Truth Tables and Logical Statements

1 Construct the truth table for the expression $P \wedge Q$. Use 1 for true and 0 for false. **[2]**

2 The statement P is true and the statement Q is false. Write down the truth value (1 or 0) of each of the following.
(a) $P \wedge Q$ **[1]**
(b) $P \vee Q$ **[1]**
(c) $P \oplus Q$ **[1]**

3 State the number of rows in the truth table of an expression that has
(a) 3 variables **[1]**
(b) 6 variables **[1]**

4 Let P be the statement "it is raining" and Q the statement "I have an umbrella".
Write the statement "it is raining and I do not have an umbrella" in symbols. **[2]**

5 Construct the truth table for the implication $P \rightarrow Q$. Use 1 for true and 0 for false. **[2]**

6 State the values of P and Q for which $P \leftrightarrow Q$ is true. **[2]**

7 The operation NAND is written $P \uparrow Q$ and means NOT (P AND Q). The operation NOR is written $P \downarrow Q$ and means NOT (P OR Q).
(a) Construct the truth table for $P \uparrow Q$. **[2]**
(b) Construct the truth table for $P \downarrow Q$. **[2]**

8 Construct the truth table for $\neg P \vee Q$.
Hence write $P \rightarrow Q$ using only the connectives $\neg$ and $\vee$. **[3]**

9 P and R are true and Q is false. Find the truth value of each expression.
(a) $(P \vee Q) \wedge \neg R$ **[1]**
(b) $(P \rightarrow Q) \vee R$ **[1]**
(c) $\neg(P \wedge Q) \leftrightarrow R$ **[2]**

10 Consider the statement "If a shape is a square then it is a rectangle."
Write down the converse, the inverse and the contrapositive of this statement, and state which one is logically equivalent to the original statement. **[4]**

11 The exclusive-or operation $P \oplus Q$ is true when exactly one of P and Q is true.
Show, using a truth table, that $P \oplus Q$ is equivalent to $(P \vee Q) \wedge \neg(P \wedge Q)$. **[4]**

12 Construct the truth table for $(P \rightarrow Q) \wedge (Q \rightarrow P)$. Use columns for $P \rightarrow Q$, $Q \rightarrow P$ and the whole expression. **[3]**

13 State whether each of the following is a tautology, a contradiction or neither.
(a) $P \vee \neg P$ **[1]**
(b) $P \wedge \neg P$ **[1]**
(c) $P \rightarrow \neg P$ **[1]**

14 Construct a truth table with columns for P, Q, R, $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$.
Hence state, giving a reason, whether the two expressions are equivalent. **[5]**

15 Use truth tables to prove the distributive law $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$. **[5]**

16 Prove that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology. **[5]**

17 A security light L comes on when the override switch S is on, or when the motion sensor M detects movement while the daylight sensor D reports darkness.
Let M, D and S each be 1 when true.
Write down a logical expression for L, and construct its truth table. **[5]**

18 Let P and Q be statements.
(a) Show, using a truth table, that $P \rightarrow Q$ is equivalent to its contrapositive $\neg Q \rightarrow \neg P$. **[4]**
(b) Explain why $P \rightarrow Q$ is not equivalent to its converse $Q \rightarrow P$. **[2]**

19 Let P and Q be statements.
(a) Show that $\neg(P \oplus Q) \equiv P \leftrightarrow Q$. **[3]**
(b) Hence, or otherwise, show that $(P \oplus Q) \oplus Q \equiv P$. **[3]**

20 The rule of a cafe is: "You can have a cake only if you have eaten your vegetables."
Let C be "you have a cake" and V be "you have eaten your vegetables".
Write the rule as a statement in symbols, and state, with a reason, the only combination of truth values of C and V for which the rule is broken. **[4]**

21 Determine whether $((P \rightarrow Q) \wedge (R \rightarrow \neg Q)) \rightarrow (P \rightarrow \neg R)$ is a tautology, a contradiction or neither. Use a truth table with intermediate columns. **[6]**

22 The truth table of a function f of P, Q and R is shown.

P	Q	R	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Write f as a disjunction of conjunctions of P, Q, R and their negations, with one conjunction for each row where f = 1. **[3]**
(b) Hence, or otherwise, show that $f \equiv (P \wedge Q) \vee (Q \wedge R) \vee (P \wedge R)$, using a truth table. **[3]**

23 A two-variable truth table has four rows, and each row can output 0 or 1.
(a) Explain why there are 16 different truth tables for two variables. **[2]**
(b) Write down, in the form 2^k, the number of different truth tables for three variables. **[2]**
(c) Find how many of the truth tables for three variables output 1 for exactly three of the eight rows. **[2]**

