

Centres of Mass of Particles

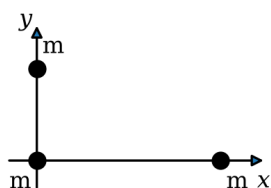
- 1 Particles of mass 2 kg and 3 kg are placed at the points $x = 1$ and $x = 4$ on a straight line (x in metres). Find the x -coordinate of their centre of mass. [2]

- 2 Particles of mass 1 kg, 2 kg and 3 kg are at $x = 0$, $x = 2$ and $x = 5$ respectively on a straight line. Find the distance of their centre of mass from the origin. [2]

- 3 Particles of mass 2 kg, 3 kg and 5 kg are at the points $(1, 3)$, $(4, 0)$ and $(-2, 2)$ respectively. Find the coordinates of the centre of mass. [2]

- 4 Particles of mass 4 kg and 6 kg are at two points A and B, where $AB = 2$ m. Find the distance of the centre of mass from A. [2]

- 5 Three particles of equal mass are placed at the points $(0, 0)$, $(6, 0)$ and $(0, 3)$. Find the coordinates of their centre of mass.

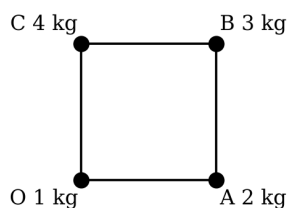


[2]

- 6 Particles of mass m and $2m$ are attached to the ends of a light rod of length 0.9 m. Find the distance of the centre of mass of the system from the particle of mass m . [2]

- 7 Particles of mass 2 kg, m kg and 4 kg are placed at $x = 0$, $x = 3$ and $x = 6$ on a straight line. The x -coordinate of their centre of mass is 3.5. Find m . [3]

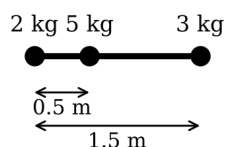
- 8 Particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the vertices O, A, B and C of a square OABC of side 4 m, as shown. Find the distance of the centre of mass from OA and from OC.



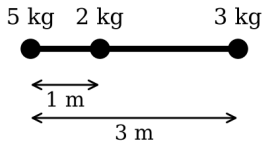
[4]

- 9 Particles of mass 3 kg and 5 kg are at $x = 0$ and $x = 4$ on a straight line. A third particle of mass 2 kg is placed at $x = p$ so that the centre of mass of the three particles is at $x = 3$. Find p . [3]

- 10 A light rod AB of length 1.5 m carries particles of mass 2 kg at A, 3 kg at B and 5 kg at the point C where $AC = 0.5$ m. Find the distance of the centre of mass of the system from A.

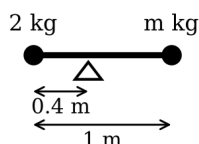


[3]

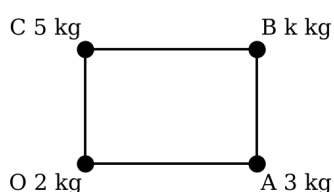
- 11** Particle A of mass 2 kg is at $x = 0$ and moves with velocity 3 m s^{-1} along the x -axis. Particle B of mass 3 kg is at $x = 10$ and moves with velocity -2 m s^{-1} .
 (a) Find the x -coordinate of the centre of mass at time t seconds. [3]
 (b) Describe the motion of the centre of mass, and explain your answer. [2]
-
- 12** Particles of mass 2 kg, 3 kg and 5 kg have position vectors $(2\mathbf{i} + \mathbf{j}) \text{ m}$, $(-\mathbf{i} + 4\mathbf{j}) \text{ m}$ and $(3\mathbf{i} - 2\mathbf{j}) \text{ m}$. Find the position vector of the centre of mass. [3]
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- 13** Particles of mass 1 kg, 2 kg and m kg are placed at $A(0, 0)$, $B(6, 0)$ and $C(0, 6)$ respectively. The y -coordinate of the centre of mass is 4. Find m , and the x -coordinate of the centre of mass. [4]
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- 14** Three particles of mass 2 kg, m kg and n kg are at $x = 0$, $x = 2$ and $x = 5$ on a straight line. The total mass is 10 kg and the centre of mass is at $x = 2.8$. Find m and n . [5]
-
- 15** A light rod AB of length 2 m carries particles of mass 2 kg at A and 3 kg at B. A third particle of mass 2 kg is placed on the rod so that the centre of mass of the system is at the midpoint of AB. Find the distance of this particle from A. [4]
-
- 16** Six particles, each of mass m , are placed at the vertices of a regular hexagon of centre O and side 0.5 m.
 (a) Write down the position of the centre of mass. [1]
 (b) One particle is removed. Find the distance of the new centre of mass from O. [2]
-
- 17** A light rod AB of length 3 m carries particles of mass 5 kg at A, 2 kg at a point C where $AC = 1 \text{ m}$, and 3 kg at B. The rod rests in a horizontal position on a single support at its centre of mass. Take $g = 9.8 \text{ m s}^{-2}$.
- 
- (a) Find the distance of the support from A. [3]
 (b) Find the magnitude of the force exerted by the support on the rod. [2]
-
- 18** Three particles A, B and C, of mass 2 kg, 3 kg and 5 kg, move with constant velocities $(3\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$, $(-\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ and $(\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$. At time $t = 0$ their position vectors are $2\mathbf{i}$, $4\mathbf{j}$ and $(-\mathbf{i} + \mathbf{j})$ metres respectively.
 (a) Find the velocity of the centre of mass. [2]
 (b) Find the position vector of the centre of mass at time $t = 0$, and at time $t = 3 \text{ s}$. [4]
-
- 19** Particles of mass 2 kg, 3 kg and 5 kg are at the points $(1, 2)$, $(4, -1)$ and (a, b) respectively. The centre of mass of the three particles is at $(3, 1)$. Find a and b . [4]
-
- 20** Particles of mass $3m$, $2m$ and m are at the vertices A, B and C of an equilateral triangle of side $6a$. Find the distance of the centre of mass from AB and from the perpendicular bisector of AB. [6]
-
- 21** Particles of mass m , $2m$ and $3m$ are placed at the vertices A, B and C of an equilateral triangle of side a . Show that the distance of the centre of mass from A is $\frac{a\sqrt{19}}{6}$. [7]
-
- 22** Three particles of equal mass are at the points with position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ relative to an origin O. Show that the centre of mass G of the three particles lies on the median from A to the midpoint M of BC, and that $AG : GM = 2 : 1$. [6]

- 23** Particles of mass 2 kg, 3 kg and 5 kg are at the points (0, 0), (4, 0) and (0, 6). A student writes: " $\bar{x} = \frac{0+4+0}{3} = 1.33$ and $\bar{y} = \frac{0+0+6}{3} = 2$, so the centre of mass is (1.33, 2)." Explain the error in the student's working, and find the correct coordinates of the centre of mass. [6]

- 24** A light rod AB of length 1 m carries a particle of mass 2 kg at A and a particle of mass m kg at B. The rod is suspended from a string attached at the point C where $AC = 0.4$ m and hangs in equilibrium with the rod horizontal. Find m and the tension in the string. A third particle of mass 1 kg is then fixed at the midpoint of AB; find the distance from A of the point where the string must be attached for the rod to hang horizontally. Take $g = 9.8 \text{ m s}^{-2}$.



- 25** Particles of mass 2 kg, 3 kg, k kg and 5 kg are placed at the vertices O, A, B and C respectively of a rectangle OABC with $OA = 6$ m and $OC = 4$ m.



- (a) Find, in terms of k , the distances of the centre of mass from OC and from OA. [4]
 (b) Given that the centre of mass is equidistant from OA and OC, find k and the distance of the centre of mass from OA. [3]

- 26** The Sun has mass 1.99×10^{30} kg and Jupiter has mass 1.90×10^{27} kg. Their centres are 7.78×10^{11} m apart. Model each as a particle. Find the distance of the centre of mass of the Sun and Jupiter from the centre of the Sun, to 3 significant figures. The radius of the Sun is 6.96×10^8 m; state whether the centre of mass lies inside the Sun. [6]

- 27** Particles of masses 1 kg, 2 kg, 3 kg, ..., n kg are placed at the points $x = 1, x = 2, x = 3, \dots, x = n$ respectively on a straight line (the particle of mass k kg is at $x = k$).

