

Activity Networks

1 (2 marks)

(a) An arc represents an activity (a task which takes time), with its duration marked

B1

(b) A node represents an event: a point in time at which activities start or finish

B1

Answer: arc = activity; node = event

2 marks

2 (2 marks)

A dummy shows a dependency (or keeps two activities distinct) and takes no time

B1

It is drawn as a dashed arc (with an arrow) and has duration 0

B1

Answer: Shows a dependency; dashed arc with duration 0

2 marks

3 (2 marks)

A and B have no activities before them, so they can start at once

B1

E can only start when C and D are both complete

B1

Answer: A and B; E

2 marks

4 (2 marks)

Activities (arcs): A, B, C, D, E, F, G, so 7

B1

Events (nodes): 1 to 6, so 6

B1

Answer: 7 activities; 6 events

2 marks

5 (2 marks)

A: none; B: none

B1

C: A; D: B, C

B1

Answer: A: -, B: -, C: A, D: B, C

2 marks

6 (2 marks)

A and B can both start at once: 2 activities

B1

For example, C and D (C needs only A and D needs only B)

B1

Answer: 2; e.g. C and D

2 marks

7 (3 marks)

A and B start from the first event

B1

C follows A, and D follows B, with both meeting at the event from which E starts

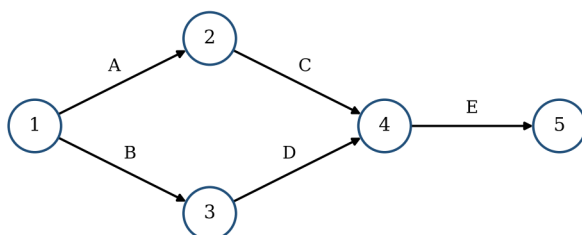
M1

E leaves that event to the final event; the network has 5 events and no dummy

A1

Answer: Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 3 to 4; E: 4 to 5

3 marks



8 (4 marks)

A and B start at the first event

B1

C and D follow A (from the end of A)

M1

E follows B

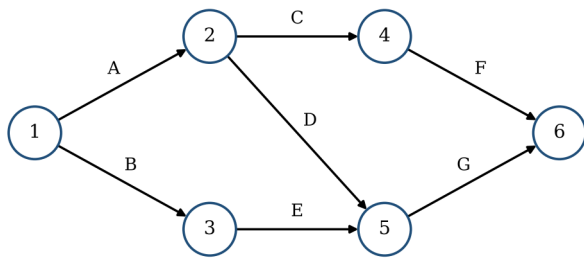
M1

F follows C; G follows both D and E (they end at one node)

A1

Answer: Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 2 to 5; E: 3 to 5; F: 4 to 6; G: 5 to 6

4 marks

**9 (3 marks)**

Q must precede both R and S, but P must precede only R

M1

If P and Q both ended at the same event, S would also have to wait for P

E1

So Q ends at a separate event from which S starts, and a dummy joins this event to the event where P ends and R starts

A1

Answer: Q ends at its own node, with a dummy to the node where R starts

3 marks

10 (4 marks)

A: none; B: none; C: A, B

B1

D: B; E: C; F: D

B1

The dummy goes from the end of B to the start of C

M1

It shows that C needs B to be complete (as well as A), while D needs B only

E1

Answer: A -, B -, C A and B, D B, E C, F D

4 marks

11 (3 marks)

Start by A or B

B1

A, C, F; A, D, G

M1

B, E, G

A1

Answer: A-C-F; A-D-G; B-E-G

3 marks

12 (3 marks)

Two activities with the same start and end events cannot be told apart by their start and end events (ambiguous, and often the network is not a proper activity network)

E1

Put right by splitting one activity's finish into a new event

M1

and joining that event to the original end event with a dummy activity

A1

Answer: Make one of the activities end at a new event, and join that event to event 3 with a dummy activity of duration 2

13 (3 marks)

A, B leave the first node

B1

C after A, D after B

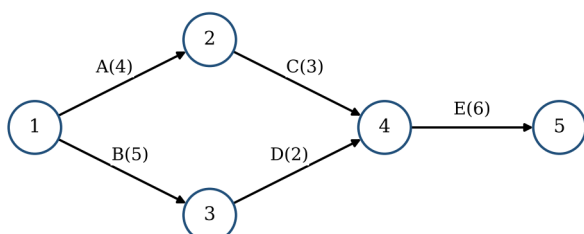
M1

C and D end at the same node, and E leaves it; durations 4, 5, 3, 2, 6 marked

A1

Answer: Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 3 to 4; E: 4 to 5

3 marks



14 (5 marks)

A and B start from the first event

B1

C needs A and B, D needs B only

M1

Event after A is the start of C; B ends at an event where D starts

M1

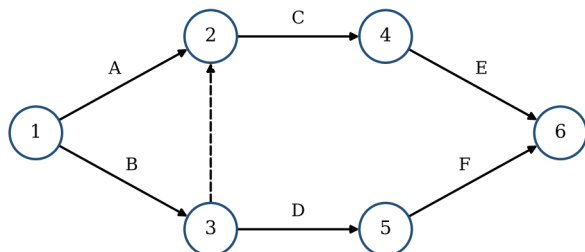
Dummy from the end of B to the start of C

A1

E follows C, F follows D, and both end at the final event

A1

Answer: Arcs (event to event): A: 1 to 2; B: 1 to 3; dummy: 3 to 2; C: 2 to 4; D: 3 to 5; E: 4 to 6; F: 5 to 6; one dummy

**15 (6 marks)**

A and B start from the first event; C follows A, D follows B

B1

F and G need both A and D, so a dummy is needed from the end of A to the event at the end of D

M1 A1

E follows C; F and G follow that event (A and D)

M1

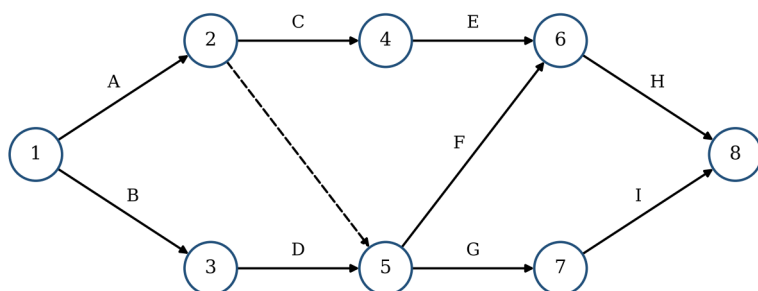
H needs E and F; I needs G; H and I end at the final event

M1

Only one dummy is needed

A1

Answer: Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 3 to 5; dummy: 2 to 5; E: 4 to 6; F: 5 to 6; G: 5 to 7; H: 6 to 8; I: 7 to 8

**16 (4 marks)**

A, B, C have no preceding activity: A: none; B: none; C: none

M1 A1

D: A; E: B; F: B; G: C

A1

H: D, E; I: F, G; J: H, I; K: F, G

A1

Answer: A: none; B: none; C: none; D: A; E: B; F: B; G: C; H: D, E; I: F, G; J: H, I; K: F, G

4 marks

17 (5 marks)

A, B, C start from the first event

B1

D, E, F and G start from the events at the ends of their preceding activities

M1

H, I and the final activities join at common events, as in the table: D: A; E: B; F: B; G: C

M1

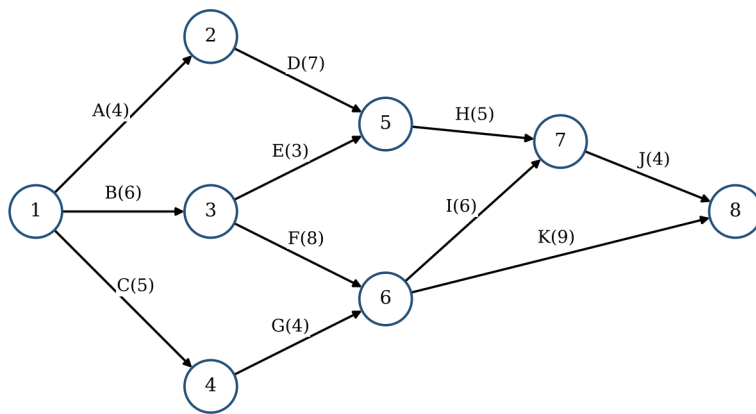
The remaining activities: H: D, E; I: F, G; J: H, I; K: F, G

A1

The network has 8 events and no dummy activity

A1

Answer: 8 events. Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 1 to 4; D: 2 to 5; E: 3 to 5; F: 3 to 6; G: 4 to 6; H: 5 to 8; I: 6 to 8

**18 (4 marks)**

X and Y both start from the event at which A and B finish

M1

A and B must meet at a common event, but Z needs only B

M1

So B ends at its own event, where Z starts, and a dummy runs from that event to the common event where A ends

A1

One dummy is needed

A1

Answer: Yes: one dummy from the end of B to the start of X and Y

4 marks

19 (4 marks)

A and B both finish at the event where C starts: no other activity needs only one of them

M1

C leads to a node from which D and E start

M1

D and E end at the event where F starts

A1

No dummy activity is needed

A1

Answer: No dummy; chain A, B then C then D, E then F

4 marks

20 (3 marks)

S needs both P and Q, but R needs only P

M1

If P and Q finished at the same event, R would also depend on Q

E1

So P must end at its own event for R, with a dummy to the event where Q ends and S starts

A1

Answer: R needs only P while S needs P and Q

3 marks

21 (3 marks)

(a) 1 arc (A)

B1

(b) 2 arcs (A and B)

B1

(c) A real arc A, and a dummy from the end of B: 1 real arc and 1 dummy arc, so 2 arcs in all, one of them a dummy

B1

Answer: 1; 2; 2 (one a dummy)

3 marks

22 (7 marks)

A and B start from the first event; C and D follow A; E and F follow B

B1

G needs only C, so C ends at its own event where G starts

M1

H needs C, D and E: a dummy from the end of C to the event where D and E finish, from which H starts

M1 A1

I needs C, D, E and F: a dummy from that event to the end of F, where I starts

M1

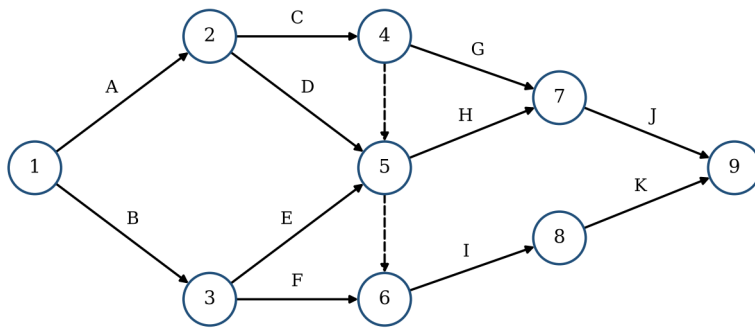
J follows G and H; K follows I; J and K end at the final event

A1

Two dummies are used

A1

Answer: 2 dummies. Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 2 to 5; E: 3 to 5; F: 3 to 6; G: 4 to 7; H: 5 to 7; I: 7 to 8; J: 7 to 8; K: 6 to 8



23 (5 marks)

From the network: C starts at the event where A ends, and a dummy joins the end of B to that event, so C needs A and B

M1 A1

D starts at the event where B ends and the dummy leaves that same event, so D needs B only

M1

E follows C only (not D) and F follows D only; these agree with the student's F but not the student's E

M1 A1

Answer: C: A, B; D: B; E: C

5 marks

24 (4 marks)

The table lists only immediately preceding activities; A is before C, and C is before D, so D cannot start before A is finished

E1

D's dependence on A is implied through C (it is a redundant dependency)

E1

Listing A as well would add nothing: it would need a dummy that does not change the times

E1

Hence A is not listed for D

B1

Answer: D depends on A only through C

4 marks

25 (7 marks)

A and B leave the first event; the end of A is where C starts

B1

D leaves the end of B, and a dummy runs from the end of B to the start of C

M1 A1

E follows C, F follows D, and both end at the final event

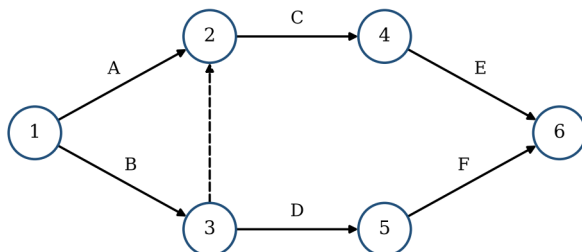
M1 A1

Paths: A, C, E; B, dummy, C, E; B, D, F (3 paths)

M1 A1

Answer: A-C-E; B-C-E; B-D-F

7 marks



26 (6 marks)

C needs A only, E needs B only and D needs both, so the starting events of C, D and E must be different

M1

A must end at an event where only A has arrived (for C), and B at an event where only B has arrived (for E)

M1

Neither of these events can be the start of D, because that event must be reached from both A and B

A1

So D's starting event needs a dummy from the end of A and a dummy from the end of B: at least 2 dummies

