

Angular Velocity

1 (2 marks)

$$(a) 30 \times \frac{2\pi}{60} = \pi$$

B1

Answer: $\pi \text{ rad s}^{-1}$

1 mark

$$(b) 150 \times \frac{2\pi}{60} = 5\pi$$

B1

Answer: $5\pi \text{ rad s}^{-1}$

1 mark

2 (2 marks)

$$v = r\omega = 2.5 \times 4$$

M1

$$= 10 \text{ m s}^{-1}$$

A1

Answer: 10 m s^{-1}

2 marks

3 (2 marks)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8}$$

M1

$$= \frac{5\pi}{2} \text{ rad s}^{-1} (7.85 \text{ rad s}^{-1})$$

A1

Answer: $\frac{5\pi}{2} \text{ rad s}^{-1}$

2 marks

4 (2 marks)

$$\omega = \frac{v}{r} = \frac{12}{3}$$

M1

$$= 4 \text{ rad s}^{-1}$$

A1

Answer: 4 rad s^{-1}

2 marks

5 (2 marks)

$$6 \text{ rad s}^{-1} = \frac{6 \times 60}{2\pi} \text{ revolutions per minute}$$

M1

$$= 57.3 \text{ revolutions per minute}$$

A1

Answer: $57.3 \text{ rev min}^{-1}$

2 marks

6 (2 marks)

$$(a) 3.5 \times 2\pi = 7\pi$$

B1

Answer: $7\pi \text{ rad}$

1 mark

$$(b) 3.5 \times 360 = 1260^\circ$$

B1

Answer: 1260°

1 mark

7 (4 marks)

$$(a) \omega = \frac{v}{r} = \frac{5}{0.35}$$

M1

$$= 14.3 \text{ rad s}^{-1}$$

A1

Answer: 14.3 rad s^{-1}

2 marks

$$(b) \text{Revolutions} = \frac{\omega \times 60}{2\pi} = \frac{14.3 \times 60}{2\pi}$$

M1

$$= 136$$

A1

Answer: 136

2 marks

8 (4 marks)

$$(a) \omega = 150 \times \frac{2\pi}{60}$$

M1

$$= 5\pi = 15.7 \text{ rad s}^{-1}$$

A1

Answer: $5\pi = 15.7 \text{ rad s}^{-1}$

2 marks

$$(b) v = r\omega = 0.6 \times 5\pi$$

M1

$$= 3\pi = 9.42 \text{ m s}^{-1}$$

A1

Answer: $3\pi = 9.42 \text{ m s}^{-1}$ **2 marks****9 (4 marks)**

(a) $\omega = \frac{2\pi}{24 \times 3600}$

M1

$= 7.27 \times 10^{-5} \text{ rad s}^{-1}$

A1**Answer: $7.27 \times 10^{-5} \text{ rad s}^{-1}$** **2 marks**

(b) $v = r\omega = 6.37 \times 10^6 \times 7.27 \times 10^{-5}$

M1

$= 463 \text{ m s}^{-1}$

A1**Answer: 463 m s^{-1}** **2 marks****10 (4 marks)**

(a) $T = \frac{2\pi r}{v} = \frac{8\pi}{6}$

M1

$= 4.19 \text{ s}$

A1**Answer: 4.19 s** **2 marks**

(b) $\omega = \frac{6}{4} = 1.5 \text{ rad s}^{-1}$

M1

$t = \frac{2}{1.5} = 1.33 \text{ s}$

A1**Answer: 1.33 s** **2 marks****11 (4 marks)**

(a) $\omega = 45 \times \frac{2\pi}{60} = \frac{3\pi}{2}$

M1

$= 4.71 \text{ rad s}^{-1}$

A1**Answer: $\frac{3\pi}{2} = 4.71 \text{ rad s}^{-1}$** **2 marks**

(b) $v_P = 0.4 \times 4.71 = 1.88 \text{ m s}^{-1}$

M1

$v_Q = 0.9 \times 4.71 = 4.24 \text{ m s}^{-1}$

A1**Answer: P: 1.88 m s^{-1} , Q: 4.24 m s^{-1}** **2 marks****12 (3 marks)**

$v_B - v_A = 5\omega - 2\omega = 3\omega$

M1

$3\omega = 3 \text{ so } \omega = 1 \text{ rad s}^{-1}$

A1

$v_A = 2 \times 1 = 2 \text{ m s}^{-1}$

A1**Answer: $\omega = 1 \text{ rad s}^{-1}$, $v_A = 2 \text{ m s}^{-1}$** **3 marks****13 (4 marks)**

(a) $270^\circ = \frac{3\pi}{2} \text{ rad}$

B1

$t = \frac{3\pi}{2 \div 0.75} = 6.28 \text{ s}$

M1**Answer: 6.28 s** **2 marks**

(b) Angle turned $= 0.75 \times 60 = 45 \text{ rad}$, and $45 \div 2\pi = 7.16$

M1

So 7 complete revolutions

A1**Answer: 7****2 marks****14 (7 marks)**

(a) $\omega = \frac{2\pi}{90} = 0.0698 \text{ rad s}^{-1}$

M1

$v = 25\omega = 1.75 \text{ m s}^{-1}$

A1**Answer: $0.0698 \text{ rad s}^{-1}$, 1.75 m s^{-1}** **2 marks****(b)** The radius to the passenger makes angle θ with the downward vertical, so the passenger is $25\cos\theta$ below the centre**M1**Height above the ground $= 27 - 25\cos\theta$ metres (check: $\theta = 0$ gives 2 m, the lowest point)**A1****Answer: The height of the passenger above the ground is $27 - 25\cos\theta$ metres****2 marks**

(c) $27 - 25\cos\theta = 40$ so $\cos\theta = -0.52$

M1

$\theta = 2.12 \text{ rad}$

A1

$t = \frac{\theta}{\omega} = 30.3 \text{ s}$

A1

Answer: 30.3 s

3 marks

15 (4 marks)

(a) $\omega_A = 150 \times \frac{2\pi}{60} = 5\pi \text{ rad s}^{-1}$

M1

Belt speed $= 0.12 \times 5\pi = 1.88 \text{ m s}^{-1}$

A1

Answer: 1.88 m s⁻¹

2 marks

(b) Same belt speed: $5\omega_B = 12 \times 5\pi$ (in cm and rad s⁻¹)

M1

$\omega_B = 12\pi \text{ rad s}^{-1} = 360 \text{ revolutions per minute}$

A1

Answer: 360 rev min⁻¹

2 marks

16 (4 marks)

Radius of orbit $r = 6370 + 400 = 6770 \text{ km}$

B1

$\omega = \frac{2\pi}{92.5 \times 60} = 1.13 \times 10^{-3} \text{ rad s}^{-1}$

M1 A1

$v = r\omega = 6770 \times 1.132 \times 10^{-3} = 7.66 \text{ km s}^{-1}$

A1

Answer: 1.13 × 10⁻³ rad s⁻¹, 7.66 km s⁻¹

4 marks

17 (5 marks)

(a) Q gains 2π radians on P: $(3.5 - 2)t = 2\pi$

M1

$t = \frac{2\pi}{1.5} = 4.19 \text{ s}$

A1

Answer: 4.19 s

2 marks

(b) $(3.5 + 2)t = 2\pi$ so $t = 1.14 \text{ s}$

M1 A1

Distance $= 3 \times 2 \times 1.14 = 6.85 \text{ m}$

A1

Answer: 1.14 s, 6.85 m

3 marks

18 (6 marks)

(a) $|\mathbf{r}|^2 = 9\cos^2 2t + 9\sin^2 2t = 9$

M1

so $|\mathbf{r}| = 3 \text{ m}$, a circle centre O of radius 3 m

A1

Answer: circle, radius 3 m

2 marks

(b) $\mathbf{v} = -6 \sin 2t \mathbf{i} + 6 \cos 2t \mathbf{j}$

M1

$|\mathbf{v}| = 6 \text{ m s}^{-1}$, and $\omega = \frac{v}{r} = 2 \text{ rad s}^{-1}$

A1

Answer: 6 m s⁻¹, 2 rad s⁻¹

2 marks

(c) $\mathbf{r} \cdot \mathbf{v} = -18 \sin 2t \cos 2t + 18 \sin 2t \cos 2t$

M1

$= 0$, so the velocity is perpendicular to $\mathbf{r}$

A1

Answer: The velocity of P is always perpendicular to r

2 marks

19 (4 marks)

(a) $\omega = 2\pi \text{ rad s}^{-1}$; $v_P = 0.4 \times 2\pi$, $v_Q = 0.1 \times 2\pi$, in the same direction

M1

Relative speed $= 0.3 \times 2\pi = 1.88 \text{ m s}^{-1}$

A1

Answer: 1.88 m s⁻¹

2 marks

(b) The velocities are in opposite directions

M1

Relative speed $= (0.4 + 0.1) \times 2\pi = 3.14 \text{ m s}^{-1}$

A1

Answer: 3.14 m s⁻¹

2 marks

20 (4 marks)

(a) Minute hand: $\frac{2\pi}{3600} = \frac{\pi}{1800}$

B1

Hour hand: $\frac{2\pi}{43200} = \frac{\pi}{21600}$

B1

Answer: $\frac{\pi}{1800}, \frac{\pi}{21600} \text{ rad s}^{-1}$

2 marks

(b) Minute tip: $12 \times \frac{\pi}{1800} = \frac{\pi}{150}$; hour tip: $8 \times \frac{\pi}{21600} = \frac{\pi}{2700}$

M1

Ratio = $\frac{\pi}{150} \div \frac{\pi}{2700} = 18$

A1

Answer: The speed of the tip of the minute hand is 18 times the speed of the tip of the hour hand

2 marks**21 (8 marks)**

(a) Minute hand: one revolution per 60 minutes; hour hand: one revolution per 720 minutes

M1

Ratio of angular speeds = $\frac{720}{60} = 12$

A1

Answer: The minute hand turns through 12 times the angle of the hour hand

2 marks

(b) Minute hand has turned 2π more than the hour hand: $(12 - 1)\omega t = 2\pi$, where ω is the hour hand's angular speed

M1

$t = \frac{2\pi}{11\omega}$ and $\omega = \frac{2\pi}{720}$ per minute, so $t = \frac{720}{11} = 65.45$ minutes

M1 A1

= 1 hour 5 minutes 27 seconds

A1

Answer: $\frac{720}{11} \text{ min} = 1 \text{ h } 5 \text{ min } 27 \text{ s}$

4 marks

(c) Overlaps occur every $\frac{720}{11}$ minutes, and $11 \times \frac{720}{11} = 720$ minutes

M1

So 11 times in each 12 hours (counting 12:00 once)

A1

Answer: 11

2 marks**22 (7 marks)**

The formula $v = r\omega$ needs ω in rad s^{-1} , but 20 is in revolutions per minute

E1

$\omega = 20 \times \frac{2\pi}{60} = \frac{2\pi}{3}$

M1 A1

$v = 4 \times \frac{2\pi}{3} = 8.38 \text{ m s}^{-1}$

M1 A1

Period = $\frac{60}{20} = 3 \text{ s}$

M1 A1

Answer: 8.38 m s^{-1} , 3 s

7 marks**23 (6 marks)**

(a) $\omega_P = \frac{8}{5} = 1.6 \text{ rad s}^{-1}$, $\omega_Q = \frac{6}{5} = 1.2 \text{ rad s}^{-1}$

M1

They approach at 2.8 rad s^{-1} and are π radians apart: $t = \frac{\pi}{2.8}$

M1

= 1.12 s

A1

Answer: 1.12 s

3 marks

(b) Second meeting when the angles add to 3π : $t = \frac{3\pi}{2.8} = 3.37 \text{ s}$

M1 A1

Distance = $8 \times 3.37 = 26.9 \text{ m}$

A1

Answer: 26.9 m

3 marks

24 (6 marks)

The chain moves the same number of teeth past each wheel in the same time

B1

$$\text{Sprocket revolutions per minute} = 90 \times \frac{48}{16} = 270$$

M1 A1

$$\omega = 270 \times \frac{2\pi}{60} = 9\pi \text{ rad s}^{-1}$$

M1

$$\text{Speed} = r\omega = 0.33 \times 9\pi$$

M1

$$= 9.33 \text{ m s}^{-1}$$

A1

Answer: 9.33 m s^{-1}

6 marks

25 (6 marks)

(a) $\omega = \frac{v}{r}$

M1

Start: $\frac{4.8}{1.5} = 3.2 \text{ rad s}^{-1}$; end: $\frac{4.8}{3.0} = 1.6 \text{ rad s}^{-1}$

A1

Answer: 3.2 rad s^{-1} , 1.6 rad s^{-1}

2 marks

(b) $3.2 \times \frac{60}{2\pi} = 30.6$

M1

$$1.6 \times \frac{60}{2\pi} = 15.3 \text{ revolutions per minute}$$

A1

Answer: 30.6 and $15.3 \text{ rev min}^{-1}$

2 marks

(c) $v = r\omega$, so with v constant, $\omega = \frac{v}{r}$

E1

As the radius of wound tape increases, ω decreases

E1

Answer: ω inversely proportional to r

2 marks

26 (7 marks)

$$T = 27.3 \times 24 \times 3600 = 2.359 \times 10^6 \text{ s}$$

M1

$$\omega = \frac{2\pi}{T} = 2.66 \times 10^{-6} \text{ rad s}^{-1}$$

M1 A1

$$v = r\omega = 3.84 \times 10^8 \times 2.66 \times 10^{-6} = 1020 \text{ m s}^{-1}$$

M1

which is 1.02 km s^{-1} to 3 s.f.

