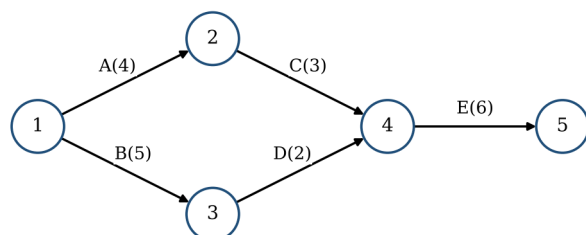


Activity Networks

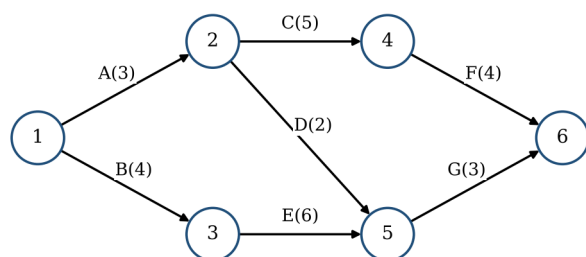
- 1 In an activity network drawn with activities on arcs, state what is represented by (a) an arc, (b) a node (event). [2]

- 2 State the purpose of a dummy activity in an activity network, and say how a dummy activity is shown. [2]

- 3 The activity network shows a project. The durations of the activities are in hours. Write down the activities that can start straight away, and the activity that cannot start until both C and D are complete.



- 4 The activity network shows a project. How many activities, and how many events (nodes), are there in the network?



- 5 A project has activities A, B, C and D. A and B can start at the same time, C needs A to be complete, and D needs both B and C to be complete. Complete the table of immediately preceding activities. [2]

- 6 A project has the following activities: A (no activity before it), B (no activity before it), C (after A) and D (after B). All four must be complete before the project ends. State how many of the activities can be done at the same time at the start, and name a pair of activities that can be carried out together later. [2]

- 7 A project is defined by the following table. Draw an activity network for the project (activities on arcs), with no dummy activities.

Activity	Preceded by
A	-
B	-
C	A
D	B
E	C, D

[3]

- 8** A project is defined by the table.
Draw an activity network for the project, using no dummies.

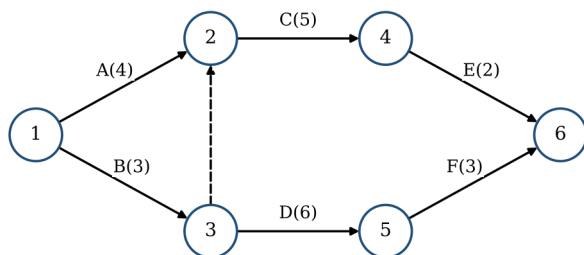
Activity	Preceded by
A	-
B	-
C	A
D	A
E	B
F	C
G	D, E

[4]

- 9** Three activities P, Q and R are such that R cannot start until P and Q are both complete, but a fourth activity S needs only Q to be complete.
Explain why a dummy activity is needed when drawing the network, and show how it is used.

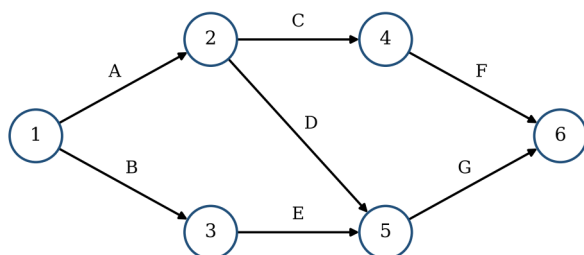
[3]

- 10** The activity network shows a project.
Complete the table showing, for each activity, the activities which immediately precede it.
Hence explain what the dummy activity shows.



[4]

- 11** The activity network shows a project.
List every path through the network from start to finish.



[3]

- 12** A student draws an activity network for a project and finds that two different activities both start at event 2 and both finish at event 3.
Explain why this is not acceptable, and say how the problem can be put right.

[3]

- 13** A project has 5 activities A to E with durations 4, 5, 3, 2 and 6 days and the following precedences: A and B have none; C follows A; D follows B; E follows C and D.
Draw the activity network, putting each activity's duration beside its letter.

[3]

- 14** A project is defined by the table.
Draw an activity network for the project, using exactly one dummy activity.

Activity	Preceded by
A	-
B	-
C	A, B
D	B
E	C
F	D

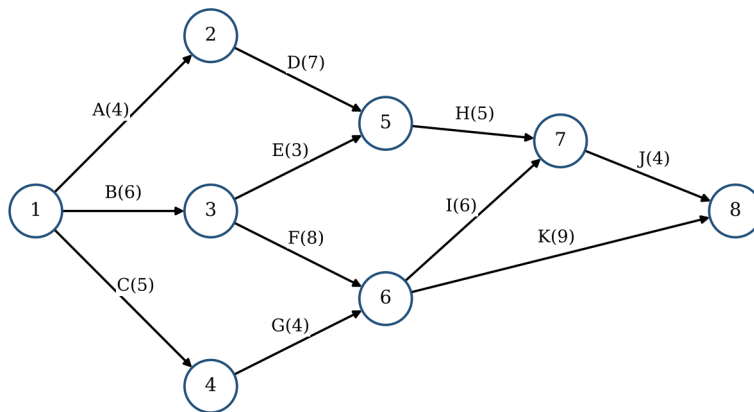
[5]

- 15** A project is defined by the table.
Draw an activity network for the project, using as few dummy activities as possible.

Activity	Preceded by
A	-
B	-
C	A
D	B
E	C
F	A, D
G	A, D
H	E, F
I	G

[6]

- 16** The activity network shows a project. The durations are in days.
Construct the table of activities with their immediately preceding activities.



[4]

- 17** A project is defined by the table (durations in days).
Draw an activity network for the project, and state the number of events in your network.

Activity	Duration	Preceded by
A	4	-
B	6	-
C	5	-
D	7	A
E	3	B
F	8	B
G	4	C
H	5	D, E
I	6	F, G
J	4	H, I
K	9	F, G

[5]

- 18** Activities X and Y both need A and B to be complete before they can start, and nothing else. A third activity Z needs only B to be complete.
Explain how this affects the way A, B, X, Y and Z are drawn, and whether a dummy activity is needed.

[4]

- 19** A builder plans a project with the following rules. Foundations (A) and Delivery of materials (B) can start together. Walls (C) need both. Roof (D) needs Walls. Plumbing (E) needs Walls only. Electrics (F) needs Roof and Plumbing.
Draw an activity network for the project, and say whether a dummy is needed.

[4]

- 20** Explain why the network for the following project needs a dummy: P and Q start together; R follows P; S follows P and Q.

[3]

- 21** Each of the following gives the activities which immediately precede activity T. In each case say how many arcs end at the event where T starts, in the simplest network without dummies: (a) T needs only A; (b) T needs A and B, which have no other successors; (c) T needs A and B, but B is also needed by another activity U (which does not need A).

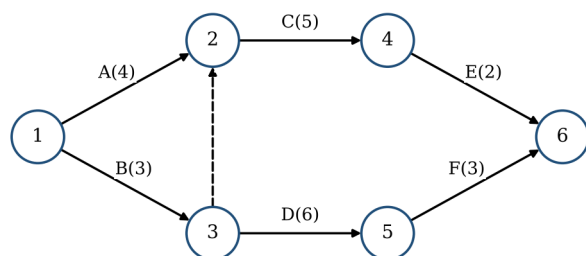
[3]

- 22** A project is defined by the table, with durations in days.
Draw an activity network for the project, using as few dummy activities as possible, and state how many dummies you needed.

Activity	Duration	Preceded by
A	4	-
B	5	-
C	3	A
D	6	A
E	2	B
F	4	B
G	5	C
H	3	C, D, E
I	6	C, D, E, F
J	4	G, H
K	5	I

[7]

- 23** A student constructs the table of preceding activities for the activity network shown. The student's table gives: C: A; D: A, B; E: C, D; F: D. Using the network, find the three entries in the student's table that are wrong, and give the corrections.



[5]

- 24** Activity C needs A and B. Activity D needs B and C. Activity E needs C only. A student says that D needs A as well, since C needs A. Explain why the student is wrong in terms of the table, and why drawing the network does not require showing this.

[4]

- 25** The table shows the activities of a project. The durations are in minutes. Draw the activity network, using exactly one dummy, then list every path through it.

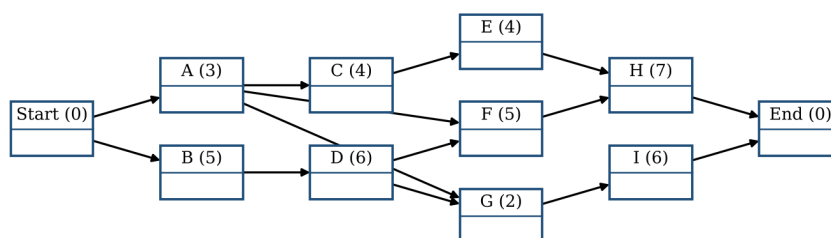
Activity	Duration	Preceded by
A	4	-
B	3	-
C	5	A, B
D	6	B
E	2	C
F	3	D

[7]

- 26** Five activities A to E have these precedences: A and B have none; C needs A only; D needs both A and B; E needs B only. Show that at least two dummy activities are needed, and draw a network that uses exactly two.

[6]

- 27** The diagram shows a project as an activity-on-node network: each box is an activity, and an arrow means that the activity at the tail must be complete before the activity at the head can start. The durations are in days.



- (a) Write down the table of immediately preceding activities for the project.
- (b) Draw an activity-on-arc network for the project, with durations on the arcs, using the smallest number of dummy activities.

[3]

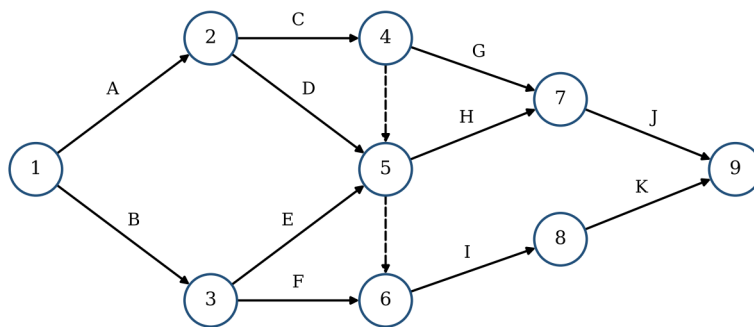
[5]

28 A project is defined by the table (durations in days).

Activity	Duration	Preceded by
A	3	-
B	5	-
C	4	-
D	6	A
E	2	B
F	7	C
G	4	D, E
H	5	B, F
I	3	D, E
J	6	G
K	2	H, I

- (a) Draw an activity network for the project, using as few dummy activities as possible. [6]
 (b) State how many paths there are from start to finish. [1]

29 The activity network shown has two dummy activities.

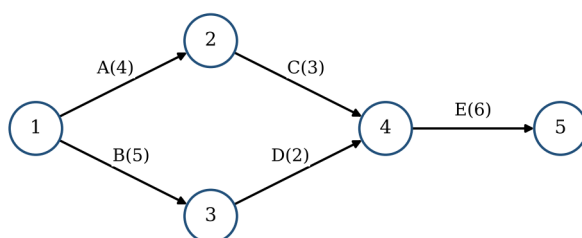


- (a) List all the paths from start to finish, using activity letters (do not list dummies). [4]
 (b) Explain why removing the dummy activity from event 4 to event 5 would change the project: state which activities would then have fewer preceding activities. [4]

30 A project table is given as follows, but it has an error: A: C; B: A; C: B; D: B. Explain what is wrong with the table, say what this means for the project, and suggest two different corrections to make a valid precedence table, giving a network for one of them. [6]

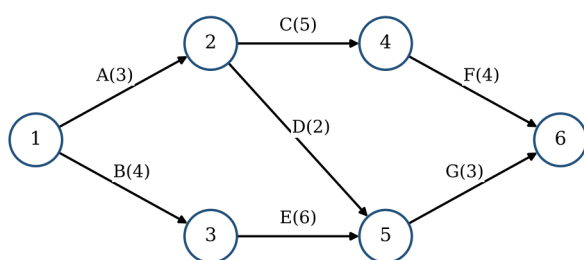
Precedence Tables

- 1** Explain what is meant by "immediately preceding activity" in a precedence table. [2]
2 Write down the table of immediately preceding activities for the activity network shown.