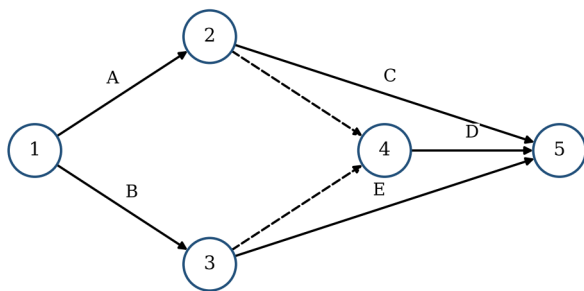
M1 A1

Network: A (1 to 2), B (1 to 3), dummies 2 to 4 and 3 to 4, C (2 to 5), D (4 to 5), E (3 to 5)

A1

Answer: At least 2 dummies. Arcs (event to event): A: 1 to 2; B: 1 to 3; dummy: 2 to 4; dummy: 3 to 4; C: 2 to 5; D: 4 to 5; E: 3 to 5

6 marks



27 (8 marks)

(a) Reading the arrows: A: none; B: none; C: A

D: B; E: C; F: A, D

G: A, D; H: E, F; I: G

Answer: A -, B -, C A, D B, E C, F A D, G A D, H E F, I G

B1

M1

A1

3 marks

(b) A and B start from the first event; C follows A; D follows B

F and G need both A and D, so a dummy is needed from the end of A to the end of D

E follows C; H follows E and F; I follows G

One dummy only, and H and I end at the final event

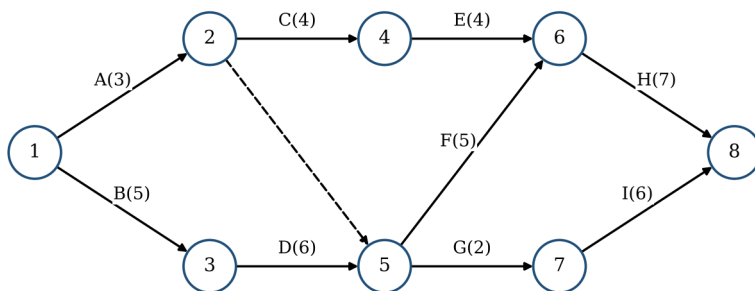
B1

M1 A1

M1

A1

Answer: One dummy. Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 2 to 4; D: 3 to 5; dummy: 2 to 5; E: 4 to 6; F: 5 to 6; G: 5 to 7; H: 6 to 8; I: 7 to 8



28 (7 marks)

(a) A, B, C from the first event; D after A; E after B; F after C

G and I need D and E: D and E end at a common event

H needs B and F: a dummy from the end of B to the end of F

J follows G only; K follows H and I

One dummy only, so the final event is reached by J and K

Answer: One dummy. Arcs (event to event): A: 1 to 2; B: 1 to 3; C: 1 to 4; D: 2 to 5; E: 3 to 5; dummy: 3 to 6; F: 4 to 6; G: 5 to 7; H: 6 to 8; I: 6 to 8; J: 7 to 9; K: 8 to 9

(b) Counting paths through the network: 6

Answer: 6

B1

M1

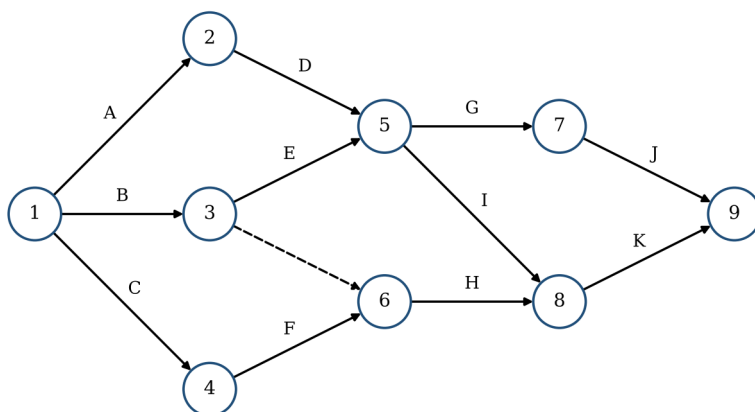
M1 A1

M1

A1

B1

1 mark



29 (8 marks)**(a)** Counting arcs systematically from the start, A then C, D; B then E, F

M1

Paths: A – C – I – K; A – C – H – J; A – C – G – J; A – D – I – K

A1

A – D – H – J; B – E – I – K; B – E – H – J; B – F – I – K

A1

Total: 8 paths

A1

Answer: 8 paths

4 marks

(b) The dummy joins the end of C to the start of H (and, via the other dummy, I)

M1

Without it, H would no longer need C, and I would no longer need C

M1 A1

Only H and I would be affected

B1

Answer: H and I would no longer need C

4 marks

30 (6 marks)

A needs C, C needs B, B needs A: a cycle

M1 A1

None of A, B or C can start, so the project can never begin (no activity starts at time 0)

E1

Correction 1: remove the requirement of A on C, so A: none, giving A, then B, then C, with D after B

M1

Correction 2: remove the requirement of B on A (B: none), so B, C, A in sequence, D after B

M1

Network for correction 1: A, then B, then C and D from the end of B

A1

Answer: A cycle of dependencies

6 marks

Precedence Tables**1 (2 marks)**

An activity which must be completed before the given activity can start

B1

and which has no other activity that must come between them (it is the last one required before)

B1

Answer: A direct prerequisite of the activity

2 marks

2 (2 marks)

A, B: none

B1

C: A; D: B; E: C, D

B1

Answer: A: none; B: none; C: A; D: B; E: C, D

2 marks

3 (1 mark)

B and C both need only A

B1

Answer: B and C

1 mark

4 (2 marks)

A and B have no preceding activity

B1

D and E both need only C, so they can be done together

B1

Answer: A, B; D and E

2 marks

5 (2 marks)

E

B1

No other activity needs E, and E needs both C and D (it comes after all others)

B1

Answer: E

2 marks

6 (2 marks)

B must be complete

B1

B needs A, so A must be complete too: A and B

B1

Answer: A and B

2 marks

7 (3 marks)

A, B: none

B1

C: A; D: A; E: B

M1

F: C; G: D, E

A1

Answer: A: none; B: none; C: A; D: A; E: B; F: C; G: D, E

3 marks

8 (3 marks)

A, B: none

B1

The dummy joins B to the start of C, so C: A, B; D: B

M1

E: C; F: D

A1

Answer: A: none; B: none; C: A, B; D: B; E: C; F: D

3 marks

9 (3 marks)

P: none; Q: P; R: P

B1

S: Q

B1

T: R, S

B1

Answer: P -, Q P, R P, S Q, T R and S

3 marks

10 (3 marks)

F needs C and D

B1

C needs A; D needs A and B

M1

So A, B, C and D must all be complete

A1

Answer: A, B, C, D

3 marks

11 (3 marks)

C needs only A, so it can start once A is done

M1

B and D are independent of C (they depend only on B), so C can run in parallel with B and D

A1

F, G come after E which needs C, so they cannot

B1

Answer: B and D

3 marks

12 (2 marks)

A needs B, B needs C, and C needs A: a cycle

M1

None of them could start, so the project could never begin

E1

Answer: Circular dependency

2 marks

13 (3 marks)

C needs B, and B needs A, so A is already required before C starts

M1 E1

Shorter table: A: none; B: A; C: B; D: C

A1

Answer: C: B only

3 marks

14 (4 marks)

A, B, C have no preceding activity

B1

D: A; E: B; F: B; G: C

M1

H: D, E; I: F, G; J: H, I; K: F, G

M1

K needs F and G, which need B and C: so B, C, F, G are complete before K

A1

Answer: A: none; B: none; C: none; D: A; E: B; F: B; G: C; H: D, E; I: F, G; J: H, I; K: F, G; before K: B, C, F, G**15 (5 marks)**

A: none; B: none; C: A; D: B

B1

E: C

B1

F: A, D; G: A, D (the dummy from the end of A to the end of D)

M1 A1

H: E, F; I: G. The dummy means that F and G need A as well as D

E1

Answer: A: none; B: none; C: A; D: B; E: C; F: A, D; G: A, D; H: E, F; I: G

5 marks

16 (4 marks)

E must be last, and A must come before C, B before D

M1

The first four are an arrangement of A, B, C, D with A before C and B before D

M1

Orders: ABCDE, ABDCE, ACBDE

A1

BACDE, BADCE, BDACE (6 orders)

A1

Answer: ABCDE, ABDCE, ACBDE, BACDE, BADCE, BDACE

4 marks

17 (4 marks)

(a) A, B, C

B1

(b) J and K

B1

(c) K needs H and I; H needs B and F; I needs D and E; D needs A; E needs B; F needs C

M1

So A, B, C, D, E, F, H, I must be complete

A1

Answer: A, B, C; J, K; A, B, C, D, E, F, H, I

4 marks

18 (3 marks)

X: B

B1

D: B, C, X

M1

B is not needed in D's list any more, as X already needs B: D: C, X

A1

Answer: X: B; D: C, X

3 marks

19 (4 marks)

C needs A and B, but D needs only B

M1

A dummy is needed from the end of B to the start of C

A1

Without it, D would depend on A as well as B

M1

so D could not start until A was complete, which changes the project

A1

Answer: Dummy from end of B to start of C

4 marks

20 (4 marks)

C needs only A but D needs both A and B

M1

so C and D cannot start from the same event

E1

A must end at C's start event, and a dummy from there to the end of B (start of D)

M1

One dummy is needed

A1

Answer: One dummy

4 marks

21 (5 marks)

A, B: none; C: A; D: A; E: B; F: B

B1

G: C

B1

H: C, D, E (using the dummy from C's end event to the end of D, E)

M1 A1

I: C, D, E, F; J: G, H; K: I. Before J: G, H, C, D, E, A, B: 7 activities

A1

Answer: A: none; B: none; C: A; D: A; E: B; F: B; G: C; H: C, D, E; I: C, D, E, F; J: G, H; K: I; 7 activities before J**22 (6 marks)**

The dummy joins the end of B to the start of C, so C needs both A and B (the student wrote C: A)

M1 A1

E starts at the end of C only, so E: C (the student included D)

M1 A1

Correct table: A: none; B: none; C: A, B; D: B; E: C; F: D

A1

The student ignored the dummy arc when reading the dependencies

E1

Answer: C: A, B and E: C

6 marks

23 (5 marks)

The conditions are: A before C, B before C, C before E, B before D, D before F

M1

Case 1, A before B: then A and B must be first and second (every other activity needs B)

M1

The other four, C, D, E, F, have C before E and D before F: $\frac{4!}{2 \times 2} = 6$ orders

A1

Case 2, B before A: B is first (nothing else can come before B). The other five have A, C, E in that order and D, F in that order: choose the places of D, F in 5 positions: $C(5,2) = 10$

M1

Total = 6 + 10 = 16

A1

Answer: 16

5 marks

24 (6 marks)

Earliest finish times: A 3, B 5, C = 3 + 4 = 7

M1

D starts at max(3, 5) = 5, finishing at 11

M1 A1

E = 7 + 2 = 9

A1

F starts at max(11, 9) = 11, finishing at 16

M1

The shortest time is 16 days, with B, D, F the longest path (5 + 6 + 5)

A1

Answer: 16 days; B, D, F

6 marks

25 (4 marks)

No activity needs F or G, so both are finishing activities E1
 F needs D and E; D needs B; E needs B and C; B and C need A M1
 Activities needed: A, B, C, D, E A1
 That is 5 activities A1
Answer: 5 4 marks

26 (4 marks)

X: A, B B1
 F: D, E, X M1
 E needs B and C, C needs A: so X's requirements A, B are already needed before E M1
 But X is not needed by E, so F must keep X: no simplification A1
Answer: X: A, B; F: D, E, X 4 marks

27 (7 marks)

(a) E needs both A and B, but C needs only A and D needs only B M1
 If A and B both ended at the start of E, then C (starting from that event) would also need B, and D would also need A M1
 So A and B cannot both end at E's starting event, and a dummy is needed E1
Answer: Dummies are needed 3 marks
(b) A must end at an event where only A has arrived (the start of C), and B at an event where only B has arrived (the start of D) M1
 Neither event can be the start of E, since E must be reached from both A and B M1
 So E starts at a new event, reached by a dummy from the end of A and a dummy from the end of B: 2 dummies A1
 C, D and E all end at the event where F starts, so no more dummies are needed A1
Answer: 2 dummies: from the end of A and from the end of B to the start of E 4 marks

28 (7 marks)

Earliest finish: A 3, B 5, C 4 B1
 $D = 3 + 6 = 9$; $E = 5 + 2 = 7$; $F = 4 + 7 = 11$ M1 A1
 G, I start at $\max(D, E) = 9$: $G = 13$; $I = 12$; H starts at $\max(B, F) = 11$: $H = 16$ M1 A1
 $J = 13 + 6 = 19$; K starts at $\max(H, I) = 16$: $K = 18$ M1
 Time 19 days; the longest path is A, D, G, J A1
Answer: 19 days; A, D, G, J 7 marks

29 (6 marks)

