Truth Tables and Logical Statements

1 (2 marks)

The expression is 1 only when both P and Q are 1 **B1**

The four rows give outputs 0, 0, 0, 1 in the order 00, 01, 10, 11 **B1**

Answer: Output column 0, 0, 0, 1 **2 marks**

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

2 (3 marks)

(a) P and Q are not both true **B1**

Answer: 0 **1 mark**

(b) At least one of P, Q is true **B1**

Answer: 1 **1 mark**

(c) Exactly one of P, Q is true **B1**

Answer: 1 **1 mark**

3 (2 marks)

(a) $2^3 = 8$ **B1**

Answer: 8 **1 mark**

(b) $2^6 = 64$ **B1**

Answer: 64 **1 mark**

4 (2 marks)

"I do not have an umbrella" is ¬Q **B1**

Joining with "and" gives P ∧ ¬Q **B1**

Answer: P ∧ ¬Q **2 marks**

5 (2 marks)

P → Q is false only when P is true and Q is false **B1**

Outputs in the order 00, 01, 10, 11 are 1, 1, 0, 1 **B1**

Answer: Output column 1, 1, 0, 1 **2 marks**

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

6 (2 marks)

P ↔ Q is true when P and Q have the same truth value **M1**

P = Q = 1 or P = Q = 0 **A1**

Answer: P = Q = 1 or P = Q = 0 **2 marks**

7 (4 marks)

(a) P ∧ Q column is 0, 0, 0, 1 **B1**

Negating gives 1, 1, 1, 0 **B1**

Answer: 1, 1, 1, 0 for rows 00, 01, 10, 11 **2 marks**

(b) P ∨ Q column is 0, 1, 1, 1 **B1**

Negating gives 1, 0, 0, 0 **B1**

Answer: 1, 0, 0, 0 for rows 00, 01, 10, 11 **2 marks**

P	Q	$P \uparrow Q$	$P \downarrow Q$
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

8 (3 marks)

Columns: ¬P = 1, 1, 0, 0 and then ¬P ∨ Q = 1, 1, 0, 1 **B1 B1**

This matches the table for P → Q, so P → Q ≡ ¬P ∨ Q **B1**

Answer: P → Q ≡ ¬P ∨ Q **3 marks**

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

9 (4 marks)

(a) P ∨ Q = 1 and ¬R = 0, so 1 ∧ 0 = 0 **B1**

Answer: 0 **1 mark**

(b) P → Q = 0, R = 1, so 0 ∨ 1 = 1 **B1**

Answer: 1 **1 mark**

(c) P ∧ Q = 0, so ¬(P ∧ Q) = 1 **M1**

1 ↔ 1 = 1 **A1**

Answer: 1 **2 marks**

10 (4 marks)

Converse: if a shape is a rectangle then it is a square **B1**

Inverse: if a shape is not a square then it is not a rectangle **B1**

Contrapositive: if a shape is not a rectangle then it is not a square **B1**

The contrapositive is equivalent to the original **B1**

Answer: Contrapositive (if not a rectangle then not a square) is equivalent **4 marks**

11 (4 marks)

Columns P ∨ Q: 0, 1, 1, 1 and P ∧ Q: 0, 0, 0, 1 **B1**

¬(P ∧ Q): 1, 1, 1, 0 **B1**

(P ∨ Q) ∧ ¬(P ∧ Q): 0, 1, 1, 0 **B1**

This is the same as the P ⊕ Q column, so the two expressions are equivalent **E1**

Answer: Both columns are 0, 1, 1, 0, so they are equivalent **4 marks**

P	Q	$P \vee Q$	$\neg(P \wedge Q)$	$(P \vee Q) \wedge \neg(P \wedge Q)$	$P \oplus Q$
0	0	0	1	0	0
0	1	1	1	1	1
1	0	1	1	1	1
1	1	1	0	0	0

12 (3 marks)

P → Q: 1, 1, 0, 1 and Q → P: 1, 0, 1, 1 **B1 B1**

(P → Q) ∧ (Q → P): 1, 0, 0, 1 **B1**

Answer: Output column 1, 0, 0, 1 (it equals P ↔ Q) **3 marks**

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

13 (3 marks)

(a) P ∨ ¬P is always 1 **B1**

Answer: Tautology **1 mark**

(b) P ∧ ¬P is always 0 **B1**

Answer: Contradiction **1 mark**

(c) If P = 1 the output is 0; if P = 0 the output is 1 **B1**

Answer: Neither **1 mark**

14 (5 marks)

Eight rows for P, Q, R **B1**

P ∧ Q column: 0, 0, 0, 0, 0, 0, 1, 1 **B1**

(P ∧ Q) ∨ R column: 0, 1, 0, 1, 0, 1, 1, 1 **B1 A1**

P ∧ (Q ∨ R) column: 0, 0, 0, 0, 0, 1, 1, 1; the columns differ (for example when P = 0, Q = 0, R = 1), so the expressions are not equivalent **E1**

Answer: Not equivalent: columns 01010111 and 00000111 differ **5 marks**

P	Q	R	$(P \wedge Q) \vee R$	$P \wedge (Q \vee R)$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

15 (5 marks)

Eight rows for P, Q, R and Q ∧ R: 0, 0, 0, 1, 0, 0, 0, 1 **B1**

P ∨ (Q ∧ R): 0, 0, 0, 1, 1, 1, 1, 1 **B1**

P ∨ Q: 0, 0, 1, 1, 1, 1, 1, 1 and P ∨ R: 0, 1, 0, 1, 1, 1, 1, 1 **B1**

(P ∨ Q) ∧ (P ∨ R): 0, 0, 0, 1, 1, 1, 1, 1 **B1**

The final columns are identical in all 8 rows, so the expressions are equivalent **E1**

Answer: Both sides give 0, 0, 0, 1, 1, 1, 1, 1 for rows 000 to 111, so the law holds **5 marks**

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

16 (5 marks)

Construct columns for P → Q, Q → R, their conjunction and P → R **M1**

P → Q: 1, 1, 1, 1, 0, 0, 1, 1 and Q → R: 1, 1, 0, 1, 1, 1, 0, 1 **A1**

Conjunction: 1, 1, 0, 1, 0, 0, 0, 1 and P → R: 1, 1, 1, 1, 0, 1, 0, 1 **A1**

Wherever the conjunction is 1, P → R is also 1 **M1**

So the whole expression is 1 in all 8 rows and is a tautology **A1**

Answer: The output is 1 in all 8 rows, so it is a tautology **5 marks**

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$	$P \to R$	$((P \to Q) \wedge (Q \to R)) \to (P \to R)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

17 (5 marks)

L = (M ∧ D) ∨ S **M1 A1**

Eight rows for M, D, S **B1**

M ∧ D column: 0, 0, 0, 0, 0, 0, 1, 1 **M1**

L column: 0, 1, 0, 1, 0, 1, 1, 1 **A1**

Answer: (M ∧ D) ∨ S; L column 0, 1, 0, 1, 0, 1, 1, 1 **5 marks**

M	D	S	$M \wedge D$	$(M \wedge D) \vee S$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

18 (6 marks)

(a) P → Q: 1, 1, 0, 1 **B1**

¬Q: 1, 0, 1, 0 and ¬P: 1, 1, 0, 0 **B1**

¬Q → ¬P: 1, 1, 0, 1 **B1**

The columns are identical, so the statements are equivalent **E1**

Answer: P → Q and ¬Q → ¬P both give 1, 1, 0, 1 for rows 00, 01, 10, 11 **4 marks**

(b) Q → P has truth table 1, 0, 1, 1 **B1**

It differs from P → Q when P = 0, Q = 1 (output 0 against 1), so they are not equivalent **E1**

Answer: Q → P gives 1, 0, 1, 1, differing from P → Q when P = 0, Q = 1 **2 marks**

P	Q	$P \to Q$	$\neg Q \to \neg P$	$Q \to P$
0	0	1	1	1
0	1	1	1	0
1	0	0	0	1
1	1	1	1	1

19 (6 marks)

(a) P ⊕ Q: 0, 1, 1, 0 **B1**

¬(P ⊕ Q): 1, 0, 0, 1 **B1**

This is the column of P ↔ Q **E1**

Answer: ¬(P ⊕ Q) and P ↔ Q both give 1, 0, 0, 1 for rows 00, 01, 10, 11 **3 marks**

(b) Rows 00, 01, 10, 11 give P ⊕ Q = 0, 1, 1, 0 **M1**

Then (P ⊕ Q) ⊕ Q = 0 ⊕ 0, 1 ⊕ 1, 1 ⊕ 0, 0 ⊕ 1 = 0, 0, 1, 1 **M1**

This equals the column of P **A1**

Answer: (P ⊕ Q) ⊕ Q gives 0, 0, 1, 1, the same column as P **3 marks**

P	Q	$P \oplus Q$	$\neg(P \oplus Q)$	$P \leftrightarrow Q$	$(P \oplus Q) \oplus Q$
0	0	0	1	1	0
0	1	1	0	0	0
1	0	1	0	0	1
1	1	0	1	1	1

20 (4 marks)

"Only if" gives C → V **M1 A1**

An implication is false only when the first part is true and the second false **M1**

C = 1 and V = 0: a cake without eating vegetables **A1**

Answer: C → V; broken when C = 1, V = 0 **4 marks**

21 (6 marks)

Eight rows with columns P → Q, ¬Q, R → ¬Q **M1 A1**

P → Q: 1, 1, 1, 1, 0, 0, 1, 1 and R → ¬Q: 1, 1, 1, 0, 1, 1, 1, 0 **A1**

Conjunction: 1, 1, 1, 0, 0, 0, 1, 0 **M1**

P → ¬R: 1, 1, 1, 1, 1, 1, 0, 0 **A1**

Whole expression is 1 in every row, so it is a tautology **A1**

Answer: The output is 1 in all 8 rows, so it is a tautology **6 marks**

P	Q	R	$P \to Q$	$R \to \neg Q$	$(P \to Q) \wedge (R \to \neg Q)$	$P \to \neg R$	$((P \to Q) \wedge (R \to \neg Q)) \to (P \to \neg R)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	0	0	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	0	0	1
1	1	0	1	1	1	1	1
1	1	1	1	0	0	0	1

22 (6 marks)

(a) f = 1 in rows 011, 101, 110, 111 **M1**

Each row gives a conjunction, for example row 011 gives ¬P ∧ Q ∧ R **A1**

f ≡ (¬P ∧ Q ∧ R) ∨ (P ∧ ¬Q ∧ R) ∨ (P ∧ Q ∧ ¬R) ∨ (P ∧ Q ∧ R) **A1**

Answer: (¬P ∧ Q ∧ R) ∨ (P ∧ ¬Q ∧ R) ∨ (P ∧ Q ∧ ¬R) ∨ (P ∧ Q ∧ R) **3 marks**

(b) P ∧ Q: 0, 0, 0, 0, 0, 0, 1, 1; Q ∧ R: 0, 0, 0, 1, 0, 0, 0, 1; P ∧ R: 0, 0, 0, 0, 0, 1, 0, 1 **M1 A1**

Disjunction: 0, 0, 0, 1, 0, 1, 1, 1, which is the column of f **A1**

Answer: Both columns are 0, 0, 0, 1, 0, 1, 1, 1 **3 marks**

P	Q	R	$P \wedge Q$	$Q \wedge R$	$P \wedge R$	$(P \wedge Q) \vee (Q \wedge R) \vee (P \wedge R)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