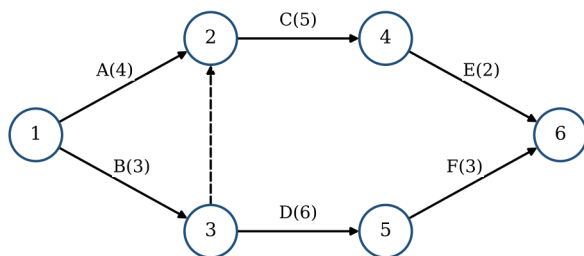
[2]

- 3** A table of immediately preceding activities is: A: none; B: A; C: A; D: B, C. State which activities can start as soon as A is complete. [1]
- 4** From the table (A: none; B: none; C: A, B; D: C; E: C), name the activities that have no preceding activity, and the activities that could be carried out at the same time as each other after C is complete. [2]
- 5** In the table below, which activity must be completed last? (A: none; B: A; C: A; D: B; E: C, D). Give a reason. [2]
- 6** Using the table A: none; B: A; C: B; D: B, find all the activities which have to be complete before D can start. [2]
- 7** The activity network shows a project.
Write down the table of immediately preceding activities.



[3]

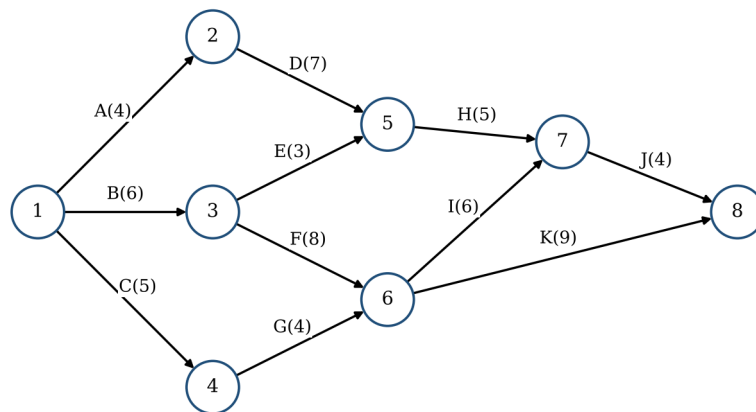
- 8** The activity network shown contains a dummy activity.
Write down the table of immediately preceding activities.



[3]

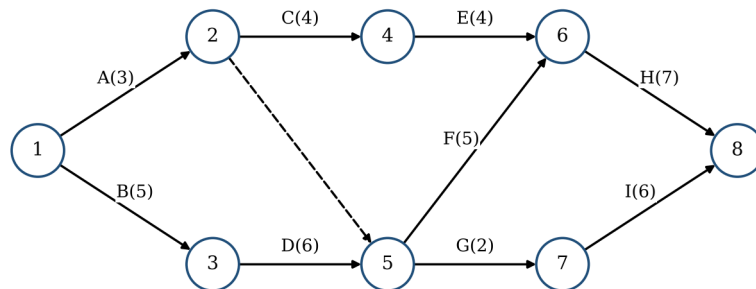
- 9** Activity P can start at once. Q and R can start once P is complete. S can start when Q is complete, and T when both R and S are complete.
Write the statements as a table of immediately preceding activities. [3]
- 10** The table gives the immediately preceding activities: A: none; B: none; C: A; D: A, B; E: C; F: C, D.
Which activities must be complete before F can start? [3]
- 11** A table of preceding activities is A: none; B: none; C: A; D: B; E: C, D; F: E; G: E. Which activities can be carried out at the same time as C? [3]
- 12** Explain why the following precedence table cannot represent a real project: A: B; B: C; C: A. [2]
- 13** A student's precedence table is: A: none; B: A; C: A, B; D: C. Explain why listing A as a preceding activity for C is unnecessary, and give the shorter table. [3]

- 14** The activity network shown represents a project. Construct the precedence table, and write down the activities that must all be complete before activity K can start.



[4]

- 15** The activity network shown contains a dummy activity. Write down the precedence table, and explain the effect of the dummy.



[5]

- 16** A project has activities A to E with the precedence table A: none; B: none; C: A; D: B; E: C, D. One person carries out the activities one at a time. List all the orders in which this can be done.

[4]

- 17** The table shows a project, with durations in days. Find (a) the activities that have no preceding activity, (b) the activities that no other activity needs (the last activities), and (c) all activities that must be complete before K can start.

Activity	Duration	Preceded by
A	3	-
B	5	-
C	4	-
D	6	A
E	2	B
F	7	C
G	4	D, E
H	5	B, F
I	3	D, E
J	6	G
K	2	H, I

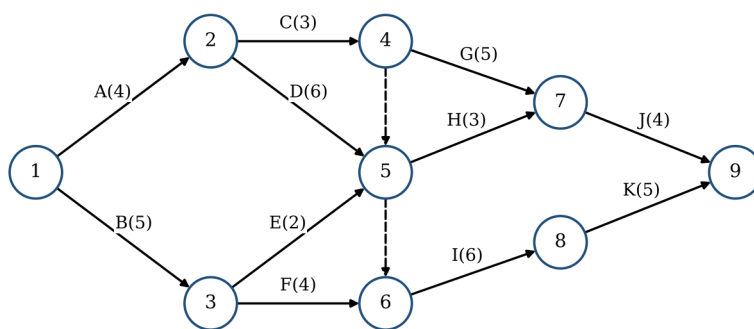
[4]

- 18** A new activity X is added to the project with the precedence table A: none; B: A; C: A; D: B, C. X needs B to be complete, and D now needs X as well as B and C. Write down the new table, and say whether any entry can now be simplified. [3]

- 19** The table is given: A: none; B: none; C: A, B; D: B; E: C; F: D. Explain why a dummy activity is needed in the activity network, and which two events it joins. State what would go wrong if the dummy were omitted and A and B both ended at the start of C and D. [4]

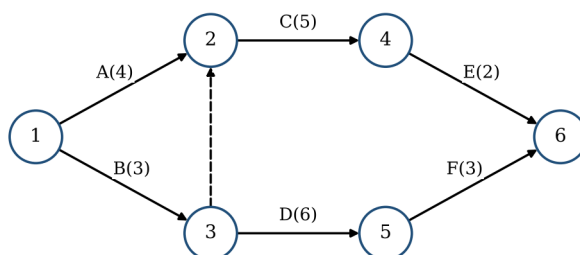
- 20** The table of immediately preceding activities is: A: none; B: none; C: A; D: A, B; E: C, D; F: D. Explain why activities C and D cannot be shown as starting from a single event in an activity-on-arc network, and write down the minimum number of dummies needed. [4]

- 21** The activity network shown has two dummy activities. Complete the precedence table, and find the number of activities which must all be complete before J can start.



[5]

- 22** A student writes the precedence table for the network shown as: A: none; B: none; C: A; D: B; E: C, D; F: D. The table has two errors. Find them, give the correct table, and explain the likely cause of the errors.



[6]

- 23** A project has the precedence table A: none; B: none; C: A, B; D: B; E: C; F: D. One person does the activities one at a time. Find the number of different orders in which the six activities can be done. [5]

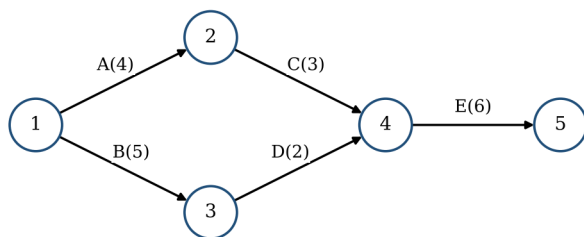
- 24** The table gives durations (days) and immediately preceding activities: A, 3, none; B, 5, none; C, 4, A; D, 6, A, B; E, 2, C; F, 5, D, E. Find the shortest possible time to complete the project and the activities which then have no spare time. [6]

- 25** A precedence table has the following rows: A: none; B: A; C: A; D: B; E: B, C; F: D, E; G: C.
Explain why G and F are both finishing activities, and find the number of activities that must be complete before F can start. [4]
- 26** A project has the precedence table A: none; B: none; C: A; D: A; E: B, C; F: D, E.
Write down the precedence table for the network which results if a new activity X, which needs A and B and is needed by F, is added. Say whether F's entry can be simplified. [4]
- 27** A precedence table has A: none; B: none; C: A; D: B; E: A, B; F: C, D, E.
(a) Explain why it is impossible to draw an activity network for this table with no dummy activity. [3]
(b) Find the smallest number of dummy activities needed, and describe where they go. [4]
- 28** A project has the precedence table below, with durations in days.
Find the shortest time in which the project can be completed, and show that the activities on the critical path are A, D, G, J.
- | Activity | Duration | Preceded by |
|----------|----------|-------------|
| A | 3 | - |
| B | 5 | - |
| C | 4 | - |
| D | 6 | A |
| E | 2 | B |
| F | 7 | C |
| G | 4 | D, E |
| H | 5 | B, F |
| I | 3 | D, E |
| J | 6 | G |
| K | 2 | H, I |
- [7]
- 29** A precedence table is valid if no activity (directly or indirectly) needs itself. A project has activities A to F with: A: none; B: A; C: B; D: C; E: D, B; F: E, C.
Show that C needs B and A, find all activities needed by F, and determine which entries in the table are redundant (could be removed without changing which activities must be complete before each activity). [6]
- 30** A project has 6 activities A to F where each activity after A needs all of the activities earlier in the alphabet (so B: A; C: A, B; D: A, B, C; and so on).
Show that the table can be reduced to a chain of immediately preceding activities, write down the reduced table, and state the number of different orders in which one person could do the activities. Generalise your answer to n activities. [5]

Earliest and Latest Event Times

- 1** In a critical path analysis, what is meant by (a) the earliest event time, (b) the latest event time? [2]
- 2** Three activities end at the same event. They finish at (earliest) times 8, 11 and 9. State the earliest event time, and explain your answer. [2]
- 3** Two activities start from the same event. The latest times at the events where they end are 20 and 24, and their durations are 6 and 3. Find the latest time at the event where they start. [3]

- 4 In the activity network shown, find the earliest event times at events 2 and 3.
(Durations are in hours.)



[2]

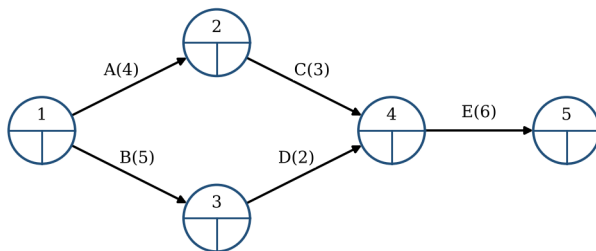
- 5 The earliest event time at the finishing event of a project is 27 days. State the minimum time for completion of the project, and the latest time at the finishing event.

[2]

- 6 Event 4 has earliest time 12 and latest time 12. Event 5 has earliest time 9 and latest time 13. Which of these events is on the critical path? Explain your answer.