C needs B, which needs A: so C needs B and A M1
 F needs E, C; E needs D, B; D needs C: all of A, B, C, D, E M1 A1
 E: D, B: D needs C which needs B, so B is redundant in E's list M1 A1
 F: E, C: E needs D which needs C, so C is redundant in F's list A1
Answer: B in E's list; C in F's list 6 marks

30 (5 marks)

C needs B, and B needs A, so A is redundant in C's list; the same applies to every earlier activity M1
 Reduced table: A: none; B: A; C: B; D: C; E: D; F: E A1
 Each activity needs the previous one, so only the order A, B, C, D, E, F is possible: 1 order M1 A1
 For n activities the activities again form a chain, so there is exactly 1 order B1
Answer: 1 order (a chain); also 1 for n activities 5 marks

Earliest and Latest Event Times

1 (2 marks)

(a) The earliest time at which all the activities leading into the event can be complete, so that activities leaving it can start B1

(b) The latest time at which the event can occur without delaying the completion of the project B1

Answer: Earliest time activities from the event can start; latest time without delaying the project 2 marks

2 (2 marks)

The earliest event time is 11 B1

All three activities must be complete before activities leaving the event can start, so we take the largest value E1

Answer: 11 2 marks

3 (3 marks)

Via the first: $20 - 6 = 14$ M1

Via the second: $24 - 3 = 21$ M1

The latest time is the smaller value, 14 A1

Answer: 14 3 marks

4 (2 marks)

Event 2: $0 + 4 = 4$ B1

Event 3: $0 + 5 = 5$ B1

Answer: 4 and 5 2 marks

5 (2 marks)

The minimum completion time is 27 days B1

The latest time at the finishing event is also 27 days B1

Answer: 27 days; 27 days 2 marks

6 (2 marks)

Event 4 B1

Its earliest and latest times are equal, so there is no slack at that event; event 5 has slack of 4 E1

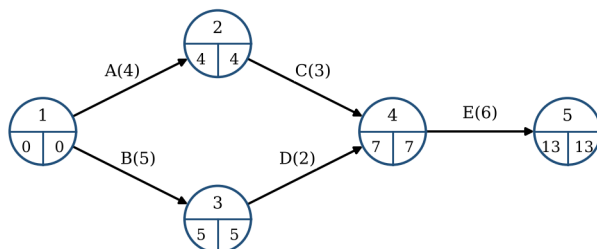
Answer: Event 4 2 marks

7 (4 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 5 = 5$; Event 4: $\max(4 + 3, 5 + 2) = \max(7, 7) = 7$; Event 5: $7 + 6 = 13$ M1 A1

Backward pass: Event 5: late time = 13 (the minimum project time); Event 4: $13 - 6 = 7$; Event 3: $7 - 2 = 5$; Event 2: $7 - 3 = 4$; Event 1: $\min(4 - 4, 5 - 5) = \min(0, 0) = 0$ A1 A1

Answer: Earliest times 1: 0, 2: 4, 3: 5, 4: 7, 5: 13; latest times 1: 0, 2: 4, 3: 5, 4: 7, 5: 13; minimum time 13 days 4 marks



8 (3 marks)

Event 1: early time 0

M1

Event 2: $0 + 3 = 3$ Event 3: $0 + 4 = 4$

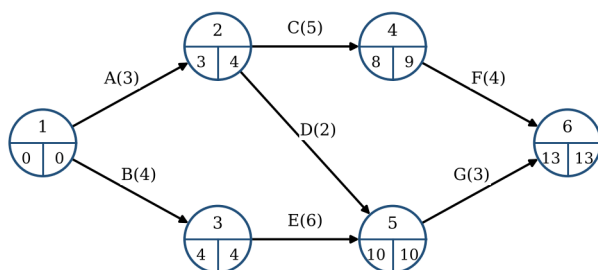
A1

Event 4: $3 + 5 = 8$ Event 5: $\max(3 + 2, 4 + 6) = \max(5, 10) = 10$ Event 6: $\max(8 + 4, 10 + 3) = \max(12, 13) = 13$

A1

Answer: Earliest times 1: 0, 2: 3, 3: 4, 4: 8, 5: 10, 6: 13; minimum time 13 hours

3 marks

**9 (3 marks)**

Event 6: late time = 13 (the minimum project time)

M1

Event 5: $13 - 3 = 10$ Event 4: $13 - 4 = 9$

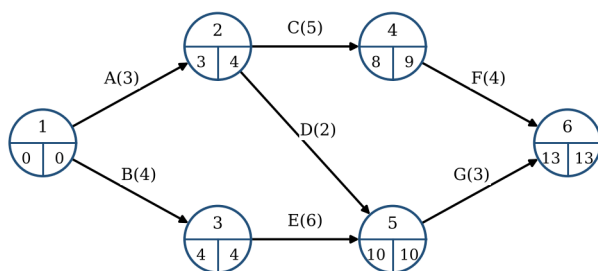
A1

Event 3: $10 - 6 = 4$ Event 2: $\min(9 - 5, 10 - 2) = \min(4, 8) = 4$ Event 1: $\min(4 - 3, 4 - 4) = \min(1, 0) = 0$

A1

Answer: 0, 4, 4, 9, 10, 13

3 marks

**10 (4 marks)**

Event 1: early time 0

M1

Event 3: $0 + 3 = 3$ Event 2: $\max(0 + 4, 3 + 0) = \max(4, 3) = 4$

A1

Event 5: $3 + 6 = 9$

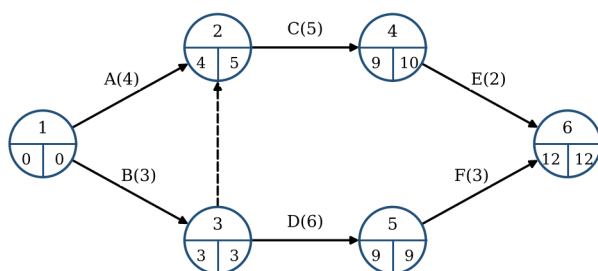
A1

Event 4: $4 + 5 = 9$ Event 6: $\max(9 + 2, 9 + 3) = \max(11, 12) = 12$

A1

Answer: Earliest times 1: 0, 2: 4, 3: 3, 4: 9, 5: 9, 6: 12; minimum time 12

4 marks



11 (3 marks)Via the first: $6 + 5 = 11$

M1

Via the dummy: $11 + 0 = 11$, so the earliest event time is 11

A1

The dummy shows that activities leaving the event also depend on the earlier event

E1

Answer: 11

3 marks

12 (3 marks)First: $19 - 6 = 13$

M1

Second: $19 - 4 = 15$

M1

The latest start event times are 13 and 15 respectively

A1

Answer: 13 and 15

3 marks

13 (4 marks)

Event 6: late time = 12 (the minimum project time)

M1

Event 4: $12 - 2 = 10$ Event 5: $12 - 3 = 9$

A1

Event 2: $10 - 5 = 5$ Event 3: $\min(5 - 0, 9 - 6) = \min(5, 3) = 3$ Event 1: $\min(5 - 4, 3 - 3) = \min(1, 0) = 0$

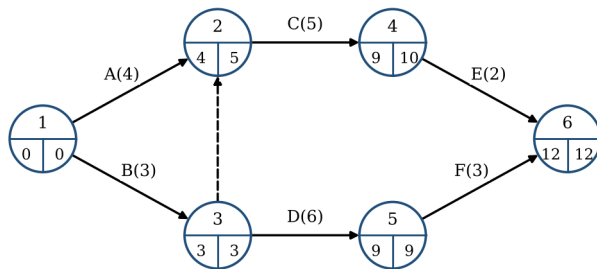
A1

Critical events (early = late): 1, 3, 5, 6

B1

Answer: 0, 5, 3, 10, 9, 12

4 marks

**14 (6 marks)**Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 6 = 6$; Event 4: $0 + 5 = 5$;

M1 A1

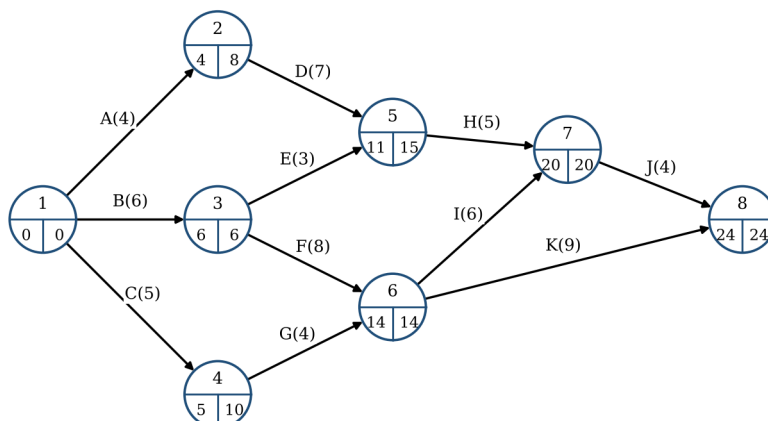
Event 5: $\max(4 + 7, 6 + 3) = \max(11, 9) = 11$; Event 6: $\max(6 + 8, 5 + 4) = \max(14, 9) = 14$;Event 7: $\max(11 + 5, 14 + 6) = \max(16, 20) = 20$; Event 8: $\max(20 + 4, 14 + 9) = \max(24, 23) = 24$ Backward pass: Event 8: late time = 24 (the minimum project time); Event 7: $24 - 4 = 20$; Event

A1 A1

6: $\min(20 - 6, 24 - 9) = \min(14, 15) = 14$; Event 5: $20 - 5 = 15$; Event 4: $14 - 4 = 10$; Event3: $\min(15 - 3, 14 - 8) = \min(12, 6) = 6$; Event 2: $15 - 7 = 8$; Event 1: $\min(8 - 4, 6 - 6, 10 - 5) = \min(4, 0, 5) = 0$

Minimum project time: 24; critical events: 1, 3, 6, 7, 8

A1 A1

Answer: Earliest times 1: 0, 2: 4, 3: 6, 4: 5, 5: 11, 6: 14, 7: 20, 8: 24; latest times 1: 0, 2: 8, 3: 6, 4: 10, 5: 15, 6: 14, 7: 20, 8: 24

15 (6 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 3 = 3$; Event 3: $0 + 5 = 5$; Event 4: $3 + 4 = 7$; Event 5: $\max(5 + 6, 3 + 0) = \max(11, 3) = 11$; Event 6: $\max(7 + 4, 11 + 5) = \max(11, 16) = 16$; Event 7: $11 + 2 = 13$; Event 8: $\max(16 + 7, 13 + 6) = \max(23, 19) = 23$

M1 A1

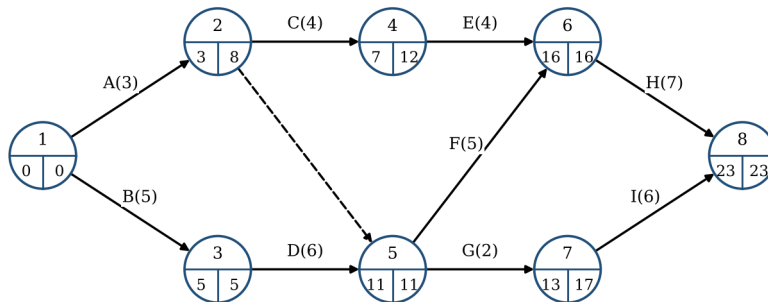
Backward pass: Event 8: late time = 23 (the minimum project time); Event 7: $23 - 6 = 17$; Event 6: $23 - 7 = 16$; Event 5: $\min(16 - 5, 17 - 2) = \min(11, 15) = 11$; Event 4: $16 - 4 = 12$; Event 3: $11 - 6 = 5$; Event 2: $\min(12 - 4, 11 - 0) = \min(8, 11) = 8$; Event 1: $\min(8 - 3, 5 - 5) = \min(5, 0) = 0$

A1 A1

Minimum project time: 23; critical events: 1, 3, 5, 6, 8

A1 A1

Answer: Earliest times 1: 0, 2: 3, 3: 5, 4: 7, 5: 11, 6: 16, 7: 13, 8: 23; latest times 1: 0, 2: 8, 3: 5, 4: 12, 5: 11, 6: 16, 7: 17, 8: 23



16 (3 marks)

Critical events have equal early and late times: 1, 3, 6, 7, 8

M1 A1

A difference of 4 shows the event could be delayed by up to 4 without delaying the project

E1

Answer: 1, 3, 6, 7, 8

3 marks

17 (4 marks)

The event before E has early time $\max(4 + 3, x + 2)$

M1

This equals 11, and $4 + 3 = 7 \neq 11$, so $x + 2 = 11$

M1 A1

$x = 9$

A1

Answer: $x = 9$

4 marks

18 (7 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 3 = 3$; Event 3: $0 + 5 = 5$; Event 4: $0 + 4 = 4$; Event 5: $\max(3 + 6, 5 + 2) = \max(9, 7) = 9$; Event 6: $\max(5 + 0, 4 + 7) = \max(5, 11) = 11$; Event 7: $9 + 4 = 13$; Event 8: $\max(11 + 5, 9 + 3) = \max(16, 12) = 16$; Event 9: $\max(13 + 6, 16 + 2) = \max(19, 18) = 19$