23 (6 marks)

(a) Each of the 4 rows has 2 possible outputs, independently — **M1**

$2^4 = 16$ — **A1**

Answer: $2^4 = 16$ — **2 marks**

(b) 8 rows, each with 2 outputs — **M1**

2^8 — **A1**

Answer: $2^8 = 256$ — **2 marks**

(c) Choose which 3 of the 8 rows output 1 — **M1**

8!/(3! 5!) = 56 — **A1**

Answer: 56 — **2 marks**

24 (6 marks)

(a) Take P = 1, Q = 0 — **M1**

¬(P ∧ Q) = ¬0 = 1 but ¬P ∧ ¬Q = 0 ∧ 1 = 0 — **A1**

Answer: P = 1, Q = 0 (outputs 1 and 0) — **2 marks**

(b) ¬(P ∧ Q) ≡ ¬P ∨ ¬Q — **B1**

¬(P ∧ Q): 1, 1, 1, 0 — **B1**

¬P ∨ ¬Q: 1 ∨ 1, 1 ∨ 0, 0 ∨ 1, 0 ∨ 0 = 1, 1, 1, 0 — **B1**

The columns are identical, so the law is proved — **E1**

Answer: ¬P ∨ ¬Q — **4 marks**

P	Q	$\neg(P \wedge Q)$	$\neg P \wedge \neg Q$	$\neg P \vee \neg Q$
0	0	1	1	1
0	1	1	0	1
1	0	1	0	1
1	1	0	0	0

25 (6 marks)

$\overline{A}$s claim is true exactly when A is a knight, so a ↔ ¬b must hold — **M1 A1**

$\overline{B}$s claim is a ↔ b, which must be true exactly when B is a knight, so b ↔ (a ↔ b) must hold — **M1 A1**

Rows (a, b): 00: a ↔ ¬b = 0; 01: 1 then b ↔ (a ↔ b) = 1 ↔ 0 = 0; 10: 1 then 0 ↔ 0 = 1; 11: 0 — **M1**

Only a = 1, b = 0 satisfies both — **A1**

Answer: A is a knight and B is a knave — **6 marks**

a	b	$a \leftrightarrow \neg b$	$b \leftrightarrow (a \leftrightarrow b)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	1

26 (6 marks)

Eight rows for P, Q, R **B1**

P ∨ Q: 0, 0, 1, 1, 1, 1, 1, 1 **B1**

¬P ∨ R: 1, 1, 1, 1, 0, 1, 0, 1 **B1**

¬Q ∨ ¬R: 1, 1, 1, 0, 1, 1, 1, 0 **B1**

Conjunction is 1 only in rows 010 and 101 **M1**

P = 0, Q = 1, R = 0 or P = 1, Q = 0, R = 1 **A1**

Answer: (P, Q, R) = (0, 1, 0) or (1, 0, 1) **6 marks**

P	Q	R	$P \vee Q$	$\neg P \vee R$	$\neg Q \vee \neg R$	$(P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee \neg R)$
0	0	0	0	1	1	0
0	0	1	0	1	1	0
0	1	0	1	1	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	0	1	0
1	1	1	1	1	0	0

27 (8 marks)

(a) Q ↔ R: 1, 0, 0, 1, 1, 0, 0, 1 **B1**

P ↔ (Q ↔ R): 0, 1, 1, 0, 1, 0, 0, 1 **M1 A1**

(P ↔ Q) ↔ R: 0, 1, 1, 0, 1, 0, 0, 1, the same column **A1**

Answer: Both sides give 0, 1, 1, 0, 1, 0, 0, 1 for rows 000 to 111 **4 marks**

(b) Q ⊕ R: 0, 1, 1, 0, 0, 1, 1, 0 **B1**

P ⊕ (Q ⊕ R): 0, 1, 1, 0, 1, 0, 0, 1; (P ⊕ Q) ⊕ R gives the same column **M1 A1**

This is 1 exactly in the rows containing one or three 1s **A1**

Answer: Both give 0, 1, 1, 0, 1, 0, 0, 1: true for an odd number of 1s **4 marks**

P	Q	R	$Q \leftrightarrow R$	$P \leftrightarrow (Q \leftrightarrow R)$	$P \leftrightarrow Q$	$(P \leftrightarrow Q) \leftrightarrow R$
0	0	0	1	0	1	0
0	0	1	0	1	1	1
0	1	0	0	1	0	1
0	1	1	1	0	0	0
1	0	0	1	1	0	1
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

P	Q	R	$Q \oplus R$	$P \oplus (Q \oplus R)$	$P \oplus Q$	$(P \oplus Q) \oplus R$
0	0	0	0	0	0	0
0	0	1	1	1	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	0
1	0	0	0	1	1	1
1	0	1	1	0	1	0
1	1	0	1	0	0	0
1	1	1	0	1	0	1

28 (8 marks)

(a) Base case: a single variable P_i has value 1 when all variables are 1 **B1**

Suppose expressions X and Y both have value 1 when all variables are 1 **M1**

Then X ∧ Y = 1 ∧ 1 = 1 **A1**

and X ∨ Y = 1 ∨ 1 = 1, so every expression built this way has value 1 **E1**

Answer: Any ∧, ∨ expression is 1 when all variables are 1 **4 marks**

(b) Any such expression in P is 1 when P = 1, but ¬P = 0 when P = 1 **M1**

So no such expression is equivalent to ¬P **E1**

Answer: Such an expression is 1 when P = 1 but ¬P = 0, so none equals ¬P **2 marks**

(c) Such an expression in P and Q has value 1 when P = Q = 1 **M1**

but P ⊕ Q = 0 when P = Q = 1, so they cannot be equivalent **A1**

Answer: Such an expression is 1 when P = Q = 1 but P ⊕ Q = 0 **2 marks**

29 (8 marks)

There are four possible culprits, so four rows **B1**

Ann guilty: statements 0, 1, 0, 1, two are true **M1 A1**

Ben guilty: statements 1, 0, 1, 1, three are true **A1**

Cal guilty: statements 0, 0, 1, 0, exactly one is true **M1 A1**

Dee guilty: statements 0, 0, 1, 1, two are true **A1**

Only Cal gives exactly one true statement, so Cal committed the offence **A1**

Answer: Cal **8 marks**

Culprit	Ann	Ben	Cal	Dee	Total true
Ann	0	1	0	1	2
Ben	1	0	1	1	3
Cal	0	0	1	0	1
Dee	0	0	1	1	2

30 (9 marks)

(a) P ↑ P = ¬(P ∧ P) **M1**

P ∧ P ≡ P, so P ↑ P ≡ ¬P **A1**

Answer: P ↑ P gives 1, 0 for P = 0, 1, so P ↑ P ≡ ¬P **2 marks**

(b) P ↑ Q = ¬(P ∧ Q) and the outer operation applies ↑ to this twice **M1**

Using part (a), X ↑ X ≡ ¬X with X = ¬(P ∧ Q) **M1**

¬¬(P ∧ Q) ≡ P ∧ Q **A1**

Answer: (P ↑ Q) ↑ (P ↑ Q) gives 0, 0, 0, 1, the column of P ∧ Q **3 marks**

(c) De Morgan: P ∨ Q ≡ ¬(¬P ∧ ¬Q) **M1**

¬P ≡ P ↑ P and ¬Q ≡ Q ↑ Q **M1**

P ∨ Q ≡ (P ↑ P) ↑ (Q ↑ Q) **A1**

Table: P ↑ P: 1, 1, 0, 0 and Q ↑ Q: 1, 0, 1, 0, whose NAND is 0, 1, 1, 1 = P ∨ Q **B1**

Answer: (P ↑ P) ↑ (Q ↑ Q) **4 marks**

P	Q	$P \uparrow P$	$P \uparrow Q$	$(P \uparrow Q) \uparrow (P \uparrow Q)$	$(P \uparrow P) \uparrow (Q \uparrow Q)$
0	0	1	1	0	0
0	1	1	1	0	1
1	0	0	1	0	1
1	1	0	0	1	1

Boolean Algebra and Simplification

1 (3 marks)

(a) Identity law: A·1 = A **B1**

Answer: A **1 mark**

(b) Complement law: $A + \overline{A} = 1$ **B1**

Answer: 1 — 1 mark
(c) Null law: $A \cdot 0 = 0$ — B1
Answer: 0 — 1 mark

2 (3 marks)
(a) Order does not matter — B1
Answer: Commutative law — 1 mark
(b) Multiplication distributes over addition — B1
Answer: Distributive law — 1 mark
(c) Brackets can be regrouped — B1
Answer: Associative law — 1 mark

3 (3 marks)
(a) Idempotent law — B1
Answer: A — 1 mark
(b) Idempotent law — B1
Answer: A — 1 mark
(c) Double complement (involution) law — B1
Answer: A — 1 mark

4 (2 marks)
(a) De Morgan: the complement of a sum is the product of the complements — B1
Answer: $\overline{A} \cdot \overline{B}$ — 1 mark
(b) De Morgan: the complement of a product is the sum of the complements — B1
Answer: $\overline{A} + \overline{B}$ — 1 mark

5 (2 marks)
(a) Distributive law — B1
Answer: $A \cdot \overline{B} + A \cdot C$ — 1 mark
(b) Take out the common factor B — B1
Answer: $B \cdot (C + \overline{D})$ — 1 mark

6 (2 marks)
(a) Swap + with · and 0 with 1 — B1
Answer: $A \cdot 1 = A$ — 1 mark
(b) Swap + and · — B1
Answer: $A \cdot (B + C) = A \cdot B + A \cdot C$ — 1 mark

7 (3 marks)
$A + A \cdot B = A \cdot 1 + A \cdot B$ (identity) — M1
$= A \cdot (1 + B)$ (distributive) — M1
$= A \cdot 1 = A$ (null, identity) — A1
Answer: $A + A \cdot B = A$ — 3 marks

8 (3 marks)
$A \cdot B + A \cdot \overline{B} = A \cdot (B + \overline{B})$ (distributive) — M1
$= A \cdot 1$ (complement) — A1
$= A$ (identity) — A1
Answer: A — 3 marks

9 (3 marks)
Distribute: $A + \overline{A} \cdot B = (A + \overline{A}) \cdot (A + B)$ (distributive law for + over ·) — M1
$= 1 \cdot (A + B)$ (complement) — A1
$= A + B$ (identity) — A1
Answer: $A + \overline{A} \cdot B = A + B$ — 3 marks

10 (3 marks)

$A \cdot (A + B) = A$ (absorption) **M1**

Then $A \cdot (\overline{A} + C) = A \cdot \overline{A} + A \cdot C$ (distributive) **M1**

$= 0 + A \cdot C = A \cdot C$ (complement, identity) **A1**

Answer: $A \cdot C$ **3 marks**

11 (3 marks)

Complement of a sum: $\overline{A \cdot \overline{B} \cdot \overline{C}}$ (De Morgan) **M1**

