

Angular Velocity

- 1 Express each angular speed in rad s^{-1} , giving exact answers in terms of π .
 - (a) 30 revolutions per minute [1]
 - (b) 150 revolutions per minute [1]
- 2 A particle moves in a circle of radius 2.5 m with constant angular speed 4 rad s^{-1} . Find the speed of the particle. [2]
- 3 A particle moves in a horizontal circle and completes one revolution every 0.8 s. Find its angular speed, giving your answer in terms of π . [2]
- 4 A particle moves in a circle of radius 3 m with constant speed 12 m s^{-1} . Find its angular speed. [2]
- 5 A turntable rotates at 6 rad s^{-1} . Convert this angular speed into revolutions per minute, giving your answer to 3 significant figures. [2]
- 6 A wheel turns through 3.5 complete revolutions.
 - (a) Write down the angle turned through in radians, in terms of π . [1]
 - (b) Write down the angle turned through in degrees. [1]
- 7 A bicycle wheel has radius 0.35 m. A point on the rim moves with constant speed 5 m s^{-1} relative to the centre of the wheel.
 - (a) Find the angular speed of the wheel. [2]
 - (b) Find the number of revolutions the wheel makes in one minute. [2]
- 8 A particle moves in a circle of radius 0.6 m, making 150 revolutions in one minute.
 - (a) Find the angular speed of the particle. [2]
 - (b) Find the speed of the particle. [2]
- 9 The Earth rotates once about its axis every 24 hours. Model the Earth as a sphere of radius $6.37 \times 10^6 \text{ m}$.
 - (a) Find the angular speed of the Earth, in rad s^{-1} , to 3 significant figures. [2]
 - (b) Find the speed of a point on the equator. [2]
- 10 A particle moves around a circle of radius 4 m with constant speed 6 m s^{-1} .
 - (a) Find the time taken to complete one revolution. [2]
 - (b) Find the time taken for the radius joining the particle to the centre to turn through 2 radians. [2]
- 11 A turntable rotates at 45 revolutions per minute. Two small coins, P and Q, rest on the turntable at distances 0.4 m and 0.9 m from the centre respectively.
 - (a) Find the angular speed of the turntable in rad s^{-1} . [2]
 - (b) Find the speeds of P and Q. [2]
- 12 Two particles A and B move in horizontal circles of radii 2 m and 5 m, with the same angular speed ω about the same vertical axis. The speed of B is 3 m s^{-1} greater than the speed of A. Find ω and the speed of A. [3]
- 13 A particle moves in a circle with constant angular speed 0.75 rad s^{-1} .
 - (a) Find the time taken to turn through an angle of 270° . [2]
 - (b) Find the number of complete revolutions made in one minute. [2]

- 14** A Ferris wheel has radius 25 m and its centre is 27 m above horizontal ground. The wheel makes one complete revolution every 90 s at constant angular speed. A passenger starts at the lowest point of the wheel.
- Find the angular speed of the wheel and the speed of the passenger. [2]
 - The passenger turns through an angle θ from the lowest point. Show that the height of the passenger above the ground is $27 - 25\cos \theta$ metres. [2]
 - Find the time taken, from the start, for the passenger first to reach a height of 40 m. [3]
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- 15** A belt passes round two wheels A and B without slipping. Wheel A has radius 12 cm and rotates at 150 revolutions per minute. Wheel B has radius 5 cm.
- Find the speed of the belt in m s^{-1} . [2]
 - Find the angular speed of wheel B in revolutions per minute. [2]
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- 16** A satellite orbits the Earth in a circle at a height of 400 km above the Earth's surface, taking 92.5 minutes for one orbit. The radius of the Earth is 6370 km. Find the angular speed of the satellite in rad s^{-1} and its speed in km s^{-1} , each to 3 significant figures. [4]
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- 17** Two particles P and Q move round the same horizontal circle of radius 3 m, with constant angular speeds 2 rad s^{-1} and 3.5 rad s^{-1} respectively. At time $t = 0$ the particles are at the same point.
- The particles move in the same direction. Find the time until they are next at the same point. [2]
 - The particles move in opposite directions. Find the time until they are next at the same point, and the distance travelled by P in that time. [3]
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- 18** At time t seconds, the position vector of a particle P relative to a fixed origin O is $\mathbf{r} = 3\cos 2t \mathbf{i} + 3 \sin 2t \mathbf{j}$, where distances are in metres.
- Show that P moves in a circle and state its radius. [2]
 - Find the speed of P and its angular speed. [2]
 - Show that the velocity of P is always perpendicular to $\mathbf{r}$. [2]
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- 19** A disc of radius 0.4 m rotates at 60 revolutions per minute about its centre. Particle P is on the rim and particle Q is on the disc 0.1 m from the centre. Find the speed of P relative to Q at an instant when
- P and Q are on the same radius, on the same side of the centre, [2]
 - P and Q are on the same diameter, on opposite sides of the centre. [2]
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- 20** The minute hand of a clock has length 12 cm and the hour hand has length 8 cm. Treat the hands as rotating at constant angular speeds.
- Find the angular speed of each hand in rad s^{-1} in terms of π . [2]
 - Show that the speed of the tip of the minute hand is 18 times the speed of the tip of the hour hand. [2]
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- 21** A clock has hour and minute hands, each turning at constant angular speed. At 12:00 the hands point in the same direction.
- Show that the minute hand turns through 12 times as large an angle as the hour hand in any given time. [2]
 - Show that the hands next point in the same direction at a time $\frac{720}{11}$ minutes after 12:00, and convert this to hours, minutes and seconds (to the nearest second). [4]
 - How many times in a 12-hour period, including the start, do the hands point in the same direction? [2]
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- 22** A fairground ride carries seats in a circle of radius 4 m, rotating at 20 revolutions per minute. A student writes: "The speed of a seat is $v = r\omega = 4 \times 20 = 80 \text{ m s}^{-1}$." Explain what is wrong with the student's working, and find the correct speed of a seat to 3 significant figures. Find also the time for one revolution. [7]

- 23** Two particles P and Q move in opposite directions round a horizontal circle of radius 5 m, with constant speeds 8 m s^{-1} and 6 m s^{-1} respectively. At time $t = 0$ they are at opposite ends of a diameter.
- (a) Find the time at which they first meet. [3]
- (b) Find the distance travelled by P before the second meeting. [3]

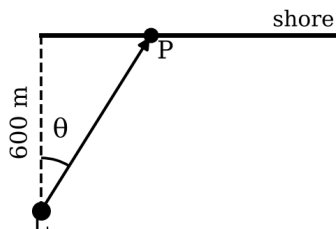
- 24** A bicycle has a front chainring with 48 teeth and a rear sprocket with 16 teeth, joined by a chain. The rear sprocket is fixed to the rear wheel, which has radius 0.33 m. The cyclist pedals at a constant 90 revolutions per minute, and the wheel rolls without slipping. Find the speed of the bicycle in m s^{-1} to 3 significant figures. [6]

- 25** In a tape recorder, tape is wound on to a reel at a constant speed of 4.8 cm s^{-1} . The radius of the wound tape is 1.5 cm at the start and 3.0 cm at the end.
- (a) Find the angular speed of the reel at the start and at the end, in rad s^{-1} . [2]
- (b) Convert both answers to revolutions per minute, to 3 significant figures. [2]
- (c) Explain why the angular speed of the reel decreases while the tape speed stays the same. [2]

- 26** The Moon moves in a circle round the Earth, of radius $3.84 \times 10^8 \text{ m}$, completing one orbit in 27.3 days. Show that the speed of the Moon is approximately 1.02 km s^{-1} , and find the angle, in degrees to 3 significant figures, through which the Moon moves in one day. [7]

- 27** A clock has an hour hand, a minute hand and a second hand, all turning at constant angular speeds and all pointing at 12 at time $t = 0$. Let t be the time in minutes after 12:00. Show that, for $0 \leq t \leq 720$, the three hands point in the same direction only when $t = 0$ and $t = 720$, and state the clock times of these two occasions. [9]

- 28** A lighthouse L stands on a small island 600 m from a straight shore. Its lamp rotates at a constant 2 revolutions per minute, producing a spot of light P on the shore. θ is the angle between the beam and the perpendicular from L to the shore.