A1

$$\text{Angle per day} = \frac{360}{27.3} = 13.2^\circ$$

M1 A1

Answer: 1.02 km s^{-1} ; 13.2° per day

7 marks

27 (9 marks)

Hour and minute hands coincide when the minute hand has gained k whole turns: $(12 - 1) \times \frac{2\pi t}{720}$

M1 A1

$$= 2\pi k, \text{ so } t = \frac{720k}{11} \text{ minutes, } k = 0, 1, \dots, 11$$

In t minutes the hour hand turns $\frac{t}{720}$ revolutions and the second hand turns t revolutions

B1

Second hand points the same way as the hour hand when $t - \frac{t}{720}$ is a whole number m : $\frac{719t}{720} = m$

M1

$$\text{so } t = \frac{720m}{719}$$

A1

Equate: $\frac{720k}{11} = \frac{720m}{719}$, so $719k = 11m$

M1

11 and 719 have no common factor (719 is prime), so 11 divides k

M1

With $0 \leq k \leq 11$ this gives $k = 0$ or $k = 11$, so $t = 0$ or $t = 720$

A1

These are 12:00 and 12:00 twelve hours later

B1

Answer: $t = 0$ and $t = 720$: both 12:00

9 marks

28 (10 marks)**(a)** Distance along shore from the foot of the perpendicular: $x = 600 \tan \theta$ **B1**

$$\frac{dx}{dt} = 600 \sec^2 \theta \frac{d\theta}{dt}$$

M1

$$\omega = 2 \times \frac{2\pi}{60} = \frac{\pi}{15} = 0.209 \text{ rad s}^{-1}$$

M1 A1

Answer: $\omega = \frac{\pi}{15} \text{ rad s}^{-1}$

4 marks**(b)** $\theta = 0$: $600 \times 0.209 = 126 \text{ m s}^{-1}$ **M1** $\theta = 60^\circ$: $\sec^2 60^\circ = 4$, so speed = 503 m s^{-1} **M1 A1****Answer:** 126 m s^{-1} , 503 m s^{-1} **3 marks****(c)** $600\omega \sec^2 \theta > 340$, so $\cos^2 \theta < \frac{600\omega}{340} = \frac{126}{340} = 0.3696$ **M1**

$$\cos \theta < 0.6079$$

M1

$$\theta > 52.6^\circ$$

A1**Answer:** 52.6° **3 marks****29 (9 marks)**

Time between frames = $\frac{1}{24} \text{ s}$

B1

Angle turned between frames = $\frac{5 \times 360}{24} = 75^\circ$ clockwise

M1 A1

Spokes are $\frac{360}{8} = 45^\circ$ apart, so positions repeat every 45°

M1

$75^\circ = 45^\circ + 30^\circ$, so the picture looks as if the wheel has moved 30° clockwise

A1

But $75^\circ = 90^\circ - 15^\circ$, so it also looks as if the wheel has moved 15° anticlockwise

M1

The smaller apparent movement is 15° anticlockwise

E1

Apparent angular speed = $15^\circ \times 24 = 360^\circ$ per second

M1

$= 1$ revolution per second anticlockwise

A1**Answer:** **1 revolution per second anticlockwise****9 marks****30 (11 marks)****(a)** Angular speeds $\frac{2\pi}{90}$ and $\frac{2\pi}{100}$ rad per minute**B1**

A gains 2π : $(\frac{2\pi}{90} - \frac{2\pi}{100})t = 2\pi$

M1

$$t = \frac{1}{\frac{1}{90} - \frac{1}{100}} = 900 \text{ minutes}$$

M1 A1

A: $900 \div 90 = 10$ orbits; B: $900 \div 100 = 9$ orbits

A1**Answer:** **900 minutes; A 10 orbits, B 9 orbits****5 marks****(b)** Relative angle π : $(\frac{2\pi}{90} - \frac{2\pi}{100})t = \pi$ **M1**

$t = 450 \text{ minutes}$

A1

Angle turned by A = $\frac{2\pi \times 450}{90} = 10\pi$, a whole number of revolutions, so OA is in its initial direction (angle 0)

M1 A1**Answer:** **450 minutes; angle 10π rad (OA is back in its initial direction)****4 marks****(c)** Angular speeds are equal and opposite, so together they cover 2π when $(\frac{2\pi}{90} + \frac{2\pi}{90})t = 2\pi$ **M1**

$t = 45 \text{ minutes}$

A1**Answer:** **45 minutes****2 marks**

Centripetal Acceleration

1 (2 marks)

$$a = \frac{v^2}{r} = \frac{64}{2}$$

M1

$$= 32 \text{ m s}^{-2}$$

A1

Answer: 32 m s⁻²

2 marks

2 (2 marks)

$$a = r\omega^2 = 0.5 \times 36$$

M1

$$= 18 \text{ m s}^{-2}$$

A1

Answer: 18 m s⁻²

2 marks

3 (2 marks)

$$a = \frac{v^2}{r} \text{ so } r = \frac{25}{10}$$

M1

$$= 2.5 \text{ m}$$

A1

Answer: 2.5 m

2 marks

4 (2 marks)

$$a = r\omega^2 \text{ so } r = \frac{12}{16}$$

M1

$$= 0.75 \text{ m}$$

A1

Answer: 0.75 m

2 marks

5 (3 marks)

(a) Towards the centre of the circle

B1

Answer: Towards the centre

1 mark

(b) $a = \frac{v^2}{r}$ and $a = r\omega^2$

B1 B1

Answer: $\frac{v^2}{r}$, $r\omega^2$

2 marks

6 (2 marks)

$$\omega = 120 \times \frac{2\pi}{60} = 4\pi \text{ rad s}^{-1}$$

M1

$$a = r\omega^2 = 0.2 \times (4\pi)^2 = 31.6 \text{ m s}^{-2}$$

A1

Answer: 31.6 m s⁻²

2 marks

7 (5 marks)

(a) $72 \text{ km h}^{-1} = \frac{72 \times 1000}{3600} = 20 \text{ m s}^{-1}$

B1

$$a = \frac{20^2}{50} = 8 \text{ m s}^{-2}$$

M1 A1

Answer: 8 m s⁻²

3 marks

(b) $\frac{8}{9.8} = 0.816$, so the acceleration is 0.816g

M1 A1

Answer: 0.816g

2 marks

8 (3 marks)

$$r = \frac{v^2}{a} = \frac{36}{12} = 3 \text{ m}$$

M1 A1

$$\omega = \frac{v}{r} = \frac{6}{3} = 2 \text{ rad s}^{-1}$$

A1

Answer: $r = 3 \text{ m}$, $\omega = 2 \text{ rad s}^{-1}$

3 marks

9 (4 marks)

$$\omega = 3000 \times \frac{2\pi}{60} = 100\pi \text{ rad s}^{-1}$$

M1

$$a = r\omega^2 = 0.15 \times (100\pi)^2 = 14800 \text{ m s}^{-2}$$

M1 A1

$$\frac{a}{g} = 1510, \text{ i.e. } 1510g$$

A1

Answer: 14800 m s⁻² (1510g)

4 marks

10 (4 marks)

$$\omega = \frac{2\pi}{86\,400} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

B1

$$a = r\omega^2 = 6.37 \times 10^6 \times (7.27 \times 10^{-5})^2 = 0.0337 \text{ m s}^{-2}$$

M1

A1

$$\frac{a}{9.8} = 0.00344$$

A1

Answer: 0.0337 m s⁻²; 0.00344 of 9.8

4 marks

11 (4 marks)

$$\omega = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad s}^{-1}$$

B1

$$r = \frac{a}{\omega^2} = \frac{6}{\frac{\pi^2}{4}} = 2.43 \text{ m}$$

M1 A1

$$v = r\omega = 2.43 \times \frac{\pi}{2} = 3.82 \text{ m s}^{-1}$$

A1

Answer: r = 2.43 m, v = 3.82 m s⁻¹

4 marks

12 (3 marks)

$$a_p = \frac{v^2}{r}$$

B1

$$a_Q = \frac{(2v)^2}{\frac{r}{2}} = \frac{8v^2}{r}$$

M1

$$\text{Ratio } a_Q : a_p = 8 : 1$$

A1

Answer: 8 : 1

3 marks

13 (5 marks)

$$(a) \omega = \frac{2\pi}{27.3 \times 86\,400} = 2.66 \times 10^{-6} \text{ rad s}^{-1}$$

B1

$$a = r\omega^2 = 3.84 \times 10^8 \times (2.66 \times 10^{-6})^2 = 0.00272 \text{ m s}^{-2}$$

M1 A1

Answer: 0.00272 m s⁻²

3 marks

$$(b) \frac{9.8}{0.00272} = 3597$$

M1

so the ratio is 1 : 3597

A1

Answer: 1 : 3597

2 marks

14 (6 marks)

$$(a) \omega = 1200 \times \frac{2\pi}{60} = 40\pi \text{ rad s}^{-1}$$

M1

$$a = 0.25 \times (40\pi)^2 = 3950 \text{ m s}^{-2} = 403g$$

M1

A1

Answer: 3950 m s⁻² = 403g

3 marks

$$(b) r\omega^2 \geq 40g \text{ so } \omega \geq \sqrt{\frac{40 \times 9.8}{0.25}} = 39.6 \text{ rad s}^{-1}$$

M1 A1

= 378 revolutions per minute

A1

Answer: 378 rev min⁻¹

3 marks

15 (6 marks)

(a) $\mathbf{v} = -a\omega \sin \omega t \mathbf{i} + a\omega \cos \omega t \mathbf{j}$

M1 A1

$|\mathbf{v}| = a\omega \sqrt{\sin^2 \omega t + \cos^2 \omega t} = a\omega$, a constant

A1

Answer: $a\omega$

3 marks

(b) $\mathbf{a} = -a\omega^2 \cos \omega t \mathbf{i} - a\omega^2 \sin \omega t \mathbf{j}$

M1

$= -\omega^2 \mathbf{r}$, so the acceleration is towards O

A1

Magnitude $= \omega^2 |\mathbf{r}| = a\omega^2$

A1

Answer: $a\omega^2$ towards O

3 marks

16 (5 marks)

(a) $\frac{v^2}{80} \leq 0.6 \times 9.8$

M1

$v \leq \sqrt{0.6 \times 9.8 \times 80} = 21.7 \text{ m s}^{-1}$

A1

$= 21.7 \times 3.6 = 78.1 \text{ km h}^{-1}$

A1

Answer: $21.7 \text{ m s}^{-1} = 78.1 \text{ km h}^{-1}$

3 marks

(b) $\frac{30^2}{r} \leq 0.6 \times 9.8$

M1

$r \geq \frac{900}{5.88} = 153 \text{ m}$

A1

Answer: 153 m

2 marks

17 (4 marks)

$a = r\omega^2$, so $\omega^2 = \frac{5}{0.8} = 6.25$

M1

$\omega = 2.5 \text{ rad s}^{-1}$

A1

$a = 0.5 \times 6.25$

M1

$= 3.12 \text{ m s}^{-2}$

A1

Answer: $\omega = 2.5 \text{ rad s}^{-1}$, $a = 3.12 \text{ m s}^{-2}$

4 marks

18 (5 marks)

$\omega = \frac{2\pi}{T}$

B1

$a = r\omega^2 = r \times \frac{4\pi^2}{T^2}$

M1

$= \frac{4\pi^2 r}{T^2}$

A1

$v = r\omega$ so $r\omega^2 = (r\omega)\omega = v\omega$

M1 A1

Answer: $a = \frac{4\pi^2 r}{T^2} = v\omega$

5 marks

19 (7 marks)

(a) Orbit radius $= 2R$

B1

$a = \frac{gR^2}{(2R)^2} = \frac{g}{4} = 2.45 \text{ m s}^{-2}$

M1 A1

Answer: 2.45 m s^{-2}

3 marks

(b) $\frac{v^2}{2R} = 2.45$, so $v = \sqrt{2.45 \times 1.274 \times 10^7} = 5590 \text{ m s}^{-1}$

M1 A1

$T = \frac{2\pi \times 2R}{v} = 14300 \text{ s} = 3.98 \text{ hours}$

M1 A1

Answer: 5590 m s^{-1} , 3.98 hours

4 marks

20 (5 marks)

(a) $\omega = 2.5 \times 2\pi = 5\pi \text{ rad s}^{-1}$

M1

$a = 0.8 \times (5\pi)^2 = 197 \text{ m s}^{-2}$

A1

Answer: 197 m s^{-2}

2 marks

(b) $0.8\omega^2 \leq 400$, so $\omega \leq \sqrt{500} = 22.4 \text{ rad s}^{-1}$

M1 A1

$$\frac{\omega}{2\pi} = 3.56 \text{ revolutions per second}$$

A1

Answer: 3.56 rev s⁻¹

3 marks

21 (8 marks)

(a) $a = \frac{10^2}{5} = 20 \text{ m s}^{-2}$

M1 A1

Answer: 20 m s⁻²

2 marks

(b) Velocities are perpendicular, each of magnitude 10

M1

$$|\Delta \mathbf{v}| = 10\sqrt{2} = 14.1 \text{ m s}^{-1}$$

M1 A1

Answer: 10√2 = 14.1 m s⁻¹

3 marks

(c) Time = $\frac{\text{quarter circumference}}{\text{speed}} = \frac{\frac{5\pi}{2}}{10} = \frac{\pi}{4} \text{ s}$

M1

$$\text{Average acceleration} = \frac{10\sqrt{2}}{\frac{\pi}{4}} = 18.0 \text{ m s}^{-2}$$

A1

The acceleration changes direction during the motion, so the vector sum of the changes is smaller than the sum of the magnitudes

E1

Answer: 18.0 m s⁻²

3 marks

22 (8 marks)

(a) $\mathbf{v} = -6 \sin 2t \mathbf{i} + 4 \cos 2t \mathbf{j}$

M1

$$\mathbf{a} = -12 \cos 2t \mathbf{i} - 8 \sin 2t \mathbf{j} = -4(3 \cos 2t \mathbf{i} + 2 \sin 2t \mathbf{j})$$

M1

= -4 $\mathbf{r}$, so the acceleration is towards O

A1

Answer: a = -4r

3 marks

(b) $|\mathbf{a}|^2 = 144 \cos^2 2t + 64 \sin^2 2t = 64 + 80 \cos^2 2t$

M1

Greatest 12 m s⁻² (cos²2t = 1), least 8 m s⁻² (cos 2t = 0)

A1 A1

First least value at 2t = $\frac{\pi}{2}$, t = $\frac{\pi}{4}$

A1

Answer: 12 and 8 m s⁻²; t = $\frac{\pi}{4}$

4 marks

(c) Speed $|\mathbf{v}| = \sqrt{36 \sin^2 2t + 16 \cos^2 2t}$ varies with t (it is 4 at t = 0 and 6 when t = $\frac{\pi}{4}$)

E1

Answer: speed not constant

1 mark

23 (6 marks)

The angular speed has to be in rad s⁻¹, not revolutions per minute

E1

$$\omega = 90 \times \frac{2\pi}{60} = 3\pi \text{ rad s}^{-1}$$

M1

$$a = 0.3 \times (3\pi)^2$$

M1

$$= 26.6 \text{ m s}^{-2}$$

A1

This is 2.72g, not 248g

B1

Direction: towards the centre of the wheel

B1

Answer: 26.6 m s⁻² towards the centre

6 marks

24 (7 marks)

The point moves in a circle of radius R cos λ about the axis, with angular speed ω

M1 A1

Acceleration = (R cos λ)ω² towards the axis

A1

The direction to the axis makes angle λ with the direction to the Earth's centre

M1

$$\text{Component towards the centre} = R\omega^2 \cos \lambda \times \cos \lambda = R\omega^2 \cos^2 \lambda$$

A1

$$R\omega^2 = 0.0337 \text{ m s}^{-2}, \text{ so } R\omega^2 \cos 51.5^\circ = 0.0210 \text{ m s}^{-2}$$

M1

$$R\omega^2 \cos^2 51.5^\circ = 0.0131 \text{ m s}^{-2}$$

A1

Answer: 0.0210 m s⁻² and 0.0131 m s⁻²

7 marks

25 (7 marks)

$$a = \frac{v^2}{r} = \frac{28^2}{1.6} \quad \text{M1}$$

$$= 490 \text{ m s}^{-2} \quad \text{A1}$$

$$\omega = \frac{v}{r} = \frac{28}{1.6} = 17.5 \text{ rad s}^{-1} \quad \text{M1 A1}$$

$$\text{Revolutions per second} = \frac{17.5}{2\pi} = 2.79 \quad \text{M1 A1}$$

$$a = \frac{490}{9.8} \text{ g} = 50\text{g} \quad \text{A1}$$

Answer: 490 m s⁻² = 50g, 17.5 rad s⁻¹, 2.79 rev s⁻¹ 7 marks

26 (7 marks)