M1 A1 A1

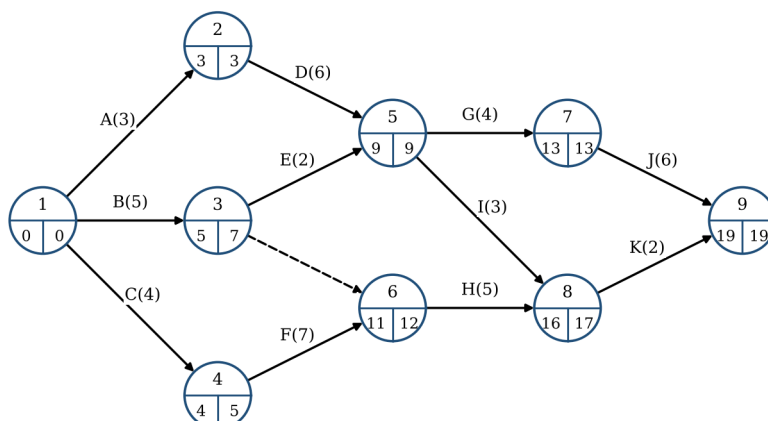
Backward pass: Event 9: late time = 19 (the minimum project time); Event 8: $19 - 2 = 17$; Event 7: $19 - 6 = 13$; Event 6: $17 - 5 = 12$; Event 5: $\min(13 - 4, 17 - 3) = \min(9, 14) = 9$; Event 4: $12 - 7 = 5$; Event 3: $\min(9 - 2, 12 - 0) = \min(7, 12) = 7$; Event 2: $9 - 6 = 3$; Event 1: $\min(3 - 3, 7 - 5, 5 - 4) = \min(0, 2, 1) = 0$

A1 A1

Minimum project time: 19; critical events: 1, 2, 5, 7, 9

A1 A1

Answer: Earliest times 1: 0, 2: 3, 3: 5, 4: 4, 5: 9, 6: 11, 7: 13, 8: 16, 9: 19; latest times 1: 0, 2: 3, 3: 7, 4: 5, 5: 9, 6: 12, 7: 13, 8: 17, 9: 19



19 (4 marks)

Backward pass from 16: event 5: 16; event 4: $16 - 6 = 10$

M1

Event 2: $10 - 3 = 7$; event 3: $10 - 2 = 8$

A1

Event 1: $\min(7 - 4, 8 - 5) = 3$

A1

Every latest time is 3 more than before, equal to the extra time allowed; no event is now critical

E1

Answer: 5: 16; 4: 10; 2: 7; 3: 8; 1: 3

4 marks

20 (4 marks)

The latest time is found using the project's finish time and the earliest time is found using the longest path from the start

M1

The earliest time at an event is the length of the longest path to it, and the finish time is at least that plus the longest path from it

M1 E1

A smaller latest time would mean that the target finish time was less than the minimum possible, so the project could not be finished on time

A1

Answer: Early $\leq$ late; otherwise the deadline is impossible

4 marks

21 (7 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 5 = 5$; Event 4: $4 + 3 = 7$; Event 5: $\max(4 + 6, 5 + 2, 7 + 0) = \max(10, 7, 7) = 10$; Event 6: $\max(5 + 4, 10 + 0) = \max(9, 10) = 10$; Event 7: $\max(7 + 5, 10 + 3) = \max(12, 13) = 13$; Event 8: $10 + 6 = 16$; Event 9: $\max(13 + 4, 16 + 5) = \max(17, 21) = 21$

M1 A1 A1

Backward pass: Event 9: late time = 21 (the minimum project time); Event 8: $21 - 5 = 16$; Event 7: $21 - 4 = 17$; Event 6: $16 - 6 = 10$; Event 5: $\min(10 - 0, 17 - 3) = \min(10, 14) = 10$; Event 4: $\min(10 - 0, 17 - 5) = \min(10, 12) = 10$; Event 3: $\min(10 - 2, 10 - 4) = \min(8, 6) = 6$; Event 2: $\min(10 - 3, 10 - 6) = \min(7, 4) = 4$; Event 1: $\min(4 - 4, 6 - 5) = \min(0, 1) = 0$

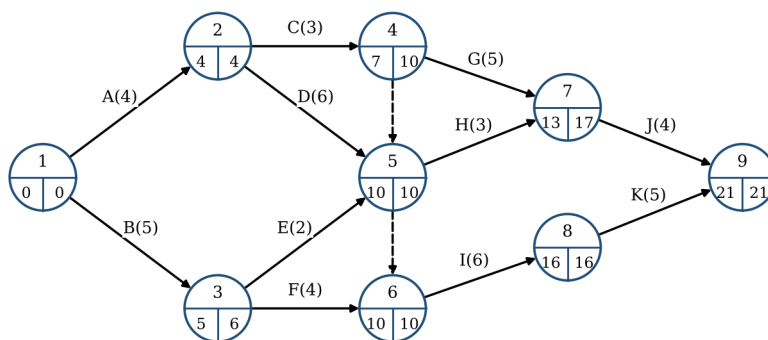
A1 A1

Minimum project time: 21; critical events: 1, 2, 5, 6, 8, 9

A1 A1

Answer: Earliest times 1: 0, 2: 4, 3: 5, 4: 7, 5: 10, 6: 10, 7: 13, 8: 16, 9: 21; latest times 1: 0, 2: 4, 3: 6, 4: 10, 5: 10, 6: 10, 7: 17, 8: 16, 9: 21

4 marks

**22 (8 marks)**

Forward pass: Event 1: early time 0; Event 2: $0 + 5 = 5$; Event 3: $0 + 4 = 4$; Event 4: $0 + 6 = 6$; Event 5: $\max(5 + 3, 4 + 7) = \max(8, 11) = 11$; Event 6: $\max(4 + 0, 6 + 2) = \max(4, 8) = 8$; Event 7: $11 + 4 = 15$; Event 8: $\max(8 + 5, 11 + 6) = \max(13, 17) = 17$; Event 9: $\max(15 + 3, 17 + 5) = \max(18, 22) = 22$

M1 A1 A1

Backward pass: Event 9: late time = 22 (the minimum project time); Event 8: $22 - 5 = 17$; Event 7: $22 - 3 = 19$; Event 6: $17 - 5 = 12$; Event 5: $\min(19 - 4, 17 - 6) = \min(15, 11) = 11$; Event 4: $12 - 2 = 10$; Event 3: $\min(11 - 7, 12 - 0) = \min(4, 12) = 4$; Event 2: $11 - 3 = 8$; Event 1: $\min(8 - 5, 4 - 4, 10 - 6) = \min(3, 0, 4) = 0$

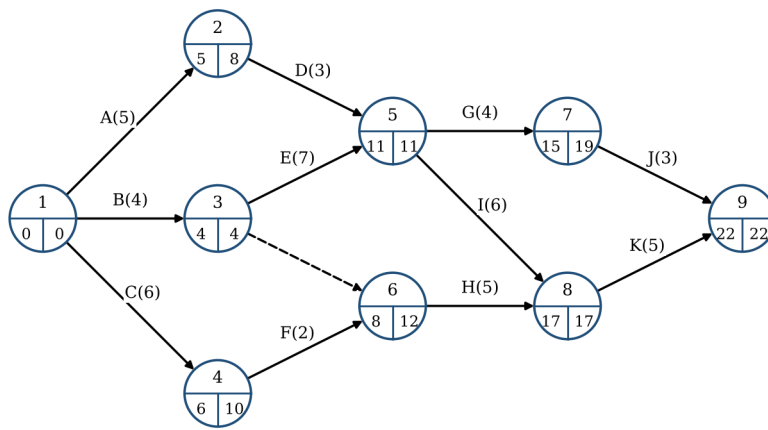
A1 A1 A1

Earliest start = early time at the start event; latest finish = late time at the end event: A: 0, 8; B: 0, 4; C: 0, 10; D: 5, 11; E: 4, 11; F: 6, 12; G: 11, 19; H: 8, 17; I: 11, 17; J: 15, 22; K: 17, 22

A1 A1

Answer: A: (0, 8); B: (0, 4); C: (0, 10); D: (5, 11); E: (4, 11); F: (6, 12); G: (11, 19); H: (8, 17); I: (11, 17); J: (15, 22); K: (17, 22)

4 marks

**23 (7 marks)**

Forward pass: Event 1: early time 0; Event 2: $0 + 5 = 5$; Event 3: $0 + 3 = 3$; Event 4: $0 + 6 = 6$;
 Event 5: $\max(5 + 4, 3 + 7) = \max(9, 10) = 10$; Event 6: $\max(3 + 2, 6 + 5) = \max(5, 11) = 11$;
 Event 7: $\max(10 + 4, 11 + 6) = \max(14, 17) = 17$; Event 8: $\max(17 + 3, 11 + 8) = \max(20, 19) = 20$

M1 A1 A1

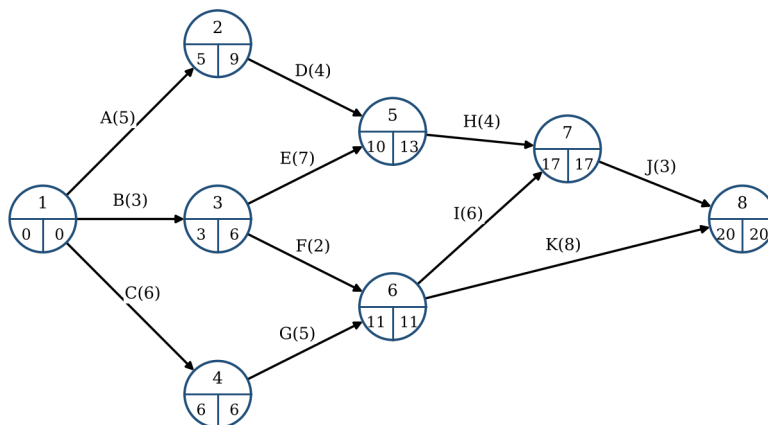
Backward pass: Event 8: late time = 20 (the minimum project time); Event 7: $20 - 3 = 17$; Event 6: $\min(17 - 6, 20 - 8) = \min(11, 12) = 11$; Event 5: $17 - 4 = 13$; Event 4: $11 - 5 = 6$; Event 3: $\min(13 - 7, 11 - 2) = \min(6, 9) = 6$; Event 2: $13 - 4 = 9$; Event 1: $\min(9 - 5, 6 - 3, 6 - 6) = \min(4, 3, 0) = 0$

A1 A1

Minimum project time: 20; critical events: 1, 4, 6, 7, 8

A1 A1

Answer: Earliest times 1: 0, 2: 5, 3: 3, 4: 6, 5: 10, 6: 11, 7: 17, 8: 20; latest times 1: 0, 2: 9, 3: 6, 4: 6, 5: 13, 6: 11, 7: 17,

**24 (6 marks)**

Event 4 has early time $4 + 3 = 7$, and event 5 has early time 10

M1

Event 7 early time = $\max(7 + y, 10 + 3) = \max(7 + y, 13)$

M1 A1

Via J (duration 4) the finishing event has time $\max(7 + y, 13) + 4$; the other route, via I and K, gives $16 + 5 = 21$

M1

The project time stays 21 provided $7 + y + 4 \leq 21$

M1

$y \leq 10$, so for $0 < y \leq 10$

A1

Answer: $0 < y \leq 10$

6 marks

25 (6 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 6 = 6$; Event 3: $0 + 2 = 2$; Event 4: $6 + 5 = 11$;
 Event 5: $\max(2 + 3, 6 + 0) = \max(5, 6) = 6$; Event 6: $\max(11 + 4, 6 + 6) = \max(15, 12) = 15$;
 Event 7: $6 + 3 = 9$; Event 8: $\max(15 + 5, 9 + 4) = \max(20, 13) = 20$

M1 A1

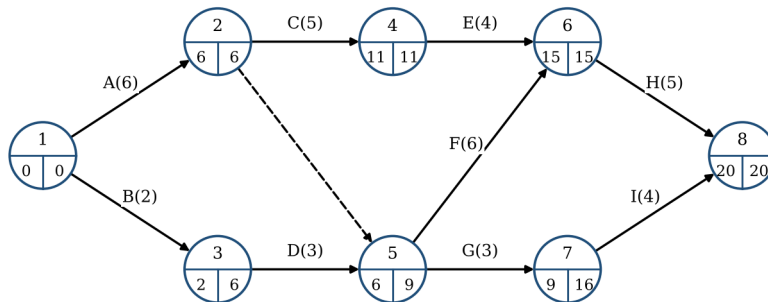
Backward pass: Event 8: late time = 20 (the minimum project time); Event 7: $20 - 4 = 16$; Event 6: $20 - 5 = 15$; Event 5: $\min(15 - 6, 16 - 3) = \min(9, 13) = 9$; Event 4: $15 - 4 = 11$; Event 3: $9 - 3 = 6$; Event 2: $\min(11 - 5, 9 - 0) = \min(6, 9) = 6$; Event 1: $\min(6 - 6, 6 - 2) = \min(0, 4) = 0$