- (a) Show that the x -coordinate of the centre of mass is $\frac{2n+1}{3}$. [5]
 (b) Find n if the centre of mass is at $x = 7$. [2]
 (c) Particles in the same pattern but placed at $x = 1, 2, 3, \dots, n$ with masses 1, 4, 9, ..., n^2 : find the centre of mass in terms of n . [3]

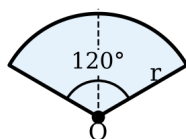
- 28** Particles of mass 1 kg, 2 kg, 3 kg and 4 kg are at the points with position vectors $(\mathbf{i})$, $(2\mathbf{j})$, $(3\mathbf{k})$ and $(2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ metres respectively, relative to an origin O.

- (a) Find the position vector of the centre of mass G. [3]
 (b) Find the distance OG, to 3 significant figures. [2]
 (c) A fifth particle of mass 10 kg is added. The centre of mass of all five particles is now at the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$. Find the position vector of the fifth particle. [4]

- 29** Two particles A and B, of masses m and $2m$, are projected simultaneously from the same point O on horizontal ground. A is projected with velocity $(6\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$ and B with velocity $(3\mathbf{i} + 12\mathbf{j}) \text{ m s}^{-1}$, where $\mathbf{i}$ is horizontal and $\mathbf{j}$ is vertically upwards. Take $g = 9.8 \text{ m s}^{-2}$. Neither particle has landed during the time considered.
- Find the horizontal and vertical components of the velocity of the centre of mass at time $t = 0$. [3]
 - Show that the path of the centre of mass has equation of the form $y = ax - bx^2$, and find the exact values of a and b . [5]
 - Find the greatest height of the centre of mass above O, to 3 significant figures. [2]
- 30** Particles of masses $m_1, m_2, \dots, m_n$ are at the points with position vectors $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$. The total mass is M and the centre of mass has position vector $\mathbf{g}$. Let $\mathbf{p}$ be any fixed point and define $I(\mathbf{p}) = \sum m_i |\mathbf{r}_i - \mathbf{p}|^2$.
- Show that $I(\mathbf{p}) = \sum m_i |\mathbf{r}_i - \mathbf{g}|^2 + M|\mathbf{g} - \mathbf{p}|^2$. [5]
 - Deduce that $I(\mathbf{p})$ is least when $\mathbf{p}$ is the centre of mass. [2]
 - Particles of mass 1 kg, 2 kg and 3 kg are at $x = 0, 1$ and 4 on a line. Find the least value of $\sum m_i (x_i - p)^2$ as p varies. [3]

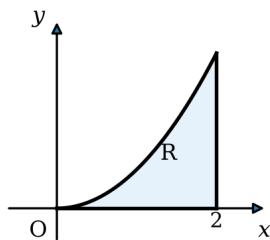
Centres of Mass of Rods and Laminae

- 1** Write down the position of the centre of mass of each of the following uniform bodies.
- A rod AB of length 0.8 m. [1]
 - A rectangular lamina ABCD with AB = 6 cm and BC = 4 cm. [1]
- 2** A uniform triangular lamina has a base of length 12 cm and perpendicular height 9 cm. Find the distance of its centre of mass from the base. [2]
- 3** A uniform semicircular lamina has radius 6 cm. Find the distance of its centre of mass from the centre of the straight edge, to 3 significant figures. [2]
- 4** A uniform wire is bent into an L shape OAB, with OA = 4 cm along the x-axis and OB = 3 cm along the y-axis, where O is the origin and angle AOB = 90° . Find the coordinates of the centre of mass of the wire. [2]
- 5** A uniform wire is bent into a semicircular arc of radius 5 cm. Find the distance of its centre of mass from the centre of the circle, to 3 significant figures. [2]
- 6** Using standard results, find the distance of the centre of mass from the plane face of
- a uniform solid hemisphere of radius 8 cm, [1]
 - a uniform solid cone of height 12 cm. [1]
- 7** A uniform wire is bent into the shape of a right-angled triangle OAB with OA = 4 cm, OB = 3 cm and AB = 5 cm, the right angle being at O. Taking O as origin with OA along the x-axis, find the coordinates of the centre of mass of the wire. [4]
- 8** A uniform lamina is in the shape of a sector of a circle of radius 10 cm and centre O, with angle 120° at O, as shown. Find the distance of its centre of mass from O, to 3 significant figures.



[3]

- 9** The region R is bounded by the curve $y = x^2$, the x-axis and the line $x = 2$, as shown. A uniform lamina is cut in the shape of R. Find the coordinates of its centre of mass.



[5]

- 10** A uniform triangular lamina has vertices at (1, 1), (7, 2) and (4, 8). Find the coordinates of its centre of mass.

[3]

- 11** A uniform lamina is in the shape of a quarter of a circle of radius 9 cm, bounded by two perpendicular straight edges OA and OB meeting at O. Find the distance of its centre of mass from each of OA and OB, to 3 significant figures.

[4]

- 12** A uniform wire is bent into an arc of a circle of centre O and radius 12 cm, which subtends an angle of 60° at O. Find the distance of the centre of mass of the arc from O, to 3 significant figures.

[3]

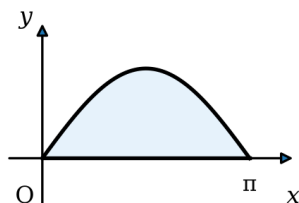
- 13** A uniform rod AB has mass 2 kg and length 1.2 m. A particle of mass 1 kg is attached to the rod at B. Find the distance of the centre of mass of the combined body from A.

[3]

- 14** A uniform wire of length $(\pi r + 2r)$ is bent into the shape of a semicircular arc together with its diameter. Show that the distance of the centre of mass from the diameter is $\frac{2r}{\pi + 2}$, and find the value of this distance when $r = 5$ cm, to 3 significant figures.

[5]

- 15** The region R is bounded by the curve $y = \sin x$ for $0 \leq x \leq \pi$ and the x-axis. A uniform lamina has the shape of R.



- (a) Find the area of R, and write down the x-coordinate of the centre of mass.

[3]

- (b) Show that the y-coordinate of the centre of mass is $\frac{\pi}{8}$.

[4]

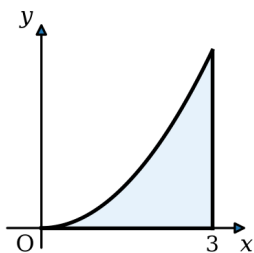
- 16** Find the coordinates of the centre of mass of a uniform lamina bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$.

[6]

- 17** A uniform wire is bent into an isosceles triangle ABC with $AB = AC = 13$ cm and $BC = 10$ cm. Find the distance of the centre of mass of the wire from BC.

[5]

- 18** The region bounded by the curve $y = x^2$, the x-axis and the line $x = 3$ is rotated through 2π radians about the x-axis to form a uniform solid.

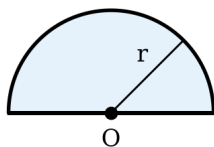


- (a) Show that the volume of the solid is $\frac{243\pi}{5}$. [2]
 (b) Find the distance of the centre of mass of the solid from the origin. [3]

- 19** A uniform lamina is bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 3$. Find the coordinates of its centre of mass, to 3 significant figures. [6]

- 20** A uniform wire is bent into the shape of the boundary of a sector of a circle of radius 10 cm and centre O: two straight radii and a circular arc, with angle 120° at O. Find the distance of the centre of mass of the wire from O, to 3 significant figures. [6]

- 21** Prove that the centre of mass of a uniform semicircular lamina of radius r is at a distance $\frac{4r}{3\pi}$ from the centre of the straight edge.

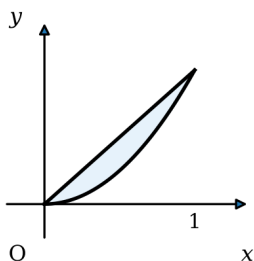


[7]

- 22** A uniform solid cone has base radius r and height h . By considering the cone as a solid of revolution, show that its centre of mass is a distance $\frac{h}{4}$ from its base. [6]

- 23** A uniform solid hemisphere has radius r . By considering it as a solid of revolution formed by rotating the region under the curve $y = \sqrt{r^2 - x^2}$, $0 \leq x \leq r$, about the x-axis, show that its centre of mass is a distance $\frac{3r}{8}$ from the centre of its plane face. [6]

- 24** The finite region R is bounded by the curves $y = x^2$ and $y = x$. A uniform lamina has the shape of R.