(a) Acceleration of the satellite = $r\omega^2$, which equals $\frac{gR^2}{r^2}$ M1

So $r^3\omega^2 = gR^2$, i.e. $r^3 = \frac{gR^2}{\omega^2}$ A1

Answer: The radius r of the orbit satisfies $r^3 = \frac{gR^2}{\omega^2}$, where ω is the angular speed of the Earth 2 marks

(b) $\omega = \frac{2\pi}{86400} \text{ rad s}^{-1}$ B1

$$r^3 = \frac{9.8 \times (6.37 \times 10^6)^2}{\omega^2}, \text{ so } r = 4.22 \times 10^7 \text{ m} \quad \text{M1 A1}$$

$$\text{Height} = 42200 - 6370 = 35800 \text{ km} \quad \text{M1 A1}$$

Answer: $r = 42200 \text{ km}$, height = 35800 km 5 marks

27 (9 marks)

Velocities at A and B each have magnitude v , and the angle between them is $\delta\theta$ (equal to the angle AOB) B1

Isosceles triangle of velocities: $|\delta\mathbf{v}| = 2v \sin \frac{\delta\theta}{2}$ M1 A1

$$\delta\theta = \omega \delta t, \text{ so } \frac{|\delta\mathbf{v}|}{\delta t} = \frac{2v \sin \frac{\delta\theta}{2}}{\delta t} \quad \text{M1}$$

$$= v \omega \times \frac{\sin \frac{\delta\theta}{2}}{\frac{\delta\theta}{2}} \quad \text{M1}$$

$$\text{As } \delta t \rightarrow 0, \frac{\sin \frac{\delta\theta}{2}}{\frac{\delta\theta}{2}} \rightarrow 1, \text{ so } a = v\omega = \frac{v^2}{r} \quad \text{M1 A1}$$

The base angles of the isosceles triangle are $90^\circ - \frac{\delta\theta}{2} \rightarrow 90^\circ$, so $\delta\mathbf{v}$ becomes perpendicular to the velocity at A M1

and hence is along the radius towards O A1

Answer: $\frac{v^2}{r}$ towards O 9 marks

28 (11 marks)

(a) $v^2/r = \frac{k}{r^2}$, so $v^2 = \frac{k}{r}$ M1 A1

$$v_A^2 : v_B^2 = \frac{1}{r_A} : \frac{1}{4r_A} = 4 : 1, \text{ so } v_A = 2v_B \quad \text{M1 A1}$$

Answer: The speed of A is twice the speed of B 4 marks

(b) $T = \frac{2\pi r}{v}$, so $\frac{T_B}{T_A} = \frac{r_B}{r_A} \times \frac{v_A}{v_B} = 4 \times 2 = 8$ M1 A1

$$\omega \propto \frac{1}{T}, \text{ so } \omega_A : \omega_B = 8 : 1 \quad \text{A1}$$

Answer: $T_B : T_A = 8 : 1$; $\omega_A : \omega_B = 8 : 1$ 3 marks

$$(c) a \propto \frac{1}{r^2}, \text{ so } \frac{a_A}{a_B} = (4)^2 = 16$$

M1 A1

Answer: 16 : 1

2 marks

$$(d) T^2 \propto r^3 \text{ (from } T = \frac{2\pi r}{v} \text{ and } v^2 = \frac{k}{r}), \text{ so } T_C = T_A(9)^{3/2}$$

$$= 27T_A$$

M1

A1

Answer: 27T_A

2 marks

29 (11 marks)

$$(a) \text{ Radial component} = \frac{v^2}{r} = \frac{2.25t^2}{2} = 1.125t^2$$

M1 A1

$$\text{Tangential component} = \frac{dv}{dt} = 1.5 \text{ m s}^{-2}$$

M1 A1

Answer: tangential 1.5 m s⁻²

4 marks

$$(b) 1.125t^2 = 1.5$$

M1

$$t^2 = \frac{4}{3}, t = 1.15 \text{ s}$$

A1

Answer: 1.15 s

2 marks

$$(c) \text{ At } t = 2: \text{ radial} = 4.5, \text{ tangential} = 1.5$$

B1

$$\text{Magnitude} = \sqrt{4.5^2 + 1.5^2} = 4.74 \text{ m s}^{-2}$$

M1 A1

$$\text{Angle} = \arctan \frac{1.5}{4.5} = 18.4^\circ$$

M1 A1

Answer: 4.74 m s⁻², 18.4° to the inward radius

5 marks

30 (11 marks)

$$(a) \text{ On AJ: } a = \frac{20^2}{40} = 10 \text{ m s}^{-2} \text{ towards C}$$

M1 A1

$$\text{On JB: } a = \frac{20^2}{100} = 4 \text{ m s}^{-2} \text{ towards C'}$$

M1 A1

Answer: 10 m s⁻² towards C; 4 m s⁻² towards C'

4 marks

$$(b) \text{ Length} = \pi \times 40 + \pi \times 100 = 140\pi \text{ m}$$

M1

$$t = \frac{140\pi}{20} = 7\pi = 22.0 \text{ s}$$

A1

Answer: 7π = 22.0 s

2 marks

(c) The centres C and C' are on opposite sides of the road, so at J the two accelerations are in opposite directions

M1 A1

$$\text{Change} = 10 + 4 = 14 \text{ m s}^{-2}$$

A1

Answer: 14 m s⁻²

3 marks

$$(d) \text{ Greatest acceleration is on the radius 40 m arc: } \frac{v^2}{40} \leq 0.5 \times 9.8$$

M1

$$v \leq \sqrt{196} = 14 \text{ m s}^{-1}$$

A1

Answer: 14 m s⁻¹

2 marks

Centripetal Force**1 (2 marks)**

$$F = \frac{mv^2}{r} = \frac{2 \times 9}{1.5}$$

M1

$$= 12 \text{ N}$$

A1

Answer: 12 N

2 marks

2 (2 marks)

$$F = mr\omega^2 = 0.4 \times 0.5 \times 36$$

M1

$$= 7.2 \text{ N}$$

A1

Answer: 7.2 N

2 marks

3 (2 marks)

$$T = \frac{mv^2}{r} = \frac{0.5 \times 16}{0.8}$$

M1

$$= 10 \text{ N}$$

A1

Answer: 10 N

2 marks

4 (2 marks)

$$\frac{mv^2}{r} = 24 \text{ so } v^2 = \frac{24 \times 2}{3} = 16$$

M1

$$v = 4 \text{ m s}^{-1}$$

A1

Answer: 4 m s⁻¹

2 marks

5 (2 marks)Friction provides the centripetal force: $F = mr\omega^2$

M1

$$= 0.02 \times 0.15 \times 64 = 0.192 \text{ N}$$

A1

Answer: 0.192 N

2 marks

6 (3 marks)

(a) Friction between the tyres and the road

B1

Answer: Friction

1 mark

(b) Gravitational attraction of the Earth

B1

Answer: Gravity

1 mark

(c) Tension in the string

B1

Answer: Tension

1 mark

7 (5 marks)

$$(a) F = \frac{mv^2}{r} = \frac{900 \times 15^2}{40}$$

M1

$$= 5060 \text{ N}$$

A1

Answer: 5060 N

2 marks

$$(b) R = mg = 900 \times 9.8 = 8820 \text{ N}$$

B1

$$\mu \geq \frac{F}{R} = \frac{5062}{8820} = 0.574$$

M1 A1

Answer: 0.574

3 marks

8 (4 marks)

$$(a) T = \frac{mv^2}{r} \leq 20$$

M1

$$v \leq \sqrt{\frac{20 \times 0.6}{0.3}} = 6.32 \text{ m s}^{-1}$$

A1

Answer: 6.32 m s⁻¹

2 marks

$$(b) \omega = \frac{v}{r} = 10.5 \text{ rad s}^{-1}$$

M1

$$= 10.5 \times \frac{60}{2\pi} = 101 \text{ revolutions per minute}$$

A1

Answer: 101 rev min⁻¹

2 marks

9 (4 marks)At the point of slipping $F = \mu R = \mu mg$, and $F = mr\omega^2$

M1

$$\mu mg = m(0.2)\omega^2, \text{ so } \omega^2 = \frac{0.4 \times 9.8}{0.2} = 19.6$$

M1

$$\omega = 4.43 \text{ rad s}^{-1}$$

A1

$$= 42.3 \text{ revolutions per minute}$$

A1

Answer: 4.43 rad s⁻¹ = 42.3 rev min⁻¹

4 marks

10 (4 marks)

$$\text{Extension} = 0.2 \text{ m}, T = \frac{\lambda e}{l} = \frac{12 \times 0.2}{0.4} = 6 \text{ N}$$

M1 A1

$$T = \frac{mv^2}{r}: 6 = \frac{0.5v^2}{0.6}$$

M1

$$v = 2.68 \text{ m s}^{-1}$$

A1

$$\text{Answer: } T = 6 \text{ N}, v = 2.68 \text{ m s}^{-1}$$

4 marks

11 (3 marks)

The hanging particle is in equilibrium, so $T = 0.6g = 5.88 \text{ N}$

B1

$$\text{For P: } T = \frac{mv^2}{r}, \text{ so } 5.88 = \frac{0.2v^2}{0.5}$$

M1

$$v^2 = 14.7, v = 3.83 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 3.83 \text{ m s}^{-1}$$

3 marks

12 (3 marks)

Gravity is the resultant force towards the centre: $F = \frac{mv^2}{r}$

M1

$$= \frac{500 \times (7.5 \times 10^3)^2}{7.0 \times 10^6}$$

M1

$$= 4020 \text{ N}$$

A1

$$\text{Answer: } 4020 \text{ N}$$

3 marks

13 (3 marks)

$$6000 = \frac{1200v^2}{60}$$

M1

$$v^2 = 300, v = 17.3 \text{ m s}^{-1}$$

A1

$$= 17.3 \times 3.6 = 62.4 \text{ km h}^{-1}$$

A1

$$\text{Answer: } 17.3 \text{ m s}^{-1} = 62.4 \text{ km h}^{-1}$$

3 marks

14 (7 marks)

$$T = \frac{8 \times 0.2}{0.5} = 3.20 \text{ N, maximum friction } \mu mg = 1.960 \text{ N}$$

M1 A1

Least ω : friction acts away from O: $T - \mu mg = mr\omega^2$

M1

$$3.20 - 1.960 = 0.28\omega^2, \text{ so } \omega = 2.10 \text{ rad s}^{-1}$$

M1 A1

Greatest ω : friction acts towards O: $T + \mu mg = mr\omega^2, \text{ so } \omega = 4.29 \text{ rad s}^{-1}$

M1 A1

$$\text{Answer: } 2.10 \text{ rad s}^{-1} \leq \omega \leq 4.29 \text{ rad s}^{-1}$$

7 marks

15 (5 marks)

B is at distance $0.5 + 0.7 = 1.2 \text{ m}$ from O

B1

$$\text{For B: } T_{AB} = m_B r_B \omega^2 = 0.3 \times 1.2 \times 9$$

M1

$$= 3.24 \text{ N}$$

A1

$$\text{For A: } T_{OA} - T_{AB} = m_A r_A \omega^2 = 0.4 \times 0.5 \times 9$$

M1

$$T_{OA} = 5.04 \text{ N}$$

A1

$$\text{Answer: } T_{AB} = 3.24 \text{ N}, T_{OA} = 5.04 \text{ N}$$

5 marks

16 (6 marks)

Let $AC = x$, so $CB = 1.2 - x$

B1

Tension in AC gives the force on A: $T = 2x\omega^2$. Tension in CB gives the force on B: $T = 3(1.2 - x)\omega^2$

M1 A1

$$2x = 3(1.2 - x), \text{ so } 5x = 3.6 \text{ and } x = 0.720 \text{ m}$$

M1 A1

$$T = 2 \times 0.720 \times 4^2 = 23.0 \text{ N}$$

A1

$$\text{Answer: } AC = 0.720 \text{ m}, T = 23.0 \text{ N}$$

6 marks

17 (6 marks)

$$(a) T = \frac{mv^2}{r} = \frac{0.25 \times 36}{1.2}$$

M1

$$= 7.50 \text{ N}$$

A1

Answer: 7.50 N

2 marks

$$(b) 12 = \frac{0.25v^2}{1.2}$$

M1

$$v = 7.59 \text{ m s}^{-1}$$

A1

Answer: 7.59 m s⁻¹

2 marks

(c) The particle moves along the tangent at speed 7.59 m s⁻¹; distance 15.2 m

M1

$$\text{Distance from O} = \sqrt{1.2^2 + (15.2)^2} = 15.2 \text{ m}$$

A1

Answer: 15.2 m

2 marks

18 (6 marks)

$$\text{Slipping when } \mu mg = mr\omega^2, \text{ i.e. } \omega^2 = \frac{\mu g}{r}$$

M1

$$A: \omega = \sqrt{\frac{0.4 \times 9.8}{0.1}} = 6.26 \text{ rad s}^{-1}$$

A1

$$B: \omega = \sqrt{\frac{0.4 \times 9.8}{0.25}} = 3.96 \text{ rad s}^{-1}$$

A1

B has the smaller slipping angular speed (the larger r), so B slips first

E1

$$\text{Speed of B} = 0.25 \times 3.96 = 0.990 \text{ m s}^{-1}$$

M1 A1

Answer: $\omega = 3.96 \text{ rad s}^{-1}$, $v = 0.990 \text{ m s}^{-1}$

6 marks

19 (6 marks)

$$(a) \frac{mv^2}{r} = \frac{GMm}{r^2}$$

M1

$$v^2 = \frac{GM}{r}, \text{ so } v = \sqrt{\frac{GM}{r}}$$

A1

Answer: The speed of the satellite is $\sqrt{\frac{GM}{r}}$

2 marks

$$(b) v = \sqrt{\frac{3.99 \times 10^{14}}{2.0 \times 10^7}} = 4470 \text{ m s}^{-1}$$