A1 A1

Minimum project time: 20; critical events: 1, 2, 4, 6, 8

A1 A1

Answer: Earliest times 1: 0, 2: 6, 3: 2, 4: 11, 5: 6, 6: 15, 7: 9, 8: 20; latest times 1: 0, 2: 6, 3: 6, 4: 11, 5: 9, 6: 15, 7: 16, 8: 20

**26 (5 marks)**

Path A, C, E = $4 + 5 + 2 = 11$; path B, C, E (via the dummy) = $3 + 5 + 2 = 10$

M1

Path B, D, F = $3 + z + 3 = 6 + z$

M1

$z = 9$: the longest is 15, so the project takes 15 days

A1

The time is 11 days when $6 + z \leq 11$, so $z \leq 5$

M1

So for $0 < z \leq 5$ the time is 11 days

A1

Answer: 15 days; $0 < z \leq 5$

5 marks

27 (8 marks)

(a) A, C, E has length $4 + x + 6$; B, D, E has length $5 + 3 + 6 = 14$

M1

Equal when $10 + x = 14$, so $x = 4$

A1

Answer: $x = 4$

2 marks

(b) Event 4 early time = $\max(4 + x, 8)$

M1

Project time = $\max(4 + x, 8) + 6$

M1

= 14 for $0 < x \leq 4$, and $10 + x$ for $x > 4$

A1

Answer: 14 for $x \leq 4$; $10 + x$ for $x > 4$

3 marks

(c) Minimum time 14; event 4 late time = $14 - 6 = 8$

M1

Event 2 late time = $8 - x = 6$, early time 4: C can start up to 2 days late

M1 A1

Answer: 6; C has 2 days of slack

3 marks

28 (7 marks)

Paths: A, C, F = $3 + 5 + 4 = 12$; A, D, G = $3 + 2 + 3 = 8$; B, E, G = $4 + x + 3 = 7 + x$

M1 A1

The minimum project time is the longest path: $\max(12, 8, 7 + x)$

M1

$7 + x > 12$ when $x > 5$

M1 A1

So the time is 12 days for $0 < x \leq 5$ and $7 + x$ days for $x > 5$

A1

Check: $x = 2$ gives 12; $x = 6$ gives 13

B1

Answer: 12 for $0 < x \leq 5$; $7 + x$ for $x > 5$

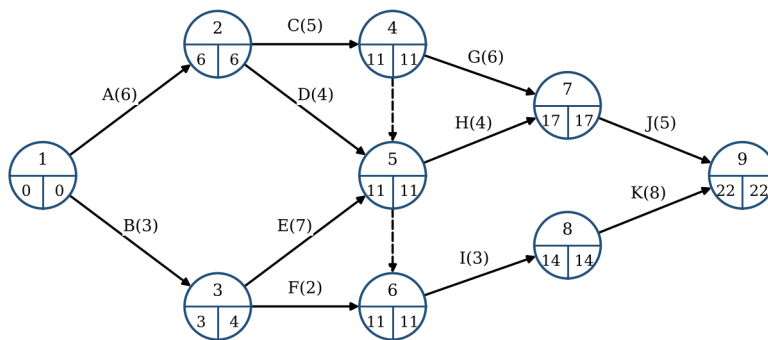
7 marks

29 (10 marks)**(a)** Event 1: early time 0**M1**Event 2: $0 + 6 = 6$ Event 3: $0 + 3 = 3$ Event 4: $6 + 5 = 11$ **A1**Event 5: $\max(6 + 4, 3 + 7, 11 + 0) = \max(10, 10, 11) = 11$ Event 6: $\max(3 + 2, 11 + 0) = \max(5, 11) = 11$ **A1**Event 7: $\max(11 + 6, 11 + 4) = \max(17, 15) = 17$ Event 8: $11 + 3 = 14$ Event 9: $\max(17 + 5, 14 + 8) = \max(22, 22) = 22$ **A1****Answer: 1: 0, 2: 6, 3: 3, 4: 11, 5: 11, 6: 11, 7: 17, 8: 14, 9: 22****4 marks****(b)** The backward pass starts from the deadline 26 at event 9, and every latest time is 4 more than in the 22-day version**M1**Using the 22-day backward pass: Event 8: $22 - 8 = 14$; Event 7: $22 - 5 = 17$; Event 6: $14 - 3 = 11$; Event 5: $\min(11 - 0, 17 - 4) = \min(11, 13) = 11$ **M1**

Adding 4 to each: event 1: 4; event 2: 10; event 3: 8; event 4: 15; event 5: 15; event 6: 15; event 7: 21; event 8: 18; event 9: 26

A1 A1**Answer: 1: 4, 2: 10, 3: 8, 4: 15, 5: 15, 6: 15, 7: 21, 8: 18, 9: 26****4 marks****(c)** Early and late times differ by 4 at events on the old critical path, and by more elsewhere**M1**

So every event has slack of at least 4 days

E1**Answer: At least 4 days****2 marks****30 (7 marks)****(a)** Let P be a longest path from the start to v , of length $e(v)$, and Q any path from v to the finish, of length q **M1**Then $e(v) + q \leq T$ (since this is a path from start to finish)**M1** $l(v) = T - (\text{longest path from } v \text{ to finish}) \geq e(v)$ **E1****Answer: $e(v) \leq l(v)$** **3 marks****(b)** $l(v) = T - q^*$ where q^* is the longest path from v to the finish**M1** $e(v) = l(v)$ gives $e(v) + q^* = T$ **M1**The longest path from the start to v , followed by the longest path from v to the finish, has length**M1** $e(v) + q^* = T$ So v lies on a path of length T , which is a critical path**E1****Answer: $e(v) + q^* = T$** **4 marks****Critical Paths****1 (2 marks)****(a)** The longest path through the activity network, whose length is the minimum time for the project**B1****(b)** An activity on a critical path: any delay to it delays the whole project**B1****Answer: Longest path; delay delays the project****2 marks**

2 (2 marks)

The minimum time is the length of the longest path: 19 days

B1

The path of length 19 is the critical path

B1

Answer: 19 days; the path of length 19

2 marks

3 (1 mark)

The project time increases by 3 days

B1

Answer: 3 days longer

1 mark

4 (2 marks)

The longer path is B, D, E

B1

Minimum completion time 12 days

B1

Answer: B, D, E; 12 days

2 marks

5 (2 marks)

Yes

B1

Two (or more) different paths can both have the greatest length, so both are critical

E1

Answer: Yes, if paths have equal greatest length

2 marks

6 (2 marks)

A critical activity has no spare time: earliest finish = $8 + 5 = 13$

M1

Latest finish = 13

A1

Answer: 13

2 marks

7 (4 marks)

Paths: B – E – G: 13; A – C – F: 12; A – D – G: 8

M1 A1

The longest is B, E, G of length 13

A1

Critical path B, E, G; minimum time 13 days

A1

Answer: B, E, G; 13 days

4 marks

8 (4 marks)

Paths: B – C – E: 13; B – D – F: 12; A – C – E: 11

M1 A1

The longest path is B – C – E with length 13

A1

Minimum time 13 days

A1

Answer: B – C – E; 13 days

4 marks

9 (3 marks)

Paths A, C, E: $4 + 3 + 6 = 13$; B, D, E: $5 + 2 + 6 = 13$

M1 A1

Both are critical; the critical activities are A, B, C, D, E

A1

Answer: A-C-E and B-D-E; all five activities

3 marks

10 (3 marks)

An activity is critical if latest finish – earliest start = duration

M1

A: $5 - 0 = 5$ (= 5), D: $11 - 5 = 6$ (= 6), G: $16 - 11 = 5$ (= 5)

A1

Critical activities: A, D, G

A1

Answer: A, D, G

3 marks

11 (3 marks)

An activity is critical if its start event and end event are both critical (early = late)

M1

and the activity's duration equals the difference between the event times

M1

Critical activities: A, D, G; time 16 days

A1

Answer: A, D, G; 16 days

3 marks

12 (3 marks)

Paths: A, C, E = 10; B, D, E = 12; critical path B, D, E

M1

D delayed by 2: B, D, E = 14

M1

New minimum time 14 days

A1

Answer: B-D-E; 14 days

3 marks

13 (4 marks)

Path B, D, F becomes $3 + 4 + 3 = 10$

M1

But path A, C, E = $4 + 5 + 2 = 11$, so the minimum time is 11 days

A1

A different path becomes the longest, so the saving is only 1 day

E1

New critical path: A, C, E

B1

Answer: 11 days; A, C, E becomes critical

4 marks

14 (6 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 6 = 6$; Event 4: $0 + 5 = 5$;

M1 A1

Event 5: $\max(4 + 7, 6 + 3) = \max(11, 9) = 11$; Event 6: $\max(6 + 8, 5 + 4) = \max(14, 9) = 14$;

Event 7: $\max(11 + 5, 14 + 6) = \max(16, 20) = 20$; Event 8: $\max(20 + 4, 14 + 9) = \max(24, 23) = 24$

Backward pass: Event 8: late time = 24 (the minimum project time); Event 7: $24 - 4 = 20$; Event

M1 A1

6: $\min(20 - 6, 24 - 9) = \min(14, 15) = 14$; Event 5: $20 - 5 = 15$; Event 4: $14 - 4 = 10$; Event

3: $\min(15 - 3, 14 - 8) = \min(12, 6) = 6$; Event 2: $15 - 7 = 8$; Event 1: $\min(8 - 4, 6 - 6, 10 - 5) = \min(4, 0, 5) = 0$

Critical activities: early = late at both ends and duration = difference: B, F, I, J

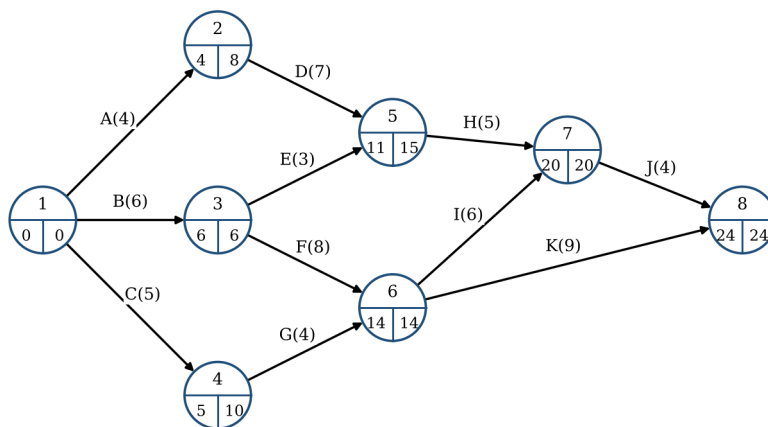
M1

Minimum time 24 days, critical path B – F – I – J

A1

Answer: B – F – I – J; 24 days

6 marks

**15 (5 marks)**

Paths with lengths: B – D – F – H: 23; B – D – G – I: 19; A – C – E – H: 18; A – F – H: 15; A – G – I: 11

M1 A1 A1

Critical path B – D – F – H

A1

Minimum time 23 days

A1

Answer: B – D – F – H; 23 days

5 marks

16 (5 marks)

Original minimum time 19 days, critical path A, D, G, J

B1

E increased from 2 to 8 days

M1

Path B, E, G, J = $5 + 8 + 4 + 6 = 23$ (was 17)

M1 A1

The new minimum time is 23 days, so the project is delayed by 4 days; B, E, G, J is now critical

A1

Answer: 23 days (4 days later)

5 marks

17 (5 marks)

All paths through I: A, D, I, K = 21; B, F, I, K = 20; A, C, I, K = 18; B, E, I, K = 18

M1

The longest path not containing I is A, D, H, J = 17

M1 A1

Shortening I by r days gives a time of $21 - r$ while this is at least 17, so up to 4 days

M1

Then the minimum project time is 17 days

A1

Answer: 4 days; 17 days

5 marks

18 (3 marks)

Paths: A, C, E = $7 + 1 + 2 = 10$; B, D, E = $3 + 6 + 2 = 11$

M1 A1

The critical path is B, D, E, so the longest activity A (7 days) is not critical (its path is 1 day shorter)

E1

Answer: A is the longest activity (7 days) but is not critical: A, C, E = 10 < B, D, E = 11

3 marks

19 (3 marks)

Both A, C, E and B, D, E have length 13

B1

Shortening C reduces only the path A, C, E

M1

B, D, E remains 13 days, so the time does not fall: shortening C alone is not enough

E1

Answer: No: B, D, E still takes 13 days

3 marks

20 (4 marks)

Critical path B, E, G has length 13; the next longest path is A, C, F with length 12

M1 A1

Whatever is shortened, the path A, C, F or B, E, G is not both reduced by 2: if the critical path alone is shortened by 2, then A, C, F = 12 remains

M1

So the project time cannot fall below 12 days, a saving of at most 1 day

A1

Answer: Saving at most 1 day

4 marks

21 (7 marks)