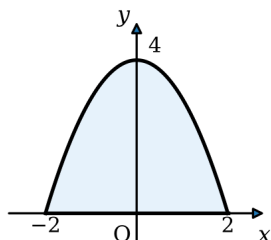


- (a) Find the area of R. [3]
 (b) Find the coordinates of the centre of mass of the lamina. [4]

- 25** A uniform wire is bent into a semicircular arc of radius 6 cm. A student writes: "The centre of mass of a semicircle is $\frac{4r}{3\pi}$ from the centre, so the distance is $\frac{4 \times 6}{3\pi} = 2.55$ cm from the centre of the diameter." Explain the error, and find the correct distance, to 3 significant figures. Show also how the distance would change if the body were a semicircular lamina of the same radius. [6]

- 26** The region bounded by the curve $y = x^2$, the y-axis and the line $y = 4$ is rotated through 2π radians about the y-axis to form a uniform solid bowl-shaped body (a paraboloid). Find the volume of the solid, and the distance of its centre of mass from the vertex, to 3 significant figures. [6]

- 27** The region R is bounded by the curve $y = 4 - x^2$ and the x-axis. A uniform lamina has the shape of R, with the x-axis as its base.



- (a) Show that the area of R is $\frac{32}{3}$ and that the centre of mass of the lamina is at the point $(0, \frac{8}{5})$. [6]
- (b) The lamina is freely suspended from the point A(2, 0) and hangs in equilibrium. Find the angle between the x-axis and the downward vertical, to the nearest degree. [3]

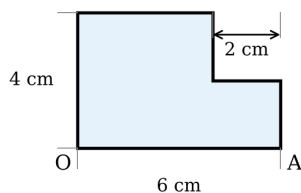
- 28** Show that the centre of mass of a uniform hemispherical shell (a thin curved surface of radius r , without its plane base) is at a distance $\frac{r}{2}$ from the centre of the base. [You may use the fact that the area of the part of a sphere of radius r between two parallel planes at distance dz apart is $2\pi r dz$.] [8]

- 29** The finite region bounded by the curve $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$, the x axis and the y axis is rotated through 2π radians about the x axis to form a uniform solid S.
- (a) Show that the volume of S is $\frac{\pi^2}{4}$. [4]
- (b) Show that the x-coordinate of the centre of mass of S is $\frac{\pi}{4} - \frac{1}{\pi}$, and find its value to 3 significant figures. [8]

- 30** A uniform lamina is bounded by the curve $y = \ln x$, the x-axis and the line $x = e$. Find the exact coordinates of its centre of mass, and give them to 3 significant figures. [9]

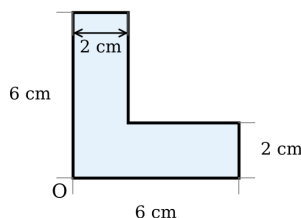
Composite Bodies

- 1 A uniform rectangular lamina OABC has $OA = 6$ cm and $OC = 4$ cm. A square of side 2 cm is cut from the corner B, leaving the lamina shown. Find the coordinates of the centre of mass of the remaining lamina, taking O as origin with OA along the x-axis.



[3]

- 2 A uniform lamina is L-shaped, made from a vertical rectangle 2 cm wide and 6 cm high and a horizontal rectangle 4 cm long and 2 cm high joined to its foot on the right (so that the overall width at the base is 6 cm), as shown. Find the coordinates of its centre of mass relative to the bottom-left corner O, with axes along the two outer edges.



[3]

- 3 A uniform solid cylinder of radius 3 cm and height 4 cm has a second uniform solid cylinder of the same material, of radius 1 cm and height 2 cm, placed on top of it with the same axis. Find the height of the centre of mass above the base of the larger cylinder, to 3 significant figures.

[3]

- 4 A uniform T-shaped lamina is made from a horizontal bar 6 cm by 2 cm on top of a vertical stem 2 cm by 4 cm, the stem being centred under the bar. Find the distance of the centre of mass from the top edge of the bar.

[3]

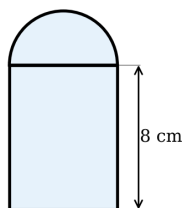
- 5 A uniform circular lamina has centre O and radius 5 cm. A circular hole of radius 2 cm is cut out, with its centre 3 cm from O. Find the distance of the centre of mass of the remaining lamina from O.

[3]

- 6 A uniform square lamina OABC of side 6 cm and mass 6 kg has a particle of mass 1 kg attached at B. Find the distance of the centre of mass of the combined body from OC.

[3]

- 7 A uniform lamina consists of a rectangle 6 cm wide and 8 cm high with a semicircle of radius 3 cm attached to one of its 6 cm sides, as shown. Find the distance of the centre of mass from the opposite side of the rectangle, to 3 significant figures.

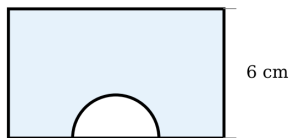


[4]

- 8** A uniform solid toy is made from a solid hemisphere of radius 6 cm and a solid cone of base radius 6 cm and height 12 cm, joined so that their plane faces coincide. Find the distance of the centre of mass from the plane face, and state which part of the toy contains it. [4]

- 9** A uniform lamina is formed by attaching an isosceles triangle, of base 8 cm and height 6 cm, to one side of a square of side 8 cm, the base of the triangle coinciding with that side. Find the distance of the centre of mass from the side of the square opposite to the triangle. [4]

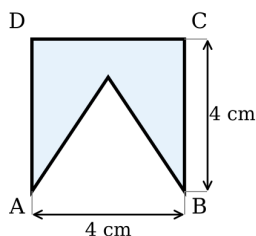
- 10** A uniform rectangular lamina is 10 cm wide and 6 cm high. A semicircle of radius 2 cm, with its diameter on the lower edge and its centre at the midpoint of that edge, is removed. Find the distance of the centre of mass of the remaining lamina from the lower edge, to 3 significant figures.



- 11** A uniform solid cylinder has radius 4 cm and height 10 cm. A cylindrical hole of radius 2 cm and depth 5 cm is drilled into the centre of one circular end, along the axis. Find the distance of the centre of mass of the remaining solid from the other (undrilled) end, to 3 significant figures. [4]

- 12** A uniform wire of total length 14 cm is bent into a U shape ABCD, with AB = 4 cm, BC = 6 cm and CD = 4 cm, where angles ABC and BCD are right angles, A and D being on the same side of BC. Find the distance of the centre of mass of the wire from BC. [3]

- 13** A uniform square lamina ABCD has side 4 cm. An isosceles triangular piece, with base AB and height 3 cm, is cut from the lamina. Find the distance of the centre of mass of the remaining lamina from AB.



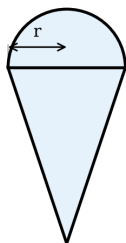
- 14** A uniform lamina is formed by removing a semicircle of radius $2a$ from a semicircle of radius $3a$. The two semicircles have the same centre O and the same diameter, so that the lamina is a semicircular ring (a thick semicircular arc). Show that the centre of mass of the lamina is at a distance $\frac{76a}{15\pi}$ from O along the axis of symmetry, and find this distance in terms of a to 3 significant figures. [6]

- 15** A uniform solid is formed from a solid cylinder of radius r and height h joined to a solid hemisphere of radius r , the plane face of the hemisphere coinciding with a circular end of the cylinder. Given that the centre of mass of the solid is at the centre of the common plane face, show that $h = \frac{r}{\sqrt{2}}$. [6]

- 16** A uniform solid cylinder has radius 4 cm and height 6 cm. A conical hole, with base radius 4 cm and height 6 cm, is hollowed out of one end of the cylinder, the base of the cone being the circular end of the cylinder. Find the distance of the centre of mass of the remaining solid from the other end, to 3 significant figures. [5]

- 17** A uniform solid hemisphere has radius r . A hemispherical hollow of radius $\frac{r}{2}$, with the same centre O and the same plane face, is removed. Show that the distance of the centre of mass of the remaining solid from O is $\frac{45r}{112}$. [6]

- 18** A uniform lamina is formed from a semicircle of radius r on top of an isosceles triangle with base equal to the diameter of the semicircle and height $3r$, as shown (an "ice cream cone" shape). Find the distance of the centre of mass from the diameter, in terms of r , to 3 significant figures, and state whether it lies in the triangle or the semicircle.