- (a) Show that the speed of P along the shore is $600\omega \sec^2\theta$, where ω is the angular speed of the lamp, and find ω in rad s^{-1} . [4]
- (b) Find the speed of P when $\theta = 0$ and when $\theta = 60^\circ$. [3]
- (c) The speed of sound is 340 m s^{-1} . Find the smallest value of θ , in degrees to 1 decimal place, for which P travels faster than sound. [3]
- 29** A wheel has 8 identical spokes, equally spaced, and rotates clockwise at a constant 5 revolutions per second. It is filmed by a camera taking 24 frames per second. Because the spokes look identical, the viewer sees each spoke as having moved to the nearest spoke position. Show that the wheel appears to rotate anticlockwise in the film, and find its apparent angular speed in revolutions per second. [9]

- 30** Two satellites A and B move in coplanar circular orbits about the Earth's centre O, in the same direction, with periods 90 minutes and 100 minutes respectively. At time $t = 0$ the satellites and O are in a straight line, with A and B on the same side of O.
- (a) Find the time until A and B are next on the same side of O in a straight line with O, and the number of orbits A and B have then completed. [5]
- (b) Find the first time at which the satellites are on opposite sides of O in a straight line with O, and the angle between OA and its initial direction then. [4]
- (c) A third satellite C is in the same orbit as A but moves in the opposite direction, starting at the same point as A. Find the time after $t = 0$ at which A and C first pass each other. [2]

Centripetal Acceleration

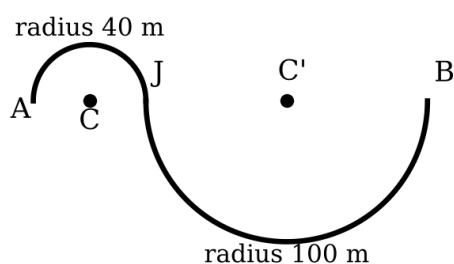
- 1** A particle moves in a circle of radius 2 m at a constant speed of 8 m s^{-1} . Find the magnitude of its acceleration. [2]
- 2** A particle moves in a circle of radius 0.5 m with constant angular speed 6 rad s^{-1} . Find the magnitude of its acceleration. [2]
- 3** A particle moves in a circle at constant speed 5 m s^{-1} with acceleration of magnitude 10 m s^{-2} . Find the radius of the circle. [2]
- 4** A particle moves in a circle with constant angular speed 4 rad s^{-1} and has acceleration of magnitude 12 m s^{-2} . Find the radius of the circle. [2]
- 5** A particle moves in a circle of radius r with constant speed v and constant angular speed ω .
- (a) State the direction of the acceleration of the particle. [1]
- (b) Write down two expressions for the magnitude of the acceleration, one in terms of v and r and one in terms of r and ω . [2]
- 6** A wheel of radius 0.2 m rotates at 120 revolutions per minute. Find the magnitude of the acceleration of a point on the rim, giving your answer to 3 significant figures. [2]
- 7** A car travels round a circular bend of radius 50 m at a constant 72 km h^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the speed of the car is 20 m s^{-1} , and find the magnitude of its acceleration. [3]
- (b) Express the acceleration as a multiple of g , to 3 significant figures. [2]
- 8** A particle moves in a circle at constant speed 6 m s^{-1} , and the magnitude of its acceleration is 12 m s^{-2} . Find the radius of the circle and the angular speed of the particle. [3]
- 9** A laboratory centrifuge spins at 3000 revolutions per minute. A sample is at a distance of 0.15 m from the axis. Find the magnitude of the acceleration of the sample, and express it as a multiple of g ($g = 9.8 \text{ m s}^{-2}$), giving both answers to 3 significant figures. [4]
- 10** The Earth rotates once every 24 hours. Model the Earth as a sphere of radius $6.37 \times 10^6 \text{ m}$. Find the magnitude of the acceleration of a point on the equator due to the Earth's rotation, giving your answer to 3 significant figures, and find what fraction of 9.8 m s^{-2} it is. [4]
- 11** A particle moves in a horizontal circle with period 4 s. The magnitude of its acceleration is 6 m s^{-2} . Find the radius of the circle and the speed of the particle, each to 3 significant figures. [4]
- 12** Particle P moves in a circle of radius r at speed v . Particle Q moves in a circle of radius $\frac{r}{2}$ at speed $2v$. Find the ratio of the magnitude of the acceleration of Q to that of P. [3]

- 13** The Moon moves in a circle of radius 3.84×10^8 m about the Earth with period 27.3 days.
- (a) Find the magnitude of the acceleration of the Moon, to 3 significant figures. [3]
- (b) Find the ratio of this acceleration to 9.8 m s^{-2} , in the form $1 : n$, with n to the nearest whole number. [2]
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- 14** The drum of a spin dryer has radius 0.25 m. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) The drum rotates at 1200 revolutions per minute. Find the magnitude of the acceleration of clothing at the inner surface of the drum, giving your answer as a multiple of g to 3 significant figures. [3]
- (b) Find the least angular speed, in revolutions per minute to 3 significant figures, for which the acceleration is at least $40g$. [3]
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- 15** A particle P moves so that its position vector relative to a fixed origin O at time t seconds is $\mathbf{r} = a \cos \omega t \mathbf{i} + a \sin \omega t \mathbf{j}$, where a and ω are positive constants.
- (a) Find the velocity of P, and show that its speed is constant. [3]
- (b) Show that the acceleration of P is $-\omega^2 \mathbf{r}$, and deduce its magnitude and direction. [3]
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- 16** The tyres of a car can provide a maximum acceleration, towards the centre of a bend, of $0.6g$, where $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the greatest speed at which the car can travel round a bend of radius 80 m. Give your answer in m s^{-1} and in km h^{-1} , to 3 significant figures. [3]
- (b) Find the smallest radius of bend that the car can take at a constant 30 m s^{-1} . [2]
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- 17** A turntable rotates at constant angular speed. A particle at a distance of 0.8 m from the axis has acceleration of magnitude 5 m s^{-2} . Find the magnitude of the acceleration of a particle at a distance of 0.5 m from the axis, and find the angular speed of the turntable in rad s^{-1} . [4]
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- 18** A particle moves in a circle of radius r with constant speed v , period T and angular speed ω . Show that its acceleration has magnitude $\frac{4\pi^2 r}{T^2}$, and show that this is also equal to $v\omega$. [5]
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- 19** A satellite moves in a circular orbit at a height above the Earth's surface equal to the radius of the Earth, $R = 6.37 \times 10^6$ m. The magnitude of the acceleration due to gravity at distance r from the centre of the Earth is $\frac{gR^2}{r^2}$, with $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the magnitude of the acceleration of the satellite. [3]
- (b) Find the speed of the satellite and the time for one orbit, in hours, each to 3 significant figures. [4]
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- 20** A stone is whirled in a horizontal circle of radius 0.8 m at a constant 2.5 revolutions per second.
- (a) Find the magnitude of the acceleration of the stone. [2]
- (b) The string can only provide a force that gives the stone an acceleration of at most 400 m s^{-2} . Find the greatest number of revolutions per second, to 3 significant figures. [3]
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- 21** A particle moves round a circle of radius 5 m with constant speed 10 m s^{-1} .
- (a) Find the magnitude of the acceleration of the particle. [2]
- (b) The particle moves from a point A to a point B a quarter of a circle later. Find the magnitude of the change in velocity between A and B. [3]
- (c) Find the magnitude of the average acceleration over this quarter circle, and explain why it is smaller than your answer to part (a). [3]

- 22** A particle P moves so that its position vector at time t is $\mathbf{r} = 3 \cos 2t \mathbf{i} + 2 \sin 2t \mathbf{j}$, relative to a fixed origin O.
- Show that the acceleration of P is always directed towards O. [3]
 - Find the greatest and least values of the magnitude of the acceleration, and the values of t for which the least value first occurs. [4]
 - Explain why P is not in uniform circular motion. [1]
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- 23** A wheel of radius 0.3 m rotates at 90 revolutions per minute. A student writes: "The acceleration of a point on the rim is $a = r\omega^2 = 0.3 \times 90^2 = 2430 \text{ m s}^{-2}$, which is 248g." Explain the error, find the correct magnitude of the acceleration to 3 significant figures, and state its direction. [6]
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- 24** Take the Earth to be a sphere of radius $R = 6.37 \times 10^6 \text{ m}$ rotating about its polar axis once every 24 hours. A point on the Earth's surface is at latitude λ north. Show that the acceleration of this point, due to the Earth's rotation, has magnitude $R\omega^2 \cos \lambda$ directed towards the axis, where ω is the angular speed of the Earth, and that its component towards the centre of the Earth is $R\omega^2 \cos^2 \lambda$. Find these two quantities for the latitude of London, 51.5° , to 3 significant figures. [7]
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- 25** A hammer thrower swings a ball in a horizontal circle of radius 1.6 m, so that the ball has a constant speed of 28 m s^{-1} just before release. Find, for this circular motion, the magnitude of the acceleration of the ball, the angular speed in rad s^{-1} , and the number of complete revolutions the ball makes per second to 3 significant figures. Express the acceleration as a multiple of g ($g = 9.8 \text{ m s}^{-2}$). [7]
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- 26** A geostationary satellite orbits above the equator and always remains above the same point of the Earth's surface. The Earth's radius is $R = 6.37 \times 10^6 \text{ m}$ and the acceleration due to gravity at distance r from the Earth's centre is $\frac{gR^2}{r^2}$, with $g = 9.8 \text{ m s}^{-2}$. The Earth rotates once every 24 hours.
- Show that the radius r of the orbit satisfies $r^3 = \frac{gR^2}{\omega^2}$, where ω is the angular speed of the Earth. [2]
 - Find r , and the height of the satellite above the Earth's surface, each in km to 3 significant figures. [5]
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- 27** A particle moves in a circle of centre O and radius r at constant speed v . In a short time δt it moves from A to B, turning through a small angle $\delta\theta$ at O. By considering the triangle of velocities at A and B, show that the magnitude of the change in velocity is $2v \sin \frac{\delta\theta}{2}$. Hence show that the magnitude of the acceleration of the particle is $\frac{v^2}{r}$, and that its direction is towards O. [9]
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- 28** Two satellites A and B move in circular orbits about the same planet. The acceleration of a satellite at distance r from the centre of the planet has magnitude $\frac{k}{r^2}$, where k is a constant. The radius of the orbit of B is 4 times the radius of the orbit of A.
- Show that the speed of A is twice the speed of B. [4]
 - Find the ratio of the periods of A and B, and the ratio of their angular speeds. [3]
 - Find the ratio of the magnitudes of their accelerations. [2]
 - A third satellite C has an orbit of radius 9 times that of A. Find the time for C to make one orbit, in terms of T_A , the period of A. [2]