Earliest starts: A, B, C: 0

B1

D (after A): 3; E (after B): 5; F (after C): 4

M1

G, I (after D, E): $\max(9, 7) = 9$; H (after B, F): $\max(5, 11) = 11$

M1 A1

J (after G): $9 + 4 = 13$; K (after H, I): $\max(11 + 5, 9 + 3) = 16$

M1

Earliest finish: J = $13 + 6 = 19$; K = $16 + 2 = 18$; minimum time 19 days

A1

Critical activities: A, D, G, J

A1

Answer: 19 days; A, D, G, J

7 marks

22 (6 marks)

Forward and backward passes give minimum time 22 days

M1 A1

Critical events: 1, 2, 4, 5, 6, 7, 8, 9

M1

Critical paths: A – C – I – K; A – C – G – J

A1 A1

Activities on every critical path: A and C

A1

Answer: A – C – I – K; A – C – G – J

6 marks

23 (6 marks)

Critical paths A, C, E and B, D, E, each 13 days: both must be reduced by 1

M1

Option 1: shorten E by 1: cost £90

A1

Option 2: shorten one of A, C and one of B, D: cheapest A (£30) and D (£35): £65

M1 A1

Other pairs cost more (A, B £70; C, B £90; C, D £85)

M1

Cheapest: shorten A and D by 1 day each, cost £65

A1

Answer: A and D by 1 day each; £65

6 marks

24 (5 marks)

Paths: B, C, E: $5 + 2 + 6 = 13$ (via the dummy); B, D, F: $5 + 4 + 3 = 12$; A, C, E: $3 + 2 + 6 = 11$

M1 A1

The longest is B, C, E at 13 days, so it is the critical path

A1

C needs both A and B: the dummy makes C wait for B (which takes longer than A)

M1

So the dummy is part of the longest chain of dependencies and any delay to B passes through it to C

E1

Answer: B, C, E; 13 days

5 marks

25 (6 marks)

Paths: C – G – I – J: 20; C – G – K: 19; B – E – H – J: 17

M1 A1

Critical path C – G – I – J, 20 days

A1

Path C, G, K has length 19 with K = 8, so it has 1 day to spare

M1

K can have duration 9 days (C, G, K = 20)

A1

If K = 10, C, G, K = 21 days and becomes the critical path

A1

Answer: C – G – I – J; K up to 9 days

6 marks

26 (5 marks)

Paths: A, C, F = $3 + 5 + 4 = 12$; B, E, G = $4 + 6 + 3 = 13$; A, D, G = $3 + y + 3 = 6 + y$

M1 A1

$y = 10$: A, D, G = $16 > 13$, so the minimum time is 16 days

A1

D is not critical while $6 + y < 13$, that is $y < 7$

M1

So D is not on any critical path for $0 < y < 7$ (when $y = 7$ it is on a second critical path)

A1

Answer: 16 days; $0 < y < 7$

5 marks

27 (12 marks)

(a) Critical path A, D, G, J (19); next longest C, F, H, K = 18

M1

Cheapest activity on the critical path is J at £30 per day

A1

Shorten J by 1 day: cost £30

A1

Answer: J by 1 day; £30

3 marks

(b) Shorten J by 2 days (£60): critical path A, D, G, J = 17; but C, F, H, K = 18 is now the longest

M1

Also shorten C by 1 day (£20) so C, F, H, K = 17 (cheapest on that path, £20)

M1 A1

Total £60 + £20 = £80

A1

Answer: J by 2 days and C by 1 day; £80

4 marks

(c) Paths of length 19, 18 and 17 (A, D, G, J; C, F, H, K; B, E, G, J) all need attention

M1

B, E, G, J = 17: shorten G or J; J by 2 saves on both A, D, G, J and B, E, G, J

M1

Plan: J by 2 days (£60), D by 1 day (£45), C by 1 day (£20), K by 1 day (£25): total £150

M1 A1

A check shows no cheaper combination reduces all three paths to 16

A1

Answer: £150: J 2 days, D 1 day, C 1 day, K 1 day

5 marks

28 (7 marks)

(a) Paths through F: B, F, I, J = $6 + x + 6 + 4 = 16 + x$; B, F, K = $6 + x + 9 = 15 + x$

M1 A1

Paths not through F: the longest is A, D, H, J = $4 + 7 + 5 + 4 = 20$

M1

Project time = $\max(20, 16 + x)$

M1

= 20 for $0 < x \leq 4$, and $16 + x$ for $x > 4$

A1

Answer: 20 for $x \leq 4$; $16 + x$ for $x > 4$

5 marks

(b) F is on a longest path only when $16 + x \geq 20$, so F is not critical for $x < 4$

M1

$x = 9$: time = $16 + 9 = 25$ days

A1

Answer: $x < 4$ (F critical for $x \geq 4$); 25 days

2 marks

29 (6 marks)

(a) B, E, H, J = $6 + x + 5 + 4 = 15 + x$

M1

For $x \leq 9$ this is at most 24, and the critical path B, F, I, J = $6 + 8 + 6 + 4 = 24$ is unaffected

M1

So the project time is 24 days for $x \leq 9$

A1

Answer: 24 days for $x \leq 9$

3 marks

(b) $15 + 12 = 27$ days on B, E, H, J, which exceeds 24

M1 A1

Other paths do not exceed 24, so the minimum time is 27 days with critical path B, E, H, J

A1

Answer: 27 days; B, E, H, J

3 marks

30 (8 marks)

(a) Shortening E by 2: $2 \times 90 = £180$, giving both paths 11 days

M1

Alternatively, each path must lose 2 days: A, C, E needs 2 days from A (max 2) and C (max 1)

M1

Cheapest on A, C, E: A by 2 days (£60); on B, D, E: B by 1 (£40) and D by 1 (£35) = £75 (B by 2 would be £80)

M1 A1

Total £60 + £75 = £135, less than £180 for E

A1

Answer: A by 2 days, B by 1 day, D by 1 day: £135

5 marks

(b) E for 1 day costs £90 and saves 1 day on both paths

M1

The second day then costs at least £30 (A) + £35 (D) = £65 more: $£155 > £135$

M1 A1

Answer: £155 > £135

3 marks

Float

1 (2 marks)

The maximum time by which an activity can be delayed without delaying the completion of the project

B1

Total float = latest finish – earliest start – duration

B1

Answer: latest finish – earliest start – duration

2 marks

2 (2 marks)

Total float = $16 - 6 - 4$

M1

= 6

A1

Answer: 6

2 marks

3 (2 marks)

0

B1

Any delay to a critical activity delays the whole project, so it has no spare time

E1

Answer: 0

2 marks

4 (2 marks)

(a) No: $2 \leq 3$

B1

(b) Yes: $5 > 3$, so the project is delayed (by 2 days if nothing else changes)

B1

Answer: (a) No; (b) yes, by 2 days

2 marks

5 (2 marks)

Float = $12 - 5 - 7$

M1

= 0, so it is critical

A1

Answer: Float 0

2 marks

6 (1 mark)

Total float = latest start – earliest start = 5

B1

Answer: 5

1 mark

7 (4 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 5 = 5$; Event 3: $0 + 4 = 4$; Event 4: $\max(5 + 3, 4 + 6) = \max(8, 10) = 10$; Event 5: $10 + 2 = 12$

M1

Backward pass: Event 5: late time = 12 (the minimum project time); Event 4: $12 - 2 = 10$; Event 3: $10 - 6 = 4$; Event 2: $10 - 3 = 7$; Event 1: $\min(7 - 5, 4 - 4) = \min(2, 0) = 0$

M1

Float = latest finish – earliest start – duration: A: $7 - 0 - 5 = 2$; B: $4 - 0 - 4 = 0$; C: $10 - 5 - 3 = 2$; D: $10 - 4 - 6 = 0$; E: $12 - 10 - 2 = 0$

M1 A1

Answer: A 2, B 0, C 2, D 0, E 0

4 marks

8 (3 marks)

C: float = $13 - 5 - 4 = 4$

M1 A1

Latest start = latest finish – duration = $13 - 4 = 9$

A1

Answer: float 4; latest start 9

3 marks

9 (4 marks)

Float = latest finish – earliest start – duration

M1

A: $5 - 0 - 5 = 0$; B: $9 - 0 - 3 = 6$; C: $13 - 5 - 4 = 4$; D: $11 - 5 - 6 = 0$; E: $11 - 3 - 2 = 6$;

A1 A1

F: $16 - 9 - 3 = 4$; G: $16 - 11 - 5 = 0$

Critical (float 0): A, D, G

A1

Answer: A 0, B 6, C 4, D 0, E 6, F 4, G 0

4 marks

10 (3 marks)

C has float 2 days

M1

Delay of 1 day is less than the float, so the project time is unchanged

A1

Remaining float = $2 - 1 = 1$ days

A1

Answer: 1 days

3 marks

11 (2 marks)

Float = latest finish – earliest start – duration

M1

The path through the activity has length at most the project time, so the float is at least 0

E1

Answer: Longest path $\geq$ any path

2 marks

12 (3 marks)

Float zero: A, D, G are critical

B1

E has float 6

M1

So E can be delayed by at most 6 days

A1

Answer: A, D, G; 6 days

3 marks

13 (3 marks)

Total float = $14 - 3 - 5 = 6$

M1 A1

Latest start = $14 - 5 = 9$

A1

Answer: float 6; latest start 9

3 marks

14 (6 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 6 = 6$; Event 4: $0 + 5 = 5$; Event 5: $\max(4 + 7, 6 + 3) = \max(11, 9) = 11$; Event 6: $\max(6 + 8, 5 + 4) = \max(14, 9) = 14$; Event 7: $\max(11 + 5, 14 + 6) = \max(16, 20) = 20$; Event 8: $\max(20 + 4, 14 + 9) = \max(24, 23) = 24$

M1 A1

Backward pass: Event 8: late time = 24 (the minimum project time); Event 7: $24 - 4 = 20$; Event 6: $\min(20 - 6, 24 - 9) = \min(14, 15) = 14$; Event 5: $20 - 5 = 15$; Event 4: $14 - 4 = 10$; Event 3: $\min(15 - 3, 14 - 8) = \min(12, 6) = 6$; Event 2: $15 - 7 = 8$; Event 1: $\min(8 - 4, 6 - 6, 10 - 5) = \min(4, 0, 5) = 0$

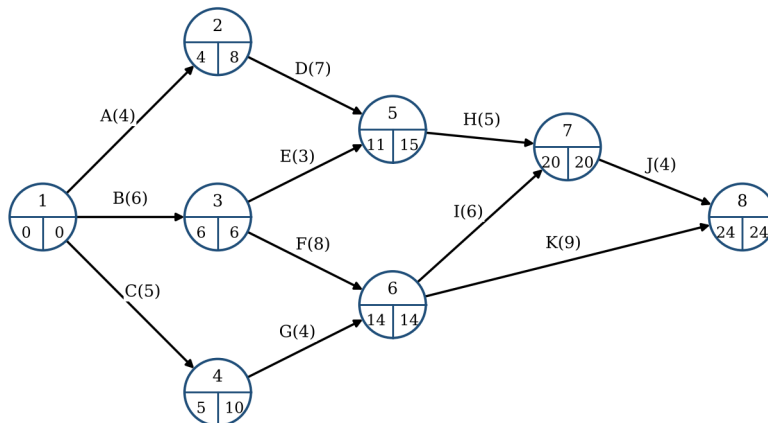
M1 A1

Float = late(end) – early(start) – duration: A: $8 - 0 - 4 = 4$; B: $6 - 0 - 6 = 0$; C: $10 - 0 - 5 = 5$; D: $15 - 4 - 7 = 4$; E: $15 - 6 - 3 = 6$; F: $14 - 6 - 8 = 0$; G: $14 - 5 - 4 = 5$; H: $20 - 11 - 5 = 4$; I: $20 - 14 - 6 = 0$; J: $24 - 20 - 4 = 0$; K: $24 - 14 - 9 = 1$

M1 A1

Answer: A 4, B 0, C 5, D 4, E 6, F 0, G 5, H 4, I 0, J 0, K 1

6 marks



15 (6 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 3 = 3$; Event 3: $0 + 5 = 5$; Event 4: $3 + 4 = 7$; Event 5: $\max(5 + 6, 3 + 0) = \max(11, 3) = 11$; Event 6: $\max(7 + 4, 11 + 5) = \max(11, 16) = 16$; Event 7: $11 + 2 = 13$; Event 8: $\max(16 + 7, 13 + 6) = \max(23, 19) = 23$

M1 A1

Backward pass: Event 8: late time = 23 (the minimum project time); Event 7: $23 - 6 = 17$; Event 6: $23 - 7 = 16$; Event 5: $\min(16 - 5, 17 - 2) = \min(11, 15) = 11$; Event 4: $16 - 4 = 12$; Event 3: $11 - 6 = 5$; Event 2: $\min(12 - 4, 11 - 0) = \min(8, 11) = 8$; Event 1: $\min(8 - 3, 5 - 5) = \min(5, 0) = 0$

M1 A1

Floats: A: $8 - 0 - 3 = 5$; B: $5 - 0 - 5 = 0$; C: $12 - 3 - 4 = 5$; D: $11 - 5 - 6 = 0$; E: $16 - 7 - 4 = 5$; F: $16 - 11 - 5 = 0$; G: $17 - 11 - 2 = 4$; H: $23 - 16 - 7 = 0$; I: $23 - 13 - 6 = 4$