M1 A1

$$T = \frac{2\pi r}{v} = 28100 \text{ s} = 7.82 \text{ hours}$$

M1 A1

Answer: 4470 m s⁻¹, 7.82 hours

4 marks

20 (6 marks)

$$F = \frac{mv^2}{r} = \frac{80 \times 64}{20} = 256 \text{ N}$$

M1 A1

$$R = mg = 784 \text{ N}, \mu \geq \frac{F}{R} = 0.327$$

M1 A1

$$\tan \theta = \frac{F}{R} = \frac{v^2}{rg} = 0.3265$$

M1

$$\theta = 18.1^\circ \approx 18^\circ$$

A1

Answer: 256 N, $\mu \geq 0.327$, $\theta \approx 18^\circ$

6 marks

21 (8 marks)

$$\text{Extension} = \frac{a}{4}, \text{ so } T = \frac{3mg \times \frac{a}{4}}{a} = \frac{3mg}{4} \quad \text{M1 A1}$$

$$\text{Maximum friction} = \mu R = \frac{1}{3}mg \quad \text{B1}$$

$$\text{Least } \omega: \text{ friction acts outwards: } T - F = m\left(\frac{5a}{4}\right)\omega^2 \quad \text{M1}$$

$$\frac{3}{4}mg - \frac{1}{3}mg = \frac{5}{12}mg = \frac{5ma}{4}\omega^2, \text{ so } \omega^2 = \frac{g}{3a} \quad \text{M1 A1}$$

$$\text{Greatest } \omega: \text{ friction acts inwards: } \frac{3}{4}mg + \frac{1}{3}mg = \frac{13}{12}mg = \frac{5ma}{4}\omega^2, \text{ so } \omega^2 = \frac{13g}{15a} \quad \text{M1 A1}$$

$$\text{Answer: } \frac{g}{3a} \leq \omega^2 \leq \frac{13g}{15a} \quad \text{8 marks}$$

22 (7 marks)

The normal reaction is $R = mg = 800 \times 9.8 = 7840 \text{ N}$, not 800 N (800 is the mass) E1

$$F_{\max} = 0.6 \times 7840 = 4704 \text{ N} \quad \text{M1 A1}$$

$$4704 = \frac{800v^2}{50}, \text{ so } v = 17.1 \text{ m s}^{-1} \quad \text{M1 A1}$$

$$= 61.7 \text{ km h}^{-1} \quad \text{A1}$$

$$\text{Radius } 100 \text{ m: } v = \sqrt{0.6 \times 9.8 \times 100} = 24.2 \text{ m s}^{-1} \quad \text{M1}$$

$$\text{Answer: } 17.1 \text{ m s}^{-1} = 61.7 \text{ km h}^{-1}; 24.2 \text{ m s}^{-1} \text{ on } 100 \text{ m} \quad \text{7 marks}$$

23 (9 marks)

$$\text{(a) For A: force from rod} = m_A r_A \omega^2 = 0.3 \times 1.2 \times 25 \quad \text{M1}$$

$$= 9 \text{ N towards O} \quad \text{A1}$$

$$\text{For B: } F_{OB} - F_{AB} = m_B r_B \omega^2 = 0.5 \times 0.4 \times 25 = 5 \quad \text{M1}$$

$$F_{OB} = 9 + 5 = 14 \text{ N} \quad \text{A1}$$

$$\text{Answer: } 9 \text{ N, } 14 \text{ N} \quad \text{4 marks}$$

(b) The rod is light, so the net force on it is zero

The particles pull the rod away from O with forces 9 N and 5 N , so the axle exerts a force of 14 N M1 A1

directed along the rod from A towards O B1

$$\text{Answer: } 14 \text{ N towards O} \quad \text{3 marks}$$

$$\text{(c) } 0.3 \times 1.2 \times \omega^2 \leq 20 \quad \text{M1}$$

$$\omega \leq \sqrt{\frac{20}{0.36}} = 7.45 \text{ rad s}^{-1} \quad \text{A1}$$

$$\text{Answer: } 7.45 \text{ rad s}^{-1} \quad \text{2 marks}$$

24 (8 marks)

$$\text{(a) Same tension } T \text{ gives the centripetal force for both: } T = 3m r_A \omega^2 = 2m r_B \omega^2 \quad \text{M1}$$

$$\text{So } r_A : r_B = 2 : 3 \quad \text{A1}$$

$$r_A + r_B = 5a \text{ so } r_A = 2a, r_B = 3a \quad \text{M1}$$

$$T = 3m \times 2a \times \omega^2 = 6ma\omega^2 \quad \text{A1}$$

$$\text{Answer: A is at distance } 2a \text{ from O, and the tension in the string is } 6ma\omega^2 \quad \text{4 marks}$$

$$\text{(b) } 6 \times 0.5 \times 0.4 \times \omega^2 \leq 30, \text{ so } \omega^2 \leq 25 \quad \text{M1}$$

$$\omega = 5 \text{ rad s}^{-1} \quad \text{A1}$$

$$v_B = r_B \omega = 3 \times 0.4 \times 5 = 6 \text{ m s}^{-1} \quad \text{M1 A1}$$

$$\text{Answer: } \omega = 5 \text{ rad s}^{-1}, v_B = 6 \text{ m s}^{-1} \quad \text{4 marks}$$

25 (8 marks)**(a)** For Q in equilibrium $T = Mg$ **B1**For P: $T = mr\omega^2 = 0.3r\omega^2$ **M1**

So $\omega^2 = \frac{Mg}{0.3r}$

A1**Answer: The angular speed ω of P satisfies $\omega^2 = \frac{Mg}{0.3r}$** **3 marks**

(b) $\omega^2 = \frac{0.9 \times 9.8}{0.3 \times 0.4} = 73.5$, $\omega = 8.57 \text{ rad s}^{-1}$

M1 A1

$v = r\omega = 3.43 \text{ m s}^{-1}$

A1

$T = \frac{2\pi}{\omega} = 0.733 \text{ s}$

M1 A1**Answer: $\omega = 8.57 \text{ rad s}^{-1}$, $v = 3.43 \text{ m s}^{-1}$, $T = 0.733 \text{ s}$** **5 marks****26 (7 marks)****(a)** Let the extension be x . $T = \frac{20x}{0.5} = 40x$ **B1**

$T = mr\omega^2 = 0.2(0.5 + x) \times 64$

M1

$40x = 6.4 + 12.8x$, so $x = \frac{6.4}{27.2} = 0.235 \text{ m}$

M1 A1**Answer: 0.235 m****4 marks****(b)** $40x = 0.2(0.5 + x)\omega^2$, so $x(40 - 0.2\omega^2) = 0.1\omega^2$ **M1**

$x = \frac{0.1\omega^2}{40 - 0.2\omega^2}$ is positive and finite only if $\omega^2 < 200$

A1As $\omega^2 \rightarrow 200$ the extension tends to infinity: the string cannot hold the particle**E1****Answer: $\omega^2 < 200$** **3 marks****27 (11 marks)****(a)** $T = 0.6g = 5.88 \text{ N}$; maximum friction $= 0.3 \times 0.4 \times 9.8 = 1.176 \text{ N}$ **B1 B1**Least ω : friction acts away from O: $T - F = mr\omega^2$, $5.88 - 1.176 = 0.2\omega^2$ **M1**

$\omega^2 = 23.52$

A1

Greatest ω : $T + F = 0.2\omega^2$, so $\omega^2 = \frac{7.056}{0.2} = 35.28$

M1 A1

$4.85 \leq \omega \leq 5.94 \text{ rad s}^{-1}$

A1**Answer: $4.85 \leq \omega \leq 5.94 \text{ rad s}^{-1}$** **7 marks****(b)** $Mg \mp \mu mg = mr\omega^2$ with the same limits**M1**

$r_{\min} = \frac{4.704}{0.4 \times 29.16} = 0.403 \text{ m}$

M1 A1

$r_{\max} = \frac{7.056}{0.4 \times 29.16} = 0.605 \text{ m}$

A1**Answer: $0.403 \text{ m} \leq r \leq 0.605 \text{ m}$** **4 marks****28 (12 marks)****(a)** The same tension T gives the centripetal force on each: $T = m r_A \omega^2 = 2m r_B \omega^2$ **M1**

So $r_A = 2r_B$; $r_A + r_B = l + e$ gives $r_A = \frac{2}{3}(l + e)$

M1 A1

$T = m \times \frac{2}{3}(l + e)\omega^2 = \frac{2m\omega^2(l + e)}{3}$

A1**Answer: $r_A = \frac{2}{3}(l + e)$** **4 marks****(b)** Hooke's law: $T = \frac{\lambda e}{l}$ **B1**

$\frac{\lambda e}{l} = \frac{2m\omega^2(l + e)}{3}$

M1

$3\lambda e = 2m\omega^2 l(l + e)$

M1

$e(3\lambda - 2m\omega^2 l) = 2m\omega^2 l^2$, giving the result

A1

$$\text{Answer: } e = \frac{2m\omega^2 l^2}{3\lambda - 2m\omega^2 l}$$

4 marks

(c) Need $3\lambda - 2m\omega^2 l > 0$, so $\omega^2 < \frac{3\lambda}{2ml}$

M1

With the data: $\omega^2 < \frac{90}{0.6} = 150$, so $\omega < 12.2 \text{ rad s}^{-1}$

A1

$$e = \frac{2 \times 0.5 \times 36 \times 0.36}{90 - 2 \times 0.5 \times 36 \times 0.6} = \frac{12.96}{68.4} = 0.189 \text{ m}$$

M1 A1

Answer: $\omega < \sqrt{\frac{3\lambda}{2ml}}$; $e = 0.189 \text{ m}$

4 marks

29 (11 marks)

(a) C is at distance $3a$: $T_3 = 3m \times 3a \times \omega^2 = 9ma\omega^2$

M1 A1

B is at distance $2a$: $T_2 - T_3 = 2m \times 2a \times \omega^2 = 4ma\omega^2$, so $T_2 = 13ma\omega^2$

M1 A1

A is at distance a : $T_1 - T_2 = m \times a \times \omega^2$, so $T_1 = 14ma\omega^2$

M1 A1

Answer: $T_2 = 13ma\omega^2$, $T_1 = 14ma\omega^2$

6 marks

(b) $\Sigma mr = m \times a + 2m \times 2a + 3m \times 3a = 14ma$, so $\omega^2 \Sigma mr = 14ma\omega^2$

B1

The string OA supplies the whole centripetal force of all three particles together

E1

Answer: $T_1 = 14ma\omega^2$

2 marks

(c) BC: $9 \times 0.5 \times 0.4 \times \omega^2 \leq 36$, so $\omega^2 \leq 20$

M1

OA: $14 \times 0.5 \times 0.4 \times \omega^2 \leq 60$, so $\omega^2 \leq 21.43$

M1

The smaller limit applies: $\omega \leq \sqrt{20} = 4.47 \text{ rad s}^{-1}$, and AB ($13 \times 0.2 \times 20 = 52 \text{ N}$) is within any limit given

A1

Answer: 4.47 rad s^{-1}

3 marks

30 (12 marks)

(a) Same force on each: $M r_E \omega^2 = m r_M \omega^2$

M1

So $M r_E = m r_M$, and $r_E + r_M = d$

M1

$$r_E = \frac{md}{M+m}$$

A1

Answer: The distance of the Earth's centre from C is $\frac{md}{M+m}$

3 marks

(b) $\frac{GMm}{d^2} = M r_E \omega^2 = \frac{Mmd}{M+m} \omega^2$

M1 A1

$$\text{So } \omega^2 = \frac{G(M+m)}{d^3}$$

A1

Answer: $\omega^2 = \frac{G(M+m)}{d^3}$

3 marks

(c) $\omega = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0435 \times 10^{24}}{(3.84 \times 10^8)^3}} = 2.67 \times 10^{-6} \text{ rad s}^{-1}$

M1 A1

$$T = \frac{2\pi}{\omega} = 27.3 \text{ days}$$

M1 A1

$$r_E = 4.67 \times 10^6 \text{ m}$$

M1

This is less than the Earth's radius ($6.37 \times 10^6 \text{ m}$), so C lies inside the Earth

E1

Answer: 27.3 days; $4.67 \times 10^6 \text{ m}$; C inside the Earth

6 marks

Horizontal Circular Motion**1 (2 marks)**

Vertically: $T \cos 30^\circ = mg$

M1

$$T = \frac{0.5 \times 9.8}{\cos 30^\circ} = 5.66 \text{ N}$$

A1

Answer: 5.66 N

2 marks

2 (2 marks)

$$T \cos \theta = mg \text{ so } \cos \theta = \frac{9.8}{12}$$

M1

$$\theta = 35.2^\circ$$

A1

Answer: 35.2°

2 marks

3 (2 marks)

$$\tan 15^\circ = \frac{v^2}{rg}$$

M1

$$v = \sqrt{60 \times 9.8 \times \tan 15^\circ} = 12.6 \text{ m s}^{-1}$$

A1

Answer: 12.6 m s⁻¹

2 marks

4 (2 marks)

$$\text{Horizontally: } T \sin \theta = m(l \sin \theta) \omega^2, \text{ so } T = ml \omega^2$$

M1

$$T = 0.4 \times 0.9 \times 25 = 9 \text{ N}$$

A1

Answer: 9 N

2 marks

5 (2 marks)

$$\text{Radius} = 1.5 \sin 40^\circ = 0.964 \text{ m}$$

B1

$$\text{Depth} = 1.5 \cos 40^\circ = 1.15 \text{ m}$$

B1

Answer: radius 0.964 m, depth 1.15 m

2 marks

6 (3 marks)

$$(a) T \cos \theta = mg$$

B1

Answer: T cos θ = mg

1 mark

$$(b) T \sin \theta = mr \omega^2$$

B1

Answer: T sin θ = mr ω²

1 mark

$$(c) \text{ Divide: } \frac{T \sin \theta}{T \cos \theta} = \frac{mr \omega^2}{mg}, \text{ so } \tan \theta = \frac{r \omega^2}{g}$$

B1

Answer: tan θ = $\frac{r \omega^2}{g}$

1 mark

7 (5 marks)

$$T \cos 40^\circ = mg, \text{ so } T = 3.84 \text{ N}$$

M1 A1

$$T \sin 40^\circ = m(0.8 \sin 40^\circ) \omega^2, \text{ so } \omega^2 = \frac{T}{0.3 \times 0.8} = \frac{g}{0.8 \cos 40^\circ}$$

M1

$$\omega = 4.00 \text{ rad s}^{-1}$$

A1

$$\text{Period} = \frac{2\pi}{\omega} = 1.57 \text{ s}$$

A1

Answer: T = 3.84 N, ω = 4.00 rad s⁻¹, period 1.57 s

5 marks

8 (4 marks)

$$R \sin \theta = \frac{mv^2}{r}, R \cos \theta = mg$$

M1

$$\tan \theta = \frac{v^2}{rg} = \frac{400}{784}$$

M1

$$\theta = 27.0^\circ$$

A1

$$R = \frac{mg}{\cos \theta} = 11000 \text{ N}$$

A1

Answer: θ = 27.0°, R = 11000 N

4 marks

9 (5 marks)

$$\text{Vertical: } F = mg; \text{ horizontal: } R = mr \omega^2$$

M1 M1

$$\text{Limiting friction: } F = \mu R, \text{ so } mg = 0.5 \times m \times 3 \omega^2$$