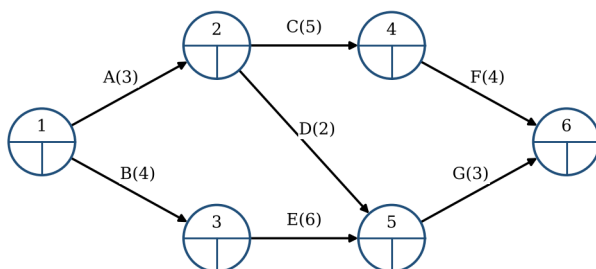
[2]

- 7 The activity network shows a project. Durations are in days.
Find the earliest and latest event times for each event, and the minimum project time.



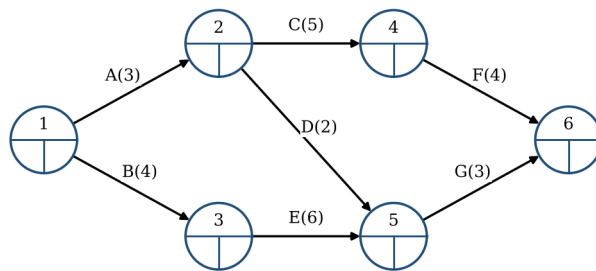
[4]

- 8 The activity network shown has durations in hours. Complete a forward pass to find the earliest event times, and state the minimum project time.



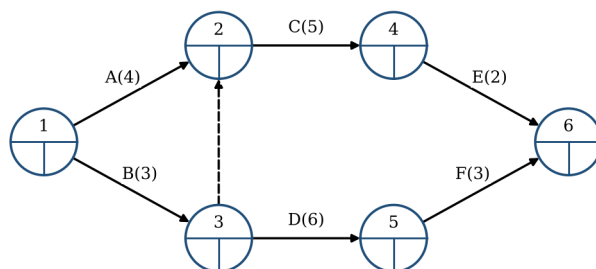
[3]

- 9** For the network shown the earliest event times are 0, 3, 4, 8, 10 and 13 for events 1 to 6 respectively, so the minimum project time is 13 hours. Carry out a backward pass to find the latest event times.



[3]

- 10** The activity network shown contains a dummy activity, which has duration 0. Find the earliest event times, and state the minimum completion time.



[4]

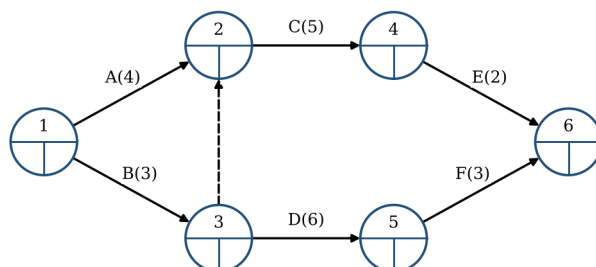
- 11** An event is reached by two activities. The earliest start of the first of them is 6 and its duration is 5. The second is a dummy from an event with earliest time 11. Find the earliest event time at that event, and say what the dummy shows.

[3]

- 12** A project has minimum completion time 19 days. The project manager wants to finish in 19 days. The latest time at the finishing event is therefore 19. An activity of duration 6 ends at this event, and another of duration 4 also ends there. Find the latest time at the start of each of these activities.

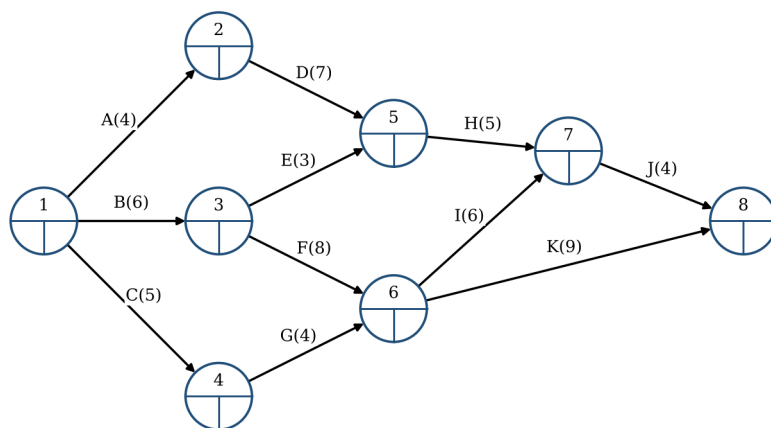
[3]

- 13** For the network shown, which contains a dummy, the earliest event times are 0, 4, 3, 9, 9 and 12 for events 1 to 6 respectively. Find the latest event times, and hence say which events are critical.



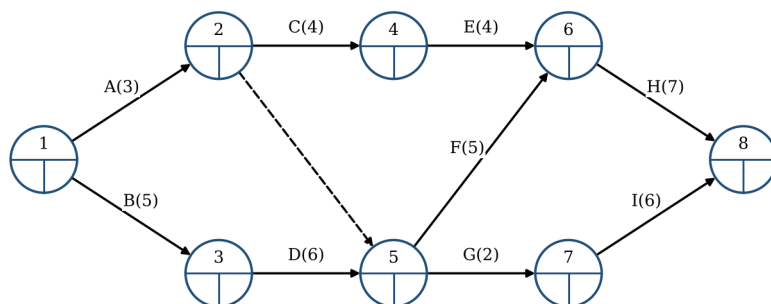
[4]

- 14** The activity network shows a project, with durations in days.
Complete the forward pass and the backward pass, writing the earliest and latest event times in the boxes, and state the minimum completion time of the project.



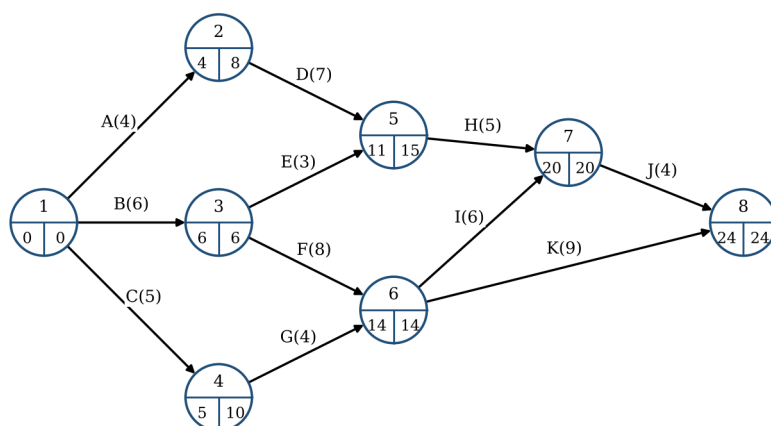
[6]

- 15** The activity network shown contains a dummy. Durations are in days.
Find the earliest and latest event times and the minimum project time.



[6]

- 16** In the network shown, the earliest and latest times are marked at each event.
Write down the critical events, and explain why an event with early time 8 and late time 12 is not critical.

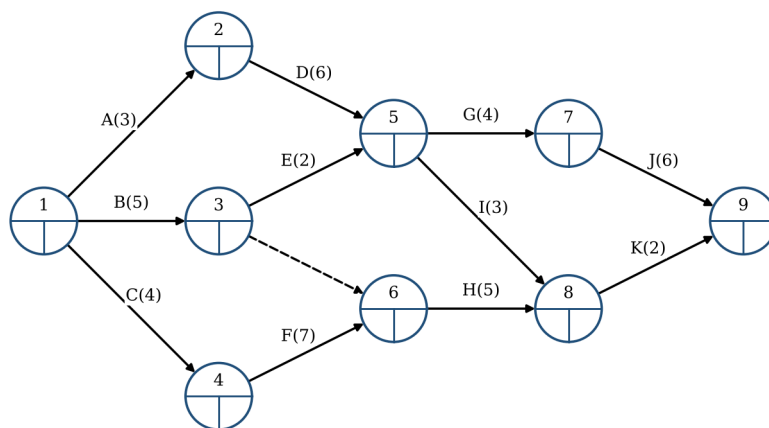


[3]

- 17** In a project, activities A and B start the project (durations 4 and x days), C needs A (duration 3), D needs B (duration 2), and E needs C and D (duration 6). The earliest time at the event before E is 11 days.
Find x.

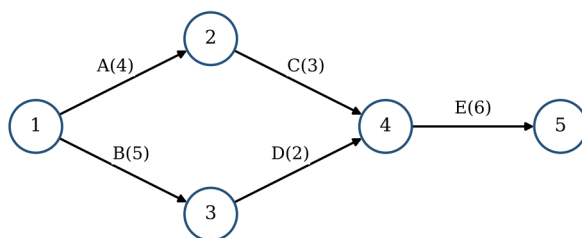
[4]

- 18** The activity network shown has durations in days.
Find the earliest and latest event times, writing them on the network, and state the minimum project time and the critical events.



[7]

- 19** For the network shown, the minimum completion time is 13 days. The project must be completed within 16 days, and the latest time at the finishing event is taken to be 16. Find the latest event times, and compare with those for a 13-day deadline.

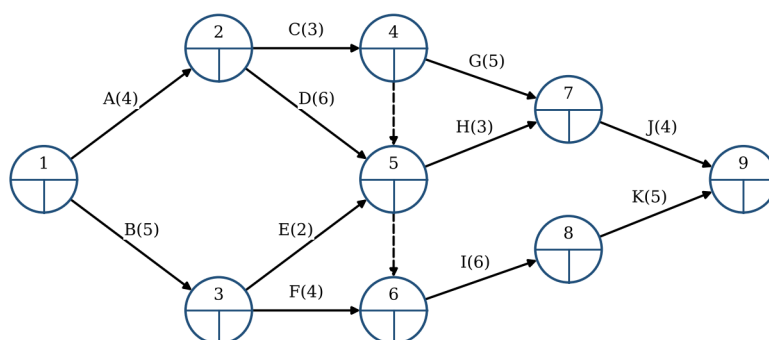


[4]

- 20** A student says: "The latest time at an event can never be smaller than the earliest time." Explain, with reference to the minimum project time, why the student is correct if the latest time at the finishing event equals the minimum project time, and say what it would mean if a latest time were smaller than an earliest time.

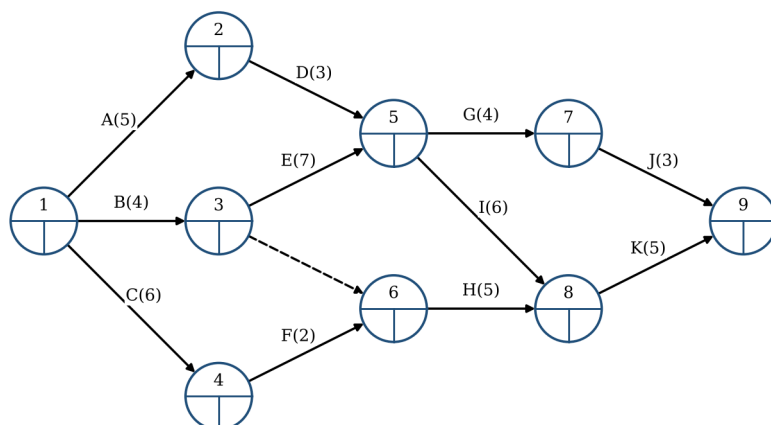
[4]

- 21** The activity network shown has two dummy activities. Durations are in days. Find the earliest and latest event times, and state the minimum project time and the critical events.