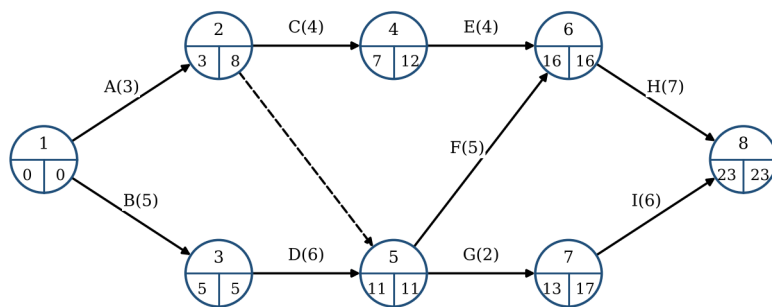
M1

Critical activities: B, D, F, H

A1

Answer: A 5, B 0, C 5, D 0, E 5, F 0, G 4, H 0, I 4

6 marks

**16 (4 marks)**

C is delayed by 3, which is less than its float of 4: on its own no effect

M1 A1

E is delayed by 7, which exceeds its float of 6 by 1

M1

So the project takes 17 days: it is affected by E but not by C

A1

Answer: Delayed by 1 day (because of E)

4 marks

17 (3 marks)

(a) Latest start = $9 + 5 = 14$

B1

(b) Latest finish = $14 + 6 = 20$

B1

(c) Earliest finish = $9 + 6 = 15$

B1

Answer: 14; 20; 15

3 marks

18 (3 marks)

Float = $20 - 8 - x = 5$

M1

$x = 7$

A1

Latest start = $20 - 7 = 13$

A1

Answer: $x = 7$; 13

3 marks

19 (4 marks)

Earliest starts: A 0, B 0, C 5, D 4, E 10

M1

Bars: A 0 to 5, B 0 to 4, C 5 to 8, D 4 to 10, E 10 to 12

A1

Float bars: A has float 2 (5 to 7), C has float 2 (8 to 10)

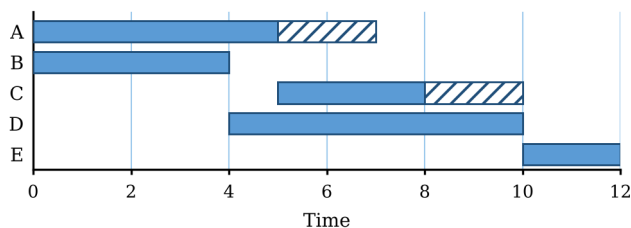
M1

B, D, E are critical with no float

A1

Answer: A 0 to 5 (float 2), B 0 to 4, C 5 to 8 (float 2), D 4 to 10, E 10 to 12; B, D, E critical

4 marks

**20 (4 marks)**

Durations read from the bars: A 5, B 4, C 3, D 6, E 2

M1 A1

Floats (hatched bars): A 2, B 0, C 2, D 0, E 0

M1

Critical activities: B, D, E

A1

Answer: Critical: B, D, E

4 marks

21 (7 marks)

Forward pass: Event 1: early time 0; Event 2: $0 + 4 = 4$; Event 3: $0 + 5 = 5$; Event 4: $4 + 3 = 7$; Event 5: $\max(4 + 6, 5 + 2, 7 + 0) = \max(10, 7, 7) = 10$; Event 6: $\max(5 + 4, 10 + 0) = \max(9, 10) = 10$; Event 7: $\max(7 + 5, 10 + 3) = \max(12, 13) = 13$; Event 8: $10 + 6 = 16$; Event 9: $\max(13 + 4, 16 + 5) = \max(17, 21) = 21$

M1 A1

Backward pass: Event 9: late time = 21 (the minimum project time); Event 8: $21 - 5 = 16$; Event 7: $21 - 4 = 17$; Event 6: $16 - 6 = 10$; Event 5: $\min(10 - 0, 17 - 3) = \min(10, 14) = 10$; Event 4: $\min(10 - 0, 17 - 5) = \min(10, 12) = 10$; Event 3: $\min(10 - 2, 10 - 4) = \min(8, 6) = 6$; Event 2: $\min(10 - 3, 10 - 6) = \min(7, 4) = 4$; Event 1: $\min(4 - 4, 6 - 5) = \min(0, 1) = 0$

M1 A1

Floats: A: $4 - 0 - 4 = 0$; B: $6 - 0 - 5 = 1$; C: $10 - 4 - 3 = 3$; D: $10 - 4 - 6 = 0$; E: $10 - 5 - 2 = 3$; F: $10 - 5 - 4 = 1$; G: $17 - 7 - 5 = 5$; H: $17 - 10 - 3 = 4$; I: $16 - 10 - 6 = 0$; J: $21 - 13 - 4 = 4$; K: $21 - 16 - 5 = 0$

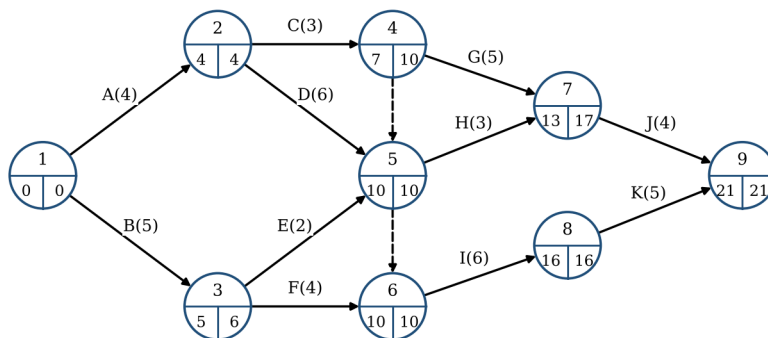
M1 A1

Critical activities: A, D, I, K

A1

Answer: A 0, B 1, C 3, D 0, E 3, F 1, G 5, H 4, I 0, J 4, K 0

7 marks

**22 (8 marks)**

Read the event times from the diagram

B1

E: total float 6; free float = $11 - 6 - 3 = 2$; independent float = $\max(0, 11 - 6 - 3) = 2$

M1 A1

G: total float 5; free float = $14 - 5 - 4 = 5$; independent float = $\max(0, 14 - 10 - 4) = 0$

M1 A1

K: total float 1; free float = $24 - 14 - 9 = 1$; independent float = $\max(0, 24 - 14 - 9) = 1$

M1 A1

Check ordering: total $\geq$ free $\geq$ independent for every activity

A1

Answer: E: 6, 2, 2; G: 5, 5, 0; K: 1, 1, 1

8 marks

23 (6 marks)

Total float of F = 4 and of H = 4

B1

Delay $3 \leq$ float 4, so the project time is still 20 days

M1 A1

The path B, D, F, H now has length $20 - 4 + 3 = 19$, so H's float is $20 - 19 = 1$

M1 A1

F and H lie on the same path, so they share the 4 days of float; using 3 days in F leaves 1 day for H

E1

Answer: unchanged; new float of H is 1

6 marks

24 (5 marks)

Total float of C = 5

B1

C's delay exceeds its float by $8 - 5 = 3$ days

M1

New project time = $24 + 3 = 27$ days

A1

C is now critical; critical path(s): C – G – I – J

M1 A1

Answer: 27 days

5 marks

25 (5 marks)

Total floats: B 2, F 1

B1

Recalculate the forward pass with B = 7 and F = 9

M1

New project time = 20 days

A1

The delays are on different paths, and each delay is partly absorbed by float, so the delays do not simply add to the project time

E1

Increase of 1 day only, compared with 4 days if the delays simply added

A1

Answer: 20 days

5 marks

26 (5 marks)

C is on the path A, C, F of length $5 + x + 3 = x + 8$

M1

Project time is unchanged while $x + 8 \leq 16$

M1

Greatest $x = 8$

A1

When $x = 8$, A, C, F has length 16, the same as A, D, G

M1

Critical paths: A, D, G and A, C, F

A1

Answer: $x = 8$

5 marks

27 (6 marks)

The longest path through H has length $24 - 4 = 20$ when $x = 5$, so length $15 + x$ in general

M1 A1

Every path not through H still has length at most 24

B1

Project time = $\max(24, 15 + x)$

M1

That is 24 for $x \leq 9$ and $15 + x$ for $x \geq 9$

A1

H is critical when $15 + x \geq 24$, so $x \geq 9$

A1

Answer: $\max(24, 15 + x)$; $x \geq 9$

6 marks

28 (5 marks)

Free float is $e(E) - e(S) - d$ and total float is $l(E) - e(S) - d$

B1

$e(E) \leq l(E)$ for every event, so free float $\leq$ total float

M1 A1

$e(S) \leq l(S)$, so $e(E) - l(S) - d \leq e(E) - e(S) - d =$ free float

M1

If $e(E) - l(S) - d$ is negative the independent float is 0, which is at most the free float (free float ≥ 0 as $e(E) \geq e(S) + d$)

A1

Answer: independent float $\leq$ free float $\leq$ total float

5 marks

29 (7 marks)

(a) Path A, C, F has length $5 + 4 + 3 = 12$

M1

Path A, D, G has length $5 + x + 5 = x + 10$

M1

Path B, E, G has length $3 + 2 + 5 = 10$, which is never the longest since $12 > 10$

B1

Project time = $\max(12, x + 10)$: 12 for $x \leq 2$, $x + 10$ for $x \geq 2$

A1

Answer: $\max(12, x + 10)$: 12 for $x \leq 2$ and $x + 10$ for $x \geq 2$

4 marks

(b) (i) $x = 1$: A, C, F (12 against 11)

B1

(ii) $x = 2$: A, C, F and A, D, G (both 12)

B1

(iii) $x = 5$: A, D, G (15 against 12)

B1

Answer: (i) A, C, F; (ii) A, C, F and A, D, G; (iii) A, D, G

3 marks

30 (6 marks)

(a) Delay F by 4: path B, D, F, H now has length 20, so the project is still 20 days

M1 A1

H then has float 0 (it is critical)

B1

Delaying H by 1 more gives project time 21, so the delays cannot both be 4

M1 A1

(b) At most 4 days in total, because F and H lie on one path with slack 4 days

E1

Answer: at most 4 days in total

6 marks

Resource Scheduling**1 (2 marks)**

A bar chart with time on the horizontal axis

B1

The height of each bar is the number of workers needed in that interval

B1

Answer: Workers against time

2 marks

2 (2 marks)

$36 \div 4$

M1

$= 9$ days

A1

Answer: 9 days

2 marks

3 (2 marks)

$$45 \div 12 = 3.75$$

M1

Round up to 4 workers

A1

Answer: 4 workers

2 marks

4 (2 marks)

Activities cannot start until their predecessors are complete

B1

So some workers may have to wait with nothing to do, and the work cannot always be shared evenly

E1

Answer: Precedence causes idle workers

2 marks

5 (3 marks)

Greatest number is 4

B1

$$\text{Total} = 3 + 3 + 4 + 4 + 2 + 2$$

M1

$$= 18 \text{ worker-days}$$

A1

Answer: 4 workers; 18 worker-days

3 marks

6 (2 marks)

$$\text{A: } 2 \times 5 = 10, \text{ B: } 3 \times 4 = 12$$

M1

Total 22 worker-days

A1

Answer: 10, 12, 22

2 marks

7 (4 marks)

$$\text{Worker-days: } 5 \times 2 + 4 \times 1 + 3 \times 2 + 6 \times 3 + 2 \times 1$$

M1

$$= 10 + 4 + 6 + 18 + 2 = 40$$

A1

$$40 \div 12 = 3.33$$

M1

So at least 4 workers

A1

Answer: 40 worker-days; 4 workers

4 marks

8 (4 marks)

Earliest starts: A 0, B 0, C 5, D 4, E 10

M1

Add the workers of the activities running in each interval

M1

Workers needed (from time to time): 0 to 4: 3; 4 to 8: 5; 8 to 10: 3; 10 to 12: 1

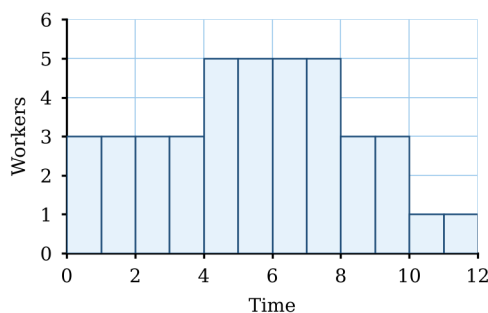
A1

Histogram drawn with these heights; greatest number 5

A1

Answer: 0 to 4: 3; 4 to 8: 5; 8 to 10: 3; 10 to 12: 1

4 marks

**9 (4 marks)**

(a) Greatest number is 5

B1

Between times 4 and 8

B1

Answer: 5 workers, between times 4 and 8

2 marks

$$\text{(b) Total} = 3 + 3 + 3 + 3 + 5 + 5 + 5 + 5 + 3 + 3 + 1 + 1$$

M1

$$= 40 \text{ worker-days}$$

A1

Answer: 40 worker-days

2 marks

10 (3 marks)(a) C (3 workers) and D (2 workers) are running: $3 + 2 = 5$

M1 A1

(b) D (2) and F (1): $2 + 1 = 3$

B1

Answer: 5; 3

3 marks

11 (3 marks)

$$59 \div 16 = 3.69\dots$$

M1 A1

This is more than 3, so 3 workers cannot do it in 16 days; at least 4 are needed

E1

Answer: At least 4

3 marks

12 (3 marks)