24 A student writes: "$\neg(P \wedge Q)$ is equivalent to $\neg P \wedge \neg Q$, by De Morgan's law."
(a) Show that the student is wrong by finding values of P and Q for which the two expressions differ. **[2]**
(b) State the correct form of De Morgan's law for $\neg(P \wedge Q)$, and prove it using a truth table. **[4]**

25 On an island every person is either a knight, who always tells the truth, or a knave, who always lies.
Person A says: "B is a knave." Person B says: "A and I are the same type."
Let a = 1 if A is a knight (0 if a knave), and b = 1 if B is a knight (0 if a knave). Construct a truth table to determine what type each person is. **[6]**

26 Find all combinations of truth values of P, Q and R for which $(P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee \neg R)$ is true. Show a full truth table with a column for each bracket. **[6]**

27 Let P, Q and R be statements.

(a) Show that $P \leftrightarrow (Q \leftrightarrow R) \equiv (P \leftrightarrow Q) \leftrightarrow R$, so that the brackets do not matter when writing $P \leftrightarrow Q \leftrightarrow R$. **[4]**

(b) Show that $P \oplus (Q \oplus R) \equiv (P \oplus Q) \oplus R$ and that it is true exactly when an odd number of P, Q, R are true. **[4]**

28 A Boolean expression is built from the variables P_1, P_2, ..., P_n using only the connectives $\wedge$ and $\vee$ (no negations and no constants).

(a) Prove, by considering how the expression is built up, that it has value 1 when every variable has value 1. **[4]**

(b) Hence show that $\neg P$ cannot be written using only $\wedge$ and $\vee$. **[2]**

(c) Show that the same argument proves that no expression using only $\wedge$ and $\vee$ is equivalent to $P \oplus Q$. **[2]**

29 Exactly one of four people, Ann, Ben, Cal and Dee, committed an offence. Each makes one statement:
Ann: "It was Ben."
Ben: "It was Ann."
Cal: "It was not Ann."
Dee: "It was not Cal."
Exactly one of the four statements is true.
Construct a table showing the truth value of each statement for each possible culprit, and hence determine who committed the offence. **[8]**

30 The NAND operation is defined by $P \uparrow Q = \neg(P \wedge Q)$.

(a) Show that $P \uparrow P \equiv \neg P$. **[2]**

(b) Show that $(P \uparrow Q) \uparrow (P \uparrow Q) \equiv P \wedge Q$. **[3]**

(c) Find an expression for $P \vee Q$ using only the NAND operation, and verify it with a truth table. **[4]**

Boolean Algebra and Simplification

1 Simplify each of the following using Boolean laws.

(a) $A \cdot 1$ **[1]**

(b) $A + \overline{A}$ **[1]**

(c) $A \cdot 0$ **[1]**

2 Name the Boolean law that each statement illustrates.

(a) $A + B = B + A$ **[1]**

(b) $A \cdot (B + C) = A \cdot B + A \cdot C$ **[1]**

(c) $A + (B + C) = (A + B) + C$ **[1]**

3 Simplify each of the following.

(a) $A + A$ **[1]**

(b) $A \cdot A \cdot A$ **[1]**

(c) $\overline{\overline{A}}$ **[1]**

4 Use De Morgan's laws to rewrite each expression so that the complement signs apply only to single variables.

(a) $\overline{A + B}$ **[1]**

(b) $\overline{A \cdot B}$ **[1]**

5 Expand the first expression and factorise the second.

(a) $A \cdot (\overline{B} + C)$ **[1]**

(b) $B \cdot C + B \cdot \overline{D}$ **[1]**

6 The dual of a Boolean identity is found by swapping + with · and 0 with 1 throughout. Write down the dual of each identity.

(a) $A + 0 = A$ **[1]**

(b) $A + B \cdot C = (A + B) \cdot (A + C)$ **[1]**

7 Use Boolean laws to show that $A + A \cdot B = A$. **[3]**

8 Simplify $A \cdot B + A \cdot \overline{B}$, stating the laws used. **[3]**

9 Show that $A + \overline{A} \cdot B = A + B$. **[3]**

10 Simplify $A \cdot (A + B) \cdot (\overline{A} + C)$. **[3]**

11 Find the complement of $A \cdot \overline{B} + C$, giving your answer as a product of sums in which complements apply only to single variables. **[3]**

12 Simplify $A \cdot B \cdot C + A \cdot B \cdot \overline{C} + A \cdot \overline{B}$. **[4]**

13 Simplify $(A + \overline{B}) \cdot (\overline{A} + \overline{B})$. **[3]**

14 Simplify $\overline{A} \cdot B \cdot C + A \cdot B \cdot C + A \cdot \overline{B} \cdot C$ to a product of C and a sum of two variables. **[5]**