- 29** A bead moves round a circular wire of radius 2 m so that its speed at time t seconds is $v = 1.5t \text{ m s}^{-1}$, for $t \geq 0$.
- Show that the component of acceleration of the bead towards the centre of the circle is $1.125t^2 \text{ m s}^{-2}$, and find the component along the tangent. [4]
 - Find the value of t at which the two components of acceleration are equal. [2]
 - Find the magnitude of the acceleration at $t = 2$, and the angle it makes with the direction towards the centre of the circle. [5]

- 30** A road bend AB is made of a semicircle AJ of radius 40 m followed immediately by a semicircle JB of radius 100 m, curving the opposite way, as shown. A car travels from A to B at a constant speed of 20 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.



- Find the magnitude of the acceleration of the car on each semicircle, and state the direction in each case. [4]
- Find the total time taken to travel from A to B. [2]
- Find the magnitude of the change in the acceleration of the car as it passes J. [3]
- The car's acceleration must not exceed $0.5g$ at any point. Find the greatest constant speed at which the bend can be driven, to 3 significant figures. [2]

Centripetal Force

- A particle of mass 2 kg moves in a horizontal circle of radius 1.5 m at a constant speed of 3 m s^{-1} . Find the magnitude of the resultant force on the particle. [2]
- A particle of mass 0.4 kg moves in a circle of radius 0.5 m with constant angular speed 6 rad s^{-1} . Find the magnitude of the resultant force on the particle. [2]
- A particle of mass 0.5 kg is attached to one end of a light inextensible string of length 0.8 m. The other end is fixed to a point O on a smooth horizontal table, and the particle moves in a circle on the table at a constant speed of 4 m s^{-1} . Find the tension in the string. [2]
- A particle of mass 3 kg moves in a circle of radius 2 m under a resultant force of magnitude 24 N directed towards the centre. Find the speed of the particle. [2]
- A coin of mass 0.02 kg rests on a horizontal turntable at a distance of 0.15 m from the axis. The turntable rotates at a constant 8 rad s^{-1} and the coin does not slip. Find the magnitude of the frictional force on the coin. [2]
- State the force, or forces, that provide the resultant force towards the centre of the circle in each of the following cases.
 - A car turns round a flat horizontal circular bend. [1]
 - The Moon orbits the Earth. [1]
 - A particle moves in a circle on a smooth horizontal table, attached to a fixed point by a string. [1]

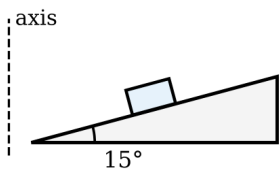
- 7** A car of mass 900 kg travels at a constant 15 m s^{-1} round a flat circular bend of radius 40 m. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the frictional force needed to keep the car moving in the circle. [2]
 (b) Find the least possible coefficient of friction between the tyres and the road. [2]
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- 8** A particle of mass 0.3 kg is attached to one end of a light inextensible string of length 0.6 m. The other end is fixed to a point O on a smooth horizontal table, and the particle moves in a circle on the table. The string breaks if the tension exceeds 20 N.
 (a) Find the greatest speed at which the particle can move in a circle without the string breaking. [2]
 (b) Find the greatest angular speed, in revolutions per minute to 3 significant figures. [2]
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- 9** A coin lies on a horizontal turntable at a distance of 0.2 m from the axis. The coefficient of friction between the coin and the turntable is 0.4. The angular speed of the turntable is slowly increased from zero. Find the angular speed, in rad s^{-1} and in revolutions per minute (both to 3 significant figures), at which the coin begins to slip. Take $g = 9.8 \text{ m s}^{-2}$. [4]
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- 10** A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.4 m and modulus of elasticity 12 N. The other end of the string is fixed to a point O on a smooth horizontal table. The particle moves in a circle on the table with the string stretched to a length of 0.6 m. Find the tension in the string and the speed of the particle. [4]
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- 11** A particle P of mass 0.2 kg moves in a circle of radius 0.5 m on a smooth horizontal table. P is attached to a light inextensible string which passes through a small smooth hole in the table and carries a particle of mass 0.6 kg hanging at rest below the table. Take $g = 9.8 \text{ m s}^{-2}$. Find the speed of P. [3]
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- 12** A satellite of mass 500 kg moves in a circular orbit of radius $7.0 \times 10^6 \text{ m}$ at a constant speed of $7.5 \times 10^3 \text{ m s}^{-1}$. Find the magnitude of the gravitational force acting on the satellite, to 3 significant figures. [3]
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- 13** A car of mass 1200 kg takes a flat circular bend of radius 60 m. The greatest frictional force that the road can exert on the car is 6000 N. Find the greatest speed at which the car can take the bend without slipping, in m s^{-1} and km h^{-1} (3 s.f.). [3]
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- 14** A particle of mass 0.4 kg lies on a rough horizontal turntable and is attached to the centre O of the turntable by a light elastic string of natural length 0.5 m and modulus of elasticity 8 N. The particle is at a distance of 0.7 m from O and moves in a circle with the turntable, without slipping. The coefficient of friction is 0.5. Find the least and greatest angular speeds of the turntable, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [7]
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- 15** Particles A and B, of masses 0.4 kg and 0.3 kg, are attached to the ends of a light inextensible string AB of length 0.7 m. A is also attached to a fixed point O on a smooth horizontal table by a light inextensible string OA of length 0.5 m. The particles move on the table in horizontal circles about O with the same constant angular speed 3 rad s^{-1} , with O, A and B in a straight line and both strings taut. Find the tension in each string. [5]
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- 16** A light rod AB of length 1.2 m carries particles of mass 2 kg at A and 3 kg at B. The rod rotates in a horizontal plane with constant angular speed 4 rad s^{-1} about a vertical axis through the point C of the rod, and the forces on the particles act along the rod. The tensions in the sections AC and CB of the rod are equal. Find the distance AC and the tension in each section. [6]
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- 17** A particle of mass 0.25 kg is attached to one end of a light inextensible string of length 1.2 m. The other end is fixed to a point O on a smooth horizontal table, and the particle moves in a circle on the table at a constant speed of 6 m s^{-1} .
- (a) Find the tension in the string. [2]
- (b) The string breaks when the tension reaches 12 N. Find the speed of the particle at which this happens. [2]
- (c) After the string breaks, the particle moves in a straight line across the table. Find its distance from O 2 seconds after the string breaks. [2]
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- 18** Two coins A and B lie on a rough horizontal turntable at distances 0.1 m and 0.25 m from the axis, and the coefficient of friction between each coin and the turntable is 0.4. The angular speed of the turntable is slowly increased from rest. Show that B slips before A, and find the angular speed at which B starts to slip and the speed of B at that instant, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [6]
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- 19** A satellite of mass m moves in a circular orbit of radius r about the Earth. The gravitational force on the satellite is $\frac{GMm}{r^2}$, where M is the mass of the Earth and G is the gravitational constant.
- (a) Show that the speed of the satellite is $\sqrt{\frac{GM}{r}}$. [2]
- (b) Given that $GM = 3.99 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, find the speed and the period of a satellite with orbit radius $2.0 \times 10^7 \text{ m}$, each to 3 significant figures. [4]
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- 20** A cyclist and bicycle, of total mass 80 kg, travel at a constant 8 m s^{-1} round a horizontal circular track of radius 20 m. Take $g = 9.8 \text{ m s}^{-2}$. Find the frictional force on the bicycle and the least coefficient of friction between the tyres and the track. The resultant of the normal reaction and the friction acts along the line from the contact point through the cyclist's centre of mass; find the angle this line makes with the vertical, to the nearest degree. [6]
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- 21** A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to the centre O of a rough horizontal turntable. P lies on the turntable at a distance $\frac{5a}{4}$ from O and moves in a circle with the turntable. The coefficient of friction between P and the turntable is $\frac{1}{3}$. The turntable rotates with constant angular speed ω and P does not slip. Show that $\frac{g}{3a} \leq \omega^2 \leq \frac{13g}{15a}$. [8]
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- 22** A car of mass 800 kg turns round a flat circular bend of radius 50 m. The coefficient of friction between the tyres and the road is 0.6. A student writes: "The friction is $F = \mu R = 0.6 \times 800 = 480 \text{ N}$. This provides the centripetal force, so $480 = \frac{800v^2}{50}$ and $v = 5.5 \text{ m s}^{-1}$." Identify the error in the working, and find the correct greatest speed of the car in m s^{-1} and km h^{-1} , to 3 significant figures. Find also the greatest speed on a bend of radius 100 m. Take $g = 9.8 \text{ m s}^{-2}$. [7]
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- 23** A light rod OA of length 1.2 m rotates in a horizontal plane about a vertical axis through O. A particle of mass 0.3 kg is attached at A and a particle of mass 0.5 kg is attached at B, where $OB = 0.4 \text{ m}$. The rod rotates with constant angular speed 5 rad s^{-1} . The forces on the rod act along its length.
- (a) Find the force exerted on the particle at A by the rod, and the force exerted on the particle at B by the part OB of the rod. [4]
- (b) Find the force exerted on the rod by the axle at O, and state its direction. [3]
- (c) The section AB of the rod can carry a force of at most 20 N. Find the greatest angular speed of the rod. [2]