M1

$$\omega = 2.56 \text{ rad s}^{-1}$$

A1

$$= 24.4 \text{ revolutions per minute}$$

A1

Answer: 2.56 rad s⁻¹ = 24.4 rev min⁻¹

5 marks

10 (4 marks)

$$L \cos \theta = mg, L \sin \theta = \frac{mv^2}{r} \quad \text{M1}$$

$$\tan \theta = \frac{v^2}{rg} = \frac{6400}{14\,700} \quad \text{M1}$$

$$\theta = 23.5^\circ \quad \text{A1}$$

$$L = \frac{4000 \times 9.8}{\cos \theta} = 42800 \text{ N} \quad \text{A1}$$

Answer: $\theta = 23.5^\circ$, $L = 42800 \text{ N}$ 4 marks

11 (4 marks)

The normal reaction R is perpendicular to the surface, at 30° to the horizontal B1

$$\text{Vertical: } R \sin 30^\circ = mg; \text{ horizontal: } R \cos 30^\circ = \frac{mv^2}{0.4} \quad \text{M1}$$

$$\text{Dividing: } \frac{1}{\tan 30^\circ} = \frac{v^2}{0.4 \times 9.8}, \text{ so } v^2 = 6.790 \quad \text{M1}$$

$$v = 2.61 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 2.61 m s^{-1} 4 marks

12 (5 marks)

P is in equilibrium, so $T = 0.6g = 5.88 \text{ N}$ (the tension is the same throughout the string over a smooth ring) B1

$$\text{For } Q: T \cos \theta = 0.2g, \text{ so } \cos \theta = \frac{0.2g}{T} = \frac{1}{3} \quad \text{M1}$$

$$\theta = 70.5^\circ \quad \text{A1}$$

$$T \sin \theta = 0.2 \times (0.9 \sin \theta) \omega^2, \text{ so } \omega^2 = \frac{T}{0.2 \times 0.9} = 32.67 \quad \text{M1}$$

$$\omega = 5.72 \text{ rad s}^{-1} \quad \text{A1}$$

Answer: $T = 5.88 \text{ N}$, $\theta = 70.5^\circ$, $\omega = 5.72 \text{ rad s}^{-1}$ 5 marks

13 (5 marks)

Radius of circle $r = 3 + 4 \sin 35^\circ = 5.29 \text{ m}$ B1

$$T \cos 35^\circ = mg, T \sin 35^\circ = \frac{mv^2}{r} \quad \text{M1}$$

$$v^2 = rg \tan 35^\circ = 36.33 \quad \text{M1}$$

$$v = 6.03 \text{ m s}^{-1} \quad \text{A1}$$

$$\omega = \frac{v}{r} = 1.14 \text{ rad s}^{-1} \quad \text{A1}$$

Answer: $v = 6.03 \text{ m s}^{-1}$, $\omega = 1.14 \text{ rad s}^{-1}$ 5 marks

14 (7 marks)

(a) The string makes angle θ with the vertical where $\cos \theta = \frac{0.4}{0.8} = \frac{1}{2}$ B1

$$\text{Horizontally: } T \sin \theta = m(0.8 \sin \theta) \omega^2, \text{ so } T = 0.5 \times 0.8 \times 16 \quad \text{M1}$$

$$= 6.40 \text{ N} \quad \text{A1}$$

Answer: 6.40 N 3 marks

(b) Vertically: $T \cos \theta + R = mg$ M1

$$R = 0.5 \times 9.8 - 6.40 \times \frac{1}{2} = 1.70 \text{ N} \quad \text{A1}$$

Answer: 1.70 N 2 marks

(c) $R = 0$: $T \cos \theta = mg$ with $T = m(0.8) \omega^2$ M1

$$\omega^2 = \frac{g}{0.8 \times 0.5} = 24.5, \text{ so } \omega = 4.95 \text{ rad s}^{-1} \quad \text{A1}$$

Answer: 4.95 rad s^{-1} 2 marks

15 (7 marks)

Greatest speed: friction acts down the slope. Resolving vertically: $R \cos 20^\circ = mg + F \sin 20^\circ$ M1

Horizontally: $R \sin 20^\circ + F \cos 20^\circ = \frac{mv^2}{r}$, with $F = 0.2R$ M1

$R(\cos 20^\circ - 0.2 \sin 20^\circ) = mg$, $R(\sin 20^\circ + 0.2 \cos 20^\circ) = \frac{mv^2}{50}$ A1

$v^2 = \frac{50g(\tan 20^\circ + 0.2)}{1 - 0.2 \tan 20^\circ} = 298.0$, $v = 17.3 \text{ m s}^{-1}$ M1 A1

Least speed: friction acts up the slope, $F = 0.2R$: $v^2 = \frac{50g(\tan 20^\circ - 0.2)}{1 + 0.2 \tan 20^\circ}$ M1

$v = 8.65 \text{ m s}^{-1}$ A1

Answer: greatest 17.3 m s^{-1} , least 8.65 m s^{-1} 7 marks

16 (7 marks)

(a) Triangle ABP is isosceles, so P is level with the midpoint of AB: vertical distance 0.4 m

$\cos \alpha = \frac{0.4}{0.5} = 0.8$, so $\sin \alpha = 0.6$, $r = 0.5 \sin \alpha = 0.3 \text{ m}$ B1

Answer: $\sin \alpha = 0.6$ for each string, where α is the angle to the vertical, and the circle has radius 0.3 m 1 mark

(b) Vertical: $(T_1 - T_2)\cos \alpha = mg$, so $T_1 - T_2 = \frac{0.4 \times 9.8}{0.8} = 4.9$ M1 A1

Horizontal: $(T_1 + T_2)\sin \alpha = mr\omega^2$, so $T_1 + T_2 = \frac{0.4 \times 0.3 \times 36}{0.6} = 7.2$ M1

$T_1 = 6.05 \text{ N}$ (AP), $T_2 = 1.15 \text{ N}$ (BP) A1

Answer: $T_{AP} = 6.05 \text{ N}$, $T_{BP} = 1.15 \text{ N}$ 4 marks

(c) BP just taut: $T_2 = 0$, so $T_1 \cos \alpha = mg$ and $T_1 \sin \alpha = mr\omega^2$ M1

$\omega^2 = \frac{g \tan \alpha}{r} = \frac{9.8 \times 0.75}{0.3} = 24.5$, $\omega = 4.95 \text{ rad s}^{-1}$ A1

Answer: 4.95 rad s^{-1} 2 marks

17 (6 marks)

(a) $\omega = 40 \times \frac{2\pi}{60} = 4.19 \text{ rad s}^{-1}$ B1

$R = mr\omega^2 = 60 \times 2.5 \times \omega^2 = 2630 \text{ N}$ M1 A1

Answer: 2630 N 3 marks

(b) $F = mg = 588 \text{ N}$ B1

$\mu \geq \frac{F}{R} = 0.223$ M1 A1

Answer: 0.223 3 marks

18 (6 marks)

Vertical: $R \cos \alpha = mg$; horizontal: $R \sin \alpha = m(0.5 \sin \alpha)\omega^2$ M1 M1

So $R = m(0.5)\omega^2$, and $\cos \alpha = \frac{g}{0.5\omega^2} = \frac{9.8}{18}$ M1 A1

$\alpha = 57.0^\circ$ A1

Depth = $0.5 \cos \alpha = 0.272 \text{ m}$ A1

Answer: $\alpha = 57.0^\circ$, depth 0.272 m 6 marks

19 (6 marks)

$R \sin 30^\circ = \frac{mv^2}{r}$, $R \cos 30^\circ = mg$, so $v^2 = rg \tan 30^\circ$ M1 A1

$v = \sqrt{25 \times 9.8 \times \tan 30^\circ} = 11.9 \text{ m s}^{-1}$ A1

$R = \frac{85 \times 9.8}{\cos 30^\circ} = 962 \text{ N}$ A1

$14 > 11.9$: the cyclist is going faster than the no-friction speed and tends to slide up the slope, so friction acts down the slope E1 B1

Answer: 11.9 m s^{-1} , 962 N; down the slope 6 marks

20 (7 marks)

$$T \cos 30^\circ = mg, \text{ so } T = \frac{0.5 \times 9.8}{\cos 30^\circ} = 5.66 \text{ N}$$

M1 A1

$$\text{Extension } e = \frac{Tl}{\lambda} = \frac{5.658 \times 0.6}{29.4} = 0.115 \text{ m}$$

M1 A1

$$\text{Length of string} = 0.7155 \text{ m, radius } r = 0.7155 \sin 30^\circ = 0.3577 \text{ m}$$

B1

$$T \sin 30^\circ = \frac{mv^2}{r} \text{ gives } v = 1.42 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } e = 0.115 \text{ m, } v = 1.42 \text{ m s}^{-1}$$

7 marks

21 (8 marks)

$$(a) T \cos \theta = mg, T \sin \theta = m(l \sin \theta) \omega^2$$

M1

$$\text{So } T = ml\omega^2, \text{ hence } \omega^2 = \frac{g}{l \cos \theta}$$

M1 A1

$$\text{Period} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

A1

$$\text{Answer: The time for one revolution is } 2\pi \sqrt{\frac{l \cos \theta}{g}}$$

4 marks

$$(b) 2\pi \sqrt{\frac{1.2 \cos 45^\circ}{9.8}} = 1.85 \text{ s}$$

M1 A1

$$\text{Answer: } 1.85 \text{ s}$$

2 marks

(c) m cancels in the equations, so the period is independent of mass
 $\cos \theta$ decreases as θ increases ($0 < \theta < 90^\circ$), so the period decreases

E1

E1

$$\text{Answer: independent of } m; \text{ decreases with } \theta$$

2 marks

22 (8 marks)

$$(a) \text{ About to slip up the slope, friction acts down the slope. Vertically: } R \cos \alpha - \mu R \sin \alpha = mg$$

M1

$$\text{Horizontally: } R \sin \alpha + \mu R \cos \alpha = \frac{mv^2}{r}$$

M1 A1

$$\text{Divide, and divide numerator and denominator by } \cos \alpha: \frac{v^2}{rg} = \frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}$$

A1

$$\text{Answer: The speed } v \text{ of the car satisfies } v^2 = \frac{rg(\tan \alpha + \mu)}{1 - \mu \tan \alpha}$$

4 marks

$$(b) v^2 = \frac{80 \times 9.8 \times (0.25 + 0.5)}{1 - 0.125} = 672$$

M1

$$v = 25.9 \text{ m s}^{-1}$$

A1

$$\text{Sliding down is limited by } v^2 \geq \frac{rg(\tan \alpha - \mu)}{1 + \mu \tan \alpha}, \text{ which is negative since } \mu > \tan \alpha$$

M1

so friction alone can hold the car even when it is at rest: no minimum speed

E1

$$\text{Answer: } 25.9 \text{ m s}^{-1}$$

4 marks

23 (7 marks)

The vertical component of tension is $T \cos \theta$ (the angle is measured from the vertical) and the horizontal component is $T \sin \theta$

E1

$$T \cos 60^\circ = mg, \text{ so } T = 3.92 \text{ N}$$

M1 A1

$$T \sin 60^\circ = m(1.0 \sin 60^\circ) \omega^2, \text{ so } \omega^2 = \frac{T}{m} = 19.6$$

M1

$$\omega = 4.43 \text{ rad s}^{-1}$$

A1

$$v = r\omega = \sin 60^\circ \times 4.43 = 3.83 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } T = 3.92 \text{ N, } \omega = 4.43 \text{ rad s}^{-1}, v = 3.83 \text{ m s}^{-1}$$

7 marks

24 (9 marks)

$$(a) 0.8^2 + 0.6^2 = 1.0^2, \text{ so angle APB} = 90^\circ \text{ (converse of Pythagoras)}$$

M1

$$\text{Area of triangle: } \frac{1}{2} \times 0.8 \times 0.6 = \frac{1}{2} \times 1.0 \times r, \text{ so } r = 0.48 \text{ m}$$

A1

$$\text{Answer: Angle APB is a right angle and the radius of the circle is } 0.48 \text{ m}$$

2 marks

$$(b) \text{ AP makes angle } \alpha \text{ with the vertical, } \sin \alpha = \frac{0.48}{0.8} = 0.6, \cos \alpha = 0.8; \text{ BP: } \sin \beta = \frac{0.48}{0.6} = 0.8, \cos \beta = 0.6$$

Vertical: $0.8T_1 - 0.6T_2 = 0.5g = 4.9$	M1
Horizontal: $0.6T_1 + 0.8T_2 = 0.5 \times 0.48 \times 36 = 8.64$	M1
$T_1 = 9.10 \text{ N}, T_2 = 3.97 \text{ N}$	A1 A1
Answer: $T_{AP} = 9.10 \text{ N}, T_{BP} = 3.97 \text{ N}$	4 marks
(c) $T_2 = 0: 0.8T_1 = 4.9$ and $0.6T_1 = 0.24\omega^2$	M1
$\omega^2 = \frac{0.6 \times 4.9}{0.8 \times 0.24} = 15.31$	M1
$\omega = 3.91 \text{ rad s}^{-1}$	A1
Answer: 3.91 rad s^{-1}	3 marks

25 (9 marks)

(a) R is at angle α to the horizontal: $R \sin\alpha = mg$, $R \cos\alpha = m\omega^2 r$, where $r = h \tan\alpha$	M1 A1
Dividing: $\frac{1}{\tan\alpha} = \frac{r\omega^2}{g}$	M1
$\omega^2 = \frac{g}{r \tan\alpha} = \frac{g}{h \tan^2\alpha}$	A1
Answer: The angular speed of P is $\omega = \sqrt{\frac{g}{h \tan^2\alpha}}$	4 marks
(b) $\omega^2 = \frac{9.8}{0.6 \times \frac{1}{3}} = 49$, $\omega = 7 \text{ rad s}^{-1}$	M1 A1
$r = 0.6 \tan 30^\circ$, $v = r\omega = 2.42 \text{ m s}^{-1}$	M1 A1
Period = $\frac{2\pi}{7} = 0.898 \text{ s}$	A1
Answer: $\omega = 7 \text{ rad s}^{-1}$, $v = 2.42 \text{ m s}^{-1}$, period 0.898 s	5 marks

26 (7 marks)

(a) Radius of circle $r = 3 + 4 \sin\theta$	B1
$T \cos\theta = mg$, $T \sin\theta = m\omega^2 r$	M1
$\tan\theta = \frac{r\omega^2}{g}$, so $\omega^2 = \frac{g \tan\theta}{3 + 4 \sin\theta}$	A1
Answer: $\omega^2 = \frac{g \tan\theta}{3 + 4 \sin\theta}$	3 marks
(b) Substitute $\omega^2 = 2.25$ in part (a)	M1
Let $f(\theta) = 9.8 \tan\theta - 2.25(3 + 4 \sin\theta)$	B1
$f(54.8^\circ) = -0.216 < 0$ and $f(55.8^\circ) = 0.223 > 0$, so by change of sign the root lies between	M1 A1
Answer: $\theta \approx 55.3^\circ$	4 marks

27 (10 marks)