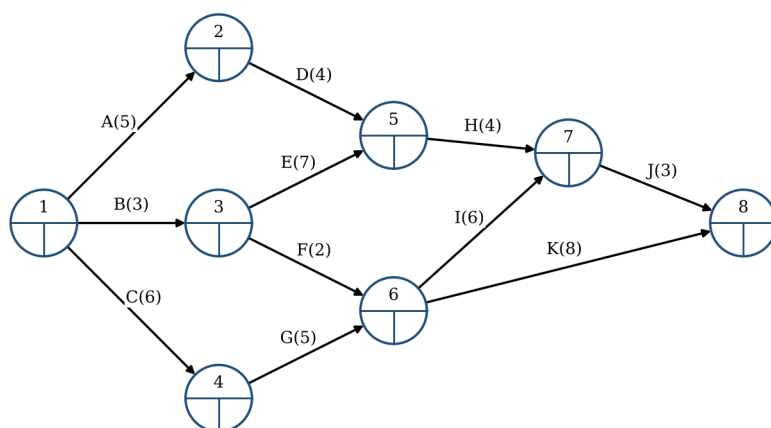
[7]

- 22** The activity network shown has durations in days.
Find the earliest and latest event times, and hence write down for each activity its earliest start time and latest finish time.



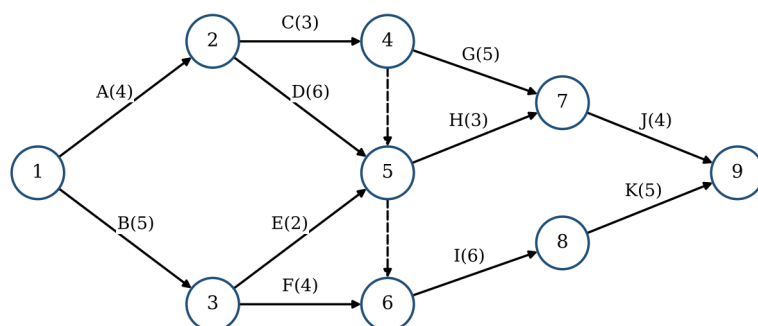
[8]

- 23** A project has the activity network shown (days). The project must be completed in the minimum possible time.
Find the earliest and latest event times and state the minimum time.



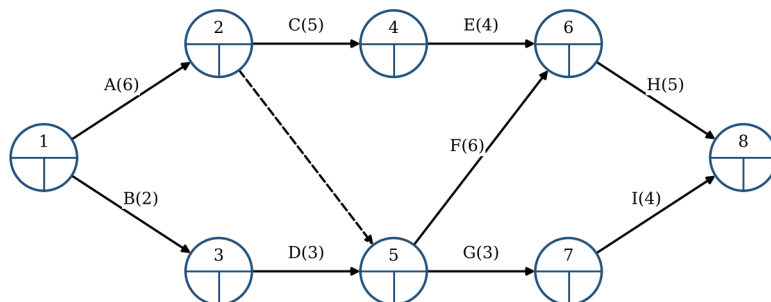
[7]

- 24** In the network shown, the duration of activity G is changed from 5 days to y days (the other durations are unchanged).
Find the range of values of y for which the minimum project time stays at 21 days.



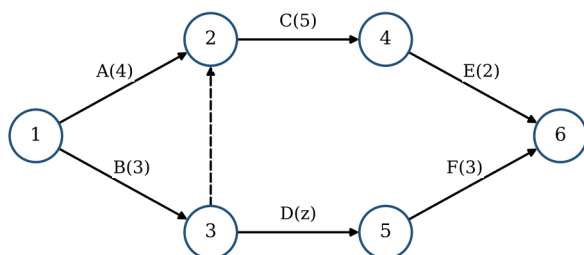
[6]

- 25** The activity network shown contains a dummy activity. Durations are in days.
Find the earliest and latest event times and state the minimum project time and critical events.



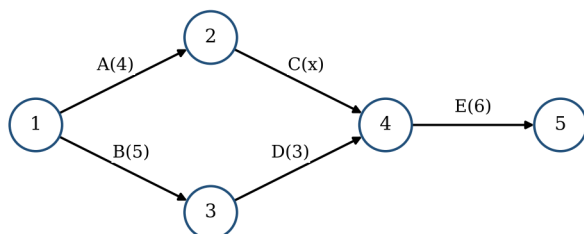
[6]

- 26** In the network shown, activity D has duration z days.
Find the minimum project time when $z = 9$, and the range of values of z for which the minimum project time is 11 days.



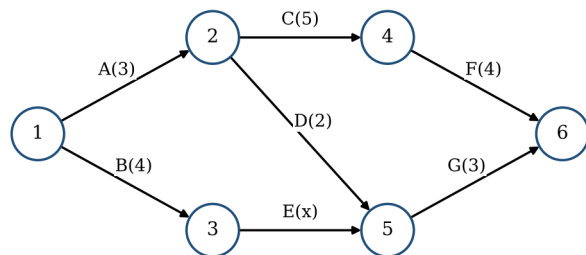
[5]

- 27** In the activity network shown, activity C has duration x days, where $x > 0$. The durations of the other activities are as marked (days).



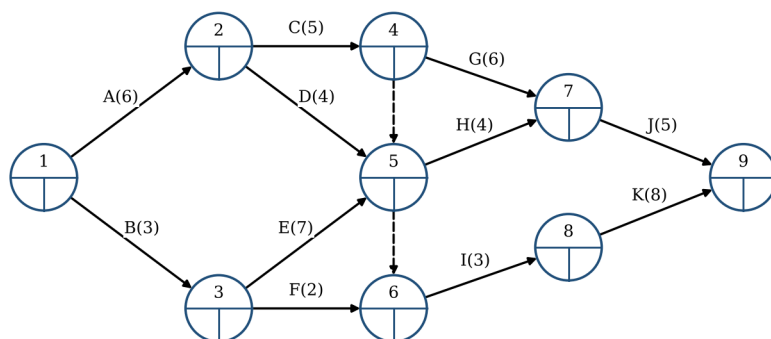
- (a) Find the value of x for which both A, C, E and B, D, E are critical paths. [2]
- (b) Find the minimum project time in terms of x for all $x > 0$, stating the values of x for which each expression applies. [3]
- (c) For $x = 2$, find the latest event time at event 2, and explain what it tells you about activity C. [3]

- 28** The activity network shown has activity E with duration x days ($x > 0$), and the other durations as marked (days).
Find the minimum project time in terms of x , giving the values of x for which each expression applies.



[7]

- 29** For the network shown the minimum project time is 22 days, but the project is only required to be completed by day 26, so the latest time at the final event is 26.



- (a) Find the earliest event times. [4]
 (b) Find the latest event times for the deadline of day 26. [4]
 (c) Explain why no event is now critical, and state the amount by which each event could be delayed. [2]

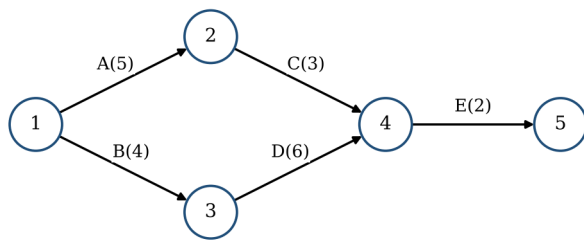
- 30** In an activity network every activity has a non-negative duration. Let $e(v)$ and $l(v)$ be the earliest and latest event times at event v when the project's deadline equals its minimum completion time T .

- (a) Prove that $e(v) \leq l(v)$ for every event v . [3]
 (b) Prove that if $e(v) = l(v)$ then v lies on a critical path (a path of length T). [4]

Critical Paths

- 1** Explain what is meant by (a) the critical path, (b) a critical activity. [2]
2 A project network has three paths from start to finish, of lengths 14, 19 and 17 days. Write down the minimum completion time of the project, and say which path is critical. [2]
3 An activity on the critical path is delayed by 3 days. State the effect on the minimum completion time of the project. [1]

- 4 The activity network shows a project. The path A, C, E has length 10 days and the path B, D, E has length 12 days. State the critical path and the minimum completion time.



[2]

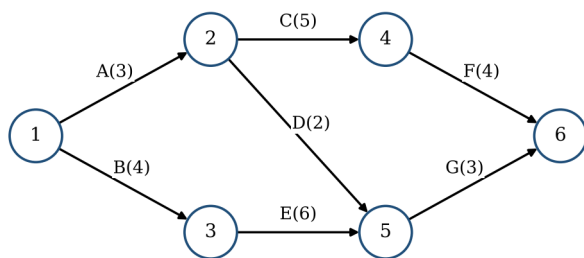
- 5 Can a project have more than one critical path? Explain your answer.

[2]

- 6 Activity X is on the critical path and has duration 5 days. Its earliest start time is 8 days. State its latest finish time.

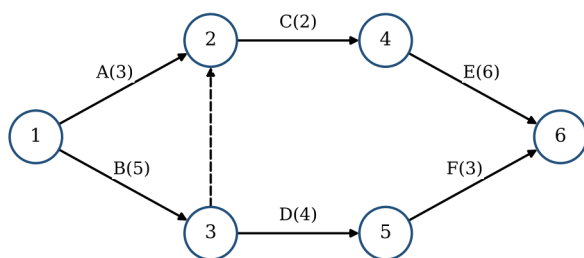
[2]

- 7 The activity network shows a project (durations in days). List all the paths from start to finish with their lengths, and state the critical path and the minimum project time.



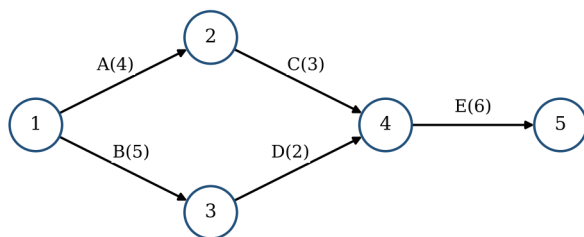
[4]

- 8 The activity network shown has a dummy activity (duration 0). Durations are in days. Find the critical path and the minimum project time by listing all the paths.



[4]

- 9 The activity network shown has two critical paths. Durations are in days. Identify both critical paths, and state which activities are critical.



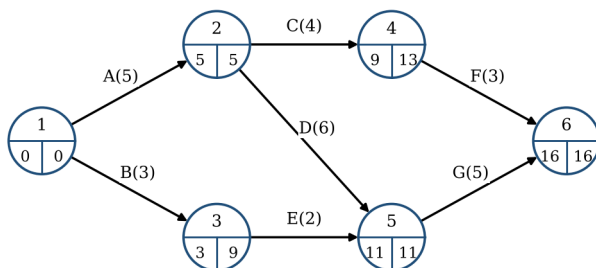
[3]

- 10 The table shows, for each activity of a project, its duration, earliest start time and latest finish time (days). Identify the critical activities, giving a reason.

Activity	Duration	Earliest start	Latest finish
A	5	0	5
B	3	0	9
C	4	5	13
D	6	5	11
E	2	3	11
F	3	9	16
G	5	11	16

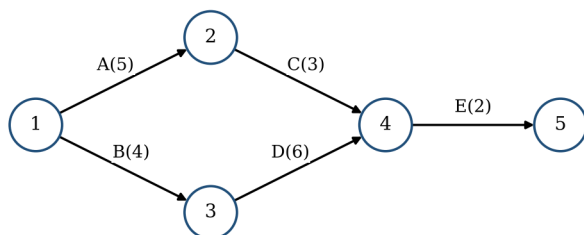
[3]

- 11 The earliest and latest event times for a project are shown in the network (days). Identify the critical activities, explaining your method, and state the minimum project time.