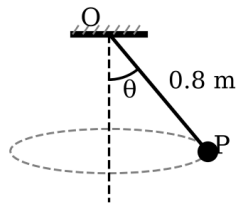
- 24** Two particles A and B, of masses $3m$ and $2m$, are joined by a light inextensible string of length $5a$. The particles move on a smooth horizontal table in horizontal circles, with the string taut, at the same constant angular speed ω about a point O of the table lying on the string between A and B.
- (a) Show that A is at distance $2a$ from O, and that the tension in the string is $6ma\omega^2$. [4]
- (b) The string breaks if the tension exceeds 30 N . Given that $m = 0.5\text{ kg}$ and $a = 0.4\text{ m}$, find the greatest possible angular speed, and the speed of B at that moment. [4]
-
- 25** A particle P of mass 0.3 kg moves in a circle of radius r on a smooth horizontal table. P is attached to a light inextensible string that passes through a small smooth hole O in the table and is attached to a particle Q of mass M hanging vertically below the hole. Take $g = 9.8\text{ m s}^{-2}$.
- (a) Show that the angular speed ω of P satisfies $\omega^2 = \frac{Mg}{0.3r}$. [3]
- (b) Given that $M = 0.9$ and $r = 0.4$, find ω , the speed of P and the time for P to make one revolution, to 3 significant figures. [5]
-
- 26** A particle of mass 0.2 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 20 N . The other end is fixed to a point O on a smooth horizontal table, and the particle moves on the table in a circle centre O with constant angular speed ω .
- (a) Given that $\omega = 8\text{ rad s}^{-1}$, find the extension of the string. [4]
- (b) Show that the motion is only possible if $\omega^2 < 200$, and explain what happens as ω^2 approaches 200 . [3]
-
- 27** A rough horizontal turntable rotates about a vertical axis through its centre O. A small smooth hole at O carries a light inextensible string. One end of the string is attached to a particle P of mass 0.4 kg lying on the turntable at distance 0.5 m from O, and the other end is attached to a particle Q of mass 0.6 kg hanging vertically below the hole. The coefficient of friction between P and the turntable is 0.3 . P remains at rest relative to the turntable and Q remains at rest. Take $g = 9.8\text{ m s}^{-2}$.
- (a) Show that the angular speed ω of the turntable satisfies $23.52 \leq \omega^2 \leq 35.28$, and find the range of possible values of ω , to 3 significant figures. [7]
- (b) The turntable now rotates at the constant angular speed $\omega = 5.4\text{ rad s}^{-1}$. Find the range of values of the distance r of P from O for which P can remain at rest relative to the turntable. [4]
-
- 28** Two particles A and B, of masses m and $2m$, are joined by a light elastic string of natural length l and modulus of elasticity λ . They move on a smooth horizontal table in circles with the same constant angular speed ω about a point O of the table lying on the string between A and B, so that the string is stretched, with extension e .
- (a) Show that A is at distance $\frac{2}{3}(l + e)$ from O, and write down an expression for the tension in terms of m , ω , l and e . [4]
- (b) Show that $e = \frac{2m\omega^2 l^2}{3\lambda - 2m\omega^2 l}$. [4]
- (c) Hence find the greatest value of ω for which the string remains in this motion, and find e when $m = 0.5\text{ kg}$, $l = 0.6\text{ m}$, $\lambda = 30\text{ N}$ and $\omega = 6\text{ rad s}^{-1}$. [4]
-
- 29** Three particles A, B and C, of masses m , $2m$ and $3m$, are attached to a light inextensible string OABC, where $OA = AB = BC = a$. The string lies straight on a smooth horizontal table and the particles move in horizontal circles about O with the same constant angular speed ω . Let T_1 , T_2 and T_3 be the tensions in OA, AB and BC.
- (a) Show that $T_3 = 9ma\omega^2$, and find T_2 and T_1 in the same form. [6]
- (b) Verify that $T_1 = \omega^2 \sum mr$, summed over all three particles, and explain why this must be so. [2]
- (c) Given that $m = 0.5\text{ kg}$, $a = 0.4\text{ m}$ and that BC can bear a tension of at most 36 N and OA at most 60 N , find the greatest angular speed of the system. [3]

- 30** The Earth (mass M) and the Moon (mass m) move in circular orbits about their common centre of mass C , with constant angular speed ω , the centres of the Earth and Moon being a constant distance d apart. The only force on each is their mutual gravitational attraction $\frac{GMm}{d^2}$.
- (a) Show that the distance of the Earth's centre from C is $\frac{md}{M+m}$. [3]
- (b) Show that $\omega^2 = \frac{G(M+m)}{d^3}$. [3]
- (c) Given $M = 5.97 \times 10^{24}$ kg, $m = 7.35 \times 10^{22}$ kg, $d = 3.84 \times 10^8$ m and $G = 6.67 \times 10^{-11}$ N m² kg⁻², find the period in days and the distance of C from the Earth's centre, each to 3 significant figures. Comment on the position of C . [6]

Horizontal Circular Motion

- 1** A particle of mass 0.5 kg is attached to one end of a light inextensible string of length 1.2 m. The other end is fixed at a point O . The particle moves in a horizontal circle, with the string making a constant angle of 30° with the downward vertical. Find the tension in the string. Take $g = 9.8$ m s⁻². [2]
- 2** A particle of mass 1 kg moves in a horizontal circle at the end of a light inextensible string, the other end of which is fixed. The tension in the string is 12 N. Find the angle between the string and the vertical. Take $g = 9.8$ m s⁻². [2]
- 3** A car travels round a circular bend of radius 60 m on a road banked at 15° to the horizontal. There is no sideways frictional force on the car. Find the speed of the car. Take $g = 9.8$ m s⁻².
- 
- [2]
- 4** A particle of mass 0.4 kg moves in a horizontal circle at the end of a light inextensible string of length 0.9 m, with angular speed 5 rad s⁻¹. The string makes an angle with the vertical. Find the tension in the string. [2]
- 5** A particle moves in a horizontal circle at the end of a light inextensible string of length 1.5 m, the other end of which is fixed at a point O . The string makes a constant angle of 40° with the vertical. Find the radius of the circle and the depth of the particle below O . [2]
- 6** A particle of mass m moves in a horizontal circle of radius r with angular speed ω at the end of a light inextensible string. The string makes an angle θ with the vertical and the tension is T .
- (a) Write down an equation for the vertical motion of the particle. [1]
- (b) Write down an equation for the horizontal motion of the particle. [1]
- (c) Hence show that $\tan \theta = \frac{r\omega^2}{g}$. [1]

- 7 A particle of mass 0.3 kg is attached to one end of a light inextensible string of length 0.8 m , the other end of which is fixed. The particle moves in a horizontal circle with the string at 40° to the downward vertical, as shown. Find the tension in the string, the angular speed of the particle and the time for one revolution, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 8 A banked circular track has radius 80 m . A car of mass 1000 kg travels round the track at a constant 20 m s^{-1} and there is no sideways frictional force. Find the angle of the banking, and the normal reaction on the car, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

[4]

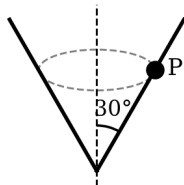
- 9 In a fairground ride, a large cylinder of radius 3 m rotates about its vertical axis. A person stands against the rough vertical inner wall; when the cylinder rotates fast enough the floor is removed and the person stays in place. The coefficient of friction between the person and the wall is 0.5 . Find the least angular speed of the cylinder, in rad s^{-1} and in revolutions per minute (3 s.f.). Take $g = 9.8 \text{ m s}^{-2}$.

