particle A of mass 0.5 kg hangs in equilibrium from a fixed point O by a light elastic string of natural length 0.6 m and modulus of elasticity 29.4 N. A second particle B, also of mass 0.5 kg, is then attached to A while A is held at rest at its equilibrium position, and the two particles are released from rest. Take $g = 9.8 \text{ m s}^{-2}$.

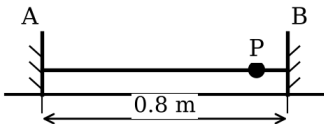
- Find the extension of the string when A hangs in equilibrium alone, and when A and B hang together in equilibrium. [3]
- Show that the subsequent motion is simple harmonic, and find its amplitude, its period and the greatest speed of the particles, to 3 significant figures. [5]

- 21** A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 19.6 N. The other end of the string is fixed at O. The particle is released from rest at a point 0.3 m vertically above the point where the string is just taut, and falls vertically. Take $g = 9.8 \text{ m s}^{-2}$.

- Find the speed of the particle when the string first becomes taut, giving your answer to 3 significant figures. [2]
- Find the equilibrium extension, and show that the amplitude of the subsequent simple harmonic motion is 0.4 m. [4]
- Find the greatest extension of the string. [1]
- Find the time from the release of the particle until it first reaches its lowest point, giving your answer to 3 significant figures. [4]

- 22** A block of mass 2 kg is on a rough horizontal table, with coefficient of friction 0.2. It is attached to a fixed point O on the table by a light spring of natural length 0.5 m and modulus of elasticity 39.2 N. The block is released from rest with the spring extended by 0.3 m, and moves towards O. Let x be the extension of the spring at time t . Take $g = 9.8 \text{ m s}^{-2}$.

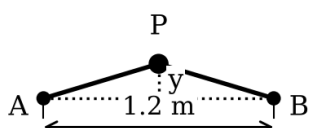
- Show that, while the block moves towards O, $\frac{d^2x}{dt^2} = -39.2(x - 0.05)$, and deduce that the motion is simple harmonic with centre $x = 0.05$. [3]
- Find the extension of the spring when the block first comes to rest, and the time taken to reach this position, giving the time to 3 significant figures. [3]
- Find the greatest speed of the block, giving your answer to 3 significant figures. [2]

- 23** A light spring AP, of stiffness 80 N m^{-1} , and a light spring PB, of stiffness 60 N m^{-1} , are attached to a particle P of mass 1.5 kg . The point A is fixed vertically above the fixed point B, and the springs are vertical, with P in equilibrium between A and B. Both springs are extended in the equilibrium position.
- P is displaced a small distance y downwards from equilibrium and released. Show that P moves in simple harmonic motion, and find the period, giving your answer to 3 significant figures. [5]
 - P is displaced 0.05 m downwards and released. Find the greatest speed of P, giving your answer to 3 significant figures. [2]
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- 24** In an experiment, a light spring hangs vertically from a fixed support with a mass $m \text{ kg}$ attached to its lower end. The mass is pulled down slightly and released, and the period of the vertical oscillations is found to be 0.400 s . A further 0.100 kg is added to the mass, and the period becomes 0.500 s . Assuming that the spring obeys Hooke's law and that the oscillations are simple harmonic, find the stiffness of the spring and the value of m , giving your answers to 3 significant figures. [6]
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- 25** A particle of mass m hangs in equilibrium from a fixed point by a light elastic string of natural length l and modulus of elasticity λ , the equilibrium extension being e . The particle is pulled down a small distance and released, and the string stays taut.
- Show that the period of the oscillations is $T = 2\pi\sqrt{\frac{e}{g}}$. [3]
 - The period is measured as 0.9 s . Find e , giving your answer to 3 significant figures. [2]
 - Given that $m = 0.4 \text{ kg}$ and $l = 0.5 \text{ m}$, find λ , giving your answer to 3 significant figures. [2]
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- 26** A particle P of mass 1.5 kg lies on a smooth horizontal table. It is attached to a fixed point O on the table by a light elastic spring of natural length 0.5 m and modulus of elasticity 49 N , and to a particle Q of mass 0.5 kg by a light inextensible string which passes over a smooth pulley at the edge of the table, so that Q hangs freely. The spring lies along the line from O to the pulley, and the string between P and the pulley is horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- Find the extension of the spring when the system is in equilibrium. [2]
 - The system is displaced a small distance x further from O and released, the string remaining taut. Show that the motion is simple harmonic and find its period, to 3 significant figures. [5]
 - Find the greatest amplitude of the motion for which the string remains taut. [3]
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- 27** A particle P of mass 0.5 kg lies on a smooth horizontal table. It is attached to a point A on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 20 N , and to a point B on the table by a light elastic string of natural length 0.5 m and modulus of elasticity 20 N . The points A, P and B lie in a straight line with $AB = 0.8 \text{ m}$, as shown. P is released from rest at the point where $AP = 0.7 \text{ m}$.
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- Show that the string BP is slack, and find the speed of P when the string AP becomes slack, giving your answer to 3 significant figures. [3]
 - Find the length of time for which both strings are slack in each crossing of the central region. [2]
 - Find the time taken for P to return to its starting point for the first time, giving your answer to 3 significant figures. [4]

- 28** Two particles A and B, of masses 2 kg and 3 kg, lie on a smooth horizontal table, connected by a light spring of natural length 0.5 m and modulus of elasticity 30 N. The particles are held at rest with the spring extended to a length of 0.7 m and then released simultaneously. Let e be the extension of the spring at time t .

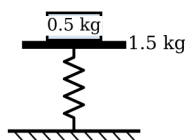
- (a) Show that $\frac{d^2e}{dt^2} = -50e$, and deduce that the extension oscillates in simple harmonic motion. [3]
- (b) Find the period of the oscillations, giving your answer to 3 significant figures. [2]
- (c) Find the greatest speed of each particle, giving your answers to 3 significant figures. [3]

- 29** A light elastic string, of natural length 1.0 m and modulus of elasticity 40 N, has its ends fixed to two points A and B, 1.2 m apart on a smooth horizontal table. A particle P of mass 0.25 kg is attached to the midpoint of the string, which is in equilibrium along AB. P is displaced a small distance y perpendicular to AB and released, as shown.



- (a) Find the tension in the string when P is at the midpoint of AB. [2]
- (b) Show that, for small y , P moves in approximately simple harmonic motion, and find the period, giving your answer to 3 significant figures. [5]
- (c) When $y = 0.05$ m, compare the restoring force from the approximate model with the exact value of the restoring force, giving your answers to 3 significant figures. [2]

- 30** A platform of mass 1.5 kg is attached to the upper end of a light vertical spring, of natural length 0.5 m and modulus of elasticity 100 N, whose lower end is fixed to the ground. A block of mass 0.5 kg rests on the platform, as shown. The system oscillates vertically in simple harmonic motion with the block remaining in contact with the platform. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that the period of the oscillations is $\frac{\pi}{5}$ s. [3]
- (b) The amplitude is 0.06 m. Find the greatest and least values of the force between the block and the platform. [3]
- (c) Find the greatest amplitude for which the block remains in contact with the platform throughout the motion. [2]
- (d) The amplitude is increased to 0.15 m. Find the further height the block rises after leaving the platform, giving your answer to 3 significant figures. [3]