[6]

- 19** A uniform solid spindle is formed by joining together, at their circular bases, two solid cones of the same material. The common base has radius 6 cm and centre O . The smaller cone has height 4 cm and the larger cone has height 10 cm.
- (a) Find the distance of the centre of mass of the spindle from O , and state which cone contains it. [4]
- (b) Hence find the distance of the centre of mass from the vertex of the larger cone. [2]

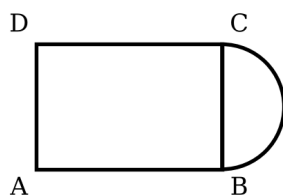
- 20** A uniform rectangular lamina $ABCD$ has $AB = 12$ cm and $AD = 8$ cm. A circular hole of radius 2 cm is cut from the lamina, its centre being 3 cm from AD and 4 cm from AB . The remaining lamina is freely suspended from A and hangs in equilibrium. Find the angle that AB makes with the downward vertical, to 3 significant figures. [6]

- 21** A uniform square lamina $OABC$ has side 12 cm. A square of side a cm is cut from the corner B , with its sides along BA and BC . The centre of mass of the remaining lamina is 5.5 cm from OC (and, by symmetry, 5.5 cm from OA). Show that $a^3 - 13a^2 + 144 = 0$, and hence find the value of a . [7]

- 22** A uniform solid is made from a solid cone of base radius r and height h joined to a solid hemisphere of radius r , with their plane faces coinciding.
- (a) Show that the centre of mass of the solid lies on the common plane face if $h = \sqrt{3} r$. [4]
- (b) State, with a reason, in which part of the solid the centre of mass lies when $h > \sqrt{3} r$. [2]

- 23** A uniform circular lamina has radius 5 cm and centre O . A circular hole of radius 2 cm, centre H , is cut from the lamina, where $OH = 3$ cm. A student writes: "The mass of the remaining lamina is proportional to $25\pi + 4\pi = 29\pi$, so the centre of mass is at a distance $\frac{4\pi \times 3}{29\pi} = 0.41$ cm from O ." Explain the error and find the correct distance and its direction relative to H . [6]

- 24** A uniform wire is bent to form the boundary of a rectangle ABCD with AB = 6 cm and BC = 4 cm, together with a semicircular arc of diameter BC lying outside the rectangle. The wire includes the side BC. Find the distance of the centre of mass of the wire from AD, to 3 significant figures.

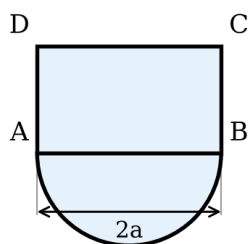


[6]

- 25** A uniform square lamina ABCD of side 12 cm has E and F as the midpoints of AB and AD respectively. The corner triangle AEF is folded along EF so that A lies on the centre of the square, and the folded triangle lies flat on top of the lamina.
- (a) Find the distances of the centre of mass of the folded lamina from AB and from AD. [6]
- (b) The folded lamina is freely suspended from the corner B and hangs in equilibrium. Find the angle between BC and the vertical, to 3 significant figures. [3]

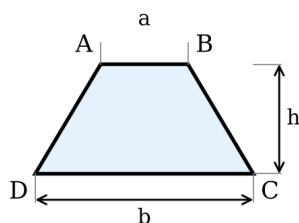
- 26** A solid hemisphere of radius r and uniform density ρ is joined to a solid cone with base radius r , height $2r$ and uniform density $k\rho$, with their plane faces coinciding. The centre of mass of the combined solid is at the centre of the common plane face. Find k . [6]

- 27** A uniform lamina consists of a rectangle ABCD with AB = $2a$ and BC = b , together with a semicircle of diameter AB attached on the side of AB away from the rectangle.



- (a) Show that the centre of mass of the lamina lies on AB if $b = a\sqrt{\frac{2}{3}}$. [5]
- (b) Find the distance of the centre of mass from AB when $a = 6$ and $b = 6$, stating on which side of AB it lies. [4]
- (c) Explain why the centre of mass is on the rectangle's side whenever $b > a\sqrt{\frac{2}{3}}$. [1]

- 28** A uniform trapezium lamina ABCD has AB parallel to DC, with AB = a , DC = $b > a$, and perpendicular distance h between AB and DC.

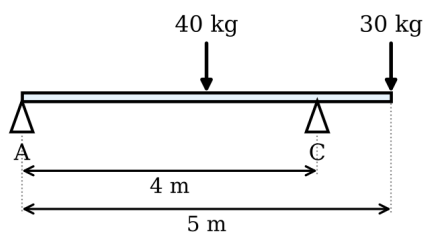


- (a) By dividing the trapezium into a rectangle and a triangle, or otherwise, show that the distance of its centre of mass from DC is $\frac{h(b + 2a)}{3(a + b)}$. [6]
- (b) Hence find the distance of the centre of mass from DC for a trapezium with AB = 4 cm, DC = 10 cm and $h = 6$ cm. [2]
- (c) Check your formula in the limiting cases $a = b$ and $a = 0$. [2]

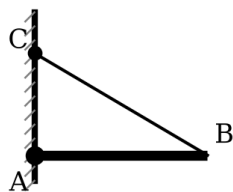
- 29** A uniform lamina is bounded by the curve $y = x^2$ and the line $y = 4$. A circular hole of radius $\frac{1}{2}$ cm, with centre on the y-axis at $(0, 3)$, is cut from the lamina (units are cm). Find the distance of the centre of mass of the remaining lamina from the x-axis, to 3 significant figures. [8]
- 30** A frustum of a uniform solid cone is made by removing, from a solid cone of height $3h$ and base radius R , the smaller cone of height h cut off by a plane parallel to the base. Show that the centre of mass of the frustum is at a distance $\frac{9h}{13}$ from the larger plane face. Find also the distance when $h = 13$ cm. [9]

Equilibrium of Rigid Bodies

- 1** A uniform beam AB of mass 20 kg and length 4 m rests horizontally on two supports, one at each end. Find the magnitude of the reaction at each support. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 2** A uniform plank AB of mass 30 kg and length 6 m rests horizontally on supports at A and at C, where $AC = 4$ m. Find the reaction at C. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 3** Two children sit on a light see-saw which balances horizontally on a pivot at its centre. One child has mass 40 kg and sits 1.5 m from the pivot. The other has mass 30 kg. Find how far from the pivot the second child sits. [2]
- 4** A uniform ladder of mass 15 kg rests with its lower end on rough horizontal ground and its upper end against a smooth vertical wall. Find the normal reaction of the ground on the ladder. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 5** A uniform rod AB of mass 4 kg is smoothly hinged to a wall at A and is held horizontal by a vertical string attached at B. Find the tension in the string. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 6** A light rod AB of length 1.2 m carries particles of mass 3 kg at A and 5 kg at B. Find the distance from A of the point at which the rod balances on a pivot. [2]
- 7** A uniform beam AB of mass 40 kg and length 5 m rests horizontally on supports at A and at C, where $AC = 4$ m. A person of mass 30 kg stands at B. Find the reactions at the two supports. Take $g = 9.8 \text{ m s}^{-2}$. [4]

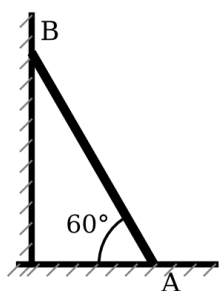


- 8** A uniform rod AB of mass 6 kg and length 1.6 m is smoothly hinged to a vertical wall at A and held horizontal by a light string BC, attached to the wall at C vertically above A. The string makes an angle of 35° with the rod. Find the tension in the string and the horizontal and vertical components of the force exerted by the hinge on the rod. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 9** A uniform ladder AB of length 5 m and mass 20 kg rests with A on rough horizontal ground and B against a smooth vertical wall. The ladder makes an angle of 60° with the ground. Find the frictional force at A and the least possible coefficient of friction between the ladder and the ground. Take $g = 9.8 \text{ m s}^{-2}$.



[5]

- 10** A uniform beam AB of mass 50 kg and length 4 m is held horizontally by two vertical ropes, one attached at each end. A load of mass 30 kg is hung from the point of the beam 1 m from A. Find the tension in each rope. Take $g = 9.8 \text{ m s}^{-2}$.

[4]

- 11** A uniform rod AB of mass 5 kg and length 2 m is smoothly hinged at A. A particle of mass 2 kg is attached at B. The rod is held horizontal by a vertical string attached at the point C of the rod where $AC = 1.5 \text{ m}$. Find the tension in the string and the magnitude and direction of the force exerted by the hinge. Take $g = 9.8 \text{ m s}^{-2}$.

[5]

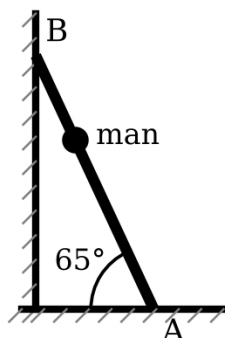
- 12** A non-uniform rod AB of length 2 m and weight 30 N has its centre of mass 0.8 m from A. It rests horizontally on supports at A and B. Find the reaction at each support.