[5]

- 10 An aircraft of mass 4000 kg flies at a constant speed of 80 m s^{-1} in a horizontal circle of radius 1500 m . The lift force L acts perpendicular to the wings, and the wings are tilted at angle θ to the horizontal. Ignoring air resistance, find θ and L , each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

[4]

- 11 A particle P moves in a horizontal circle of radius 0.4 m on the smooth inner surface of a hollow cone with a vertical axis, as shown. The semi-vertical angle of the cone is 30° . Find the speed of P . Take $g = 9.8 \text{ m s}^{-2}$.

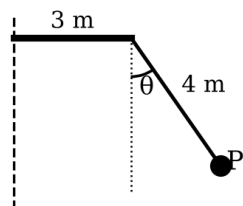


[4]

- 12 Two particles P and Q , of masses 0.6 kg and 0.2 kg , are attached to the ends of a light inextensible string which passes through a small smooth fixed ring R . P hangs in equilibrium below the ring while Q moves in a horizontal circle with constant angular speed, the part RQ of the string having length 0.9 m and making an angle θ with the vertical. Find the tension in the string, the value of θ and the angular speed of Q . Take $g = 9.8 \text{ m s}^{-2}$.

[5]

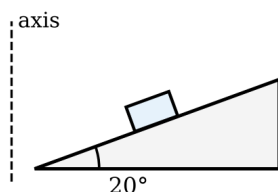
- 13** In a fairground ride, a seat P is suspended by a light chain of length 4 m from a point on a horizontal arm that rotates about a vertical axis, the point being 3 m from the axis. When the arm rotates at constant angular speed the chain makes an angle of 35° with the vertical, as shown. Find the speed of P, and the angular speed of the arm, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

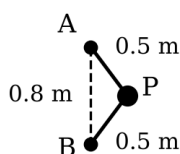
- 14** A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 0.8 m. The other end of the string is fixed at a point 0.4 m above a smooth horizontal table. P moves in a horizontal circle on the table, with the string taut, at constant angular speed 4 rad s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- Find the tension in the string. [3]
 - Find the normal reaction of the table on P. [2]
 - Find the greatest angular speed for which P remains in contact with the table. [2]

- 15** A car moves round a circular bend of radius 50 m on a road banked at 20° to the horizontal. The coefficient of friction between the car's tyres and the road is 0.2. Find the greatest speed and the least speed at which the car can travel round the bend without slipping sideways, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



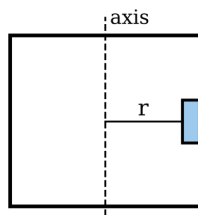
[7]

- 16** A particle P of mass 0.4 kg is attached to the ends of two light inextensible strings, AP and BP, each of length 0.5 m. The other ends are attached to fixed points A and B, with A vertically above B and $AB = 0.8 \text{ m}$. P moves in a horizontal circle with both strings taut, at constant angular speed ω . Take $g = 9.8 \text{ m s}^{-2}$.



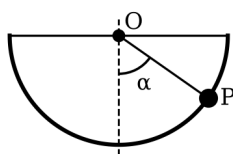
- Show that $\sin \alpha = 0.6$ for each string, where α is the angle to the vertical, and that the circle has radius 0.3 m. [1]
- Given that $\omega = 6 \text{ rad s}^{-1}$, find the tension in each string. [4]
- Find the least value of ω for which both strings remain taut. [2]

- 17** In a rotor ride, a cylinder of radius 2.5 m rotates about its vertical axis at a constant 40 revolutions per minute. A person of mass 60 kg is pressed against the inner wall and does not slip. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find the normal reaction of the wall on the person. [3]
 (b) Find the smallest coefficient of friction between the person and the wall for the person not to slip. [3]

- 18** A particle P moves in a horizontal circle on the smooth inner surface of a hemispherical bowl of radius 0.5 m, whose axis is vertical, with constant angular speed 6 rad s^{-1} . The radius OP makes angle α with the downward vertical, where O is the centre of the bowl. Find α and the depth of P below O, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

- 19** A cyclist and bicycle, of total mass 85 kg, move round a circular track of radius 25 m banked at 30° to the horizontal. Find the speed at which the cyclist can ride so that there is no tendency to slip sideways, and the normal reaction at that speed, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. The cyclist rides at 14 m s^{-1} without slipping; state whether the frictional force acts up or down the slope, with a reason. [6]

- 20** A particle of mass 0.5 kg hangs from a fixed point O by a light elastic string of natural length 0.6 m and modulus of elasticity 29.4 N. The particle moves in a horizontal circle with the string making an angle of 30° with the vertical. Find the extension of the string and the speed of the particle, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [7]

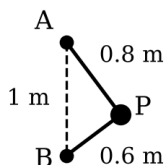
- 21** A particle moves in a horizontal circle at the end of a light inextensible string of length l , the other end being fixed at O. The string makes a constant angle θ with the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Show that the time for one revolution is $2\pi\sqrt{\frac{l \cos \theta}{g}}$. [4]
 (b) Find the time for one revolution when $l = 1.2 \text{ m}$ and $\theta = 45^\circ$. [2]
 (c) Explain why the time for one revolution does not depend on the mass of the particle, and why it decreases as θ increases. [2]

- 22** A car moves in a horizontal circle of radius r on a road banked at an angle α to the horizontal. The coefficient of friction between the tyres and the road is μ , where $\mu \tan \alpha < 1$. The car is about to slip up the slope.

- (a) Show that the speed v of the car satisfies $v^2 = \frac{rg(\tan \alpha + \mu)}{1 - \mu \tan \alpha}$. [4]
 (b) Given that $\tan \alpha = 0.25$, $\mu = 0.5$ and $r = 80 \text{ m}$, find the greatest speed, in m s^{-1} to 3 significant figures, and explain why there is no least speed for which the car avoids sliding down the slope. Take $g = 9.8 \text{ m s}^{-2}$. [4]

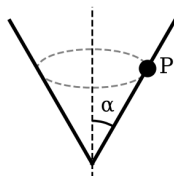
- 23** A particle of mass 0.2 kg is attached to one end of a light inextensible string of length 1.0 m. The other end is fixed at O and the particle moves in a horizontal circle with the string at 60° to the downward vertical. A student writes: "Resolving vertically, $T \sin 60^\circ = mg$; horizontally, $T \cos 60^\circ = m\omega^2 r$." Identify the error, and find the correct tension, angular speed and speed of the particle, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [7]

- 24** A particle P of mass 0.5 kg is attached to the ends of two light inextensible strings AP and BP. A is vertically above B with $AB = 1.0 \text{ m}$, $AP = 0.8 \text{ m}$ and $BP = 0.6 \text{ m}$. P moves in a horizontal circle, with both strings taut, at constant angular speed ω . Take $g = 9.8 \text{ m s}^{-2}$.



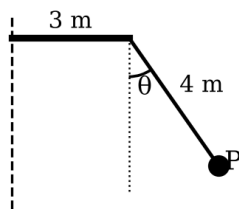
- (a) Show that angle APB is a right angle and that the radius of the circle is 0.48 m. [2]
 (b) Find the tensions in the strings when $\omega = 6 \text{ rad s}^{-1}$. [4]
 (c) Find the least value of ω for which both strings are taut. [3]

- 25** A particle P moves in a horizontal circle on the smooth inner surface of a hollow cone with vertical axis, at a height h above the vertex. The semi-vertical angle of the cone is α . Take $g = 9.8 \text{ m s}^{-2}$.



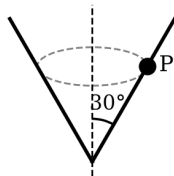
- (a) Show that the angular speed of P is $\omega = \sqrt{\frac{g}{h \tan^2 \alpha}}$. [4]
 (b) Given that $h = 0.6 \text{ m}$ and $\alpha = 30^\circ$, show that $\omega = 7 \text{ rad s}^{-1}$, and find the speed of P and the time for one revolution. [5]

- 26** In a fairground ride, a seat P of mass m is suspended by a light chain of length 4 m from a point on a horizontal arm, 3 m from the vertical axis of rotation. The arm rotates about the axis with constant angular speed ω , and the chain makes an angle θ with the vertical, as shown. Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that $\omega^2 = \frac{g \tan \theta}{3 + 4 \sin \theta}$. [3]
 (b) When $\omega = 1.5 \text{ rad s}^{-1}$, show that θ satisfies $9.8 \tan \theta = 2.25(3 + 4 \sin \theta)$, and show that θ lies between 54.8° and 55.8° . [4]

- 27** A particle P of mass m moves in a horizontal circle of radius 0.5 m on the rough inner surface of a hollow cone with vertical axis and semi-vertical angle 30° , so that the surface is inclined at 60° to the horizontal. The coefficient of friction between P and the cone is 0.2 , and P moves without slipping up or down the surface. Show that the angular speed ω of P satisfies $\omega_{\min}^2 \leq \omega^2 \leq \omega_{\max}^2$, find these two limits and the corresponding range for ω , to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

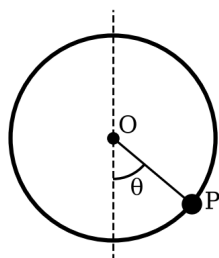


[10]