Complement of the product: $\overline{A \cdot \overline{B}} = \overline{A} + \overline{\overline{B}} = \overline{A} + B$ (De Morgan, double complement) **M1**

Result: $(\overline{A} + B) \cdot \overline{C}$ **A1**

Answer: $(\overline{A} + B) \cdot \overline{C}$ **3 marks**

12 (4 marks)

$A \cdot B \cdot C + A \cdot B \cdot \overline{C} + A \cdot \overline{B} = A \cdot B \cdot (C + \overline{C}) + A \cdot \overline{B}$ (distributive) **M1**

$= A \cdot B + A \cdot \overline{B}$ (complement, identity) **A1**

$= A \cdot (B + \overline{B})$ (distributive) **M1**

$= A$ (complement, identity) **A1**

Answer: A **4 marks**

13 (3 marks)

$(A + \overline{B}) \cdot (\overline{A} + \overline{B}) = A \cdot \overline{A} + \overline{B}$ (distributive, taking out the common term $\overline{B}$) **M1**

$= 0 + \overline{B}$ (complement) **A1**

$= \overline{B}$ **A1**

Answer: $\overline{B}$ **3 marks**

14 (5 marks)

$F = \overline{A} \cdot B \cdot C + A \cdot B \cdot C + A \cdot \overline{B} \cdot C$

$= B \cdot C \cdot (\overline{A} + A) + A \cdot \overline{B} \cdot C$ (distributive on the first two terms) **M1**

$= B \cdot C + A \cdot \overline{B} \cdot C$ (complement and identity) **A1**

$= C \cdot (B + A \cdot \overline{B})$ (factorise C) **M1**

$= C \cdot (B + A)$ ($X + \overline{X} \cdot Y = X + Y$ with $X = B$, $Y = A$) **M1 A1**

Answer: $C \cdot (A + B)$ **5 marks**

15 (5 marks)

$(A + B) \cdot (\overline{A} + C) = A \cdot \overline{A} + A \cdot C + \overline{A} \cdot B + B \cdot C$ (expand) **M1**

$= 0 + A \cdot C + \overline{A} \cdot B + B \cdot C$ (complement) **A1**

Write $B \cdot C = B \cdot C \cdot (A + \overline{A}) = A \cdot B \cdot C + \overline{A} \cdot B \cdot C$ **M1**

Then $A \cdot C + A \cdot B \cdot C = A \cdot C$ and $\overline{A} \cdot B + \overline{A} \cdot B \cdot C = \overline{A} \cdot B$ (absorption) **M1**

So the expression reduces to $A \cdot C + \overline{A} \cdot B$ **A1**

Answer: $(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot B$ **5 marks**

16 (5 marks)

$A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C + B \cdot C \cdot (A + \overline{A})$ (complement, identity) **M1**

$= A \cdot B + \overline{A} \cdot C + A \cdot B \cdot C + \overline{A} \cdot B \cdot C$ (distributive) **M1**

$= A \cdot B + A \cdot B \cdot C + \overline{A} \cdot C + \overline{A} \cdot B \cdot C$ (commutative) **A1**

$A \cdot B + A \cdot B \cdot C = A \cdot B \cdot (1 + C) = A \cdot B$ **M1**

Similarly $\overline{A} \cdot C + \overline{A} \cdot B \cdot C = \overline{A} \cdot C$, giving $A \cdot B + \overline{A} \cdot C$ **A1**

Answer: $A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$ **5 marks**

17 (5 marks)

(a) $A \oplus A = A \cdot \overline{A} + \overline{A} \cdot A$ **M1**

$= 0 + 0 = 0$ (complement, idempotent) **A1**

Answer: $A \oplus A = 0$ **2 marks**

(b) $A \oplus 1 = A\cdot\overline{1} + \overline{A}\cdot 1 = A\cdot 0 + \overline{A}\cdot 1$ (complement of 1) **M1**

$= 0 + \overline{A} = \overline{A}$ (null, identity) **A1**

Answer: $A \oplus 1 = \overline{A}$ **2 marks**

(c) $A \oplus 0 = A\cdot\overline{0} + \overline{A}\cdot 0 = A\cdot 1 + 0 = A$ **B1**

Answer: $A \oplus 0 = A$ **1 mark**

18 (4 marks)

$\overline{A + B}\cdot(\overline{A} + \overline{B}) = \overline{A}\cdot\overline{B}\cdot(\overline{A} + \overline{B})$ (De Morgan) **M1**

$= \overline{A}\cdot\overline{B}\cdot\overline{A} + \overline{A}\cdot\overline{B}\cdot\overline{B}$ (distributive) **M1**

$= \overline{A}\cdot\overline{B} + \overline{A}\cdot\overline{B}$ (idempotent) **A1**

$= \overline{A}\cdot\overline{B}$ (idempotent) **A1**

Answer: $\overline{A}\cdot\overline{B}$ **4 marks**

19 (4 marks)

Take two factors: $(A + B)\cdot(A + C) = A + B\cdot C$ (distributive) **M1**

Then $(A + B\cdot C)\cdot(A + D) = A + B\cdot C\cdot D$ (the same law again, with BC in place of B and D in place of C) **M1 A1**

Hence $(A + B)\cdot(A + C)\cdot(A + D) = A + B\cdot C\cdot D$ **A1**

Answer: $(A + B)\cdot(A + C)\cdot(A + D) = A + B\cdot C\cdot D$ **4 marks**

20 (5 marks)

$\overline{F} = \overline{A\cdot B}\cdot\overline{\overline{A}\cdot C}$ (De Morgan) **M1**

$= (\overline{A} + \overline{B})\cdot(A + \overline{C})$ (De Morgan, double complement) **A1**

$= \overline{A}\cdot A + \overline{A}\cdot\overline{C} + \overline{B}\cdot A + \overline{B}\cdot\overline{C}$ (expand) **M1**

$= \overline{A}\cdot\overline{C} + A\cdot\overline{B} + \overline{B}\cdot\overline{C}$ (complement, identity) **A1**

The consensus term $\overline{B}\cdot\overline{C}$ is redundant, so $\overline{F} = \overline{A}\cdot\overline{C} + A\cdot\overline{B}$ **A1**

Answer: $\overline{A}\cdot\overline{C} + A\cdot\overline{B}$ **5 marks**

21 (6 marks)

$F = \overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + A\cdot\overline{B}\cdot\overline{C} + A\cdot\overline{B}\cdot C + A\cdot B\cdot C$

Group the four terms containing $\overline{B}$: $\overline{B}\cdot(\overline{A}\cdot\overline{C} + \overline{A}\cdot C + A\cdot\overline{C} + A\cdot C)$ **M1**

$= \overline{B}\cdot(\overline{A}\cdot(\overline{C} + C) + A\cdot(\overline{C} + C)) + A\cdot B\cdot C$ (factorise) **M1 A1**

$= \overline{B}\cdot(\overline{A} + A) + A\cdot B\cdot C$ (complement, identity) **M1**

$= \overline{B} + A\cdot B\cdot C$ (complement, identity) **A1**

$= \overline{B} + A\cdot C$ ($X + \overline{X}\cdot Y = X + Y$ with $X = \overline{B}$ and $Y = A\cdot C$) **A1**

Answer: $\overline{B} + A\cdot C$ **6 marks**

22 (6 marks)

LHS $= A\cdot(B\cdot\overline{C} + \overline{B}\cdot C) = A\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot C$ (distributive) **M1 A1**

RHS $= A\cdot B\cdot\overline{A\cdot C} + \overline{A\cdot B}\cdot A\cdot C$ (definition of $\oplus$) **M1**

$= A\cdot B\cdot(\overline{A} + \overline{C}) + (\overline{A} + \overline{B})\cdot A\cdot C$ (De Morgan) **M1**

$= A\cdot B\cdot\overline{C} + A\cdot B\cdot\overline{A} + A\cdot C\cdot\overline{A} + A\cdot\overline{B}\cdot C$ (expand; $A\cdot\overline{A} = 0$) **M1**

$= A\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot C =$ LHS **A1**

Answer: $A\cdot(B \oplus C) = A\cdot B \oplus A\cdot C$ **6 marks**

23 (6 marks)

Write $X \rightarrow Y$ as $\overline{X} + Y$. The expression is $F = \overline{(\overline{A} + B)\cdot(\overline{B} + C)} + \overline{A} + C$ **B1**

$= \overline{\overline{A} + B} + \overline{\overline{B} + C} + \overline{A} + C$ (De Morgan) **M1**

$= A\cdot\overline{B} + B\cdot\overline{C} + \overline{A} + C$ (De Morgan again) **A1**

Now $\overline{A} + A\cdot\overline{B} = \overline{A} + \overline{B}$ and $C + B\cdot\overline{C} = C + B$ ($A + \overline{A}\cdot X = A + X$) **M1**

$F = \overline{A} + \overline{B} + C + B$ **A1**

$= \overline{A} + C + B + \overline{B} = 1$ (complement, null) **A1**

Answer: The expression simplifies to 1, so the statement is always true **6 marks**

24 (6 marks)

(a) Take $A = 0, B = 0, C = 1$: $A\cdot B = 0 = A\cdot C$ but $B \neq C$ **B1**

Take $A = 1, B = 0, C = 1$: $A + B = 1 = A + C$ but $B \neq C$ **B1**

Answer: A = 0, B = 0, C = 1 and A = 1, B = 0, C = 1 **2 marks**

(b) $B = B\cdot(A + B)$ (absorption) **M1**

$= B\cdot(A + C)$ (using $A + B = A + C$) **M1**

$= A\cdot B + B\cdot C = A\cdot C + B\cdot C$ (using $A\cdot B = A\cdot C$) **M1**

$= C\cdot(A + B) = C\cdot(A + C) = C$ (using $A + B = A + C$, absorption) **A1**

Answer: B = C **4 marks**

25 (6 marks)

(a) Add A to both sides with $\oplus$: $A \oplus (A \oplus X) = A \oplus 1$ **M1**

$(A \oplus A) \oplus X = 0 \oplus X = X$, and $A \oplus 1 = \overline{A}$, so $X = \overline{A}$ **A1**

Answer: $\overline{A}$ **2 marks**

(b) As in (a), $X = A \oplus A\cdot B$ **M1**

$= A\cdot\overline{A\cdot B} + \overline{A}\cdot A\cdot B$ (definition $X \oplus Y = X\cdot\overline{Y} + \overline{X}\cdot Y$) **M1**

$= A\cdot(\overline{A} + \overline{B}) + 0 = A\cdot\overline{B}$ (De Morgan, complement) **M1 A1**

Answer: $A\cdot\overline{B}$ **4 marks**

26 (6 marks)

(a) $A = 1$: $1\cdot B + \overline{B}\cdot C + 0\cdot\overline{C} = B + \overline{B}\cdot C$ **M1**

$= B + C$ ($X + \overline{X}\cdot Y = X + Y$) **A1**

$A = 0$: $\overline{B}\cdot C + \overline{C} = \overline{B} + \overline{C}$ ($\overline{X}\cdot Y + \overline{Y} = \overline{X} + \overline{Y}$) **A1**