[3]

- 13** A uniform plank AB of mass 25 kg and length 4 m rests horizontally on two supports at C and D, where $AC = 0.5 \text{ m}$ and $BD = 1 \text{ m}$. Find the reaction at each support. Take $g = 9.8 \text{ m s}^{-2}$.

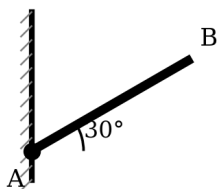
[4]

- 14** A uniform ladder AB of length 6 m and mass 20 kg rests with A on rough horizontal ground and B against a smooth vertical wall, at 65° to the horizontal. A man of mass 80 kg stands on the ladder at a point 4 m from A, and the ladder is about to slip. Find the coefficient of friction between the ladder and the ground, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



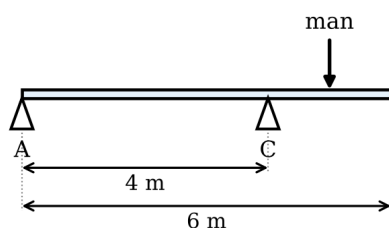
[6]

- 15** A uniform rod AB of mass 8 kg and length 2 m is smoothly hinged to a vertical wall at A. It is held in equilibrium, inclined at 30° above the horizontal, by a horizontal string attached at B and to the wall. Find the tension in the string, and the magnitude and direction of the force exerted by the hinge on the rod. Take $g = 9.8 \text{ m s}^{-2}$.



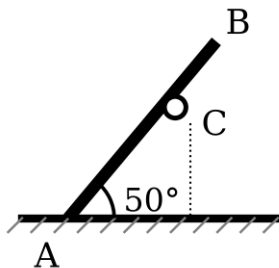
[7]

- 16** A uniform beam AB of mass 40 kg and length 6 m rests horizontally on supports at A and at C, where $AC = 4 \text{ m}$. A man of mass 60 kg walks from A towards B along the beam. Find how far from A he is when the beam is about to tilt. Take $g = 9.8 \text{ m s}^{-2}$.



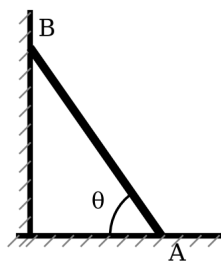
[5]

- 17** A uniform rod AB of length 3 m and mass 8 kg rests with end A on rough horizontal ground, in contact with a smooth horizontal peg at C, where AC = 2 m. The rod makes an angle of 50° with the horizontal, as shown. Find the reaction at the peg, and the least value of the coefficient of friction at A for the rod to remain in equilibrium, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[7]

- 18** A uniform ladder of mass m rests with its lower end on rough horizontal ground, the coefficient of friction being 0.4, and its upper end against a smooth vertical wall. The ladder makes an angle θ with the horizontal.

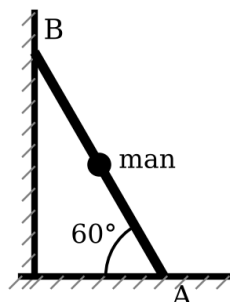


- (a) Show that the ladder can rest in equilibrium without anyone on it only if $\tan \theta \geq \frac{1}{2\mu}$, and find the least value of θ , to 3 significant figures. [4]
- (b) A man of mass $2m$ stands at the top of the ladder. Find the least value of θ for equilibrium, to 3 significant figures. [3]

- 19** A uniform rod AB of mass 40 kg and length 4 m has its end A in contact with a rough vertical wall. The rod is held in equilibrium in a horizontal position by a light string joining B to a point C of the wall vertically above A, the string making an angle of 30° with the rod. Find the tension in the string, and the least possible value of the coefficient of friction between the rod and the wall, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$. [6]

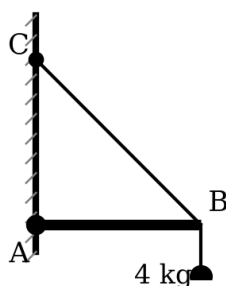
- 20** A uniform rectangular gate of weight 200 N and width 2 m is held in a vertical plane by two hinges A and B, with A vertically above B and AB = 1 m. The lower hinge B supports the entire weight of the gate and the hinges exert no vertical force at A. Find the horizontal components of the forces at A and at B, and the magnitude of the resultant force at B. [6]

- 21** A uniform ladder AB of length 5 m and mass 15 kg rests with A on rough horizontal ground, the coefficient of friction being 0.3, and with B against a smooth vertical wall. The ladder makes an angle of 60° with the horizontal. A man of mass 75 kg climbs the ladder. Find how far along the ladder from A he can climb before it slips, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



[8]

- 22** A uniform rod AB of mass 10 kg and length 2 m is smoothly hinged to a vertical wall at A. A particle of mass 4 kg is attached to the rod at B. The rod is held horizontal by a light string BC, where C is on the wall 2 m vertically above A. Find the tension in the string, and the magnitude and direction of the force exerted by the hinge on the rod, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.

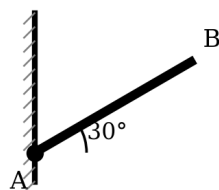


[7]

- 23** A uniform ladder of mass 12 kg and length 4 m rests with its lower end A on rough horizontal ground, the coefficient of friction being 0.5, and its upper end B against a smooth vertical wall. The ladder makes an angle of 60° with the ground. A student writes: "The friction is $F = \mu R = 0.5 \times 12g = 58.8 \text{ N}$, so the wall reaction is 58.8 N, and the ladder is in equilibrium." Explain the error in this reasoning, find the frictional force actually acting, and find the smallest angle with the horizontal at which the ladder can rest in equilibrium. Take $g = 9.8 \text{ m s}^{-2}$.

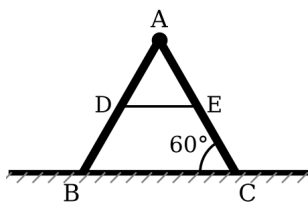
[6]

- 24** A uniform rod AB of weight 40 N and length 2 m is smoothly hinged at A to a fixed support. A force P, perpendicular to the rod and in the vertical plane containing the rod, is applied at B so that the rod is in equilibrium inclined at 30° above the horizontal. Find P, and the magnitude of the force exerted by the hinge on the rod, to 3 significant figures.



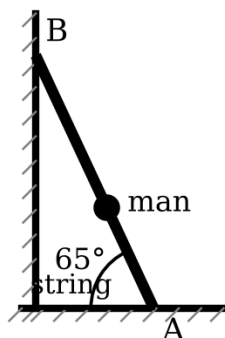
[7]

- 25** Two uniform ladders AB and AC, each of mass 10 kg and length 3 m, are smoothly hinged together at A and stand on a smooth horizontal floor with B and C on the floor, forming a step-ladder. A light horizontal rope DE joins the midpoints D and E of the ladders, and each ladder makes an angle of 60° with the floor, as shown. Find the tension in the rope, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



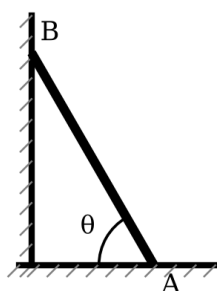
[7]

- 26** A uniform ladder of mass 20 kg and length 5 m rests with its upper end against a smooth vertical wall and its lower end A on a smooth horizontal floor. The ladder makes an angle of 65° with the floor and is prevented from slipping by a horizontal light string attached at A and to the base of the wall. The string can withstand a tension of at most 120 N. A man of mass 70 kg climbs the ladder. Find the greatest distance along the ladder from A that he can climb, to 3 significant figures. Take $g = 9.8 \text{ m s}^{-2}$.



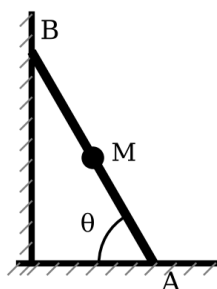
[7]

- 27** A uniform ladder AB of mass m rests in limiting equilibrium with A on rough horizontal ground, coefficient of friction μ_1 , and B against a rough vertical wall, coefficient of friction μ_2 . The ladder makes an angle θ with the horizontal. Show that $\tan \theta = \frac{1 - \mu_1 \mu_2}{2\mu_1}$, and find θ when $\mu_1 = 0.4$ and $\mu_2 = 0.25$, to 3 significant figures.