15 Prove the identity $(A + B) \cdot (\overline{A} + C) = A \cdot C + \overline{A} \cdot B$. **[5]**

16 Prove the consensus theorem $A \cdot B + \overline{A} \cdot C + B \cdot C = A \cdot B + \overline{A} \cdot C$. **[5]**

17 The exclusive-or operation is defined by $A \oplus B = A \cdot \overline{B} + \overline{A} \cdot B$. Use Boolean laws to show that

(a) $A \oplus A = 0$ **[2]**

(b) $A \oplus 1 = \overline{A}$ **[2]**

(c) $A \oplus 0 = A$ **[1]**

18 Simplify $\overline{A + B} \cdot (\overline{A} + \overline{B})$. **[4]**

19 Show that $(A + B) \cdot (A + C) \cdot (A + D) = A + B \cdot C \cdot D$. **[4]**

20 Find the complement of $A \cdot B + \overline{A} \cdot C$ and simplify it to a sum of two products in which complements apply only to single variables. **[5]**

21 Simplify $\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot \overline{B} \cdot C + A \cdot B \cdot C$ as far as possible, showing the laws used. **[6]**

22 Using the definition $X \oplus Y = X \cdot \overline{Y} + \overline{X} \cdot Y$, prove that $A \cdot (B \oplus C) = A \cdot B \oplus A \cdot C$. **[6]**

23 The implication $X \rightarrow Y$ can be written in Boolean algebra as $\overline{X} + Y$.
Show, using Boolean algebra, that the statement "if $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$" is always true, that is that $\overline{(\overline{A} + B) \cdot (\overline{B} + C)} + \overline{A} + C = 1$. **[6]**

24 In ordinary algebra, $ax = ay$ with $a \neq 0$ implies $x = y$. Investigate whether the corresponding statements hold in Boolean algebra.

(a) Show, by giving values, that $A \cdot B = A \cdot C$ does not imply $B = C$, and that $A + B = A + C$ does not imply $B = C$. **[2]**

(b) Show that if both $A \cdot B = A \cdot C$ and $A + B = A + C$ hold, then $B = C$. **[4]**

25 You may use the facts that $\oplus$ is commutative and associative, and that $X \oplus X = 0$ and $X \oplus 0 = X$ for every X.

(a) Given that $A \oplus X = 1$, show that $X = \overline{A}$. **[2]**

(b) Given that $A \oplus X = A \cdot B$, find X in its simplest form. **[4]**

26 Let $F = A \cdot B + \overline{B} \cdot C + \overline{A} \cdot \overline{C}$.

(a) Find the simplest expressions for the function obtained by putting A = 1 in F, and the function obtained by putting A = 0. **[3]**

(b) Hence show that $F = A \cdot (B + C) + \overline{A} \cdot (\overline{B} + \overline{C})$. **[3]**

27 Take as axioms: commutative laws, distributive laws (both forms), identity laws ($X + 0 = X$, $X \cdot 1 = X$) and complement laws ($X + \overline{X} = 1$, $X \cdot \overline{X} = 0$). Using only these axioms, prove:

(a) $A + A = A$ **[2]**

(b) $A + 1 = 1$ **[3]**

(c) $A + A \cdot B = A$ **[3]**

28 Use the axioms and the fact that, for any X and Y: if $X + Y = 1$ and $X \cdot Y = 0$ then Y is the unique complement of X, that is $Y = \overline{X}$.

(a) Prove that complements are unique: if $A + X = 1$ and $A \cdot X = 0$ then $X = \overline{A}$. **[4]**

(b) Hence prove De Morgan's law $\overline{A + B} = \overline{A} \cdot \overline{B}$ by showing that $\overline{A} \cdot \overline{B}$ is the complement of A + B. **[4]**

29 The consensus theorem has the dual form $(A + B) \cdot (\overline{A} + C) \cdot (B + C) = (A + B) \cdot (\overline{A} + C)$.

(a) Prove this result by expanding the left-hand side. **[4]**

(b) Hence simplify $(\overline{P} + Q) \cdot (P + R) \cdot (Q + R) \cdot (\overline{Q} + S)$. **[4]**

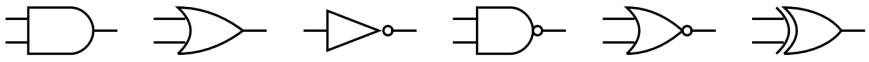
30 You may use the facts that $\oplus$ is commutative and associative, that $X \oplus X = 0$, $X \oplus 0 = X$, and that $X \cdot (Y \oplus Z) = X \cdot Y \oplus X \cdot Z$.