- 28** A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 1.2 m, the other end of which is fixed at O. P is initially at rest hanging vertically below O. It is then set moving so that it travels in a horizontal circle with the string at 60° to the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$.
- Show that the speed of P in the circular motion is 4.2 m s^{-1} . [3]
 - Find the total work done in setting P into this motion from rest in the lowest position. [4]
 - Show that, for a general angle θ , the work done is $mg(1 - \cos\theta + \frac{1}{2}\sin\theta \tan\theta)$. [2]

- 29** A car travels round a circular banked track of radius 60 m, inclined at an angle α to the horizontal. The coefficient of friction between the tyres and the track is μ . The greatest speed at which the car can go round without slipping is 18.4 m s^{-1} and the least speed is 10.2 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.
- Show that, if $\mu = \tan\lambda$, the greatest speed v_1 satisfies $\frac{v_1^2}{rg} = \tan(\alpha + \lambda)$, and find a corresponding result for the least speed v_2 . [4]
 - Hence find α and μ , to 2 significant figures. [4]

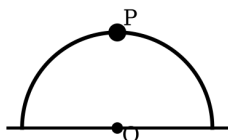
- 30** A small bead P of mass m is threaded on a smooth circular wire of radius a and centre O. The wire is fixed in a vertical plane and rotates about its vertical diameter with constant angular speed ω . P is in equilibrium relative to the wire at a position where OP makes an angle θ with the downward vertical, with $0 < \theta < \pi$.



- Show that $\cos\theta = \frac{g}{a\omega^2}$, and deduce that such a position exists only if $\omega^2 > \frac{g}{a}$. [5]
- Given $a = 0.4$ m and $\omega = 8 \text{ rad s}^{-1}$, find θ and the magnitude of the reaction when $m = 0.2$ kg. [4]
- For $\omega^2 < \frac{g}{a}$, explain what positions of equilibrium relative to the wire are possible, and say whether the lowest point is possible for $\omega^2 > \frac{g}{a}$. [2]

Vertical Circular Motion

- 1 A particle of mass 0.5 kg is attached to one end of a light inextensible string of length 0.8 m , the other end being fixed. The particle moves in a vertical circle and passes through the lowest point with speed 6 m s^{-1} . Find the tension in the string at this point. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 2 A particle of mass 0.2 kg moves in a vertical circle of radius 0.6 m on the end of a light inextensible string. At the highest point its speed is 4 m s^{-1} . Find the tension in the string at the highest point. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 3 A smooth bead is projected with speed 7 m s^{-1} from the lowest point of a smooth vertical circular wire. Find its speed when it has risen 1.2 m above the lowest point. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 4 A particle moves in a vertical circle of radius 0.9 m on the end of a light inextensible string. Find the least speed it can have at the highest point if the string is to remain taut there. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 5 A particle on a light inextensible string of length 1.5 m moves in a vertical circle. Find the least speed at the lowest point for the particle to complete the circle. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 6 A small bead is threaded on a smooth vertical circular wire of radius 0.4 m . Find the least speed at the lowest point for the bead to reach the highest point. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 7 A particle of mass 0.3 kg is attached to one end of a light inextensible string of length 0.8 m , the other end being fixed at O . The particle is projected from the point below O with speed 5.4 m s^{-1} , and moves in a vertical circle. Find the speed of the particle and the tension in the string when the string is horizontal. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 8 A roller coaster car of mass 400 kg travels round a vertical circular loop of radius 8 m on a smooth track. At the top of the loop its speed is 11 m s^{-1} . Find the normal reaction of the track on the car at the top, and the speed of the car at the bottom of the loop, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 9 A particle of mass 0.5 kg moves in a vertical circle of radius 1.0 m on the end of a light inextensible string, with speed 8 m s^{-1} at the lowest point. Find the tension in the string at the highest point and show that the particle completes the circle. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 10 A small bead of mass 0.1 kg is threaded on a smooth circular wire of radius 0.5 m , fixed in a vertical plane. The bead is projected from the lowest point with speed 5 m s^{-1} . Find the speed of the bead at the highest point, and the magnitude and direction of the reaction of the wire there. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- 11 A particle of mass 0.4 kg is at the highest point of the outside of a fixed smooth sphere of radius 2 m , moving with speed 1 m s^{-1} . Find the normal reaction of the sphere on the particle at this point, and the greatest speed it could have there without leaving the surface immediately. Take $g = 9.8 \text{ m s}^{-2}$. [4]



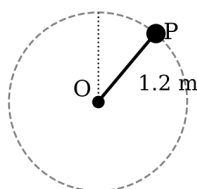
- 12** A pendulum bob of mass 0.5 kg is attached to a light inextensible string of length 0.9 m . The bob is released from rest with the string taut and horizontal. Find the speed of the bob at its lowest point and the tension in the string at that point. Take $g = 9.8 \text{ m s}^{-2}$. [4]

- 13** A car of mass 1200 kg travels over the top of a humpback bridge, the cross-section of the road at the top being an arc of a circle of radius 40 m . Find the normal reaction of the road on the car when its speed at the top is 15 m s^{-1} , and the speed at which the car would just leave the road at the top. Take $g = 9.8 \text{ m s}^{-2}$.



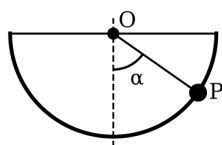
[4]

- 14** A particle of mass m is attached to one end of a light inextensible string of length 1.2 m , the other end being fixed at O . The particle is projected from the lowest point with speed 7 m s^{-1} . Show that the particle does not complete a vertical circle, and find the angle that the string makes with the upward vertical at the instant it becomes slack, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

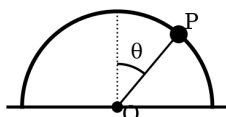
- 15** A particle of mass 0.2 kg is released from rest at the rim of a smooth hemispherical bowl of radius 0.6 m , with its axis vertical, the particle being level with the centre O of the bowl. Find the speed of the particle and the normal reaction of the bowl at the lowest point. Find also the normal reaction when OP makes an angle of 60° with the downward vertical. Take $g = 9.8 \text{ m s}^{-2}$.



[6]

- 16** A car of mass 900 kg moves on a smooth vertical circular loop of radius 6 m , with the car treated as a particle moving on the inside of the loop. Find the least speed at the lowest point for the car to stay in contact with the track all the way round, and the normal reaction at the lowest point when it has this speed. Take $g = 9.8 \text{ m s}^{-2}$. [6]

- 17** A particle P is placed at the highest point of the outside of a fixed smooth sphere of radius $a = 1.5 \text{ m}$ and centre O , and is slightly disturbed so that it moves from rest on the surface. Show that P leaves the surface when OP makes an angle θ with the upward vertical, where $\cos\theta = \frac{2}{3}$, and find the speed of P at that instant. Take $g = 9.8 \text{ m s}^{-2}$.



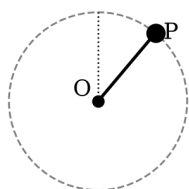
[5]

- 18** A particle of mass 0.3 kg is attached to one end of a light rigid rod of length 0.7 m . The other end of the rod is fixed at O , and the rod can rotate freely in a vertical plane. The particle is projected from the lowest point with speed 6 m s^{-1} . Find the speed of the particle at the highest point, and the force in the rod at that point, stating whether the rod is in tension or thrust. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 19** A particle P is projected horizontally with speed 2 m s^{-1} from the highest point of the outer surface of a fixed smooth sphere of radius 1.5 m and centre O . P moves on the surface of the sphere and leaves it when OP makes an angle θ with the upward vertical. Find θ , and the speed of P when it leaves the surface, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [6]

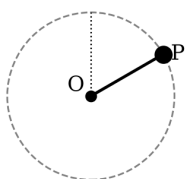
- 20** A particle P of mass 1.5 kg is attached to one end of a light rigid rod OP of length 0.2 m . The rod rotates in a vertical plane about a fixed horizontal axis through O , so that P moves in a vertical circle at a constant 300 revolutions per minute. Find the greatest and least magnitudes of the force in the rod. Take $g = 9.8 \text{ m s}^{-2}$. [6]

- 21** A particle P of mass m is attached to one end of a light inextensible string of length 1.2 m . The other end is fixed at O . P is projected from the lowest point with speed 7 m s^{-1} . The string becomes slack when OP makes an angle θ with the upward vertical, where $\cos\theta = 0.7222$ (to 4 s.f.). Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Show that the speed of P when the string becomes slack satisfies $v^2 = 1.2g \cos\theta$, and find v . [2]
 (b) Find the greatest height above O reached by P afterwards. [2]

- 22** A particle P moves on the smooth inner surface of a hollow sphere of radius 0.8 m and centre O . It is projected horizontally from the lowest point with speed u and leaves the surface when OP makes an angle of 30° above the horizontal. Find u and the speed of P at the instant it leaves the surface, each to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