Answer: B + C and $\overline{B} + \overline{C}$ **3 marks**

(b) Shannon expansion: $F = A\cdot F(1) + \overline{A}\cdot F(0)$ **M1**

$A\cdot F(1) + \overline{A}\cdot F(0) = A\cdot(B + C) + \overline{A}\cdot(\overline{B} + \overline{C})$ **A1**

Check: for $A = 1$ the right side is $B + C$ and for $A = 0$ it is $\overline{B} + \overline{C}$, matching part (a) **A1**

Answer: $F = A\cdot(B + C) + \overline{A}\cdot(\overline{B} + \overline{C})$ **3 marks**

27 (8 marks)

(a) $A + A = (A + A)\cdot 1 = (A + A)\cdot(A + \overline{A})$ (identity, complement) **M1**

$= A + A\cdot\overline{A} = A + 0 = A$ (distributive, complement, identity) **A1**

Answer: A + A = A **2 marks**

(b) $A + 1 = (A + 1)\cdot 1$ (identity) **M1**

$= (A + 1)\cdot(A + \overline{A}) = A + 1\cdot\overline{A}$ (complement, distributive) **M1**

$= A + \overline{A} = 1$ (identity, complement) **A1**

Answer: A + 1 = 1 **3 marks**

(c) $A + A\cdot B = A\cdot 1 + A\cdot B$ (identity) **M1**

$= A\cdot(1 + B)$ (distributive) **M1**

$= A\cdot 1 = A$ (part (b), identity) **A1**

Answer: A + A·B = A **3 marks**

28 (8 marks)

(a) $X = X \cdot 1 = X \cdot (A + \overline{A})$ (identity, complement) **M1**

$= X \cdot A + X \cdot \overline{A} = 0 + X \cdot \overline{A}$ (distributive, using $A \cdot X = 0$) **M1**

$= A \cdot \overline{A} + X \cdot \overline{A} = (A + X) \cdot \overline{A}$ (complement, distributive) **M1**

$= 1 \cdot \overline{A} = \overline{A}$ **A1**

Answer: $\overline{A}$ is the only X with A + X = 1 and A·X = 0 **4 marks**

(b) $(A + B) + \overline{A} \cdot \overline{B} = (A + B + \overline{A}) \cdot (A + B + \overline{B})$ (distributive) **M1**

$= (B + 1) \cdot (A + 1) = 1 \cdot 1 = 1$ **A1**

$(A + B) \cdot \overline{A} \cdot \overline{B} = A \cdot \overline{A} \cdot \overline{B} + B \cdot \overline{A} \cdot \overline{B} = 0 + 0 = 0$ **M1**

By part (a), $\overline{A} \cdot \overline{B} = \overline{A + B}$ **A1**

Answer: $\overline{A} \cdot \overline{B}$ is the complement of A + B, so $\overline{A + B} = \overline{A} \cdot \overline{B}$ **4 marks**

29 (8 marks)

(a) $(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot B + B \cdot C$ (expand, $A \cdot \overline{A} = 0$) **M1 A1**

Multiply by (B + C): $(A \cdot C + \overline{A} \cdot B + B \cdot C) \cdot (B + C)$ expands to $A \cdot B \cdot C + A \cdot C + \overline{A} \cdot B + \overline{A} \cdot B \cdot C + B \cdot C$ **M1**

$A \cdot B \cdot C$ is absorbed by $A \cdot C$, $\overline{A} \cdot B \cdot C$ is absorbed by $\overline{A} \cdot B$, and $B \cdot C$ is the consensus term and is redundant, leaving $A \cdot C + \overline{A} \cdot B = (A + B) \cdot (\overline{A} + C)$ **A1**

Answer: $(A + B) \cdot (\overline{A} + C) \cdot (B + C) = (A + B) \cdot (\overline{A} + C)$ **4 marks**

(b) Let $A = \overline{P}$, $B = Q$ and $C = R$. Then $(\overline{P} + Q) \cdot (P + R) \cdot (Q + R)$ has the form $(A + B)(\overline{A} + C)(B + C)$ **M1**

By part (a) the factor Q + R is redundant: $(\overline{P} + Q) \cdot (P + R) \cdot (Q + R) = (\overline{P} + Q) \cdot (P + R)$ **A1**

The remaining factor $\overline{Q} + S$ is unchanged **M1**

Result: $(\overline{P} + Q) \cdot (P + R) \cdot (\overline{Q} + S)$ **A1**

Answer: $(\overline{P} + Q) \cdot (P + R) \cdot (\overline{Q} + S)$ **4 marks**

30 (9 marks)

(a) Let $X = A \oplus B = A \cdot \overline{B} + \overline{A} \cdot B$. Then $A \oplus B \oplus A \cdot B = X \oplus A \cdot B = X \cdot \overline{A \cdot B} + \overline{X} \cdot A \cdot B$ **M1**

$X \cdot A \cdot B = A \cdot \overline{B} \cdot A \cdot B + \overline{A} \cdot B \cdot A \cdot B = 0$, so $X \cdot \overline{A \cdot B} = X$ **M1 A1**

Also $\overline{X} = A \cdot B + \overline{A} \cdot \overline{B}$, so $\overline{X} \cdot A \cdot B = A \cdot B$ **M1**

Hence $A \oplus B \oplus A \cdot B = X + A \cdot B = A \cdot \overline{B} + \overline{A} \cdot B + A \cdot B = A + B$ **A1**

Answer: $A + B = A \oplus B \oplus A \cdot B$ **5 marks**

(b) $A + B + C = (A + B) + C = (A + B) \oplus C \oplus (A + B) \cdot C$ (part (a)) **M1**

$= (A \oplus B \oplus A \cdot B) \oplus C \oplus (A \cdot C + B \cdot C)$ **M1**

$A \cdot C + B \cdot C = A \cdot C \oplus B \cdot C \oplus A \cdot B \cdot C$ (part (a)) **M1**

$A + B + C = A \oplus B \oplus C \oplus A \cdot B \oplus A \cdot C \oplus B \cdot C \oplus A \cdot B \cdot C$ **A1**

Answer: $A \oplus B \oplus C \oplus A \cdot B \oplus A \cdot C \oplus B \cdot C \oplus A \cdot B \cdot C$ **4 marks**

Logic Gates and Circuits

1 (3 marks)

(a) AND and (b) OR **B1**

(c) NOT and (d) NAND **B1**

(e) NOR and (f) XOR (exclusive-or) **B1**

Answer: AND, OR, NOT, NAND, NOR, XOR **3 marks**

2 (2 marks)

B passes through a NOT gate, giving $\overline{B}$ **B1**

The AND gate gives $Q = A \cdot \overline{B}$ **B1**

Answer: $A \cdot \overline{B}$ **2 marks**

3 (2 marks)

Output = $\overline{A + B}$ **B1**

This is the NOR gate **B1**

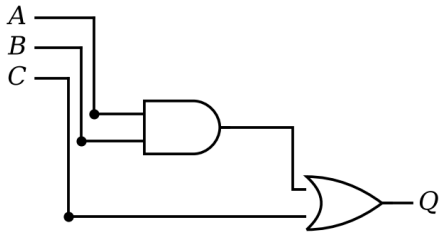
Answer: $\overline{A + B}$; NOR gate **2 marks**

4 (2 marks)

One AND gate with inputs A and B **B1**

Its output and C feed an OR gate, whose output is Q **B1**

Answer: AND gate then OR gate **2 marks**



5 (2 marks)

(a) $\overline{B} = 1$, so $Q = 1 \cdot 1 = 1$ **B1**

Answer: Q = 1 **1 mark**

(b) $\overline{B} = 0$, so $Q = 1 \cdot 0 = 0$ **B1**

Answer: Q = 0 **1 mark**

6 (2 marks)

(a) Switches in series: both must be closed **B1**

Answer: A·B **1 mark**

(b) Switches in parallel: at least one must be closed **B1**

Answer: A + B **1 mark**

7 (3 marks)

The OR gate output is A + B **B1**

C passes through a NOT gate to give $\overline{C}$ **B1**

$Q = (A + B) \cdot \overline{C}$ **B1**

Answer: $(A + B) \cdot \overline{C}$ **3 marks**

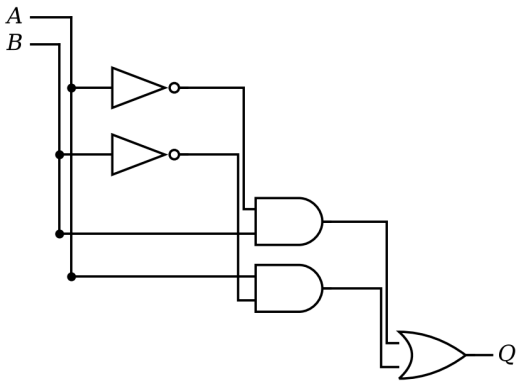
8 (3 marks)

NOT gates on A and on B **B1**

Two AND gates, one combining $\overline{A}$ with B, the other A with $\overline{B}$ **B1**

An OR gate combining the two AND outputs **B1**

Answer: AND($\overline{A}$, B) and AND(A, $\overline{B}$) feed an OR gate **3 marks**



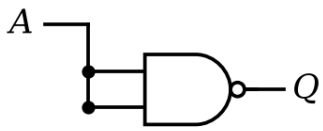
9 (3 marks)

Join both inputs of the NAND gate to the input A — **B1**

Output = $\overline{A \cdot A} = \overline{A} \cdot \overline{A} = \overline{A}$ — **M1**

Table: A = 0 gives 1 and A = 1 gives 0, which is the NOT gate — **A1**

Answer: A ↑ A = $\overline{A}$, so joining both inputs gives a NOT gate — **3 marks**



10 (5 marks)

(a) Sum: 0, 1, 1, 0 — **B1**

Carry: 0, 0, 0, 1 — **B1**

Answer: S: 0, 1, 1, 0 and C: 0, 0, 0, 1 — **2 marks**

(b) S = A ⊕ B and C = A·B — **B1**

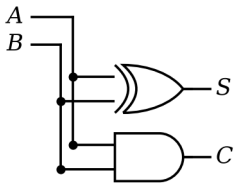
Answer: S = A ⊕ B, C = A·B — **1 mark**

(c) XOR gate on A and B gives S — **B1**

AND gate on A and B gives C — **B1**

Answer: XOR gate on A, B gives S; AND gate on A, B gives C — **2 marks**

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



11 (4 marks)

First NOR output $\overline{A + B}$ = 1, 0, 0, 0 for AB = 00, 01, 10, 11 (C ignored) — **M1**

Second NOR: $\overline{X + C}$ where X is the first output — **M1**

Q = $(A + B) \cdot \overline{C}$ — **A1**

Q = 1 only for ABC = 010, 100, 110 — **A1**

Answer: $(A + B) \cdot \overline{C}$; Q = 1 for ABC = 010, 100, 110 — **4 marks**

A	B	C	$\overline{A+B}$	$\overline{\overline{A+B}+C}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	1
1	1	1	0	0

12 (3 marks)

Parallel pair A, B in series with C — **M1**

$L = (A + B)\cdot C$ — **A1**

Lamp on for ABC = 101, 011, 111 — **A1**

Answer: (A + B)·C; ABC = 101, 011, 111 — **3 marks**

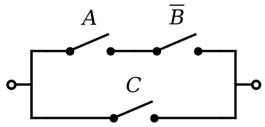
13 (3 marks)