(a) Show that $A + B = A \oplus B \oplus A \cdot B$, using the definition $X \oplus Y = X \cdot \overline{Y} + \overline{X} \cdot Y$. **[5]**

(b) Hence express A + B + C using only $\oplus$ and $\cdot$. **[4]**

Logic Gates and Circuits

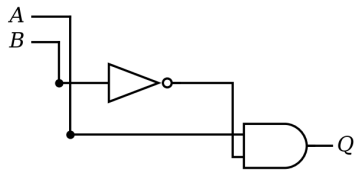
1 Name each of the six logic gates shown.



(a) (b) (c) (d) (e) (f)

[3]

2 Write down a Boolean expression for the output Q of the circuit shown.

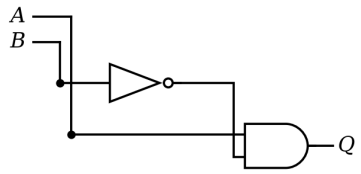


[2]

3 A NOT gate is connected to the output of an OR gate with inputs A and B.
Write down a Boolean expression for the final output and name the single logic gate that is equivalent to this combination. **[2]**

4 Draw a logic circuit, using AND, OR and NOT gates, that produces the output $Q = A \cdot B + C$. **[2]**

5 Find the value of the output Q of the circuit shown when



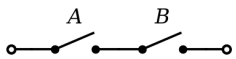
(a) A = 1 and B = 0, **[1]**

(b) A = 1 and B = 1. **[1]**

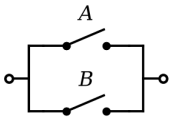
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6 A lamp L is on when current can flow through the switching circuit. A switch is closed when its input is 1. Write down a Boolean expression for L in each circuit.

(a) Circuit 1 **[1]**

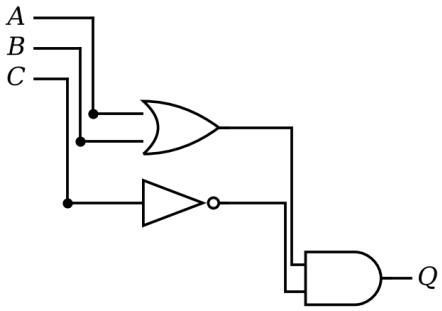


(b) Circuit 2 **[1]**



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7 Write down a Boolean expression for the output Q of the circuit shown.



[3]

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8 Draw a logic circuit, using only AND, OR and NOT gates, for the expression $Q = \overline{A}\cdot B + A\cdot\overline{B}$. **[3]**

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9 Explain, with a truth table, how a single NAND gate can be used as a NOT gate. **[3]**

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10 A half adder adds two binary digits A and B and produces a sum digit S and a carry digit C.

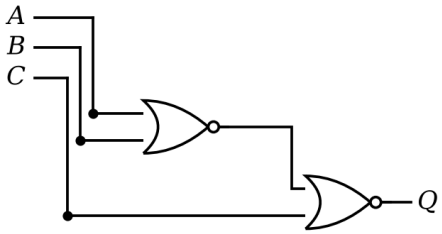
(a) Construct the truth table for S and C. **[2]**

(b) Write down Boolean expressions for S and C. **[1]**

(c) Draw a circuit for the half adder using an XOR gate and an AND gate. **[2]**

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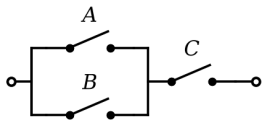
11 The circuit shown contains two NOR gates.
Construct a truth table for the output Q and write Q as a simple Boolean expression.



[4]

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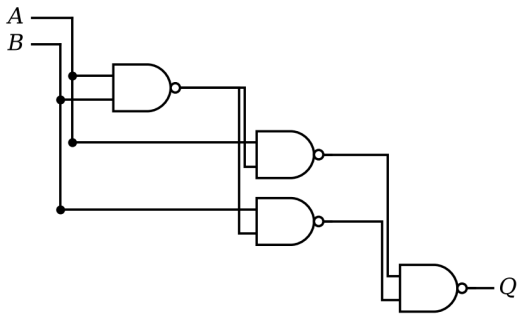
12 The lamp L in the switching circuit shown lights when current flows. A switch is closed when its input is 1.
Write down a Boolean expression for L and state all the inputs ABC for which the lamp is on.