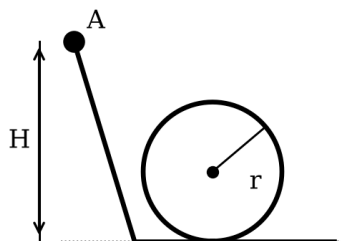


[8]

- 23** A particle of mass m is attached to one end of a light inextensible string of length 0.8 m , the other end being fixed at O . The particle is projected from the lowest point with speed 6 m s^{-1} .
 A student writes: "At the highest point, $T + mg = \frac{mv^2}{r}$ with $v = 6$, so $T = m(45 - 9.8) = 35.2m$, which is positive, so the particle completes the circle." Explain the errors in this working, find the speed the particle would have at the highest point if the string stayed taut, and state with a reason whether the particle completes the circle. Take $g = 9.8 \text{ m s}^{-2}$. [8]

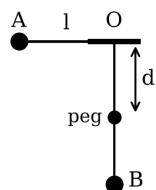
- 24** A pendulum bob of mass m hangs from a fixed point O , 3 m above horizontal ground, on a light inextensible string of length 1.2 m . The bob is released from rest with the string horizontal. The string breaks when the tension in it reaches $2.5mg$. Show that the string breaks when it makes an angle θ with the downward vertical with $\cos\theta = \frac{5}{6}$, and find the horizontal distance of the point where the bob hits the ground from the point vertically below O , to 3 significant figures. Take $g = 9.8\text{ m s}^{-2}$. [8]

- 25** A small smooth car is released from rest at the point A on a smooth track at a height H above the lowest point of a vertical circular loop of radius r . It then travels round the inside of the loop. Take $g = 9.8\text{ m s}^{-2}$.



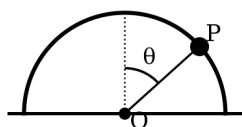
- Show that the car can complete the loop only if $H \geq \frac{5r}{2}$. [3]
- Given that $r = 6\text{ m}$, find the least value of H , and the normal reaction at the top of the loop, as a multiple of mg , when $H = 18\text{ m}$. [4]
- Explain why the mass of the car does not affect the minimum starting height. [1]

- 26** A particle P of mass m is attached to one end of a light inextensible string of length l . The other end is fixed at O . P is released from rest at A , with the string taut and horizontal. When the string becomes vertical it catches on a small smooth peg at a distance d below O , and P then moves in a vertical circle about the peg, as shown.



- Show that P can complete the circle about the peg only if $d \geq \frac{3l}{5}$. [4]
- Given that $l = 1.5\text{ m}$, $d = 0.9\text{ m}$ and $m = 0.4\text{ kg}$, find the tension in the string immediately before and immediately after it meets the peg. Take $g = 9.8\text{ m s}^{-2}$. [4]

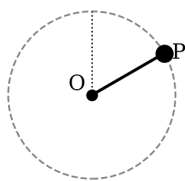
- 27** A smooth solid hemisphere of radius a is fixed with its plane face on horizontal ground and its curved surface upwards; O is the centre of the plane face. A particle P is placed at the highest point of the curved surface and is given a very small horizontal displacement, so that it slides on the surface. P leaves the surface when OP makes an angle θ with the upward vertical.



- Show that $\cos\theta = \frac{2}{3}$, and that the speed of P when it leaves the surface is $\sqrt{\frac{2ga}{3}}$. [4]
- Find the time taken for P to reach the ground after leaving the surface, in terms of a and g , and show that its speed on hitting the ground is $\sqrt{2ga}$. [4]
- Find the horizontal distance of the point where P hits the ground from O , in terms of a . [2]

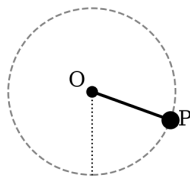
- 28** A particle of mass m moves in a complete vertical circle of radius r on the end of a light inextensible string. Its speed at the lowest point is u . Let T_B and T_T be the tensions at the lowest and highest points.
- (a) Show that $T_B = m\frac{u^2}{r} + mg$ and $T_T = m\frac{u^2}{r} - 5mg$. [4]
- (b) Show that $T_B - T_T = 6mg$ whatever the value of u . [2]
- (c) Given that $T_B = 8T_T$, show that $u^2 = \frac{41gr}{7}$, and verify that the particle does complete the circle. [4]
- (d) Find u for a circle of radius 0.9 m, to 3 significant figures, with $g = 9.8 \text{ m s}^{-2}$. [1]

- 29** A particle P moves on the smooth inner surface of a hollow sphere of radius a and centre O . It is projected horizontally from the lowest point with speed u . Suppose P leaves the surface at the point where OP makes an angle φ above the horizontal.



- (a) Show that $u^2 = ga(2 + 3 \sin \varphi)$. [4]
- (b) After leaving the surface P moves as a projectile. Show that P passes through O if $\sin \varphi = \frac{1}{\sqrt{3}}$. [6]
- (c) Find the value of u in this case, as a multiple of $\sqrt{ga}$, to 3 significant figures. [2]

- 30** A particle P of mass m is projected horizontally with speed u from the lowest point of the inside of a smooth hollow sphere of radius a and centre O . Let R be the normal reaction when OP makes an angle θ with the downward vertical, $0 \leq \theta \leq \pi$.



- (a) Show that $R = \frac{mu^2}{a} + mg(3\cos\theta - 2)$. [3]
- (b) Show that if $u^2 \leq 2ga$ then P oscillates without leaving the surface, and that if $u^2 \geq 5ga$ it completes full circles. [4]
- (c) For $2ga < u^2 < 5ga$ show that P leaves the surface, and find u when $a = 0.5 \text{ m}$ and P leaves the surface at $\theta = 120^\circ$. Take $g = 9.8 \text{ m s}^{-2}$. [4]

Circular Motion Modelling

- 1** A stone of mass 0.3 kg is whirled in a horizontal circle on the end of a string. State two modelling assumptions that are made in treating the motion as that of a particle moving in a horizontal circle with constant speed. [2]
- 2** The drum of a washing machine has radius 0.25 m and spins at 1000 revolutions per minute. Find the speed of a sock pressed against the drum, to 3 significant figures. [2]
- 3** A satellite moves in a circular orbit of radius $6.7 \times 10^6 \text{ m}$ and completes one orbit in 90 minutes. Find its speed, to 3 significant figures. [2]