Product $A\cdot\overline{B}$ means switches A and $\overline{B}$ in series — **B1**

Sum with C means a parallel branch containing C — **B1**

Complemented switch $\overline{B}$ is a normally closed switch operated by B — **B1**

Answer: A in series with normally closed B, in parallel with C — **3 marks**



14 (5 marks)

(a) Let $X = \overline{A\cdot B}$ — **M1**

$Q = (A \uparrow X) \uparrow (B \uparrow X)$, that is $Q = \overline{\overline{A\cdot X}\cdot\overline{B\cdot X}}$ — **A1**

Answer: Q = (A ↑ X) ↑ (B ↑ X) with X = A ↑ B — **2 marks**

(b) $X = \overline{A} + \overline{B}$; $A \uparrow X = \overline{A\cdot(\overline{A} + \overline{B})} = \overline{A\cdot\overline{B}} = \overline{A} + B$ — **M1**

$B \uparrow X = \overline{B\cdot(\overline{A} + \overline{B})} = \overline{\overline{A}\cdot B} = A + \overline{B}$ — **M1**

$Q = \overline{(\overline{A} + B)\cdot(A + \overline{B})} = A\cdot\overline{B} + \overline{A}\cdot B = A \oplus B$ — **A1**

Answer: Q = A ⊕ B — **3 marks**

15 (6 marks)

(a) Number of 1s among A, B, C is 0, 1, 1, 2, 1, 2, 2, 3 for the rows in order — **M1**

S is the parity: 0, 1, 1, 0, 1, 0, 0, 1 — **A1**

K is 1 when at least two inputs are 1: 0, 0, 0, 1, 0, 1, 1, 1 — **A1**

Answer: S: 0, 1, 1, 0, 1, 0, 0, 1; K: 0, 0, 0, 1, 0, 1, 1, 1 — **3 marks**

(b) S is 1 when an odd number of inputs are 1, which is A ⊕ B ⊕ C — **B1**

If A and B are both 1 then K = 1 whatever C is: term A·B — **M1**

If exactly one of A, B is 1 then K = C: term C·(A ⊕ B); if neither is 1, K = 0 — **A1**

Answer: S = A ⊕ B ⊕ C and K = A·B + C·(A ⊕ B) — **3 marks**

A	B	C	S	K
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

16 (5 marks)

(a) S = (A ⊕ B) ⊕ C — **B1**

K = A·B + (A ⊕ B)·C — **B1**

Answer: S = (A ⊕ B) ⊕ C, K = A·B + (A ⊕ B)·C — **2 marks**

(b) Check A = 1, B = 1, C = 0: S = 0 ⊕ 0 = 0, K = 1 (1 + 1 + 0 = 10 in binary) — **M1**

Check A = 1, B = 1, C = 1: S = 0 ⊕ 1 = 1, K = 1 (1 + 1 + 1 = 11 in binary) — **M1**

In general S is the parity of A, B, C and K is 1 when at least two are 1, as in the full adder table — **A1**

Answer: S is the parity of A, B, C; K = 1 when two or more inputs are 1 — **3 marks**

17 (5 marks)

(a) $A \downarrow A = \overline{A + A}$ **M1**

$= \overline{A}$ by the idempotent law **A1**

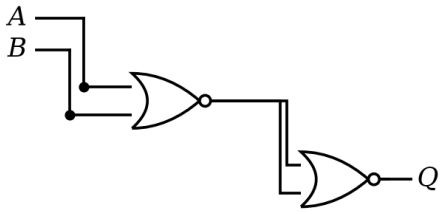
Answer: $A \downarrow A = \overline{A}$ **2 marks**

(b) $A + B = \overline{\overline{A + B}}$ **M1**

$A \vee B = (A \downarrow B) \downarrow (A \downarrow B)$, using part (a) for the final NOT **M1**

Circuit: one NOR gate for A, B followed by a NOR gate with both inputs joined to its output **A1**

Answer: $(A \downarrow B) \downarrow (A \downarrow B)$ **3 marks**



18 (5 marks)

$Q = A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B$ **B1**

$A \cdot B + A \cdot \overline{B} = A$, so $Q = A + \overline{A} \cdot B$ **M1 A1**

$= A + B$ $(A + \overline{A} \cdot B = A + B)$ **A1**

The original circuit has 6 gates (2 NOT, 3 AND, 1 OR); the simplified circuit has 1 OR gate, saving 5 gates **B1**

Answer: A + B; 5 gates saved **5 marks**

19 (5 marks)

(a) $Q = (A + B) \cdot (A + C)$ **B1**

$= A \cdot A + A \cdot C + B \cdot A + B \cdot C = A + A \cdot C + A \cdot B + B \cdot C$ **M1**

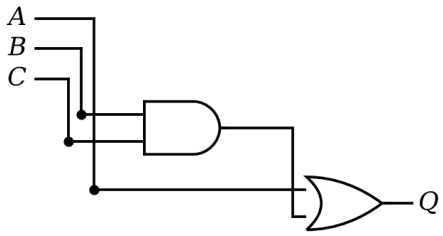
$= A + B \cdot C$ (absorption) **A1**

Answer: $A + B \cdot C$ **3 marks**

(b) An AND gate on B and C **B1**

Its output and A feed an OR gate **B1**

Answer: AND on B, C then OR with A **2 marks**



20 (5 marks)

(a) L changes when either switch changes: 0, 1, 1, 0 **B1**

Answer: L: 0, 1, 1, 0 for AB = 00, 01, 10, 11 **1 mark**

(b) L = 1 for AB = 01 and 10 **M1**

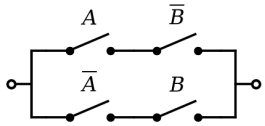
$L = A \cdot \overline{B} + \overline{A} \cdot B$ **A1**

Answer: $A \cdot \overline{B} + \overline{A} \cdot B$ **2 marks**

(c) Two parallel branches **B1**

Branch 1: A in series with $\overline{B}$ (normally closed); branch 2: $\overline{A}$ in series with B **B1**

Answer: A with normally closed B, in parallel with normally closed A with B **2 marks**



21 (6 marks)

(a) Two binary numbers are equal exactly when their digits in each position agree — **E1**

Answer: E = 1 exactly when A = C and B = D — **1 mark**

(b) A ⊕ C = 0 exactly when A = C, and B ⊕ D = 0 exactly when B = D — **M1**

E = 1 when both are 0 — **M1**

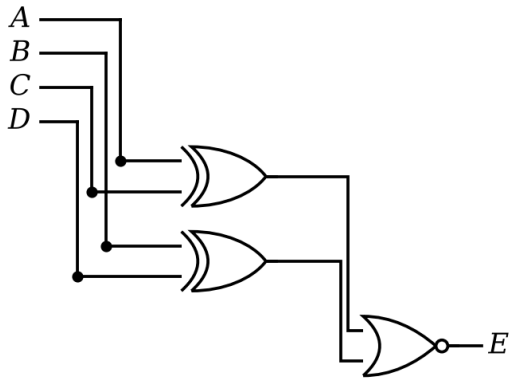
NOR of two signals is 1 exactly when both are 0, so E = (A ⊕ C) ↓ (B ⊕ D) — **A1**

Answer: E = (A ⊕ C) ↓ (B ⊕ D) — **3 marks**

(c) Two XOR gates, on (A, C) and (B, D) — **B1**

Both outputs feed one NOR gate giving E — **B1**

Answer: XOR gates on (A, C) and (B, D), both outputs into a NOR gate — **2 marks**



22 (7 marks)

(a) When S = 0: $\overline{S}\cdot A + S\cdot B = 1\cdot A + 0\cdot B = A$ — **M1**

When S = 1: $\overline{S}\cdot A + S\cdot B = 0\cdot A + 1\cdot B = B$ — **A1**

Answer: Q = $\overline{S}$·A + S·B — **2 marks**

(b) X ↑ Y = $\overline{X} + \overline{Y}$ — **B1**

$\overline{S}$ ↑ A = S + $\overline{A}$ and S ↑ B = $\overline{S} + \overline{B}$ — **M1**

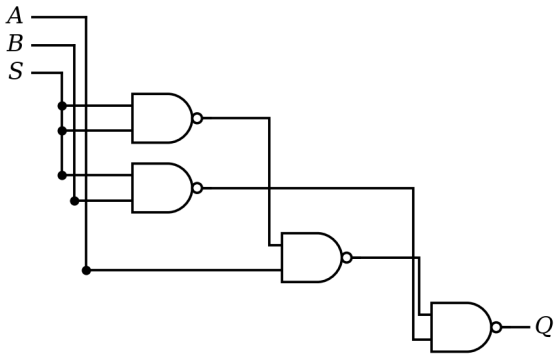
$(S + \overline{A}) \uparrow (\overline{S} + \overline{B}) = \overline{(S + \overline{A})\cdot(\overline{S} + \overline{B})} = \overline{S}\cdot A + S\cdot B$ — **A1**

Answer: Q = ($\overline{S}$ ↑ A) ↑ (S ↑ B) — **3 marks**

(c) NAND with both inputs S gives $\overline{S}$ — **B1**

Two NAND gates for $\overline{S}$ with A and S with B, and a final NAND — **B1**

Answer: NAND(S, S) = $\overline{S}$; NAND($\overline{S}$, A), NAND(S, B), final NAND — **2 marks**



23 (6 marks)

(a) $F = \overline{\overline{A \cdot \overline{B} + \overline{A} \cdot C}}$ (double complement) **M1**

$= \overline{\overline{A \cdot \overline{B}} \cdot \overline{\overline{A} \cdot C}}$ (De Morgan on the inner sum) **M1**

The inner terms are $\overline{A \cdot \overline{B}} = A \uparrow \overline{B}$ and $\overline{\overline{A} \cdot C} = \overline{A} \uparrow C$, so $F = (A \uparrow \overline{B}) \uparrow (\overline{A} \uparrow C)$ **A1**

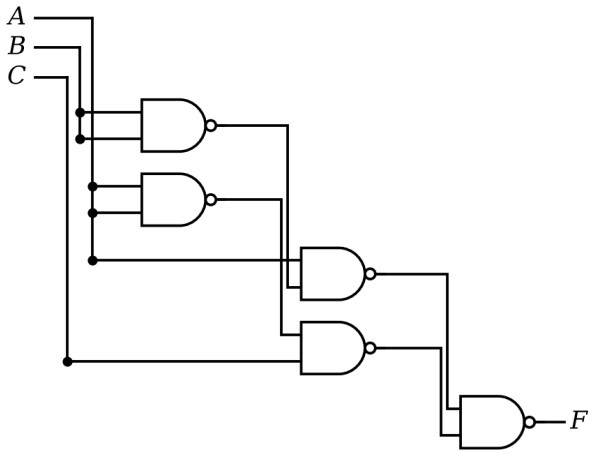
Answer: $F = (A \uparrow \overline{B}) \uparrow (\overline{A} \uparrow C)$ **3 marks**

(b) $\overline{B} = B \uparrow B$ and $\overline{A} = A \uparrow A$, two gates **B1**