[3]

13 Draw a switching circuit with switches that is on when $L = A\cdot\overline{B} + C$. Switches may be normally open (A) or normally closed ($\overline{A}$). **[3]**

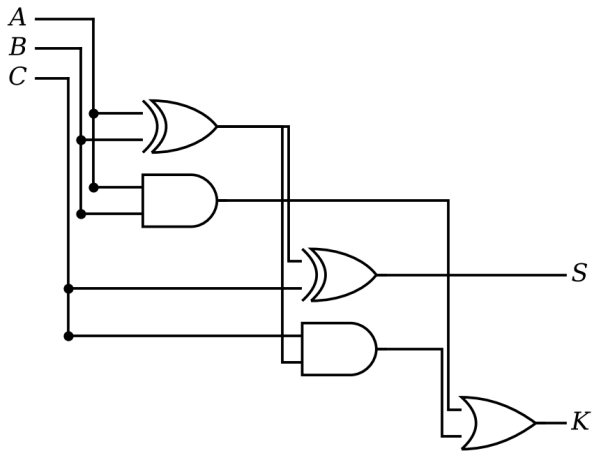
14 The circuit shown is built from four NAND gates.



(a) Write down an expression for Q in terms of A and B. **[2]**
(b) Show that $Q = A \oplus B$. **[3]**

15 A full adder adds three binary digits A, B and C (C is the carry in) and gives a sum digit S and a carry out digit K.
(a) Construct the truth table for S and K. **[3]**
(b) Show that $S = A \oplus B \oplus C$ and $K = A\cdot B + C\cdot(A \oplus B)$. **[3]**

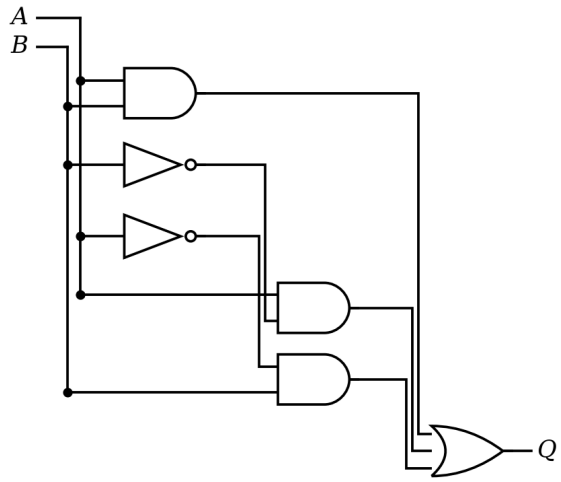
16 The circuit shown is made of two half adders (each an XOR gate and an AND gate) and an OR gate. Inputs are A, B and C.



(a) Write down expressions for S and K. **[2]**
(b) Hence explain why the circuit is a full adder. **[3]**

17 The NOR gate gives $A \downarrow B = \overline{A + B}$.
(a) Show that $A \downarrow A = \overline{A}$. **[2]**
(b) Hence show how to make an OR gate from NOR gates only, and draw the circuit. **[3]**

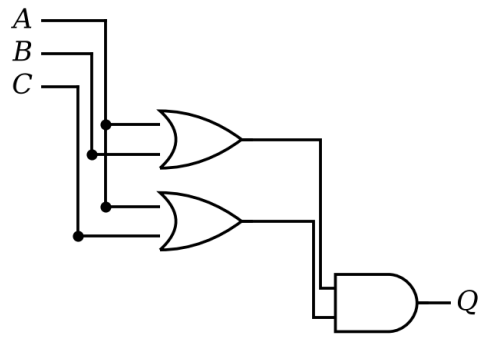
18 The circuit shown is built from NOT, AND and OR gates.
Write down the expression for Q, simplify it, and state how many gates are saved by using the simplified expression.



[5]

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19 The circuit shown has inputs A, B and C.



(a) Write down an expression for Q and simplify it. **[3]**
(b) Draw an equivalent circuit using only two gates. **[2]**

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20 A landing light L is controlled by two switches A and B, one upstairs and one downstairs. Changing either switch changes the state of the light. The light is off when A = B = 0.
(a) Construct a truth table for L. **[1]**
(b) Write down a Boolean expression for L. **[2]**
(c) Draw a switching circuit for L. **[2]**

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21 A comparator has inputs A, B (a two-digit binary number AB, with A the more significant digit) and C, D (the binary number CD). Its output E is 1 exactly when the two numbers are equal.
(a) Explain why E = 1 exactly when A = C and B = D. **[1]**
(b) Show that $E = (A \oplus C) \downarrow (B \oplus D)$, where $\oplus$ is the XOR operation and $X \downarrow Y = \overline{X + Y}$ is the NOR operation. **[3]**
(c) Draw the circuit, using two XOR gates and one NOR gate. **[2]**