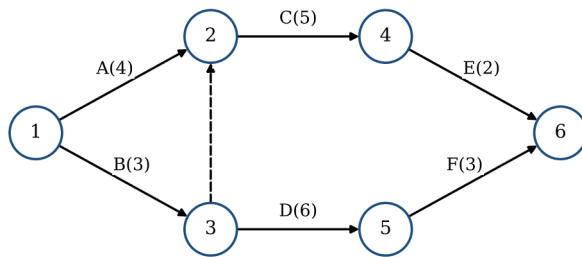
[3]

- 12 In the activity network shown, A, B, C, D and E have durations 5, 4, 3, 6 and 2 days. Find the critical path. Activity D is delayed by 2 days: find the new minimum completion time.



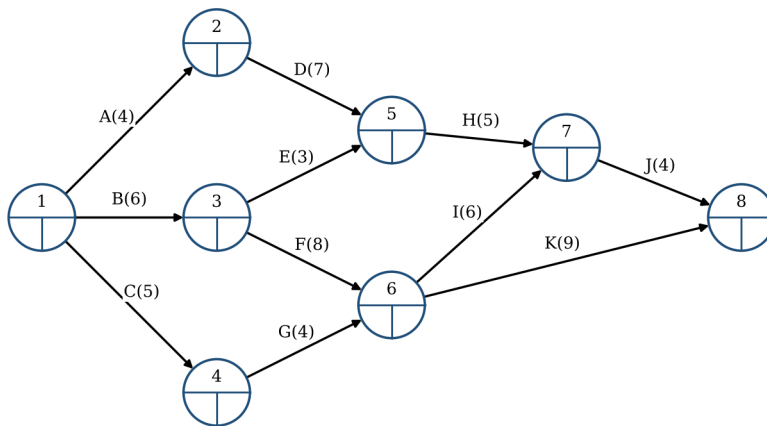
[3]

- 13** The activity network shown has critical path B, D, F. The minimum project time is 12 days. Activity D is shortened by 2 days. Find the new minimum project time, and explain why it is not 10 days.



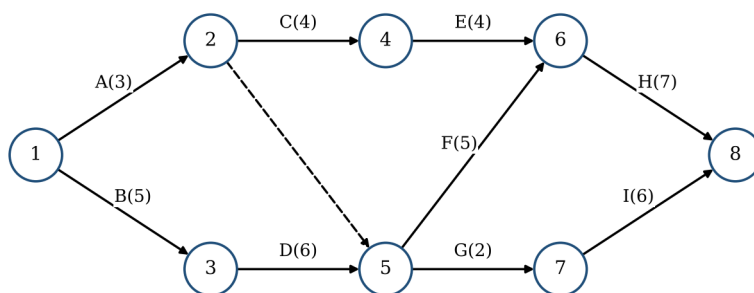
[4]

- 14** The activity network shown has durations in days. Find the earliest and latest event times, hence identify the critical activities, and state the minimum project time.



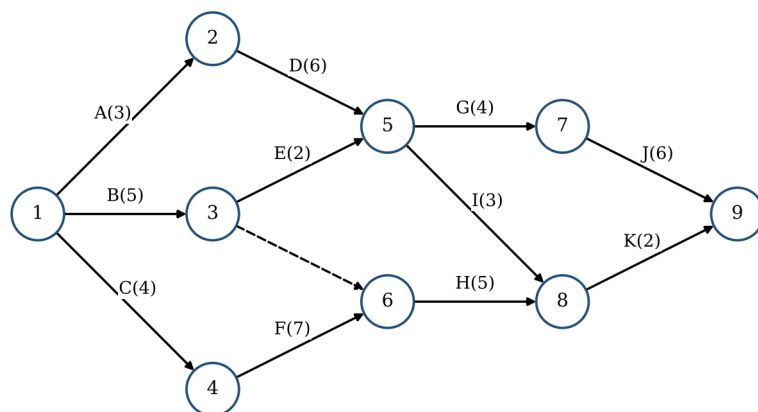
[6]

- 15** The activity network shown has a dummy. Durations are in days. List all the paths, and state the critical path and minimum project time.



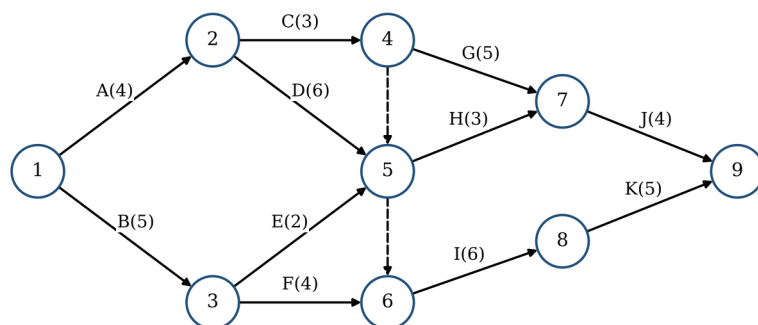
[5]

- 16** The activity network shown has durations in days. The activity E has its duration increased by 6 days.
Find the effect on the minimum project time. Show your working.



[5]

- 17** A project has the activity network shown (days). Activity I (6 days) is on the critical path A, D, I, K, which has length 21 days.
Find by how many days activity I can be shortened before the minimum project time stops falling, and state the minimum project time then.

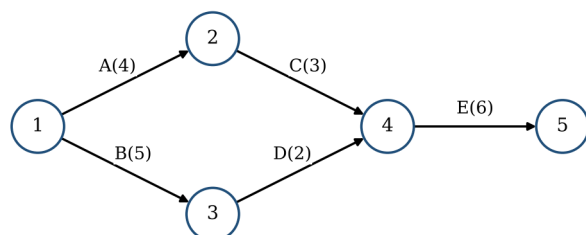


[5]

- 18** A student says: "The activity with the longest duration must be critical." A project has activities A to E, where A and B have nothing before them, C needs A, D needs B, and E needs both C and D. The durations are A 7, B 3, C 1, D 6 and E 2 days.
Show that the student is wrong.

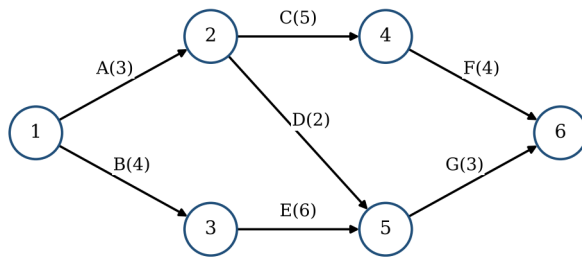
[3]

- 19** The activity network shown has two critical paths.
State whether the minimum project time can be reduced by shortening activity C alone, and explain your answer.



[3]

- 20** The activity network shown has durations in days.
Show that no single activity can be shortened so as to reduce the minimum project time by 2 days.



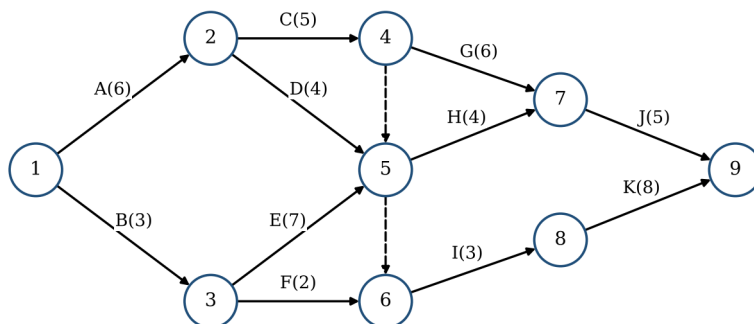
[4]

- 21** The table shows the activities of a project with durations (days) and immediately preceding activities.
Find the earliest start time of each activity, the minimum project time and the critical activities.

Activity	Duration	Preceded by
A	3	-
B	5	-
C	4	-
D	6	A
E	2	B
F	7	C
G	4	D, E
H	5	B, F
I	3	D, E
J	6	G
K	2	H, I

[7]

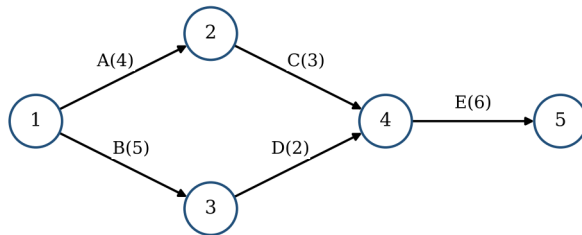
- 22** The activity network shown has durations in days.
Find all the critical paths and the activities which lie on every critical path.



[6]

- 23** The activity network shown has two critical paths and minimum time 13 days. The time can be reduced by shortening activities, at the cost shown per day saved (up to the maximum reduction).

Find the cheapest way to reduce the project time to 12 days, and its cost.

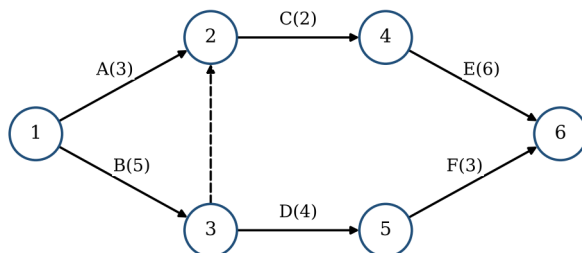


Activity	Max cut (days)	Cost per day (£)
A	2	30
B	2	40
C	1	50
D	1	35
E	2	90

[6]

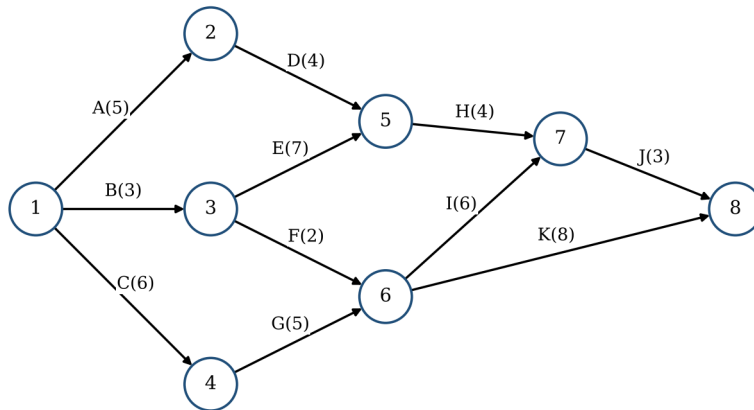
- 24** In the activity network shown the critical path is B, C, E, passing through the dummy activity (duration 0).

Show that this is the critical path, and explain why the dummy activity is on the critical path although it has duration 0.



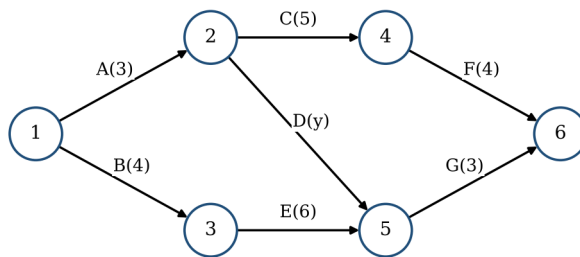
[5]

- 25** A project has the activity network shown (days).
Find the critical path. Find the greatest duration that activity K could have without increasing the minimum project time, and say which path becomes critical if K takes 1 day longer than this.