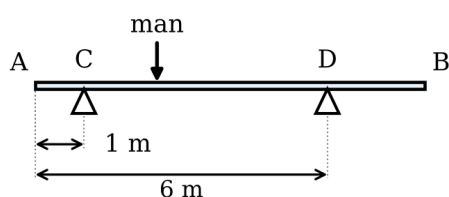
[10]

- 28** A uniform ladder of mass m and length L rests with its lower end A on rough horizontal ground, coefficient of friction μ , and its upper end B against a smooth vertical wall. The ladder makes an angle θ with the horizontal. A man of mass M climbs the ladder.



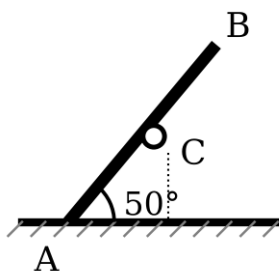
- (a) Show that, when the man is at a distance x from A, the frictional force at A is $\frac{(mL + 2Mx)g \cot \theta}{2L}$. [4]
- (b) Show that the man can reach the top of the ladder without it slipping if $\tan \theta \geq \frac{m + 2M}{2\mu(m + M)}$. [5]
- (c) Given that $m = 15$ kg, $M = 60$ kg and $\mu = 0.35$, find the least angle θ , to 3 significant figures. [2]

- 29** A uniform beam AB of mass 50 kg and length 8 m rests horizontally on two supports at C and D, where AC = 1 m and AD = 6 m. A man of mass 80 kg walks along the beam from A towards B. When he is at a distance x metres from A, the reactions at C and D are R_C and R_D . Take $g = 9.8 \text{ m s}^{-2}$.



- (a) Find, for $1 \leq x \leq 6$, expressions for R_C and R_D in terms of x . [5]
- (b) Find the greatest distance from A at which the man can stand without the beam tilting. [3]
- (c) Find the greatest mass of a man who could walk the whole length of the beam to B without it tilting. [2]

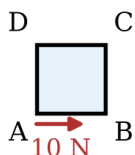
- 30** A uniform rod AB of mass m and length $2a$ rests with its lower end A on rough horizontal ground, coefficient of friction μ , and rests against a smooth horizontal peg C at height h above the ground. The rod makes an angle θ with the horizontal. Show that the rod can rest in equilibrium only if $\mu \geq \frac{a \cos \theta \sin^2 \theta}{h - a \cos^2 \theta \sin \theta}$. Hence find the least value of μ for $a = 1.5$ m, $h = 1.2$ m and $\theta = 50^\circ$, to 3 significant figures.



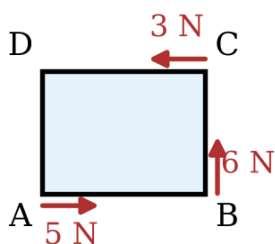
[10]

Moments and Couples

- 1 A force of magnitude 12 N acts at a perpendicular distance of 0.5 m from a point O. Find the moment of the force about O. [2]
- 2 A force of 8 N acts at one end A of a light rod AB of length 0.6 m, at an angle of 30° to the rod. Find the moment of the force about B. [2]
- 3 A couple consists of two parallel forces, each of magnitude 15 N, acting in opposite directions along lines 0.4 m apart. Find the moment of the couple. [2]
- 4 A couple of moment 4.5 N m is formed by two equal and opposite forces of 15 N. Find the perpendicular distance between their lines of action. [2]
- 5 A force $F = (3i + 4j)$ N acts at the point with position vector $(2i + j)$ m. Find the moment of the force about the origin, stating its sense. [2]
- 6 A force of 10 N acts along the side AB of a square ABCD of side 2 m, in the direction from A to B. Find its moment about C. [2]

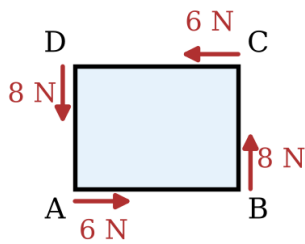


- 7 Forces of 5 N, 6 N and 3 N act along the sides AB, BC and CD of a rectangle ABCD, in the directions A to B, B to C and C to D respectively, where $AB = 4$ m and $BC = 3$ m. Find the magnitude of the resultant force and the angle it makes with AB, and the total moment of the forces about A. [5]



- 8 A force $F = (4i - 3j)$ N acts at the point A with position vector $(i + 2j)$ m.
 - (a) Find its moment about the origin O. [2]
 - (b) Find its moment about the point B with position vector $3i$ m. [2]

- 9 Forces of 6 N, 8 N, 6 N and 8 N act along the sides AB, BC, CD and DA of a rectangle ABCD, in the directions A to B, B to C, C to D and D to A. Given that $AB = 4$ m and $BC = 3$ m, show that the forces are equivalent to a couple and find its moment.



[5]

- 10 A uniform rod AB of mass 3 kg and length 2 m is smoothly hinged at A and held in equilibrium in a horizontal position by a couple. Find the moment of the couple. Take $g = 9.8 \text{ m s}^{-2}$.

[3]

- 11 A force of 12 N and a couple of moment 18 N m together act on a body. Show that they are equivalent to a single force of 12 N, and find the distance of its line of action from the point at which the original force acts.

[4]

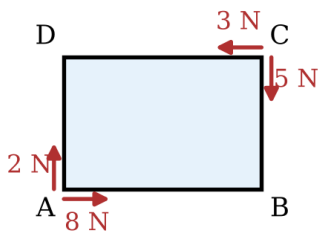
- 12 Three forces, each of magnitude 6 N, act along the sides AB, BC and CA of an equilateral triangle ABC of side 2 m, in the directions A to B, B to C and C to A. Show that they are equivalent to a couple and find its moment in exact form.

[5]

- 13 Parallel forces of 12 N, 9 N and 4 N act downwards on a light rod AB at distances 0 m, 1 m and 3 m from A respectively. Find the magnitude of the resultant force and its distance from A.

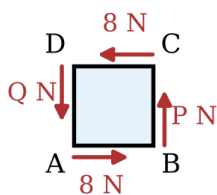
[4]

- 14 Four forces act on a rectangle ABCD with $AB = 6$ m and $BC = 4$ m: 8 N along AB (A to B), 5 N along CB (C to B), 3 N along CD (C to D) and 2 N along AD (A to D). Find the magnitude and direction of the resultant, and the point at which its line of action crosses AB.



[7]

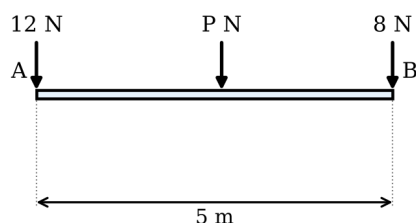
- 15 Forces of 8 N, P N, 8 N and Q N act along the sides AB, BC, CD and DA of a square ABCD of side 2 m, in the directions A to B, B to C, C to D and D to A. A clockwise couple of moment 24 N m is also applied to the square, which is in equilibrium. Find P and Q.



[5]

- 16** A force $F = (6\mathbf{i} - 2\mathbf{j})$ N acts at the point with position vector $(3\mathbf{i} + 4\mathbf{j})$ m. Find the moment of F about the origin, and find the position vector of the point where the line of action of F crosses the x axis. [5]

- 17** Three parallel forces act downwards on a light rod AB of length 5 m: 12 N at A, P N at a point C, and 8 N at B. The resultant of the three forces is 30 N, acting at a point 2.2 m from A. Find P and the distance AC.

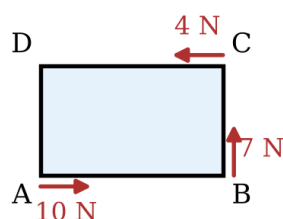


- 18** A uniform rod AB of mass 2.4 kg and length 1.5 m is smoothly hinged at A. A couple acts on the rod, which is in equilibrium inclined at 40° above the horizontal. Find the moment of the couple and the magnitude and direction of the force exerted by the hinge on the rod. Take $g = 9.8 \text{ m s}^{-2}$. [5]

- 19** A rod AB of length 1.5 m has forces acting on it, all perpendicular to the rod: 20 N at A, 20 N at B in the opposite direction, and 30 N at the midpoint M in the same direction as the force at A. Show that the system reduces to a single force and find its magnitude and the distance of its line of action from A. [6]

- 20** Three forces $F_1 = (2\mathbf{i} + a\mathbf{j})$ N, $F_2 = (b\mathbf{i} - 3\mathbf{j})$ N and $F_3 = (-\mathbf{i} + 2\mathbf{j})$ N act on a rigid body at the points with position vectors $\mathbf{i}$ m, $2\mathbf{j}$ m and $(3\mathbf{i} + \mathbf{j})$ m respectively, where a and b are constants. The three forces are equivalent to a couple. [3]
 (a) Find the values of a and b . [3]
 (b) Find the moment of the couple about the origin, stating its sense. [4]

- 21** Forces of 10 N, 7 N and 4 N act along AB (A to B), BC (B to C) and CD (C to D) of a rectangle ABCD, where $AB = 5$ m and $BC = 3$ m.