Compare each bar with 4

M1

The bars of height 5 are those from time 4 to time 8

A1

Too many workers between times 4 and 8

A1

Answer: Times 4 to 8

3 marks

13 (3 marks)

$$\text{Worker-days} = 8 + 15 + 3 + 12 + 8 + 6 = 52$$

M1 A1

$$52 \div 13 = 4, \text{ so at least 4 workers}$$

A1

Answer: 52 worker-days; 4 workers

3 marks

14 (4 marks)

Earliest starts: A 0, B 0, C 5, D 5, E 3, F 9, G 11

M1

Add the workers of the activities running in each interval

M1

Workers needed (from time to time): 0 to 3: 3; 3 to 5: 4; 5 to 9: 5; 9 to 11: 3; 11 to 12: 4; 12 to 16: 3

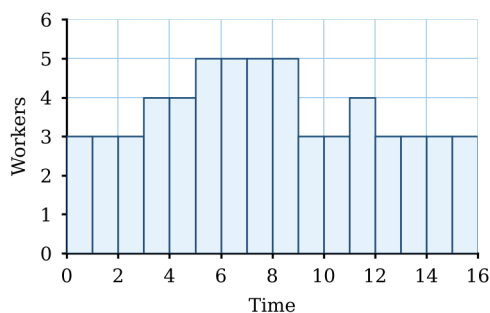
A1

Histogram drawn with these heights; greatest number 5

A1

Answer: 0 to 3: 3; 3 to 5: 4; 5 to 9: 5; 9 to 11: 3; 11 to 12: 4; 12 to 16: 3

4 marks

**15 (4 marks)**

Earliest starts: A 0, B 0, C 5, D 5, E 8, F 11

M1

Add the workers of the activities running in each interval

M1

Workers needed (from time to time): 0 to 4: 5; 4 to 8: 3; 8 to 11: 4; 11 to 12: 5; 12 to 13: 3

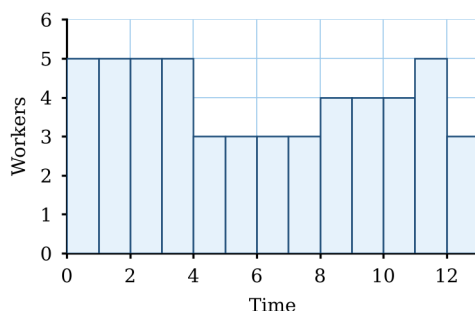
A1

Histogram drawn with these heights; greatest number 5

A1

Answer: 0 to 4: 5; 4 to 8: 3; 8 to 11: 4; 11 to 12: 5; 12 to 13: 3

4 marks

**16 (5 marks)**

Earliest starts: A 0, B 0, C 2, D 2, E 8; the peak of 4 workers is in the first 2 days (A 3 + B 1)

M1 A1

B and D each have total float 2: B now runs from 2 to 4 and D from 4 to 8

M1

Workers in each interval: 0 to 8: 3; 8 to 12: 1

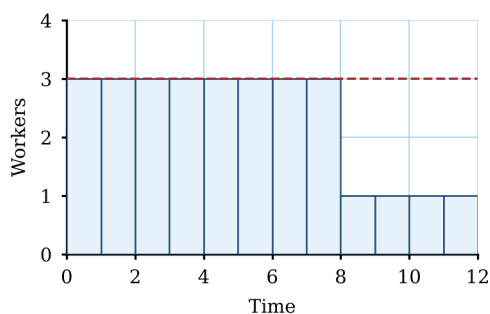
M1

Never more than 3, and the finish is still time 12

A1

Answer: At most 3 workers

5 marks

**17 (4 marks)**

(a) Worker-days 59, and $59 \div 16 = 3.69$

M1

So at least 4 workers

A1

Answer: 4 workers

2 marks

(b) The earliest-start histogram has greatest height 5

M1

So 5 workers are enough

A1

Answer: 5 workers are enough

2 marks

18 (3 marks)

Total = $5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 5 + 3 = 52$

M1 A1

$52 \div 13 = 4$, so at least 4 workers

A1

Answer: 52; 4

3 marks

19 (4 marks)

(a) Worker-days: $4 \times 2 + 6 \times 3 + 5 \times 1 + 7 \times 2 + 3 \times 2 + 8 \times 3 + 4 \times 1 + 5 \times 2 + 6 \times 3 + 4 \times 1 + 9 \times 2$

M1

$= 8 + 18 + 5 + 14 + 6 + 24 + 4 + 10 + 18 + 4 + 18 = 129$

A1

Answer: 129 worker-days

2 marks

(b) $129 \div 24 = 5.375$

M1

At least 6 workers

A1

Answer: 6 workers

2 marks

20 (4 marks)

Earliest starts: A 0, B 0, C 2, D 2, E 8

M1

Bars drawn for each activity from its start for its duration

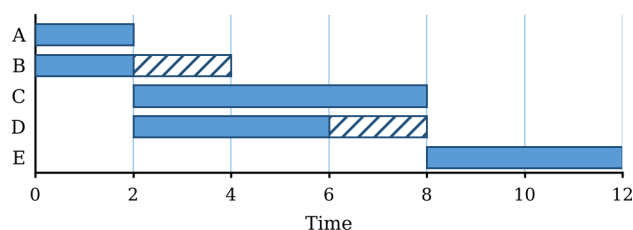
A1

Floats: B 2 and D 2, all others 0

M1 A1

Answer: Earliest starts: A 0, B 0, C 2, D 2, E 8; floats B 2, D 2 (others 0)

4 marks

**21 (7 marks)**

(a) D needs all 3 workers, so any other activity would need workers who are not available

B1

Answer: D uses all 3 workers

1 mark

(b) A (2 workers) and C (2) cannot run alongside D, and E (1) cannot either, since $3 + 1 > 3$

M1

C needs A to be complete first, and E needs both C and D, so A, D, C, E are forced into separate periods

M1

Time at least $5 + 6 + 3 + 2 = 16$ days

M1 A1

A schedule: A 0 to 5 with B 0 to 4 (3 workers), D 5 to 11, C 11 to 14, E 14 to 16

M1

At most 3 workers at any time and the project takes 16 days

A1

Answer: At least 16 days; e.g. A 0, B 0, D 5, C 11, E 14

6 marks

22 (5 marks)

Start times (days): A 0, B 0, C 5, D 9, E 3, F 9, G 15

M1 A1

No activity starts before all of its predecessors have finished

B1

Workers needed: 0 to 3: 3; 3 to 5: 4; 5 to 12: 3; 12 to 15: 2; 15 to 20: 3

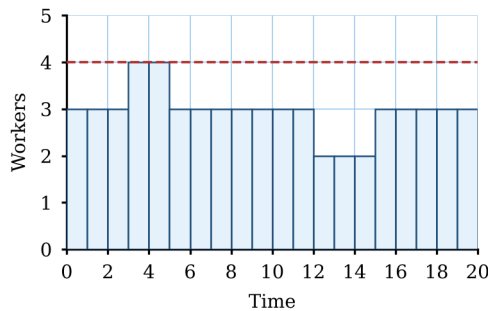
M1

At most 4 at all times, and the last activity ends at day 20

A1

Answer: Completed in 20 days

5 marks

**23 (5 marks)**

For 16 days, the critical activities A, D, G must run at times 0 to 5, 5 to 11 and 11 to 16

B1

C must start after A (time 5 or later), and F (3 days) follows C, so C must finish by time 13

M1

During times 5 to 11, D needs 2 workers and C needs 3: $2 + 3 = 5 > 4$

M1

After time 11, only 2 days remain before time 13, but C takes 4 days (and G uses 3 workers)

M1

So C cannot be scheduled: 16 days is impossible with 4 workers

A1

Answer: Impossible in 16 days

5 marks

24 (6 marks)

(a) B, D, E are critical, so D must run from time 4 to time 10

M1 A1

C must start after A finishes (time 5 or later) and finish by time 10, when E starts

M1

So C overlaps D completely: $3 + 2 = 5$ workers needed, which is more than 4

A1

Answer: D and C overlap, needing 5 workers

4 marks

(b) With every activity at its earliest start the greatest number of workers is 5

M1

So 5 workers are enough

A1

Answer: 5 workers are enough

2 marks

25 (5 marks)

Total float of each activity: A 0, B 4, C 0, D 10, E 0, F 10

M1

Delay non-critical activities to avoid overlaps of the 3-worker activities

M1

One schedule: A 0, B 0, C 7, D 7, E 13, F 12

A1

Workers in each interval: 0 to 3: 4; 3 to 7: 3; 7 to 12: 4; 12 to 14: 3; 14 to 20: 1

M1

At most 4 workers at all times; finish at day 20

A1

Answer: At most 4 workers

5 marks

26 (5 marks)

Start times: A 5, B 0, C 9, D 5, E 13, F 11

M1 A1

Predecessors are complete before each activity starts

B1

Workers needed: 0 to 5: 3; 5 to 9: 4; 9 to 11: 3; 11 to 12: 4; 12 to 13: 3; 13 to 17: 2

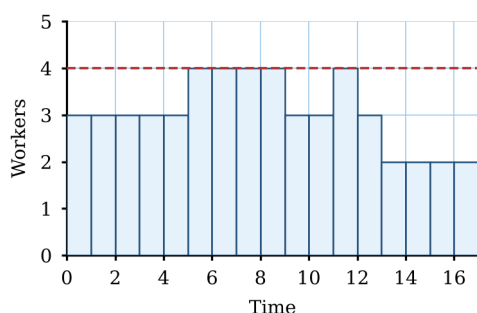
M1

Never more than 4, finishing at day 17

A1

Answer: 17 days

5 marks



27 (7 marks)

(a) The longest path gives 24 days and the worker-days give $129 \div 5 = 25.8$

M1

So at least 26 days

A1

Answer: 26 days

2 marks

(b) Latest start times: A 4, B 0, C 5, D 8, E 12, F 6, G 10, H 15, I 14, J 20, K 15

M1

Applying the rule gives start times A 0, B 0, C 4, D 9, E 14, F 6, G 14, H 17, I 18, J 24, K 22

M1 A1

Completion time 31 days

A1

Answer: 31 days

4 marks

(c) 31 is above the lower bound of 26: the rule does not give the best schedule, and a better schedule (29 days) exists

E1

Answer: The rule does not give the best schedule

1 mark

28 (6 marks)

Start times: A 0, B 0, C 0, D 4, E 6, F 6, G 9, H 11, I 14, J 20, K 14

M1 A1

Check predecessors: every activity starts at or after the latest finish of its predecessors

M1

Workers needed: 0 to 5: 6; 5 to 6: 5; 6 to 9: 7; 9 to 13: 6; 13 to 14: 5; 14 to 16: 7; 16 to 20: 5; 20 to 23: 3; 23 to 24: 1

M1

At most 7 workers at all times

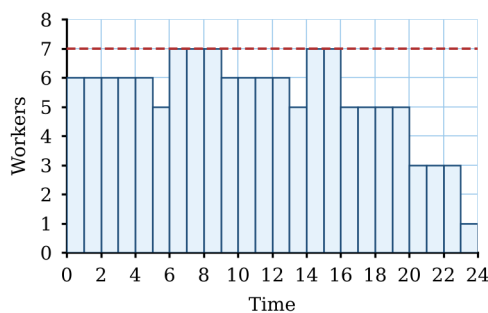
A1

Finish at day 24, equal to the minimum project time

A1

Answer: 24 days

6 marks

**29 (5 marks)**

(a) With k workers, at most k worker-days of work are done per day, so the time is at least $W \div k$

M1 A1

The activities on the longest path must be done one after another, so the time is at least T

B1

Answer: Time $\geq \max(T, W \div k)$

3 marks

(b) With 4 workers the bound is $\max(12, 40 \div 4) = \max(12, 10) = 12$ days

M1

Precedence and the numbers of workers each activity needs may force more than 4 workers to be wanted at once (or leave workers idle), so the bound need not be achievable

E1

Answer: 12 days; the bound may not be achievable because of precedence and worker requirements

2 marks

30 (5 marks)

Cost = $90 \times (\text{workers} \times \text{days})$: 3 workers: $3 \times 48 = 144$, 4 workers: $4 \times 40 = 160$, 5 workers: $5 \times$

M1 A1

$29 = 145$, 6 workers: $6 \times 27 = 162$, 7 workers: $7 \times 24 = 168$

Lowest is 3 workers: 144 worker-days, so £12 960

M1 A1

3 workers take 48 days; more workers finish much sooner (24 days with 7), which may matter more than cost

E1

Answer: 3 workers, £12 960

5 marks