Normal reaction R perpendicular to the surface: components $R \sin 60^\circ$ towards the axis, $R \cos 60^\circ$ upwards	B1
At the greatest ω , P is about to slip up the surface, so friction $F = 0.2R$ acts down the surface, with components $F \cos 60^\circ$ towards the axis and $F \sin 60^\circ$ downwards	M1
Vertical: $R \cos 60^\circ = mg + 0.2R \sin 60^\circ$	M1 A1
Horizontal: $R \sin 60^\circ + 0.2R \cos 60^\circ = m(0.5)\omega^2$	M1
Divide: $0.5\omega^2/g = \frac{\tan 60^\circ + 0.2}{1 - 0.2 \tan 60^\circ}$, so $\omega^2_{\max} = 57.94$	A1
At the least ω friction acts up the surface; vertical: $R \cos 60^\circ + 0.2R \sin 60^\circ = mg$, horizontal: $R \sin 60^\circ - 0.2R \cos 60^\circ = 0.5m\omega^2$	M1
$\omega^2_{\min} = \frac{g(\tan 60^\circ - 0.2)}{0.5(1 + 0.2 \tan 60^\circ)} = 22.30$	M1 A1
$4.72 \leq \omega \leq 7.61 \text{ rad s}^{-1}$	A1
Answer: $4.72 \leq \omega \leq 7.61 \text{ rad s}^{-1}$	10 marks

28 (9 marks)

(a) $T \cos 60^\circ = mg$, $T \sin 60^\circ = \frac{mv^2}{r}$, $r = 1.2 \sin 60^\circ$ M1

$$v^2 = rg \tan 60^\circ = 1.2 \sin 60^\circ \times 9.8 \times \tan 60^\circ$$
M1

$$= 1.2 \times 9.8 \times 1.5 = 17.64, \text{ so } v = 4.2 \text{ m s}^{-1}$$
A1

Answer: The speed of P in the circular motion is 4.2 m s^{-1} 3 marks

(b) Kinetic energy gained $= \frac{1}{2} \times 0.5 \times 17.64 = 4.41 \text{ J}$ M1 A1

Height gained $= 1.2(1 - \cos 60^\circ) = 0.6 \text{ m}$, so PE gained $= 0.5 \times 9.8 \times 0.6 = 2.94 \text{ J}$ M1

Work done $= 4.41 + 2.94 = 7.35 \text{ J}$ A1

Answer: 7.35 J 4 marks

(c) $KE = \frac{1}{2}mv^2 = \frac{1}{2}mgl \sin \theta \tan \theta$ (using $v^2 = rg \tan \theta$, $r = l \sin \theta$) M1

Work $= KE + PE = mgl(1 - \cos \theta) + \frac{1}{2}mgl \sin \theta \tan \theta$ A1

Answer: Work done $= mgl(1 - \cos \theta) + \frac{1}{2}mgl \sin \theta \tan \theta$ 2 marks

29 (8 marks)

(a) Greatest speed: $\frac{v_1^2}{rg} = \frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} = \frac{\tan \alpha + \tan \lambda}{1 - \tan \alpha \tan \lambda}$ M1 A1

$= \tan(\alpha + \lambda)$ A1

Least speed: $\frac{v_2^2}{rg} = \frac{\tan \alpha - \mu}{1 + \mu \tan \alpha} = \tan(\alpha - \lambda)$ A1

Answer: $\frac{v_1^2}{rg} = \tan(\alpha + \lambda)$ and $\frac{v_2^2}{rg} = \tan(\alpha - \lambda)$ 4 marks

(b) $\alpha + \lambda = \arctan \frac{18.4^2}{588} = 30^\circ$ M1 A1

$\alpha - \lambda = \arctan \frac{10.2^2}{588} = 10^\circ$ M1

$\alpha = 20^\circ \approx 20^\circ$, $\lambda = 10^\circ$ A1

$\mu = \tan \lambda = 0.176 \approx 0.18$

Answer: $\alpha \approx 20^\circ$, $\mu \approx 0.18$ 4 marks

30 (11 marks)

(a) Normal reaction R along the radius PO. Vertical: $R \cos \theta = mg$ M1

Horizontal: $R \sin \theta = m(a \sin \theta)\omega^2$, so $R = ma\omega^2 (\sin \theta \neq 0)$ M1 A1

$\cos \theta = \frac{mg}{R} = \frac{g}{a\omega^2}$ A1

Since $\cos \theta < 1$, we need $\omega^2 > \frac{g}{a}$ E1

Answer: $\cos \theta = \frac{g}{a\omega^2}$ 5 marks

(b) $\cos \theta = \frac{9.8}{0.4 \times 64} = 0.3828$, $\theta = 67.5^\circ$ M1 A1

$R = ma\omega^2 = 0.2 \times 0.4 \times 64 = 5.12 \text{ N}$ M1 A1

Answer: $\theta = 67.5^\circ$, $R = 5.12 \text{ N}$ 4 marks

(c) For $\omega^2 \leq \frac{g}{a}$ no angle satisfies $\cos \theta = \frac{g}{a\omega^2}$, so the only equilibrium positions are on the axis: E1

$\theta = 0$ or $\theta = \pi$

On the axis the bead has no horizontal acceleration, so the lowest point (and highest point) are E1

always positions of equilibrium, for every ω , including $\omega^2 > \frac{g}{a}$

Answer: $\theta = 0$ and π always; off-axis only if $\omega^2 > \frac{g}{a}$ 2 marks

Vertical Circular Motion

1 (2 marks)

$$T - mg = \frac{mv^2}{r} \quad \text{M1}$$

$$T = 0.5 \times 9.8 + \frac{0.5 \times 36}{0.8} = 27.4 \text{ N} \quad \text{A1}$$

Answer: 27.4 N 2 marks

2 (2 marks)

$$T + mg = \frac{mv^2}{r} \quad \text{M1}$$

$$T = \frac{0.2 \times 16}{0.6} - 0.2 \times 9.8 = 3.37 \text{ N} \quad \text{A1}$$

Answer: 3.37 N 2 marks

3 (2 marks)

$$\frac{1}{2}mv^2 = \frac{1}{2}m(49) - mg(1.2) \quad \text{M1}$$

$$v = \sqrt{49 - 2 \times 9.8 \times 1.2} = 5.05 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 5.05 m s⁻¹ 2 marks

4 (2 marks)

$$T = 0 \text{ at the limit: } mg = \frac{mv^2}{r} \quad \text{M1}$$

$$v = \sqrt{9.8 \times 0.9} = 2.97 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 2.97 m s⁻¹ 2 marks

5 (2 marks)

$$u^2 = 5gr \quad \text{M1}$$

$$u = \sqrt{5 \times 9.8 \times 1.5} = 8.57 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 8.57 m s⁻¹ 2 marks

6 (2 marks)

$$\text{Energy: } \frac{1}{2}mu^2 \geq mg(2r) \text{ (speed may be zero at the top)} \quad \text{M1}$$

$$u = \sqrt{4 \times 9.8 \times 0.4} = 3.96 \text{ m s}^{-1} \quad \text{A1}$$

Answer: 3.96 m s⁻¹ 2 marks

7 (4 marks)

$$\text{Energy: } \frac{1}{2}mv^2 = \frac{1}{2}m(5.4)^2 - mg(0.8) \quad \text{M1}$$

$$v^2 = 29.16 - 15.68 = 13.48, v = 3.67 \text{ m s}^{-1} \quad \text{A1}$$

$$\text{String horizontal: } T = \frac{mv^2}{r} \text{ (weight has no component along the string)} \quad \text{M1}$$

$$T = \frac{0.3 \times 13.48}{0.8} = 5.06 \text{ N} \quad \text{A1}$$

Answer: v = 3.67 m s⁻¹, T = 5.06 N 4 marks

8 (4 marks)

$$\text{At the top: } R + mg = \frac{mv^2}{r} \quad \text{M1}$$

$$R = \frac{400 \times 121}{8} - 3920 = 2130 \text{ N} \quad \text{A1}$$

$$\text{Energy: } \frac{1}{2}v_B^2 = \frac{1}{2}(121) + g(16) \quad \text{M1}$$

$$v_B = \sqrt{121 + 313.6} = 20.8 \text{ m s}^{-1} \quad \text{A1}$$

Answer: $R = 2130 \text{ N}$, $v = 20.8 \text{ m s}^{-1}$ 4 marks

9 (4 marks)

$$\text{Energy: } v^2 = 64 - 2g(2r) = 24.80 \quad \text{M1 A1}$$

$$\text{At the top: } T + mg = \frac{mv^2}{r}, \text{ so } T = 0.5 \times 24.80 - 4.9 = 7.50 \text{ N} \quad \text{M1}$$

$T > 0$ at the top (and the tension is smallest there), so the string stays taut and the circle is completed E1

Answer: 7.50 N 4 marks

10 (4 marks)

$$\text{Energy: } v^2 = 25 - 4 \times 9.8 \times 0.5 = 5.400, v = 2.32 \text{ m s}^{-1} \quad \text{M1 A1}$$

$$\text{At the top, resultant force towards the centre: } mg + R = \frac{mv^2}{r}, R \text{ measured downwards} \quad \text{M1}$$

$$R = \frac{0.1 \times 5.400}{0.5} - 0.98 = 0.1000 \text{ N, towards the centre (downwards)} \quad \text{A1}$$

Answer: $v = 2.32 \text{ m s}^{-1}$; $R = 0.1000 \text{ N downwards}$ 4 marks

11 (4 marks)

$$mg - R = \frac{mv^2}{r} \quad \text{M1}$$

$$R = 3.92 - \frac{0.4 \times 1}{2} = 3.72 \text{ N} \quad \text{A1}$$

$$R \geq 0: v^2 \leq gr = 19.6 \quad \text{M1}$$

$$v \leq 4.43 \text{ m s}^{-1} \quad \text{A1}$$

Answer: $R = 3.72 \text{ N}$; $v \leq 4.43 \text{ m s}^{-1}$ 4 marks

12 (4 marks)

$$\text{Energy: } \frac{1}{2}mv^2 = mg(0.9) \quad \text{M1}$$

$$v = \sqrt{2 \times 9.8 \times 0.9} = 4.20 \text{ m s}^{-1} \quad \text{A1}$$

$$T - mg = \frac{mv^2}{r} \quad \text{M1}$$

$$T = 4.9 + \frac{0.5 \times 17.64}{0.9} = 14.7 \text{ N} \quad \text{A1}$$

Answer: $v = 4.20 \text{ m s}^{-1}$, $T = 14.7 \text{ N}$ 4 marks

13 (4 marks)

$$mg - R = \frac{mv^2}{r} \quad \text{M1}$$

$$R = 1200 \times (9.8 - \frac{225}{40}) = 5010 \text{ N} \quad \text{A1}$$

$$R = 0: v^2 = gr = 392 \quad \text{M1}$$

$$v = 19.8 \text{ m s}^{-1} \quad \text{A1}$$

Answer: $R = 5010 \text{ N}$; 19.8 m s^{-1} 4 marks

14 (6 marks)

Completing the circle needs $u^2 \geq 5gr = 58.8$ but $u^2 = 49 < 58.8$, so the string goes slack M1 A1

Let θ be the angle with the upward vertical: $v^2 = 49 - 2g(1.2)(1 + \cos\theta)$ M1

$$T = 0: mg \cos\theta = \frac{mv^2}{1.2}, \text{ so } v^2 = 1.2g \cos\theta$$
 M1

$$1.2g \cos\theta = 49 - 23.52 - 23.52\cos\theta, \text{ so } 35.28\cos\theta = 25.48$$
 M1

$$\cos\theta = 0.7222, \theta = 43.8^\circ$$
 A1

Answer: $\theta = 43.8^\circ$ 6 marks

15 (6 marks)

Lowest point: $\frac{1}{2}mv^2 = mg(0.6)$, $v = 3.43 \text{ m s}^{-1}$ M1 A1

$$R - mg = \frac{mv^2}{r}, \text{ so } R = 3mg = 5.88 \text{ N}$$
 M1 A1

At 60° : $v^2 = 2g(0.6 \cos 60^\circ) = 5.880$ M1

$$R - mg \cos 60^\circ = \frac{mv^2}{r}, \text{ so } R = 2.94 \text{ N}$$
 A1

Answer: $v = 3.43 \text{ m s}^{-1}$, $R = 5.88 \text{ N}$; at 60° , $R = 2.94 \text{ N}$ 6 marks

16 (6 marks)

At the top at the limit: $R = 0$, so $v_T^2 = gr = 58.8$ M1 A1

Energy: $u^2 = v_T^2 + 4gr = 5gr = 294$, $u = 17.1 \text{ m s}^{-1}$ M1 A1

Bottom: $R - mg = \frac{mu^2}{r} = 5mg$, so $R = 6mg$ M1

$$R = 6 \times 900 \times 9.8 = 52920 \text{ N}$$
 A1

Answer: $u = 17.1 \text{ m s}^{-1}$, $R = 52920 \text{ N}$ 6 marks

17 (5 marks)

Energy: $\frac{1}{2}mv^2 = mga(1 - \cos\theta)$ M1

Radial equation: $mg \cos\theta - R = \frac{mv^2}{a}$ M1

$R = 0$: $g \cos\theta = 2g(1 - \cos\theta)$ M1

$$3\cos\theta = 2, \cos\theta = \frac{2}{3}$$
 A1

$$v^2 = ga \cos\theta = 9.8 \times 1.5 \times \frac{2}{3} = 9.8, v = 3.13 \text{ m s}^{-1}$$
 A1

Answer: $\theta = 48.2^\circ$, $v = 3.13 \text{ m s}^{-1}$ 5 marks

18 (5 marks)

Energy: $v^2 = 36 - 4 \times 9.8 \times 0.7 = 8.560$ M1

$$v = 2.93 \text{ m s}^{-1}$$
 A1

Suppose the rod force F acts downwards (tension): $F + mg = \frac{mv^2}{r}$ M1

$$F = \frac{0.3 \times 8.560}{0.7} - 2.94 = 0.729 \text{ N}$$
 A1

$F > 0$, so the rod is in tension E1

Answer: $v = 2.93 \text{ m s}^{-1}$; 0.729 N tension 5 marks

19 (6 marks)

Energy: $\frac{1}{2}mv^2 = \frac{1}{2}m(2)^2 + mg(1.5)(1 - \cos\theta)$ M1

Radial equation, with $R = 0$ on leaving: $mg \cos\theta = \frac{mv^2}{1.5}$, so $v^2 = 1.5g \cos\theta$ M1 A1

$1.5g \cos\theta = 4 + 3g(1 - \cos\theta)$, so $4.5g \cos\theta = 4 + 3g$ and $\cos\theta = 0.7574$ M1

$\theta = 40.8^\circ$ A1

$v = \sqrt{1.5 \times 9.8 \times 0.7574} = 3.34 \text{ m s}^{-1}$ A1

Answer: $\theta = 40.8^\circ$, $v = 3.34 \text{ m s}^{-1}$ 6 marks

20 (6 marks)

$\omega = 300 \times \frac{2\pi}{60} = 31.4 \text{ rad s}^{-1}$, so $mr\omega^2 = 296 \text{ N}$ M1 A1