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22 A 2-to-1 multiplexer has data inputs A and B and a select input S. The output Q equals A when S = 0 and B when S = 1.
(a) Show that $Q = \overline{S}\cdot A + S\cdot B$. **[2]**
(b) Show that $Q = (\overline{S} \uparrow A) \uparrow (S \uparrow B)$, where $\uparrow$ is the NAND operation. **[3]**
(c) Draw the circuit using four NAND gates, where one of them is used as a NOT gate to produce $\overline{S}$. **[2]**

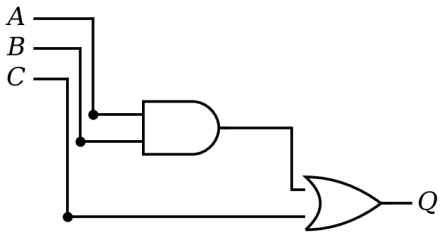
23 Design a circuit for $F = A\cdot\overline{B} + \overline{A}\cdot C$ using NAND gates only.

(a) Show, using a double complement and De Morgan's law, that $F = (A \uparrow \overline{B}) \uparrow (\overline{A} \uparrow C)$. **[3]**

(b) Hence draw the circuit, using NAND gates with their inputs joined to give $\overline{A}$ and $\overline{B}$, and state the total number of NAND gates required. **[3]**

24 A 4-bit ripple-carry adder is made from four full adders. The carry out of each full adder is the carry in of the next one to the left, and the first carry in is 0.
The adder is given the binary numbers A = 0110 and B = 0111.
By considering each full adder in turn, from right to left, find the carry out of every stage, the 4-bit sum and the final carry out. Check your answer in decimal. **[6]**

25 The circuit shown should give $Q = A\cdot B + C$. Testing shows that the output of the AND gate may be stuck permanently at 0 or permanently at 1.



(a) Find the output Q if the AND gate output is stuck at 0, and the combinations ABC for which Q then differs from the correct output. **[3]**

(b) Find the output Q if the AND gate output is stuck at 1, and the combinations ABC for which Q then differs from the correct output. **[3]**

26 An even-parity bit P for four data bits A, B, C, D is $P = A \oplus B \oplus C \oplus D$, so that the total number of 1s in A, B, C, D, P is even.

(a) Show that P = 1 exactly when an odd number of A, B, C, D equal 1. **[2]**

(b) Draw two different circuits for P using three XOR gates each: one in which the gates are connected in a chain, one in which they form a balanced tree. **[2]**

(c) Each gate has a delay of 5 ns. Find the time after the inputs change until the output is certain to be correct for each circuit. **[2]**

27 A full subtractor computes $A - B - C$, where C is the borrow in. It outputs a difference digit D and a borrow out digit W: W = 1 when the subtraction needs a borrow from the next column (that is when $A - B - C < 0$).

(a) Construct the truth table for D and W. **[3]**

(b) Show that $D = A \oplus B \oplus C$ and that $W = \overline{A}\cdot B + \overline{A \oplus B}\cdot C$. **[4]**

(c) Draw a circuit for D and W, using XOR gates for the combination $A \oplus B$. **[3]**

28 The NAND operation is $X \uparrow Y = \overline{X\cdot Y}$.

(a) Let $U = A \uparrow B$. Show that $(A \uparrow U) \uparrow (B \uparrow U) = A \oplus B$ and that $U \uparrow U = A\cdot B$. Hence draw a half adder using five NAND gates. **[5]**

(b) A full adder is built from two half adders in which the second half adder has inputs $X = A \oplus B$ and C, together with a NAND gate to combine the two carries. Let $V = X \uparrow C$. Show that the outputs can be written $S = (X \uparrow V) \uparrow (C \uparrow V)$ and $K = U \uparrow V$, and hence that the full adder needs 9 NAND gates. **[5]**

29 A 2-bit multiplier takes two 2-bit binary numbers AB and CD (with A and C the more significant digits) and outputs the 4-bit product $P_3P_2P_1P_0$.

(a) Show that the least significant digit is $P_0 = B \cdot D$ and that $P_1 = A \cdot D \oplus B \cdot C$, where the second term arises from the two partial products $A \cdot D$ and $B \cdot C$ that have weight 2. **[3]**

(b) The carry from the weight-2 addition is $A \cdot D \cdot B \cdot C$. Show that $P_2 = A \cdot C \oplus A \cdot D \cdot B \cdot C$ and simplify it, and find P_3. **[4]**

(c) Draw the circuit, and use it to find the product of 11 and 10 in binary, checking with decimal. **[3]**

30 In an adder, bit i has inputs a_i and b_i. Define the generate signal $g_i = a_i \cdot b_i$ and the propagate signal $p_i = a_i \oplus b_i$. The carry out of bit i is $c_{i+1} = g_i + p_i \cdot c_i$.