- (a) Find the magnitude and direction of the resultant. [3]
 (b) Find where the line of action of the resultant meets the line AD, extended if necessary. [4]
 (c) A fourth force is added at A to make the system equivalent to a couple. Find this force and the moment of the couple. [2]
- 22** A student says: "The forces in a couple are equal and opposite so they cancel. Hence a couple has no effect, and its moment about a point is zero." The forces of a couple are 12 N, acting 0.5 m apart. [2]
 (a) Explain two errors in the student's statement. [2]
 (b) Show that the moment of the couple about any point is the same, by finding it about a point on the line of one force and about a point midway between the lines. [4]

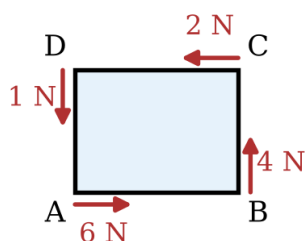
- 23** Forces of 8 N, 5 N and 3 N act along the sides AB, BC and CA of an equilateral triangle ABC of side 4 m, in the directions A to B, B to C and C to A. Find the magnitude of the resultant and the angle it makes with AB, and the perpendicular distance of its line of action from A, giving answers to 3 significant figures. [8]

- 24** A uniform rod AB of mass 6 kg and length 2 m is smoothly hinged at A. A couple of constant moment 20 N m acts on the rod in a vertical plane, opposing the moment of the weight. Take $g = 9.8 \text{ m s}^{-2}$.
 (a) Find the angle that the rod makes with the horizontal in equilibrium. [3]
 (b) State, with a reason, the direction and magnitude of the force of the hinge on the rod. [2]
 (c) Find the greatest couple that can hold the rod in equilibrium at some angle, and explain the situation if the couple is greater than this. [3]

- 25** Forces F_1 , F_2 , F_3 act on a rigid body with $F_1 + F_2 + F_3 = 0$. Show that the total moment of the three forces about a point P is independent of the position of P, and explain why this means that a system of forces with zero resultant is equivalent to a couple. [6]

- 26** Three forces act in the x-y plane, with distances in metres and forces in newtons: $F_1 = (3\mathbf{i} - 2\mathbf{j})$ at the point (1, 2); $F_2 = (-\mathbf{i} + 4\mathbf{j})$ at the point (4, 0); $F_3 = (2\mathbf{i} + \mathbf{j})$ at the point (0, 3). Find the resultant force, the total moment about the origin O, the equation of the line of action of the resultant, and the point where this line meets the y axis. [8]

- 27** Forces of 6 N, 4 N, 2 N and 1 N act along AB (A to B), BC (B to C), CD (C to D) and DA (D to A) of a rectangle ABCD, with AB = 4 m and BC = 3 m. Take A as the origin, AB along the x axis and AD along the y axis.

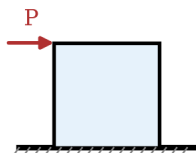


- (a) Show that the forces are equivalent to a single force and find its magnitude and direction. [3]
 (b) Find the equation of the line of action of the resultant. [3]
 (c) Find the points where the line of action meets AB and AD (extended if necessary). [2]
 (d) A single further force is applied at A so that the whole system is in equilibrium. Find its magnitude and direction, and the moment of the additional couple required. [3]
- 28** Two forces of magnitudes P and Q act in opposite directions along parallel lines a distance d apart, with $P > Q$.
 (a) Show that the resultant is a force of magnitude $P - Q$, and that its line of action lies at distance $\frac{Qd}{P - Q}$ from the line of P, on the side of Q, beyond the line of Q. [6]
 (b) Forces of 14 N and 9 N act in opposite directions along parallel lines 0.6 m apart. Find the resultant force and the position of its line of action. [3]
 (c) Explain what happens to the line of action as Q approaches P, and state what the limit represents. [2]
- 29** A uniform rectangular lamina ABCD of mass 12 kg has AB = 0.8 m and BC = 0.6 m. It is smoothly hinged at A so that it can rotate in a vertical plane. A couple of moment 20 N m acts on the lamina, which rests in equilibrium with the diagonal AC inclined at an acute angle θ to the downward vertical through A. Find θ , to 3 significant figures, the force exerted by the hinge, and the greatest couple for which equilibrium is possible. Take $g = 9.8 \text{ m s}^{-2}$. [10]

- 30** A light square frame ABCD of side 0.5 m lies on a smooth horizontal table. Forces act in its plane: 10 N along AB (A to B), 10 N along CD (C to D), 6 N along BC (B to C) and X N along DA (D to A).
- (a) Show that the moment of the system about A does not depend on X. [4]
- (b) Find the value of X for which the system reduces to a couple, and the moment of this couple. [3]
- (c) For $X = 2$, find the magnitude of the resultant force and the distance of its line of action from A. [4]

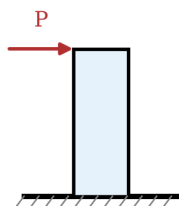
Toppling and Sliding

- 1** A uniform cuboid has a rectangular base of width 0.4 m and height 1.2 m. It stands on a rough horizontal table. It is tilted about one bottom edge. Find the angle through which it can be tilted from the vertical before it topples. [2]
- 2** A block of mass 5 kg rests on a rough horizontal surface, the coefficient of friction being 0.4. A horizontal force P is applied to the block. Find the least value of P which makes the block slide. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 3** A block rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the block and the plane is 0.5. Determine whether the block slides. [2]
- 4** A block of mass 12 kg rests on a rough horizontal surface with coefficient of friction 0.3. A horizontal force of 30 N is applied to it. Find the frictional force acting on the block. Take $g = 9.8 \text{ m s}^{-2}$. [2]
- 5** A uniform cube of side 0.4 m and mass 20 kg rests on a rough horizontal floor. A horizontal force P is applied at the middle of a top edge, perpendicular to that edge, and is increased until the cube is about to tip about the opposite bottom edge. Find P. Take $g = 9.8 \text{ m s}^{-2}$.



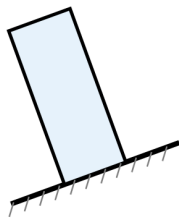
[2]

- 6** A uniform solid cylinder of radius 0.1 m and height 0.5 m stands with its base on a rough plane. The plane is slowly tilted. Assuming the cylinder does not slide, find the angle of inclination at which it topples. [2]
- 7** A uniform cuboid of mass 15 kg has a rectangular base 0.3 m wide and height 0.8 m. It rests on a rough horizontal plane, the coefficient of friction being 0.4. A horizontal force P is applied at the top, in the vertical plane through the centre of mass and perpendicular to the width, and is slowly increased. Find the value of P at which the cuboid starts to move, and state whether it slides or topples. Take $g = 9.8 \text{ m s}^{-2}$.



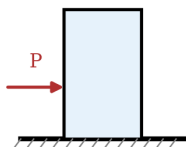
[5]

- 8** A uniform rectangular block has base 0.2 m wide (measured down the slope) and height 0.5 m. It rests on a rough inclined plane, the coefficient of friction being 0.6. The incline is slowly increased. Find the angle at which toppling would occur and the angle at which sliding would occur, and state which happens first.



[5]

- 9** A uniform block of mass m , width 0.6 m and height 1.0 m rests on a rough horizontal floor. A horizontal force P is applied at a point on the block 0.4 m above the floor. Find the least value of the coefficient of friction between the block and the floor which allows the block to topple, without sliding, when P is increased.



[5]

- 10** A uniform cube rests on a rough plane, with coefficient of friction 0.8, which is slowly tilted. Find the angle at which the cube topples and the angle at which it slides, and say which happens first.

[4]

- 11** A block of mass 10 kg rests on a rough horizontal floor with coefficient of friction 0.35. It is pulled by a rope inclined at 30° above the horizontal. Find the tension at which the block is about to slide. Take $g = 9.8 \text{ m s}^{-2}$.

[5]

- 12** A uniform rectangular lamina ABCD has $AB = 0.4 \text{ m}$ and $BC = 0.9 \text{ m}$. It rests on a rough plane inclined at angle θ to the horizontal with the side AB in contact with the plane. The plane is rough enough to prevent sliding. Find the greatest value of θ for which the lamina does not topple, to 3 significant figures.