Gates $A \uparrow \overline{B}$, $\overline{A} \uparrow C$ and the final gate, three more gates **B1**

Total 5 NAND gates **B1**

Answer: 5 NAND gates (two joined as NOT gates and three for the main circuit) **3 marks**



24 (6 marks)

Stage 1: digits 0, 1 and carry in 0: S = 1, carry out 0 **M1 A1**

Stage 2: digits 1, 1 and carry in 0: S = 0, carry out 1 **M1 A1**

Stage 3: digits 1, 1 and carry in 1: S = 1, carry out 1; Stage 4: digits 0, 0 and carry in 1: S = 1, carry out 0 **M1**

Sum = 1101, final carry out 0; check: 6 + 7 = 13 = 1101 in binary **A1**

Answer: Sum 1101, final carry 0 (carries out of stages 1 to 4: 0, 1, 1, 0) **6 marks**

Stage (right to left)	A	B	Carry in	Sum	Carry out
1	0	1	0	1	0
2	1	1	0	0	1
3	1	1	1	1	1
4	0	0	1	1	0

25 (6 marks)

(a) $Q = 0 + C = C$ **M1**

The output differs only when $A \cdot B = 1$ and $C = 0$ **M1**

That is for ABC = 110 **A1**

Answer: Q = C; differs for ABC = 110 **3 marks**

(b) $Q = 1 + C = 1$ **M1**

The output differs when the correct output is 0, that is when $A \cdot B = 0$ and $C = 0$ **M1**

That is for ABC = 000, 010, 100 **A1**

Answer: Q = 1; differs for ABC = 000, 010, 100 **3 marks**

26 (6 marks)

(a) XOR of bits is the sum of the bits modulo 2 **M1**

which is 1 exactly when the number of 1s is odd **A1**

Answer: P = 1 exactly when an odd number of A, B, C, D equal 1 **2 marks**

(b) Chain: $((A \oplus B) \oplus C) \oplus D$ **B1**

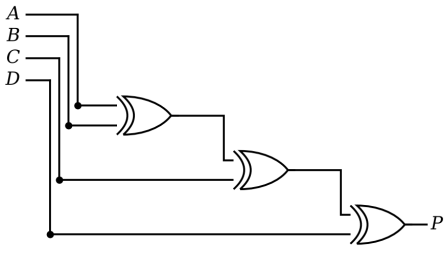
Tree: $(A \oplus B) \oplus (C \oplus D)$ **B1**

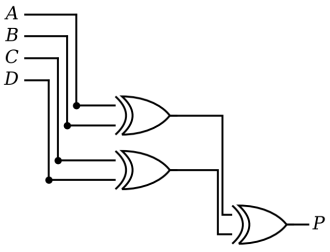
Answer: Chain: ((A ⊕ B) ⊕ C) ⊕ D. Tree: (A ⊕ B) ⊕ (C ⊕ D) **2 marks**

(c) Chain: signal passes through 3 gates, 15 ns **B1**

Tree: signal passes through 2 gates, 10 ns **B1**

Answer: Chain 15 ns; tree 10 ns **2 marks**





27 (10 marks)

(a) A − B − C takes values 0, −1, −1, −2, 1, 0, 0, −1 for the rows in order **M1**

D is the result modulo 2: 0, 1, 1, 0, 1, 0, 0, 1 **A1**

W is 1 when the value is negative: 0, 1, 1, 1, 0, 0, 0, 1 **A1**

Answer: D: 0, 1, 1, 0, 1, 0, 0, 1; W: 0, 1, 1, 1, 0, 0, 0, 1 **3 marks**

(b) D is the parity of A, B, C, which is A ⊕ B ⊕ C **B1**

If A = 0 and B = 1 then W = 1 whatever C is: term $\overline{A}\cdot B$ **M1**

If A = B then A − B = 0 and W = C: term $\overline{A \oplus B}\cdot C$; if A = 1 and B = 0 then W = 0 **M1 A1**

Answer: D = A ⊕ B ⊕ C and W = $\overline{A}\cdot B + \overline{A \oplus B}\cdot C$ **4 marks**

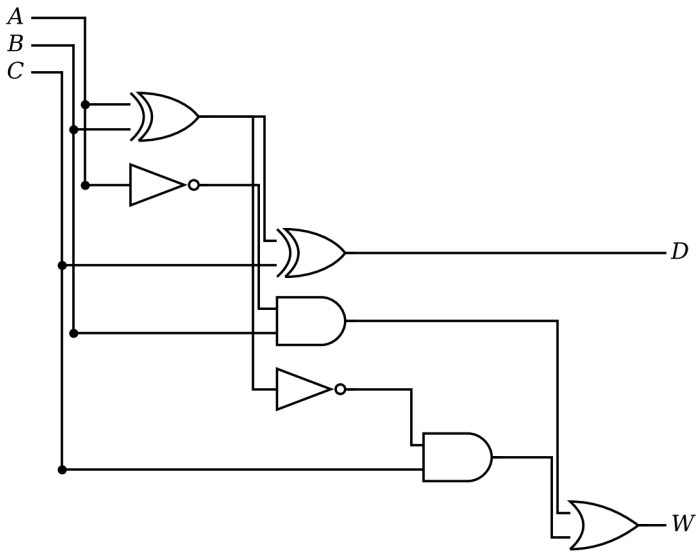
(c) First XOR gate on A and B, second XOR with C gives D **B1**

AND gate on $\overline{A}$ and B; NOT of A ⊕ B and C into another AND gate **B1**

OR gate gives W, sharing the XOR output **B1**

Answer: X = A ⊕ B, D = X ⊕ C; W = $\overline{A}\cdot B + \overline{X}\cdot C$ **3 marks**

A	B	C	D	W
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



28 (10 marks)

(a) $A \uparrow U = \overline{A \cdot (\overline{A} + \overline{B})} = \overline{A} + B$ and $B \uparrow U = A + \overline{B}$ **M1 A1**

$(A \uparrow U) \uparrow (B \uparrow U) = \overline{(\overline{A} + B) \cdot (A + \overline{B})} = A \cdot \overline{B} + \overline{A} \cdot B = A \oplus B$ **M1 A1**

$U \uparrow U = \overline{U} = A \cdot B$ (NOT built from NAND with joined inputs) **A1**

Answer: S = (A ↑ U) ↑ (B ↑ U) and C = U ↑ U: 5 gates **5 marks**

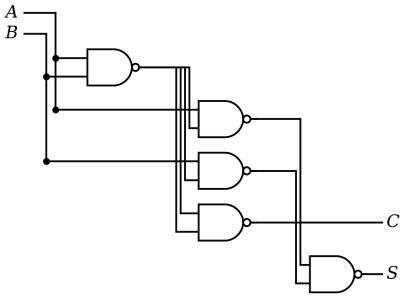
(b) The second half adder gives sum $X \oplus C = (X \uparrow V) \uparrow (C \uparrow V)$, by part (a) with X, C in place of A, B **M1 A1**

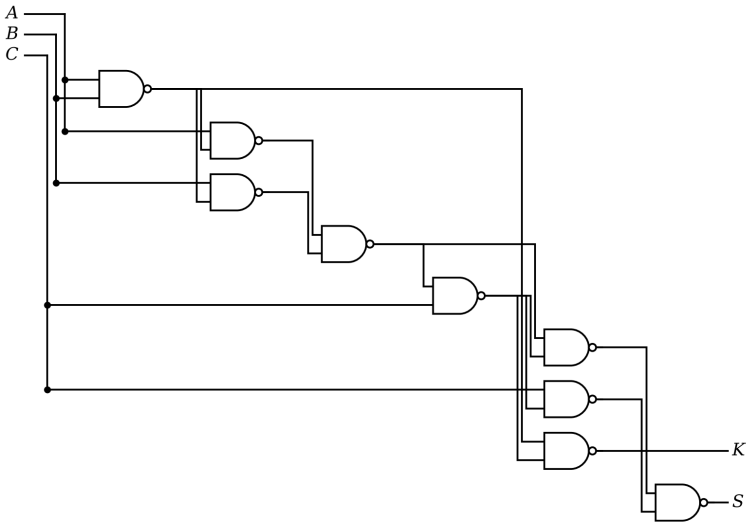
Its carry is $X \cdot C = \overline{V}$; the first carry is $A \cdot B = \overline{U}$ **M1**

$K = A \cdot B + X \cdot C = \overline{U} + \overline{V} = \overline{U \cdot V} = U \uparrow V$ **A1**

Gates: U, A ↑ U, B ↑ U, X, V, X ↑ V, C ↑ V, S, K = 9 **A1**

Answer: 9 NAND gates **5 marks**





29 (10 marks)

(a) $AB \times CD = (2A + B)(2C + D) = 4AC + 2(AD + BC) + BD$ **M1**

Weight 1: only $B \cdot D$, so $P_0 = B \cdot D$ **A1**

Weight 2: $A \cdot D + B \cdot C$ adds with sum digit $A \cdot D \oplus B \cdot C = P_1$ **A1**

Answer: $P_0 = B \cdot D$ and $P_1 = A \cdot D \oplus B \cdot C$ **3 marks**

(b) Weight 4: the term $A \cdot C$ plus the carry $A \cdot B \cdot C \cdot D$,so $P_2 = A \cdot C \oplus A \cdot D \cdot B \cdot C$ **M1**

$= A \cdot C \cdot \overline{B \cdot D}$ because $A \cdot C \oplus A \cdot C \cdot B \cdot D = A \cdot C \cdot (1 \oplus B \cdot D)$ **M1 A1**

The final carry is $A \cdot C \cdot A \cdot B \cdot C \cdot D = A \cdot B \cdot C \cdot D = P_3$ **A1**

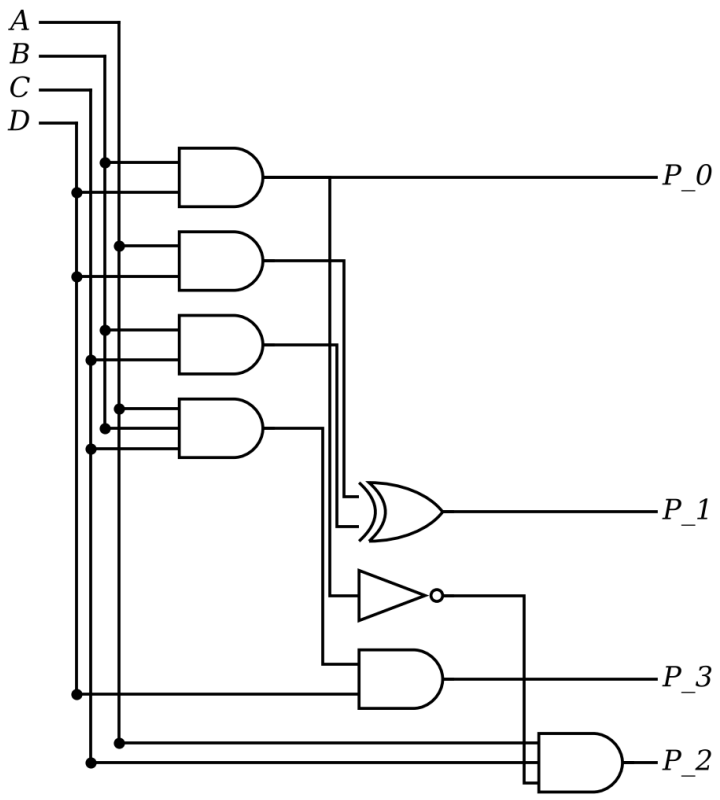
Answer: $A \cdot C \cdot \overline{B \cdot D}$; $P_3 = A \cdot B \cdot C \cdot D$ **4 marks**