(a) Explain in words why $c_{i+1} = g_i + p_i \cdot c_i$. **[2]**

(b) Show that $c_2 = g_1 + p_1 \cdot g_0 + p_1 \cdot p_0 \cdot c_0$. **[3]**

(c) In a ripple-carry adder each gate causes a delay of 1 unit, and c_{i+1} is obtained from c_i through one AND gate and one OR gate. The g_i and p_i are available after 1 unit. Find the time at which c_4 is available in a ripple-carry adder, and compare with a two-level look-ahead circuit that computes c_4 directly from the g_i, p_i and c_0 in an AND level then an OR level. **[4]**

Karnaugh Maps and Minimal Forms

1 Draw the Karnaugh map for the function $F = A + \overline{B}$ of two variables A and B. **[2]**

2 The Karnaugh map of a function F(A, B) is shown. Write down the simplest expression for F.

A \ B	**0**	**1**
0	1	1
1	0	0

[2]

3 Write $F = \overline{A} \cdot \overline{B} \cdot C + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$ as a sum of minterms in the form $\Sigma m(...)$, where the minterm number is the binary number ABC. **[3]**

4 State the number of cells in a Karnaugh map for

(a) 3 variables **[1]**

(b) 4 variables **[1]**

(c) Explain why the columns of a 4-variable map are labelled 00, 01, 11, 10 rather than 00, 01, 10, 11. **[1]**

5 The cells of a Karnaugh map are looped in groups of 1, 2, 4, 8, ... cells.
State why groups of 3 or 6 cells are not allowed, and how many variables disappear from the product term when a group of 2^k cells is looped. **[2]**

6 For a function of A, B, C write down the product term represented by

(a) the single cell ABC = 101, **[1]**

(b) the pair of adjacent cells ABC = 101 and ABC = 111. **[1]**

7 Draw the Karnaugh map for $F = A \cdot B + \overline{A} \cdot \overline{B}$ and explain why the map shows that F cannot be simplified. **[3]**

8 Use the Karnaugh map to find the simplest expression for F.

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	0

[3]

9 The truth table of F is shown. Plot F on a Karnaugh map and hence find its simplest expression.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

[4]

10 The Karnaugh map of F(A, B, C, D) is shown. Find the simplest expression for F.

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	1	0	0
10	0	0	0	0

[4]

11 F(A, B, C, D) = Σm(0, 2, 8, 10). Draw the Karnaugh map, loop the 1s and write down the simplest expression for F. State what is unusual about the loop. **[3]**

12 The Karnaugh map of F(A, B, C) is shown. Find the simplest expression for F (a) as a sum of products and (b) as a product of sums.

A \ BC	00	01	11	10
0	1	1	1	1
1	1	0	0	0

(a) Sum of products **[2]**

(b) Product of sums **[2]**

13 F(A, B, C) = Σm(1, 2, 4, 7). Write F as a product of maxterms, Π M(...), and give the product of sums. **[3]**

14 The Karnaugh map of F(A, B, C) is shown. Loop the 1s and find the simplest expression for F, explaining why there is no simpler one.

A \ *BC*	**00**	**01**	**11**	**10**
0	0	1	1	1
1	0	1	1	0

[4]

15 F(A, B, C, D) = Σm(0, 1, 2, 5, 8, 9, 10, 14). Draw the Karnaugh map and find the simplest sum of products, showing each loop. **[6]**

16 A circuit has four inputs A, B, C, D representing a binary-coded decimal digit 0 to 9 (A is the most significant digit). The output F is 1 when the digit is greater than 5. The inputs 1010 to 1111 never occur.

(a) Draw the Karnaugh map for F, marking the combinations that never occur with X. **[2]**

(b) Use the map to find the simplest expression for F. **[3]**

17 F(A, B, C, D) = Σm(4, 5, 7, 9, 10, 11, 13, 14, 15). Find the minimal sum of products and the minimal product of sums for F, and state how many literals each uses. **[6]**

18 F(A, B, C, D) = Σm(1, 3, 7, 11, 15) with don't care conditions d(0, 2). Draw the map and find the simplest expression for F. **[5]**

19 A burglar alarm F has four inputs: A (tamper), B (system armed), C (door open) and D (window open), each 1 when true. The alarm sounds if there is tampering, or if the system is armed and either the door or the window is open.

(a) Plot F on a Karnaugh map. **[2]**

(b) Find the simplest Boolean expression for F. **[4]**

20 A hall light is controlled by three switches A, B and C. Changing the position of any one switch changes the state of the light. The light is off when A = B = C = 0.