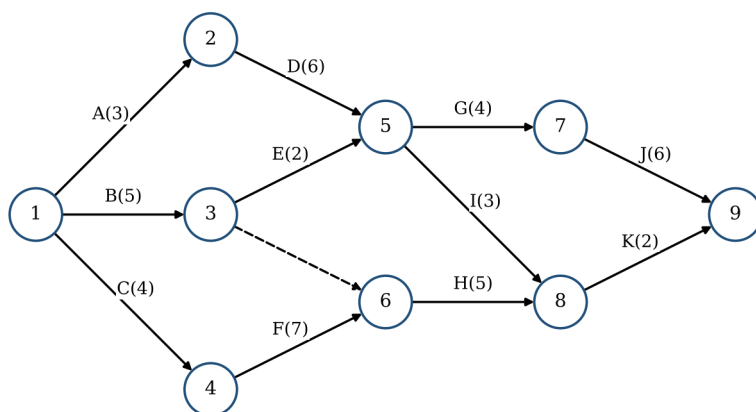
[6]

- 26** In the activity network shown, activity D has duration y days (the other durations are as marked, in days).
Find the minimum project time when $y = 10$, and the range of values of y for which D is not on any critical path.



[5]

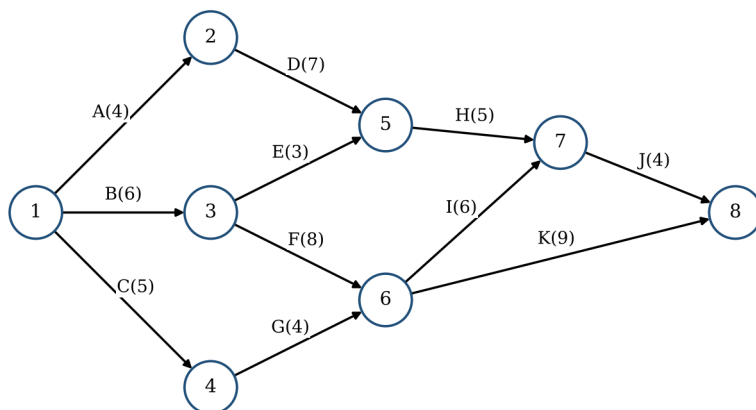
- 27** The activity network shown has minimum project time 19 days. Some activities can be shortened, at the cost per day shown in the table, up to the maximum reduction (days). The other activities cannot be shortened.



Activity	Max cut (days)	Cost per day (£)
A	1	60
C	1	20
D	2	45
F	2	35
G	2	55
H	1	40
J	2	30
K	1	25

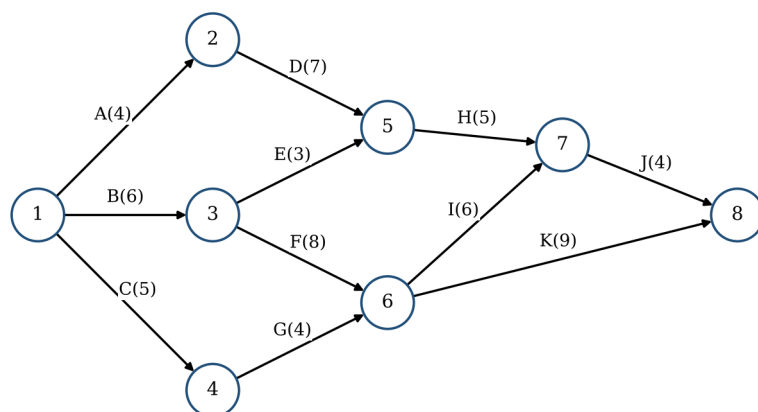
- (a) Find the cheapest way of reducing the project time to 18 days, and the cost. [3]
 (b) Find the cheapest way of reducing the project time to 17 days, and the cost. [4]
 (c) Find the cheapest way of reducing the project time to 16 days, and the cost. [5]

- 28** The activity network shown has durations in days. The duration of activity F is changed to x days, the others being unchanged.



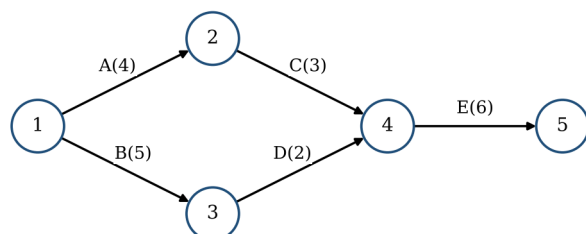
- (a) Find the minimum project time in terms of x , for $x > 0$. [5]
 (b) Hence state the range of x for which F is not critical, and find the minimum project time for $x = 9$. [2]

- 29** For the network shown, activity E has duration x days (instead of 3), the other durations unchanged (days).



- (a) Show that the minimum project time is 24 days when $x \leq 9$. [3]
 (b) Find the minimum project time for $x = 12$, and state the new critical path. [3]

- 30** The activity network shown has two critical paths and minimum time 13 days. Activities can be shortened at the costs per day shown in the table, up to the maximum reductions.



Activity	Max cut (days)	Cost per day (£)
A	2	30
B	2	40
C	1	50
D	1	35
E	2	90

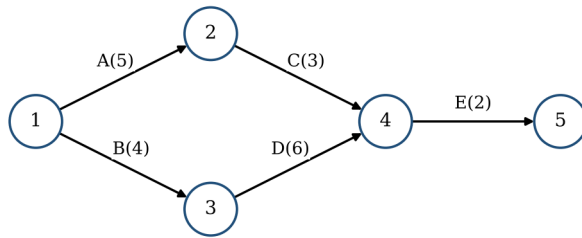
- (a) Show that shortening E by 2 days costs £180, and find the cheaper way of reaching 11 days. [5]
 (b) Explain why a mixed plan using E for one day and other activities for the second day is not cheaper. [3]

Float

- Define the total float of an activity, and write down a formula for it. [2]
- An activity has earliest start time 6, duration 4 and latest finish time 16. Find its total float. [2]
- What is the total float of an activity on the critical path? Explain your answer. [2]
- An activity has total float 3 days. State whether the project is delayed if the activity takes (a) 2 days longer than planned, (b) 5 days longer than planned. [2]
- An activity has earliest start time 5, duration 7 and latest finish time 12. Show that it is critical. [2]

6 The latest start time of an activity is 9 and its earliest start time is 4. State its total float. [1]

7 The activity network shown has durations in days. The minimum project time is 12 days along B, D, E.
Find the total float of each activity.



[4]

8 The table gives the duration, earliest start time and latest finish time (days) of each activity of a project, and the total float.

Check the total float of activity C and find the latest start time of C.

Activity	Duration	Earliest start	Latest finish	Total float
A	5	0	5	0
B	3	0	9	6
C	4	5	13	4
D	6	5	11	0
E	2	3	11	6
F	3	9	16	4
G	5	11	16	0

[3]

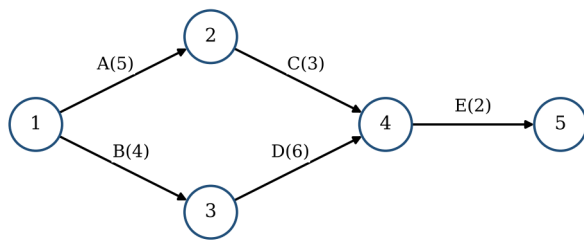
9 For each of the activities in the table, the duration, earliest start time and latest finish time are given (days).

Complete the table by finding the total float of each activity, and hence list the critical activities.

Activity	Duration	Earliest start	Latest finish
A	5	0	5
B	3	0	9
C	4	5	13
D	6	5	11
E	2	3	11
F	3	9	16
G	5	11	16

[4]

- 10** In the activity network shown (durations in days), activity C starts as early as possible but then takes 4 days instead of 3. Show that the project is not delayed, and find how many days of float remain for C.

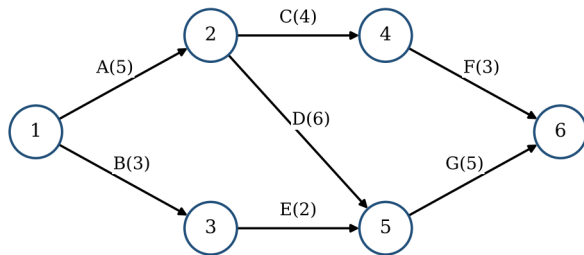


[3]

- 11** Explain why the total float of an activity cannot be negative in an activity network drawn for the minimum project time.

[2]

- 12** The activity network shown has total floats: A 0, B 6, C 4, D 0, E 6, F 4, G 0 (days). The minimum project time is 16 days. Write down the critical activities and state the greatest delay that activity E could have without increasing the project time.

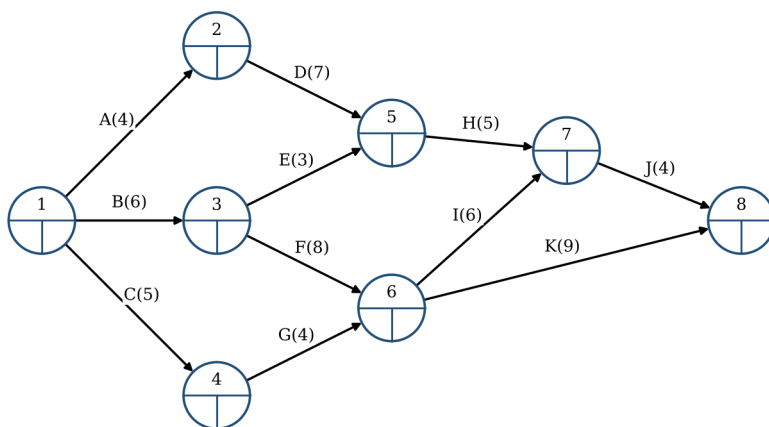


[3]

- 13** An activity has earliest start time 3, duration 5 and latest finish time 14. Find its total float and its latest start time.

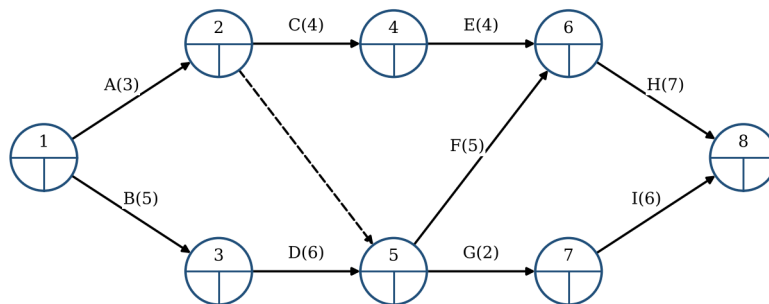
[3]

- 14** The activity network shown has durations in days. Find the total float of each activity, using the earliest and latest event times (shown in the answer).



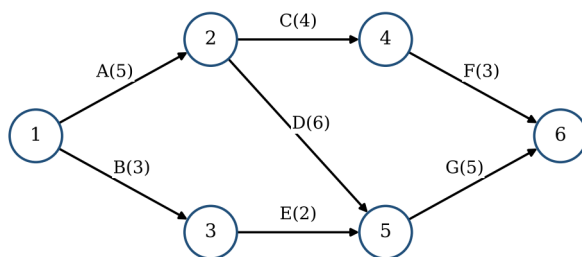
[6]

- 15** The activity network shown has a dummy activity. Durations are in days. Find the total float of each activity and list the critical activities.



[6]

- 16** The total floats of the activities of a project are A 0, B 6, C 4, D 0, E 6, F 4, G 0 (days). Activity C takes 3 days longer than planned and activity E takes 7 days longer than planned, all other activities taking the planned time. State, with reasons, whether the minimum project time of 16 days is affected.



[4]

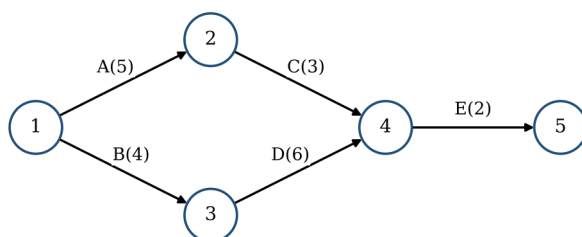
- 17** An activity has duration 6 days and earliest start time 9 days. Its total float is 5 days. Find (a) its latest start time, (b) its latest finish time, (c) the earliest time at which it can finish.