Lowest point: the rod pulls P upwards: $T - mg = mr\omega^2$ M1

$T = 296 + 14.7 = 311 \text{ N}$, the greatest force A1

Highest point: the rod pulls P downwards: $T + mg = mr\omega^2$ M1

$T = 296 - 14.7 = 281 \text{ N}$, the least force A1

Answer: greatest 311 N (at the lowest point), least 281 N (at the highest point) 6 marks

21 (7 marks)

(a) $T = 0$: $mg \cos\theta = \frac{mv^2}{1.2}$ M1

$v^2 = 1.2 \times 9.8 \times 0.7222 = 8.493$, $v = 2.91 \text{ m s}^{-1}$ A1

Answer: $v = 2.91 \text{ m s}^{-1}$ 2 marks

(b) Velocity of P at that instant is perpendicular to OP, so its upward component is $v \sin\theta$ M1

Further rise = $\frac{v^2 \sin^2\theta}{2g} = \frac{1.2 \cos\theta \sin^2\theta}{2} = 0.207 \text{ m}$ M1 A1

Height of P above O at slackening = $1.2 \cos\theta = 0.8667 \text{ m}$ M1

Greatest height above O = 1.07 m A1

Answer: 1.07 m 5 marks

22 (8 marks)

P is at height $0.8 + 0.8 \sin 30^\circ$ above the lowest point B1

Energy: $v^2 = u^2 - 2g(0.8)(1 + \sin 30^\circ)$ M1 A1

Leaves surface: $R = 0$, and the radial equation is $R + mg \sin 30^\circ = \frac{mv^2}{0.8}$, so $v^2 = 0.8g \sin 30^\circ =$ M1 A1

3.920

$u^2 = 3.92 + 23.52 = 27.44$ M1

$u = 5.24 \text{ m s}^{-1}$, $v = 1.98 \text{ m s}^{-1}$ A1 A1

Answer: $u = 5.24 \text{ m s}^{-1}$, $v = 1.98 \text{ m s}^{-1}$ 8 marks

23 (8 marks)

$v = 6$ is the speed at the lowest point, not at the highest point E1

Energy: $v^2 = 36 - 4g(0.8) = 4.640$ M1 A1

At the top, $T + mg = \frac{mv^2}{r}$, so $T = m(\frac{4.640}{0.8} - 9.8) = -4 \text{ m}$ M1 A1

T would be negative (a string cannot push), so the string cannot remain taut E1

Alternatively $u^2 = 36 < 5gr = 39.2$ M1

So the particle does not complete the circle A1

Answer: No: the string slackens before the top 8 marks

24 (8 marks)

Energy: $v^2 = 2gl \cos\theta$ M1

$T - mg \cos\theta = \frac{mv^2}{l} = 2mg \cos\theta$, so $T = 3mg \cos\theta$ M1 A1

$3mg \cos\theta = 2.5mg$ gives $\cos\theta = \frac{5}{6}$ A1

$v^2 = 2 \times 9.8 \times 1.2 \times \frac{5}{6} = 19.6$, $v = 4.43 \text{ m s}^{-1}$, directed at $\theta = 33.6^\circ$ below the horizontal M1

Height of the bob above the ground $= 3 - 1.2 \times \frac{5}{6} = 2 \text{ m}$ B1

$2 = v \sin\theta t + \frac{1}{2}gt^2$, giving $t = 0.436 \text{ s}$ M1

The bob moves towards the vertical through O, so it starts $1.2 \sin\theta$ from that line and travels $v \cos\theta t$ towards it: distance $= |1.2 \sin\theta - v \cos\theta t| = 0.946 \text{ m}$ (on the far side of the vertical) A1

Answer: 0.946 m 8 marks

25 (8 marks)

(a) At the top of the loop: $R + mg = \frac{mv^2}{r}$, with $R \geq 0$, so $v^2 \geq gr$ M1

Energy: $\frac{1}{2}mv^2 = mg(H - 2r)$ M1

$2g(H - 2r) \geq gr$, so $H \geq \frac{5r}{2}$ A1

Answer: The car can complete the loop only if $H \geq \frac{5r}{2}$ 3 marks

(b) Least $H = \frac{5 \times 6}{2} = 15 \text{ m}$ B1

$v^2 = 2g(18 - 12) = 12g$ M1

$R = \frac{mv^2}{r} - mg = m(2g) - mg = mg$ M1 A1

Answer: 15 m; R = mg 4 marks

(c) The mass cancels from the energy equation and from the condition $R \geq 0$ E1

Answer: m cancels 1 mark

26 (8 marks)

(a) Speed at the lowest point B: $v^2 = 2gl$ M1

Radius about the peg $= l - d$. At the top of this circle: $v_T^2 = v^2 - 4g(l - d) \geq g(l - d)$ M1 A1

$2gl \geq 5g(l - d)$, so $d \geq \frac{3l}{5}$ A1

Answer: P can complete the circle about the peg only if $d \geq \frac{3l}{5}$ 4 marks

(b) Before: $T - mg = \frac{mv^2}{l} = 2mg$, $T = 3mg = 11.8 \text{ N}$ M1 A1

After: radius 0.6 m , $T - mg = \frac{m \times 2g(1.5)}{0.6} = 5mg$ M1

$T = 6mg = 23.5 \text{ N}$ A1

Answer: 11.8 N before, 23.5 N after 4 marks

27 (10 marks)

(a) Energy: $v^2 = 2ga(1 - \cos\theta)$ M1

Radial: $mg \cos\theta - R = \frac{mv^2}{a}$; $R = 0$ gives $v^2 = ga \cos\theta$ M1

$ga \cos\theta = 2ga - 2ga \cos\theta$, so $\cos\theta = \frac{2}{3}$ A1

$v^2 = \frac{2ga}{3}$ A1

Answer: $\cos\theta = \frac{2}{3}$, and the speed of P when it leaves the surface is $\sqrt{\frac{2ga}{3}}$ 4 marks

(b) After leaving, P moves as a projectile with velocity v directed at angle θ below the horizontal, M1

starting at height $a \cos \theta = \frac{2a}{3}$

Energy from the top: $\frac{1}{2}mv_G^2 = mga$, so $v_G = \sqrt{2ga}$ A1

Vertical motion: $\frac{2a}{3} = v \sin \theta t + \frac{1}{2}gt^2$, where $v \sin \theta = \sqrt{\frac{2ga}{3}} \times \frac{\sqrt{5}}{3}$ M1

$$t = 0.6967 \sqrt{\frac{a}{g}}$$

A1

Answer: $t = 0.6967 \sqrt{\frac{a}{g}}$

4 marks

(c) Horizontal distance at leaving = $a \sin \theta = \frac{\sqrt{5}}{3}a$

Further distance = $v \cos \theta \times t = 0.3792 a$

M1

Total = $1.125 a$

A1

Answer: $1.12a$

2 marks

28 (11 marks)

(a) Bottom: $T_B - mg = \frac{mu^2}{r}$

M1 A1

Energy: $v_T^2 = u^2 - 4gr$

M1

Top: $T_T + mg = \frac{mv_T^2}{r}$, so $T_T = m \frac{u^2}{r} - 5mg$

A1

Answer: $T_B = m \frac{u^2}{r} + mg$ and $T_T = m \frac{u^2}{r} - 5mg$

4 marks

(b) Subtract: $(m \frac{u^2}{r} + mg) - (m \frac{u^2}{r} - 5mg) = 6mg$

M1 A1

Answer: $6mg$

2 marks

(c) $m \frac{u^2}{r} + mg = 8m \frac{u^2}{r} - 40mg$

M1

$$41g = 7 \frac{u^2}{r}, \text{ so } u^2 = \frac{41gr}{7}$$

A1

$\frac{41}{7} \approx 5.86 > 5$, so $u^2 > 5gr$: $T_T = m g(\frac{41}{7} - 5) > 0$ and the circle is completed

M1 A1

Answer: $u^2 = \frac{41gr}{7}$

4 marks

(d) $u = \sqrt{\frac{41 \times 9.8 \times 0.9}{7}} = 7.19 \text{ m s}^{-1}$

A1

Answer: 7.19 m s^{-1}

1 mark

29 (12 marks)

(a) Radial equation at P: $R + mg \sin \phi = \frac{mv^2}{a}$; $R = 0$ so $v^2 = ga \sin \phi$

M1 A1

Energy: $v^2 = u^2 - 2ga(1 + \sin \phi)$

M1

$u^2 = ga \sin \phi + 2ga + 2ga \sin \phi = ga(2 + 3 \sin \phi)$

A1

Answer: $u^2 = ga(2 + 3 \sin \phi)$

4 marks

(b) Take O as origin. P leaves at $(a \cos \phi, a \sin \phi)$ with velocity $v(-\sin \phi, \cos \phi)$

Horizontal: $a \cos \phi = v t \sin \phi$, so $t = \frac{a \cos \phi}{v \sin \phi}$

M1

Vertical: $-a \sin \phi = v t \cos \phi - \frac{1}{2}gt^2$

M1

Substitute t and $v^2 = ga \sin \phi$: $-\sin \phi = \frac{\cos^2 \phi}{\sin \phi} - \frac{\cos^2 \phi}{2 \sin^3 \phi}$

M1 A1

Multiply by $2 \sin^3 \phi$ and use $\cos^2 \phi = 1 - \sin^2 \phi$: $3 \sin^2 \phi = 1$

M1

$$\sin \phi = \frac{1}{\sqrt{3}}$$

A1

Answer: $\sin \phi = \frac{1}{\sqrt{3}}$

6 marks

(c) $u^2 = ga(2 + \frac{3}{\sqrt{3}}) = ga(2 + \sqrt{3})$

M1

$$u = 1.93 \sqrt{ga}$$

Answer: $1.93\sqrt{ga}$

A1
2 marks

30 (11 marks)

(a) Energy: $v^2 = u^2 - 2ga(1 - \cos\theta)$

M1

Radial: $R - mg \cos\theta = \frac{mv^2}{a}$

M1

$$R = \frac{mu^2}{a} - 2mg(1 - \cos\theta) + mg \cos\theta = \frac{mu^2}{a} + mg(3\cos\theta - 2)$$

A1

Answer: $R = \frac{mu^2}{a} + mg(3\cos\theta - 2)$

3 marks

(b) $u^2 \leq 2ga$: $v = 0$ occurs at $\cos\theta = 1 - \frac{u^2}{2ga} \geq 0$, i.e. at or below the level of O

M1

There $R - mg \cos\theta = 0$, so $R = mg \cos\theta \geq 0$, and R decreases as θ increases, so $R \geq 0$ throughout

A1

$u^2 \geq 5ga$: minimum R is at $\theta = \pi$: $R = \frac{mu^2}{a} - 5mg \geq 0$

M1 A1

Answer: $u^2 \leq 2ga$: P oscillates without leaving the surface; $u^2 \geq 5ga$: P completes circles

4 marks

(c) R is least at $\theta = \pi$, where it is negative if $u^2 < 5ga$, so R reaches zero before the top

E1

$R = 0$ at $\theta = 120^\circ$: $\frac{u^2}{0.5} + 9.8(3\cos 120^\circ - 2) = 0$

M1

$$\frac{u^2}{0.5} = 9.8 \times 3.5 = 34.3$$

M1

$u^2 = 17.15$, $u = 4.14 \text{ m s}^{-1}$

A1

Answer: $u = 4.14 \text{ m s}^{-1}$

4 marks

Circular Motion Modelling

1 (2 marks)

The stone is modelled as a particle (point mass), so its size and rotation are ignored

B1

The string is light and inextensible and there is no air resistance, so speed is constant

B1

Answer: particle; light inextensible string, no air resistance

2 marks

2 (2 marks)

$$\omega = 1000 \times \frac{2\pi}{60} = 105 \text{ rad s}^{-1}$$

M1

$v = 0.25\omega = 26.2 \text{ m s}^{-1}$

A1

Answer: 26.2 m s^{-1}

2 marks

3 (2 marks)

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 6.7 \times 10^6}{90 \times 60}$$

M1

$= 7800 \text{ m s}^{-1}$

A1

Answer: 7800 m s^{-1}

2 marks

4 (2 marks)

(a) $v = \frac{2\pi \times 2}{5} = 2.51 \text{ m s}^{-1}$

B1

Answer: 2.51 m s^{-1}

1 mark

(b) $a = \frac{v^2}{r} = 3.16 \text{ m s}^{-2}$

B1

Answer: 3.16 m s^{-2}

1 mark

5 (2 marks)

$$\mu mg = \frac{mv^2}{r}$$

M1

$$v = \sqrt{0.5 \times 9.8 \times 40} = 14 \text{ m s}^{-1}$$

A1

Answer: 14 m s⁻¹

2 marks

6 (2 marks)

$$a = \frac{v^2}{r} = \frac{576}{2} = 288 \text{ m s}^{-2}$$

M1

$$\frac{288}{9.8} = 29.4, \text{ so about 29.4 times } g$$

A1

Answer: 288 m s⁻², 29.4 times g

2 marks

7 (5 marks)

$$\text{Vertical: } T \cos 50^\circ = mg, \text{ so } T = 3.05 \text{ N}$$

M1 A1

$$\text{Radius} = 1.5 \sin 50^\circ = 1.15 \text{ m}, T \sin 50^\circ = \frac{mv^2}{r}, \text{ so } v^2 = rg \tan 50^\circ$$

M1

$$v = 3.66 \text{ m s}^{-1}$$

A1

Assumption: the ball is a particle, or the rope is light/air resistance is negligible

B1

Answer: T = 3.05 N, v = 3.66 m s⁻¹

5 marks

8 (4 marks)

$$\frac{v^2}{r} = \frac{gR^2}{r^2}, \text{ so } v^2 = \frac{gR^2}{r}$$

M1

$$v = \sqrt{\frac{9.8 \times (6.37 \times 10^6)^2}{3.84 \times 10^8}} = 1020 \text{ m s}^{-1}$$

A1

$$T = \frac{2\pi r}{v} = 2.37 \times 10^6 \text{ s}$$

M1

$$= 27.4 \text{ days}$$

A1

Answer: v = 1020 m s⁻¹, T = 27.4 days

4 marks

9 (4 marks)

$$\omega = 4000 \times \frac{2\pi}{60} = 419 \text{ rad s}^{-1}$$

M1

$$F = m\omega^2 = 0.005 \times 0.12 \times \omega^2 = 105 \text{ N}$$

M1 A1

$$\text{Weight} = 0.005 \times 9.8 = 0.049 \text{ N, so } F = 2150 \times \text{weight}$$

A1

Answer: 105 N = 2150 times the weight

4 marks

10 (3 marks)

$$\text{Vertical: } R = mg; \text{ horizontal: } F = \frac{mv^2}{r}$$

M1

$$\tan \theta = \frac{F}{R} = \frac{v^2}{rg} = \frac{36}{147}$$

M1

$$\theta = 13.8^\circ$$

A1

Answer: 13.8°

3 marks

11 (4 marks)

$$\text{Limiting friction: } \mu mg = m\omega^2 r$$

M1

$$\omega^2 = \frac{0.35 \times 9.8}{2} = 1.715, \omega = 1.31 \text{ rad s}^{-1}$$