- 4** A child sits on the edge of a roundabout of radius 2 m, which makes one revolution every 5 s.
 (a) Find the speed of the child. [1]
 (b) Find the magnitude of the child's acceleration. [1]
-
- 5** A car travels round a flat circular bend of radius 40 m. The coefficient of friction between the tyres and the road is 0.5. Find the greatest speed at which the car can take the bend without slipping. Take $g = 9.8 \text{ m s}^{-2}$. [2]
-
- 6** A hammer thrower swings a hammer in a horizontal circle of radius 2 m so that the hammer has a speed of 24 m s^{-1} . Find the magnitude of the acceleration of the hammer, and the number of times the acceleration is greater than g . Take $g = 9.8 \text{ m s}^{-2}$. [2]
-
- 7** A tetherball of mass 0.2 kg is attached by a light inextensible rope of length 1.5 m to the top of a vertical pole. The ball moves in a horizontal circle with the rope making a constant angle of 50° with the pole. Find the tension in the rope and the speed of the ball, to 3 significant figures. State one modelling assumption. Take $g = 9.8 \text{ m s}^{-2}$. [5]
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- 8** The Moon moves in a circular orbit of radius $3.84 \times 10^8 \text{ m}$ about the centre of the Earth. At distance r from the Earth's centre the gravitational acceleration is $\frac{gR^2}{r^2}$, where $R = 6.37 \times 10^6 \text{ m}$ is the radius of the Earth and $g = 9.8 \text{ m s}^{-2}$. Find the speed of the Moon and the time for one orbit in days, to 3 significant figures. [4]
-
- 9** A laboratory centrifuge spins at 4000 revolutions per minute. A sample of mass 5 grams is at a distance of 0.12 m from the axis. Find the resultant force on the sample, and express it as a multiple of the weight of the sample, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 10** A cyclist rides round a flat circular track of radius 15 m at a constant speed of 6 m s^{-1} . Model the cyclist and bicycle as a particle. The resultant contact force from the ground acts along the line from the contact point to the particle. Find the angle that this line makes with the vertical, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [3]
-
- 11** A child sits on the rim of a rough roundabout of radius 2 m. The coefficient of friction between the child and the roundabout is 0.35. Find the greatest number of revolutions per minute at which the roundabout can turn without the child slipping, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-
- 12** Model the Earth as a sphere of radius $6.37 \times 10^6 \text{ m}$ rotating once every 24 hours. A person of mass 70 kg stands on a set of scales at a point on the equator. Find the reading of the scales (the normal reaction) in newtons, to 4 significant figures, and the difference from the reading at the North Pole, where there is no effect of the rotation. Take $g = 9.8 \text{ m s}^{-2}$. [5]
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- 13** A stone of mass 0.4 kg is whirled in a horizontal circle on a light string. The string will break if the tension exceeds 80 N. Find the greatest speed of the stone when the radius of the circle is 1.0 m, and when it is 0.5 m. Explain briefly why a shorter string allows a smaller maximum speed. [4]
-
- 14** Mars rotates once about its axis every 24.6 hours (88 620 s). A satellite is to be placed in a circular orbit above the equator of Mars so that it stays above the same point of the surface. The gravitational acceleration at distance r from the centre of Mars is $\frac{k}{r^2}$, where $k = 4.28 \times 10^{13} \text{ m}^3 \text{ s}^{-2}$. Find the radius of the orbit and the speed of the satellite, each to 3 significant figures. [6]
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- 15** In a "wall of death" fairground ride a motorcyclist rides in a horizontal circle of radius 5 m on the inside of a vertical cylindrical wall. The coefficient of friction between the tyres and the wall is 0.6. Model the motorcycle and rider, of total mass 200 kg, as a particle. Find the least speed at which the rider can travel without slipping down the wall, and the normal reaction at that speed. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 16** A hammer of mass 7.26 kg is whirled in a horizontal circle of radius 1.8 m, taking 0.7 s to complete one revolution. The hammer is modelled as a particle at the end of a light inextensible wire. Because the weight of the hammer must be supported, the wire slopes slightly downwards from the thrower to the hammer. Find the horizontal component of the tension, the tension in the wire and the angle the wire makes with the horizontal, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [6]
- 17** A space station is modelled as a thin ring of radius 100 m rotating about its axis, so that people standing on the inside of the rim experience an "artificial gravity" equal to the force that would be given by an acceleration $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the angular speed of the station in rad s^{-1} and in revolutions per minute, and the time for one revolution. [3]
 (b) Find the apparent weight (as a fraction of normal weight) of a person standing at a distance of 60 m from the axis, on a platform fixed to the station. [2]
- 18** A car travels round a circular bend of radius 100 m on a road banked at an angle α to the horizontal, where $\tan \alpha = 0.2$. The coefficient of friction between the tyres and the road is 0.4. Find the greatest speed at which the car can travel without slipping sideways, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 19** The drum of a spin dryer has radius 0.3 m and rotates about a vertical axis. A garment is pressed against the vertical inner surface of the drum. The coefficient of friction between the garment and the drum is 0.3. Find the least angular speed of the drum, in revolutions per minute to 3 significant figures, for the garment not to slide down. Take $g = 9.8 \text{ m s}^{-2}$. [5]
- 20** The International Space Station moves in a circular orbit of radius $6.78 \times 10^6 \text{ m}$ at a speed of $7.66 \times 10^3 \text{ m s}^{-1}$. Find the acceleration of the station, and explain, using the equation of motion of an astronaut of mass m inside the station, why the astronaut experiences "weightlessness" although gravity still acts. Take the gravitational acceleration at this height to be the same as the acceleration of the station. [4]
- 21** A satellite of mass m moves in a circular orbit of radius r about a planet of mass M with period T . The gravitational force is $\frac{GMm}{r^2}$, where G is the universal gravitational constant.
 Show that $M = \frac{4\pi^2 r^3}{GT^2}$. Hence estimate the mass of the Earth from the Moon's orbit, which has radius $3.84 \times 10^8 \text{ m}$ and period 27.3 days, using $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. Give your answer to 3 significant figures, and state one modelling assumption. [7]
- 22** A Ferris wheel of radius 20 m rotates at a constant angular speed, making one revolution every 60 s. A passenger of mass 60 kg sits on a seat which remains horizontal. Model the passenger as a particle. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the normal reaction of the seat on the passenger at the highest point and at the lowest point, to 3 significant figures. [5]
 (b) Find the time for one revolution at which a passenger at the highest point would just lose contact with the seat. [3]

- 23** A student estimates the acceleration of the Earth in its orbit about the Sun. She models the orbit as a circle of radius $1.5 \times 10^{11} \text{ m}$ and the period as 365 days, and writes: " $\omega = \frac{2\pi}{365} = 0.0172 \text{ rad s}^{-1}$, so $a = r\omega^2 = 1.5 \times 10^{11} \times (0.0172)^2 = 4.4 \times 10^7 \text{ m s}^{-2}$." Explain the error, find the correct acceleration to 3 significant figures, and use it with $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ to find the mass of the Sun, given that the Sun's gravitational acceleration at distance r is $\frac{GM}{r^2}$. [8]
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- 24** The drum of a tumble dryer has radius 0.4 m and rotates about a horizontal axis. A sock of mass 0.2 kg is modelled as a particle in contact with the inner surface of the drum. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the least angular speed, in revolutions per minute to 3 significant figures, for which the sock can remain in contact with the drum when at the highest point of the drum. [3]
- (b) The drum rotates at a constant 60 revolutions per minute and the sock moves with the drum. Find the normal reaction of the drum on the sock at the highest point and at the lowest point, to 3 significant figures. [5]
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- 25** A hammer of mass 7.26 kg is whirled by an athlete in a circle of radius 1.9 m and released with speed 26 m s^{-1} at a height of 1.9 m above horizontal ground, moving at 42° above the horizontal. Take $g = 9.8 \text{ m s}^{-2}$.
- (a) Find the force needed to keep the hammer moving in a circle of radius 1.9 m at this speed, to 3 significant figures. [2]
- (b) After release, the hammer is modelled as a projectile. Find the horizontal distance it travels before hitting the ground, and its speed on landing, to 3 significant figures. [5]
-
- 26** Model the Earth as a sphere of radius $6.37 \times 10^6 \text{ m}$ rotating once every 24 hours. A person of mass 70 kg stands at rest on the ground at latitude 60° N . Find the acceleration of the person due to the rotation of the Earth, the component of that acceleration towards the centre of the Earth, and the normal reaction of the ground if the ground is horizontal, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$ to be the gravitational acceleration at the surface. [7]
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- 27** A satellite A moves in a circular orbit of radius $6.7 \times 10^6 \text{ m}$ about the Earth's centre. A second satellite B moves in a circular orbit of radius $4.22 \times 10^7 \text{ m}$ in the same plane and in the same direction. Model the gravitational acceleration at distance r from the centre of the Earth as $\frac{gR^2}{r^2}$, where $R = 6.37 \times 10^6 \text{ m}$ and $g = 9.8 \text{ m s}^{-2}$.
- (a) Show that the speed of a satellite in a circular orbit of radius r is $R\sqrt{\frac{g}{r}}$, and find the speeds of A and B to 3 significant figures. [4]
- (b) Show that the period of the satellite is $\frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$, and find the periods of A and B in hours to 3 significant figures. [4]
- (c) At one instant A and B are on the same radius, on the same side of the Earth. Find the time until this next happens, in hours to 3 significant figures. [3]

- 28** A particle at rest on the equator of a uniform spherical planet of radius R and density ρ is just about to leave the surface because of the planet's rotation. The planet rotates with period T . The gravitational acceleration at the surface is $g = \frac{GM}{R^2}$, where M is the mass of the planet and G is the universal gravitational constant.
- Show that $T = 2\pi\sqrt{\frac{R}{g}}$. [2]
 - Show that $T = \sqrt{\frac{3\pi}{G\rho}}$, independent of the radius R . [4]
 - Use the result with $\rho = 5500 \text{ kg m}^{-3}$, the mean density of the Earth, to find T in minutes, and comment on the value. [2]
 - A pulsar rotates once every 1.5 milliseconds. Find the least density the star must have for its equatorial surface not to fly apart. [2]
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- 29** Tarzan, of mass 80 kg, swings on a vine of length 10 m, fixed at a point O , from a platform. He starts from rest with the vine taut and making an angle of 55° with the downward vertical. Model Tarzan as a particle and the vine as light and inextensible. Take $g = 9.8 \text{ m s}^{-2}$.
- Find his speed at the lowest point of the swing. [2]
 - The vine can withstand a tension of at most 1500 N. Show that it does not break. [3]
 - Tarzan lets go of the vine when it makes an angle of 20° with the downward vertical on the other side of the lowest point, while still moving upwards. Find his speed at that instant and the horizontal distance he then travels before reaching the level of the lowest point of his swing, to 3 significant figures. [6]
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- 30** The designer of a roller coaster wants the reaction of the track on a passenger to be no more than $4mg$ at the bottom of a vertical loop, where m is the mass of the passenger, and the passenger to stay in contact at the top. The passenger is modelled as a particle moving on a smooth track.
- The loop is a circle of radius 6 m. Show that the reaction at the bottom is at least $6mg$ if contact is maintained at the top, and deduce that a circular loop cannot meet the design requirement. [4]
 - The designer therefore makes the radius of curvature of the track r_T at the top smaller than the radius of curvature r_B at the bottom, with a vertical height of 12 m between the lowest and highest points. Show that the requirement is met if $r_B \geq \frac{r_T + 24}{3}$. [4]
 - Given that $r_T = 4 \text{ m}$, find the least value of r_B , and the speed of the passenger at the top of the loop in this case, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [4]
-