(c) Circuit as shown, using AND gates and one XOR gate **B1**

$A = 1, B = 1, C = 1, D = 0$: $P_0 = 0$, $P_1 = A \cdot D \oplus B \cdot C = 0 \oplus 1 = 1$ **M1**

$P_2 = A \cdot C \overline{B \cdot D} = 1 \cdot \overline{1 \cdot 0} = 1$ and $P_3 = 0$, so 0110; check $3 \times 2 = 6$ **A1**

Answer: 0110 (decimal 6) **3 marks**



30 (9 marks)

(a) A carry is generated if both digits are 1 ($g_i = 1$) **E1**

Otherwise a carry in is passed on only if exactly one digit is 1 ($p_i = 1$) **E1**

Answer: Carry if both digits are 1 (g_i) or one is 1 and $c_i = 1$ ($p_i \cdot c_i$) **2 marks**

(b) $c_1 = g_0 + p_0 \cdot c_0$ **B1**

$c_2 = g_1 + p_1 \cdot c_1 = g_1 + p_1 \cdot (g_0 + p_0 \cdot c_0)$ **M1**

$= g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot c_0$ **A1**

Answer: $c_2 = g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot c_0$ **3 marks**

(c) g_i and p_i are ready at time 1 **B1**

Ripple: c_1 at 1 + 2 = 3, c_2 at 5, c_3 at 7, c_4 at 9 units **M1 A1**

Look-ahead: AND level then OR level after the g, p signals gives 1 + 2 = 3 units **A1**

Answer: Ripple 9 units; look-ahead 3 units **4 marks**

Karnaugh Maps and Minimal Forms

1 (2 marks)

F = 1 when A = 1 (cells 10, 11) and when B = 0 (cells 00, 10) **M1**

Map: row A = 0 has 1, 0; row A = 1 has 1, 1 **A1**

Answer: Ones in cells AB = 00, 10, 11 **2 marks**

A \ B	**0**	**1**
0	1	0
1	1	1

2 (2 marks)

The two 1s are adjacent and form a loop where A = 0 — **M1**

$F = \overline{A}$ — **A1**

Answer: $\overline{A}$ — **2 marks**

3 (3 marks)

$\overline{A}\cdot\overline{B}\cdot C$ is 001, so m_1 — **B1**

$A\cdot B\cdot\overline{C}$ is 110, so m_6 — **B1**

$A\cdot B\cdot C$ is 111, so m_7 — **B1**

Answer: $\Sigma m(1, 6, 7)$ — **3 marks**

4 (3 marks)

(a) $2^3 = 8$ — **B1**

Answer: 8 — **1 mark**

(b) $2^4 = 16$ — **B1**

Answer: 16 — **1 mark**

(c) Neighbouring cells must differ in exactly one variable (Gray code) — **E1**

Answer: Neighbouring cells differ in exactly one variable — **1 mark**

5 (2 marks)

A group of 2^k adjacent cells has k variables taking both values, so the group size must be a power of 2 — **E1**

k variables disappear — **B1**

Answer: Groups are powers of 2; k variables disappear — **2 marks**

6 (2 marks)

(a) A = 1, B = 0, C = 1 — **B1**

Answer: $A\cdot\overline{B}\cdot C$ — **1 mark**

(b) B takes both values so it disappears — **B1**

Answer: $A\cdot C$ — **1 mark**

7 (3 marks)

Ones at AB = 00 and 11 — **B1**

These cells touch only at a corner, so no two 1s are adjacent — **M1**

No loops larger than one cell exist, so F cannot be simplified (it is XNOR) — **A1**

Answer: No adjacent 1s — **3 marks**

A \ B	**0**	**1**
0	1	0
1	0	1

8 (3 marks)

Loop the pair m_1, m_3 (A = 0, C = 1) — **M1**

Loop the pair m_3, m_7 (B = 1, C = 1) — **M1**

$F = B\cdot C + \overline{A}\cdot C$ — **A1**

Answer: $B\cdot C + \overline{A}\cdot C$ — **3 marks**

A \ BC	**00**	**01**	**11**	**10**
0	0	1	1	0
1	0	0	1	0

9 (4 marks)

Ones at 000, 001, 101, 111 **B1**

Pair m_0, m_1: $\overline{A}\cdot\overline{B}$ **M1**

Pair m_5, m_7: $A\cdot C$ **M1**

$F = \overline{A}\cdot\overline{B} + A\cdot C$ **A1**

Answer: $\overline{A}\cdot\overline{B} + A\cdot C$ **4 marks**

A \ BC	00	01	11	10
0	1	1	0	0
1	0	1	1	0

10 (4 marks)

Loop of four cells m_4, m_5, m_6, m_7: $\overline{A}\cdot B$ **M1 A1**

Loop m_5, m_{13}: $B\cdot\overline{C}\cdot D$ **M1**

$F = \overline{A}\cdot B + B\cdot\overline{C}\cdot D$ **A1**

Answer: $\overline{A}\cdot B + B\cdot\overline{C}\cdot D$ **4 marks**

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	1	0	0
10	0	0	0	0

11 (3 marks)

Ones in the four corners of the map **B1**

The four corners form one loop of four cells because the map wraps around both horizontally and vertically **M1**

$F = \overline{B}\cdot\overline{D}$ **A1**

Answer: $\overline{B}\cdot\overline{D}$ **3 marks**

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

12 (4 marks)

(a) Loop the four 1s with A = 0, and the pair m_0, m_4 (B = 0, C = 0) **M1**

$F = \overline{A} + \overline{B}\cdot\overline{C}$ **A1**

Answer: $\overline{A} + \overline{B}\cdot\overline{C}$ **2 marks**

(b) Loop the 0s: pair m_5, m_7 gives $A\cdot C$ so factor $\overline{A} + \overline{C}$; pair m_6, m_7 gives factor $\overline{A} + \overline{B}$ **M1**

$F = (\overline{A} + \overline{C})\cdot(\overline{A} + \overline{B})$ **A1**

Answer: $(\overline{A} + \overline{C})\cdot(\overline{A} + \overline{B})$ **2 marks**

A \ BC	00	01	11	10
0	1	1	1	1
1	1	0	0	0

13 (3 marks)

The maxterms are the numbers not in the list: 0, 3, 5, 6 **M1**

$F = \Pi M(0, 3, 5, 6)$ **A1**

$F = (A + B + C)\cdot(A + \overline{B} + \overline{C})\cdot(\overline{A} + B + \overline{C})\cdot(\overline{A} + \overline{B} + C)$ **A1**

Answer: $\Pi M(0, 3, 5, 6) = (A + B + C)\cdot(A + \overline{B} + \overline{C})\cdot(\overline{A} + B + \overline{C})\cdot(\overline{A} + \overline{B} + C)$ **3 marks**

14 (4 marks)

Loop of four cells m_1, m_3, m_5, m_7: C **M1 A1**

Cell m_2 can only be paired with m_3: $\overline{A}\cdot B$ **M1**

$F = C + \overline{A}\cdot B$; every loop is as large as possible and both are needed **A1**

Answer: $C + \overline{A}\cdot B$ **4 marks**

A \ BC	00	01	11	10
0	0	1	1	1
1	0	1	1	0

15 (6 marks)

Plot the eight 1s **B1**

Quad m_0, m_2, m_8, m_{10}: $\overline{B}\cdot\overline{D}$ **M1 A1**

Quad m_0, m_1, m_8, m_9: $\overline{B}\cdot\overline{C}$ **A1**

m_5 pairs with m_1: $\overline{A}\cdot\overline{C}\cdot D$; m_{14} pairs with m_{10}: $A\cdot C\cdot\overline{D}$ **M1**

$F = \overline{B}\cdot\overline{D} + \overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{C}\cdot D + A\cdot C\cdot\overline{D}$ **A1**

Answer: $\overline{B}\cdot\overline{D} + \overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{C}\cdot D + A\cdot C\cdot\overline{D}$ **6 marks**

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	1
10	1	1	0	1

16 (5 marks)

(a) Ones at 6, 7, 8, 9 **B1**

X at 10 to 15 **B1**

Answer: Ones at 6, 7, 8, 9; X at 10 to 15; zeros at 0 to 5 **2 marks**

(b) Loop 8, 9, 10, 11, 12, 13, 14, 15 using X cells: A **M1 A1**

Loop 6, 7, 14, 15 using X cells: B·C **A1**

Answer: A + B·C **3 marks**

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	X	X	X	X
10	1	1	X	X

17 (6 marks)

Draw the map: 1s as listed, 0s at 0, 1, 2, 3, 6, 8, 12 **B1**

Loop the 1s: F = B·D + A·D + A·C + $\overline{A}$·B$\overline{C}$ **M1 A1**

Loop the 0s and complement each group: F = (A + $\overline{C}$ + D)·(A + B)·($\overline{A}$ + C + D) **M1 A1**

Literals: SOP 9, POS 8 **B1**

Answer: B·D + A·D + A·C + $\overline{A}$·B$\overline{C}$ (9 literals); (A + $\overline{C}$ + D)·(A + B)·($\overline{A}$ + C + D) (8) **6 marks**

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	0	1	1	1
10	0	1	1	1

18 (5 marks)

Map with ones at 1, 3, 7, 11, 15 and X at 0 and 2 **B1**

Loop 0, 1, 2, 3 using the X cells at 0 and 2: $\overline{A}$·$\overline{B}$ **M1 A1**

Loop 3, 7, 11, 15: C·D **M1**

F = C·D + $\overline{A}$·$\overline{B}$ **A1**

Answer: C·D + $\overline{A}$·$\overline{B}$ **5 marks**

AB \ CD	00	01	11	10
00	X	1	1	X
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

19 (6 marks)

(a) F = 1 whenever A = 1 (8 cells) **B1**

and when B = 1 with C = 1 or D = 1 (cells 0110, 0111, 0101) **B1**

Answer: Ones at 5, 6, 7 and 8 to 15 **2 marks**

(b) Loop of eight cells A = 1: A **M1 A1**

Loops m_5, m_7, m_{13}, m_{15}: B·D; and m_6, m_7, m_{14}, m_{15}: B·C **M1**

F = A + B·D + B·C **A1**

Answer: A + B·D + B·C **4 marks**

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	1	1	1	1
10	1	1	1	1

20 (5 marks)

(a) Changing one switch flips the output, so the output equals the parity of A, B, C — **M1**

Ones at 001, 010, 100, 111 — **A1**

Answer: Ones at m1, m2, m4, m7 — **2 marks**

(b) The ones form a chessboard pattern: no two are adjacent — **E1**

So every minterm is its own loop — **M1**

$F = \overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot C$ — **A1**

Answer: $\overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot C$ — **3 marks**