M1 A1

$$= 12.5 \text{ revolutions per minute}$$

A1

Answer: 12.5 rev min⁻¹

4 marks

12 (5 marks)

Acceleration towards the Earth's centre = $r\omega^2 = 0.0337 \text{ m s}^{-2}$

M1 A1

$mg - R = m(r\omega^2)$, so $R = 70(9.8 - 0.0337)$

M1

= 683.6 N

A1

North Pole reading is 686 N, so the difference is 2.36 N

B1

Answer: 683.6 N; 2.36 N less than at the pole

5 marks

13 (4 marks)

$$T = \frac{mv^2}{r} \leq 80, \text{ so } v \leq \sqrt{\frac{80r}{m}}$$

M1

$$r = 1.0: v \leq 14.1 \text{ m s}^{-1}$$

A1

$$r = 0.5: v \leq 10 \text{ m s}^{-1}$$

A1

For a given tension, $v^2 \propto r$, so a smaller radius gives a smaller maximum speed

E1

Answer: 14.1 m s⁻¹ and 10 m s⁻¹

4 marks

14 (6 marks)

The satellite has period 88 620 s, so $\omega = \frac{2\pi}{88\,620}$

B1

$$r\omega^2 = \frac{k}{r^2}, \text{ so } r^3 = \frac{k}{\omega^2} = \frac{4.28 \times 10^{13} \times 88\,620^2}{4\pi^2}$$

M1 A1

$$r = 2.04 \times 10^7 \text{ m}$$

A1

$$v = r\omega = \frac{2\pi r}{88\,620} = 1450 \text{ m s}^{-1}$$

M1 A1

Answer: $r = 2.04 \times 10^7 \text{ m}$, $v = 1450 \text{ m s}^{-1}$

6 marks

15 (5 marks)

Vertical: $F = mg$; horizontal: $R = \frac{mv^2}{r}$

M1

$$\text{Limiting: } mg = \mu \frac{mv^2}{r}$$

M1

$$v^2 = \frac{gr}{\mu} = 81.67, v = 9.04 \text{ m s}^{-1}$$

A1

$$R = \frac{mg}{\mu} = \frac{200 \times 9.8}{0.6} = 3270 \text{ N}$$

M1 A1

Answer: $v = 9.04 \text{ m s}^{-1}$, $R = 3270 \text{ N}$

5 marks

16 (6 marks)

$$\omega = \frac{2\pi}{0.7} = 8.98 \text{ rad s}^{-1}$$

B1

Horizontal component = $mr\omega^2 = 7.26 \times 1.8 \times \omega^2 = 1050 \text{ N}$

M1 A1

Vertical component = $mg = 71.1 \text{ N}$

B1

$$T = \sqrt{(1050)^2 + (71.1)^2} = 1060 \text{ N}$$

M1

$$\tan(\text{angle}) = \frac{71.1}{1050}, \text{ angle} = 3.87^\circ$$

A1

Answer: 1050 N, 1060 N, 3.87°

6 marks

17 (5 marks)

(a) Reaction $R = mr\omega^2$ must equal mg : $\omega^2 = \frac{g}{r} = 0.098$

M1

$$\omega = 0.313 \text{ rad s}^{-1} = 2.99 \text{ revolutions per minute}$$

A1

$$\text{Period} = \frac{2\pi}{\omega} = 20.1 \text{ s}$$

A1

Answer: 0.313 rad s⁻¹ = 2.99 rev min⁻¹; 20.1 s

3 marks

(b) $R = mr\omega^2 = m \times 60 \times 0.098$

M1

= 5.88m = 0.6mg, so the fraction is 0.6

A1

Answer: 0.6

2 marks

18 (5 marks)

Greatest speed: friction acts down the slope. $R \cos \alpha - 0.4R \sin \alpha = mg$

M1

$$R \sin \alpha + 0.4R \cos \alpha = \frac{mv^2}{r}$$

M1

Divide: $\frac{v^2}{rg} = \frac{\tan \alpha + 0.4}{1 - 0.4 \tan \alpha} = \frac{0.6}{0.92}$

M1 A1

$$v^2 = 639.1, v = 25.3 \text{ m s}^{-1}$$

A1

Answer: 25.3 m s^{-1}

5 marks

19 (5 marks)

Vertical: $F = mg$; horizontal: $R = mr\omega^2$

M1

$$F \leq \mu R: mg \leq 0.3 \times m \times 0.3 \times \omega^2$$

M1

$$\omega^2 \geq \frac{9.8}{0.09} = 108.9$$

A1

$$\omega \geq 10.4 \text{ rad s}^{-1}$$

A1

= 99.6 revolutions per minute

A1

Answer: $\geq 99.6 \text{ rev min}^{-1}$

5 marks

20 (4 marks)

$$a = \frac{v^2}{r} = \frac{(7.66 \times 10^3)^2}{6.78 \times 10^6}$$

M1

= 8.65 m s^{-2} (about 8.65, towards the Earth's centre)

A1

The astronaut moves in the same circle: $mg_h + R = ma$ with $a = g_h$

M1

Hence $R = 0$: the floor of the station does not push on the astronaut, so there is no sensation of weight

E1

Answer: 8.65 m s^{-2} ; $R = 0$

4 marks

21 (7 marks)

$$\frac{GMm}{r^2} = mr\omega^2$$

M1

$$\omega = \frac{2\pi}{T}: \frac{GM}{r^2} = r \frac{4\pi^2}{T^2}$$

M1

$$M = \frac{4\pi^2 r^3}{GT^2}$$

A1

$$T = 27.3 \times 86\,400 = 2.36 \times 10^6 \text{ s}$$

B1

$$M = \frac{4\pi^2 \times (3.84 \times 10^8)^3}{6.67 \times 10^{-11} \times T^2} = 6.02 \times 10^{24} \text{ kg}$$

M1 A1

Assumption: the Moon's orbit is circular, and the Moon's mass is negligible compared with the Earth's

B1

Answer: $6.02 \times 10^{24} \text{ kg}$

7 marks

22 (8 marks)

$$(a) \omega = \frac{2\pi}{60} = 0.1047 \text{ rad s}^{-1}, r\omega^2 = 0.2193 \text{ m s}^{-2}$$

B1

Top: $mg - R = mr\omega^2$, so $R = 60(9.8 - 0.2193) = 575 \text{ N}$

M1 A1

Bottom: $R - mg = mr\omega^2$, so $R = 601 \text{ N}$

M1 A1

Answer: top 575 N, bottom 601 N

5 marks

(b) $R = 0$ at the top: $r\omega^2 = g$

M1

$$\omega = \sqrt{\frac{g}{r}} = 0.7 \text{ rad s}^{-1}, \text{ so } T = \frac{2\pi}{0.7} = 8.98 \text{ s}$$

M1 A1

Answer: 8.98 s

3 marks

23 (8 marks)

The student used a period of 365 days, but ω must be in rad s^{-1} , so the period must be in seconds

E1

$$T = 365 \times 86\,400 = 3.15 \times 10^7 \text{ s}$$

B1

$$\omega = \frac{2\pi}{T} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

M1 A1

$$a = r\omega^2 = 0.00595 \text{ m s}^{-2}$$

M1 A1

$$\frac{GM}{r^2} = a, \text{ so } M = \frac{ar^2}{G} = 2.01 \times 10^{30} \text{ kg}$$

M1 A1

Answer: $a = 0.00595 \text{ m s}^{-2}$, $M = 2.01 \times 10^{30} \text{ kg}$

8 marks

24 (8 marks)

(a) At the top, $R + mg = mr\omega^2$, and $R \geq 0$

M1

$$\omega^2 \geq \frac{g}{r} = 24.5, \text{ so } \omega \geq 4.95 \text{ rad s}^{-1}$$

A1

= 47.3 revolutions per minute

A1

Answer: $47.3 \text{ rev min}^{-1}$

3 marks

(b) $\omega = 2\pi \text{ rad s}^{-1}$, so $r\omega^2 = 15.79 \text{ m s}^{-2}$

B1

Top: $R = m(r\omega^2 - g) = 0.2(15.79 - 9.8) = 1.20 \text{ N}$

M1 A1

Bottom: $R = m(r\omega^2 + g) = 5.12 \text{ N}$

M1 A1

Answer: top 1.20 N, bottom 5.12 N

5 marks

25 (7 marks)

$$(a) F = \frac{mv^2}{r} = \frac{7.26 \times 676}{1.9}$$

M1

$$= 2580 \text{ N}$$

A1

Answer: 2580 N

2 marks

(b) Vertical: $-1.9 = 26 \sin 42^\circ t - 4.9t^2$, with $26 \sin 42^\circ = 17.40$

M1

$$t = 3.66 \text{ s}$$

A1

$$\text{Horizontal distance} = 26 \cos 42^\circ \times t = 70.7 \text{ m}$$

M1 A1

$$\text{Energy: } v^2 = 26^2 + 2 \times 9.8 \times 1.9, v = 26.7 \text{ m s}^{-1}$$

A1

Answer: 70.7 m, 26.7 m s⁻¹

5 marks

26 (7 marks)

The person moves in a circle of radius $R \cos 60^\circ$ about the axis

M1

$$a = R\omega^2 \cos 60^\circ = 0.0337 \times 0.5 = 0.0168 \text{ m s}^{-2}$$

M1 A1

$$\text{Component towards the Earth's centre} = a \cos 60^\circ = 0.00842 \text{ m s}^{-2}$$

M1 A1

$$mg - R \approx m \times 0.00842, \text{ so } R = 70(9.8 - 0.00842) = 685.4 \text{ N}$$

M1 A1

Answer: $a = 0.0168 \text{ m s}^{-2}$, towards centre 0.00842 m s^{-2} , $R = 685.4 \text{ N}$

7 marks

27 (11 marks)

$$(a) \frac{v^2}{r} = \frac{gR^2}{r^2}, \text{ so } v^2 = \frac{gR^2}{r}$$

M1

$$v = R\sqrt{\frac{g}{r}}$$

A1

$$v_A = 7700 \text{ m s}^{-1}, v_B = 3070 \text{ m s}^{-1}$$

M1 A1

Answer: $v_A = 7700 \text{ m s}^{-1}$, $v_B = 3070 \text{ m s}^{-1}$

4 marks

$$(b) T = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$

M1

$$= \frac{2\pi}{R} \sqrt{\frac{r^3}{g}}$$

A1

$$T_A = 1.52 \text{ h}, T_B = 24.0 \text{ h}$$

M1 A1

Answer: $T_A = 1.52 \text{ h}$, $T_B = 24.0 \text{ h}$

4 marks

$$(c) \text{Relative angular speed: } \frac{2\pi}{T_A} - \frac{2\pi}{T_B}$$

M1

$$\left(\left(\frac{1}{T_A}\right) - \left(\frac{1}{T_B}\right)\right)t = 1, \text{ so } t = 1.62 \text{ h}$$

M1 A1

Answer: 1.62 h

3 marks

28 (10 marks)

(a) The normal reaction is zero, so the whole of gravity provides the centripetal force: $mg = mR\omega^2$

M1

$$\omega^2 = \frac{g}{R}, T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

A1

$$\text{Answer: } T = 2\pi\sqrt{\frac{R}{g}}$$

2 marks

(b) $M = \frac{4}{3}\pi R^3 \rho$

B1

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$$

M1

$$T = 2\pi\sqrt{\frac{R}{\frac{4}{3}\pi G\rho R}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

M1 A1

$$\text{Answer: } T = \sqrt{\frac{3\pi}{G\rho}} \text{ (independent of } R)$$

4 marks

(c) $T = \sqrt{\frac{3\pi}{6.67 \times 10^{-11} \times 5500}} = 5070 \text{ s}$

M1

= 84.5 minutes: the Earth would have to spin about 17 times faster than it does for objects on the equator to be weightless

A1

Answer: 84.5 min

2 marks

(d) $\rho = \frac{3\pi}{GT^2} = \frac{3\pi}{6.67 \times 10^{-11} \times (1.5 \times 10^{-3})^2}$

M1

$$= 6.28 \times 10^{16} \text{ kg m}^{-3}$$

A1

$$\text{Answer: } 6.28 \times 10^{16} \text{ kg m}^{-3}$$

2 marks

29 (11 marks)

(a) Energy: $\frac{1}{2}mv^2 = mg \times 10(1 - \cos 55^\circ)$

M1

$$v = 9.14 \text{ m s}^{-1}$$

A1

$$\text{Answer: } 9.14 \text{ m s}^{-1}$$

2 marks

(b) Greatest tension is at the lowest point: $T - mg = \frac{mv^2}{L}$

M1

$$T = 80 \times 9.8 + \frac{80 \times 83.58}{10} = 1450 \text{ N}$$

A1

1450 < 1500, so the vine does not break

E1

Answer: 1450 N, safe

3 marks

(c) Energy: $v^2 = 2g \times 10(\cos 20^\circ - \cos 55^\circ) = 71.76$, $v = 8.47 \text{ m s}^{-1}$

M1 A1

Velocity is perpendicular to the vine, at 20° above the horizontal: $v_x = 7.960$, $v_y = 2.897$

M1

Height of release above the lowest point = $10(1 - \cos 20^\circ) = 0.6031 \text{ m}$

B1

$$-0.6031 = 2.897t - 4.9t^2, \text{ so } t = 0.7544 \text{ s}$$

M1

$$\text{Horizontal distance} = 7.960 \times 0.7544 = 6.01 \text{ m}$$

A1

$$\text{Answer: } 8.47 \text{ m s}^{-1}; 6.01 \text{ m}$$

6 marks

30 (12 marks)

(a) Top, contact: $v_T^2 \geq gr$

$$\text{Energy: } v_B^2 = v_T^2 + 4gr \geq 5gr$$

M1 A1

$$\text{Bottom: } R - mg = \frac{mv_B^2}{r} \geq 5mg, \text{ so } R \geq 6mg > 4mg$$

M1 A1

$$\text{Answer: } R \geq 6mg \text{ at the bottom}$$

4 marks

(b) Top, just in contact: $v_T^2 = gr$

$$\text{Energy: } v_B^2 = gr + 2g(12)$$

M1

$$\text{Bottom: } R = mg + \frac{mv_B^2}{r_B} = mg1 + \frac{r_T + 24}{r_B} \leq 4mg$$

M1 A1

$$r_B \geq \frac{r_T + 24}{3}$$

A1

$$\text{Answer: The requirement is met if } r_B \geq \frac{r_T + 24}{3}$$

4 marks

$$\text{(c) } r_B \geq \frac{4 + 24}{3} = 9.33 \text{ m}$$

M1 A1

$$v_T = \sqrt{g r_T} = \sqrt{9.8 \times 4} = 6.26 \text{ m s}^{-1}$$

M1 A1

$$\text{Answer: } r_B \geq 9.33 \text{ m, } v_T = 6.26 \text{ m s}^{-1}$$

4 marks