[3]

- 18** The table gives the duration, earliest start time and latest finish time (days) of an activity P: duration x , earliest start 8, latest finish 20, and total float 5. Find x , and then the latest start time of P.

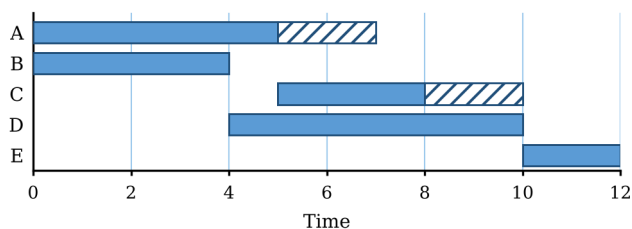
[3]

- 19** Draw a cascade chart for the project shown, in which each activity is started at its earliest time, with float shown as a separate bar after the activity. The durations are in days.



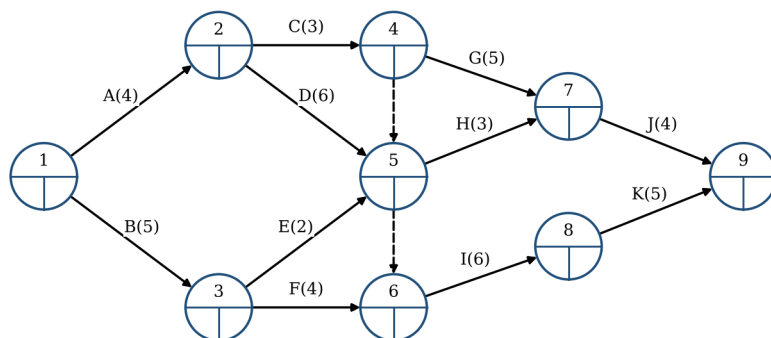
[4]

- 20** A cascade chart shows the activities of a project, each starting at its earliest time, with float shown by the hatched bars. Read from the chart the duration and float of each activity, and write down the critical activities.



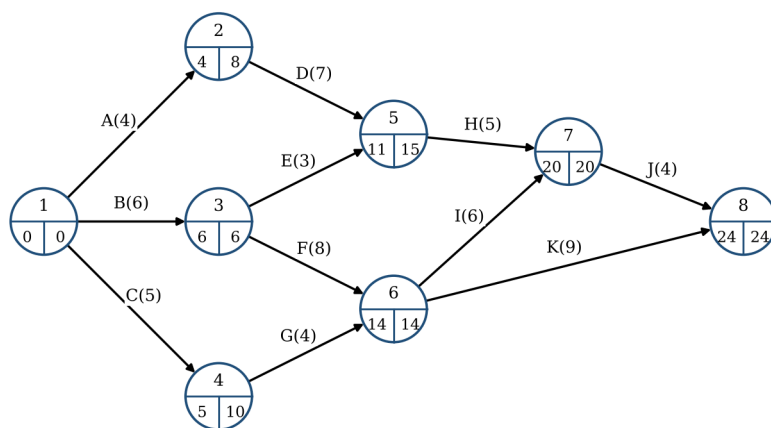
[4]

- 21** The activity network shown has two dummy activities. Durations are in days. Find the total float of each activity, and state which activities are critical.



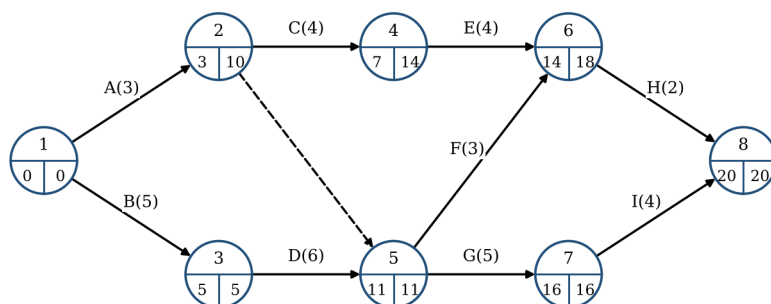
[7]

- 22** The activity network shown has earliest and latest event times marked. The free float of an activity is (earliest time at its end event) – (earliest time at its start event) – duration. The independent float is (earliest time at its end event) – (latest time at its start event) – duration, or 0 if this is negative. Find the total float, the free float and the independent float of activities E, G, K.



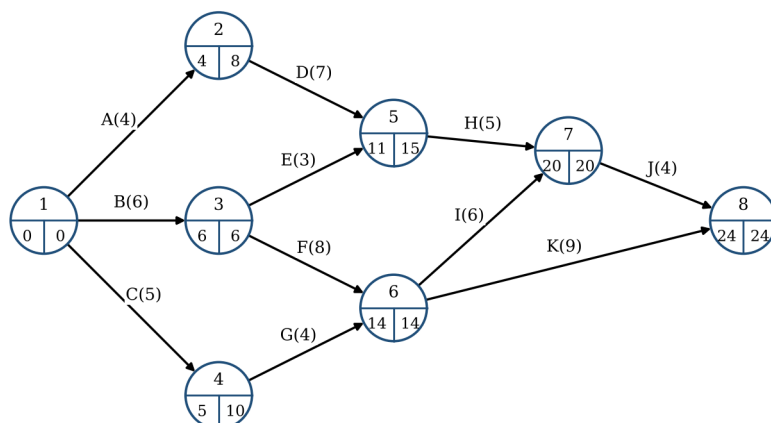
[8]

- 23** In the activity network shown, activities F and H are consecutive and neither is critical. The minimum project time is 20 days.
Activity F is delayed by 3 days.
Show that the project time is unchanged, find the new total float of H, and explain why the float of H has changed.



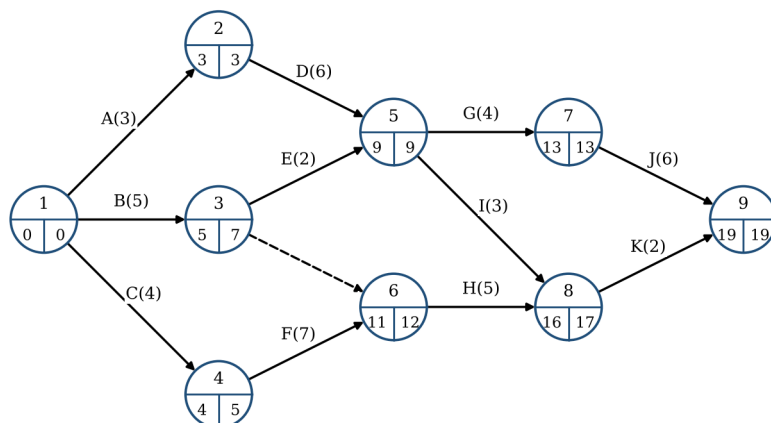
[6]

- 24** In the activity network shown, durations are in days and the minimum project time is 24 days.
Activity C takes 8 days longer than planned.
Find the new minimum project time and state the new critical path(s).



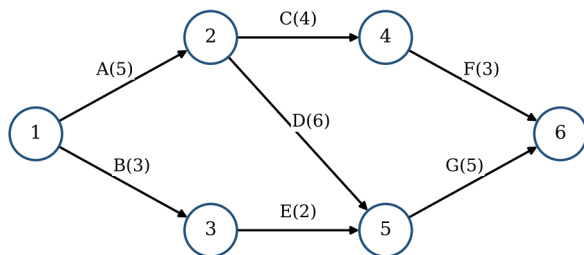
[5]

- 25** In the activity network shown, durations are in days and the minimum project time is 19 days.
Activities B and F are each delayed by 2 days.
Find the new minimum project time, and explain why it is not simply $19 + 2 + 2$.



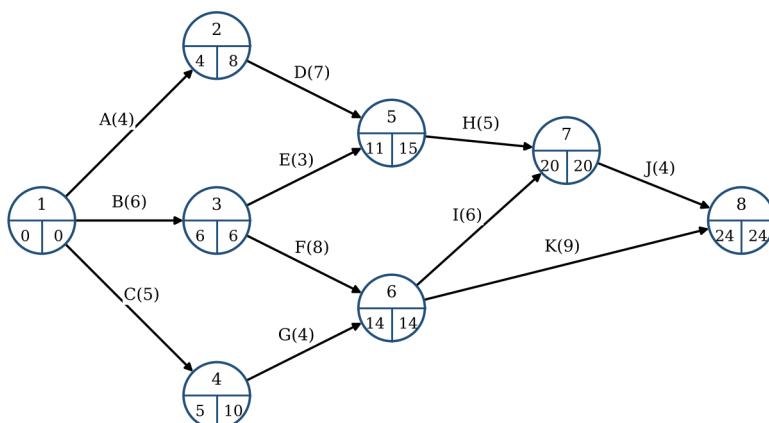
[5]

- 26** In the activity network shown, durations are in days and the minimum project time is 16 days. The duration of activity C is changed to x days. Find the greatest value of x for which the minimum project time is unchanged, and the critical paths when x takes this value.



[5]

- 27** In the activity network shown, the duration of activity H is x days (it is shown as 5 days). All other durations are in days. The minimum project time is 24 days when $x = 5$, and H has total float 4 days. Find the minimum project time in terms of x , and find the range of values of x for which H is critical.

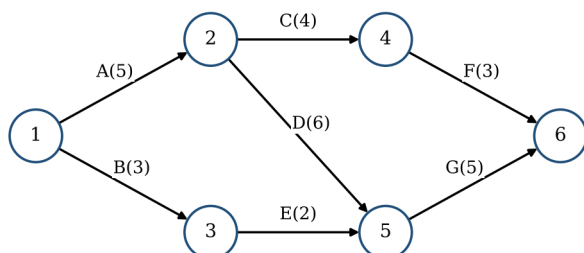


[6]

- 28** For an activity with start event S and end event E , let e and l denote earliest and latest event times, and let d be the duration. Prove that $(\text{independent float}) \leq (\text{free float}) \leq (\text{total float})$ for every activity, where $\text{total float} = l(E) - e(S) - d$, $\text{free float} = e(E) - e(S) - d$ and $\text{independent float} = \max(0, e(E) - l(S) - d)$.

[5]

- 29** In the activity network shown the duration of activity D is x days (it is shown as 6 days), where $x \geq 0$. Other durations are as shown.

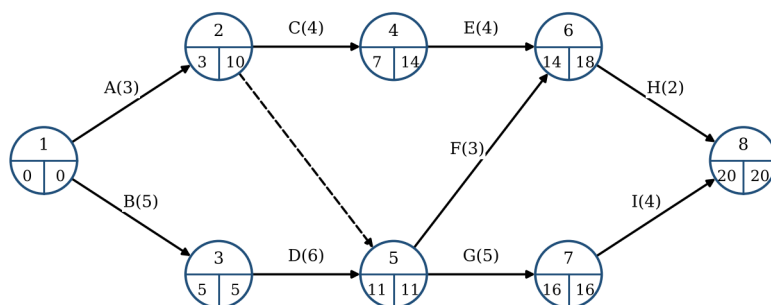


- (a) Find the minimum project time in terms of x .
 (b) State the critical path(s) when (i) $x = 1$, (ii) $x = 2$, (iii) $x = 5$.