[4]

- 13** A uniform solid cylinder of radius 5 cm and height 20 cm rests on a rough plane inclined at θ with its base on the plane. Find the least value of the coefficient of friction for which the cylinder topples before it slides as θ is increased.

[4]

- 14** A uniform solid cone has base radius 4 cm and height 12 cm. It rests with its base on a rough plane which is slowly tilted. Assuming no sliding, find the angle of tilt at which the cone topples, to 3 significant figures. State the least coefficient of friction needed for it to topple before it slides.

[5]

- 15** A uniform rectangular block of mass 20 kg has width 0.4 m and height 1.2 m, and stands on a rough horizontal floor with coefficient of friction 0.3. A force P is applied at the top edge, directed downwards at 20° below the horizontal, pushing towards the far edge. P is increased until the block is about to tip about the far bottom edge. Find P , and determine whether the block would slide before tipping. Take $g = 9.8 \text{ m s}^{-2}$.

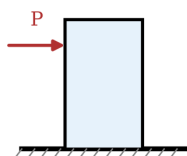
[6]

- 16** A uniform block of width 0.5 m stands on a rough horizontal floor, the coefficient of friction being 0.4. A horizontal force P is applied to the block at height h above the floor, in the vertical plane through the centre of mass. Find the greatest value of h for which the block slides before it topples. [5]

- 17** A uniform block of mass 6 kg has a rectangular cross-section of width 0.4 m and height 0.6 m. A particle of mass 2 kg is attached to the block at the top corner nearest to the lower edge of the slope. The block is placed on a rough plane inclined at θ to the horizontal, with its width measured down the slope. Find the greatest value of θ for which the block does not topple, to 3 significant figures. [5]

- 18** A uniform semicircular lamina of radius r has its diameter in contact with a rough plane inclined at θ to the horizontal, the curved edge uppermost, with the plane of the lamina vertical. The lamina is prevented from sliding. Find the greatest value of θ for which the lamina does not topple, to 3 significant figures. You may use that the centre of mass is $\frac{4r}{3\pi}$ from the diameter. [5]

- 19** A uniform rectangular block of mass 20 kg has width 0.6 m and height 1.0 m and rests on a rough horizontal floor with coefficient of friction 0.5. A horizontal force P is applied at a point 0.8 m above the floor and is gradually increased. Find the value of P at which the block first moves, and describe the motion. Take $g = 9.8 \text{ m s}^{-2}$.



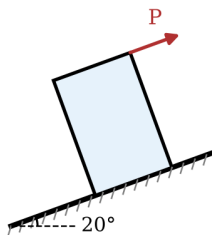
- 20** Explain why the angle at which a uniform block topples on an inclined plane does not depend on its mass, and find this angle for a block of base width 0.4 m (down the slope) and height 0.5 m, to 3 significant figures. [4]

- 21** A uniform solid hemisphere of radius r rests with its curved surface on a rough plane inclined at θ to the horizontal. The centre of mass G is at distance $\frac{3r}{8}$ from the centre O of the plane face.
- Show that the hemisphere can rest in equilibrium with its plane face inclined at ϕ to the horizontal where $\sin\phi = \frac{8}{3} \sin\theta$. [4]
 - Find the greatest angle of the plane for which this is possible, and find ϕ when $\theta = 15^\circ$, to 3 significant figures. [3]
 - State the least coefficient of friction needed at $\theta = 15^\circ$. [2]

- 22** A student writes: "A uniform block of width 0.3 m and height 0.9 m on a rough plane topples when $\tan\theta = 0.15/0.9$, so $\theta = 9.5^\circ$."
- Identify and correct the mistake. [3]
 - The coefficient of friction between the block and the plane is 0.25. Say what happens as the plane is tilted. [3]

- 23** A uniform cube A of side 0.4 m and mass 12 kg rests on a rough plane. A uniform cube B of side 0.2 m and mass 3 kg rests on top of A, with the lower-slope faces of the cubes flush. The coefficient of friction between A and the plane is 0.6 and B does not move relative to A. The plane is slowly tilted. Find the angle at which the arrangement topples and the angle at which it slides, and state which occurs first. [7]

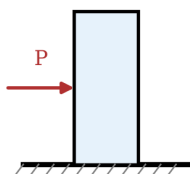
- 24** A uniform rectangular block of mass 10 kg has width 0.4 m (along the slope) and height 0.6 m. It rests on a rough plane inclined at 20° to the horizontal, with coefficient of friction 0.7. A force P acts parallel to the line of greatest slope, up the slope, at the top of the block, in the vertical plane through the centre of mass. P is slowly increased from zero. Find the value of P at which the block moves, and state whether it slides or topples. Take $g = 9.8 \text{ m s}^{-2}$.



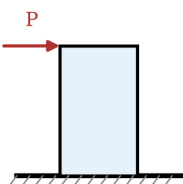
[7]

- 25** A solid is formed by joining a uniform solid cylinder of radius 0.1 m and height 0.3 m to a uniform solid cone of the same material, radius 0.1 m and height 0.2 m, with the base of the cone on the top face of the cylinder. The solid rests with the base of the cylinder on a rough plane. (The centre of mass of a cone is at a quarter of its height from its base.)
- Show that the centre of mass is 0.186 m above the base of the cylinder. [4]
 - The plane is tilted, with no sliding. Find the angle at which the solid topples. [2]
 - Find the least coefficient of friction that allows toppling before sliding. [2]

- 26** A uniform block of mass 30 kg has width 0.5 m and height 1.2 m. It rests on a rough horizontal floor, the coefficient of friction being 0.4. A horizontal force P acts on the block at a height x metres above the floor, in the vertical plane through the centre of mass. Take $g = 9.8 \text{ m s}^{-2}$.



- Show that the block slides before it topples if $x < 0.625$. [4]
 - Find P and the type of motion when $x = 1.0$ m. [2]
 - Find P and the type of motion when $x = 0.5$ m. [2]
- 27** A uniform rectangular block of mass m has width $2a$ and height $2b$. It rests on a rough horizontal floor, the coefficient of friction being μ . A horizontal force P is applied at the top of the block, in the vertical plane through the centre of mass.



- Show that the block topples before it slides if $\mu > \frac{a}{2b}$. [4]
- The block has $a = 0.3$ m, $b = 0.5$ m. Find the range of μ for which toppling occurs first, and find the minimum force needed to tip a block of mass 40 kg. Take $g = 9.8 \text{ m s}^{-2}$. [4]
- The force is now applied at height kb above the floor ($0 < k < 2$). Show that, for the same block, sliding occurs first if $\mu < \frac{a}{kb}$, and find the least k for which toppling occurs first when $\mu = 0.5$. [3]

- 28** A uniform lamina in the shape of an equilateral triangle ABC of side 0.6 m and mass $3m$ rests with BC on a rough plane inclined at θ to the horizontal, with C higher up the slope. The plane is rough enough to prevent sliding.
- (a) Show that the lamina topples when $\theta = 60^\circ$ if no other mass is attached. [4]
 - (b) A particle of mass m is fixed at A. Find the greatest angle for the plane for which the lamina does not topple, to 3 significant figures. [5]
 - (c) Find the range of coefficients of friction for which the lamina topples rather than slides. [2]
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- 29** A uniform solid hemisphere of radius r and mass m rests with its curved surface on a rough horizontal plane. A horizontal force P is applied at the highest point of the rim of the plane face, and the hemisphere rests in equilibrium with the plane face inclined at ϕ to the horizontal. The centre of mass is $\frac{3r}{8}$ from the centre O of the plane face.
- (a) Show that $P = \frac{3mg \sin \phi}{8(1 + \sin \phi)}$. [6]
 - (b) Find P when $\phi = 30^\circ$ and $m = 2$ kg. Take $g = 9.8 \text{ m s}^{-2}$. [2]
 - (c) Show that equilibrium is possible for every ϕ up to 90° provided that the coefficient of friction exceeds a particular value, and find that value. [4]
-
- 30** A uniform solid cone of base radius r and height h rests with its base on a rough plane inclined at θ to the horizontal. The plane is rough enough to prevent sliding.
- (a) Show that the cone topples when $\tan \theta = \frac{4r}{h}$. [3]
 - (b) For $r = 5$ cm and $h = 12$ cm find the toppling angle, to 3 significant figures, and the least coefficient of friction needed for the cone to topple rather than slide. [3]
 - (c) A particle of mass equal to a quarter of the mass of the cone is attached to the rim of the base at the point highest up the slope. Find the new toppling angle. [6]
-