(a) Show that the light is on exactly when an odd number of the switches are 1, and plot it on a Karnaugh map. **[2]**

(b) Explain why the map shows that no simplification is possible, and write down the simplest sum of products. **[3]**

21 F(A, B, C, D) = Σm(0, 2, 5, 7, 8, 10, 13, 15).

(a) Draw the Karnaugh map and loop the 1s. **[2]**

(b) Hence find the simplest expression for F. **[2]**

(c) Show that F is equal to $\overline{B \oplus D}$ and state what F is in words. **[2]**

22 F(A, B, C) = Σm(0, 1, 3, 4, 6, 7). Show that F has two different minimal sums of products, and give both. **[6]**

23 A seven-segment display shows the decimal digits 0 to 9 from a four-digit binary input ABCD (A most significant). Segment a (the top bar) is lit for the digits 0, 2, 3, 5, 6, 7, 8 and 9, and is off for 1 and 4. The inputs 1010 to 1111 never occur.

(a) Plot the function F for segment a on a Karnaugh map, using X for the inputs that never occur. **[3]**

(b) Hence find the simplest expression for F. **[5]**

24 A committee has four members: the chairman A and three others B, C, D. A motion passes if at least three members vote for it, or if exactly two vote for it and the chairman is one of them (the chairman has a casting vote). Each member votes 1 for and 0 against.

(a) Draw the Karnaugh map for the output F that is 1 when the motion passes. **[3]**

(b) Find the simplest expression for F. **[4]**

25 $F(A, B, C, D) = \Sigma m(4, 5, 6, 7, 9, 11, 13, 15)$.

(a) Use a Karnaugh map to find the simplest sum of products for F. **[3]**

(b) Write F in a form using only NAND operations ($X \uparrow Y = \overline{X \cdot Y}$), using the fact that a sum of products equals the NAND of the NANDs of its terms. **[2]**

(c) State the number of NAND gates needed if $\overline{A}$ is formed from A using a NAND gate with its inputs joined. **[1]**

26 A student draws the Karnaugh map for $F(A, B, C) = \Sigma m(0, 2, 4, 6)$ with the columns for BC labelled in the order 00, 01, 10, 11, as shown, and then writes $F = \overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot B\cdot\overline{C} + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot\overline{C}$.

A \ BC	00	01	10	11
0	1	0	1	0
1	1	0	1	0

(a) Explain the student's mistake. **[2]**

(b) Draw the correct map and find the simplest expression for F. **[3]**

(c) Show that the student's expression is equal to $\overline{C}$ by Boolean algebra. **[1]**

27 $F(A, B, C, D, E) = \Sigma m(4, 5, 11, 15, 20, 21, 22, 23, 24, 27, 28, 30, 31)$, where the minterm number is the binary number ABCDE. A five-variable function can be plotted on two four-variable maps, one for A = 0 and one for A = 1 (cells in the same position on the two maps are adjacent).

(a) Draw the two Karnaugh maps for F, for B, C, D, E. **[3]**

(b) Find the simplest sum of products for F, identifying any loop that uses both maps. **[7]**

28 A code converter changes a binary-coded decimal digit ABCD (0000 to 1001, A most significant) into the excess-3 code WXYZ, the binary form of (digit + 3). For example the digit 7 is 0111 and gives 1010. The inputs 1010 to 1111 never occur.

(a) Write down the output WXYZ for the digits 0 and 9. **[2]**

(b) Use Karnaugh maps with don't care conditions to find the simplest expressions for W and for X. **[6]**

(c) Show that $Y = C\cdot D + \overline{C}\cdot\overline{D}$ and $Z = \overline{D}$, and check all four expressions for the input 0111. **[2]**

29 A circuit takes a four-digit binary number ABCD (A most significant), representing 0 to 15, and outputs F = 1 when the number is prime. The primes up to 15 are 2, 3, 5, 7, 11 and 13.

(a) Draw the Karnaugh map for F. **[2]**

(b) Find a minimal sum of products for F, and state a second sum of products with the same number of terms and literals. **[5]**

(c) Find a minimal product of sums for F and compare the number of literals with your sum of products. **[3]**

30 $F(A, B, C) = A\cdot B + \overline{A}\cdot C$.

(a) Draw the Karnaugh map for F and show the two loops that correspond to the two terms of F. **[3]**

(b) Suppose B = C = 1 and A changes from 1 to 0. Explain, with reference to the map, why the output of a circuit built directly from F may briefly drop to 0 (a static hazard). **[3]**

(c) Show that adding the term $B\cdot C$ does not change the function, and explain how this removes the hazard. **[2]**