A \ BC	00	01	11	10
0	0	1	0	1
1	1	0	1	0

21 (6 marks)

(a) Plot the eight 1s — **B1**

Corners (0, 2, 8, 10) form a loop; the central four (5, 7, 13, 15) form a loop — **B1**

Answer: Loops of four: 0, 2, 8, 10 and 5, 7, 13, 15 — **2 marks**

(b) Corner loop $\overline{B}\cdot\overline{D}$, central loop $B\cdot D$ — **M1**

$F = \overline{B}\cdot\overline{D} + B\cdot D$ — **A1**

Answer: $\overline{B}\cdot\overline{D} + B\cdot D$ — **2 marks**

(c) $B \oplus D = 1$ when $B \neq D$, so $\overline{B \oplus D} = 1$ when $B = D$, that is $B\cdot D + \overline{B}\cdot\overline{D}$ — **M1**

F is 1 exactly when B and D are equal (XNOR of B and D), regardless of A and C — **A1**

Answer: B and D equal — **2 marks**

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

22 (6 marks)

Plot the six 1s; each 1 lies in exactly two possible pairs — **B1**

The pairs are m_0m_1, m_1m_3, m_3m_7, m_7m_6, m_6m_4, m_4m_0 — **M1**

All six cells are covered by choosing alternate pairs, so three pairs are needed — **M1 A1**

First choice: $\overline{B}\cdot\overline{C} + \overline{A}\cdot C + A\cdot B$ — **A1**

Second choice: $B\cdot C + \overline{A}\cdot\overline{B} + A\cdot\overline{C}$; both use three terms of two literals, so both are minimal — **A1**

Answer: $\overline{B}\cdot\overline{C} + \overline{A}\cdot C + A\cdot B$ or $B\cdot C + \overline{A}\cdot\overline{B} + A\cdot\overline{C}$ — **6 marks**

A \ BC	00	01	11	10
0	1	1	1	0
1	1	0	1	1

A \ BC	00	01	11	10
0	1	1	1	0
1	1	0	1	1

23 (8 marks)

(a) Ones at 0, 2, 3, 5, 6, 7, 8, 9 **B1**

Zeros at 1 and 4 **B1**

X at 10 to 15 **B1**

Answer: Ones at 0, 2, 3, 5, 6, 7, 8, 9; zeros at 1 and 4; X at 10 to 15 **3 marks**

(b) Loop 8, 9, 10, 11, 12, 13, 14, 15 (using X): A **M1 A1**

Loop 2, 3, 6, 7, 10, 11, 14, 15: C **M1**

Loops 0, 8 with the X cells 2, 10: $\overline{B}\cdot\overline{D}$; 5, 7, 13, 15: B·D **M1**

F = C + A + $\overline{B}\cdot\overline{D}$ + B·D **A1**

Answer: C + A + $\overline{B}\cdot\overline{D}$ + B·D **5 marks**

AB \ CD	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

24 (7 marks)

(a) F = 1 for all combinations with three or four votes for **B1**

and for two votes including A: 1100, 1010, 1001 **B1**

Ones at m_7, m_{11}, m_{13}, m_{14}, m_{15} and m_9, m_{10}, m_{12} **B1**

Answer: Ones at 7, 9, 10, 11, 12, 13, 14, 15 **3 marks**

(b) Loop 12, 13, 14, 15: A·B **M1**

Loops 10, 11, 14, 15: A·C and 9, 11, 13, 15: A·D **M1 A1**

Cell 7 only pairs with 15: B·C·D; F = A·D + A·C + A·B + B·C·D **A1**

Answer: A·D + A·C + A·B + B·C·D **4 marks**

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	1	1	1

25 (6 marks)

(a) Loop of four 4, 5, 6, 7: $\overline{A}\cdot B$ **M1 A1**

Loop of four 9, 11, 13, 15: A·D; F = $\overline{A}\cdot B$ + A·D **A1**

Answer: $\overline{A}\cdot B$ + A·D **3 marks**

(b) F = $\overline{\overline{\overline{A}\cdot B}\cdot\overline{A\cdot D}}$ **M1**

F = ($\overline{A}$ ↑ B) ↑ (A ↑ D) **A1**

Answer: F = ($\overline{A}$ ↑ B) ↑ (A ↑ D) **2 marks**

(c) Gates: A ↑ A, $\overline{A}$ ↑ B, A ↑ D and the final gate = 4 **B1**

Answer: 4 **1 mark**

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	1	1	0
10	0	1	1	0

26 (6 marks)

(a) The columns must be in Gray code order 00, 01, 11, 10 **E1**

In the student's order the cells 00 and 10 are not adjacent, so the pairs cannot be seen **E1**

Answer: Columns not in Gray code order **2 marks**

(b) Correct column order 00, 01, 11, 10 puts all four 1s in the columns 00 and 10, which are adjacent through wrap-around **M1**

They form one loop of four cells **A1**

$F = \overline{C}$ **A1**

Answer: $\overline{C}$ **3 marks**

(c) $\overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot\overline{C} = \overline{C}\cdot(\overline{A}\cdot\overline{B} + \overline{A}\cdot B + A\cdot\overline{B} + A\cdot B) = \overline{C}\cdot 1 = \overline{C}$ **A1**

Answer: The student's expression equals $\overline{C}$ **1 mark**

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

27 (10 marks)

(a) Minterms 0 to 15 go on the map for A = 0: ones at 4, 5, 11, 15 **B1**

Minterms 16 to 31 go on the map for A = 1, in cell number m − 16: ones at 4, 5, 6, 7, 8, 11, 12, 14, 15 **B1 B1**

Answer: A = 0: 4, 5, 11, 15. A = 1: 4, 5, 6, 7, 8, 11, 12, 14, 15 (cell m − 16) **3 marks**

(b) Loops 4, 5, 20, 21 (the same cells on both maps): $\overline{B}\cdot C\cdot\overline{D}$ **M1 A1**

Loop 15, 31 across both maps with 11, 27: $B\cdot D\cdot E$ **M1 A1**

Loop 20 to 23 with 28, 30 covers A = 1: $A\cdot C\cdot D$ **M1**

Cells 24 and 28 on the A = 1 map: $A\cdot B\cdot\overline{D}\cdot\overline{E}$ **A1**

$F = \overline{B}\cdot C\overline{D} + B\cdot D\cdot E + A\cdot C\cdot D + A\cdot B\cdot\overline{D}\cdot\overline{E}$ **A1**

Answer: $\overline{B}\cdot C\cdot\overline{D} + B\cdot D\cdot E + A\cdot C\cdot D + A\cdot B\cdot\overline{D}\cdot\overline{E}$ **7 marks**

BC \ DE	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	1	0
10	0	0	1	0

BC \ DE	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	0	1	1
10	1	0	1	0

28 (10 marks)

(a) Digit 0 gives 3 = 0011 **B1**

Digit 9 gives 12 = 1100 **B1**

Answer: 0011 and 1100 **2 marks**

(b) W = 1 for digits 5 to 9; X = 1 for the digits 1, 2, 3, 4, 9 **B1 B1**

W: loop 8 to 15 (A), loop 5, 7, 13, 15 (B·D), loop 6, 7, 14, 15 (B·C) **M1 A1**

X: loops $\overline{B}\cdot D$ (1, 3, 9, 11, using X cells), $\overline{B}\cdot C$ (2, 3, 10, 11) and $B\cdot\overline{C}\cdot\overline{D}$ (4, 12 using X cell 12) **M1 A1**

Answer: $W = A + B\cdot D + B\cdot C$, $X = \overline{B}\cdot D + \overline{B}\cdot C + B\cdot\overline{C}\cdot\overline{D}$ **6 marks**

(c) Y = 1 for the digits 0, 3, 4, 7, 8 (using X) and Z = 1 for the even digits, giving $C\cdot D + \overline{C}\cdot\overline{D}$ and $\overline{D}$ **M1**

For 0111: W = 0 + 1·1 + 1·1 = 1, X = 0 + 0 + 0 = 0, Y = 1, Z = 0, so 1010 **A1**

Answer: $Y = C\cdot D + \overline{C}\cdot\overline{D}$, $Z = \overline{D}$ **2 marks**

Digit	ABCD	WXYZ
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	X	X	X	X
10	0	1	X	X

29 (10 marks)

(a) Ones at 2, 3, 5, 7, 11, 13 **B1**

Zeros elsewhere (0, 1 and 4 are not prime) **B1**

Answer: Ones at 2, 3, 5, 7, 11, 13; zeros elsewhere **2 marks**

(b) Prime implicant loops: $\overline{B}\cdot C\cdot D$, $B\cdot\overline{C}\cdot D$, $\overline{A}\cdot C\cdot D$, $\overline{A}\cdot\overline{B}\cdot C$, $\overline{A}\cdot B\cdot D$ **M1**

Cells 13 and 7 force their loops, but cells 2 and 11 can each be covered in two ways, so a choice has to be made **M1**

One minimal form: $\overline{B}\cdot C\cdot D + B\cdot\overline{C}\cdot D + \overline{A}\cdot C\cdot D + \overline{A}\cdot\overline{B}\cdot C$ **M1 A1**

Another: $\overline{B}\cdot C\cdot D + B\cdot\overline{C}\cdot D + \overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot D$ **A1**

Answer: $\overline{B}\cdot C\cdot D + B\cdot\overline{C}\cdot D + \overline{A}\cdot C\cdot D + \overline{A}\cdot\overline{B}\cdot C$ (or $\overline{B}\cdot C\cdot D + B\cdot\overline{C}\cdot D + \overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot D$) **5 marks**

(c) Loop the zeros and complement: $(B + C)\cdot(\overline{B} + D)\cdot(\overline{A} + D)\cdot(\overline{A} + \overline{B} + \overline{C})$ **M1 A1**

Literals: product of sums 9, sum of products 12 **A1**

Answer: $(B + C)\cdot(\overline{B} + D)\cdot(\overline{A} + D)\cdot(\overline{A} + \overline{B} + \overline{C})$ (9 literals against 12) **3 marks**

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

30 (8 marks)

(a) Ones at 001, 011, 110, 111 **B1**

Pair m_6, m_7 gives A·B **M1**

Pair m_1, m_3 gives $\overline{A}\cdot C$; the pairs do not overlap or touch edge to edge **A1**

Answer: Ones at 1, 3, 6, 7: pair 6, 7 gives A·B and pair 1, 3 gives $\overline{A}\cdot C$ **3 marks**

(b) Cells 111 and 011 are adjacent but lie in different loops **M1**

When A changes, the term A·B switches off while $\overline{A}$·C switches on, and if the first switches off before the second is on, both terms are 0 for a moment **M1 E1**

Answer: 011 and 111 are adjacent but in different loops, so a glitch to 0 can occur **3 marks**

(c) The term B·C is the consensus of A·B and $\overline{A}$·C: A·B + $\overline{A}$·C + B·C = A·B + $\overline{A}$·C **M1**

On the map B·C is a third loop covering 011 and 111 together, so the output stays 1 during the change **A1**

Answer: A·B + $\overline{A}$·C + B·C = A·B + $\overline{A}$·C; the extra loop B·C covers 011 and 111 together **2 marks**

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	1

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	1