[4]

[3]

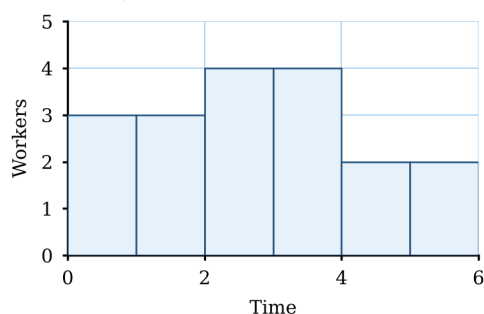
- 30** In the activity network shown, F and H each have total float 4 days, and the minimum project time is 20 days.
- (a) A student says: "F can be delayed by 4 days and H can be delayed by 4 days, so the two together can be delayed by 8 days without delaying the project." Show that this is wrong.
- (b) State the greatest total delay that can be shared between F and H without extending the project, and explain why.



[6]

Resource Scheduling

- | | | |
|---|--|-----|
| 1 | Describe a resource histogram for a project. | [2] |
| 2 | A project needs 36 worker-days of work in total. Find the shortest possible time to complete it with 4 workers, if the work could be shared perfectly. | [2] |
| 3 | A project needs 45 worker-days of work in total and its longest path is 12 days. Find the least number of workers needed to complete it in 12 days. | [2] |
| 4 | Give one reason why the lower bound (total worker-days divided by the number of days) may not be achievable in practice. | [2] |
| 5 | The resource histogram shows the number of workers needed in each day of a project. State the greatest number of workers needed at one time, and find the total number of worker-days. | |



[3]

- 6** Activity A needs 2 workers for 5 days and activity B needs 3 workers for 4 days. Find the number of worker-days for each activity and the total. [2]

- 7 The table shows the activities of a project with their durations, immediate predecessors and the number of workers each needs. The minimum completion time of the project is 12 days. Find the total number of worker-days, and the least number of workers needed to complete the project in 12 days if the work could be shared perfectly.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	4	-	1
C	3	A	2
D	6	B	3
E	2	C, D	1

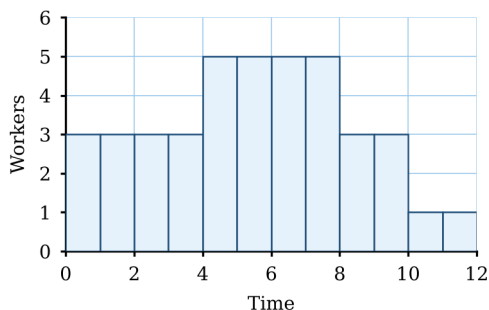
[4]

- 8 A project has the activities, durations (days), precedences and numbers of workers given in the table. Each activity starts as early as possible. Find the number of workers needed in each interval of the project and draw the resource histogram.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	4	-	1
C	3	A	2
D	6	B	3
E	2	C, D	1

[4]

- 9 The resource histogram shows the number of workers needed in each day when every activity of a project starts as early as possible.

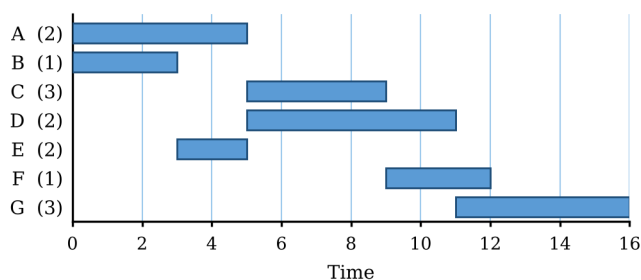


- (a) State the greatest number of workers needed, and the times at which this happens.
 (b) Find the total number of worker-days.

[2]

[2]

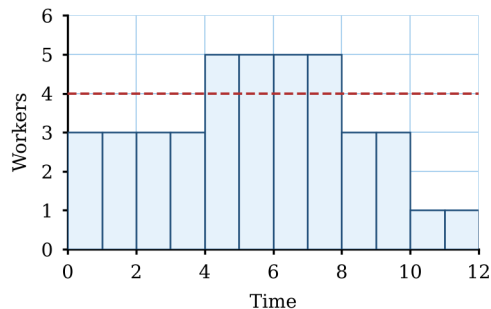
- 10 The cascade chart shows a project with every activity starting as early as possible. The number of workers needed by each activity is shown in brackets after its label. Find the total number of workers needed (a) between times 5 and 6, (b) between times 9 and 10.



[3]

- 11** A project has minimum completion time 16 days and needs a total of 59 worker-days of work. Show that at least 4 workers are needed to complete the project in 16 days. [3]

- 12** The resource histogram shows the number of workers needed in each day of a project when each activity starts as early as possible. There are only 4 workers available. Find the times during which the schedule needs more than 4 workers. [3]



- 13** The table gives a project with its durations, precedences and numbers of workers. The minimum completion time is 13 days. Find the total number of worker-days, and hence the least number of workers needed to complete the project in 13 days. [3]

Activity	Duration	Preceded by	Workers
A	4	-	2
B	5	-	3
C	3	A, B	1
D	6	B	2
E	4	C	2
F	2	D	3

- 14** A project has the activities, durations (days), precedences and numbers of workers given in the table. Each activity starts as early as possible. Find the number of workers needed in each interval of the project and draw the resource histogram. State the greatest number of workers needed at any time. [4]

Activity	Duration	Preceded by	Workers
A	5	-	2
B	3	-	1
C	4	A	3
D	6	A	2
E	2	B	2
F	3	C	1
G	5	D, E	3

- 15** A project has the activities shown in the table. Each activity starts as early as possible. Find the number of workers needed in each interval of the project and draw the resource histogram. State the greatest number of workers needed at any time.

Activity	Duration	Preceded by	Workers
A	4	-	2
B	5	-	3
C	3	A, B	1
D	6	B	2
E	4	C	2
F	2	D	3

[4]

- 16** The table shows a project. Its minimum completion time is 12 days, and the schedule with every activity starting as early as possible needs up to 4 workers. Show that by delaying activities B and D by 2 days each, the project can still be completed in 12 days using at most 3 workers.

Activity	Duration	Preceded by	Workers
A	2	-	3
B	2	-	1
C	6	A	2
D	4	B	1
E	4	C, D	1

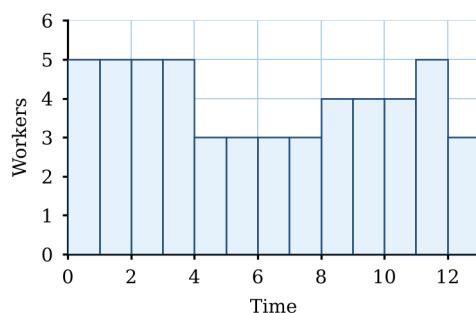
[5]

- 17** The table gives a project whose minimum completion time is 16 days.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	3	-	1
C	4	A	3
D	6	A	2
E	2	B	2
F	3	C	1
G	5	D, E	3

- (a) Find the least number of workers that could possibly complete the project in 16 days. [2]
 (b) Show that 5 workers are enough when every activity starts as early as possible. [2]

- 18** The resource histogram shows the number of workers needed each day for a project, with each activity starting as early as possible. The minimum completion time is 13 days. Find the total number of worker-days and the least number of workers that could complete the project in 13 days.



[3]

- 19** The table gives the activities of a project. The minimum completion time is 24 days.

Activity	Duration	Preceded by	Workers
A	4	-	2
B	6	-	3
C	5	-	1
D	7	A	2
E	3	B	2
F	8	B	3
G	4	C	1
H	5	D, E	2
I	6	F, G	3
J	4	H, I	1
K	9	F, G	2

- (a) Find the total number of worker-days. [2]
 (b) Find the least number of workers that could complete the project in 24 days. [2]

- 20** The table shows a project with the number of workers each activity needs. Draw a cascade chart for the project with every activity starting as early as possible, showing the floats.

Activity	Duration	Preceded by	Workers
A	2	-	3
B	2	-	1
C	6	A	2
D	4	B	1
E	4	C, D	1

[4]

- 21** The table gives a project in which activities A to E need 2, 1, 2, 3 and 1 workers respectively. Only 3 workers are available.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	4	-	1
C	3	A	2
D	6	B	3
E	2	C, D	1

- (a) Explain why no other activity can take place at the same time as D. [1]
 (b) Show that the project cannot be completed in fewer than 16 days, and give a schedule that takes 16 days. [6]

- 22** The table gives a project with the number of workers each activity needs. Only 4 workers are available.
Find a schedule that completes the project in 20 days using at most 4 workers at any time, and show it on a resource histogram.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	3	-	1
C	4	A	3
D	6	A	2
E	2	B	2
F	3	C	1
G	5	D, E	3

[5]

- 23** The table shows a project in which the minimum completion time is 16 days if there are enough workers (the critical activities are A, D and G). Only 4 workers are available.
Show that the project cannot be completed in 16 days with only 4 workers.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	3	-	1
C	4	A	3
D	6	A	2
E	2	B	2
F	3	C	1
G	5	D, E	3

[5]

- 24** The table gives a project with minimum completion time 12 days (critical activities B, D, E). Activities A to E need 2, 1, 2, 3 and 1 workers respectively.

Activity	Duration	Preceded by	Workers
A	5	-	2
B	4	-	1
C	3	A	2
D	6	B	3
E	2	C, D	1

- (a) Show that 4 workers are not enough to complete the project in 12 days.
(b) Show that 5 workers are enough.

[4]

[2]

- 25** The table gives a project that takes at least 20 days. Each activity starting as early as possible needs up to 6 workers. Find start times for the activities so that the project is still completed in 20 days but needs at most 4 workers at any time. (Activities may not start before their predecessors have finished.)

Activity	Duration	Preceded by	Workers
A	7	-	3
B	3	-	1
C	6	A, B	1
D	5	B	3
E	7	C	1
F	2	D	2

[5]

- 26** The table gives a project in which only 4 workers are available at any time. Find a schedule that completes the project in 17 days, and show that it never needs more than 4 workers.

Activity	Duration	Preceded by	Workers
A	4	-	2
B	5	-	3
C	3	A, B	1
D	6	B	2
E	4	C	2
F	2	D	3

[5]

- 27** The table gives a project with the numbers of workers each activity needs. Only 5 workers are available.

Activity	Duration	Preceded by	Workers
A	4	-	2
B	6	-	3
C	5	-	1
D	7	A	2
E	3	B	2
F	8	B	3
G	4	C	1
H	5	D, E	2
I	6	F, G	3
J	4	H, I	1
K	9	F, G	2

- (a) Find a lower bound for the completion time. [2]
- (b) At each integer time, activities whose predecessors are all complete are started in order of increasing latest start time (alphabetical order if tied), if enough workers are free. Apply this rule and find the completion time. [4]
- (c) Compare your answers to (a) and (b), and comment. [1]

- 28** The table gives the activities of a project. When every activity starts as early as possible, up to 8 workers are needed at once, but only 7 are available. Find a schedule using at most 7 workers that still completes the project in the minimum time of 24 days, and verify it.

Activity	Duration	Preceded by	Workers
A	4	-	2
B	6	-	3
C	5	-	1
D	7	A	2
E	3	B	2
F	8	B	3
G	4	C	1
H	5	D, E	2
I	6	F, G	3
J	4	H, I	1
K	9	F, G	2

[6]

- 29** A project needs W worker-days of work in total and its longest path is T days.
- (a) Prove that with k workers the completion time is at least the larger of T and $W \div k$. [3]
- (b) A project has 40 worker-days of work in total and a longest path of 12 days. Find the lower bound for the completion time with 4 workers, and give a reason why the project might not be completed in this time. [2]

- 30** A project needs 129 worker-days in total and has a minimum completion time of 24 days. The shortest completion times possible with different numbers of workers are shown in the table. Each worker is paid for every day of the project, at £90 per day. Find the number of workers that gives the lowest total labour cost, and the cost. Give a reason why the company might choose a different number of workers.

Workers	Completion time (days)
3	48
4	40
5	29
6	27
7	24

[5]